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Computational moodeling of discharging RLC circuits

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Contents

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Abstract

This report presents the results of a simulation study of RLC circuits with different configurations, including series, parallel, and mixed configurations. The objective of the study was to investigate the frequency and decay rate of each configuration, which are key parameters that determine the circuit's behaviour.

The simulations were carried out using a circuit simulation software, and the results were analysed to determine the circuit's response to different input signals. The frequency and decay rate were calculated using Fourier analysis, which allowed for a detailed characterization of each circuit configuration.

The results show that the frequency and decay rate of the RLC circuits depend strongly on the values of the circuit elements (resistor, inductor, and capacitor) and the configuration of the circuit. The series configuration exhibited a higher natural frequency and slower decay rate compared to the parallel configuration. The mixed configuration showed an intermediate behaviour between the series and parallel configurations.

These findings have important implications for the design and analysis of RLC circuits, as they provide insights into how the circuit's behaviour can be manipulated by varying the circuit parameters and configuration. The results of this study can be used to optimize the performance of RLC circuits in a wide range of applications, including in electronics, power systems, and telecommunications.
Keywords: RLC circuit, second order circuits, numerical methods, data analysis.

1. Introduction

1.1. Objectives

- Measurement of the evolution of the current and voltage that characterise the discharging RLC circuit in parallel and series.
- Computational simulation of the tested circuits and compare the results with the experimental data.
- Obtain and compare the type of solution, as well as the defining parameters, such as natural frequency of the system and the dampening factor.

1.2. The RLC circuit

An RLC circuit is an electrical circuit that consists of a resistor, an inductor, and a capacitor connected together. These circuits are used in a wide range of applications, including in electronics, power systems, and telecommunications. In this report, we will explore the behaviour and characteristics of RLC circuits, their applications, and the mathematical models used to describe their behaviour. We will also discuss the effects of varying the parameters of an RLC circuit, such as the resistance, capacitance, and inductance, on the circuit's performance. Understanding RLC circuits is essential for anyone interested in electronics or electrical engineering, and this report aims to provide a comprehensive overview of the topic.

An RLC circuit is a type of electrical circuit that includes a resistor (R), an inductor (L), and a capacitor (C) that are connected together in series or parallel. These circuits exhibit complex behaviours, which can be described by mathematical models.

The behaviour of an RLC circuit is governed by the interplay between the three circuit elements. The resistor restricts the flow of current in the circuit, the inductor opposes changes in current by inducing an electromotive force (EMF), and the capacitor stores and releases electrical charge.

When an RLC circuit is excited with an electrical signal, such as a voltage or a current, it responds with a natural frequency that depends on the values of R, L, and C. The circuit's natural frequency determines how quickly it can transfer energy between the inductor and the capacitor, and is a key parameter in the circuit's behaviour.

The behaviour of RLC circuits can be analysed using differential equations and complex numbers. These equations describe how the voltage and current in the circuit change over time, and can be solved to predict the circuit's behaviour under different conditions.

Overall, understanding the theory behind RLC circuits is essential for designing and analysing complex electrical systems, and is a fundamental concept in electrical engineering and physics.

RLC circuits are often modelled with a second order differential equation, for that reason, these type of circuits are often called second order circuits. In consumer devices, RLC circuits are found in a combination of two basic configurations: series and parallel.

RLC in series

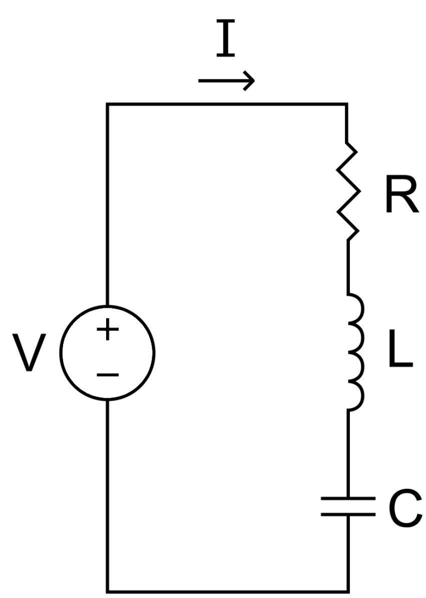

Figure 1: An example RLC circuit in a series configuration without a voltage source.

When the circuit is charged to a certain voltage and then disconnected from the voltage source, the capacitor starts to discharge through the resistor and inductor. The theoretical behavior of the discharging process can be described using Kirchhoff's laws and differential equations.

