Efficient soft demapping method for high order modulation schemes

EFFICIENT SOFT DEMAPPING METHOD FOR HIGH ORDER MODULATION SCHEMES

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Abstract— In this paper we introduce an efficient soft demapping method for high order modulation schemes combined with iterative decoder. To reduce the demapping complexity, we employ a simple demapping method using a decision threshold instead of using an exhaustive Euclidean distance estimation method. The proposed demapping process needs a reduced number of computing operations and our simulation results show that the proposed demapping method produce the performance approximating to the exhaustive estimation method at a bit error rate (BER) range from 10^{-5} to 10^{-6}.

I. Introduction

The concept of turbo codes using the parallel concatenated convolutional codes was introduced by Berrou *et al* [1]. Decoding using an iterative process with soft input soft output decoders shows exceptional performance close to the Shannon limit. Soon after, Pyndiah *et al*. first presented block turbo codes [2] using product codes [3]. By virtue of the outstanding performance of the turbo codes, they have been applied to many areas. For example, convolutional turbo codes have been adopted as standards in many wireless communication areas [4]. Moreover, because of several advantages of block turbo codes including flexibility in modification, they are known to be applicable to many areas implicating satellite communications. The wireless LAN 802.16a standard adopted block turbo codes [5].

However, all those advantages of turbo codes should be accompanied with accurate estimation of soft demapping of the received symbols. This would be more important in the system using high order modulation because extracting soft decision input from a received symbol includes many complex computing operations.

As recent applications required high spectral efficiency transmission systems, iterative decoding algorithm for high order modulation has become an essential field of research in digital communications. In Digital Video Broadcasting standard for Return Channel via Satellite (DVB-RCS), 4-PSK convolutional codes for turbo codes are used [6], and it was extended for triple-binary circular recursive systematic convolutional turbo codes [7].

Motivated by these situations, we propose an efficient soft decision demapping method for high order modulation scheme. In a conventional method, we have to estimate distances between the received symbols and all possible point in the signal constellation, and this process includes a large number of log likelihood estimations [8]. We use a simple decision threshold to estimate soft input to the decoder instead of the exhaustive estimation method. The proposed metho[1] highly reduces computational complexity but produces almost the same decoding performances of the conventional exhaustive estimation method.

In section II, we briefly present the concept of the conventional soft demapping method for M-ary modulation scheme. Section III proposes a novel soft demapping method using a decision threshold. We compare the computational complexity of the proposed algorithm with the conventional demapping algorithms in section IV. Section V investigates the performance of the soft demapping algorithms and demonstrates the simulation results on an Additive White Gaussian Noise (AWGN) channel. We draw the conclusion in Section IV.

II. Conventional soft demapping method

We can achieve a high spectral efficiency using high order modulation schemes, and can compensate the reduced power efficiency using turbo codes. Figure 1 shows the turbo code system with a high order modulation scheme.

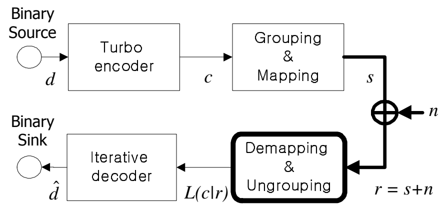

Figure 1. Baseband model of turbo code system with a high order modulation

The *Turbo encoder* processes binary signal, d and produces the encoded signal, c. Then, the *Grouping & Mapping* block takes M coded bits $c_{1,\ldots,M-1} \in \{0,1\}$ and maps to a complex symbol s using the mapping function, $s = map(c_{1,\ldots,M-1})$. AWGN is added on the channel. At the receiver, the *Demapping & Ungrouping* block demodulates the complex channel symbols, and extracts M soft outputs for a received symbol using a log likelihood ratio calculation. The *Iterative decoder* estimates the transmitted bits using soft input from the *Demapping & Ungrouping* block. Because the iterative decoder needs channel reliability (soft input) of each M coded bit, the demapper has to extract a channel reliability of each coded bit from the complex channel symbol, s.

