Deep Dynamical Modeling and Control of Unsteady Fluid Flows

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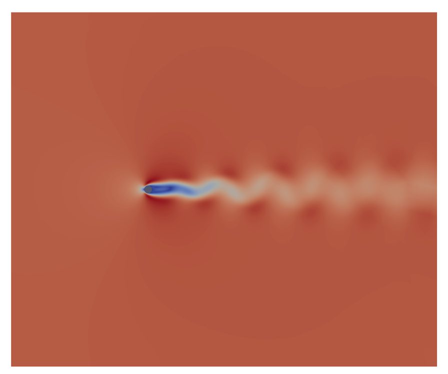
Problem Overview

- Many control techniques prove ineffective when applied to fluid flows due to the nonlinear nature of the Navier-Stokes equations.
- Recent advances in computational fluid dynamics (CFD) have enabled the simulation of complex fluid flows with high accuracy, opening the possibility of using learning-based approaches to facilitate controller design.

In this work, we consider data from 2-dimensional simulations of airflow over a cylinder with vortex shedding at a Reynolds number of 50.

Goal: Learn a data-driven model for the forced dynamics, then use the model to design a controller for suppressing vortex shedding.



Modeling Dynamical Systems

- Our goal is to construct models for discrete-time dynamical systems, which have the general form $x_{t+1} = F(x_t)$.
- If we assume a system is linear, then the dynamics are governed by $x_{t+1} = Ax_t$.

Question: How can we find the value of A if we only have access to time snapshots of a given system?

❶ Construct state vectors x_t from time snapshots.

❷ Build matrices X and Y:

$$X = \begin{bmatrix} | & & | \\ x_1 & \dots & x_{T-1} \\ | & & | \end{bmatrix}, \quad Y = \begin{bmatrix} | & & | \\ x_2 & \dots & x_T \\ | & & | \end{bmatrix}.$$

❸ Perform least-squares to find matrix A such that $Y \approx AX$.

- If the true system dynamics are nonlinear, then linear models may not be a good fit.
- However, **Koopman theory** suggests there may exist a state mapping $g(x_t)$ under which the dynamics are approximately linear $g(x_{t+1}) = Ag(x_t)$.
- Unfortunately, for most systems it is not obvious what this state mapping should be.

Approach: Use deep neural networks to automatically find appropriate state mappings $g(x_t)$.

Training Procedure

We extract data from a 128×256 grid near the cylinder to construct image-like inputs, where different input channels correspond to different physical quantities.

Density | x-Momentum | y-Momentum | Energy

The **Deep Koopman model** is a neural network autoencoder trained to generate mappings $g(x_t)$ that can be propagated forward in a linear manner.

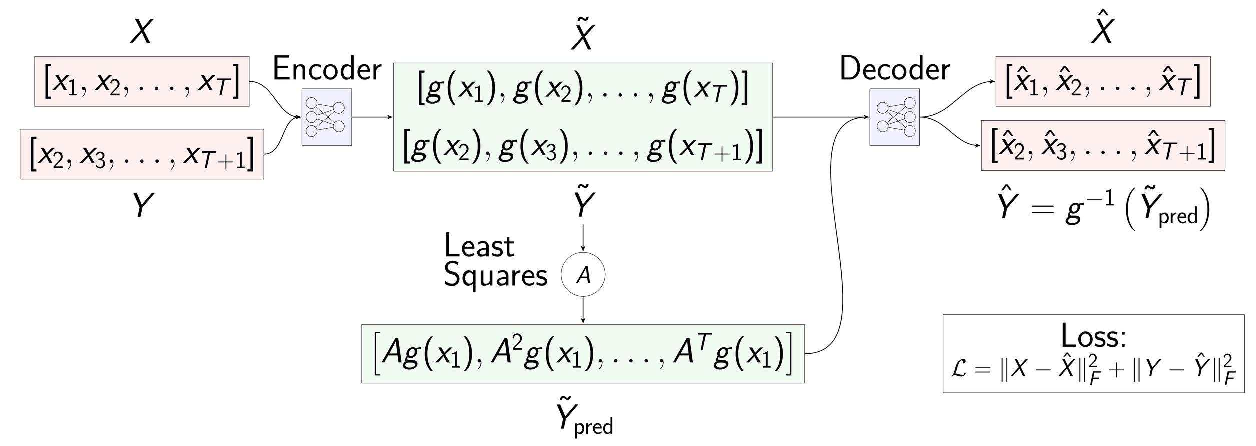
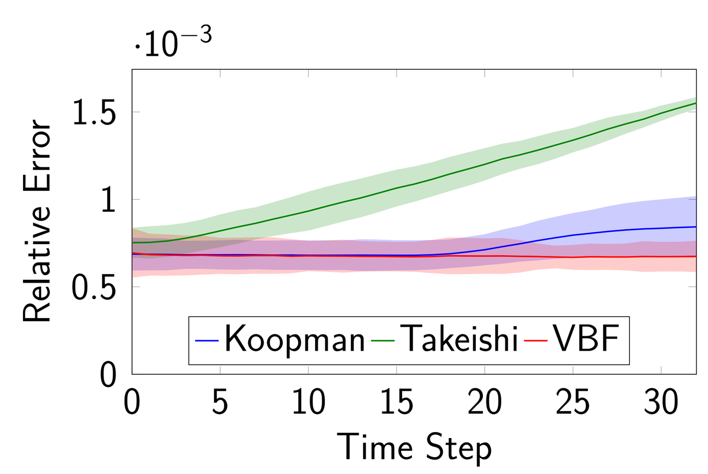


