A Benchmark Suite for Designing Combinational Logic Circuits via Metaheuristics

Lucas Augusto Müller de Souza, José Eduardo H. da Silva,
Luciano Jerez Chaves, and Heder Soares Bernardino.

This supplementary material presents additional tabular results that were obtained by Cartesian Genetic Programming (CGP) when solving the proposed benchmark problems.

The goal of the proposed benchmark is to optimize the number of transistors of the final solution. However, when the metaheuristic is initialized with a non-feasible solution (like the random initialization of the population for the "-R" methods), it is also possible to evaluate the performance of the algorithms during the search for the first feasible solution. This is shown in tables 1 to 3, that present the information about the number of evaluations, the number of transistors, and the size and depth circuit-complexity metrics of the first feasible solution obtained by SAM-R and SG-R algorithms. Note that SAM-E and PM-E algorithms are already initialized with the baseline feasible solution.

The remaining tables (4 to 12) show the information about the final solution obtained by the four search methods. This includes the size and depth circuit-complexity metrics, the CPU execution time, and the p-values of all algorithms for each problem. Information about the number of transistors, the success rate, and the relative reduction for all algorithms are available in tables 3 to 5 in the article.

In all tables, the best results are highlighted in boldface, except for the p-values, where the boldface indicates the results with a statistical difference. Missing information (-) for SAM-R and SG-R algorithms indicates that the method didn't obtain a feasible solution for the respective problem.

Table 1: The number of objective function evaluations to find the first feasible solution for the SAM-R and SG-R methods. This table also shows the percentage of the evaluation budged used to find the first feasible solution and the p-values of the algorithms. It is possible to see that, for 16 of the 22 problems with at least one feasible solution, the SG-R method has the lowest number of evaluations (both in the median and mean) required to find the first feasible solution.

Problem	Method	Objective Function Evaluations							% eval.	p-value
		Best	Q1	Median	Q3	Worst	Mean	Stdev		
C17	SAM-R	**541.00**	**1889.00**	**2373.00**	3733.00	**15385.00**	**3302.76**	3.08E+03	**0.03**	1.00E-00
	SG-R	1221.00	2029.00	3005.00	**3665.00**	15833.00	3680.04	**2.94E+03**	0.73	
cm42a	SAM-R	**14977.00**	31761.00	43229.00	53877.00	94449.00	45702.76	2.02E+04	0.36	2.88E-01
	SG-R	15237.00	**23301.00**	**31013.00**	**39785.00**	**74321.00**	**34081.00**	**1.33E+04**	**0.27**	
cm82a	SAM-R	4213.00	**11377.00**	**20085.00**	32933.00	**111697.00**	**28149.16**	**2.46E+04**	**0.35**	1.00E-00
	SG-R	**3345.00**	13125.00	21253.00	**32485.00**	132805.00	30386.28	2.94E+04	0.38	
cm138a	SAM-R	91677.00	210009.00	265941.00	405473.00	798257.00	333323.40	1.86E+05	1.74	9.17E-01
	SG-R	**47021.00**	**161609.00**	**234853.00**	**324709.00**	**444297.00**	**237318.12**	**1.15E+05**	**1.24**	
decod	SAM-R	393377.00	572217.00	683077.00	952613.00	1300929.00	779161.48	2.58E+05	6.49	**4.79E-02**
	SG-R	**174473.00**	**403597.00**	**550285.00**	**669189.00**	**880637.00**	**535477.16**	**1.77E+05**	**4.46**	
f51m	SAM-R	**436065.00**	2314637.00	3343513.00	5421109.00	**11605525.00**	4127082.60	2.94E+06	21.49	1.00E-00
	SG-R	869073.00	**1347501.00**	**2047477.00**	**3715721.00**	13699469.00	**2990684.68**	**2.69E+06**	**15.58**	
