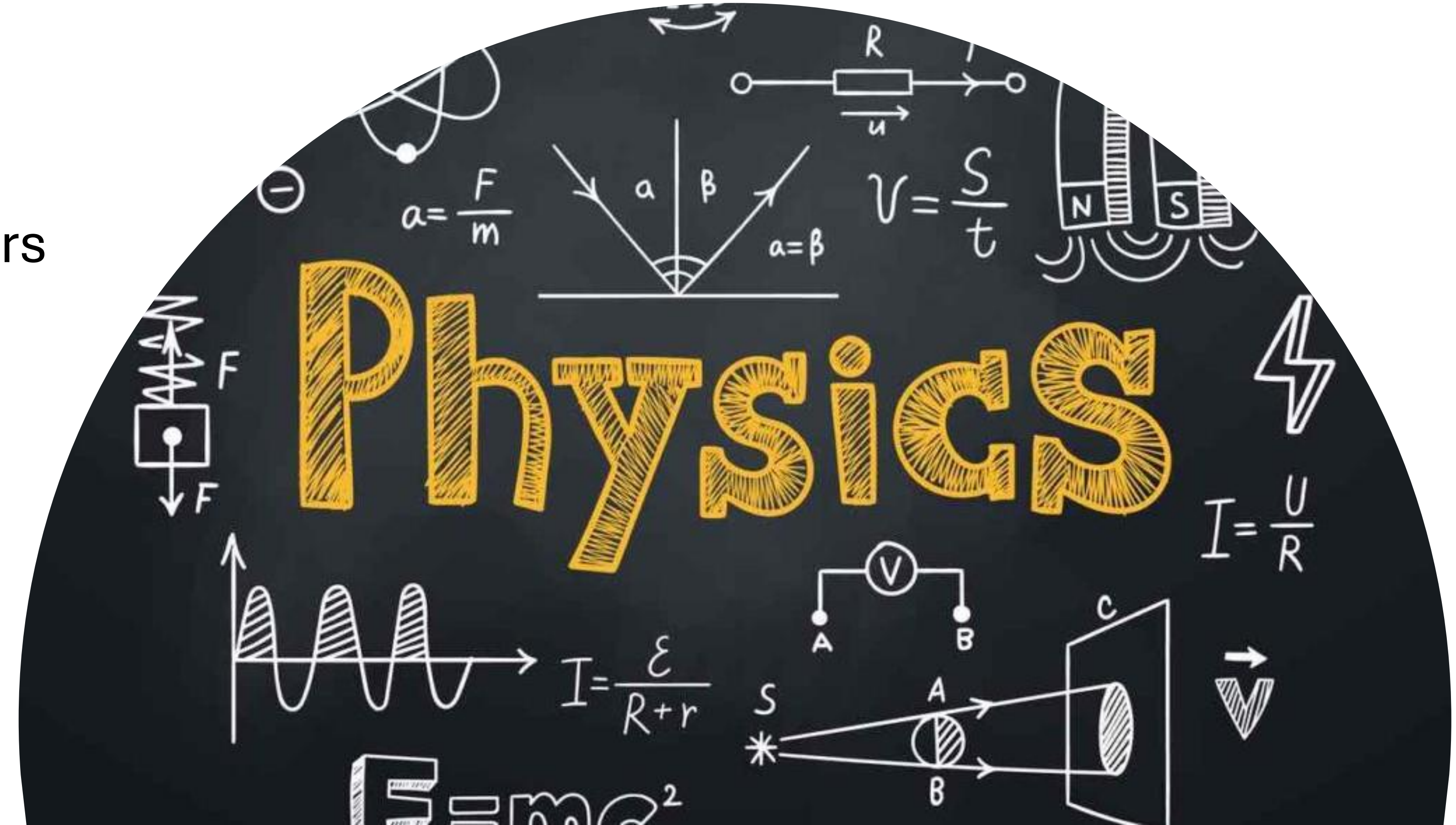
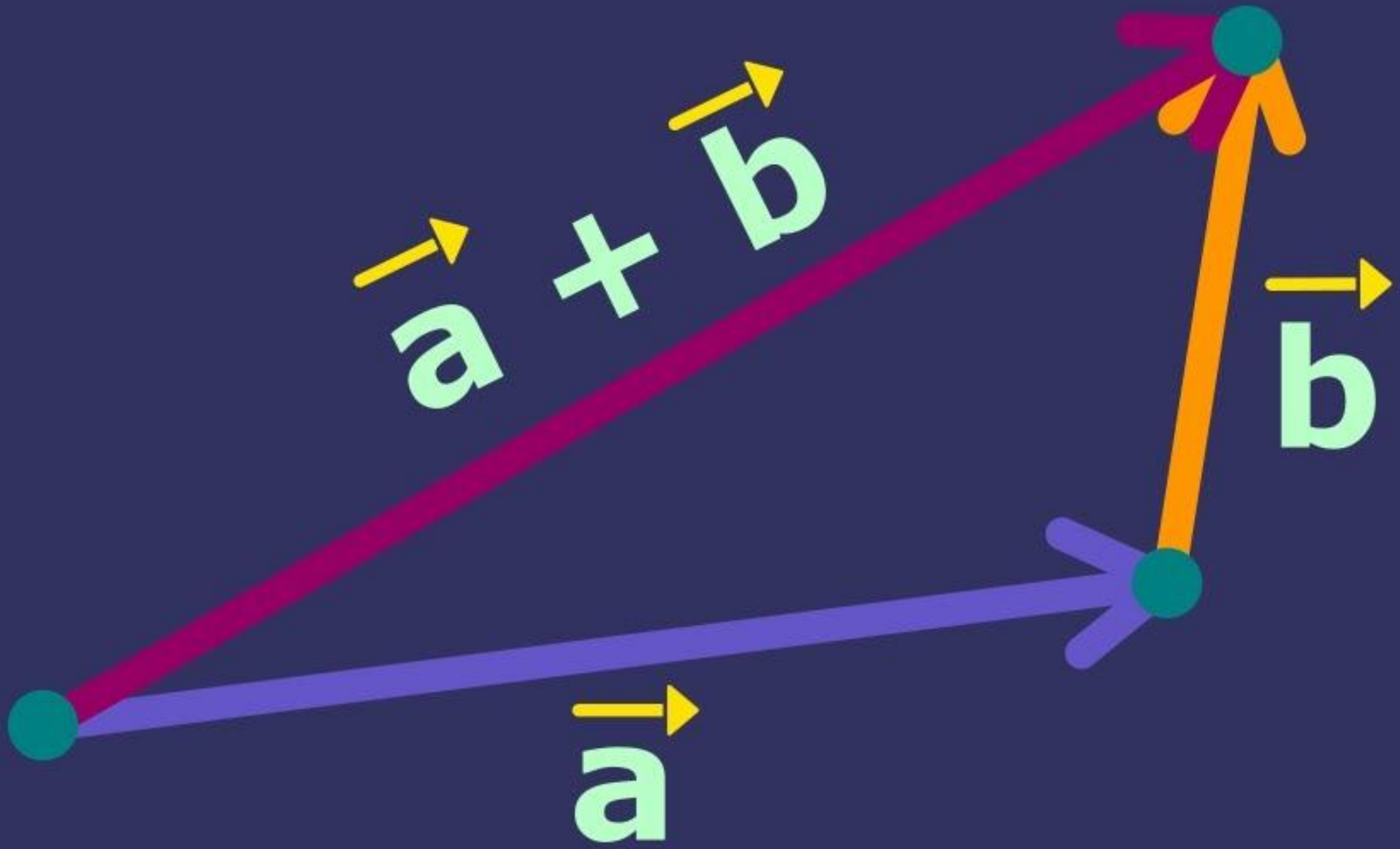
Physics 1
GAP 121

