Conservative Forces and Potential Energy

- The conservative force is related to the potential energy function through

$$F_x = -\frac{dU}{dx}$$

- The *x* component of a conservative force acting on an object within a system equals the negative of the potential energy of the system with respect to *x*

Conservative Forces and Potential Energy – Check

- Look at the case of a deformed spring

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

  - This is Hooke's Law

Isolated Systems

Conservation of Mechanical Energy of an Isolated System

- The isolated system means no energy transfer to or from the system, thus
- $W_{\text{applied}}=0$
- From the work-kinetic energy theorem

$$W = \Delta K$$

But $W = W_{\text{applied}} + W_{\text{field}} = W_{\text{field}} = -\Delta U$, therefore

$$-\Delta U = \Delta K$$

Conservation of Mechanical Energy of an Isolated System

- Thus for an isolated system, we have

$$\Delta U + \Delta K = 0$$

$$U_f - U_i + K_f - K_i = 0$$

$$K_f + U_f = K_i + U_i$$

$$E_f = E_i$$

Conservation of Mechanical Energy

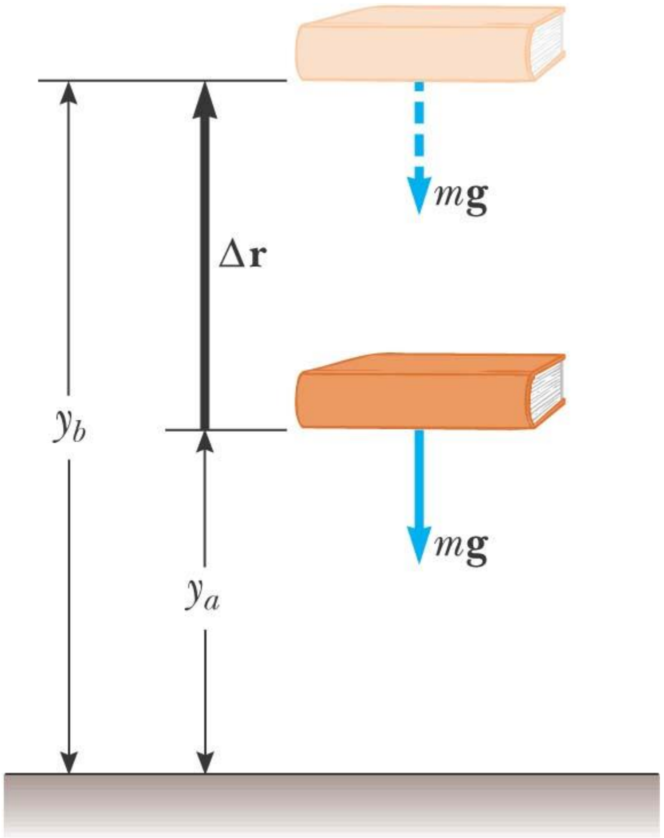
- The mechanical energy of a system is the algebraic sum of the kinetic and potential energies in the system
  - $E_{mech} = K + U$
- The statement of Conservation of Mechanical Energy for an isolated system is

$$K_f + U_f = K_i + U_i$$

  - An isolated system is one for which there are no energy transfers across the boundary

