Review 2

Motion and Kinematics

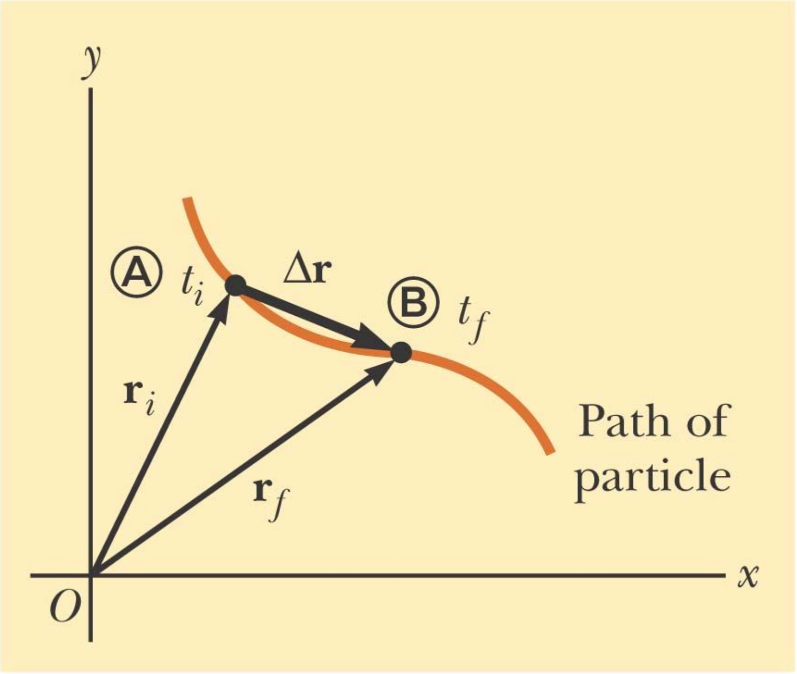
Kinematics

- Describes motion while ignoring the agents that caused the motion
- Will use the particle model
  - A particle is a point-like object, has mass but infinitesimal size

Translational Motion

Position and Displacement

- The position of an object is described by its position vector, **r**
- The **displacement** of the object is defined as the ***change in its position***
  - $\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$

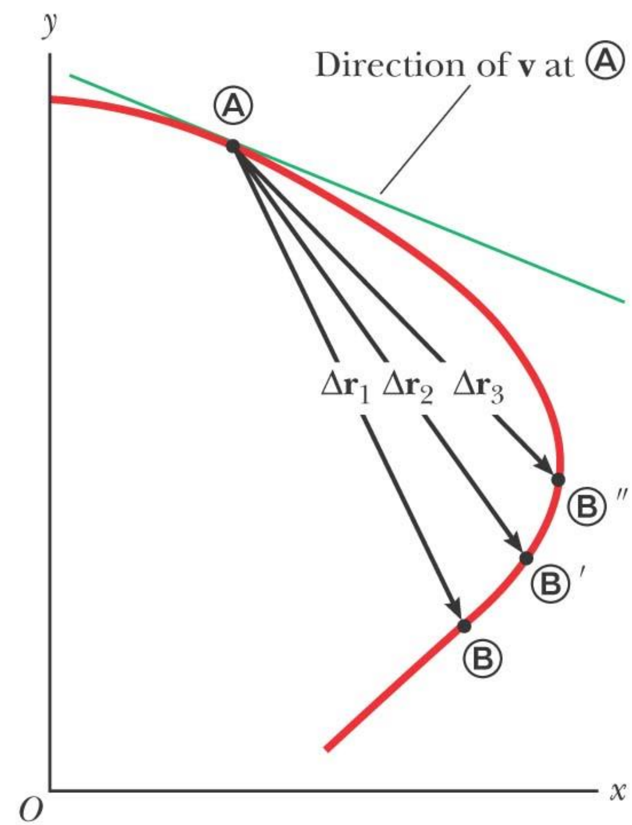


Average Velocity

- The average velocity is the ratio of the displacement to the time interval for the displacement

$$\overline{\mathbf{v}} = \frac{\Delta\mathbf{r}}{\Delta t}$$

  - The direction of the average velocity is the direction of the displacement vector, $\Delta\mathbf{r}$