For M-ary modulation scheme, the demapper needs to calculate log-likelihood ratios on the coded bits $c_1, c_2,\ldots,c_M$ for each incoming received symbol. The channel

[1] After we submitted this paper, we discovered that almost the same algorithm was already proposed for different applications [9][10].

information of the coded bit c_k conditioned on the received symbol r can be calculated as follows.

$$L(c_k|r) = \ln \frac{\Pr(c_k = 1|r)}{\Pr(c_k = 0|r)} = \ln \frac{\sum_{i=0}^{2^{M-1}-1} \Pr\left(c_k = 1, c_{j, j=0,\cdots,M-1, j\neq k} \equiv bin(i) \middle| r\right)}{\sum_{i=0}^{2^{M-1}-1} \Pr\left(c_k = 0, c_{j, j=0,\cdots,M-1, j\neq k} \equiv bin(i) \middle| r\right)}, \quad (1)$$

where $c_{j, j=0, \ldots, M-1, j\neq k} \equiv bin(i)$ denotes the joint event of the variables $c_{j,\ j=0,\ \ldots,\ M-1,\ j\neq k}$ having values of zero or one according to the binary decomposition of i.

For the complex AWGN channel, the conditional probability density function can be represented as

$$\Pr(s|r) = \frac{1}{2\pi\sigma^2} \exp\left[-\frac{1}{2\sigma^2}\|s - r\|^2\right], \quad (2)$$

where σ^2 is the noise variance that is double-sided power spectral density of $N_0/2$, s is a transmitted symbol and r is a received symbol. Using (2) we can rewrite (1) as follows.

$$L(c_k|r) = \ln \frac{\sum_{i=0}^{2^{M-1}-1} \exp\left[-\frac{1}{2\sigma^2}\left(r - s_{k(1,i)}\right)^2\right]}{\sum_{i=0}^{2^{M-1}-1} \exp\left[-\frac{1}{2\sigma^2}\left(r - s_{k(0,i)}\right)^2\right]}, \quad (3)$$

where $s_{k\,(1,i)}$ and $s_{k\,(0,i)}$ represent the mapped symbol corresponding to the kth coded bit of one and zero respectively. That is $s_{k\,(1,i)} = map(c_k = 1,\ c_{j,\ j=0,\ldots,M-1,\ j\neq k} \equiv bin(i))$, $s_{k\,(0,i)} = map\ (c_k = 0,\ c_{j,\ j=0,\ldots,M-1,\ j\neq k} \equiv bin(i))$. We refer to this exhaustive estimation as Ex_{est}.

We can apply the max-log approximation of (4) to reduce the computational complexity and to avoid saturation problems of reliability values caused by high E_b/N_0 or a large number of iterations.

$$\ln\left(e^{\delta_1} + e^{\delta_2} + e^{\delta_3} + \cdots + e^{\delta_n}\right) \approx \max_{i\in\{1,\ldots n\}} \delta_i \cdot \quad (4)$$

Now, we can implement the soft demapper with reduced complexity as follows

$$L(c_k|r) = \max_{i=0,1,\ldots,2^{M-1}-1}\left[\frac{1}{\sigma^2}\left(r \cdot s_{k(1,i)} - \frac{1}{2}s^2{}_{k(1,i)}\right)\right] - \max_{i=0,1,\ldots,2^{M-1}-1}\left[\frac{1}{\sigma^2}\left(r \cdot s_{k(1,i)} - \frac{1}{2}s^2{}_{k(1,i)}\right)\right]. \quad (5)$$

Let x be the in-phase component and y be the quadrature component of the complex channel symbol, *i.e.* $r = (x,\ y)$. For the kth coded bit, c_k, $z_{k(0)} = (x_0,\ y_0)$ is the nearest point in the constellation from the received symbol among the points whose kth bits are zero, and $z_{k(1)} = (x_1\ ,\ y_1)$ is the nearest point from the received symbol among the points whose kth bit is one. Figure 2 illustrates this. By using the above notation, we can express (5) as follows.

$$L(c_k|r) = -\frac{1}{2\sigma^2}\left[\left(-2xx_1 - 2yy_1 + x_1^2 + y_1^2\right) - \left(-2xx_0 - 2yy_0 + x_0^2 + y_0^2\right)\right] \cdot \quad (6)$$