When the switch is closed and the circuit is charged, the capacitor stores energy in the form of electric charge, and the voltage across the capacitor increases according to the equation $Vc(t) = V(1 - e^(-t/RC))$, where V is the initial voltage, t is the time elapsed since the switch was closed, R is the resistance, and C is the capacitance. When the switch is opened and the capacitor starts to discharge, the energy stored in the capacitor is dissipated as heat in the resistor and magnetic energy in the inductor. The differential equation governing the discharging behaviour of the circuit is given by $Ldi/dt + Ri + 1/C \int idt = 0$, where i is the current flowing through the circuit at time t, L is the inductance, R is the resistance, and C is the capacitance. This equation can be solved using the method of integrating factors to obtain an expression for the current as a function of time.

The solution to the differential equation shows that the current through the circuit decreases exponentially over time, as the energy stored in the capacitor is dissipated. The time constant of the circuit is given by RC, and represents the time it takes for the voltage across the capacitor to decrease to 37% of its initial value. The voltage across the capacitor can be calculated using the equation $Vc(t) = V(1 - e^(-t/RC))$, where V is the initial voltage and R and C are the resistance and capacitance, respectively.

In summary, a discharging RLCS circuit in series exhibits exponential decay of the current over time, as the energy stored in the capacitor is dissipated through the resistor and inductor. The behavior of the circuit can be described using Kirchhoff's laws and a differential equation, and the time constant of the circuit can be calculated using the resistance and capacitance.

RLC circuits in parallel

1.3. The Runge-Kutta 4 method

The Runge-Kutta 4 method is a numerical algorithm used to solve ordinary differential equations (ODEs) by approximating the solution at discrete time steps. The method is a fourth-order, explicit method, meaning that it uses information at the current time step and previous time steps to estimate the solution at the next time step.

The basic idea behind the Runge-Kutta 4 method is to estimate the derivative of the solution at four different points within the time step, and then use a weighted average of these

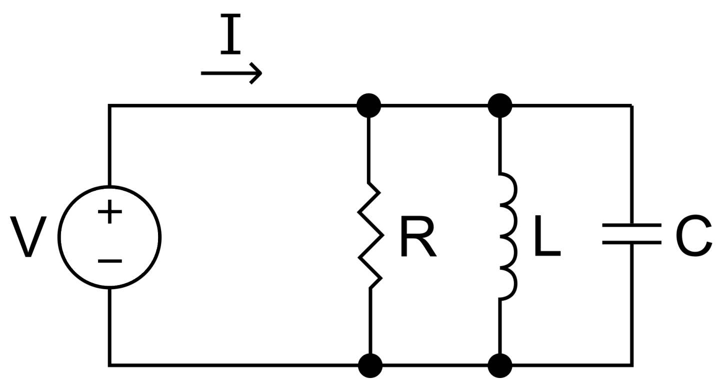

Figure 2: An example RLC circuit in a parallel configuration without a voltage source.

estimates to compute the approximate solution at the end of the time step. This weighted average is called the Runge-Kutta formula, and is calculated using a set of coefficients that depend on the order of the method.

The Runge-Kutta 4 method is widely used because it provides a good balance between accuracy and computational efficiency. It is more accurate than lower-order methods such as the Euler method, and can be used to solve a wide range of ODEs with relatively small time steps.

To use the Runge-Kutta 4 method, the ODE must first be converted to a system of first-order ODEs. The method then proceeds by iterating through the time steps, using the Runge-Kutta formula to estimate the solution at each time step. The accuracy of the method can be improved by decreasing the size of the time step, although this can come at the cost of increased computational complexity.

Overall, the Runge-Kutta 4 method is a powerful tool for solving ODEs numerically, and is widely used in a variety of fields, including physics, engineering, and computer science.

Following [**burden**] we'll first give and introduction to the Runge-Kuta method of order 4 for a single first order equation, then we'll briefly introduce how to reduce a second order equation to two first order equations to finally introduce the Runge-Kuta method of order 4 for a system of six first order equations.

Runge-Kutta method of order 4

Recall that a first order differential equation has the form:

$$y' = f(t, y), \quad \text{for} \quad a \leq t \leq b, \quad \text{subject to} \quad y(a) = \alpha \tag{1}$$

And recall that, supposing that this initial value problem (IVP) is well posed, we will be able to use the Runge-Kutta method of order four to approximate the solution y in this interval.