Instantaneous Velocity

- The instantaneous velocity is the limit of the average velocity as Δt approaches zero
  - The direction of the instantaneous velocity is along a line that is tangent to the path of the particle's direction of motion

$$\mathbf{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}$$

Instantaneous Velocity, cont

- The direction of the instantaneous velocity vector at any point in a particle's path is along a line tangent to the path at that point and in the direction of motion
- The magnitude of the instantaneous velocity vector is the speed
  - The speed is a scalar quantity

Average Acceleration

- The average acceleration of a particle as it moves is defined as the change in the instantaneous velocity vector divided by the time interval during which that change occurs. The average acceleration is a vector quantity directed along $\Delta\mathbf{v}$

$$\bar{\mathbf{a}} = \frac{\mathbf{v}_{\mathrm{f}} - \mathbf{v}_i}{t_f - t_i} = \frac{\Delta\mathbf{v}}{\Delta t}$$

Instantaneous Acceleration

- The instantaneous acceleration is the limit of the average acceleration as Δt approaches zero

$$\mathbf{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$$

Special Case
Motion in One Dimension

Displacement

- Defined as the change in position during some time interval
  - Represented as Δx

    $\Delta x = x_f - x_i$
  - SI units are meters (m) Δx can be positive or negative
- Different than distance – the length of a path followed by a particle

Instantaneous Velocity, equations

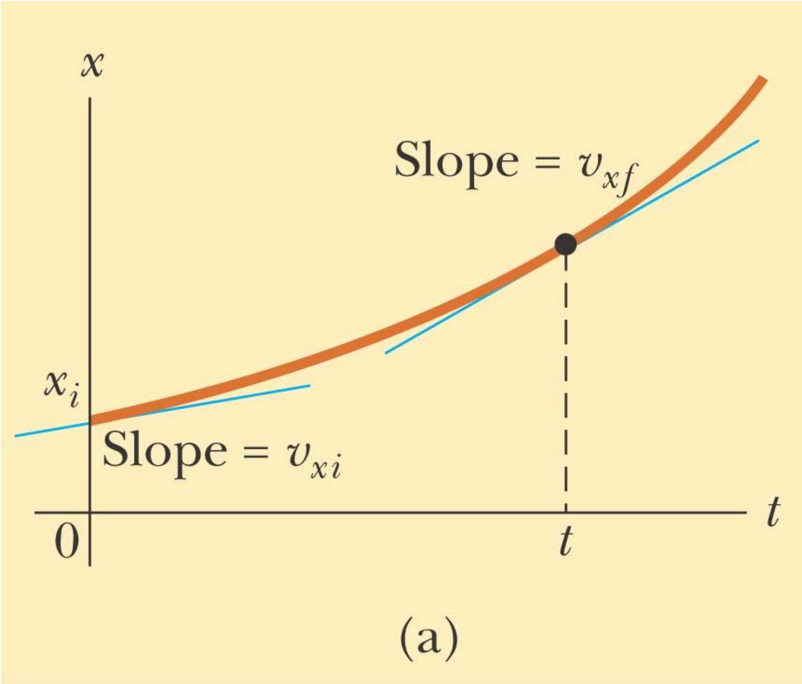
- The general equation for instantaneous velocity is

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- The instantaneous velocity can be positive, negative, or zero

Graphical Look at Motion – displacement – time curve

- The slope of the curve is the velocity
- The curved line indicates the velocity is changing
  - Therefore, there is an acceleration



Instantaneous Speed

- The instantaneous speed is the magnitude of the instantaneous velocity
- Remember that the average speed is <u>not</u> the magnitude of the average velocity

Average Acceleration

- Acceleration is the rate of change of the velocity

$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{\Delta t}$$

- Dimensions are L/T^2
- SI units are m/s^2

Instantaneous Acceleration

- The instantaneous acceleration is the limit of the average acceleration as Δt approaches 0

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2 x}{dt^2}$$

Kinematic Equations -- summary

Table 2.2

Kinematic Equations for Motion of a Particle Under Constant Acceleration	
Equation	**Information Given by Equation**
$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

Note: Motion is along the *x* axis.