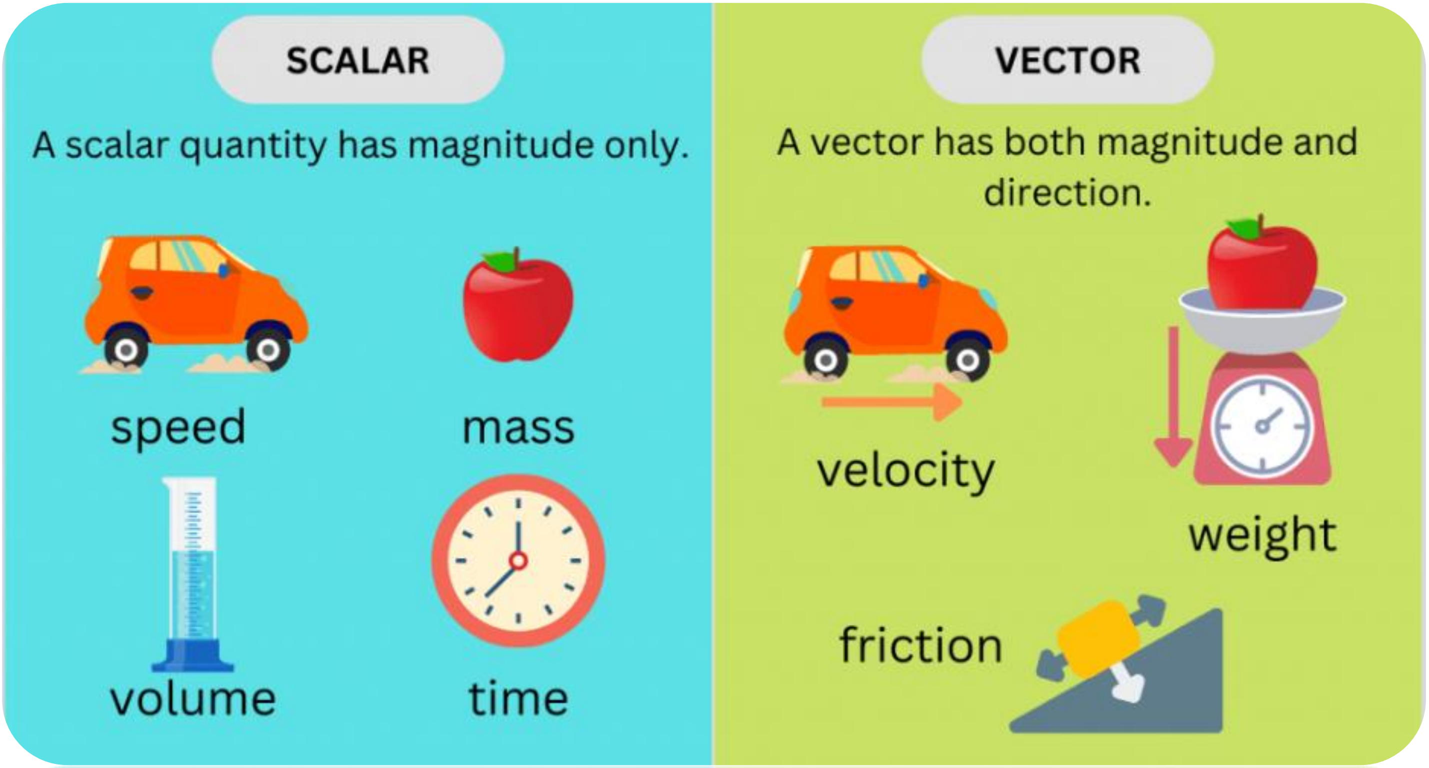
Revision on part 1 (vectors and circular motion)



Vectors



Vectors VS Scalars



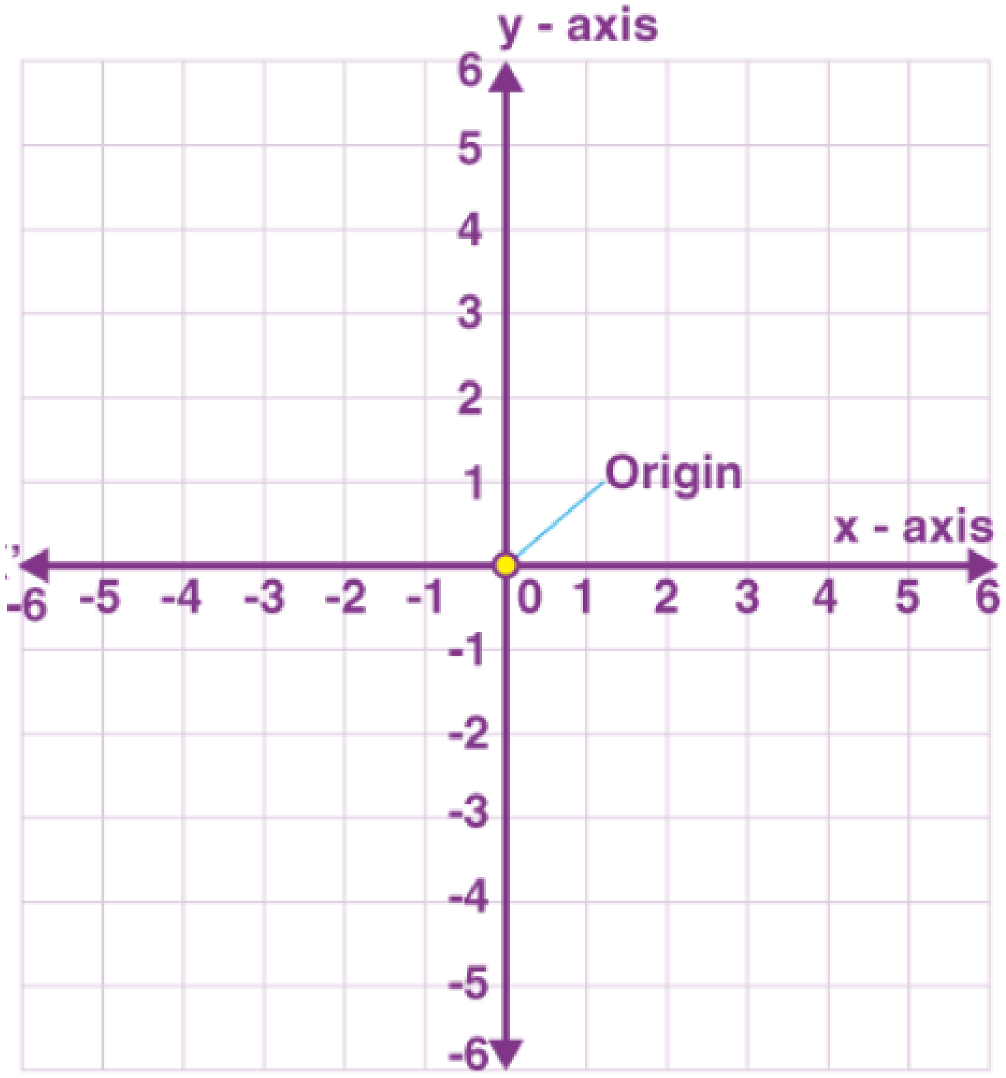
Coordinate Systems

- Used to describe the position of a point in space.
- Coordinate system consists of

