Energy

Energy
and
Energy Transfer

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Introduction to Energy

- The concept of energy is one of the most important topics in science
- Every physical process that occurs in the Universe involves energy and energy transfers or transformations
- Energy is not easily defined

Systems

- A *system* is a small portion of the Universe
  - We will ignore the details of the rest of the Universe
- A critical skill is to identify the system

Valid System

- A valid system may
  - be a single object or particle
  - be a collection of objects or particles
  - be a region of space
  - vary in size and shape

Environment

- There is a *system boundary* around the system
  - The boundary is an imaginary surface
  - It does not necessarily correspond to a physical boundary
- The boundary divides the system from the *environment*
  - The environment is the rest of the Universe

Work

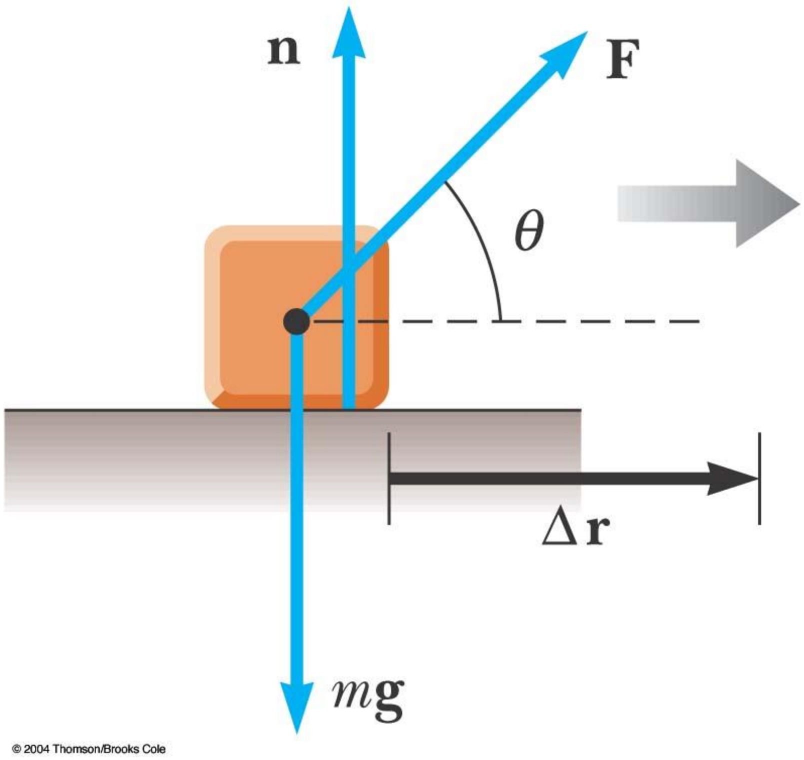
- The work, W, done on a system by an agent exerting a constant force on the system is the product of the magnitude, F, of the force, the magnitude Δr of the displacement of the point of application of the force, and cos θ, where θ is the angle between the force and the displacement vectors

Work, cont.

- $W = F \Delta r \cos \theta$
  - The displacement is that of the point of application of the force
  - A force does no work on the object if the force does not move through a displacement
  - The work done by a force on a moving object is zero when the force applied is perpendicular to the displacement of its point of application

Work Example

- The normal force, n, and the gravitational force, *m* g, do no work on the object
  - $\cos \theta = \cos 90° = 0$
- The force **F** does do work on the object



More About Work

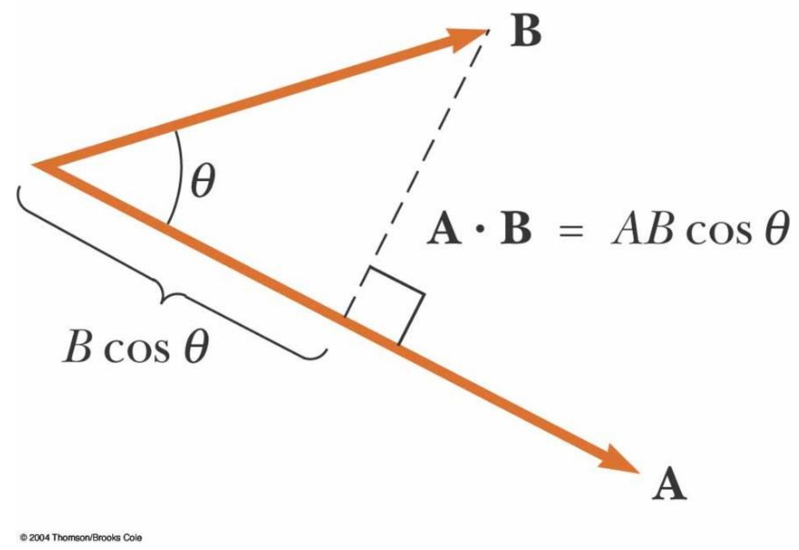
- The system and the environment must be determined when dealing with work
  - The environment does work on the system
    - Work **by** the environment **on** the system
- The sign of the work depends on the direction of **F** relative to Δ**r**
  - Work is positive when projection of **F** onto Δ**r** is in the same direction as the displacement
  - Work is negative when the projection is in the opposite direction