Remark

- Various changes in a particle's motion may produce an acceleration
  - The magnitude of the velocity vector may change
  - The direction of the velocity vector may change
    - Even if the magnitude remains constant
  - Both may change simultaneously

Kinematic Equations for Two-Dimensional Motion

- When the two-dimensional motion has a constant acceleration, a series of equations can be developed that describe the motion
- These equations will be similar to those of one-dimensional kinematics

Kinematic Equations, 2

- Position vector $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$
- Velocity $\mathbf{v} = \dfrac{d\mathbf{r}}{dt} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$
  - Since acceleration is constant, we can also find an expression for the velocity as a function of time: $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t$

Kinematic Equations, 3

- The position vector can also be expressed as a function of time:
  - $\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2$
  - This indicates that the position vector is the sum of three other vectors:
    - The initial position vector
    - The displacement resulting from $\mathbf{v}_i\, t$
    - The displacement resulting from $\frac{1}{2}\mathbf{a}t^2$

Kinematic Equations, Components

- The equations for final velocity and final position are vector equations, therefore they may also be written in component form
- This shows that two-dimensional motion at constant acceleration is equivalent to two independent motions
  - One motion in the *x*-direction and the other in the *y*-direction

Kinematic Equations, Component Equations

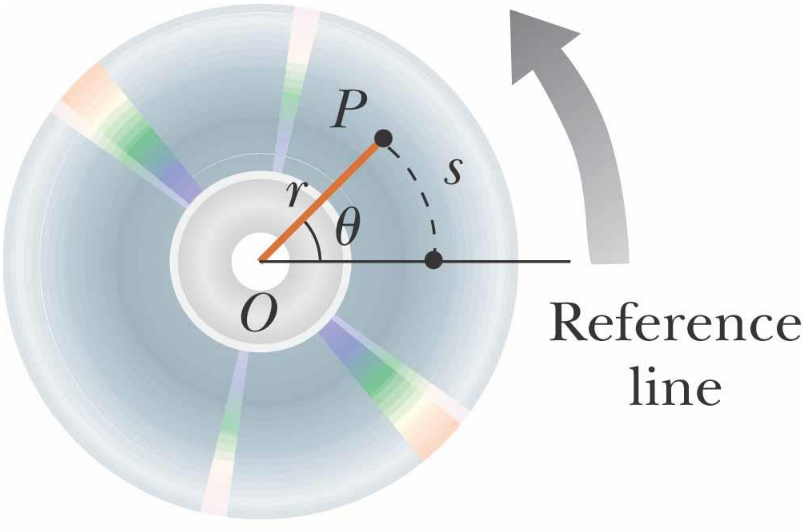
- $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t$ becomes
  - $v_{xf} = v_{xi} + a_x t$ and
  - $v_{yf} = v_{yi} + a_y t$
- $\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2$ becomes
  - $x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$ and
  - $y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$

2 Angular Motion

- Angular position, velocity and acceleration

Angular Position

- As the particle moves, the only coordinate that changes is θ
- As the particle moves through θ, it moves though an arc length s.
- The arc length and r are related:
  - $s = \theta\, r$



