Sign Restrictions and Bayesian VARs Lecture Notes June 2006, ZEI, Bonn University

Harald Uhlig
Humboldt University Berlin,
CentER, Bundesbank, CEPR

uhlig@wiwi.hu-berlin.de, June 11, 2006

Outline

- Economics and Vector Autoregressions
- Identifying VARs: traditional identification
- The new approach: sign restrictions
- Application 1: the effects of monetary policy on output
- Application 2: the effects of fiscal policy on output
- Application 3: exchange rate dynamics
- Conclusions

Economics and Vector Autoregressions

- Many research questions in macroeconomics are about the dynamic effects of changes in the economy:
  - What happens to GNP, to inflation or to exchange rates, when monetary policy is changed?
  - What happens to GNP, when fiscal policy is changed?
- The holy grail: to settle on a convincing theory, which explains the observed data, and which delivers quantitative answers to these questions.
- A key question: what does the data say?

Vector Autoregressions

- Vector autoregressions as a tool to answer these research questions empirically were popularized by Sims (1980) in his classic paper, "Macroeconomics and Reality", Econometrica, 48, 1-48.

- Sims (1980) on the econometric models of the time: "*the connection between ... models and reality - the style in which 'identification' is achieved for these models - is inappropriate, to the point at which claims for identification in these models cannot be taken seriously.*"

- Thus: VARs.

Vector Autoregressions

- A VAR in **"reduced"** form:

$$
\begin{aligned}
Y_t &= B(L)Y_{t-1} + u_t \qquad (1) \\
&= B_{(1)}Y_{t-1} + B_{(2)}Y_{t-2} + \ldots + B_{(l)}Y_{t-l} + u_t,\ E[u_t u_t'] = \Sigma
\end{aligned}
$$

Advantages of VARs

- Expectation-driven theories imply that any variable useful for forecasting is useful for decision making and can therefore impact current economic variables.
- The VAR takes this into account by not including all variables at all lags (up to l) in all equations.
- So far, no identification is necessary, just the choice of variables, the lag-length and the choice of deterministic regressors. This increases the credibility of communicating the statistical results.

Co-Integration

- A large literature investigates methods for distinguishing between stationary and non-stationary components of the VARs.

- As a result, one often sees a VAR rewritten as an error correction mechanism,

$$\Delta Y_t = c + \alpha\beta' Y_{t-1} + D_{(1)}\Delta Y_{t-1} + \ldots + D_{(l-1)}\Delta Y_{t-l+1} + u_t$$

- Here, we focus on the level representation (1) of the VAR, because
  - the issue of cointegration is unrelated to the focus here, so we shall not worry about the best way of presenting the reduced-form VAR,
  - we use a Bayesian methodology, in which there is no stringent need to distinguish between stationary and unit roots.

Impulse and Propagation

- The raw estimation results for a VAR are rarely interesting. Alternatively, one can represent a VAR as **responses** to **impulses**.

- Formally, consider the moving average representation,

$$
\begin{aligned}
Y_t &= (I - B(L))^{-1} u_{t-1} \\
&= \sum_{s=0}^{\infty} C_s u_{t-s}
\end{aligned}
$$

  The impulse response to the i-th one-step ahead prediction error is given by the i-th column of the C_s.

- ... however, responses to one-step ahead prediction errors are rarely economically interesting.

Outline

- Economics and Vector Autoregressions
- Identifying VARs: traditional identification
- The new approach: sign restrictions
- Application 1: the effects of monetary policy on output
- Application 2: the effects of fiscal policy on output
- Application 3: exchange rate dynamics
- Conclusions

Identifying VARs: traditional identification

- To interpret the VAR in an economically meaningful way, one needs to disentangle u_t into **"structural"** shocks v_t, like e.g. monetary policy shocks, productivity shocks, etc.,

$$u_t = Av_t$$

- Let $a = A_i$ be the i-th column of A. The impulse response of all variables at horizon s to the i-th structural shock $v_t' = e_i$ is then given by

$$r_s = C_s a$$

Identification

- Assuming $E[v_t v_t'] = I$, the only restriction on A is

$$\Sigma = E[u_t u_t'] = AE[v_t v_t']A' = AA'$$

- So, one needs (at least) $m(m-1)/2$ additional identifying assumptions to identify v_t. Possibilities:

1. **Cholesky**-decomposition. See Sims.
2. **"Structural"** VAR: Impose equations in $u_{i,t}$ and $v_{j,t}$. Often: impose zero restrictions on entries in A or A^{-1}. See Bernanke, Blanchard-Watson, Sims.
3. Decompose into **permanent** ("supply") shocks and transitory ("demand") shocks. See Blanchard-Quah, Gali.