First we will need to pick a number of $n + 1$ equally spaced points in the interval $[a, b]$ in order to define the step size $h > 0$, the initial t_0 and the starting value y_0 as:

$$\begin{aligned} h &= \frac{b - a}{n} \\ t_0 &= a \\ y_0 &= y(a) = \alpha \end{aligned}$$

This now defines in which points of the interval we'll be approximating the function. This points are defined as:

$$t_{n+1} = t_n + h \quad \vee \quad t_{n+1} = t_0 + nh = a + nh \tag{2}$$

Now we need to compute what is pretty much the essence of this method, the ki's coefficients, defined as:

$$\begin{aligned} k_1 &= hf(t_n, y_n) \\ k_2 &= hf\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right) \\ k_3 &= hf\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right) \\ k_4 &= hf\ (t_n + h, y_n + k_3) \end{aligned}$$

Note that this computations are actually values of the function f, which we may also interpret as slopes of y, on different parts of the interval between t_n and t_{n+1}. Note that k_1 is the slope of y at the beginning of the interval, k_2 is the slope at the midpoint of the interval but with k_1, k_3 is also the value of y in the midpoint but now with k_2, and k_4 is the slope at the end of the interval using k_3 and so, for each interval we get values of how the function behaves, we now define the approximation on this interval with a weighted average as:

$$y_{n+1} = y_n + \frac{1}{6}\Big(k_1 + 2k_2 + 2k_3 + k_4\Big)$$

These are the ideas of this method and should not be mistaken as either a pseudocode or an algorithm. It's important to say that this method has a local truncation error $O(h^4)$ provided that the solution y has five continuous derivatives i.e. $y \in C^5$.

Second Order Differential Equations

Consider the following second order IVP problem:

$$y'' = f(t, y, y'), \quad \text{for} \quad a \leq t \leq b, \quad \text{subject to} \quad y(a) = \alpha_1,\ y'(a) = \alpha_2$$

To attack this kind of problem we can reduce the second order IVP into a system of 2 first order equations. We do this basically by re-naming variables. By assigning the labels $u_1(t) = y(t)$, $u_2(t) = y'(t)$ we get the following system:

$$\begin{aligned} \dot{u}_1 &= u_2 & u_1(a) &= \alpha_1 \\ \dot{u_2} &= f(t, u_1, u_2) & u_2(a) &= \alpha_2 \end{aligned}$$

Making a problem with an unknown solution into a solvable problem.

Runge-Kutta method for a system of 2 first order ODE's

Now we'll give the algorithm of the RK4 method for a IVP system of 2 first order ODE's.

Consider a first order IVP system of 2 equations with the form:

$$u_j' = f_j(t, u_1, u_2), \quad a \leq t \leq b, \quad \text{with} \quad u_j(a) = \alpha_j$$

for $j = 1, 2$ at $n + 1$ equally spaced points in the interval $[a, b]$:

IN endpoints a, b; integer n, initial conditions α_1, α_2.

OUT approximations w_j to $u_j(t)$ at the $n + 1$ values of t.

Step 1 set $h = (b - a)/n$;
t=a.

Step 2 For $j = 1, 2$ set $w_j = \alpha_j$

Step 3 **OUT** (t, w_1, w_2)

Step 4 For $i = 1, ..., n$ do steps 5-11 .

Step 5 For $j = 1, 2$ set
$$k_{1,j} = hf_j(t, w_1, w_2)$$

Step 6 For $j = 1, 2$ set
$$k_{2,j} = hf_j\left(t + \frac{h}{2}, w_1 + \frac{1}{2}k_{1,1}, w_w + \frac{1}{2}k_{1,w}\right)$$

Step 7 For $j = 1, 2$ set
$$k_{3,j} = hf_j\left(t + \frac{h}{2}, w_1 + \frac{1}{2}k_{2,1}, w_2 + \frac{1}{2}k_{2,2}\right)$$

Step 8 For $j = 1, 2$ set
$$k_{4,j} = hf_j\left(t + \frac{h}{2}, w_1 + \frac{1}{2}k_{3,1}, w_2 + \frac{1}{2}k_{3,2}\right)$$

Step 9 For $j = 1, 2$ set
$$w_j = w_j + \frac{1}{6}\left(k_{1,j} + 2k_{2,j} + 2k_{3,j} + k_{4,j}\right)$$

Step 10 Set $t = a + ih$

OUT (t, w_1, w_2).

Step 12 STOP

1.4. Least squares

Linear problem

The least squares method is a mathematical technique used to find the best-fit line or curve that describes the relationship between two or more variables. It is commonly used in regression analysis to determine the relationship between a dependent variable and one or more independent variables.

The basic idea behind the least squares method is to minimize the sum of the squared differences between the observed data points and the predicted values of the dependent variable based on the independent variable(s). In other words, the method seeks to find the line or curve that minimizes the sum of the squared errors between the predicted values and the actual observed values.

To use the least squares method, the data points are first plotted on a graph, and a line or curve is drawn that represents the relationship between the variables. The line or curve is then adjusted iteratively until the sum of the squared errors is minimized.