majority	SAM-R	**557.00**	**3137.00**	**5457.00**	**9129.00**	24053.00	**6174.12**	**4.90E+03**	**0.08**	1.00E-00
	SG-R	885.00	3853.00	5661.00	12557.00	**19377.00**	7605.96	5.36E+03	0.10	
z4ml	SA M-R	**21129.00**	122533.00	200429.00	303501.00	482493.00	221805.32	1.23E+05	1.32	1.00E-00
	SG-R	37181.00	**70309.00**	**163353.00**	**258665.00**	**463189.00**	**172815.08**	**1.12E+05**	**1.03**	
9symml	SAM-R	**260965.00**	**555709.00**	1274929.00	2861877.00	9904525.00	1894035.72	2.07E+06	8.77	1.00E-00
	SG-R	287257.00	700026.00	**1028541.00**	**2511969.00**	**4211033.00**	**1624326.17**	**1.23E+06**	**7.52**	
alu2	SAM-R	**7789925.00**	**9587184.00**	**11384443.00**	**13181702.00**	14978961.00	**11384443.00**	3.59E+06	**71.15**	1.00E-00
	SG-R	14686325.00	14686325.00	14686325.00	14686325.00	**14686325.00**	14686325.00	**0.00E+00**	91.79	
alu4	SAM-R	4586621.00	**9375589.00**	**11991005.00**	18784375.00	29996997.00	**13909170.74**	6.81E+06	**41.40**	1.00E-00
	SG-R	**3992309.00**	11196071.00	15097929.00	**17484821.00**	**29470241.00**	15349400.37	**6.46E+06**	45.68	
cm85a	SAM-R	243661.00	377333.00	541665.00	1117569.00	1692469.00	717483.08	4.52E+05	4.08	8.99E-01
	SG-R	**115893.00**	**280477.00**	**431705.00**	**574533.00**	**948761.00**	**451483.40**	**2.04E+05**	**2.57**	
cm151a	SAM-R	37385.00	79497.00	101497.00	151505.00	**226145.00**	112819.88	**5.15E+04**	0.39	5.13E-01
	SG-R	**27673.00**	**64197.00**	**81129.00**	**93809.00**	313189.00	**86299.08**	5.39E+04	**0.30**	
cm162a	SAM-R	**107533.00**	**313641.00**	553757.00	**855689.00**	2164481.00	685180.68	4.95E+05	1.53	1.00E-00
	SG-R	108257.00	432441.00	**551581.00**	877985.00	**1610085.00**	**668600.52**	**3.73E+05**	**1.49**	
cu	SAM-R	1370425.00	2865457.00	3867989.00	7267673.00	16369765.00	5455837.48	3.73E+06	12.19	**2.67E-03**
	SG-R	**669965.00**	**1366277.00**	**2297385.00**	**2626785.00**	**8138933.00**	**2352364.52**	**1.57E+06**	**5.25**	
x2	SAM-R	576325.00	1407977.00	2473437.00	3398381.00	4854321.00	2445051.72	1.19E+06	10.19	2.41E-01
	SG-R	**296413.00**	**879709.00**	**1617541.00**	**2099813.00**	**3526977.00**	**1610590.28**	**8.64E+05**	**6.71**	
cc	SAM-R	1882389.00	2284765.00	3380073.00	5839385.00	8951901.00	4112475.24	2.11E+06	8.16	1.00E-00
	SG-R	**1298045.00**	**2126313.00**	**2933933.00**	**4230937.00**	**7358309.00**	**3279860.84**	**1.51E+06**	**6.51**	
cmb	SAM-R	-	-	-	-	-	-	-	-	-
	SG-R	-	-	-	-	-	-	-	-	
cordic	SAM-R	-	-	-	-	-	-	-	-	-
	SG-R	-	-	-	-	-	-	-	-	
frg1	SAM-R	1853877.00	**4076977.00**	5990665.00	**7069533.00**	17084045.00	6354018.44	3.48E+06	9.46	1.00E-00
	SG-R	**1775573.00**	4489533.00	**5703125.00**	8200213.00	**15155957.00**	**6231536.20**	**3.01E+06**	**9.27**	
pm1	SAM-R	**1791037.00**	**4032489.00**	9084849.00	**11387865.00**	**14881761.00**	**8185932.84**	**4.06E+06**	**21.32**	1.00E-00
	SG-R	4663713.00	7030979.00	**8821889.00**	12244145.00	33652349.00	10811699.67	6.46E+06	28.16	
sct	SAM-R	10387637.00	13780521.00	21271377.00	32387513.00	43430237.00	23233395.44	**1.00E+07**	50.95	1.00E-00
	SG-R	**4950309.00**	**11930974.00**	**17676179.00**	**27486698.00**	**36316169.00**	**19439902.45**	**1.00E+07**	**42.63**	
t481	SAM-R	50925.00	**123813.00**	**215753.00**	607685.00	1782545.00	423432.36	4.33E+05	1.65	1.00E-00
	SG-R	**32785.00**	138909.00	223365.00	**422949.00**	**1439421.00**	**341220.52**	**2.94E+05**	**1.33**	
tcon	SAM-R	86449.00	190485.00	287345.00	370529.00	652245.00	298886.12	1.49E+05	0.73	**3.18E-04**
	SG-R	**47813.00**	**90365.00**	**122233.00**	**168149.00**	**270469.00**	**135175.88**	**6.06E+04**	**0.33**	
vda	SAM-R	-	-	-	-	-	-	-	-	-
	SG-R	-	-	-	-	-	-	-	-	

