Potential Energy

Prof. Salah Gamal

Systems with Multiple Particles

- Consider a system of objects in equilibrium with each others. The objects are interacting among themselves and are exerting forces on each element of the system. Let us focus on one particular object of the system. This object is affected by the forces from all other objects. Thus we can think of our system as this one particular object plus the force field $\mathbf{F}_{field}$ due to all other objects.

Systems with Multiple Particles, cont.

- To change the position of the object with respect to all other objects, that is, to take the object from position A to position B we apply an external applied force $\mathbf{F}_{app}$ to take the object from position A to position B. The object moves from A to B under the influence of two forces: the applied force, $\mathbf{F}_{app}$ and the field force $\mathbf{F}_{field}$. From the work kinetic energy theorem, we have:

Systems with Multiple Particles, cont.

$$W_{\text{total}} = \Delta K$$

$$W_{\text{app}} + W_{\text{field}} = \Delta K$$

But as the initial and final velocities are zero, $\Delta K = 0$, and therefore,

$$W_{\text{app}} = - W_{\text{field}}$$

As work is done on the system there is energy transfer from the environment to the system, and the system energy in configuration B is greater than in configuration A. Let ΔE be the change in system energy. Thus, $\Delta E = W_{\text{app}}$, and thereby, $\Delta E = - W_{\text{field}}$

Systems with Multiple Particles, cont.
Potential Energy U

- Furthermore, if W_{field} is independent of the path from A to B (the field force is called in this case **conservative**), ΔE is constant (independent of the path) and depends only on the initial and final configurations of the system. This particular energy associated with the system configuration in the presence of conservative force field is called the potential energy U for a given configuration of the system . Thus, we can write
- $$\Delta U = U_B - U_A$$

Potential Energy

- Thus,

$$U_B - U_A = -W_{\text{field}}$$

where U_A and U_B are the energies associated with the system configurations A and B, respectively. For a given configuration we cannot calculate the potential energy U; only we can calculate the difference in potential energy of two configurations.

However, we may take arbitrary $U_A = 0$ and calculate the potential energy U_B of configuration B as $U_B = -W_{\text{field}}$.

The potential energy function $U(x,y,z)$

- It is often convenient to establish some particular location (x_A, y_A, z_A) of one member of a system as representing a reference configuration and measure all potential energy differences with respect to it. We can then define the potential energy function as

$$U(x,y,z) = -\int_{\mathbf{r}_A}^{\mathbf{r}} \mathbf{F}_{\text{field}} \cdot d\,\mathbf{r} + U(x_A, y_A, z_A)$$

The value of U_A is often taken to be zero for the reference configuration. It really does not matter what value we assign to U_A because any nonzero value merely shifts U(x,y,z) by a constant amount and only the *change* in potential energy is physically meaningful.

