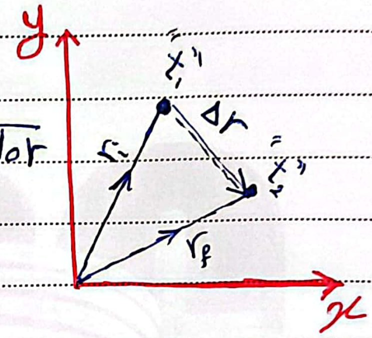
# "Motion and Kinematics" Section 2

## → Position & Displacement

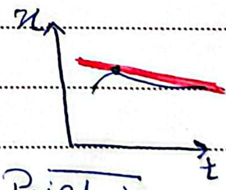



* the Position of an object is discribed by its Position Vector "$\vec{r}$"

* $\boxed{\Delta\vec{r} = \vec{r}_f - \vec{r}_i}$ "change in Potion"

## → Avg. Velocity

$\vec{V} = \dfrac{\Delta\vec{r}}{\Delta t}$ , the direction of avg. Velocity is the direction of displacement Vector $\Delta\vec{r}$.




## → Ins. Velocity :-

direction of ins. Velo. Vector at any Point in Particle Path is along a line Tangent To the Path at that Point & in the direction of motion

mag. of inst. Velocity Vector is Speed. ↳"scalar"

→ "avg. Acc."  $\boxed{\vec{a} = \frac{\Delta \vec{V}}{\Delta t}}$

↳ rate of change of instantaneous velocity

→ "ins. acc."  $\vec{a} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

↳ limit of avg. acc. as "$\Delta t \to 0$"

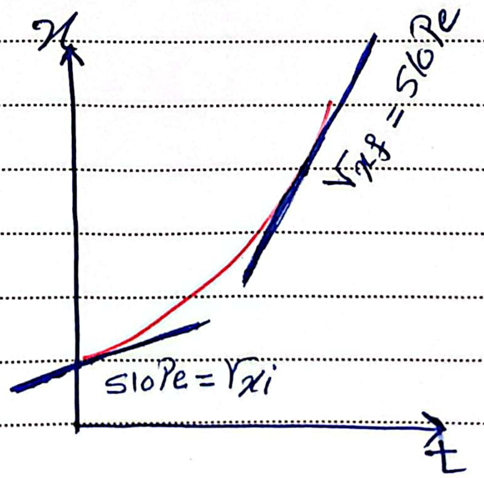
⇒ Motion in "1D"

Displacement: change in position

$$\Delta x = x_f - x_i$$

* instant. velocity

$$\boxed{V_x = \frac{dx}{dt}}$$



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## ⟹ Kinematic Equations

* $V_f = V_i + at$ ⟶ (1)

* $x_f - x_i = V_i t + \frac{1}{2} a t^2$ ⟶ (2)

* $V_{xf}^2 = V_i^2 + 2a\Delta x$ ⟶ (3)

## ⟹ For 2D

* Position vector $\boxed{\vec{r} = x\hat{i} + y\hat{j}}$

* Velocity $\boxed{\vec{V} = \frac{d\vec{r}}{dt} = V_x\hat{i} + V_y\hat{j}}$

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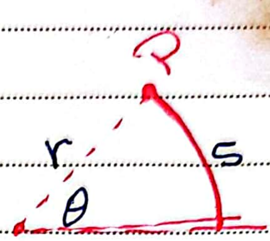
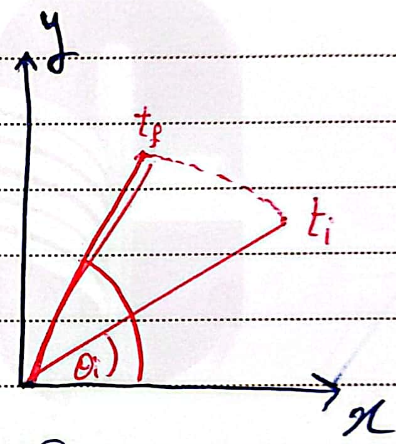
## Angular motion




$S = \theta r$

↳ arc length

## * Angular Displacement




$$\Delta\theta = \theta_f - \theta_i$$

* $\boxed{\vec{\omega} = \frac{\Delta\theta}{\Delta t}}$ "rad/sec" "$s^{-1}$"

→ avg. angular speed of rotating rigid object

$\boxed{\omega = \frac{d\theta}{dt}}$ → instantaneous angular speed.