Units of Work

- Work is a scalar quantity
- The unit of work is a joule (J)
  - 1 joule = 1 newton · 1 meter
  - J = N · m

Scalar Product of Two Vectors

- The scalar product of two vectors is written as $\mathbf{A} \cdot \mathbf{B}$
  - It is also called the dot product
- $\mathbf{A} \cdot \mathbf{B} = A\,B\cos\,\theta$
  - θ is the angle *between* A and B



Dot Products of Unit Vectors

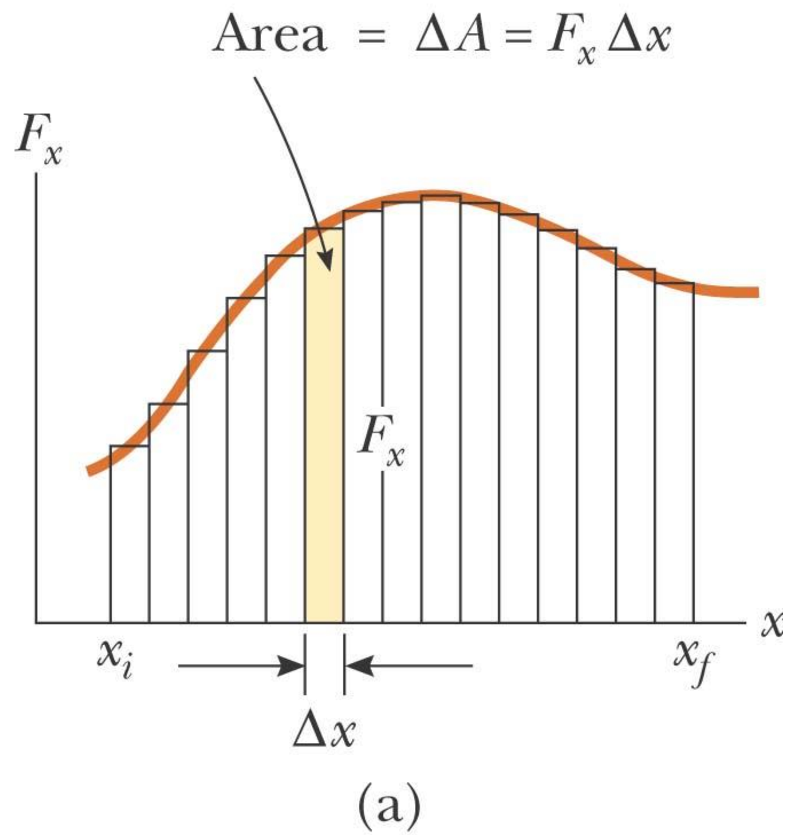
- $$\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1$$
  $$\hat{i}\cdot\hat{j}=\hat{i}\cdot\hat{k}=\hat{j}\cdot\hat{k}=0$$

- Using component form with **A** and **B**:
  $$A=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$$
  $$B=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$$
  $$A\cdot B=A_xB_x+A_yB_y+A_zB_z$$