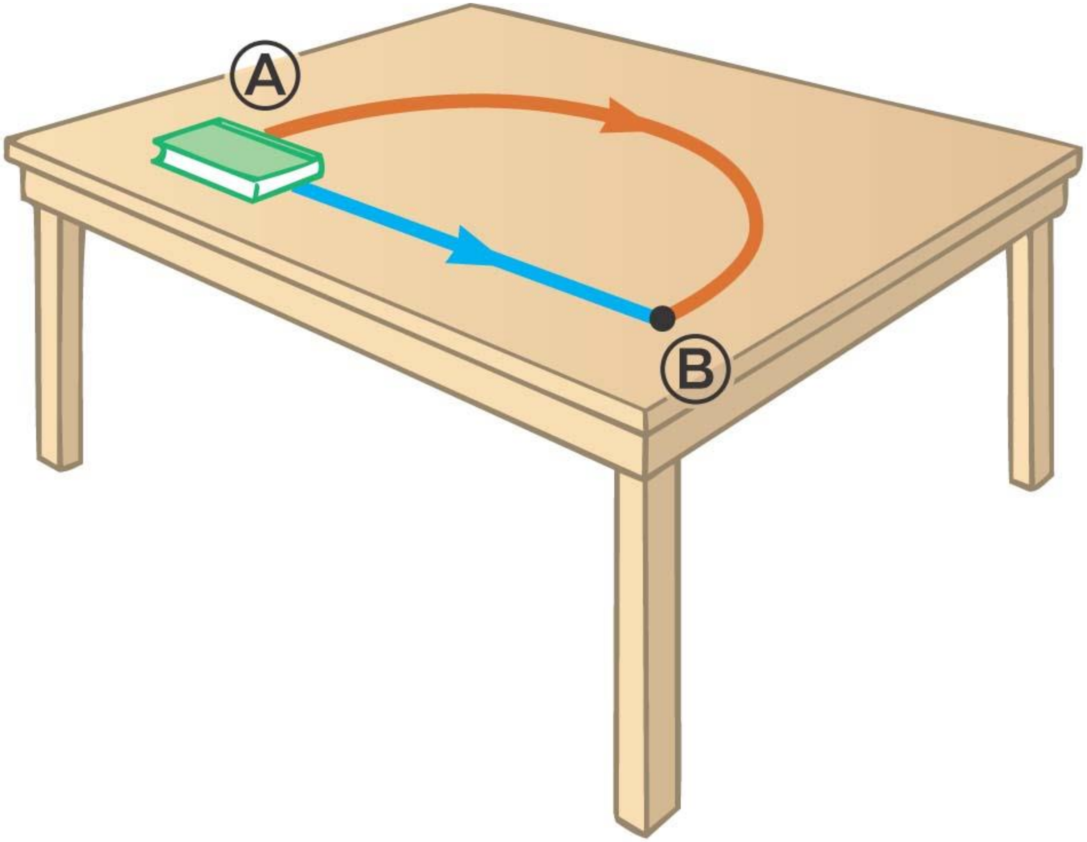
Potential Energy

- Potential energy is the energy associated with the configuration of a system of objects that exert *conservative* forces on each other. In this case, we say that one object is in the field of force of other objects. Examples of force fields are the gravitational force between masses and the electric force between charges.

Conservative Forces

- The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle
- The work done by a conservative force on a particle moving through any closed path is zero
  - A closed path is one in which the beginning and ending points are the same

Conservative Forces



Conservative Forces, cont

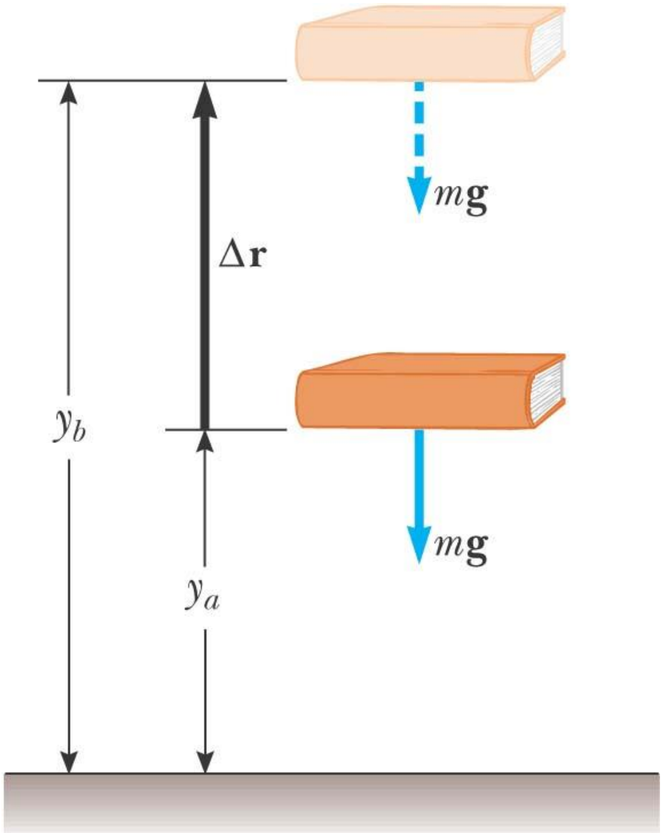
- Examples of conservative forces:
  - Gravity
  - Spring force
- We can associate a potential energy for a system with any conservative force acting between members of the system
  - This can be done only for conservative forces
  - In general: $W_C = - \Delta U$

