recall Thm 3.5

if $v_1, \ldots, v_n$ is a basis of V and $T: V \to W$ is linear

then the values of $Tv_1, \ldots, Tv_n$ determine the values

of T on arbitrary vectors in V.

$Tv_1, \ldots, Tv_n$ 决定了 Tv $\forall v \in V$

Def matrix

$$A = \begin{pmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & & \vdots \\ A_{m1} & \cdots & A_{mn} \end{pmatrix}$$

3.32 Def Suppose $T \in \mathcal{L}(V, W)$ and

$v_1, \ldots, v_n$ is a basis of V and

$w_1, \ldots, w_n$ is a basis of W.

The matrix of T with respect to these bases

is the $m \times n$ matrix $\mathcal{M}(T)$

whose entries A_{jk} are defined by

$$Tv_k = A_{1k} w_1 + \cdots + A_{mk} w_m$$

insight: $Tv_k \in W$, W中任一向量可由W的一个基线性表出.

(represent as a linear combination)

3.33 Example

Suppose $T \in \mathcal{L}(\mathbb{F}^2, \mathbb{F}^3)$ is defined by $T(x, y) = (x+3y,\ 2x+5y,\ 7x+9y)$

Find the matrix of T with respect to the standard bases of $\mathbb{F}^2$ and $\mathbb{F}^3$

Solution

$$\underset{e_1=(1,0)}{T(1,0)} = (\underset{A_{11}}{1},\ \underset{A_{21}}{2},\ \underset{A_{31}}{7})$$

$$\underset{e_2=(0,1)}{T(0,1)} = (\underset{A_{12}}{3},\ \underset{A_{22}}{5},\ \underset{A_{32}}{9})$$

$$\mathcal{M}(T) = \begin{pmatrix} 1 & 3 \\ 2 & 5 \\ 7 & 9 \end{pmatrix}$$