1) a fixed reference point called the origin

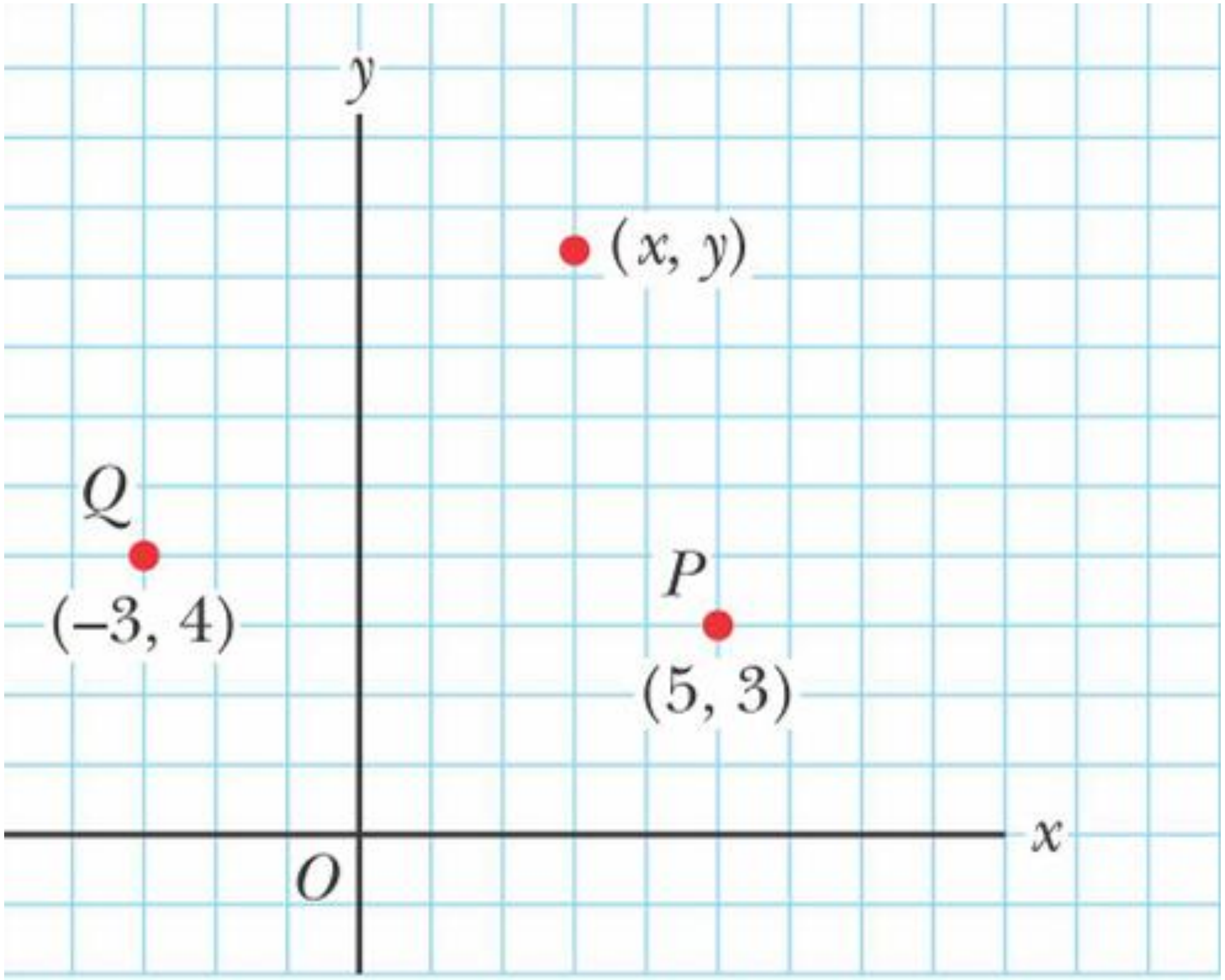
2) specific axes with scales and labels

3) instructions on how to label a point relative to the origin and the axes



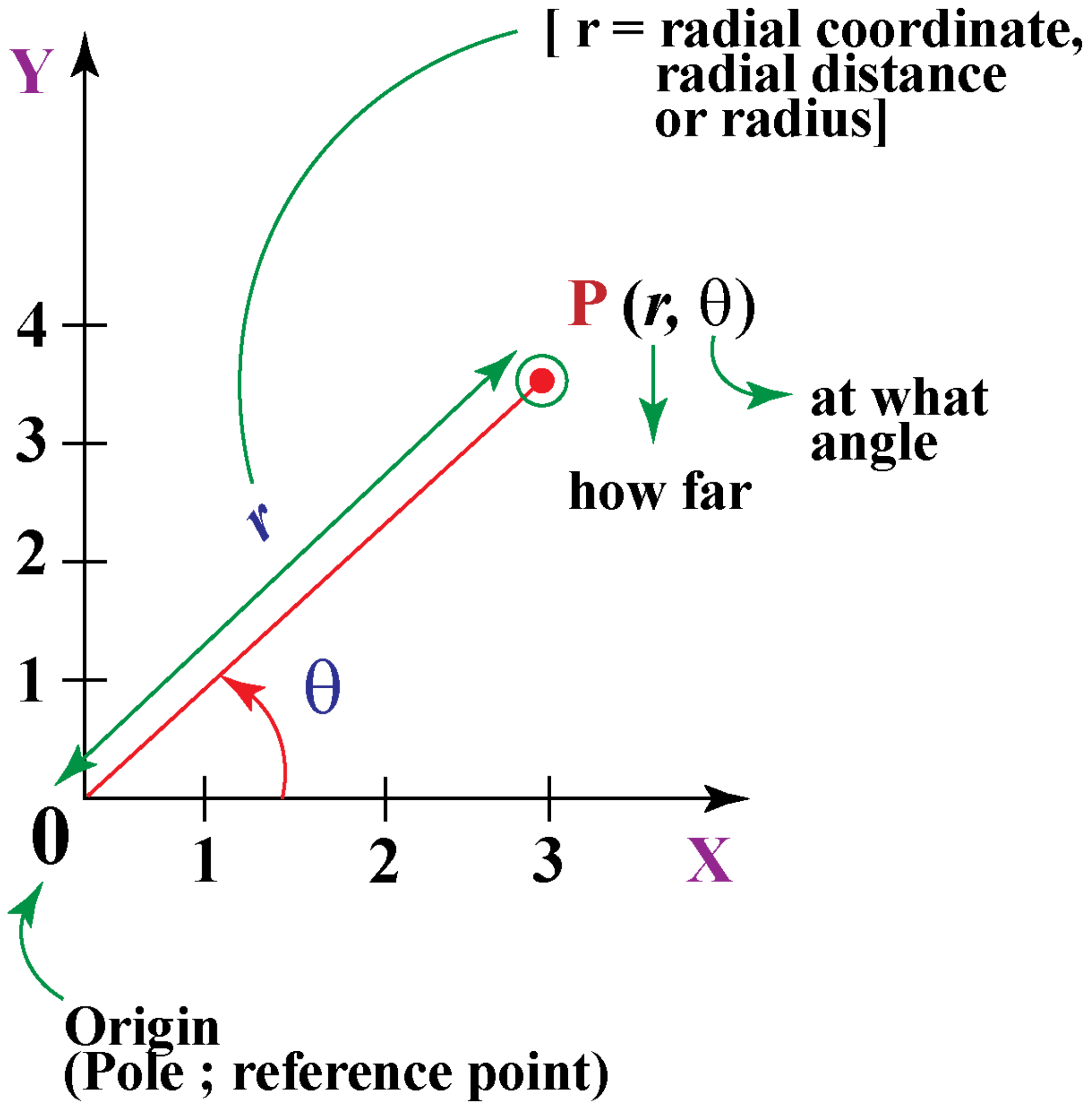
Cartesian Coordinate System

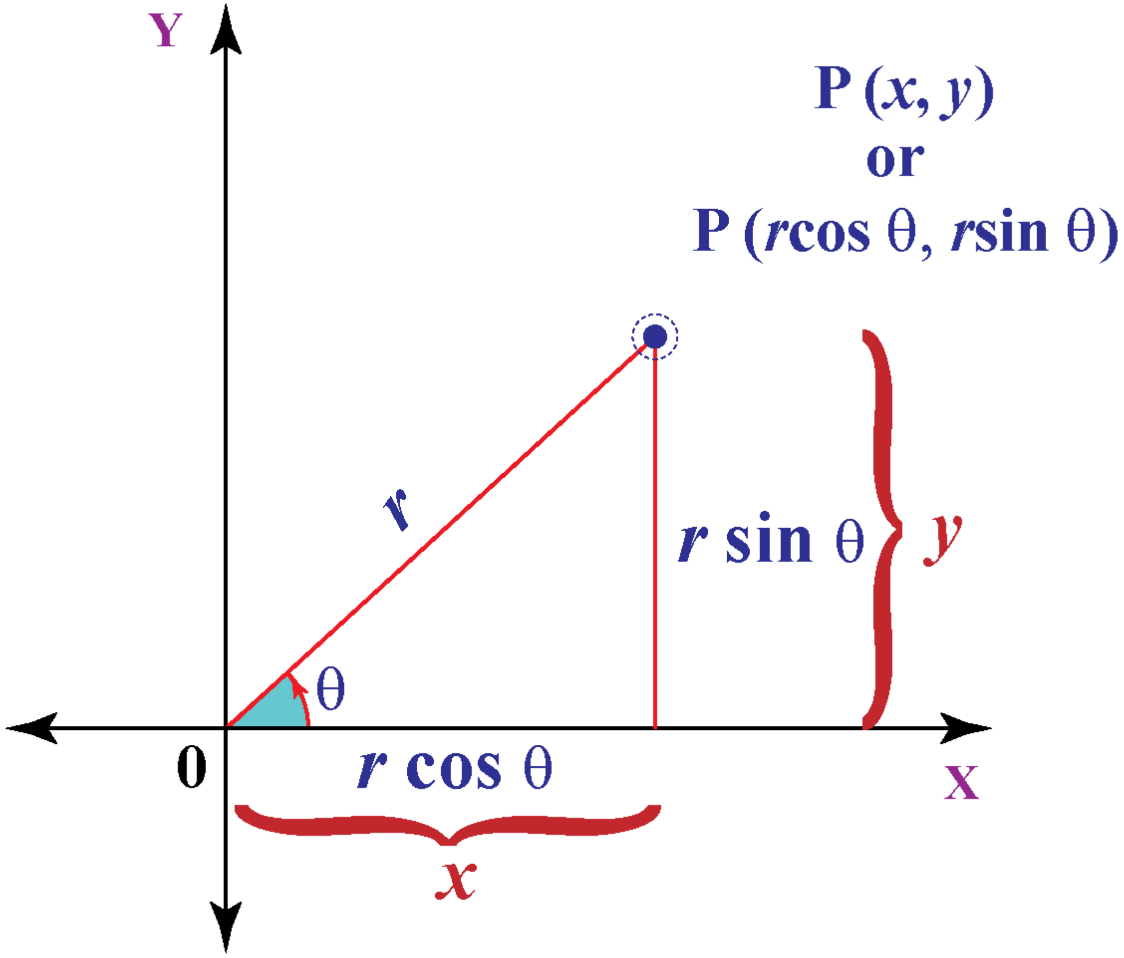
- Also called rectangular coordinate system
- x- and y-axes intersect at the origin
- Points are labeled (x,y)



Polar Coordinate System

- Origin and reference line are noted
- Point is distance r from the origin in the direction of angle θ
- Points are labeled (r, θ)





Cartesian to Polar Coordinates and vice versa

- Polar to Cartesian Coordinate
- x = r cos(Θ) & y = r sin (Θ)

- Cartesian to Polar Coordinates
- $r = \sqrt[2]{x^2 + y^2}$ & tan (Θ) = y/x

1) A displacement vector lying in the xy plane has a magnitude of 50.0 m and is directed at an angle of 60 degree to the positive x axis. What are the rectangular components of this vector?

solution

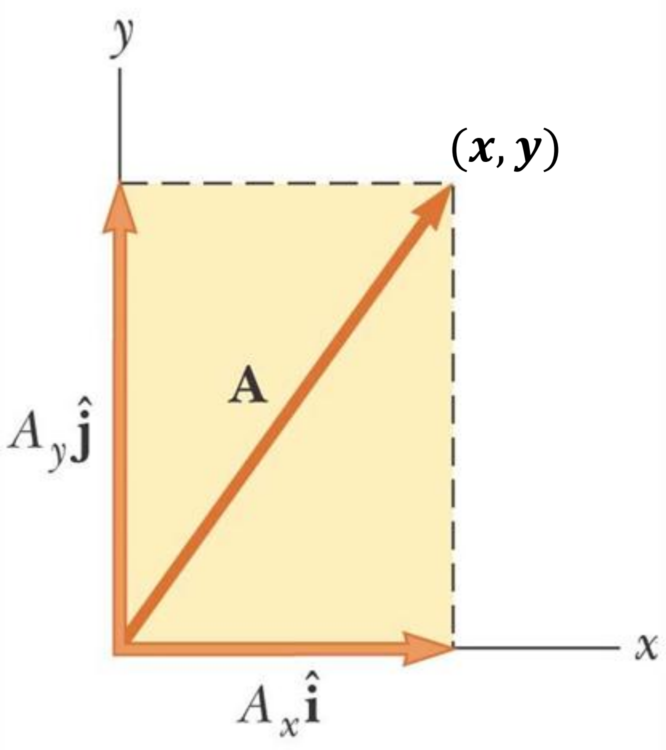
$$x = rcos(\theta) = 50\cos(60) = 25\,m \;\&\; y = rsin(\theta) = 50\sin 60 = 43.3\,m$$

2) A fly lands on one wall of a room. The lower left-hand corner of the wall is selected as the origin of a two-dimensional Cartesian coordinate system. If the fly is located at the point having coordinates (2.00, 1.00) m, (a) how far is it from the corner of the room? (b) What is its location in polar coordinates?

