ECT 113
Information Technology

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Lecture 3

Numbering system

what is Numbering System ?

- ***Numbering System*** *is the method of writing or expressing a number and using it to perform mathematical calculations.*

  - *There are various types of number system in mathematics or computer system:*
    - *Decimal*
    - *Binary*
    - *Octal*
    - *Hexadecimal*

Decimal Numbering System

- **The Decimal System** is composed of 10 numerals or symbols.
- These 10 symbols are 0,1,2,3,4,5,6,7,8,9: using these symbols as digits of a number, we can express any quantity.
- The decimal system, also called the base-10 system because it has 10 digits.
- the decimal system is a positional-value system in which the value of a digit depends on its position.

Decimal Numbering System

- **For example:** consider the decimal number **453**.

- Digit 4 actually represents 4 hundreds, the 5 represent 5 tens, and the 3 represent 3 units.

- In essence, the 4 carries the most weight of the three digits; it is referred to as *the most significant digit (MSD)*.

- The 3 carries the least weight and is called the *least significant digit (LSD).*

Decimal Numbering System

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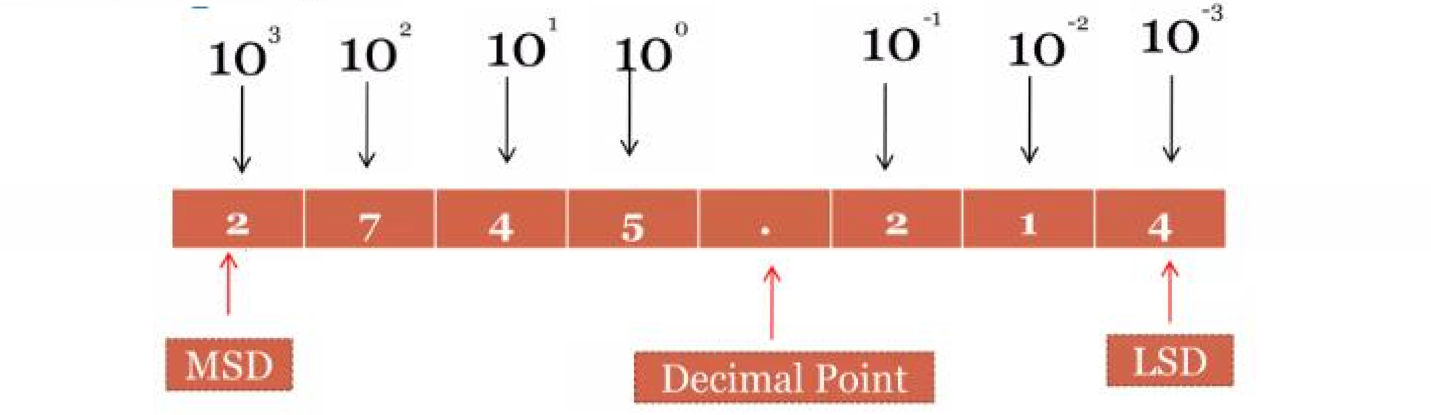
- **For example:** consider the decimal number **27.35**.
- This number is actually equal to 2 tens plus 7 units plus 3 tenth plus 5 hundredth or (2*10 + 7*1 + 3*0.1 + 5*0.01).
- The decimal point is used to separate the integer and fractional parts of the number.

Decimal Numbering System

- Moreover, the various positions relative to the decimal point carry weights that can be expressed as powers of 10.

- Example : 2745.214



$$= (2\text{x } 10^3)+(7\text{x } 10^2)+(4\text{x } 10^1)+(5\text{x } 10^0)+(2\text{x } 10^{-1})+(1\text{x } 10^{-2})+(4\text{x } 10^{-3})$$

Decimal Numbering System

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- In general any number is simply the sum of product of each digit value and its positional value.

- Unfortunately, Decimal number system does not lend itself to convinient implementation in digital system.