The least squares method is widely used in fields such as statistics, engineering, and physics, and can be used to analyse a wide range of data sets, including experimental data, survey data, and financial data. The method is robust and flexible, and can be used to fit linear and non-linear relationships between variables.

Overall, the least squares method is a powerful tool for analysing data and finding the best-fit line or curve that describes the relationship between variables. It is widely used in a variety of fields and has many practical applications.

Non-linear problem

$$y = f(x; \beta) + \epsilon, \quad f(X) - \text{not linear} \tag{3}$$

$$r_i = y_i - f(x_i; \beta) \tag{4}$$

$$L = \sum_i r_i^2, \quad \min_\beta L, \quad f(x; \beta) \neq x^T \beta \tag{5}$$

$$\boldsymbol{\nabla}_\beta = 0 \tag{6}$$

The system of equations is not linear.

This can be solved iteratively with the Newton-Raphson method, with an initial guess.

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \tag{7}$$

$$x_{i+1} = x_i - \frac{f'(x_i)}{f''(x_i)} = x_i - (\nabla_x^2 f)^{-1} \boldsymbol{\nabla}_x f \tag{8}$$

The calculation of the Hessian can be complicated, so it if often approximated.

$$\nabla_{\beta_j} L = \sum_i 2r_i \cdot \frac{\partial r_i}{\partial \beta_j} = -2 \sum_i r_i \cdot \frac{\partial f_i}{\partial \beta_j} = -2 \cdot J^T \cdot r \tag{9}$$

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial \beta_1} & \cdots & \frac{\partial f_1}{\partial \beta_p} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial \beta_1} & \cdots & d\frac{\partial f_n}{\partial \beta_p} \end{pmatrix}, \qquad r = \begin{pmatrix} r_1 \\ \vdots \\ r_n \end{pmatrix} \tag{10}$$

$$\nabla^2_{\beta_j \beta_k} L = -2 \sum_i \left(-\frac{\partial f_i}{\partial \beta_k} \cdot \frac{\partial f_i}{\partial \beta_j} + r_j \cancel{\frac{\partial^2 f_i}{\partial \beta_j \partial \beta_k}} \right) \approx \sum_i \frac{\partial f_i}{\partial \beta_k} \frac{\partial f_i}{\partial \beta_j} = 2 \cdot J^T \cdot J \tag{11}$$

$$\beta_{t+1} = \beta_t - \left(\nabla_\beta^2 L \right)^{-1} \nabla_\beta L \approx \beta_t - \left(2 \cdot J_t^T \cdot J_t \right)^{-1} \left(-2 \cdot J_t^T \cdot r_t \right) \tag{12}$$

$$\boxed{\beta_{t+1} \approxeq \beta_t + \left(J_t^T \cdot J_t \right)^{-1} \left(\cdot J_t^T \cdot r_t \right)} \tag{13}$$

2. Methodology

2.1. Circuit

To measure the voltage and current of an RLC circuit in both series and parallel configurations, the following methodology can be used:

Prepare the RLC circuit by connecting the components in the desired configuration (series or parallel).

Power on the circuit and wait for it to reach a steady-state.

Use a multimeter to measure the voltage across the resistor (Vr), capacitor (Vc), and inductor (VL) in the circuit. To measure the voltage in parallel, connect the multimeter in parallel with the component being measured. To measure the voltage in series, connect the multimeter in series with the component being measured.

Use a second multimeter to measure the current flowing through the circuit. To measure the current in series, connect the multimeter in series with the circuit. To measure the current in parallel, connect the multimeter in parallel with the circuit.

Record the voltage and current measurements in a table, along with any relevant information about the circuit configuration and component values.

Repeat steps 3-5 for different values of the frequency or other relevant variables, to observe changes in the behavior of the circuit.

Analyze the data to calculate the impedance of the circuit, using the equation Z = V/I, where Z is the impedance, V is the voltage, and I is the current. For series circuits, the impedance is equal to the sum of the resistance, inductive reactance, and capacitive reactance. For parallel circuits, the impedance is equal to the reciprocal of the sum of the reciprocals of the resistance, inductive reactance, and capacitive reactance.

Plot the data on a graph to visualize the behavior of the circuit, and compare the results for different circuit configurations and component values.

Overall, the methodology for measuring the voltage and current of an RLC circuit involves using multimeters to measure the voltage and current, recording the measurements, and analyzing the data to calculate the impedance and visualize the behavior of the circuit.

2.2. Simulation

To perform computational simulation of RLC circuits, the following methodology can be used:

Choose a suitable software package for simulating RLC circuits, such as LTspice, Multisim, or PSpice.