$r = \sqrt[2]{x^2 + y^2} = \sqrt[2]{2^2 + 1^2} = \sqrt{5}$ m and also tan (Ɵ) = y/x = ½ so (θ)= 26.6

Vector Notation (2D)

- For 2D vector $\boldsymbol{A} = A_X\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}} = (\boldsymbol{x}, \boldsymbol{y})$
- $A_X = \boldsymbol{r}\,\boldsymbol{cos}(\boldsymbol{\theta})$ & $A_y = \boldsymbol{rsin}(\boldsymbol{\theta})$
- For 2D vector $\boldsymbol{B} = B_X\hat{\boldsymbol{\imath}} + B_y\hat{\boldsymbol{\jmath}} = (\boldsymbol{x}, \boldsymbol{y})$
- $\boldsymbol{A} + \boldsymbol{B} = (A_X + B_X)\,\hat{\boldsymbol{\imath}} + \left(A_y + B_y\right)\,\hat{\boldsymbol{\jmath}}$
- $|\mathbf{A} + \mathbf{B}| = \sqrt{(A_X + B_X)^2 + (A_y + B_y)^2}$
- $\theta = tan^{-1}\left(\frac{A_y+B_y}{A_X+B_X}\right)$



- Q1) A force $\mathbf{F_1}$ of magnitude 6.00 N acts at the origin in a direction 30.0° above the positive x axis. A second force $\mathbf{F_2}$ of magnitude 5.00 N acts at the origin in the direction of the positive y axis. Find the magnitude and direction of the resultant force $\mathbf{F_1} + \mathbf{F_2}$.
- Solution

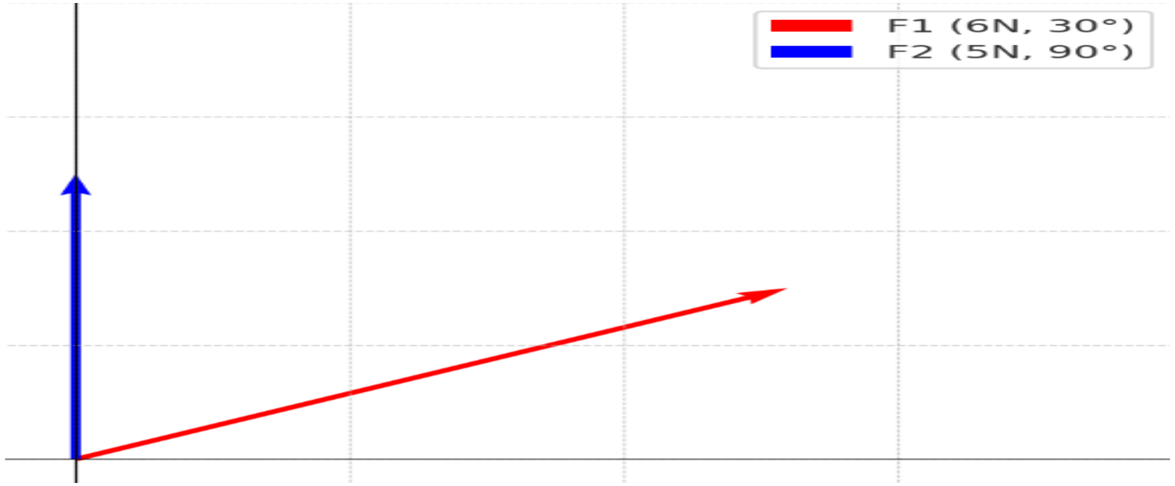
As we For 2D vector $\boldsymbol{F1} = F_{1X}\hat{\boldsymbol{\imath}} + F_{1y}\widehat{\boldsymbol{j}}$ and $F_{1X} = \boldsymbol{r}\,\boldsymbol{cos}(\boldsymbol{\theta}) = \boldsymbol{6} \times \boldsymbol{cos}(\boldsymbol{30}) = \boldsymbol{3}\sqrt{\boldsymbol{3}}$ &