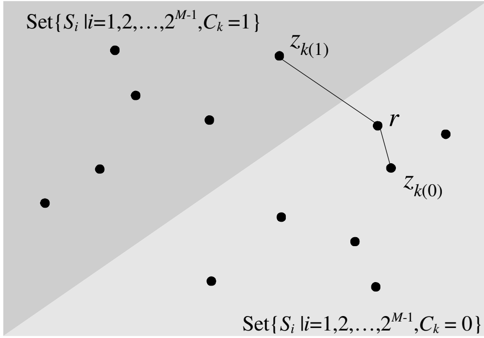

Figure 2. The demapping method using a nearest constellation

If we apply the max-log approximation explained above, we do not need exponential and log operations. Although this can reduce computational complexity, we still have to estimate all possible Euclidean distances to search the nearest constellation points as shown in Figure 2. We refer to this reduced search method using the nearest points as $\mathrm{Ne}_{\mathrm{est}}$. We further reduce the computational complexity by adopting a simple threshold comparison mechanism, and thus a single distance estimation process accomplishes the soft demapping. In the next section we explain it in more detail.

III. Soft demapping method using a decision threshold

One of the most important factors in turbo coded high order modulation scheme is reducing the demapping complexity of the exhaustive distance estimation method. We propose a novel soft demapping method using a decision threshold that exhibits reasonable demapping complexity but practically negligible loss in accuracy.

The decision threshold for the kth coded bit, DTH_k, here we mean by the threshold which divides the constellation plane into 2 regions by the value of the kth bit in the symbol points. Referring to Figure 3, DTH_1 divides the conste

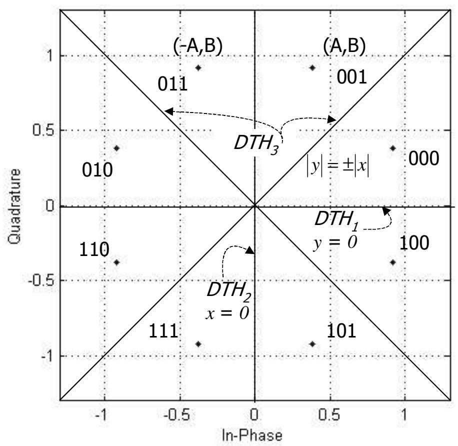

Figure 3. Gray coded 8PSK constellation and the decision thresholds

In the region above DTH_1 the first bit of all points are 0, while in the region below DTH_1 the first bit of all points are 1. Similarly, DTH_2 divides the constellation plane into left and right region by the second bit value of the symbols.

Now we can easily estimate soft demapping values by using DTH_k. The channel reliability of each consisting bit is the distance from the received symbol and the decision threshold. Therefore, this method does not need exhaustive Euclidean distance estimations, but needs only a single simple distance estimation. That is,

$$L(c_k|r) = \frac{2}{\sigma^2}[r - DTH_k] = \frac{2}{\sigma^2} sign(r - DTH_k)\cdot\|r - DTH_k\|\cdot \quad (7)$$

We refer to this estimation method using DTH_k as DT_{est}. In many digital modulation systems, we generally use the symbol mapping scheme using Gray encoding for the best BER performance at the same symbol error rate. In the Gray encoding, the adjacent signal differ by one binary digit. By using this property, we can employ a simple decision threshold.

The Gray coded 16QAM constellation is given in Figure 4 as an example. The exhaustive demapping method, Ex_{est} must estimate the Euclidean distances between the received symbol r and all constellation points. Therefore, we need 16 distance estimations in this example. The demapping method using max-log approximation, Ne_{est} considers the Euclidean distance between the received symbol and the nearest constellation point $z_{k(0)}$ or $z_{k(1)}$. Although this is much simpler than the Ex_{est}, we still need 16 distance estimations to find the nearest point from the received symbol. The proposed method, DT_{est} can obtain the channel reliability with just a single distance estimation between DTH_k and the received symbol r.