Create a circuit schematic using the software, with the components arranged in the desired configuration (series or parallel).

Assign appropriate values for the resistance, inductance, and capacitance of the components in the circuit.

Set up the simulation parameters, such as the time step and simulation time.

Run the simulation and collect data on the voltage and current at various points in the circuit, as well as other relevant parameters such as power dissipation and frequency response.

Analyze the simulation data using software tools, such as MATLAB or Python, to calculate the impedance of the circuit and visualize the behavior of the circuit.

Compare the results of the simulation with theoretical calculations, to validate the accuracy of the simulation.

Perform sensitivity analysis by varying the values of the components in the circuit to observe how changes affect the behavior of the circuit.

Document the simulation results and include them in the report, along with relevant graphs and tables.

Overall, the methodology for simulating RLC circuits involves creating a circuit schematic, assigning component values, running a simulation, analyzing the data, and validating the results. By using software tools to simulate RLC circuits, it is possible to study the behavior of circuits under different conditions and to gain a deeper understanding of their characteristics.

Método de diferencias finitas para problemas lineales

Following [**burden**] we'll first give and introduction to the Runge-Kuta method of order 4 for a single first order equation, then we'll briefly introduce how to reduce a second order equation to two first order equations to finally introduce the Runge-Kuta method of order 4 for a system of six first order equations.

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And recall that, supposing that this initial value problem (IVP) is well posed, we will be able to use the Runge-Kutta method of order four to approximate the solution y in this interval.

First we will need to pick a number of $n + 1$ equally spaced points in the interval $[a, b]$ in order to define the step size $h > 0$, the initial t_0 and the starting value y_0 as:

$$\begin{aligned} h &= \frac{b - a}{n} \\ t_0 &= a \\ y_0 &= y(a) = \alpha \end{aligned}$$

This now defines in which points of the interval we'll be approximating the function. This points are defined as:

$$t_{n+1} = t_n + h \quad \vee \quad t_{n+1} = t_0 + nh = a + nh \tag{14}$$

Now we need to compute what is pretty much the essence of this method, the ki's coefficients, defined as:

$$\begin{aligned} k_1 &= hf(t_n, y_n) \\ k_2 &= hf\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right) \\ k_3 &= hf\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right) \\ k_4 &= hf\left(t_n + h, y_n + k_3\right) \end{aligned}$$

Note that this computations are actually values of the function f, which we may also interpret as slopes of y, on different parts of the interval between t_n and t_{n+1}. Note that k_1 is the slope of y at the beginning of the interval, k_2 is the slope at the midpoint of the interval but with k_1, k_3 is also the value of y in the midpoint but now with k_2, and k_4 is the slope at the end of the interval using k_3 and so, for each interval we get values of how the function behaves, we now define the approximation on this interval with a weighted average as:

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Second Order Differential Equations

Consider the following second order IVP problem:

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To attack this kind of problem we can reduce the second order IVP into a system of 2 first order equations. We do this basically by re-naming variables. By assigning the labels $u_1(t) = y(t)$, $u_2(t) = y'(t)$ we get the following system:

$$\begin{aligned} \dot{u}_1 &= u_2 & u_1(a) &= \alpha_1 \\ \dot{u_2} &= f(t, u_1, u_2) & u_2(a) &= \alpha_2 \end{aligned}$$

Making a problem with an unknown solution into a solvable problem.

Runge-Kutta method for a system of 2 first order ODE's

Now we'll give the algorithm of the RK4 method for a IVP system of 2 first order ODE's.

Consider a first order IVP system of 2 equations with the form:

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for $j = 1, 2$ at $n + 1$ equally spaced points in the interval $[a, b]$:

IN endpoints a, b; integer n, initial conditions α_1, α_2.

OUT approximations w_j to $u_j(t)$ at the $n + 1$ values of t.

Step 1 set $h = (b - a)/n$;
t=a.

Step 2 For $j = 1, 2$ set $w_j = \alpha_j$

Step 3 **OUT** (t, w_1, w_2)

Step 4 For $i = 1, ..., n$ do steps 5-11 .

Step 5 For $j = 1, 2$ set
$$k_{1,j} = hf_j(t, w_1, w_2)$$

Step 6 For $j = 1, 2$ set
$$k_{2,j} = hf_j\left(t + \frac{h}{2}, w_1 + \frac{1}{2}k_{1,1}, w_w + \frac{1}{2}k_{1,w}\right)$$

Step 7 For $j = 1, 2$ set
$$k_{3,j} = hf_j\left(t + \frac{h}{2}, w_1 + \frac{1}{2}k_{2,1}, w_2 + \frac{1}{2}k_{2,2}\right)$$

Step 8 For $j = 1, 2$ set
$$k_{4,j} = hf_j\left(t + \frac{h}{2}, w_1 + \frac{1}{2}k_{3,1}, w_2 + \frac{1}{2}k_{3,2}\right)$$

Step 9 For $j = 1, 2$ set
$$w_j = w_j + \frac{1}{6}\left(k_{1,j} + 2k_{2,j} + 2k_{3,j} + k_{4,j}\right)$$

Step 10 Set $t = a + ih$

OUT (t, w_1, w_2).