Table 2: The number of transistors in the first feasible solution found by the SAM-R and SG-R methods, including the p-values of the algorithms. It is possible to see that, for 18 of the 22 problems with at least one feasible solution, the SAM-R method has the lowest median number of transistors in the first feasible solution.

Problem	**Method**	**First Feasible Solution**							**p-value**
		Best	**Q1**	**Median**	**Q3**	**Worst**	**Mean**	**Stdev**	
C17	SAM-R	**15.00**	33.00	47.00	62.00	83.00	48.92	1.72E+01	2.60E-01
	SG-R	20.00	**32.00**	**42.00**	**57.00**	**67.00**	**43.52**	**1.47E+01**	
cm42a	SAM-R	104.00	112.00	122.00	129.00	150.00	121.52	**1.07E+01**	2.36E-01
	SG-R	**94.00**	**109.00**	**115.00**	**127.00**	**149.00**	**117.84**	1.22E+01	
cm82a	SAM-R	48.00	**64.00**	**69.00**	**75.00**	**89.00**	**69.84**	**9.76E+00**	8.74E-02
	SG-R	**46.00**	68.00	76.00	87.00	116.00	76.84	1.61E+01	
cm138a	SAM-R	**76.00**	**88.00**	**102.00**	**113.00**	**129.00**	**102.52**	1.48E+01	7.40E-02
	SG-R	79.00	100.00	113.00	120.00	133.00	110.08	**1.32E+01**	
decod	SAM-R	**113.00**	**132.00**	**139.00**	**144.00**	**163.00**	**139.40**	1.25E+01	1.77E-01
	SG-R	125.00	132.00	146.00	151.00	170.00	143.68	**1.16E+01**	
f51m	SAM-R	**93.00**	**112.00**	**120.00**	**128.00**	**161.00**	**122.68**	1.65E+01	**8.53E-03**
	SG-R	111.00	128.00	134.00	143.00	164.00	134.28	**1.39E+01**	
majority	SAM-R	**22.00**	32.00	**36.00**	55.00	87.00	44.60	1.75E+01	3.72E-01
	SG-R	23.00	**30.00**	37.00	**47.00**	**66.00**	**38.96**	**1.15E+01**	
z4ml	SAM-R	67.00	**79.00**	**86.00**	**96.00**	**112.00**	**88.16**	**1.18E+01**	6.34E-01
	SG-R	**57.00**	81.00	88.00	103.00	124.00	89.88	1.62E+01	
9symml	SAM-R	**95.00**	**154.00**	**175.00**	251.00	**332.00**	**202.24**	6.96E+01	4.41E-01
	SG-R	118.00	170.50	207.00	**240.25**	341.00	210.50	**5.17E+01**	
alu2	SAM-R	**393.00**	**404.50**	**416.00**	**427.50**	**439.00**	**416.00**	2.30E+01	2.21E-01
	SG-R	489.00	489.00	489.00	489.00	489.00	489.00	**0.00E+00**	
alu4	SAM-R	**222.00**	**294.00**	**345.00**	**390.00**	**469.00**	**341.22**	**6.97E+01**	**4.29E-03**
	SG-R	268.00	375.00	408.00	477.50	549.00	416.11	7.89E+01	
cm85a	SAM-R	**70.00**	**119.00**	**141.00**	**176.00**	**298.00**	**151.96**	**5.15E+01**	**1.56E-03**
	SG-R	88.00	192.00	214.00	228.00	301.00	203.48	5.53E+01	
cm151a	SAM-R	**78.00**	**124.00**	**150.00**	195.00	**282.00**	159.00	5.03E+01	9.77E-01
	SG-R	84.00	124.00	159.00	**168.00**	294.00	**158.04**	**4.43E+01**	
cm162a	SAM-R	**110.00**	183.00	**209.00**	251.00	**304.00**	211.00	5.35E+01	1.00E+00
	SG-R	133.00	**181.00**	214.00	**242.00**	309.00	**209.40**	**4.41E+01**	
cu	SAM-R	**126.00**	**215.00**	**238.00**	**254.00**	**383.00**	**241.20**	**5.86E+01**	**4.34E-03**
	SG-R	192.00	238.00	284.00	359.00	406.00	300.88	6.82E+01	
x2	SAM-R	**155.00**	**190.00**	**212.00**	**251.00**	374.00	**224.80**	5.17E+01	**2.63E-02**
	SG-R	165.00	220.00	241.00	283.00	**350.00**	253.76	**4.86E+01**	
cc	SAM-R	**375.00**	**463.00**	**513.00**	**554.00**	**665.00**	**519.04**	**7.42E+01**	**5.67E-05**
	SG-R	410.00	553.00	644.00	705.00	827.00	637.72	9.79E+01	
cmb	SAM-R	-	-	-	-	-	-	-	-
	SG-R	-	-	-	-	-	-	-	
cordic	SAM-R	-	-	-	-	-	-	-	-
	SG-R	-	-	-	-	-	-	-	
frg1	SAM-R	**107.00**	139.00	158.00	176.00	**242.00**	160.36	**3.05E+01**	6.91E-01
	SG-R	108.00	**131.00**	**154.00**	**175.00**	242.00	**157.60**	3.37E+01	
pm1	SAM-R	**155.00**	**214.00**	**256.00**	**307.00**	**379.00**	**266.56**	**5.41E+01**	**4.10E-03**
	SG-R	201.00	284.75	315.50	355.50	496.00	328.92	6.55E+01	
sct	SAM-R	**308.00**	**364.25**	**423.00**	**458.00**	**597.00**	**423.11**	**7.84E+01**	**2.43E-03**
	SG-R	386.00	450.00	510.00	567.00	758.00	515.18	8.69E+01	
t481	SAM-R	**60.00**	107.00	147.00	**193.00**	413.00	**159.60**	**7.21E+01**	8.08E-01
	SG-R	62.00	**96.00**	**128.00**	215.00	**378.00**	162.08	8.51E+01	
tcon	SAM-R	**345.00**	**450.00**	**478.00**	**545.00**	**660.00**	**501.40**	7.71E+01	**5.21E-05**
	SG-R	469.00	555.00	613.00	643.00	761.00	603.36	**6.65E+01**	
vda	SAM-R	-	-	-	-	-	-	-	-
	SG-R	-	-	-	-	-	-	-	

Table 3: The mean number of each type of logic gate, the total number of gates, and the circuit depth of the first feasible solution obtained by SAM-R and SG-R methods.