$F_{1y} = \boldsymbol{rsin}(\boldsymbol{\theta}) = \boldsymbol{6sin}(\boldsymbol{30}) = \boldsymbol{3}$ so $\boldsymbol{F1} = \boldsymbol{3}\sqrt{\boldsymbol{3}}\,\hat{\boldsymbol{\imath}} + \boldsymbol{3}\,\widehat{\boldsymbol{j}}$

By the same way $\boldsymbol{F2} = F_{2X}\hat{\boldsymbol{\imath}} + F_{2y}\widehat{\boldsymbol{j}}$ and $F_{2X} = \boldsymbol{r}\,\boldsymbol{cos}(\boldsymbol{\theta}) = \boldsymbol{5} \times \boldsymbol{cos}(\boldsymbol{90}) = 0$ &

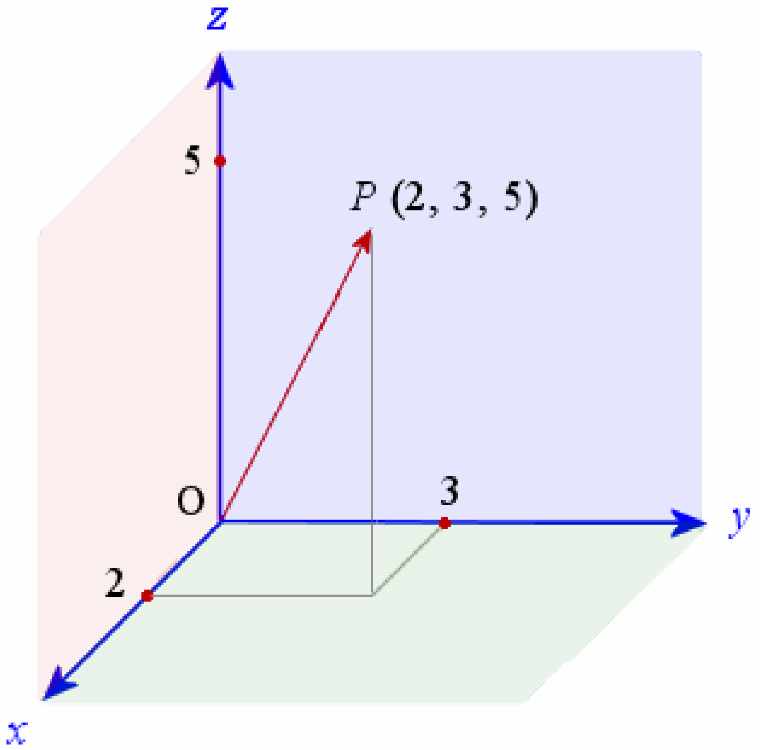
$F_{2y} = \boldsymbol{rsin}(\boldsymbol{\theta}) = \boldsymbol{5sin}(\boldsymbol{90}) = \boldsymbol{5}$ so $\boldsymbol{F2} = \boldsymbol{0}\hat{\imath} + \boldsymbol{5}\,\widehat{\boldsymbol{j}}$

Then $\mathbf{F_1} + \mathbf{F_2} = \boldsymbol{3}\sqrt{\boldsymbol{3}}\,\hat{\boldsymbol{\imath}} + \boldsymbol{8}\,\widehat{\boldsymbol{j}}$ so $|\mathbf{F_1} + \mathbf{F_2}| = \sqrt{(3\sqrt{3})^2 + (8)^2} = \sqrt{91}$ $and\ \theta = tan^{-1}\left(\frac{8}{3\sqrt{3}}\right) = 57$

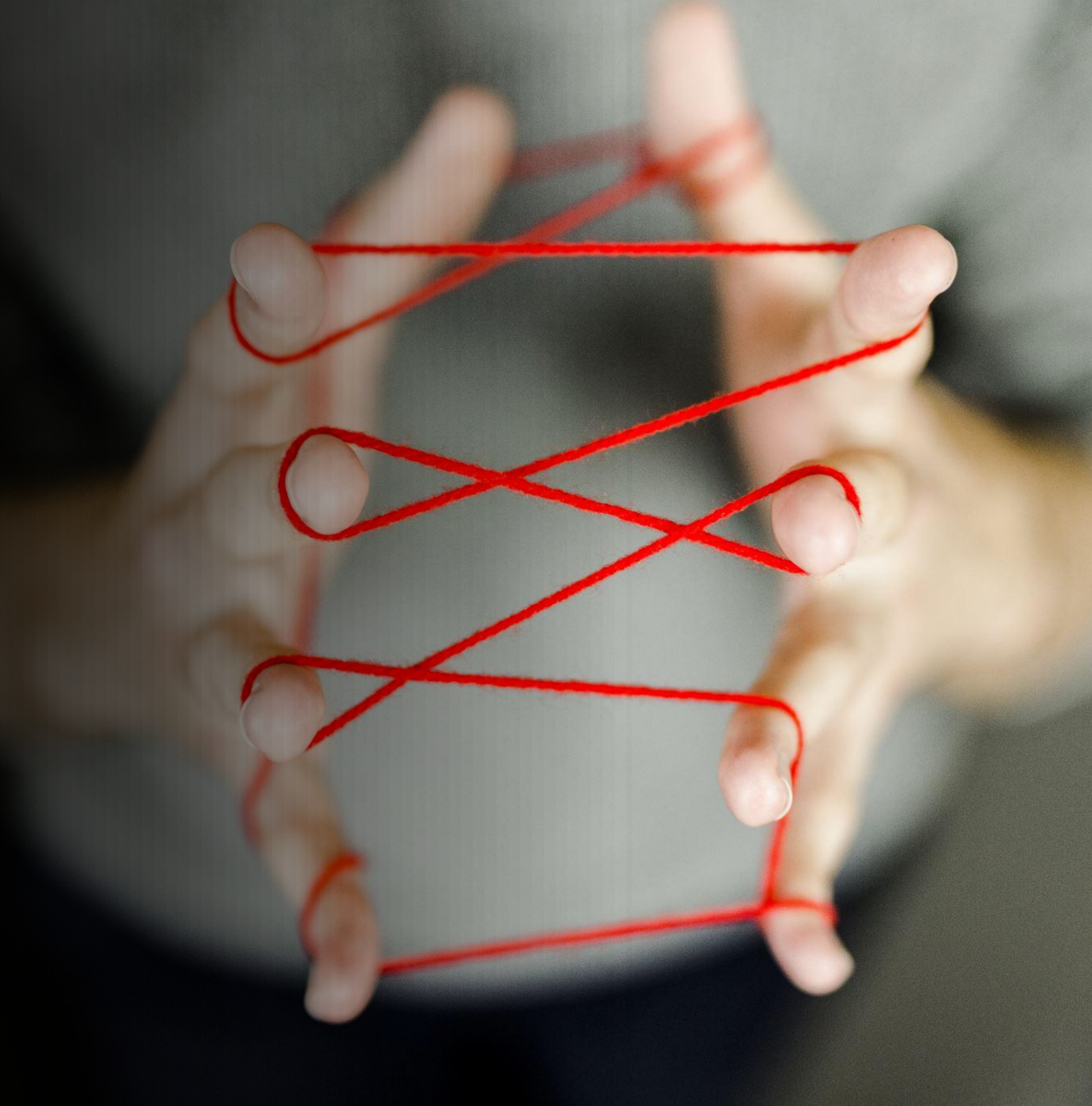


Vector Notation (3D)

