Review 2

Motion and Kinematics (Cont.)

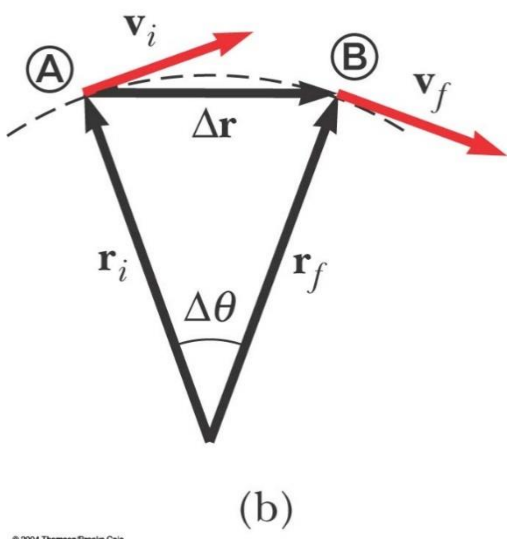
Uniform and Non-uniform Circular Motion

Uniform Circular Motion

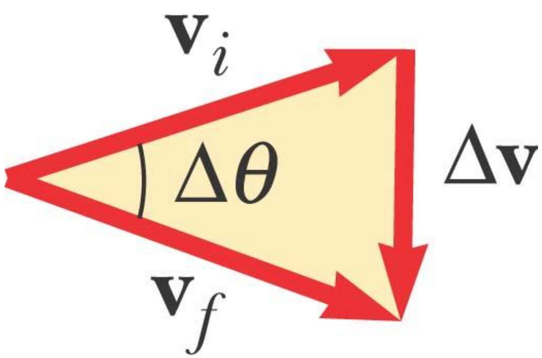
- ***Uniform circular motion*** occurs when an object moves in a circular path with a constant speed
- An acceleration exists since the *direction* of the motion is changing
  - This change in velocity is related to an acceleration
- The velocity vector is always tangent to the path of the object

Changing Velocity in Uniform Circular Motion

- The change in the velocity vector is due to the change in direction



- The vector diagram shows $\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$



Centripetal Acceleration

- The acceleration is always perpendicular to the path of the motion
- The acceleration always points toward the center of the circle of motion
- This acceleration is called the ***centripetal acceleration***

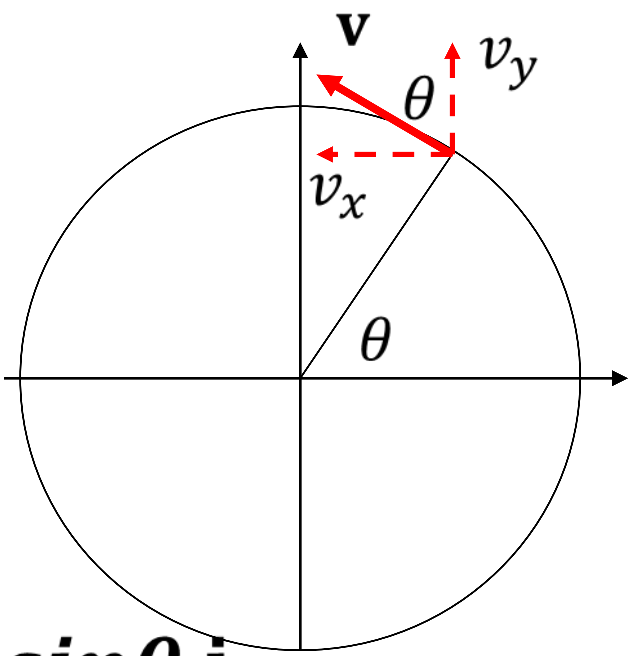
Centripetal Acceleration, cont

- The magnitude of the centripetal acceleration vector is given by

$$a_C = \frac{v^2}{r}$$

- The direction of the centripetal acceleration vector is always changing, to stay directed toward the center of the circle of motion

