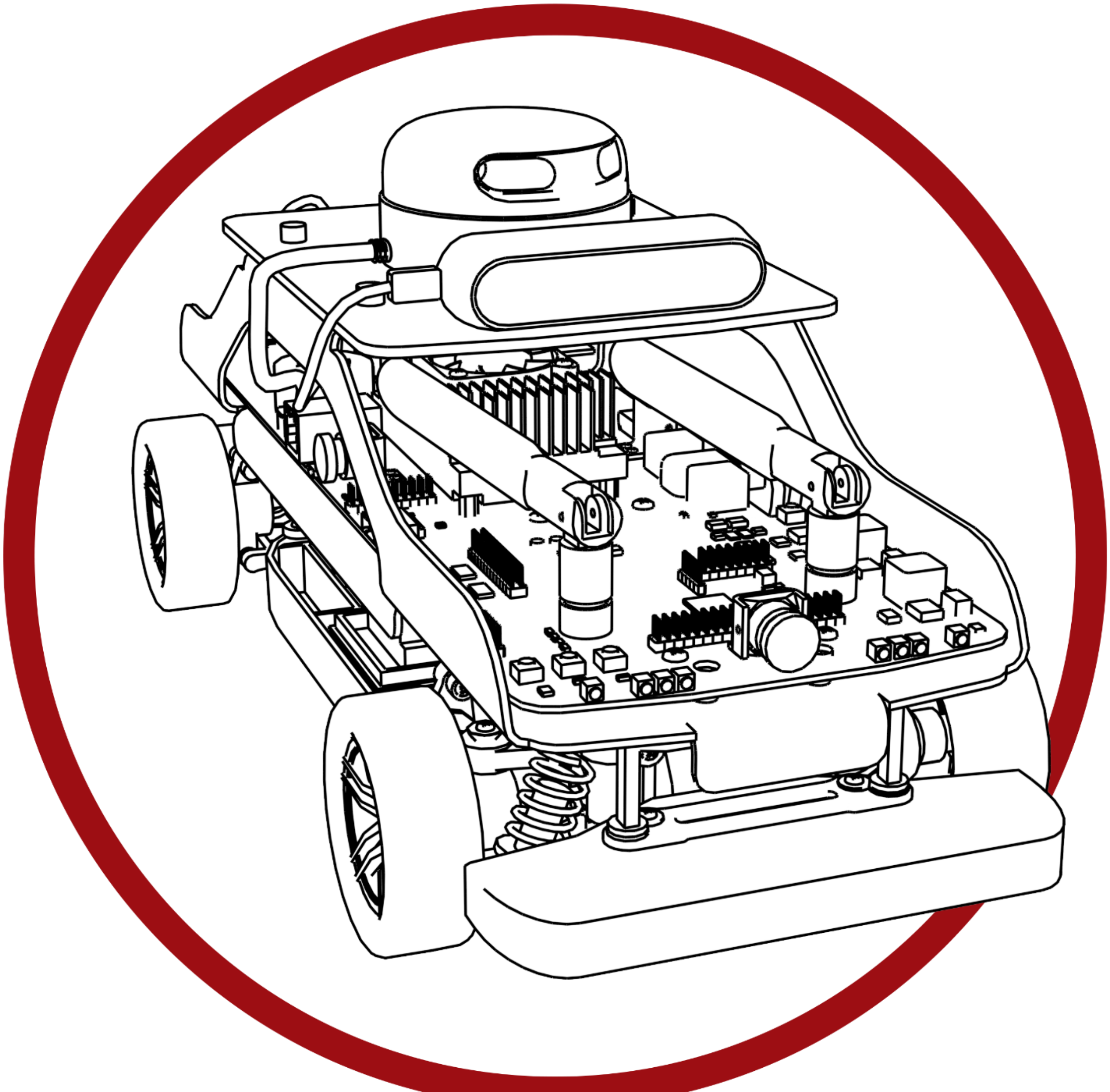
Self-Driving Car Research Studio



3 DOF Bicycle Model

V 1.1 (November 2020)

Table of Contents

Frames of reference

The Inertial frame **{I}** is a right-handed one used as a reference point for the QCar's pose (position and orientation). Localization techniques (AprilTags, optical localization, SLAM etc.) provide the pose of the vehicle in this frame. The Body frame **{B}** is attached to the vehicle, and rotates/translates with the vehicle. The frames are displayed in Figure 1 below. Depending on the steering command δ, the vehicle turns about the turning point **{O}** with a turning radius R.