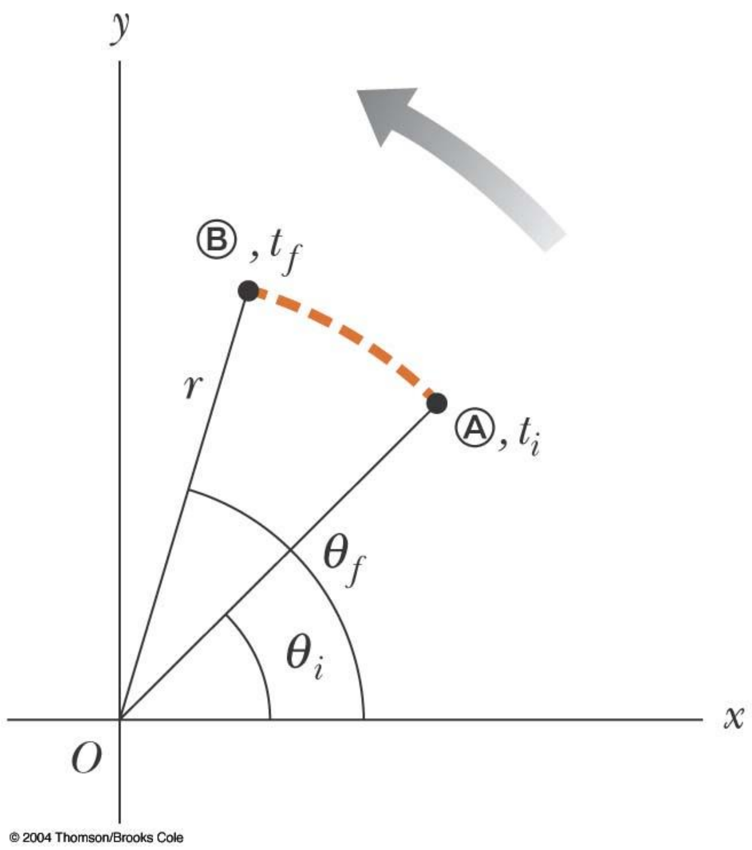
(b)

Angular Displacement

- The *angular displacement* is defined as the angle the object rotates through during some time interval

$$\Delta\theta = \theta_f - \theta_i$$

- This is the angle that the reference line of length *r* sweeps out



Average Angular Speed

- The average angular speed, ω, of a rotating rigid object is the ratio of the angular displacement to the time interval

$$\bar{\omega} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

Instantaneous Angular Speed

- The *instantaneous* angular speed is defined as the limit of the average speed as the time interval approaches zero

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$

Angular Speed, final

- Units of angular speed are radians/sec
  - rad/s or s^{-1} since radians have no dimensions
- Angular speed will be positive if θ is increasing (counterclockwise)
- Angular speed will be negative if θ is decreasing (clockwise)

Average Angular Acceleration

- The average angular acceleration, α, of an object is defined as the ratio of the change in the angular speed to the time it takes for the object to undergo the change:

$$\bar{\alpha} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous Angular Acceleration

- The instantaneous angular acceleration is defined as the limit of the average angular acceleration as the time goes to 0