The Centripetal Acceleration



- $\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j}$
- $\boldsymbol{v_x = -v\, sin\theta}$
- $\boldsymbol{v_y} = \boldsymbol{v} \cos\boldsymbol{\theta}$
- $\mathbf{a} = \frac{d\mathbf{v}}{\boldsymbol{dt}} = \frac{dv_x}{dt}\mathbf{i} + \frac{dv_y}{dt}\mathbf{j}$
- $\quad = -v\,\frac{d\theta}{dt}\cos\boldsymbol{\theta}\,\mathbf{i}\, - v\,\frac{d\theta}{dt}\,\boldsymbol{sin\theta}\,\mathbf{j}$
- $\quad = -v\,\omega\,\cos\boldsymbol{\theta}\,\mathbf{i}\, - v\,\omega\,\boldsymbol{sin\theta}\,\mathbf{j}$

- $\mathbf{a} = -v\,\omega\,\cos\boldsymbol{\theta}\,\mathbf{i} - v\,\omega\,\boldsymbol{sin\theta}\,\mathbf{j}$

- The direction of this acceleration is toward the center and its magnitude is $a = |\mathbf{a}|$
- $a = \sqrt{v^2\omega^2 cos^2\theta + v^2\omega^2 sin^2\theta}$
- $a = v\ \omega\ \sqrt{cos^2\theta + sin^2\theta}\ = v\ \omega = v\ \frac{v}{r} = \frac{v^2}{r}$
- *Because* $v = \omega\ r$

Period

- The ***period***, *T*, is the time required for one complete revolution
- The speed of the particle would be the circumference of the circle of motion divided by the period
- Therefore, the period is $T \equiv \frac{2\pi r}{v}$

Period (Cont.)

- Therefore, the period is $$T \equiv \frac{2\pi r}{v}$$

- But $v = \omega r$
- Thus $T = \frac{2\pi}{\omega}$ or $\omega = \frac{2\pi}{T}$

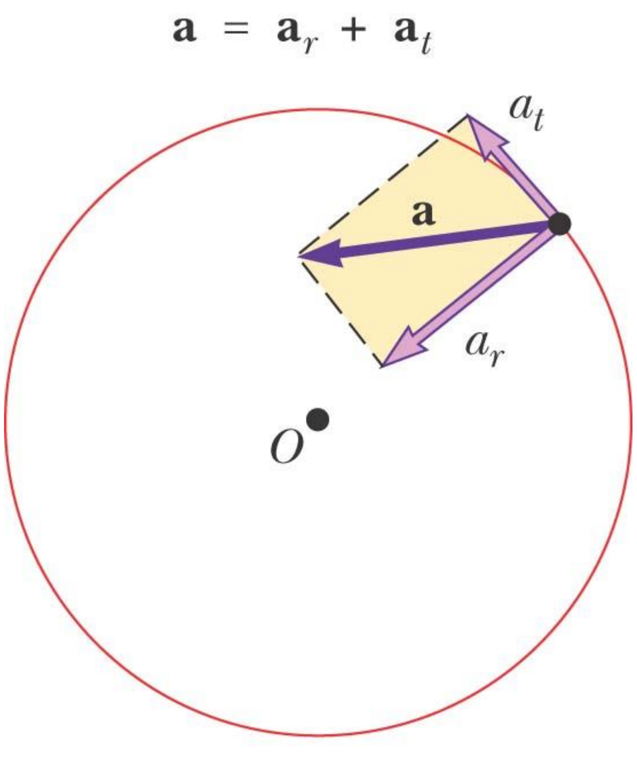
- Where ω is the angular speed

Non-uniform Circular Motion Tangential Acceleration

- The magnitude of the velocity could also be changing
- In this case, there would be a ***tangential acceleration***

Total Acceleration

- The tangential acceleration causes the change in the speed of the particle
- The radial acceleration comes from a change in the direction of the velocity vector



(b)

Total Acceleration, equations

- The tangential acceleration: $a_t = \dfrac{d\left|\mathbf{v}\right|}{dt}$
- The radial acceleration: $a_r = -a_C = -\dfrac{v^2}{r}$
- The total acceleration:
  - Magnitude $a = \sqrt{a_r^2 + a_t^2}$