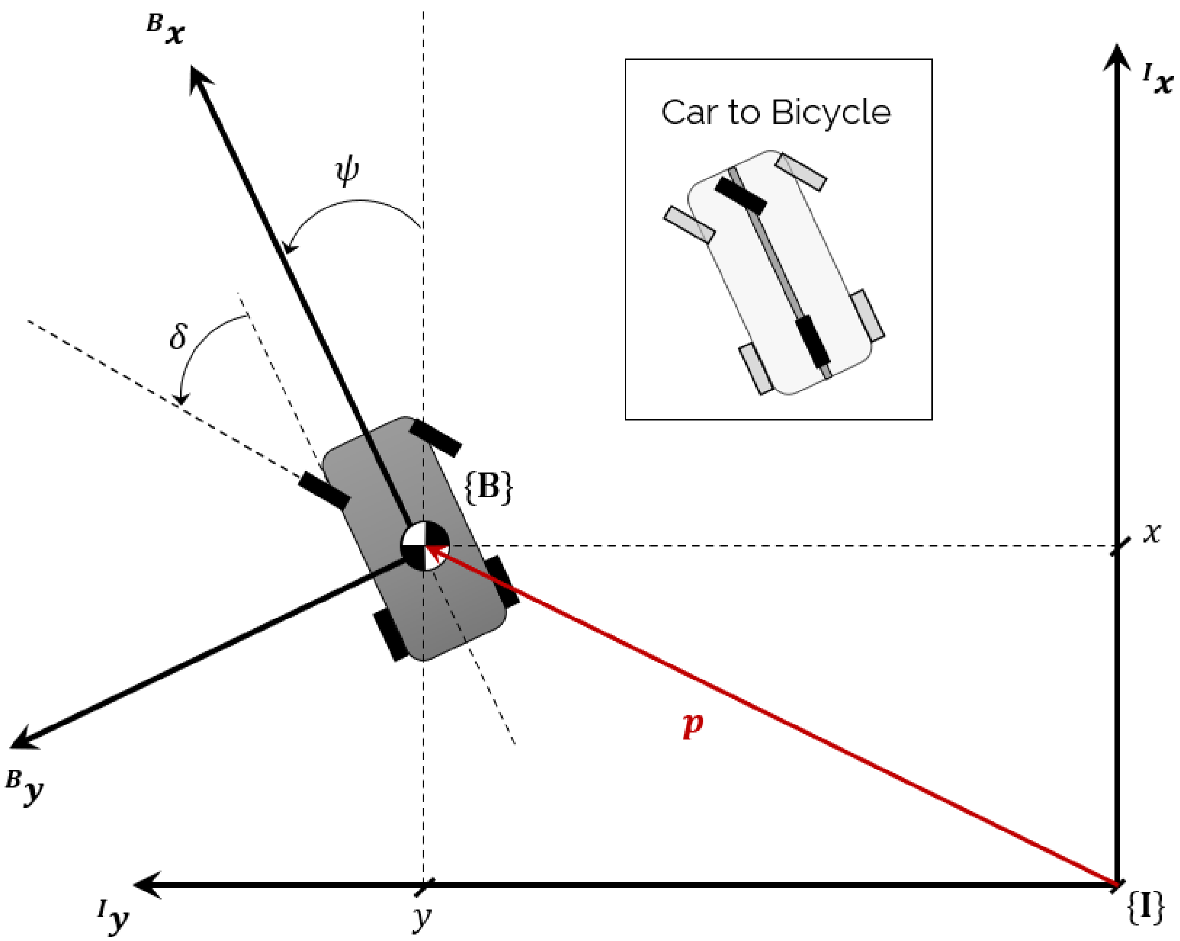


Figure 1. QCar frames of reference

Bicycle Model

The pose of the vehicle as shown in Figure 1 in the inertial frame is described by,

$$^{\mathbf{I}}\mathbf{p} = \begin{bmatrix} x \\ y \\ \psi \end{bmatrix} \tag{1}$$

Where x and y represent the planar position and ψ is the yaw orientation. In this simplified model, the height z as well as the roll ϕ and pitch θ are considered to be 0. Consider the bicycle model corresponding to the car as shown in Figure 2.