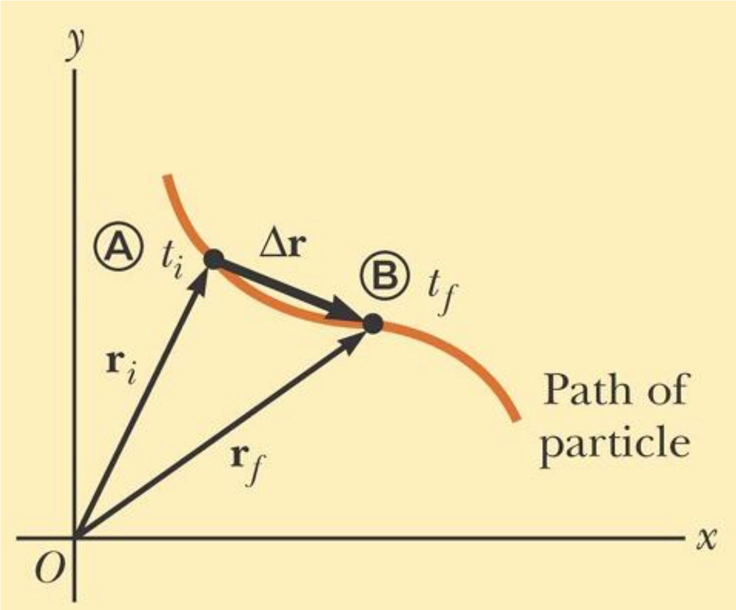
- For 3D vector $\boldsymbol{A} = A_X\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}} + A_z\hat{k} = (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$
- For 3D vector $\boldsymbol{B} = B_X\hat{\boldsymbol{\imath}} + B_y\hat{\boldsymbol{\jmath}} \;\; B_z\hat{k} = (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$
- $\boldsymbol{A} + \boldsymbol{B} = (A_X + B_X)\,\hat{\boldsymbol{\imath}} + \left(A_y + B_y\right)\,\hat{\boldsymbol{\jmath}} + (A_z + B_z)\,\hat{k}$
- $|\mathbf{A} + \mathbf{B}| = \sqrt{(A_X + B_X)^2 + \left(A_y + B_y\right)^2 + (A_z + B_z)^2}$
- $\theta = cos^{-1}\left(\frac{A_X + B_X}{|\mathbf{A} + \mathbf{B}|}\right)$



Motion and Kinematics



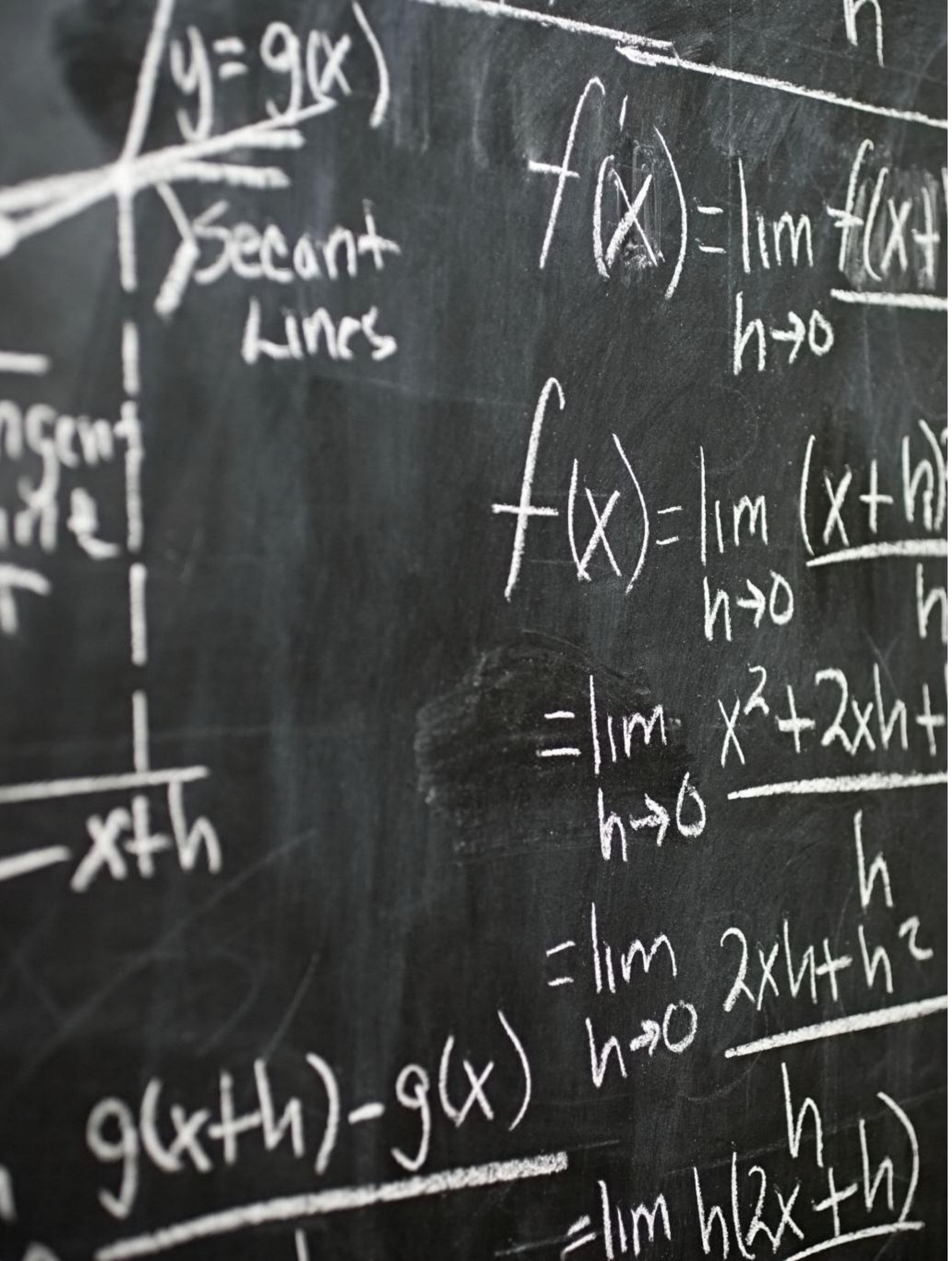
Translational Motion



- The position of an object is described by its position vector, r
- The displacement of the object is defined as the change in its position
- $\Delta \boldsymbol{r} = \boldsymbol{r_f} - \boldsymbol{r_i}$
- The average velocity is the ratio of the displacement to the time interval for the displacement
- $\boldsymbol{v} = \frac{\Delta \boldsymbol{r}}{\Delta t}$
- The average acceleration of a particle as it moves is defined as the change in the instantaneous velocity vector divided by the time interval during which that change occurs. $\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t}$

Instantaneous Velocity and Acceleration

- The instantaneous velocity is the limit of the average velocity as Δt approaches zero
- $\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt}$
- The direction of the instantaneous velocity is along a line that is tangent to the path of the particle's direction of motion
- The magnitude of the instantaneous velocity vector is the speed (scalar)
- The instantaneous acceleration is the limit of the average acceleration as Δt approaches zero
- $\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt}$