- It is very difficult to design electronics equipment so that it can work with 10 different voltage levels (0-9).

- For this reason, almost every digital system uses the binary number system as the basic number to design electronics circuits that operate with only two voltage levels.

Binary Numbering System

- In Binary system, there are only two symbols or possible digit values, 0 and 1.

- Even so, this base-2 system can be used to represent any quantity that can be represented in decimal or other number systems.

- The binary system is also a positional-value system, where in each binary digit has its own value or weight expressed as a power of 2.

Binary Numbering System

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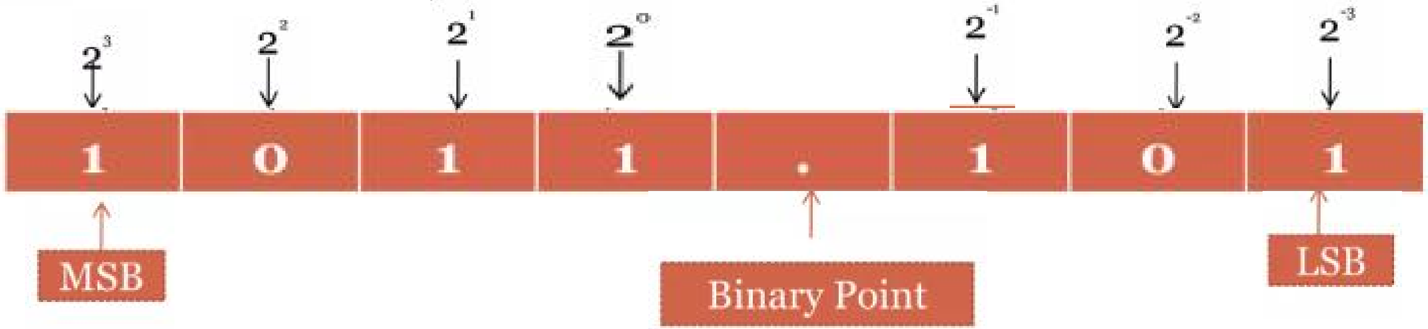
- Example:



- Here, places to the left of the binary point (counterpart of the decimal point) are positive power of 2 and places to the right are negative power of 2.

- Exercise : Find the equivalent in the decimal system for the number 1011.101_2

Answer : 11.625_{10} (How?)

Binary Numbering System

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$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$ LSB	Decimal equivalent
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7
1	0	0	0	8
1	0	0	1	9
1	0	1	0	10
1	0	1	1	11
1	1	0	0	12
1	1	0	1	13
1	1	1	0	14
1	1	1	1	15

Binary Numbering System

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- **Example:**

  What is the largest number that can be represented using 8 bits?

- **Solution :**

$$2^N - 1 = 2^8 - 1$$

$$= 255_{10}$$

$$= 11111111_2$$

Binary to Decimal Conversion

- A Binary number can be converted to decimal by multiplying the weight of each position with the binary digit and adding together.
- Example :

Convert the Binary number 10110_2 to its Decimal equivalent.

- Solution:

Binary number $1\,0\,1\,1\,0_2$

$$2^4 + 2^3 + 2^2 + 2^1 + 2^0$$

$$(2^4 \text{ x } 1)+(2^3 \text{ x } 0)+(2^2 \text{ x } 1)+(2^1 \text{ x } 1)+(2^0 \text{ x } 0)$$

$$= 16 + 0 + 4 + 2 + 0$$

$$= 22_{10}$$

Binary to Decimal Conversion

- **Example :**

Convert the Fractional Binary Number 101.102 to its Decimal equivalent.

- Solution:

Binary Number = 1 0 1 . 1 0

Power of 2 position = $2^2 \quad 2^1 \quad 2^0 \;.\; 2^{-1} \quad 2^{-2}$

$= (2^2 \times 1)+(2^1 \times 0)+(2^0 \times 1) \,.\, (2^{-1} \times 1)+(2^{-2} \times 0)$

Decimal Value = 4 + 0 + 1 . 0.5 + 0

$= 5.5_{10}$

Decimal to Binary Conversion

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- Example :

Convert Decimal 20_{10} to its Binary equivalent.