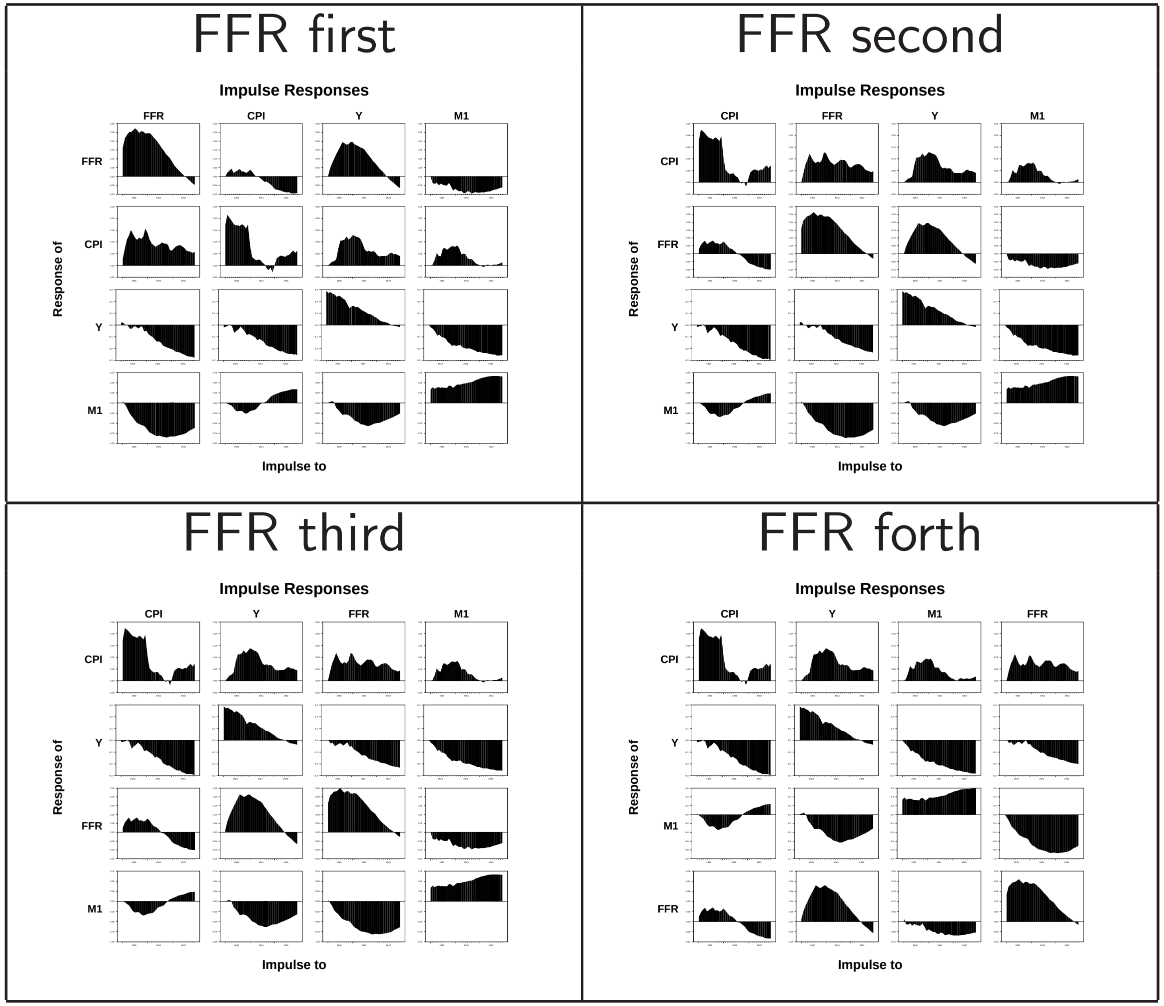
See surveys in Christiano-Eichenbaum-Evans, Leeper-Sims-Zha, Favero, others.