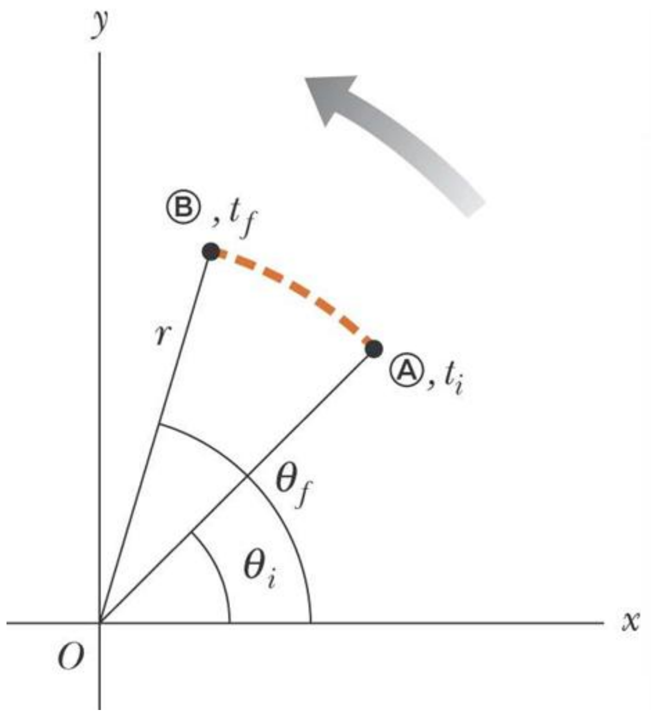
Kinematic Equations for Two-Dimensional Motion

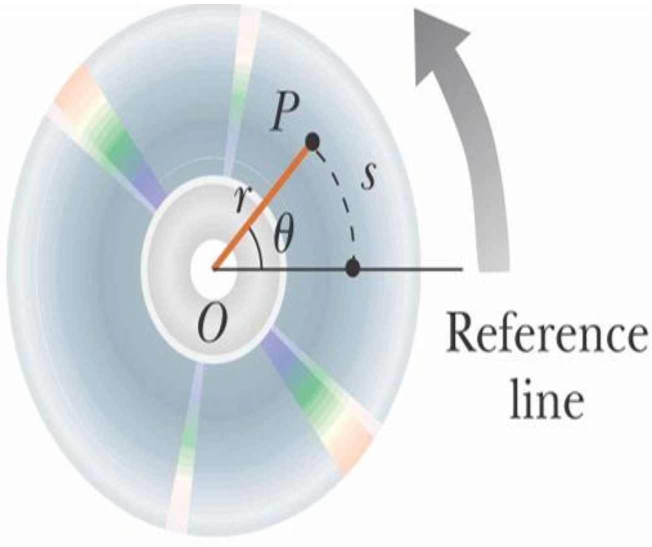
- Position vector $\boldsymbol{r} = \boldsymbol{x}\hat{\boldsymbol{\imath}} + \boldsymbol{y}\hat{\boldsymbol{\jmath}}$ and note that x,y are function of time
- Velocity $\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} = \ v_x\ \hat{\boldsymbol{\imath}} + v_y\ \hat{\boldsymbol{\jmath}}$
- Acceleration $\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt}$

- Q2) The vector position of a particle varies in time according to the expression $\boldsymbol{r} = t^3\hat{\imath} + 20\hat{\jmath}$ (a) Find expressions for the velocity and acceleration as functions of time. (b) Determine the particle's position and velocity at t = 1.00 s.
- Solution
- $\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} = \frac{d}{dt}(t^3\hat{\imath} + 20\hat{\jmath}) = 3\,t^2\hat{\imath}$
- $\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt} = \frac{d}{dt}(3\,t^2\hat{\imath}) = 3\times 2\,t^1\hat{\imath} = 6\text{ t }\hat{\imath}$
- $r(t = 1) = 1^3\hat{\imath} + 20\hat{\jmath} = 1\hat{\imath} + 20\hat{\jmath}$
- $v(t = 1) = 3 \times 1^2\,\hat{\imath} = 3\,\hat{\imath}$
- Q3) The vector position of a particle varies in time according to the expression $\boldsymbol{r} = t^2\hat{\imath} + 20t\,\hat{\jmath}$ (a) Find expressions for the velocity and acceleration as functions of time. (b) Determine the particle's position and velocity at t = 1.5 s.
- a) $\boldsymbol{v} = 2t\,\hat{\imath} + 20\,\hat{\jmath}$ and $\boldsymbol{a} = 2\,\hat{\imath}$ b) $r(t = 1) = \frac{9}{4}\hat{\imath} + 30\hat{\jmath}$ & $v(t = 1) = 3\,\hat{\imath} + 20\,\widehat{j}$
- Q4) The vector position of a particle varies in time according to the expression $\boldsymbol{r} = t^2\hat{\imath} + 20\hat{\jmath}$ (a) Find expressions for the velocity and acceleration as functions of time. (b) Determine the particle's position and velocity at t = 2.00 s.
- a) $\boldsymbol{v} = 2t\,\hat{\imath}$ and $\boldsymbol{a} = 2\,\hat{\imath}$ b) $r(t = 2) = 4\hat{\imath} + 20\hat{\jmath}$ and $v(t = 2) = 4\,\hat{\imath}$

Angular Motion

- The arc length and r are related:
- $S = r \times \theta$
- $\Delta\theta = \theta_f - \theta_i$
- $\omega = \frac{\Delta\theta}{\Delta t}$ (Average Angular Speed)
- $\omega = \frac{d\theta}{dt}$ (Instantaneous Angular Speed)
- $\alpha = \frac{\Delta\omega}{\Delta t}$ (Average Angular Acceleration)
- $\alpha = \frac{d\omega}{dt}$ (Instantaneous Angular Acceleration)





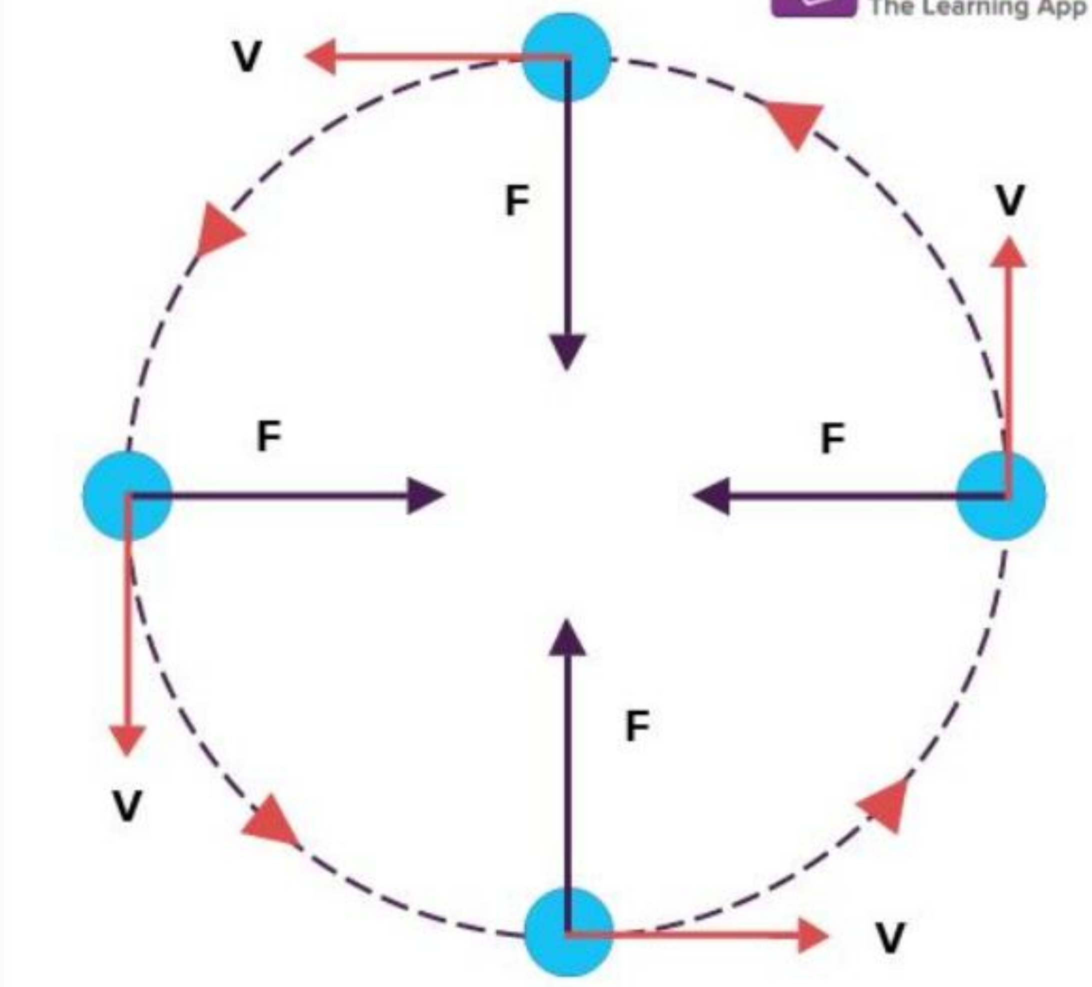
Rotational Motion About Fixed Axis	Linear Motion
$\omega_f = \omega_i + \alpha t$	$v_f = v_i + at$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	$x_f = x_i + v_i t + \frac{1}{2}at^2$
$\omega_f{}^2 = \omega_i{}^2 + 2\alpha(\theta_f - \theta_i)$	$v_f{}^2 = v_i{}^2 + 2a(x_f - x_i)$
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$	$x_f = x_i + \frac{1}{2}(v_i + v_f)t$