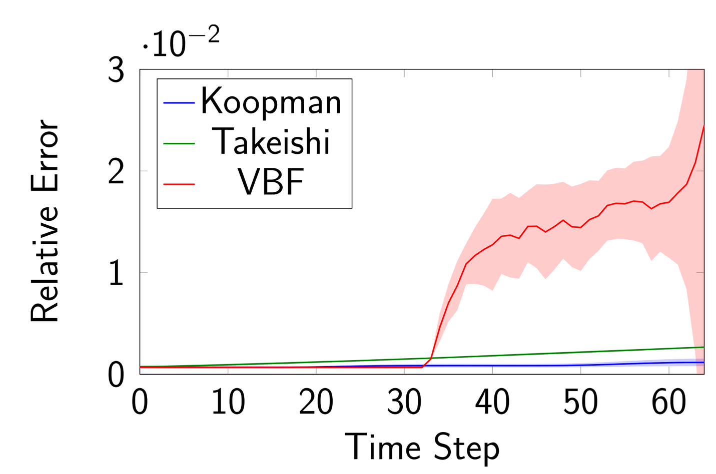
Figure: Modified form of algorithm presented in *Learning Koopman Invariant Subspaces for Dynamic Mode Decomposition* by N. Takeishi *et al.* (NIPS 2017).

Evaluation

- Deep Koopman modeling of unforced dynamics is benchmarked against the model proposed by Takeishi *et al.* and the Variational Bayes Filter (VBF).
- Deep Koopman model produces more stable predictions, with mean prediction errors below 0.2% over a prediction horizon of 128 time steps.



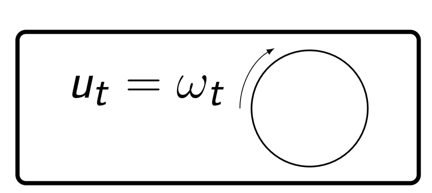
32-step predictions



64-step predictions

Forced Dynamics

We now allow the cylinder to rotate and aim to learn a model for the forced dynamics of the system.



We now find the A-matrix through:

$$A = (\tilde{Y} - B\Gamma)\tilde{X}^{\dagger}$$

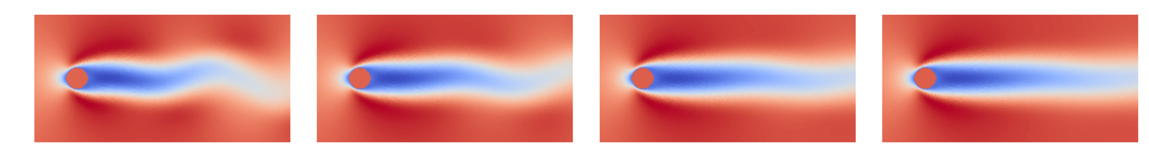
where $\Gamma = [u_1, \dots, u_T]$ and B-matrix is learned.

Model Predictive Control

- We select control inputs at each time step by solving a quadratic program.
- We set x_{goal} to be a snapshot of steady flow at a Reynolds number of 45.