Example: What is a monetary policy shock?

- A shock to the Federal Funds Rate,
  - ... that is ordered last in a Cholesky decomposition
  - ... that has no permanent effect on output
  - ... that solves a set of structural equations, sorting out demand from supply shocks.
  - ...

- Often, the problem is: the **reactions** of the other variables do not look "as they should".

Example: FFR with CPI infl, real GNP, M1, 1986-2001



Typical puzzles

- **The liquidity puzzle:** when identifying monetary policy shocks as surprise increases in the stock of money, interest rates tend to go down, not up.

- **The price puzzle:** after a contractionary monetary policy shock, even with interest rates going up and money supply going down, inflation goes up rather than down.

What to do?

- Accept the results as they come out or
- continue the specification search, until one set of identifying restrictions delivers the "right result" or
- **impose the "right results" as part of the identifying restrictions**
- Continuation of Sims agenda: avoid unreasonable identication restrictions.

Outline

- Economics and Vector Autoregressions
- Identifying VARs: traditional identification
- The new approach: sign restrictions
- Application 1: the effects of monetary policy on output
- Application 2: the effects of fiscal policy on output
- Application 3: exchange rate dynamics
- Conclusions

The new approach: sign restrictions: A new, emerging literature

- Mark Dwyer (1997), "Dynamic Response Priors for Discriminating Structural Vector Autoregressions," draft, UCLA.
- Jon Faust (1998), "The Robustness of Identified VAR Conclusions about Money," Carnegie-Rochester, 49(0), Dec., 207-44. Uhlig (1998): comment.
- Canova, Fabio and Gianni de Nicolo (2002), "Monetary Disturbances matter for business fluctuations in the G-7," Journal of Monetary Economics, September, 49(6), 1131-59.
- Uhlig, Harald (2005), "What are the effects of monetary policy shocks on output?", Journal of Monetary Economics, forthcoming.

Precursors

- Blanchard, Olivier Jean, "A Traditional Interpretation of Macroeconomic Fluctuations," American Economic Review, vol. 79, no. 5 (Dec. 1989), 1146-1164.
- Leamer, ...