Problem	Method	First Feasible Solution								
		AND	**OR**	**NOT**	**NAND**	**NOR**	**XOR**	**XNOR**	**Total**	**Depth**
C17	SAM-R	3.84	3.92	3.20	**3.84**	**2.84**	2.72	2.88	23.24	9.44
	SG-R	**3.52**	**3.24**	**3.00**	4.00	3.08	**2.32**	**2.24**	**21.40**	**9.08**
cm42a	SAM-R	**6.24**	12.72	**5.68**	**10.08**	**7.00**	7.32	7.20	56.24	**11.72**
	SG-R	6.52	**10.24**	6.36	11.52	7.88	**7.20**	**6.36**	**56.08**	12.16
cm82a	SAM-R	**3.12**	**3.20**	**3.56**	**3.76**	**3.04**	**5.72**	**6.48**	**28.88**	**10.28**
	SG-R	3.44	3.84	4.60	3.80	3.52	6.56	6.72	32.48	10.48
cm138a	SAM-R	**6.52**	10.48	**4.80**	**10.12**	8.48	**4.84**	**5.12**	**50.36**	12.32
	SG-R	7.00	**10.28**	5.48	11.12	**7.48**	5.44	6.00	52.80	**12.20**
decod	SAM-R	**14.72**	**7.56**	**5.56**	**9.60**	15.68	7.84	**7.72**	**68.68**	**12.12**
	SG-R	15.80	7.96	6.84	9.68	**14.56**	**7.64**	8.12	70.60	12.52
f51m	SAM-R	**3.84**	**4.20**	**5.80**	5.04	**3.84**	**12.80**	**12.12**	**47.64**	**12.52**
	SG-R	6.04	5.88	6.24	**4.48**	4.68	12.96	12.92	53.20	12.80
majority	SAM-R	3.12	**3.28**	2.36	3.48	**2.92**	2.52	3.00	20.68	9.80
	SG-R	**3.08**	3.40	**1.72**	**2.64**	3.20	**1.96**	**2.48**	**18.48**	**9.20**
z4ml	SAM-R	4.36	**3.84**	5.16	**3.60**	4.24	**8.36**	**7.52**	37.08	11.24
	SG-R	**3.52**	4.40	**4.88**	**3.60**	**3.20**	9.08	7.88	**36.56**	**10.92**
9symml	SAM-R	12.88	**12.32**	**9.20**	**12.44**	12.40	15.76	**14.52**	**89.52**	**17.72**
	SG-R	**12.29**	13.62	9.38	14.29	**11.29**	**15.58**	15.67	92.12	18.33
alu2	SAM-R	**28.50**	**20.00**	**17.50**	**31.50**	31.50	**31.00**	**28.50**	**188.50**	**20.50**
	SG-R	36.00	27.00	23.00	35.00	**28.00**	38.00	32.00	219.00	21.00
alu4	SAM-R	**22.13**	**21.61**	**24.09**	**24.00**	**22.26**	**23.22**	**22.43**	**159.74**	**18.65**
	SG-R	26.42	26.42	28.53	29.58	28.32	28.32	27.37	194.95	20.37
cm85a	SAM-R	**11.16**	**11.04**	**11.36**	**10.24**	**9.20**	**9.32**	**9.64**	**71.96**	**15.96**
	SG-R	13.56	15.84	14.36	15.52	14.16	12.00	12.28	97.72	16.96
cm151a	SAM-R	11.96	12.96	9.88	11.92	**12.88**	**8.48**	9.28	77.36	16.64
	SG-R	**11.04**	**12.72**	**9.40**	**11.80**	13.16	9.24	**9.16**	**76.52**	**16.60**
cm162a	SAM-R	**16.84**	**13.64**	**14.96**	16.80	**12.84**	12.80	12.56	**100.44**	**15.20**
	SG-R	**16.84**	13.76	16.48	**15.80**	14.52	**12.48**	**12.04**	101.92	16.20
cu	SAM-R	**19.80**	**18.48**	**19.48**	**17.24**	**19.88**	**13.04**	**12.92**	**120.84**	**16.68**
	SG-R	24.84	21.88	21.40	20.12	23.48	17.20	17.68	146.60	17.48
x2	SAM-R	**16.44**	18.56	**15.84**	**17.20**	**15.52**	**13.36**	**12.24**	**109.16**	17.32
	SG-R	18.96	**18.48**	17.80	19.96	16.56	15.40	14.60	121.76	**16.80**
cc	SAM-R	**39.68**	**32.28**	**39.92**	**36.56**	**37.96**	**31.24**	**32.60**	**250.24**	**19.08**
	SG-R	48.92	41.68	48.60	41.40	43.52	41.28	39.44	304.84	21.04
cmb	SAM-R	-	-	-	-	-	-	-	-	-
	SG-R	-	-	-	-	-	-	-	-	-
cordic	SAM-R	-	-	-	-	-	-	-	-	-
	SG-R	-	-	-	-	-	-	-	-	-
frg1	SAM-R	20.80	**13.20**	12.16	17.68	13.56	4.56	**4.40**	86.36	16.32
	SG-R	**20.36**	13.80	**11.64**	**16.96**	**12.00**	**4.28**	4.72	**83.76**	**15.84**
pm1	SAM-R	**21.92**	**18.28**	**24.64**	**21.68**	**18.48**	**14.88**	**13.76**	**133.64**	**16.52**
	SG-R	24.75	22.88	27.00	26.46	24.21	16.96	19.67	161.92	17.12
sct	SAM-R	**30.17**	**29.44**	**33.61**	**30.00**	**28.50**	**25.11**	**26.61**	**203.44**	**19.94**
	SG-R	33.82	35.59	41.73	35.77	33.73	33.91	31.91	246.45	21.45
t481	SAM-R	**9.36**	**8.48**	**12.88**	10.56	**9.88**	12.12	10.92	**74.20**	15.20
	SG-R	9.96	10.08	13.60	**9.80**	10.00	**11.92**	**10.76**	76.12	**14.96**
tcon	SAM-R	**33.80**	**32.60**	**37.24**	**34.68**	**34.32**	**32.48**	**32.56**	**237.68**	**19.28**
	SG-R	43.40	39.20	46.52	42.36	39.96	38.16	38.12	287.72	20.56
vda	SAM-R	-	-	-	-	-	-	-	-	-
	SG-R	-	-	-	-	-	-	-	-	-

Table 4: The mean number of each type of logic gate, the total number of gates, and the circuit depth of the final feasible solution obtained by each method for the problems in group 1.