Relationship Between Angular and Linear Quantities in Rotation

- $S = r \times \theta$
- $v = r \times \omega$ The linear velocity is always tangent to the circular path (tangential velocity)
- $a = r \times \alpha$ The tangential acceleration is the derivative of the tangential velocity
- Every point on the rotating object has the same angular motion
- Every point on the rotating object does not have the same linear motion due to the dependance of r.

Uniform Circular Motion

- Uniform circular motion occurs when an object moves in a circular path with a constant speed
- An acceleration exists since the direction of the motion is changing
- The acceleration is always perpendicular to the path of the motion
- The acceleration always points toward the center of the circle of motion
- This acceleration is called the **centripetal acceleration** $\boldsymbol{a_c = \frac{v^2}{r}}$

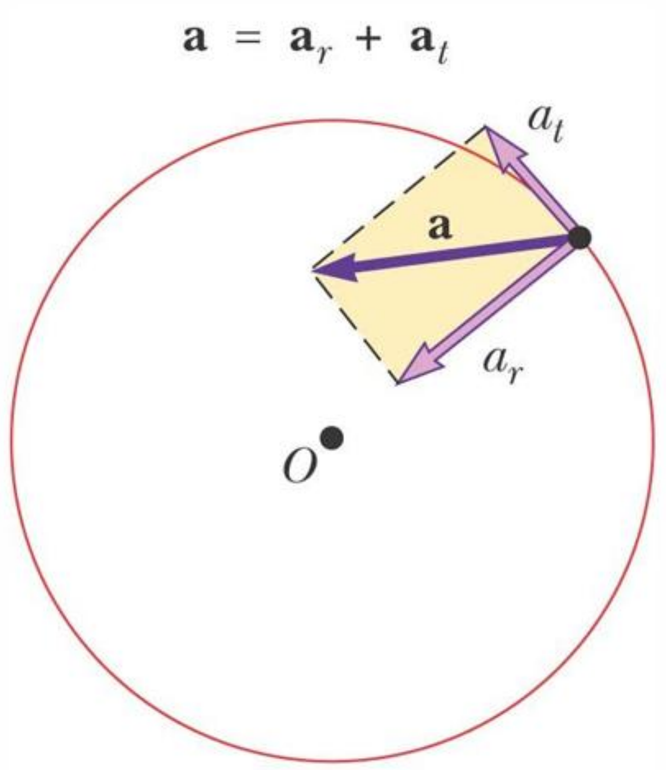


Period

- The period, T, is the time required for one complete revolution
- The speed of the particle would be the circumference of the circle of motion divided by the period
- $T = \frac{2\pi r}{v} = \frac{1}{\mathrm{f}} = \frac{2\pi}{\omega}$ (ω: angular speed)

Non-uniform Circular Motion Tangential Acceleration

- The magnitude of the velocity could also be changing
- In this case, there would be a tangential acceleration (causes the change in the speed of the particle)
- The radial acceleration comes from a change in the direction of the velocity vector



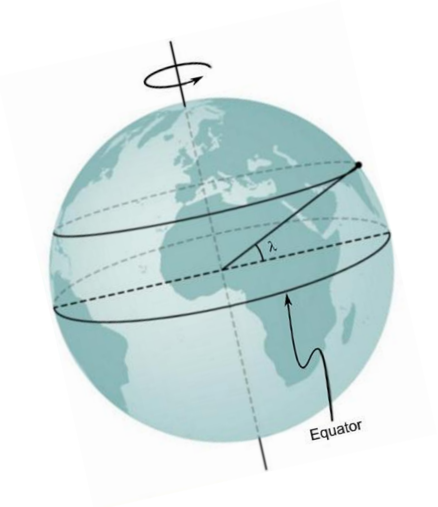
The tangential acceleration: $a_t = \frac{d|\mathbf{v}|}{dt}$

The radial acceleration: $a_r = -a_C = -\frac{v^2}{r}$

The total acceleration:

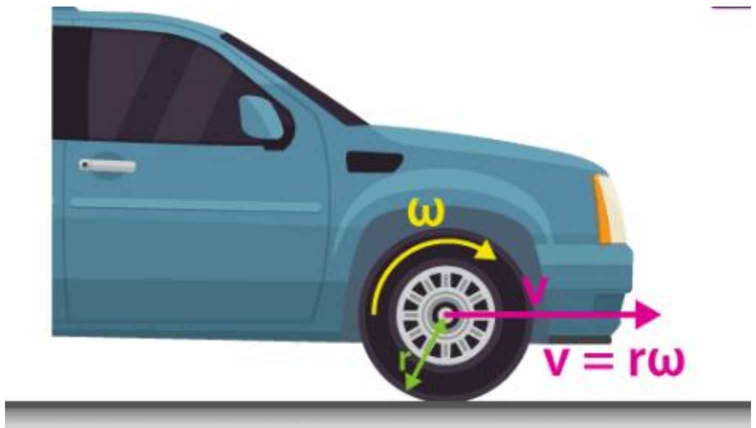
- Magnitude $a = \sqrt{a_r^2 + a_t^2}$

- Q1) Compute the radial acceleration of a point on the surface of the Earth at the equator, due to the rotation of the Earth about its axis. The radius of the earth is 6.37×10^6 m.



- $\boldsymbol{sol}.\ \boldsymbol{a} = \frac{\boldsymbol{v}^2}{\boldsymbol{r}}$
- $v = \frac{2\pi r}{T} = \frac{2\pi \times 6.37 \times 10^6}{24 \times 60 \times 60} = 463.24 \frac{m}{s}$
- $\boldsymbol{a} = \frac{\boldsymbol{v}^2}{\boldsymbol{r}}$ = 0.336 m/s^2

- Q2) A tire 0.500 m in radius rotates at a constant rate of 200 rev/min. Find the speed and acceleration of a small stone lodged in the tread of the tire (on its outer edge).



- $f = \frac{200}{60} = \frac{10}{3}$
- $v = r \times \omega = 0.5 \times \left(2\pi \ \times \frac{10}{3}\right) = 10.47 \frac{m}{s}$
- $\boldsymbol{a} = \frac{\boldsymbol{v}^2}{\boldsymbol{r}}$ = 219 m/s^2

- Q3) An automobile whose speed is increasing at a rate of 0.600 m/s^2 travels along a circular road of radius 20.0 m. When the instantaneous speed of the automobile is 4.00 m/s, find (a) the tangential acceleration component, (b) the centripetal acceleration component, and (c) the magnitude and direction of the total acceleration.
- Sol.
- A) $a_t = 0.6\,\frac{m}{s^2}$
- B) $\boldsymbol{a_c} = \frac{\boldsymbol{v^2}}{\boldsymbol{r}} = \frac{4^2}{20} = 0.8\,\frac{m}{s^2}$ (towards the center)
- C) $a = \sqrt{{a_t}^2 + {\boldsymbol{a_c}}^2} = 1\,\frac{m}{s^2}$ $\theta = tan^{-1}\left(\frac{\boldsymbol{a_c}}{a_t}\right) = 53.1$ ((inward to path))