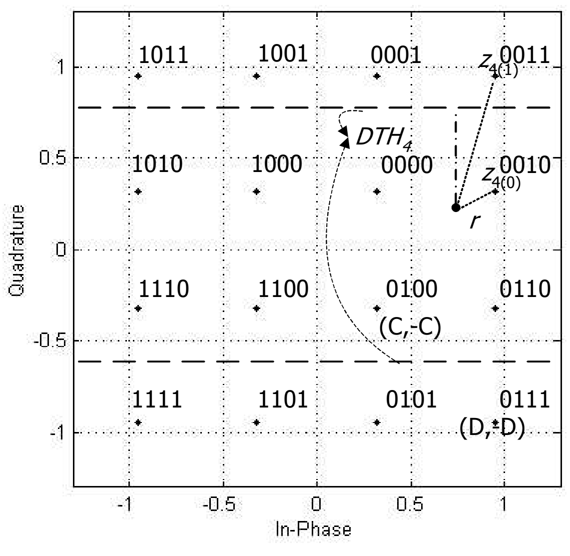

Figure 4. Gray coded 16QAM constellation and the decision threshold for the 4th coded bit

We now investigate several examples of estimating reliability values using DT_{est}, and also derive decision thresholds, DTH_k . In our method, the soft demapping values are represented as compact closed forms so that we can directly estimate reliability values from the received symbol values without any distance comparisons.

For QPSK modulation scheme, there are two bits in a symbol. This can be regarded as each consisting bit is sent to In phase (I) and Quadature phase (Q) channel respectively. Therefore, the received value in each channel would be the channel reliability value itself. This means that, in the constellation plane, x and y axes are decision thresholds.

For 8PSK modulation example of Figure 3, we will derive the expressions of reliability values for three consisting bits using Ne_{est} first, before we derive reliability expressions using DT_{est}. Since we used Gray encoding in Figure 3, we can express the reliability values using (6) as follows.

$$L(c_1|r) = \frac{2}{\sigma^2}\left[\frac{1}{2}y\cdot(y_1 - y_0)\right], \quad (8\text{-}1)$$

$$L(c_2|r) = \frac{2}{\sigma^2}\left[\frac{1}{2}x\cdot(x_1 - x_0)\right], \quad (8\text{-}2)$$

$$L(c_3|r) = \frac{2}{\sigma^2}\cdot\frac{1}{2}\left[x\cdot(x_1 - x_0) + y\cdot(y_1 - y_0)\right]. \quad (8\text{-}3)$$

If we use two positive constants A and B ($A < B$) which represent the amplitudes of I and Q components of an 8PSK symbol as shown in Figure 3, we can rewrite (8-1) to (8-3) as follows.

$$L(c_1|r) = \frac{2}{\sigma^2}K_1 y, \qquad K_1 = \begin{cases} A & \text{if } |y| > |x| \\ (A+B)/2 & \text{otherwise} \end{cases}, \quad (9\text{-}1)$$

$$L(c_2|r) = \frac{2}{\sigma^2}K_2 x, \qquad K_2 = \begin{cases} A & \text{if } |x| > |y| \\ (A+B)/2 & \text{otherwise} \end{cases}, \quad (9\text{-}2)$$

$$L(c_3|r) = -\frac{2}{\sigma^2}K_3(|x| - |y|), \qquad K_3 = \left(\frac{B - A}{2}\right). \quad (9\text{-}3)$$

Now we derive reliability expressions using DT_{est}. As we can see in Figure 3, the decision threshold for each coded bits can be set as DTH_1: $y = 0$, DTH_2: $x = 0$, DTH_3: $|y| = \pm|x|$. By applying these decision thresholds, we can express the reliability values for three consisting bit as

$$L(c_1|r) = \frac{2}{\sigma^2}[y - 0], \quad (10\text{-}1)$$

$$L(c_2|r) = \frac{2}{\sigma^2}[x - 0], \quad (10\text{-}2)$$

$$L(c_3|r) = \frac{2}{\sigma^2}[|x| - |y|]. \quad (10\text{-}3)$$

As we can see in (8-1) to (10-3), reliability expression using Ne_{est} and Dt_{est} are numerically equivalent.