Problem	Method	Final Feasible Solution								
		AND	**OR**	**NOT**	**NAND**	**NOR**	**XOR**	**XNOR**	**Total**	**Depth**
C17	SAM-R	**1.68**	0.32	0.08	1.24	3.24	**0.00**	**0.00**	**6.56**	4.96
	SG-R	1.80	**0.24**	**0.20**	0.92	3.56	0.08	**0.00**	6.80	5.16
	SAM-E	2.12	0.48	**0.20**	**0.00**	4.20	**0.00**	**0.00**	7.00	5.28
	PM-E	2.36	1.00	0.40	0.28	**2.72**	**0.00**	**0.00**	6.76	**4.80**
cm42a	SAM-R	0.76	3.88	0.84	8.12	6.36	0.20	0.04	**20.20**	**6.28**
	SG-R	1.32	**3.56**	**0.72**	8.20	5.96	0.40	0.28	20.44	6.68
	SAM-E	**0.40**	8.40	1.76	5.24	5.72	**0.00**	**0.00**	21.52	6.44
	PM-E	0.68	8.80	2.36	**4.56**	**5.32**	**0.00**	**0.00**	21.72	6.48
cm82a	SAM-R	**0.92**	**0.56**	1.00	1.16	**3.48**	4.32	1.16	12.60	7.88
	SG-R	1.08	1.00	1.08	1.24	3.60	4.12	0.92	13.04	**7.68**
	SAM-E	1.92	1.48	0.92	0.04	6.28	**2.32**	0.52	13.48	8.72
	PM-E	1.96	1.16	**0.64**	**0.00**	4.24	3.08	**0.48**	**11.56**	8.12
cm138a	SAM-R	1.48	**3.36**	2.00	6.88	**5.68**	0.20	0.16	19.76	8.20
	SG-R	1.48	**3.36**	**1.68**	6.84	6.04	0.20	0.12	**19.72**	**7.88**
	SAM-E	0.32	11.84	2.92	**2.80**	5.96	0.04	**0.00**	23.88	8.44
	PM-E	**0.24**	11.52	3.32	3.04	**5.68**	**0.00**	**0.00**	23.80	8.24
decod	SAM-R	**5.68**	2.56	3.12	4.16	18.88	0.72	1.04	**36.16**	8.36
	SG-R	7.68	2.84	**2.68**	3.84	18.44	0.80	1.12	37.40	9.60
	SAM-E	28.96	**0.12**	3.12	0.40	5.92	**0.00**	**0.00**	38.52	7.64
	PM-E	30.04	0.28	3.24	**0.16**	**4.08**	0.16	**0.00**	37.96	**7.56**
f51m	SAM-R	**2.80**	**2.96**	**3.72**	3.68	4.36	14.00	5.16	**36.68**	**11.60**
	SG-R	4.24	4.28	3.92	3.24	5.48	13.52	6.88	41.56	12.00
	SAM-E	133.80	64.84	8.12	0.08	8.28	0.60	0.52	216.24	23.80
	PM-E	246.00	69.00	8.00	**0.00**	**0.00**	**0.00**	**0.00**	323.00	29.00
majority	SAM-R	**0.76**	1.20	0.08	1.72	4.48	0.28	0.04	8.56	**5.76**
	SG-R	1.28	**1.16**	0.32	1.20	4.80	0.20	0.04	9.00	6.12
	SAM-E	2.92	1.96	0.08	0.60	2.96	**0.00**	**0.00**	**8.52**	6.16
	PM-E	4.28	3.04	**0.04**	**0.40**	**1.44**	**0.00**	**0.00**	9.20	6.48
z4ml	SAM-R	1.80	**1.36**	1.84	1.84	4.32	7.28	2.52	20.96	**9.76**
	SG-R	**1.72**	1.56	**1.60**	1.52	4.04	8.08	2.16	**20.68**	10.12
	SAM-E	31.12	18.04	4.56	0.16	6.56	0.92	0.48	61.84	17.40
	PM-E	193.00	55.00	7.00	**0.00**	**0.00**	**0.00**	**0.00**	255.00	33.00

Table 5: The mean number of each type of logic gate, the total number of gates, and the circuit depth of the final feasible solution obtained by each method for the problems in group 2.

Problem	Method	Final Feasible Solution								
		AND	**OR**	**NOT**	**NAND**	**NOR**	**XOR**	**XNOR**	**Total**	**Depth**
9symml	SAM-R	8.00	**7.92**	**3.96**	7.96	12.68	11.88	7.20	59.60	**16.12**
	SG-R	**7.79**	8.71	4.25	8.79	11.42	10.33	7.62	**58.92**	16.17
	SAM-E	73.64	46.20	8.20	0.32	13.64	0.20	0.04	142.24	22.60
	PM-E	430.00	85.00	9.00	**0.00**	**0.00**	**0.00**	**0.00**	524.00	92.00
alu2	SAM-R	**21.00**	**21.00**	15.00	24.00	31.00	24.50	19.00	**155.50**	20.50
	SG-R	29.00	30.00	21.00	34.00	25.00	24.00	22.00	185.00	**19.00**
	SAM-E	218.04	98.56	13.96	0.16	9.00	0.04	0.04	339.80	64.08
	PM-E	750.00	140.00	**10.00**	**0.00**	**0.00**	**0.00**	**0.00**	900.00	74.00
alu4	SAM-R	-	-	-	-	-	-	-	-	-
	SG-R	-	-	-	-	-	-	-	-	-
	SAM-E	**3657.60**	659.44	26.12	4.80	9.08	0.96	0.72	**4358.72**	451.60
	PM-E	4521.00	**640.00**	**14.00**	**0.00**	**0.00**	**0.00**	**0.00**	5175.00	**200.00**
cm85a	SAM-R	4.24	**3.88**	3.24	4.04	9.96	2.80	1.48	**29.64**	**12.32**
	SG-R	**3.16**	4.56	3.80	5.20	9.76	2.68	1.60	30.76	12.44
	SAM-E	18.96	10.04	**3.20**	0.04	7.88	**0.00**	**0.00**	40.12	18.00
	PM-E	256.00	45.00	8.00	**0.00**	**0.00**	**0.00**	**0.00**	309.00	25.00
cm151a	SAM-R	**3.48**	2.92	2.84	4.28	13.96	1.60	0.76	29.84	11.88
	SG-R	4.52	**2.88**	2.32	3.76	13.72	1.32	0.60	**29.12**	**11.68**
	SAM-E	14.72	5.08	1.64	**0.36**	10.04	**0.00**	**0.00**	31.84	12.00
	PM-E	16.04	4.80	**1.56**	1.16	**7.88**	**0.00**	**0.00**	31.44	12.20
cm162a	SAM-R	6.76	**3.88**	4.60	5.84	11.76	3.48	1.56	37.88	**11.12**
	SG-R	**6.28**	4.96	4.80	6.16	11.04	3.48	1.12	**37.84**	11.60
	SAM-E	12.08	12.12	**4.04**	**0.52**	11.20	**0.48**	0.04	40.48	12.24
	PM-E	12.64	12.28	4.32	0.88	**10.40**	0.68	**0.00**	41.20	12.52
cu	SAM-R	**7.20**	4.84	4.92	4.44	17.00	2.36	1.08	**41.84**	**10.96**
	SG-R	8.52	**4.12**	5.12	5.68	15.76	2.36	0.76	42.32	11.48
	SAM-E	19.72	5.80	**3.76**	**0.08**	16.80	0.04	**0.00**	46.20	12.36
	PM-E	21.60	6.04	4.00	0.84	**14.40**	**0.00**	**0.00**	46.88	12.84
x2	SAM-R	**4.44**	6.44	4.36	6.72	11.28	1.96	1.52	**36.72**	11.80
	SG-R	6.04	**5.04**	4.72	6.72	12.12	2.00	0.84	37.48	**11.64**
	SAM-E	9.08	13.52	**3.88**	**1.96**	10.24	**0.24**	**0.04**	38.96	12.72
	PM-E	8.48	13.52	4.16	**1.96**	**9.44**	0.60	0.08	38.24	12.92