Gravitational Potential Energy

- Gravitational Potential Energy is associated with an object at a given distance above Earth's surface.
- Take the y-axis vertical upward. The gravitational force near the surface of earth on an object of mass *m* is given by
- $$\mathbf{F}_g = -mg\,\mathbf{j}$$

$$\Delta\mathbf{r} = \Delta y\,\hat{\mathbf{j}}$$

- $W_{field} = \mathbf{F}_g \cdot \Delta\mathbf{r} = -mg\,(y_f - y_i)$



Gravitational Potential Energy, cont

- As $\Delta U_g = -W_{field}$, we have,
- $U_{gf} - U_{gi} = mg\,(y_f - y_i)$

We can set $U_{gi} = 0$ and take the origin of the y-axis at y_i. In this case,

$$U_g\,(y) = mgy$$

Gravitational Potential Energy, final

- The gravitational potential energy depends only on the vertical height of the object above Earth's surface
- In solving problems, you must choose a reference configuration for which the gravitational potential energy is set equal to some reference value, normally zero
  - The choice is arbitrary because you normally need the *difference* in potential energy, which is independent of the choice of reference configuration

Elastic Potential Energy

- ***Elastic Potential Energy*** is associated with a spring
- The force the spring exerts (on a block, for example) is $F_s = - kx$
- The work done by the field (spring) is
- $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$
- $\Delta U = -W_s = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$

Elastic Potential Energy

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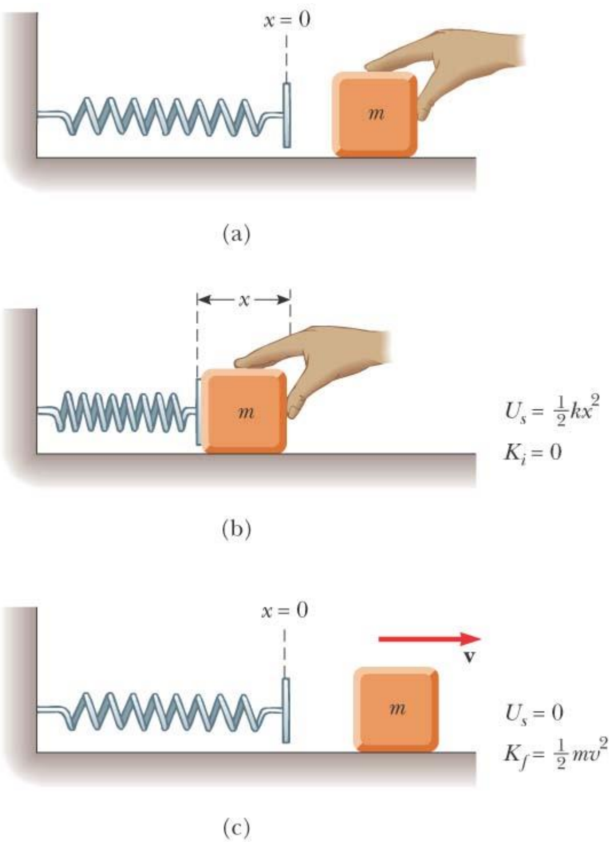
- $U_f - U_i = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$
- If x_i=0 and let $U_i = 0$, then
- $$U(x) = \frac{1}{2}kx^2$$

Elastic Potential Energy, cont

- This expression is the elastic potential energy: $U_s = ½\ kx^2$
- The elastic potential energy can be thought of as the energy stored in the deformed spring
- The stored potential energy can be converted into kinetic energy



Elastic Potential Energy, final

- The elastic potential energy stored in a spring is zero whenever the spring is not deformed ($U = 0$ when $x = 0$)
  - The energy is stored in the spring only when the spring is stretched or compressed
- The elastic potential energy is a maximum when the spring has reached its maximum extension or compression
- The elastic potential energy is always positive
  - x^2 will always be positive