Solution:

2	20	remainder of	0
2	10	remainder of	0
2	5	remainder of	1
2	2	remainder of	0
2	1	remainder of	1
	0		

$1\ 0\ 1\ 0\ 0_2$

Decimal to Binary Conversion

- Below example show the steps to convert decimal fraction 0.625 to its binary equivalent.

Step 1 : 0.625 will be multiply by 2 ($0.625 \times 2 = 1.25$)

Step 2 : The integer part will be the MSB in the binary result

$0.25 \quad x \ 2 \ = 0.50$

$0.5 \quad x \ 2 \ = 1.00$ LSB

carried MSB ← 1 0 1

$= 0.625_{10} = .101_2$

Binary Addition

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- Adding of two binary numbers follows same as addition of two decimal numbers.
- Some times binary addition is very much easier then Decimal or any other number system addition, because in binary you deal with only 2 numbers.
- There are mainly 4 rules should be followed in the process of addition in binary numbers:

		sum	carryout
Rule 1 :	0 + 0 =	0	0
Rule 2 :	0 + 1 =	1	0
Rule 3 :	1 + 0 =	1	0
Rule 4 :	1 + 1 =	0	1

Binary Addition

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Example:

Perform Binary Addition for $101_2 + 010_2$

Solution:

$$\begin{array}{ccc} 1 & 0 & 1_2 \\ 0 & 1 & 0_2 \; + \\ \hline 1 & 1 & 1 \end{array}$$

Exercise:

Perform Binary Addition for $1011_2 + 0111_2$

Answer: $10\,01\,0_2$

Signed Binary Numbers

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- Sign bit (+)

A4	A3	A2	A1	A 0
0	1	1	0	1

= +13

B4	B3	B2	B1	B0
1	1	1	0	1

= - 13

Representing Signed Number

Signed Binary Numbers

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Example:

- Express the Decimal number -46 in 8 bit Signed magnitude system

True Binary number for +46

= 00101110

Change the sign bit to 1 and remain unchanged magnitude nits

= 10101110 = -46

Octal Numbering System

- Octal number has eight possible symbols: 0 , 1 , 2 , 3 , 4 , 5 , 6 , 7 and used to express binary numbers, which is called as base of 8 number system or Radix of 8.

  - Figure: illustrated how it decrease with negative power of 8:

$$8^5\ 8^4\ 8^3\ 8^2\ 8^1\ 8^0\ .\ \overrightarrow{8^{-1}\ 8^{-2}\ 8^{-3}\ 8^{-4}\ 8^{-5}}$$

Decrease with negative power of 8

Octal to Binary Conversion

- Any octal number can be represent by 3 bit binary number, such as 000_2 to represent 0_8 and 111_2 to represent 7_8

Example:

Convert 435_8 to its Binary equivalent.

Solution:

$$\begin{array}{ccc} 4 & 3 & 5_8 \\ \downarrow & \downarrow & \downarrow \\ 100 & 011 & 101 \end{array} = 100011101_2$$

Octal to Binary Conversion

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- Exercise:

Convert 54.7_8 to its Binary equivalent.

Solution :

$$\begin{array}{cccc} 5 & 4 & . & 7_8 \\ \downarrow & \downarrow & & \downarrow \\ 101 & 100 & . & 111 \end{array} \quad = 101100.111_2$$

Binary to Octal Conversion

- This is the reverse form of the octal to binary conversion.
- First, the Binary number should be divided into group of three from LSB.
- Then each three-bit binary number is converted to an Octal form.

Example:

Convert 100101011_2 to its equivalent Octal number.