$\vec{\alpha}, \vec{\omega}$ → +ve inc. in $\theta$

→ -ve dec. in $\theta$

$\boxed{\vec{\alpha} = \frac{\Delta\omega}{\Delta t}}$

$\boxed{\alpha = \frac{d\omega}{dt}}$

# * Rotational Kinematic :-

* $\omega_f = \omega_i + \alpha t$

* $\theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$

* $\omega_f^2 = \omega_i^2 + 2\alpha \Delta\theta$

## → Relationship between Angular & linear Quantites in Rotation

* Every Point on the rotating object has the same "angular motion" & "does not" have the same "linear motion"

Displacements

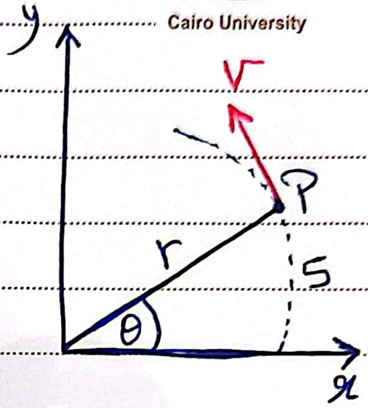
$s = \theta r$

Speeds

$v = \omega r$

accelerations $a = \alpha r$

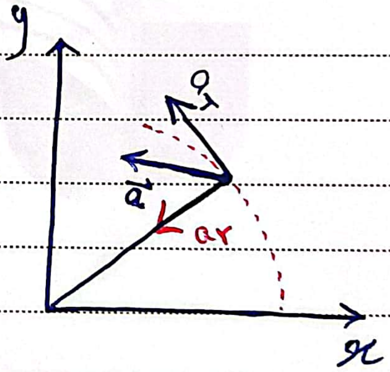
## "Speed Comparison"




* Linear Velocity always Tangent To circular Path "Tangential Velocity"

$$\vec{V} = \frac{dS}{dt} = r\frac{d\theta}{dt} = \vec{r} \times \vec{\omega}$$

## Acceleration Comparison:




* Tang. acc. is the derivative of Tang. Velocity

$$\vec{a_t} = \frac{dV}{dt} = r\frac{d\omega}{dt} = \vec{r} \times \vec{\alpha}$$

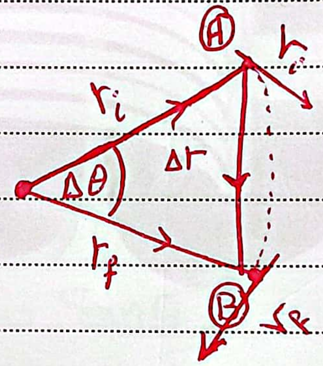
All Points on the rigid object depend on (have the same angular speed but not the same $V_T$, Bec: $V_T$ depend on "r"

## uniform circular motion :-

* occurs when Body moves in cir. Path with cons. speed. ∴ direction changes so acc. exists

* the change of velocity in uniform motion is a change in direction.

$\Delta v = v_f - v_i$



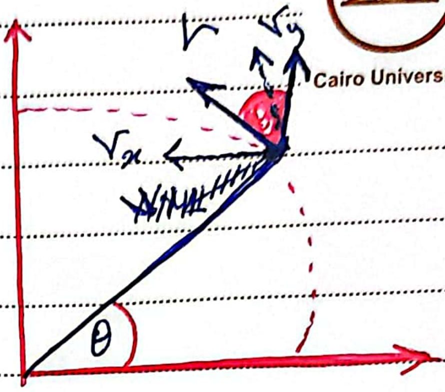

## Centripetal ACC.

→ acc. always ⊥ to the Path of motion

$a_c = v^2/r$

→ mag.

→ direction always changing to stay directed toward the center of the circle of motion.




$V = V_x \hat{i} + V_y \hat{j}$

$V_x = -V \sin\theta$

$V_y = V \cos\theta$

$$\vec{a} = \frac{dV}{dt} = \frac{dV_x}{dt}\hat{i} + \frac{dV_y}{dt}\hat{j}$$

$$= -V\cos\theta \frac{d\theta}{dt}\hat{i} - V\sin\theta \frac{d\theta}{dt}\hat{j}$$

$$\vec{a} = -V\omega\cos\theta\,\hat{i} - V\omega\sin\theta\,\hat{j}$$

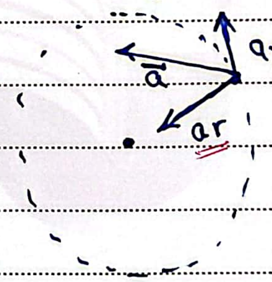
$$|\vec{a}| = V\omega = V \times V/r = V^2/r$$

direction towards the center

* "Period" $\Longrightarrow$ Time for one compelete Revolution

$\boxed{T = \frac{2\pi r}{V}}$ as $V = r \times \omega$ $T = \frac{2\pi}{\omega}$, $\omega = \frac{2\pi}{T}$

* non-uniform circular motion
  * the mag. of Velocity is Changing
  * so there would be "Tangential acc."
    ↳ cause change in speed
  * the radiar acc. comes from changing the direction of Velocity Vector.




* $\boxed{a_T = \frac{d|\vec{V}|}{dt}}$
* $\boxed{a_r = -a_c = -V^2/r}$
* Total acc. $\boxed{a = \sqrt{a_r^2 + a_t^2}}$