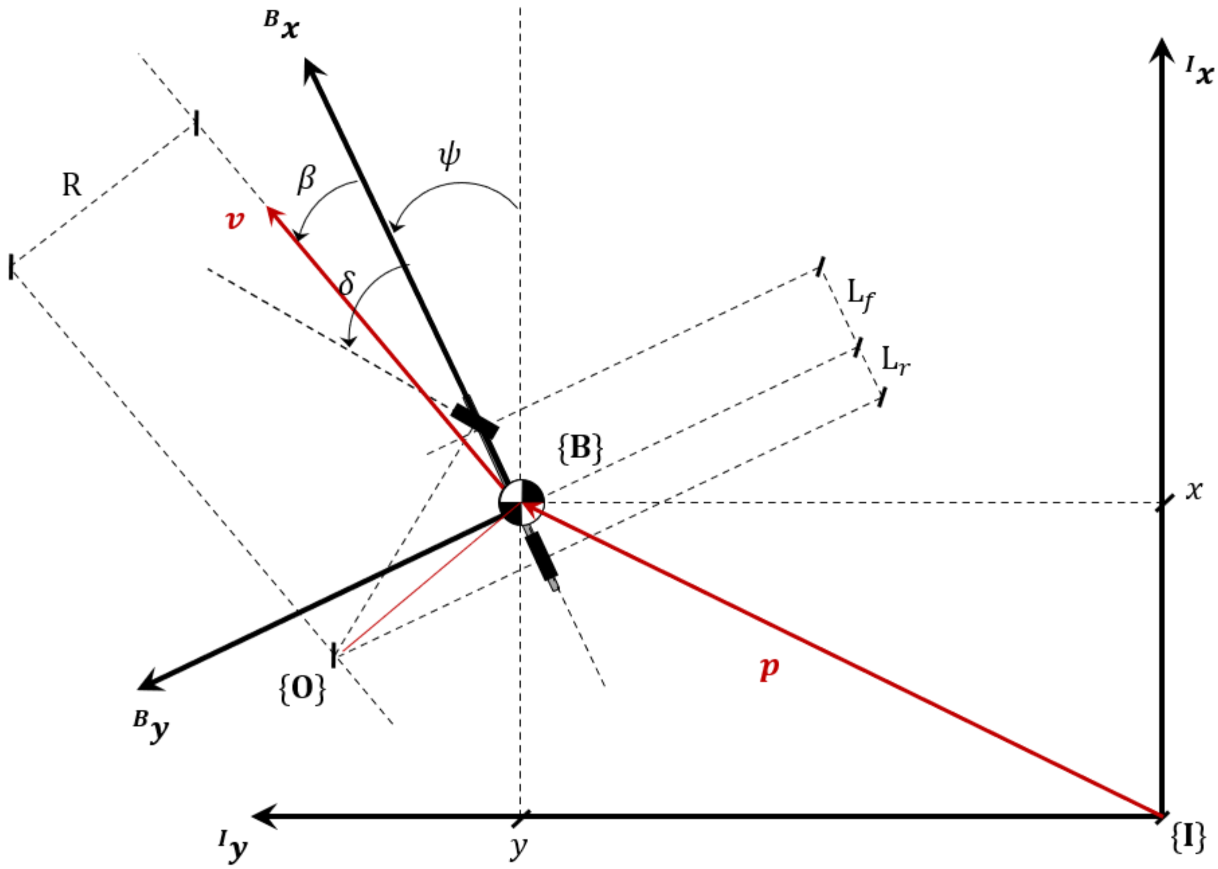


Figure 2. QCar frames of reference

In the body frame {B}, the steering angle δ represents the steering command, and the sideslip angle β represents the orientation of the velocity $^{\mathbf{I}}\mathbf{v}$ vector. The sideslip angle β can be derived by analyzing the triangle LNO as shown in Figure 3.