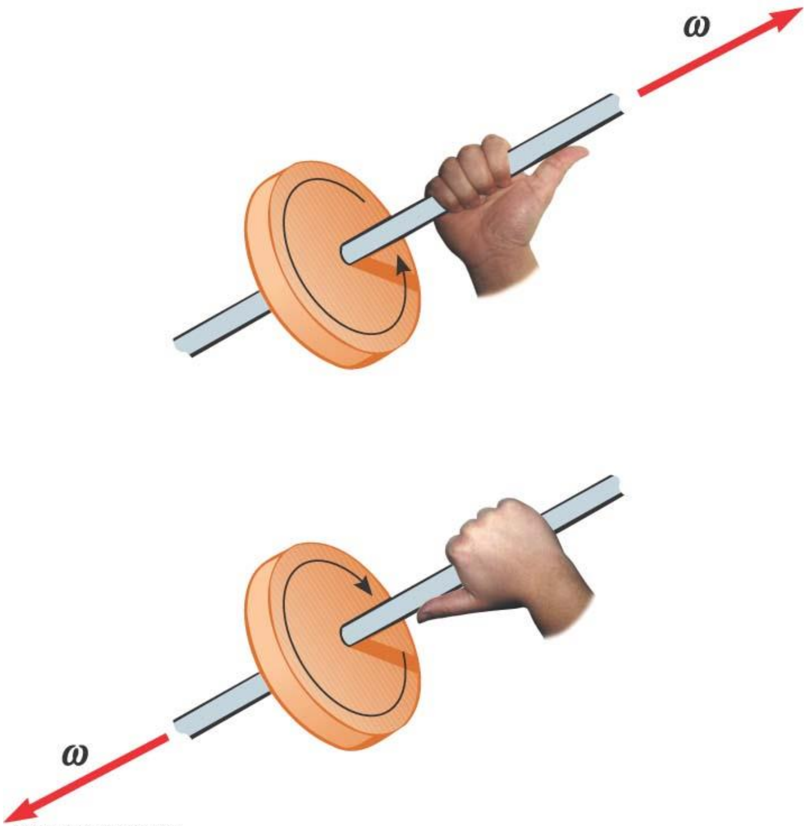
$$\alpha \equiv \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt}$$

Angular Acceleration, final

- Units of angular acceleration are rad/s² or s^{-2} since radians have no dimensions
- Angular acceleration will be positive if an object rotating counterclockwise is speeding up
- Angular acceleration will also be positive if an object rotating clockwise is slowing down

Directions, details

- Strictly speaking, the speed and acceleration (ω, α) are the magnitudes of the velocity and acceleration vectors
- The directions are actually given by the right-hand rule



Rotational Kinematics

- Under **constant angular acceleration**, we can describe the motion of the rigid object using a set of kinematic equations
  - These are similar to the kinematic equations for linear motion
  - The rotational equations have the same mathematical form as the linear equations

Rotational Kinematic Equations

$$\omega_f = \omega_i + \alpha t$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$

$$\theta_f = \theta_i + \frac{1}{2}\left(\omega_i + \omega_f\right)t$$

Comparison Between Rotational and Linear Equations

Table 10.1

Kinematic Equations for Rotational and Linear Motion Under Constant Acceleration

Rotational Motion About Fixed Axis	Linear Motion
$\omega_f = \omega_i + \alpha t$	$v_f = v_i + at$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	$x_f = x_i + v_i t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	$v_f^2 = v_i^2 + 2a(x_f - x_i)$
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$	$x_f = x_i + \frac{1}{2}(v_i + v_f)t$

Relationship Between Angular and Linear Quantities in Rotation

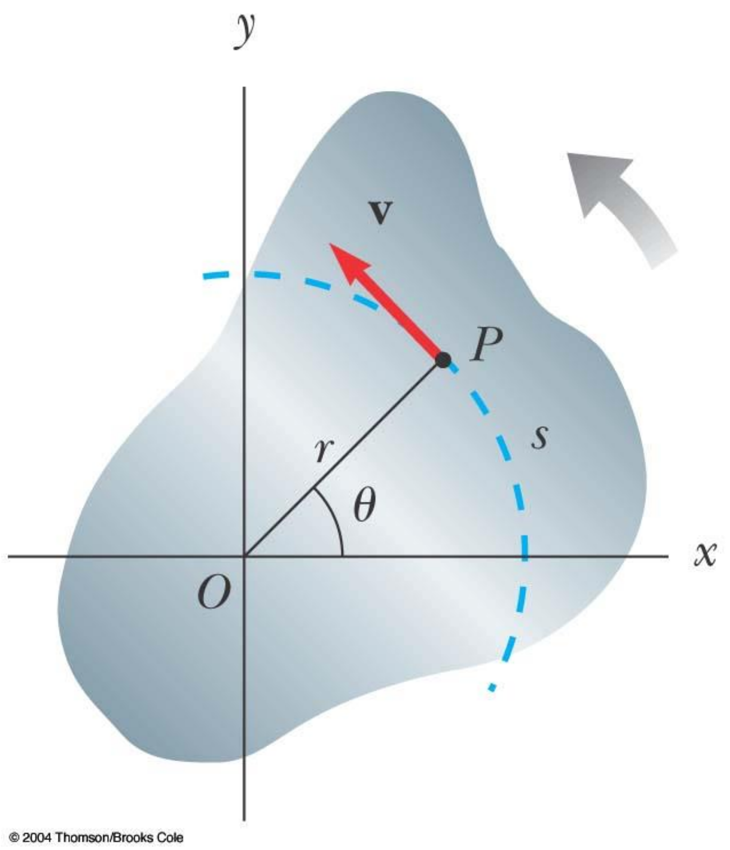
- Displacements
  $$s = \theta r$$
- Speeds
  $$v = \omega r$$
- Accelerations
  $$a = \alpha r$$