Using 16QAM example of Figure 4, the first and third bits of symbols are symmetric with respect to x-axis and the second and forth bit are symmetric with y-axis. We can derive the expressions of reliability values for the consisting bits using Ne_{est} as follows.

$$L(c_{1,3}|r) = \frac{2}{\sigma^2}\cdot\frac{1}{2}\left[x\cdot(x_1 - x_0) + \left(\frac{-x_1^2 + x_0^2}{2}\right)\right], \quad (11\text{-}1)$$

$$L(c_{2,4}|r) = \frac{2}{\sigma^2}\cdot\frac{1}{2}\left[y\cdot(y_1 - y_0) + \left(\frac{-y_1^2 + y_0^2}{2}\right)\right]. \quad (11\text{-}2)$$

If we use two positive constants C and D ($C < D$), which represent the amplitudes of I and Q components of a 16QAM, symbol as shown in Figure 4, we can rewrite (11-1) to (11-2) as follows.

$$L(c_1|r) = \begin{cases} \frac{2}{\sigma^2} K_4 x, & \text{if } |x| < \left(\frac{C+D}{2}\right) \\ \frac{2}{\sigma^2} K_5 sign(x)\left(|x| - \frac{D-C}{2}\right), & \text{otherwise} \end{cases} \quad (12\text{-}1)$$

$$L(c_2|r) = \begin{cases} \frac{2}{\sigma^2} K_4 y, & \text{if } |y| < \left(\frac{C+D}{2}\right) \\ \frac{2}{\sigma^2} K_5 sign(y)\left(|y| - \frac{D-C}{2}\right), & \text{otherwise} \end{cases} \quad (12\text{-}2)$$

$$L(c_3|r) = \frac{2}{\sigma^2} K_6\left(|x| - \frac{C+D}{2}\right), \quad (12\text{-}3)$$

$$L(c_4|r) = \frac{2}{\sigma^2} K_6\left(|y| - \frac{C+D}{2}\right), \quad (12\text{-}4)$$

where $K_4 = C / 2$, $K_5 = (C + D) / 2$, $K_6 = (C - D) / 2$.

We can also represent reliability values for four consisting bits as a compact form. The decision thresholds are as follows; DTH_1: $x = 0$ DTH_2: $y = 0$, DTH_3: $|x| = (C + D) / 2$, DTH_4: $|y| = (C + D) / 2$.

$$L(c_1|r) = \frac{2}{\sigma^2}[x - 0], \quad (13\text{-}1)$$

$$L(c_2|r) = \frac{2}{\sigma^2}[y - 0], \quad (13\text{-}2)$$

$$L(c_3|r) = \frac{2}{\sigma^2}\left[|x| - \frac{C+D}{2}\right], \quad (13\text{-}3)$$

$$L(c_4|r) = \frac{2}{\sigma^2}\left[|y| - \frac{C+D}{2}\right]. \quad (13\text{-}4)$$

As for the 8PSK and 16QAM example, for lager order modulation, the reliability values of DT_{est} are numerically equivalent to that of Ne_{est} are.

IV. Complexity comparison

Table I compares the computational complexity of three demapping methods investigated in the previous section and shows the number of computations required to estimate the channel reliability.

To calculate the channel reliability, Ex_{est} groups constellation points by their consisting bit values. After then, we must calculate the $M{\cdot}2^M$ Euclidean distances between a received symbol and the constellation points. This method also includes several complex operations (exponential and log operations).

In the Ne_{est}, the exponential and log operations are eliminated by considering only nearest term for nominator and denominator of (5). However, we still have to estimate Euclidean distances and search the minimum among them.

The proposed DT_{est} does not need any Euclidean distance estimation. Instead, we need to find the decision threshold of each coded bit, but this can be done prior to the system operation and thus can be saved in the ROM. Therefore the demapper simply estimate reliability values using a simple equation for each bit. Therefore, as the modulation order M is larger, we can further reduce computational complexity.

If we only consider the distance estimation operations, DT_{est} needs only $1/2^M$ computational complexity compared to Ne_{est} and Ex_{est}. Therefore, even with 8PSK scheme, DT_{est} needs only 12.5% of complexity, and Figure 5 demonstrates complexity comparison with number of distance estimations.