Step 12 STOP

2.3. Parameter calculations

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3. Results

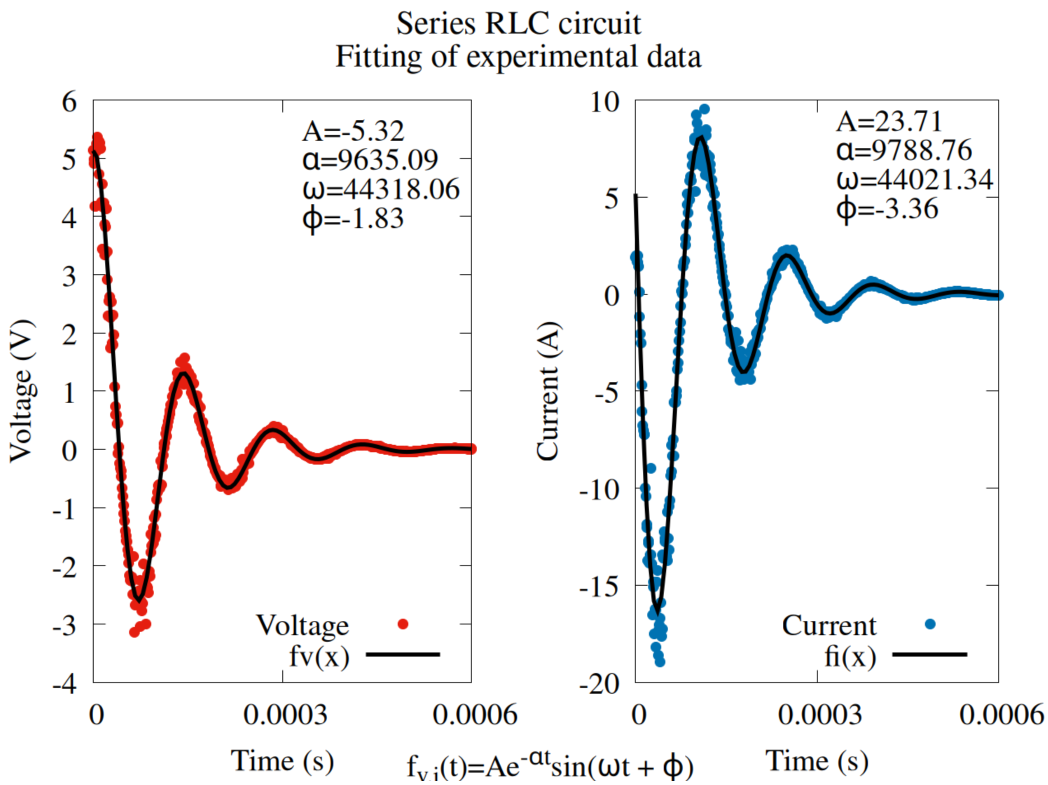

Figure 3: Experimental data and fitting for the series RLC circuit.

3.1. Series RLC circuit

4. Analysis

4.1. Parallel RLC circuit

5. Conclusion

Conclusion .[1]