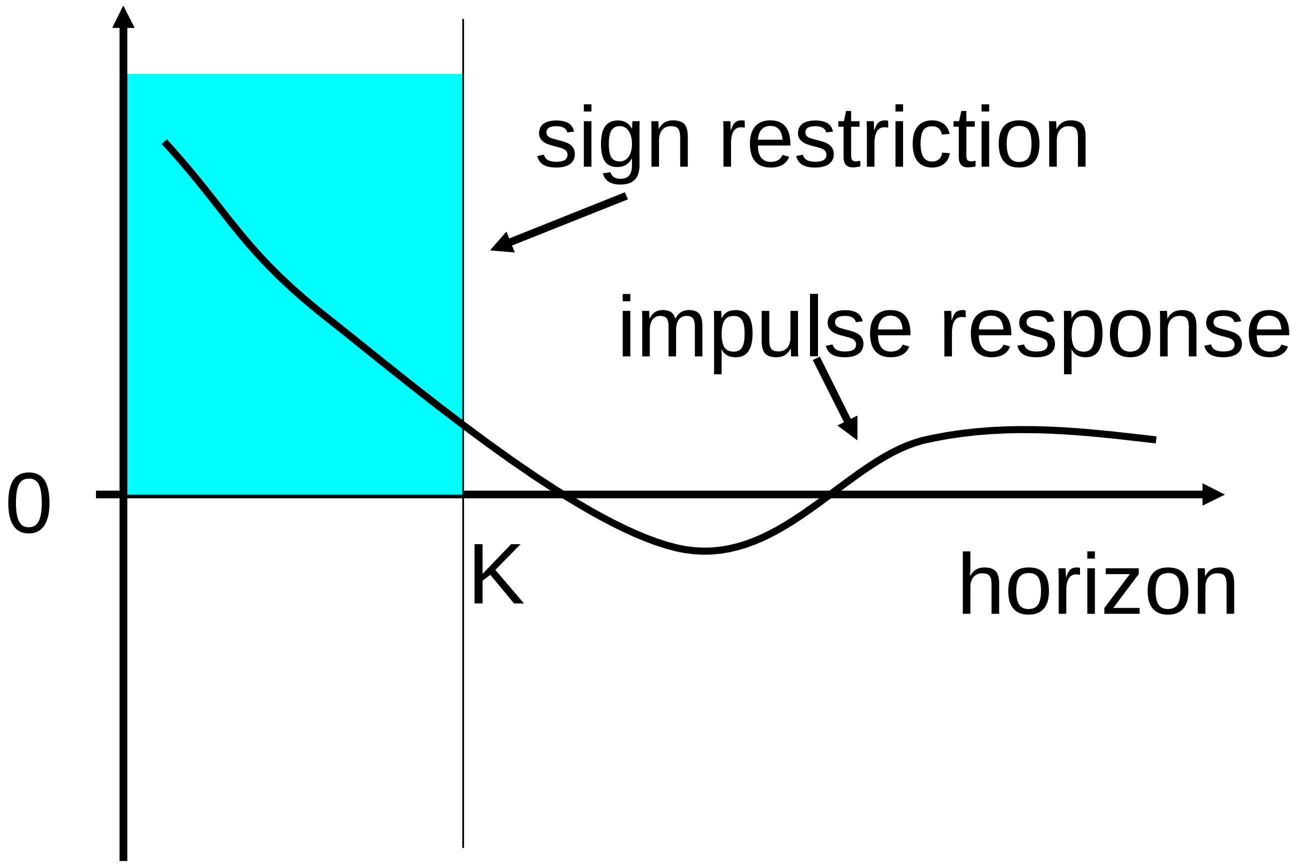
Example: monetary policy shocks

The conventional wisdom says: after a contractionary monetary policy shock,

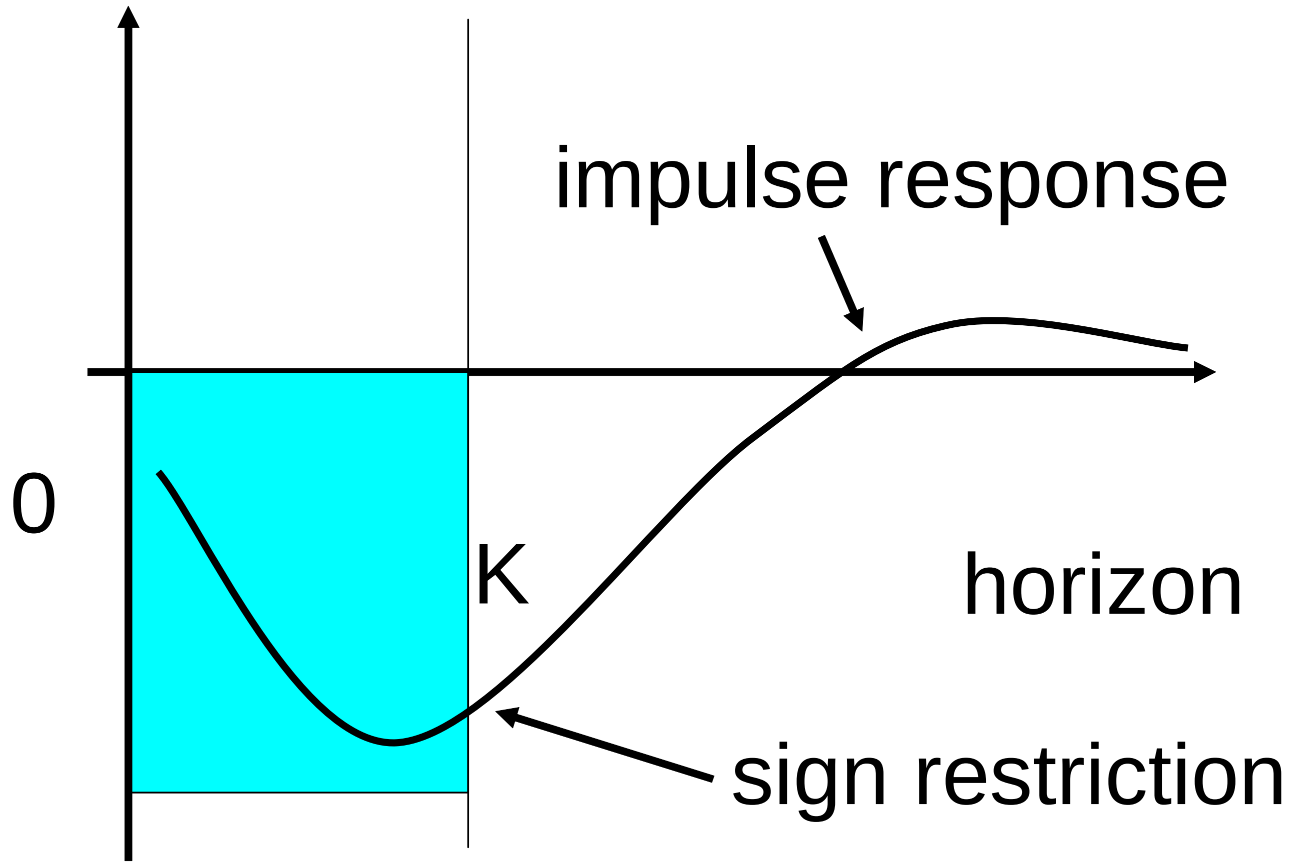
- Interest rates go up
- Inflation goes down
- Nonborrowed reserves go down
- real output goes down