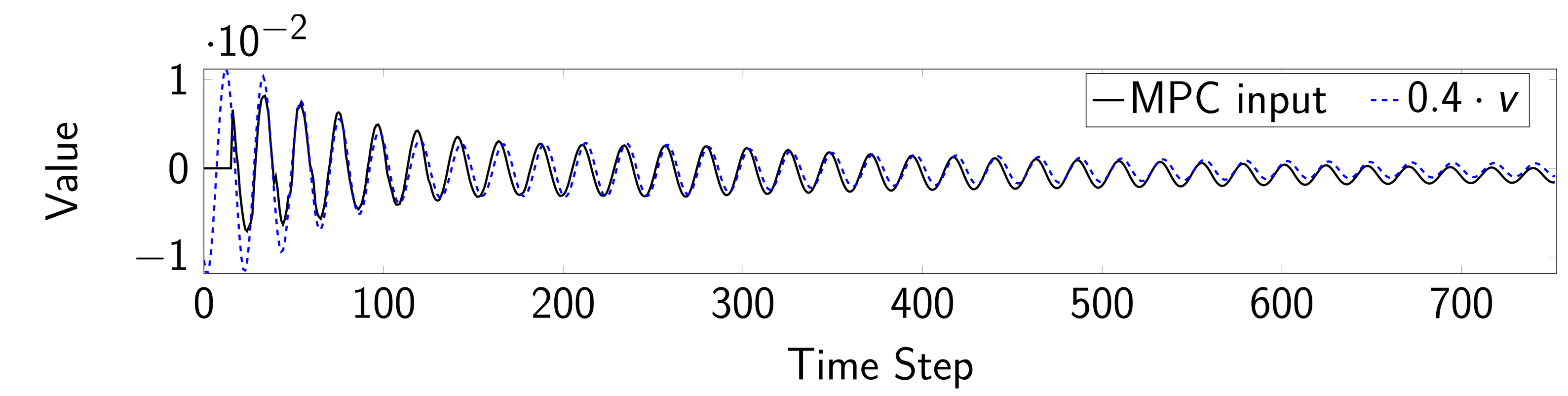
$$\text{minimize} \quad \sum_{t=1}^{T} (c_t - g(x_{\text{goal}}))^{\top} Q (c_t - g(x_{\text{goal}})) + \sum_{t=1}^{T-1} u_t^{\top} R u_t$$
$$\text{subject to} \quad c_1 = g(x_1),\ c_{t+1} = Ac_t + Bu_t,\ u_{\min} \preceq u_t \preceq u_{\max}$$

MPC is shown to be effective at **suppressing vortex shedding**.



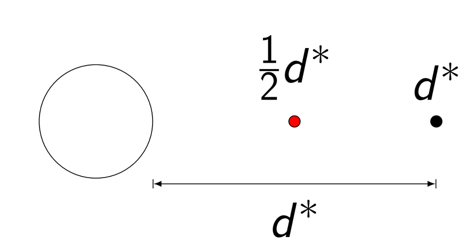
$t = 0$ | $t = 100$ | $t = 400$ | $t = 700$

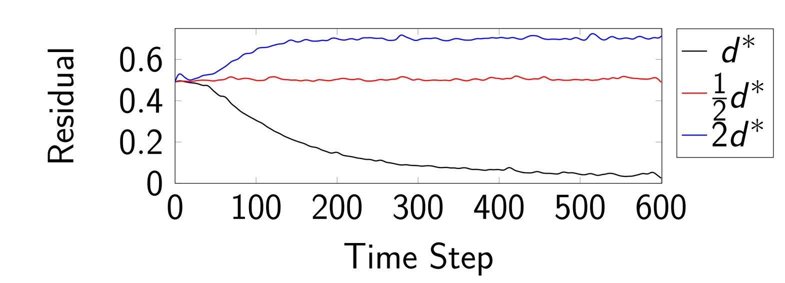
MPC actions prove to be similar to actions from a **proportional control** law based on downstream y-velocity measurements.



Proportional Control

Proportional controller based on measurements at identified location suppresses vortex shedding; other measurement locations are ineffective.





Conclusions

- We presented a method for data-driven modeling and control of airflow over a cylinder for suppressing vortex shedding.
- Flow controllers may be hard to design, but could be easy to implement. Data-driven modeling may facilitate the discovery of effective control laws.

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