TABLE I. COMPARISON OF THE COMPLEXITY OF SOFT DEMAPPING METHODS PER MODULATION SYMBOL (NA; NOT APPLICABLE)

Operations \ Demapping Method	**Ex_{est}**	**Ne_{est}**	**DT_{est}**
Number of distance estimations	$M{\cdot}2^M$	$M{\cdot}2^M$	M
Exponential estimations	$M{\cdot}2^M$	NA	NA
Log estimations	M	NA	NA
Searching the minimum distance	NA	$M{\cdot}2$	NA

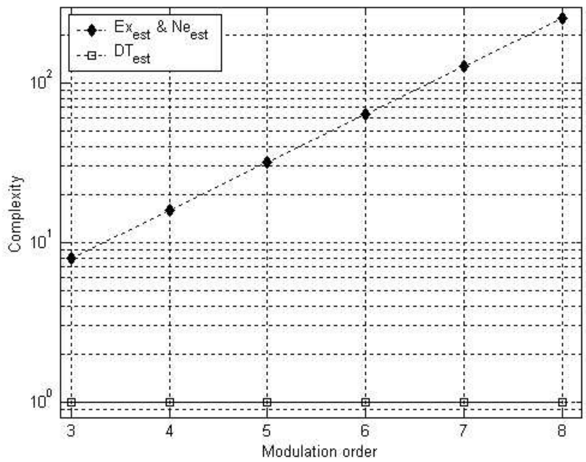

Figure 5. Complexity comparison with number of distance estimations

V. Simulation results

We implemented the proposed demapping algorithm and performed simulation on an AWGN channel. We used a modified soft output Viterbi algorithm in the iterative decoder [11]. In the previous section, we showed that the result of DT_{est} is numerically equivalent to that of Ne_{est} except the scaling factor. Normalization of the soft output values is one of the important factors in the SOVA based decoder, and thus we optimized the normalization factor for the adopted soft demapping schemes.

Figure 6 and 7 compares the performance of soft demapping algorithms. We used a 2D block turbo code with a (31,25) extended BCH code.

As we can see in Figure 6 and 7, the DT_{est} gives almost the same performance as the Ne_{est} and Ex_{est} does. In other words, the proposed demapper DT_{est} can achieve the same

BER performance with much lower computational complexity than the conventional demapper Ex_{est} and Ne_{est}.

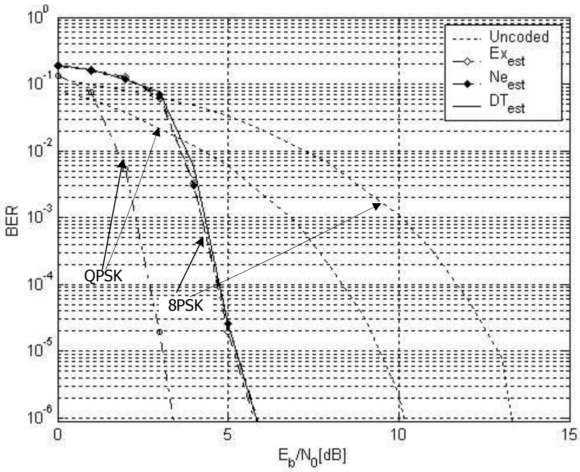

Figure 6. BER performance comparison of the soft demapping algorithms with QPSK and 8PSK modualtion schemes

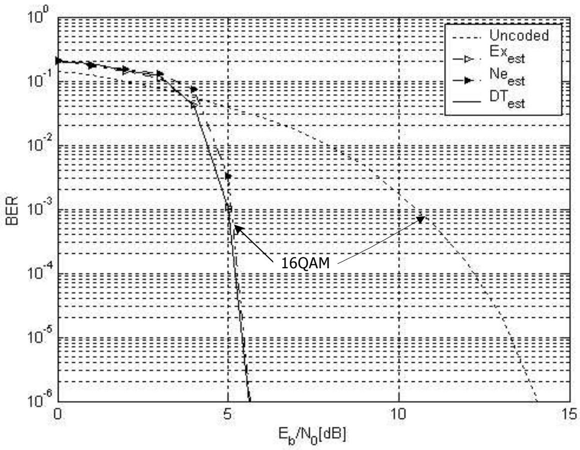

Figure 7. BER performance comparison of the soft demapping algorithms with 16QAM modualtion scheme

VI. Conclusion

We have introduced a soft demapping method using a decision threshold for high order modulation schemes. The proposed algorithm reduces computational complexity not only by eliminating complex log and exponential operations but also by reducing the number of distance estimations itself. The proposed algorithm always needs only a single distance estimation operation, and thus we can get larger complexity reduction with higher modulation order. Our simulation results using block turbo codes reveal that the proposed method achieves BER performance approximating to that of the exhaustive estimation demapping algorithm.

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