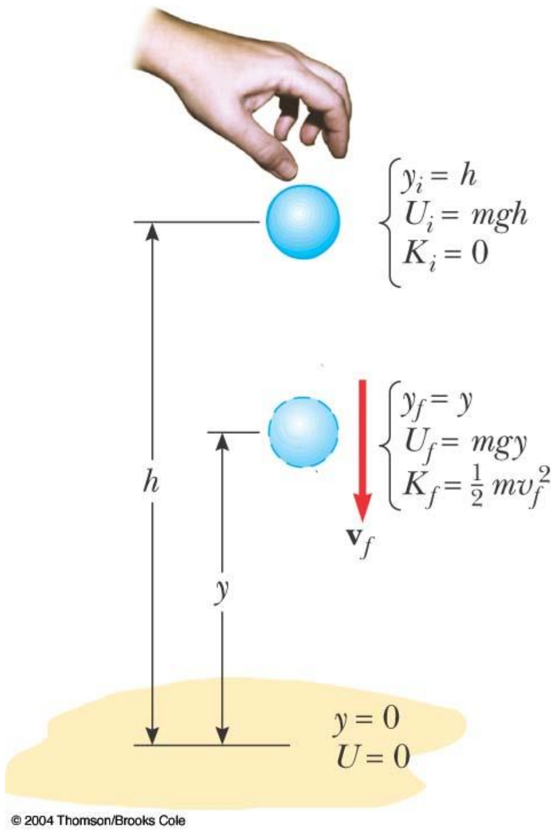
Conservation of Mechanical Energy, example

- Look at the work done by the book as it falls from some height to a lower height
- $W_{\text{on book}} = \Delta K_{\text{book}}$
- Also, $W = mgy_b - mgy_a$
- So, $\Delta K = -\Delta U_g$



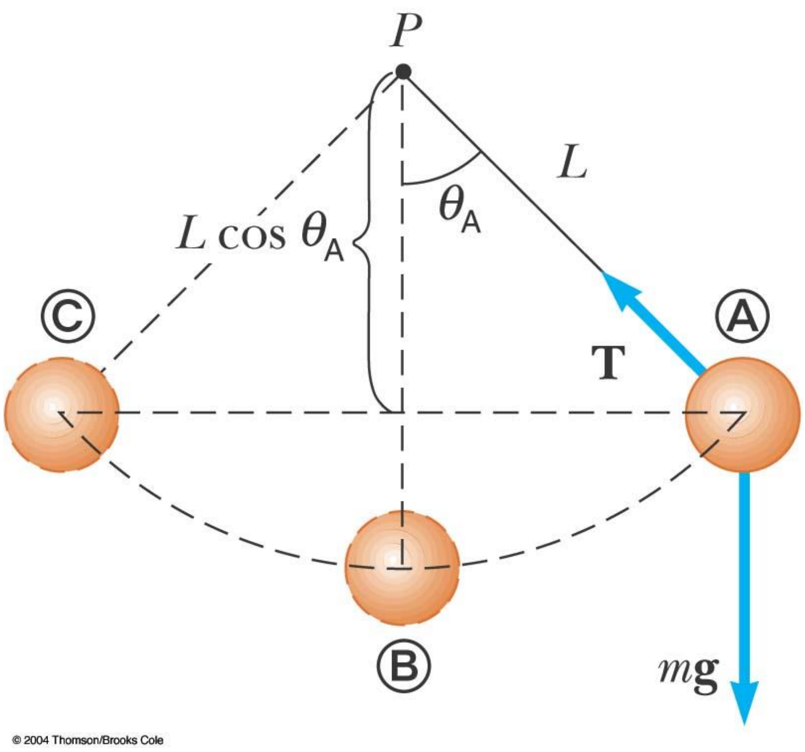
Conservation of Energy, Example 1 (Drop a Ball)

- Initial conditions:
  - $E_i = K_i + U_i = mgh$
  - The ball is dropped, so $K_i = 0$
- The configuration for zero potential energy is the ground
- Conservation rules applied at some point y above the ground gives
  - $\frac{1}{2}\, mv_f^2 + mgy = mgh$



Conservation of Energy, Example 2 (Pendulum)

- As the pendulum swings, there is a continuous change between potential and kinetic energies
- At A, the energy is potential
- At B, all of the potential energy at A is transformed into kinetic energy
  - Let zero potential energy be at B
- At C, the kinetic energy has been transformed back into potential energy



Conservation of Energy, Example 3 (Spring Gun)

- Choose point A as the initial point and C as the final point
- $E_A = E_C$
  - $K_A + U_{gA} + U_{sA} = K_A + U_{gA} + U_{sA}$
  - $\frac{1}{2} kx^2 = mgh$