Table 6: The mean number of each type of logic gate, the total number of gates, and the circuit depth of the final feasible solution obtained by each method for the problems in group 3.

Problem	Method	Final Feasible Solution								
		AND	**OR**	**NOT**	**NAND**	**NOR**	**XOR**	**XNOR**	**Total**	**Depth**
cc	SAM-R	**10.40**	**5.84**	9.96	5.48	16.88	2.40	2.32	53.28	9.44
	SG-R	11.40	6.40	12.32	5.32	16.16	2.36	2.44	56.40	9.80
	SAM-E	28.68	11.12	6.96	**0.20**	4.88	**0.00**	**0.00**	**51.84**	8.88
	PM-E	33.60	12.44	**6.92**	0.88	**2.68**	0.20	**0.00**	56.72	**8.76**
cmb	SAM-R	-	-	-	-	-	-	-	-	-
	SG-R	-	-	-	-	-	-	-	-	-
	SAM-E	11.92	16.72	5.08	1.48	5.60	0.20	0.00	41.00	13.16
	PM-E	11.88	15.92	4.76	1.64	4.48	0.32	0.00	39.00	11.76
cordic	SAM-R	-	-	-	-	-	-	-	-	-
	SG-R	-	-	-	-	-	-	-	-	-
	SAM-E	**5908.56**	**680.92**	57.32	18.12	32.24	6.36	4.36	**6707.88**	**537.60**
	PM-E	12911.00	912.00	**23.00**	**0.00**	**0.00**	**0.00**	**0.00**	13846.00	788.00
frg1	SAM-R	**17.08**	**7.52**	4.48	12.44	13.32	0.80	0.48	56.12	13.76
	SG-R	17.16	8.28	**4.36**	12.56	12.48	0.72	0.24	**55.80**	**13.20**
	SAM-E	107.88	53.00	15.84	0.28	16.16	**0.00**	**0.00**	193.16	25.00
	PM-E	673.00	116.00	27.00	**0.00**	**0.00**	**0.00**	**0.00**	816.00	116.00
pm1	SAM-R	7.64	**3.20**	**8.96**	6.16	8.64	1.60	0.36	**36.56**	**9.36**
	SG-R	**7.58**	4.29	10.25	5.38	9.67	1.25	0.29	38.71	10.00
	SAM-E	17.44	14.20	14.68	1.56	3.32	0.08	**0.00**	51.28	11.04
	PM-E	876.00	158.00	16.00	**0.00**	**0.00**	**0.00**	**0.00**	1050.00	45.00
sct	SAM-R	7.72	**6.39**	11.50	7.00	13.44	5.44	3.56	**55.06**	**14.44**
	SG-R	**7.45**	7.73	14.09	5.82	13.68	5.45	3.82	58.05	15.14
	SAM-E	21.52	18.92	**7.28**	0.12	16.56	0.48	0.32	65.20	16.24
	PM-E	164.48	59.04	15.96	**0.08**	**0.08**	**0.04**	**0.04**	239.72	25.32
t481	SAM-R	2.83	2.90	**7.67**	3.13	6.47	6.43	3.03	32.47	**9.83**
	SG-R	**2.33**	**2.77**	7.77	3.23	6.90	5.93	3.03	**31.97**	10.27
	SAM-E	21.12	12.16	14.32	0.28	8.00	0.36	0.28	56.52	16.16
	PM-E	4271.00	480.00	16.00	**0.00**	**0.00**	**0.00**	**0.00**	4767.00	489.00
tcon	SAM-R	**3.08**	**2.20**	4.12	2.40	13.80	2.12	1.04	28.76	5.04
	SG-R	3.60	3.24	4.60	2.96	11.88	1.84	1.08	29.20	5.00
	SAM-E	13.36	7.00	**1.92**	**0.00**	3.96	**0.00**	**0.00**	**26.24**	**4.36**
	PM-E	13.68	7.04	2.08	0.84	**2.72**	**0.00**	**0.00**	26.36	4.52
vda	SAM-R	-	-	-	-	-	-	-	-	-
	SG-R	-	-	-	-	-	-	-	-	-
	SAM-E	**3284.38**	775.92	41.92	12.00	18.42	2.63	1.42	**4136.67**	298.42
	PM-E	4657.00	**749.00**	**17.00**	**0.00**	**0.00**	**0.00**	**0.00**	5423.00	**61.00**

Table 7: The CPU time spent by each method for the problems in group 1. The computational environment used for the experiments with SAM-R and SG-R methods consists of two Intel Xeon E5-2660 v4 @ 50520 CPUs 2.00GHz processors with 14 cores each, totaling 28 cores and 224GB DDR4 DIMMs. The computational environment used for the experiments with PM-E and SAM-E methods consists of two Intel Xeon E5520 processors at 2.27GHz with 4 cores each, totaling 8 cores and 24 GB DDR3 DIMMs. Note that the CPU time cannot be compared due to the executions in different computational environments.