Work Done by a Varying Force

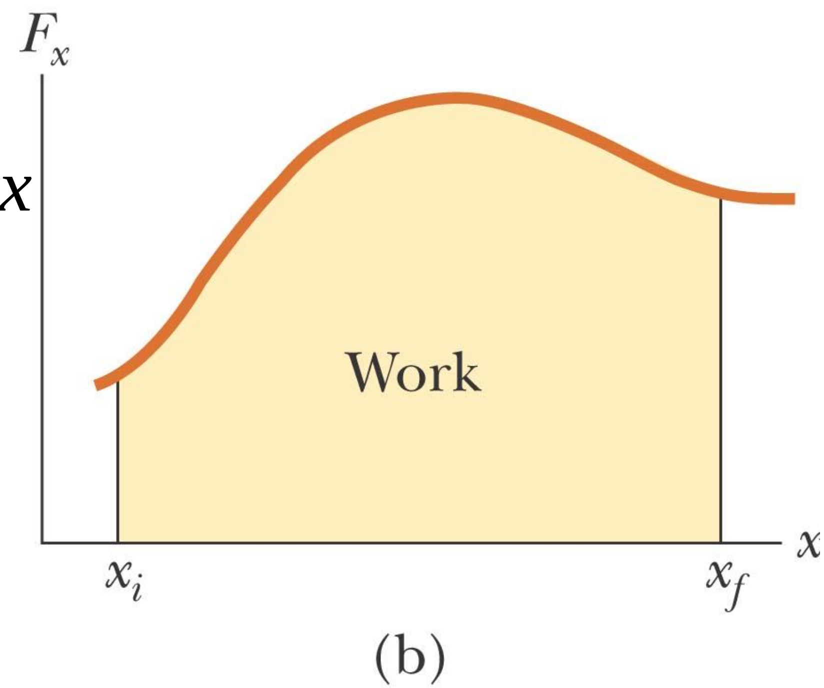
- Assume that during a very small displacement, Δx, F is constant
- For that displacement, $W \sim F \Delta x$
- For all of the intervals,

$$W \approx \sum_{x_i}^{x_f} F_x \Delta x$$



Work Done by a Varying Force, cont

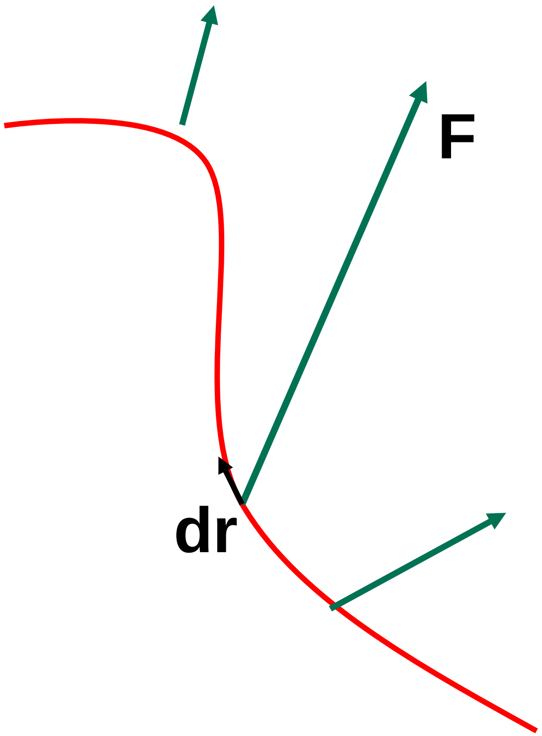
- $$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$
- Therefore, $W = \int_{x_i}^{x_f} F_x dx$
- The work done is equal to the area under the curve



- $dW = \mathbf{F}.\mathbf{dr}$

- $\boldsymbol{W} = \int_{\mathbf{r}_A}^{\mathbf{r}_B} \mathbf{F}.\mathbf{dr}$

- $\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$

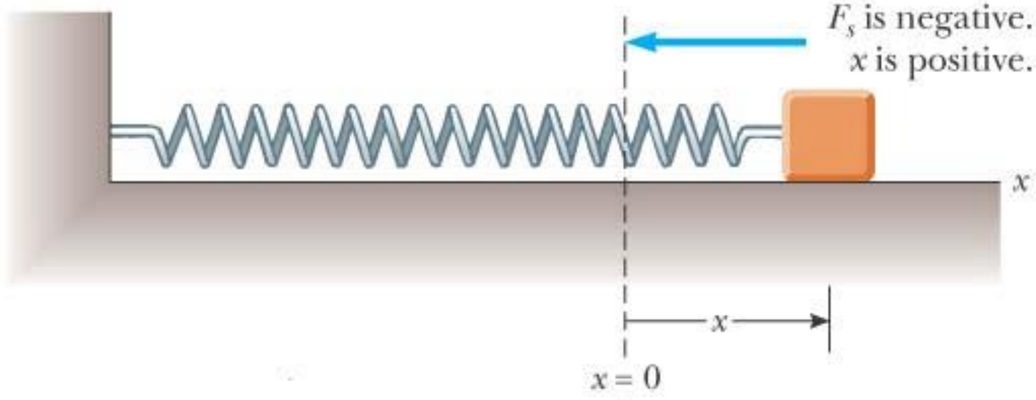


Work Done by Multiple Forces, cont.