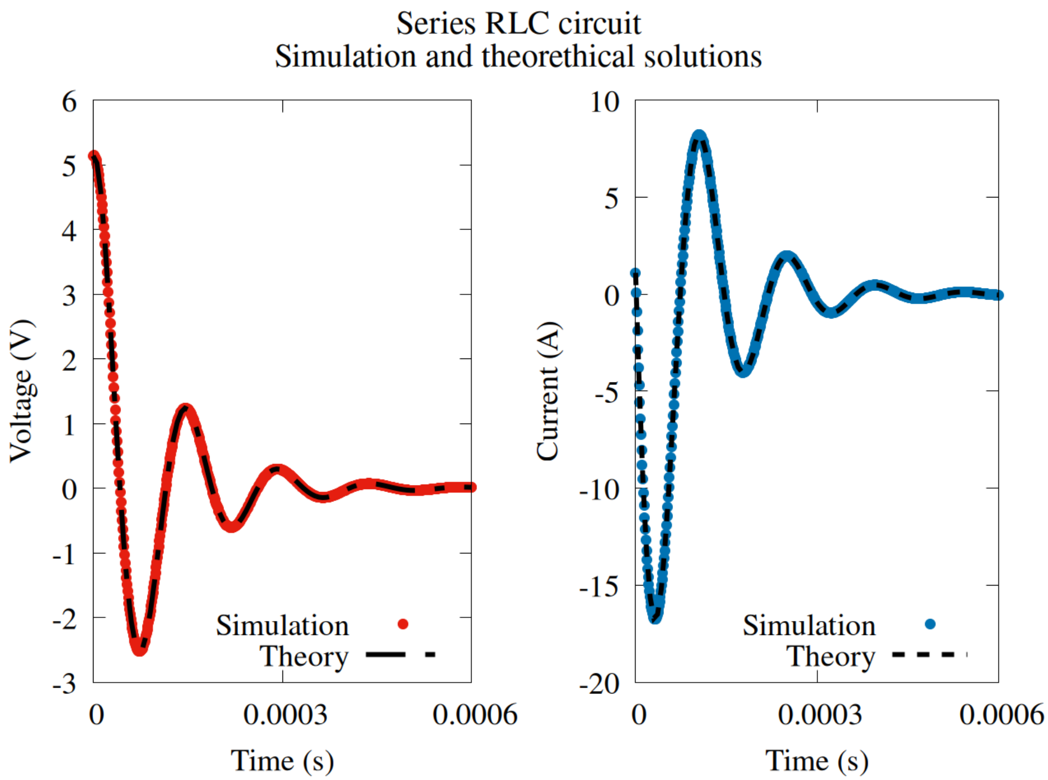

Figure 4: Experimental data and fitting for the series RLC circuit.

References

[1] Leonard S Bobrow. *Análisis de circuitos eléctricos*. 621.3 B663a. Edit. Iberoamericana, 1983.

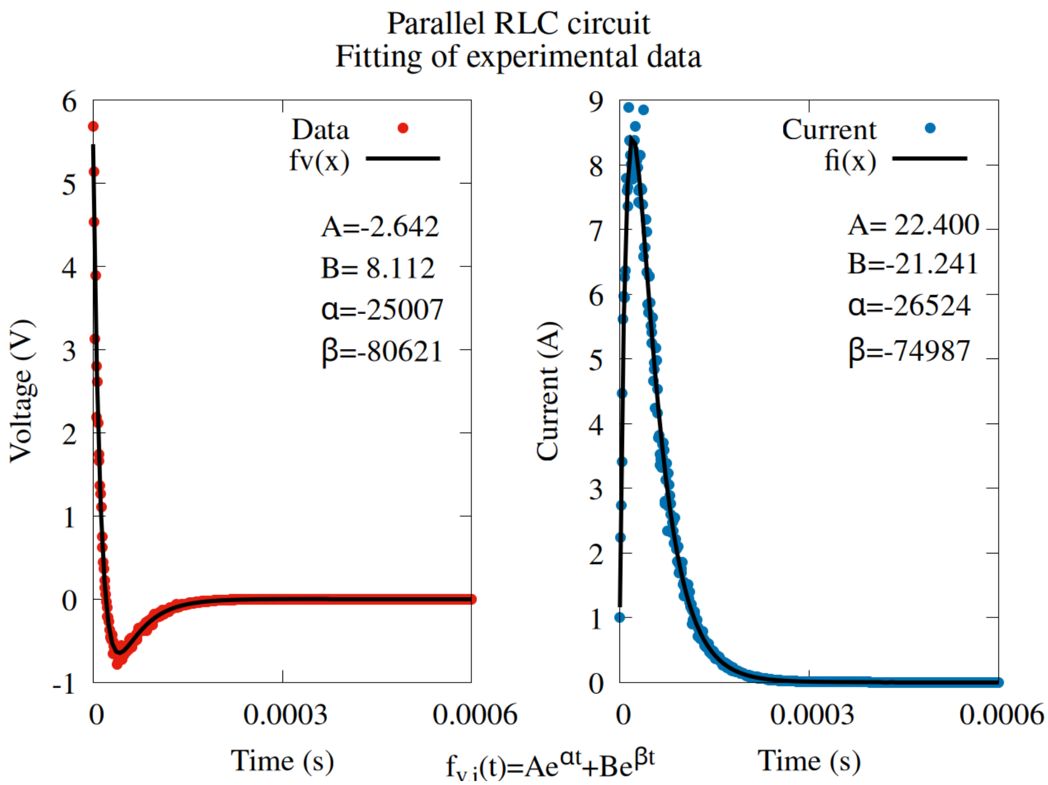

Figure 5: Experimental data and fitting for the parallel RLC circuit.