Problem	**Method**	**Execution time (s)**	
		Mean	**Stdev**
C17	SAM-R	46.13	7.16E+00
	SG-R	40.23	6.88E+00
	SAM-E	20.15	2.08E+00
	PM-E	**14.10**	**1.68E+00**
cm42a	SAM-R	**120.40**	3.78E+01
	SG-R	124.56	3.33E+01
	SAM-E	160.05	1.77E+01
	PM-E	158.63	**1.22E+01**
cm82a	SAM-R	45.46	**1.26E+01**
	SG-R	**42.46**	1.35E+01
	SAM-E	84.19	2.54E+01
	PM-E	66.10	1.52E+01
cm138a	SAM-R	229.93	5.10E+01
	SG-R	**224.74**	5.47E+01
	SAM-E	323.55	6.46E+01
	PM-E	296.15	**3.49E+01**
decod	SAM-R	299.94	7.74E+01
	SG-R	365.16	1.51E+02
	SAM-E	270.71	5.29E+01
	PM-E	**233.24**	**3.29E+01**
f51m	SAM-R	**354.14**	1.03E+02
	SG-R	377.63	**9.92E+01**
	SAM-E	1995.98	2.40E+02
	PM-E	3317.60	2.49E+02
majority	SAM-R	24.71	4.87E+00
	SG-R	20.87	4.22E+00
	SAM-E	16.30	**1.55E+00**
	PM-E	**14.31**	1.89E+00
z4ml	SAM-R	171.72	**4.40E+01**
	SG-R	**165.93**	6.29E+01
	SAM-E	1054.02	4.82E+02
	PM-E	1773.45	1.28E+02

Table 8: The CPU time spent by each method for the problems in group 2. The computational environment used for the experiments with SAM-R and SG-R methods consists of two Intel Xeon E5-2660 v4 @ 50520 CPUs 2.00GHz processors with 14 cores each, totaling 28 cores and 224GB DDR4 DIMMs. The computational environment used for the experiments with PM-E and SAM-E methods consists of two Intel Xeon E5520 processors at 2.27GHz with 4 cores each, totaling 8 cores and 24 GB DDR3 DIMMs. Note that the CPU time cannot be compared due to the executions in different computational environments.

Problem	**Method**	**Execution time (s)**	
		Mean	**Stdev**
9symml	SAM-R	**2309.80**	1.03E+03
	SG-R	2462.72	1.15E+03
	SAM-E	4684.54	7.88E+02
	PM-E	5625.68	**4.17E+02**
alu2	SAM-R	**7264.68**	2.20E+03
	SG-R	7940.65	2.23E+03
	SAM-E	106322.62	2.55E+04
	PM-E	10414.20	**1.26E+03**
alu4	SAM-R	**31333.04**	1.25E+04
	SG-R	31968.91	1.31E+04
	SAM-E	604800.00	**0.00E+00**
	PM-E	214032.18	8.06E+04
cm85a	SAM-R	751.98	**9.54E+01**
	SG-R	**730.57**	1.80E+02
	SAM-E	1652.11	6.88E+02
	PM-E	2334.73	2.01E+02
cm151a	SAM-R	3506.51	6.27E+02
	SG-R	3203.77	7.71E+02
	SAM-E	**1892.28**	**3.22E+02**
	PM-E	3317.33	4.88E+02
cm162a	SAM-R	2199.23	3.46E+02
	SG-R	2115.61	3.76E+02
	SAM-E	**1388.39**	**1.56E+02**
	PM-E	1924.53	2.42E+02
cu	SAM-R	2782.84	5.34E+02
	SG-R	2309.90	6.18E+02
	SAM-E	**1774.73**	**2.17E+02**
	PM-E	2369.41	2.69E+02
x2	SAM-R	1100.20	1.82E+02
	SG-R	1022.47	1.64E+02
	SAM-E	**630.05**	**8.91E+01**
	PM-E	827.97	1.14E+02

Table 9: The CPU time spent by each method for the problems in group 3. The computational environment used for the experiments with SAM-R and SG-R methods consists of two Intel Xeon E5-2660 v4 @ 50520 CPUs 2.00GHz processors with 14 cores each, totaling 28 cores and 224GB DDR4 DIMMs. The computational environment used for the experiments with PM-E and SAM-E methods consists of two Intel Xeon E5520 processors at 2.27GHz with 4 cores each, totaling 8 cores and 24 GB DDR3 DIMMs. Note that the CPU time cannot be compared due to the executions in different computational environments.