- Every point on the rotating object has the same angular motion
- Every point on the rotating object does *not* have the same linear motion

Speed Comparison

- The linear velocity is always tangent to the circular path
  - called the tangential velocity
- The magnitude is defined by the tangential speed

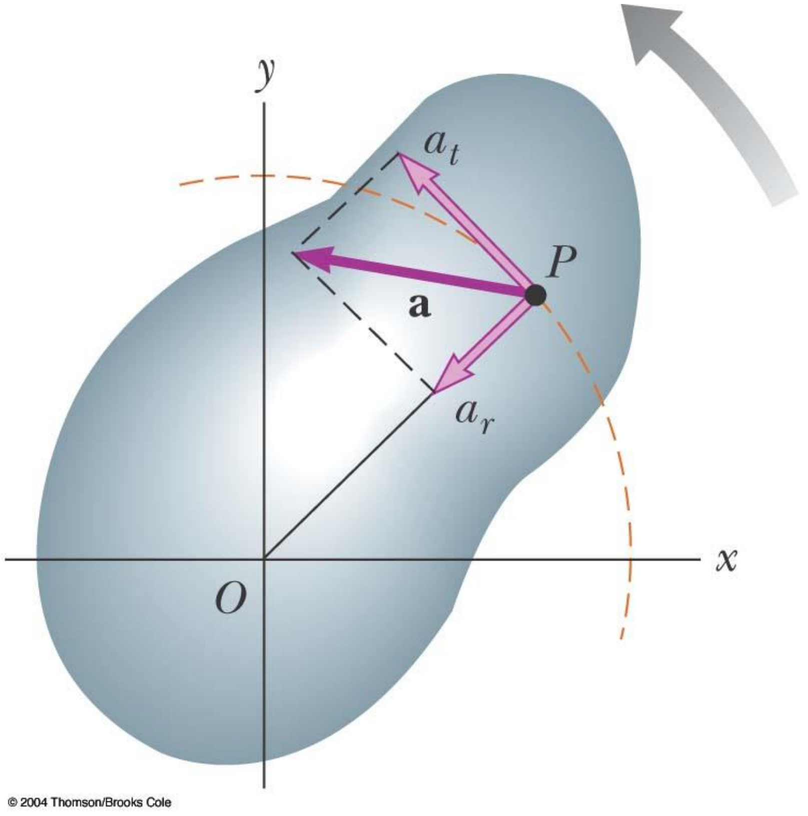
$$v = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega$$



Acceleration Comparison

- The tangential acceleration is the derivative of the tangential velocity

$$a_t = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha$$



Speed and Acceleration Note

- All points on the rigid object will have the same *angular speed*, but **not** the same *tangential speed*
- All points on the rigid object will have the same *angular acceleration*, but **not** the same *tangential acceleration*
- The tangential quantities depend on *r*, and *r* is not the same for all points on the object