Let's impose that (or a subset) in order to identify the monetary policy shock!

Sign restrictions on interest rates



Sign restrictions on prices



Impulse vectors

- Recall:

$$u_t = Av_t,\ AA' = \Sigma,\ Y_t = \sum_{s=0}^{\infty} C_s u_{t-s}$$

- Recall: let $a = A_i$ be the i-th column of A. The impulse response of all variables at horizon s to the i-th structural shock $v_t' = e_i$ is then given by

$$r_s = C_s a$$

- **Definition 1.** *The vector $a \in \mathbf{R}^m$ is called an* **impulse vector**, *iff there is some matrix A, so that $AA' = \Sigma$ and so that a is a column of A.*

- Let $\tilde{A}\tilde{A}' = \Sigma$ be some decomposition. Then, $a \in \mathbf{R}^m$ is an impulse vector, if and only if for some vector α of unit length, $a = A\alpha$.

Identification

- Task: find e.g. the monetary policy shock impulse vector.

- Identifying restrictions: the **responses** of a subset of the variables. This can mean
  - a particular **shape** of the response (Dwyer, 1997),
  - the **instantaneous response**, i.e. the sign of the entries in the impulse vector (Faust, 1998),
  - sign restrictions on the **correlations of impulse responses**, i.e. sign restrictions on the product of the impulse response of e.g. variable 1 at horizon j_1 and variable 2 at horizon j_2 (Canova-de Nicolo, 2002).
  - sign restrictions on the **impulse response for horizons** $k = 0, \ldots, K$ after the shock (Uhlig, 2005),

- Additional restrictions (long-run neutrality, some partial ordering) can be imposed without problem.

Imposing Sign Restrictions: A Bayesian Approach

- Form a prior for the reduced-form VAR. Using the data, form the posterior.
- Take a draw (B, Σ) from the posterior. Calculate the Cholesky decomposition $\Sigma = AA'$.
- Take a draw α from the unit sphere (dimensions = number of variables). Calculate $a = A\alpha$.
- Calculate the impulse responses to a. If they satisfy the sign restrictions, keep the joint draw (B, Σ, a), otherwise discard.
- Given the draws kept, calculate statistics of interest.

Summary of the Methodology

- Sign restrictions offer a new way of identifying shocks in a VAR.
- They can replace a hidden "specification search" by explicit identification.
- They are useful for solving problems of traditional identification.
- With Bayesian VARs, they are easy to implement.