Problem	**Method**	**Execution time (s)**	
		Mean	**Stdev**
cc	SAM-R	11878.37	2.76E+03
	SG-R	14096.77	3.14E+03
	SAM-E	**1925.81**	**1.80E+02**
	PM-E	3058.44	3.16E+02
cmb	SAM-R	5960.31	1.26E+03
	SG-R	9117.83	2.27E+03
	SAM-E	**664.53**	**1.23E+02**
	PM-E	721.09	1.63E+02
cordic	SAM-R	9334.73	2.01E+03
	SG-R	**8176.20**	2.06E+03
	SAM-E	528868.07	1.44E+05
	PM-E	604800.00	**0.00E+00**
frg1	SAM-R	13431.62	3.62E+03
	SG-R	**13090.45**	**3.01E+03**
	SAM-E	37334.75	5.19E+03
	PM-E	191578.47	1.36E+04
pm1	SAM-R	**4543.29**	**8.74E+02**
	SG-R	5076.47	1.76E+03
	SAM-E	6579.89	9.52E+02
	PM-E	26939.52	2.73E+03
t481	SAM-R	1761.05	**5.36E+02**
	SG-R	**1571.19**	7.51E+02
	SAM-E	57143.44	1.09E+04
	PM-E	78730.24	5.986E+03
tcon	SAM-R	6034.98	1.22E+03
	SG-R	6839.18	1.81E+03
	SAM-E	**361.93**	**3.03E+01**
	PM-E	380.79	4.62E+01
vda	SAM-R	199463.62	8.67E+04E
	SG-R	**194848.54**	6.77E+04
	SAM-E	604800.00	**0.00E+00**
	PM-E	570818.80	2.00+04

Table 10: Dunn's p-values of all methods for each problem in group 1. The values with a statistical difference (p-value < 0.05) are highlighted in boldface.

Problem	Method	Method		
		SG-R	PM-E	SAM-E
C17	SAM-R	7.87E-01	**2.70E-02**	1.90E-01
	SG-R		**1.31E-02**	2.97E-01
	PM-E			**4.26E-04**
cm42a	SAM-R	9.16E-02	**1.85E-03**	**8.63E-03**
	SG-R		2.44E-01	4.98E-01
	PM-E			5.51E-01
cm82a	SAM-R	9.63E-01	**9.08E-06**	**1.05E-5**
	SG-R		**7.08E-06**	**8.24E-06**
	PM-E			9.69E-01
cm138a	SAM-R	8.64E-01	**1.75E-09**	**8.03E-10**
	SG-R		**5.07E-10**	**2.26E-10**
	PM-E			8.78E-01
decod	SAM-R	2.45E-01	**1.03E-12**	**1.56E-10**
	SG-R		**7.30E-09**	**4.28E-07**
	PM-E			3.73E-01
f51m	SAM-R	2.65E-01	**1.62E-24**	**5.58E-08**
	SG-R		**4.16E-19**	**3.40E-05**
	PM-E			**4.51E-09**
majority	SAM-R	6.24E-01	**4.04E-12**	**6.04E-03**
	SG-R		**1.90E-10**	**2.93E-02**
	PM-E			**2.87E-07**
z4ml	SAM-R	9.55E-01	**1.66E-18**	**1.20E-03**
	SG-R		**9.31E-19**	**9.52E-04**
	PM-E			**1.17E-11**

Table 11: Dunn's p-values of all methods for each problem in group 2. The values with a statistical difference (p-value < 0.05) are highlighted in boldface.

Problem	Method	Method		
		SG-R	PM-E	SAM-E
9symml	SAM-R	9.33E-01	**1.32E-14**	**5.64E-06**
	SG-R		**4.67E-14**	**1.04E-05**
	PM-E			**1.55E-03**
alu2	SAM-R	9.65E-01	**1.46E-04**	1.83E-01
	SG-R		**7.84E-03**	3.69E-01
	PM-E			**6.03E-19**
alu4	SAM-R	2.99E-01	**3.25E-16**	**1.01E-06**
	SG-R		**2.19E-11**	**3.37E-04**
	PM-E			**8.26E-04**
cm85a	SAM-R	6.21E-01	**1.16E-17**	**1.38E-02**
	SG-R		**1.39E-15**	5.87E-02
	PM-E			**8.30E-14**
cm151a	SAM-R	8.83E-01	**3.76E-08**	**3.31E-05**
	SG-R		**1.42E-08**	**1.56E-05**
	PM-E			9.81E-02
cm162a	SAM-R	8.11E-01	4.96E-01	5.48E-01
	SG-R		3.39E-01	7.45E-01
	PM-E			1.17E-01
cu	SAM-R	4.28E-01	**1.71E-07**	**4.36E-04**
	SG-R		**9.21E-06**	**6.45E-03**
	PM-E			8.71E-02
x2	SAM-R	8.37E-01	7.60E-02	**1.45E-02**
	SG-R		1.24E-01	**2.73E-02**
	PM-E			4.11E-01

Table 12: Dunn's p-values of all methods for each problem in group 3. The values with a statistical difference (p-value < 0.05) are highlighted in boldface.

Problem	**Method**	**Method**		
		SG-R	PM-E	SAM-E
cc	SAM-R	7.64E-02	**3.39E-11**	2.39E-01
	SG-R		**4.60E-06**	3.85E-01
	PM-E			**2.47E-11**
cmb	SAM-R			
	SG-R			
	PM-E			**6.89E-03**
cordic	SAM-R			
	SG-R			
	PM-E			**1.69E-10**
frg1	SAM-R	9.69E-01	**1.88E-14**	**4.49E-06**
	SG-R		**1.38E-14**	**3.72E-06**
	PM-E			**2.13E-03**
pm1	SAM-R	4.84E-01	**1.51E-23**	**2.27E-07**
	SG-R		**1.30E-19**	**1.70E-05**
	PM-E			**3.44E-09**
sct	SAM-R	3.64E-01	**2.98E-15**	**1.75E-02**
	SG-R		**1.96E-13**	1.54E-01
	PM-E			**3.42E-14**
t481	SAM-R	9.31E-01	**5.12E-13**	**7.04E-04**
	SG-R		**9.60E-13**	**9.63E-04**
	PM-E			**1.09E-04**
tcon	SAM-R	2.02E-01	**1.45E-03**	7.72E-02
	SG-R		5.61E-02	6.22E-01
	PM-E			1.56E-01
vda	SAM-R			
	SG-R			
	PM-E			**2.18E-10**