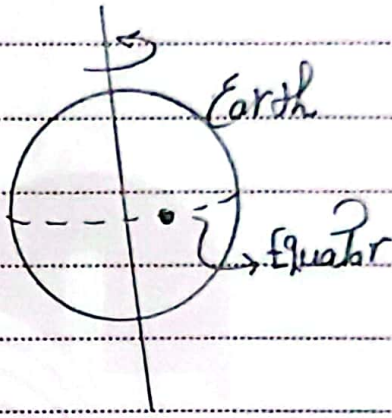
Sol. of GAP "121" Midterm

Q1) Find $a_r = ?$, $R = 6.37 \times 10^6$ m

$$a_r = \frac{V^2}{R} \quad , \quad \vec{V} = \vec{R} \times \vec{\omega} \quad , \quad \omega = \frac{2\pi}{T}$$



$$V = \frac{2\pi R}{T} = \frac{2 \times \pi \times 6.37 \times 10^6}{24 \times 60 \times 60}$$

$$V \approx 463.24 \text{ m/s}$$

$$\therefore a_r = \frac{V^2}{R} \approx 0.034 \text{ m/s}^2$$

Q2 $\vec{r} = t^3 \hat{i} + 20 \hat{j}$, a) $\vec{V}$ & $\vec{a}$ b) $r(2) = ?$

Sol: a) $\vec{V} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(t^3 \hat{i} + 20\hat{j}) = 3t^2 \hat{i}$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}(3t^2 \hat{i}) = 6t\hat{i}$$

b) $r(2) = 2^3 \hat{i} + 20\hat{j} = 8\hat{i} + 20\hat{j}$

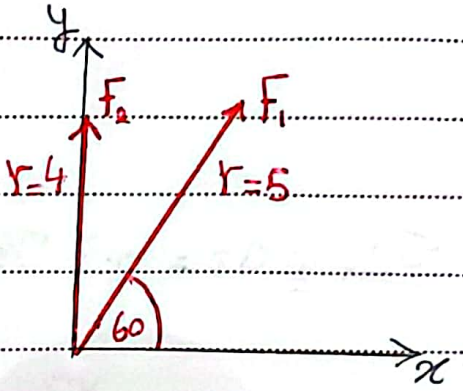
Q3)

$f = \frac{100 \text{ rev}}{\text{min}}$, find Speed & acceleration

Sol: $\vec{V} = \vec{r} \times \vec{\omega} = \vec{r} \times 2\pi f = 2\pi r f = 2 \times \pi \times 0.4 \times \frac{100}{60} = \frac{4\pi}{3}$ m/s

$a = \frac{V^2}{r} \approx 43.86 \text{ m/s}^2$

Q4) Find $|\vec{F}_1 + \vec{F}_2|$ & $\theta(\vec{F}_1 + \vec{F}_2)$



Sol: $\vec{F}_1 = F_{1x}\hat{i} + F_{1y}\hat{j}$

$= r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j}$

$= 5\cos 60\,\hat{i} + 5\sin 60\,\hat{j}$

$\vec{F}_1 = \frac{5}{2}\hat{i} + \frac{5\sqrt{3}}{2}\hat{j}$ ---- (1)

By the same way

$F_2 = F_{2x}\hat{i} + F_{2y}\hat{j}$

$= r\cos\theta\,\hat{i} + r\sin\theta\,\hat{i}$

$= 4\cos 90\,\hat{i} + 4\sin 90\,\hat{j}$

$\vec{F}_2 = 0\hat{i} + 4\hat{j}$

Then $\vec{F}_1 + \vec{F}_2 = \frac{5}{2}\hat{i} + \frac{8+5\sqrt{3}}{2}\hat{j}$

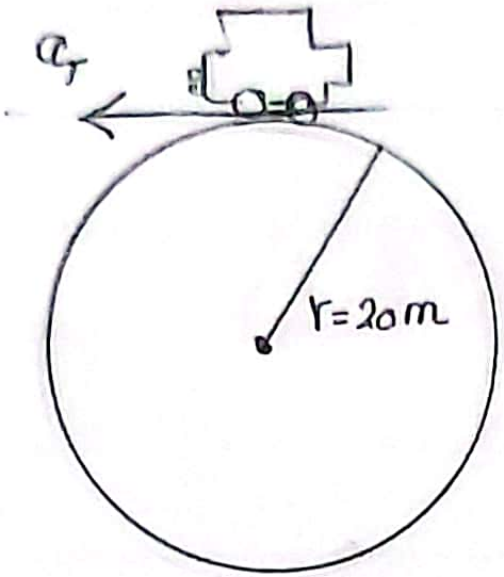
$$|\vec{F}_1 + \vec{F}_2| = \sqrt{(F_{1x} + F_{2x})^2 + (F_{1y} + F_{2y})^2}$$

$$|\vec{F}_1 + \vec{F}_2| = \sqrt{(5/2)^2 + \left(\frac{8+5\sqrt{3}}{2}\right)^2} \simeq 8.7 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{F_{1y} + F_{2y}}{F_{1x} + F_{2x}}\right) \simeq 73.3^\circ$$

Q5)

$v = 4\ m/s$ $\quad a_T = 0.7\ m/s^2$



a) $a_T = ?$

b) $a_c = ?$

c) $|a_T| = ?$

Sol: a) $a_T = 0.7\ m/s^2$

b) $a_c = v^2/R = 4^2/20 = 4/5\ m/s^2$ (towards center)

c) $|a| = \sqrt{a_T^2 + a_c^2} = \sqrt{0.7^2 + (4/5^2)} = \frac{\sqrt{113}}{10} \simeq 1.06\ m/s^2$