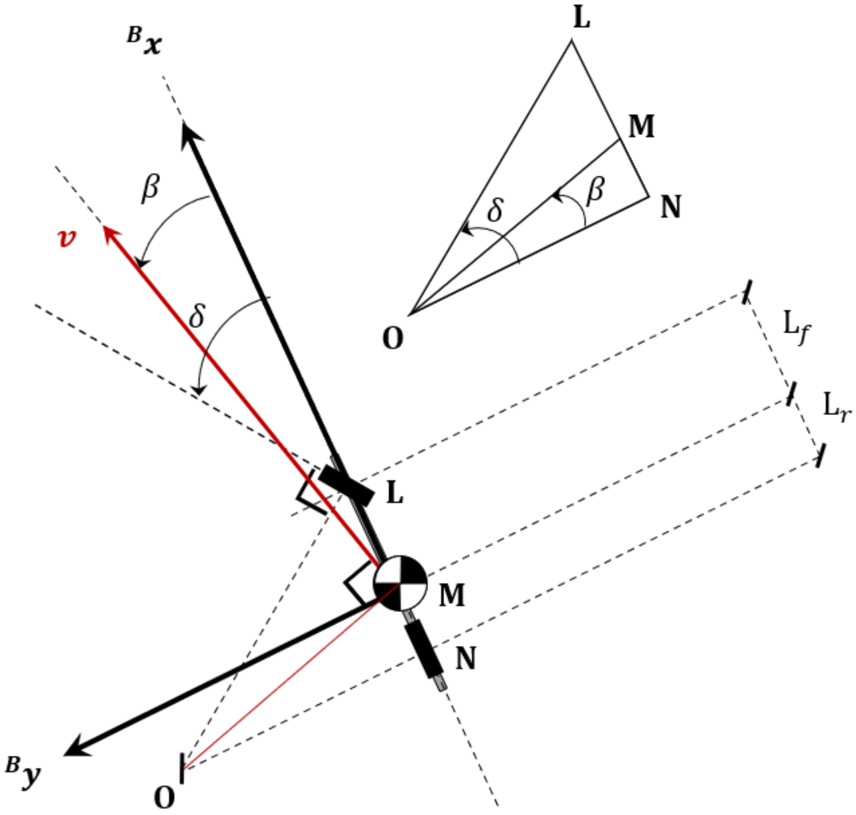


Figure 3. sideslip angle derivation

In triangle LNO, we get,

$$\tan\delta = \frac{LN}{ON}$$
$$\tan\beta = \frac{MN}{ON} \tag{2}$$

Combining the two equations yields,

$$\frac{1}{ON} = \frac{\tan\delta}{LN} = \frac{\tan\beta}{MN} \tag{3}$$

Replacing the lengths and isolating for β yields

$$\beta = \tan^{-1}\left(\frac{L_r}{L_r + L_f}\tan\delta\right) \tag{4}$$

In triangle LNO, the turning radius R is represented by the length OM as,

$$R = OM = \frac{ON}{\cos\beta} = \frac{\frac{LN}{\tan\delta}}{\cos\beta} = \frac{L_f + L_r}{\cos\beta \quad \tan\delta} \tag{3}$$

Given the measured speed v of the vehicle (from encoder speed), and the turning radius R, the angular rate of the vehicle is given by

$$\dot{\psi} = \frac{v}{R} \tag{4}$$

In addition, the velocity of the vehicle in inertial frame is then,

$$^{\mathbf{I}}\mathbf{v} = \begin{bmatrix} v\cos(\beta + \psi) \\ v\sin(\beta + \psi) \\ \dot{\psi} \end{bmatrix} \tag{5}$$

Keep in mind that this poses a problem. Equation 5 requires the yaw angle ψ, which is not provided by equation 4. Simply integrating this will lead to errors due to integration drift. With localization techniques such as April tags localization, LIDAR based SLAM or optical localization providing the pose $^{\mathbf{I}}\mathbf{p}$, a complementary filter can be used to combine them and the current yaw estimate can be fed back. See the Complementary Filters support documentation for more information.

Application to the QCar

Given the parameters of the QCar, the minimum turning radius can be estimated as follows. Referring to the **System Hardware** module of the QCar **User Guide**, the maximum steering angle for the QCar is about 30˚ and the wheelbase is 0.256m. If we assume the centre of gravity of the QCar is close to the centre of the vehicle chassis such that $L_f = L_r$, then the sideslip angle β is estimated to be:

$$\beta = \tan^{-1}\left(\frac{1}{2}\tan 30^\circ\right) = 16.1^\circ$$

The corresponding turning radius is estimated to be:

$$R = \frac{0.256\text{m}}{\cos 16.1^\circ \quad \tan 30^\circ} \approx 0.462\text{m}$$