Solution:

$$\begin{array}{ccc} 100 & 101 & 011 \\ \downarrow & \downarrow & \downarrow \\ 4 & 5 & 3 \end{array} \qquad = 4\,5\,3_8$$

Binary to Octal Conversion

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Example:

Convert 11010110_2 to its equivalent Octal number.

Solution:

$$\begin{array}{ccc} 011 & 010 & 110 \\ \downarrow & \downarrow & \downarrow \\ 3 & 2 & 6_8 \end{array} \quad = 326_8$$

NOTE that a 0 was placed to the left of the MSB to produce even groups of 3 Bits.

Octal to Decimal Conversion

- Octal number can be converted to decimal by multiplying the weight of each position with the octal number and adding together.

- Example:

Convert the Octal number 257_8 to its decimal equivalent

- Solution:

$$257_8 = (2 \times 8^2) + (5 \times 8^1) + (7 \times 8^0)$$
$$= (2 \times 64) + (5 \times 8) + (7 \times 1)$$
$$= 175_{10}$$

Decimal to Octal Conversion

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- Here we can apply the same method done in decimal to binary conversion. Dividing the decimal number by 8 can do conversion to octal.

Example :

Convert 97_{10} to its Octal equivalent.

Solution :

8	97	+ remainder of	1
8	12	+ remainder of	4
8	1	+ remainder of	1
	0		

$1\ 4\ 1_8$

Hexadecimal Numbering System

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- Hexadecimal number system is called as base 16 number system.
- It uses 10 decimal numbers and 6 alphabetic characters to represent all 16 possible symbols.
- Table below, shows Hexadecimal numbers with its equivalent in decimal and Binary.

Hexadecimal Numbering System

Hexadecimal Number	Binary Number	Decimal Nmber
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
A	1010	10
B	1011	11
C	1100	12
D	1101	13
E	1110	14
F	1111	15

Hexadecimal to Binary Conversion

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- Hexadecimal number can be represent in Binary form by using 4 bits for each hexadecimal number.

  0_{16} can be written in binary = 0000

  7_{16} in binary can be written = 0111

  A_{16} in binary can be written = 1010

Example:

Convert the Hexadecimal $A5_{16}$ to its Binary equivalent.

Solution :

$$\begin{array}{cc} A & 5_{16} \\ \downarrow & \downarrow \\ 1010 & 0101_2 \end{array} = 10100101_2$$

Hexadecimal to Binary Conversion

Exercise:

Convert the Hexadecimal $9F2_{16}$ to its Binary equivalent.

Solution:

$$
\begin{array}{cccccc}
9 & F & 2 & & \\
\downarrow & \downarrow & \downarrow & & \\
1001 & 1111 & 0010 & = & 100111110010_2
\end{array}
$$

Binary to Hexadecimal Conversion

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- **Example:**

Convert the Binary 1011011011111010_2 to its equivalent Hexadecimal number.

Solution:

1011	0110	1111	1010	
↓	↓	↓	↓	
B	6	F	A	$= B6FA_{16}$

Hexadecimal to Decimal Conversion

- Hexadecimal number can be converted to decimal by multiplying the weight of each position of the hexadecimal number (power of 16) and adding togather.

Example:

Convert the Hexadecimal number 327_{16} to its Decimal Equivalent.

Solution :

$$327_{16} = (3 \times 16^2) + (2 \times 16^1) + (7 \times 16^0)$$
$$= (3 \times 256) + (2 \times 16) + (7 \times 1)$$
$$= \mathbf{807_{10}}$$

Decimal to Hexadecimal Conversion

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- Here we can apply the same method done in Decimal to Binary conversion.
- Since we need to convert to Hexadecimal, so we have to divide the Decimal number by 16.

- Example :

Convert the Decimal 382_{10} to its Hexadecimal equivalent.

Solution:

$16 \,|\, 382$ + remainder of 14

$16 \,|\, 23$ + remainder of 7

0

$7 \quad E_{16}$

THANK YOU