The total work is equal to the algebraic sum of the work done by the individual forces

$$W_{\text{net}} = \sum W_{\text{by individual forces}}$$

Hooke's Law



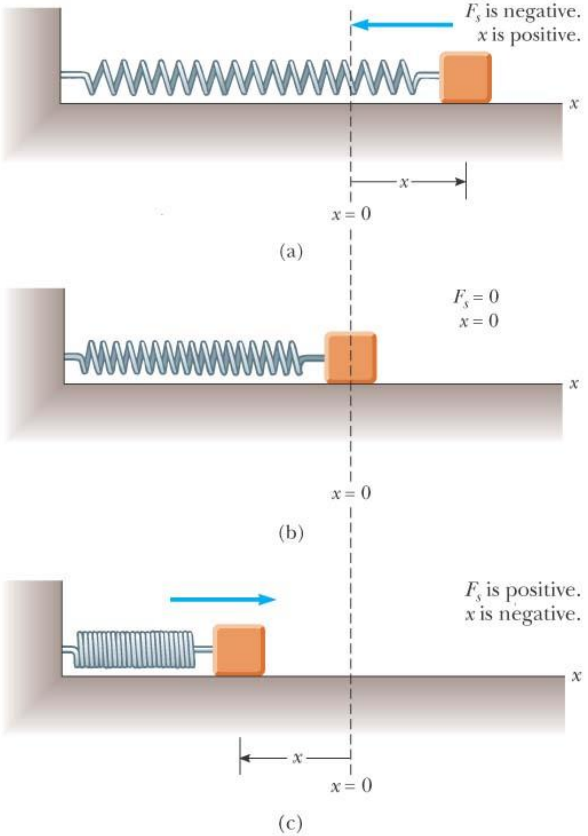
- The force exerted by the spring is

  $$F_s = - kx$$

  - x is the position of the block with respect to the equilibrium position ($x = 0$)
  - k is called the spring constant or force constant and measures the stiffness of the spring
- This is called Hooke's Law

Hooke's Law, cont.

- When *x* is positive (spring is stretched), *F* is negative
- When *x* is 0 (at the equilibrium position), *F* is 0
- When *x* is negative (spring is compressed), *F* is positive



Hooke's Law, final

- The force exerted by the spring is always directed opposite to the displacement from equilibrium
- F is called the *restoring force*
- If the block is released it will oscillate back and forth between $-x$ and x

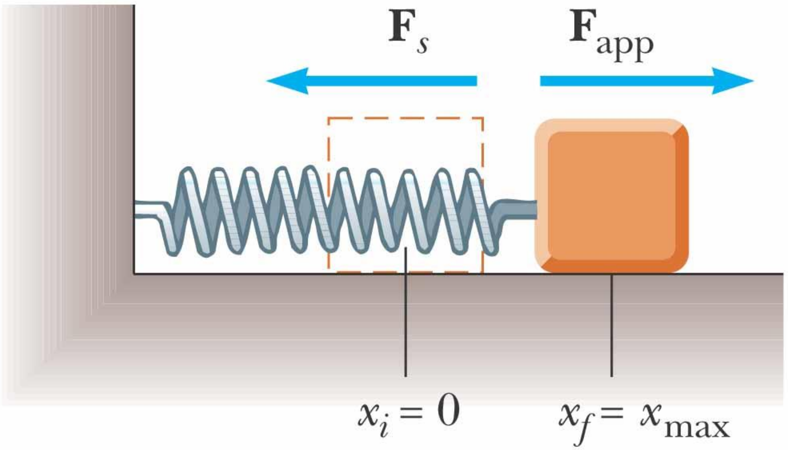
Work Done by a Spring

- Identify the block as the system
- Calculate the work as the block moves from x_1 to x_2
- $W_s = \int_{x_1}^{x_2}(-kx)dx = -k\int_{x_1}^{x_2} xdx$
- $= \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$
- If $x_1 = 0$, $W_s = -\frac{1}{2}kx^2$

and is always negative.