A. Computational code (Python)

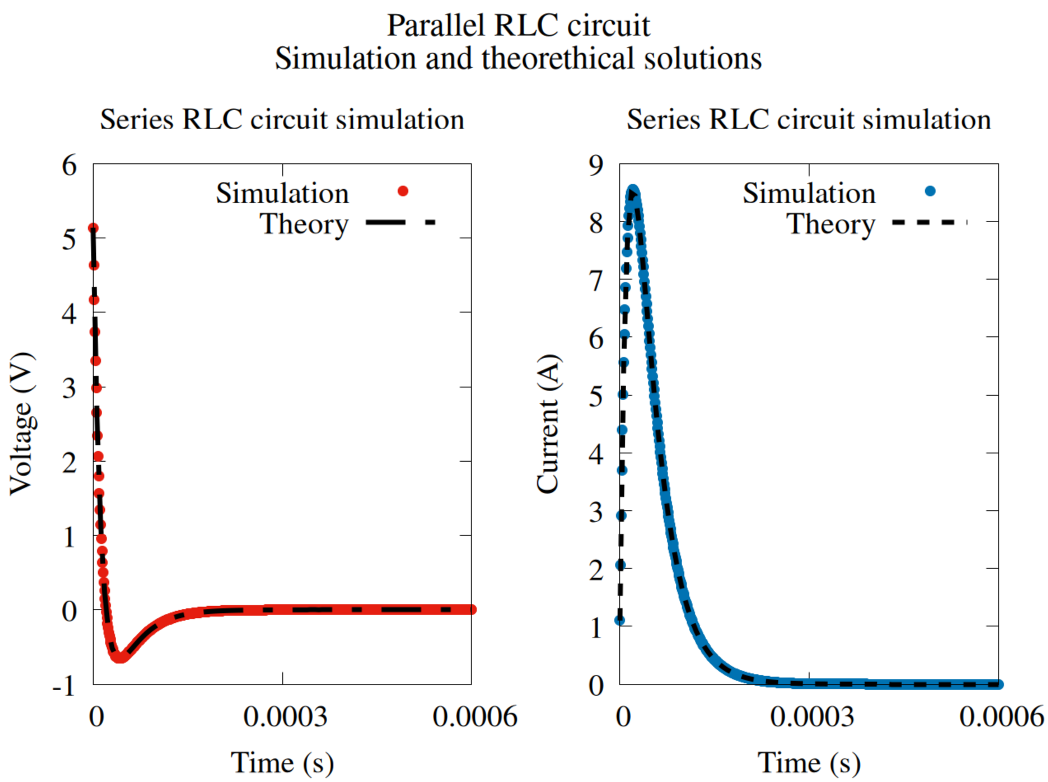

Figure 6: Experimental data and fitting for the parallel RLC circuit.

	Parameter:			
	A	α	ω	ϕ
Analytical reference	5.333	9803.922	43227.2699	1.301
Experimental error (%)	-0.206	-1.722	2.523	0.173
Simulation error (%)	4.947×10^{-12}	-1.293×10^{-05}	-1.570×10^{-06}	8.738×10^{-12}

Table 1: Parameters of the analytical evolution of the voltage through a capacitor for the simple problem of series RLC circuit with values for the components of $R = 0.1\,\Omega$, $L = 5.1\,\mu H$, and $C = 9.98\,\mu F$; as well as the initial conditions $V_0 = 5.14V$ and $I_0 = 1.1A$. The circuit is classified as underdamped, so its function is of the form $V(t) = Ae^{-\alpha t}\sin(\omega t + \phi)$. The error generated by the parameters found using the Newton-Ramphson method for data fitting by non linear least squares of both datasets, obtained by experimental measurements and computer simulation, are also included. In both cases the resulting error is minimal.

	Parameter:			
	A	α	B	β
Analytical reference	23.309	-26748.191	-22.209	-73452.209
Experimental error (%)	-3.899	-0.835	-4.358	2.090
Simulation error (%)	0.541	0.115	0.567	-0.314

Table 2: Parameters of the analytical evolution of the current through an inductor for the simple problem of a RLC circuit in parallel configuration, with values for the components of $R = 0.1\,\Omega$, $L = 5.1\,\mu H$, and $C = 9.98\,\mu F$; as well as the initial conditions $V_0 = 5.14V$ and $I_0 = 1.1A$. This is an overdamped circuit, so the function is of the form $I(t) = Ae^{\alpha} + Be^{\beta}$. The error generated by the parameters found using the Newton-Ramphson method for data fitting by non linear least squares of both datasets, obtained by experimental measurements and computer simulation, are also included. In both cases the resulting error is minimal.