Spring with an Applied Force

- Suppose an external agent, F_{app}, stretches the spring
- The applied force is equal and opposite to the spring force
- $F_{app} = -F_s = -(-kx) = kx$
- Work done by F_{app} is equal to ½ kx^2_{max}



Kinetic Energy

The work done in changing the velocity

- Calculating the work:

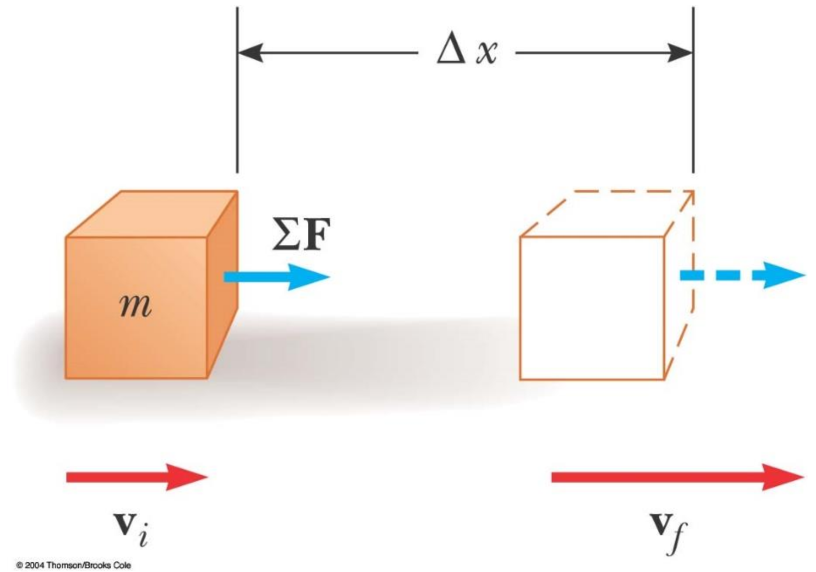
$$W = \int_{x_i}^{x_f} \sum F \, dx = \int_{x_i}^{x_f} ma \, dx$$

$$W = \int_{v_i}^{v_f} mv \, dv$$

$$\sum W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

Because

$$a \, dx = \frac{dv}{dt} dx = v \frac{dx}{dt} = v dv$$



Work-Kinetic Energy Theorem

- We can define the kinetic energy as

$$K = \tfrac{1}{2}\, mv^2$$

- Thus

$$W = K_f - K_i = \Delta K$$

which is called the kinetic energy theorem

Nonisolated System

- A *nonisolated system* is one that interacts with or is influenced by its environment
  - An *isolated system* would not interact with its environment
- The Work-Kinetic Energy Theorem is applied to nonisolated systems

Work Is An Energy Transfer

- If a system interacts with its environment, this interaction can be described as a transfer of energy across the system boundary
  - This will result in a change in the amount of energy stored in the system

Work Is An Energy Transfer, cont.

- If the work is done on a system and it is positive, energy is transferred to the system
- If the work done on the system is negative, energy is transferred from the system

Power

- The time rate of energy transfer is called ***power***
- The average power is given by

$$\overline{P} = \frac{W}{\Delta t}$$

when the method of energy transfer is work

Instantaneous Power

- The ***instantaneous power*** is the limiting value of the average power as Δt approaches zero

$$P = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt}$$

- This can also be written as

$$P = \frac{dW}{dt} = F \cdot \frac{dr}{dt} = F \cdot v$$

Power Generalized

- Power can be related to any type of energy transfer
- In general, power can be expressed as

$$P = \frac{dE}{dt}$$

- *dE*/*dt* is the rate rate at which energy is crossing the boundary of the system for a given transfer mechanism

Units of Power

- The SI unit of power is called the watt
  - 1 watt = 1 joule / second = 1 kg · m^2 / s^2
- A unit of power in the US Customary system is horsepower
  - 1 hp = 746 W
- Units of power can also be used to express units of work or energy
  - 1 kWh = (1000 W)(3600 s) = 3.6 $\times 10^6$ J