Minimax and Alpha/Beta Pruning for Tic-Tac-Toe

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Abstract

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The problem with the Minimax algorithm is that it spends too much time evaluating moves that are worse than already considered moves. It is interesting to resolve this for improving the performance of games with many possible moves. This solution presents an implementation of Alpha Beta Pruning which drastically reduced the amount of considered moves in the Minimax algorithm. From the solution the conclusion is that Alpha Beta Pruning could significantly improve performance in games that require many moves to win.

Contents

1 Introduction

This paper is about the Minimax algorithm and Alpha Beta pruning, which make up the core of a former assignment "Tic-Tac-Toe using Minimax" for the subject Data Science. This project was chosen over other projects because the topics were among the more interesting to research further.
One of the objectives of this paper is to shed some light on the Minimax Algorithm, how it works, why this is interesting along with an implentation of it. The main objective is showing how to optimize this algorithm further by implementing Alpha/Beta pruning and comparing it to the bare Minimax Algorithm. This will be accomplished by describing the logic behind the topics and showing implementations from the Tic-Tac-Toe assignment, that will be modified for the purpose of this paper. This paper is not meant to prove anything new, but is simply a result of a fascination with Alpha Beta Pruning. The report is split in sections explaining about Minimax alone and its implementation and then with Alpha Beta Pruning also following up with its implementation.

2 Minimax Algorithm

2.1 Introduction to Minimax

The Minimax algorithm is a recursive algorithm which is used in turn-based games like chess and tic-tac-toe, to make the AI player consider all possible moves and choose the most rational one.

2.2 Explanation of Minimax

2.2.1 About Minimax

First, it is important to understand the meaning behind recursive code. When code is recursive, it for example means that it's a function that calls itself with updated arguments until it reaches some condition the function wants to return. In the Minimax algorithm, the function is made to call itself in order to keep looking at more possible future moves, and ultimately decides which move is the best move to make on the AI's current board.

2.2.2 Why is it called Minimax

The idea behind minimax is that the algorithm expects the game to be a turn-based 2-player game, and the algorithm defines these 2 players as the minimizer and the maximizer. This simply means that the algorithm always expects the minimizer to pick the move with the lowest score, and the maximizer to do the opposite. Here is an example: Here the maximizer is the human player and the minimizer is the AI player in this scenario.

Consider the Tic-Tac-Toe board in figure 1.

In this scenario the AI player goes first as "X" and tries to figure out the most rational move of the 9 moves possible. It does this by predicting what the opposing player, maximizer would then do on his next move and so on until it has found the fastest way to win for that one move. Basically, the algorithm runs a simulated game on each of the current board's possible moves, against the maximizer, and when it is done with one move it backtracks back to the current board with that moves score value, to run a new simulation on the next move. When it has simulated how to win by all the possible moves it will then make the move that will help it win the fastest.

Consider again another example in figure 2.

The Minimax tree shows an example of how the minimax works. In this scenario the maximizer goes first, and has 2 possible moves. The maximizer would therefore explore the first move first, marked -10. At this point the algorithm has not simulated this move yet and therefore does not know that value yet. So first it just tries the moves leftmost in the tree until it hits the depth 4 which is

Figure 1: An example of a Tic-Tac-Toe board

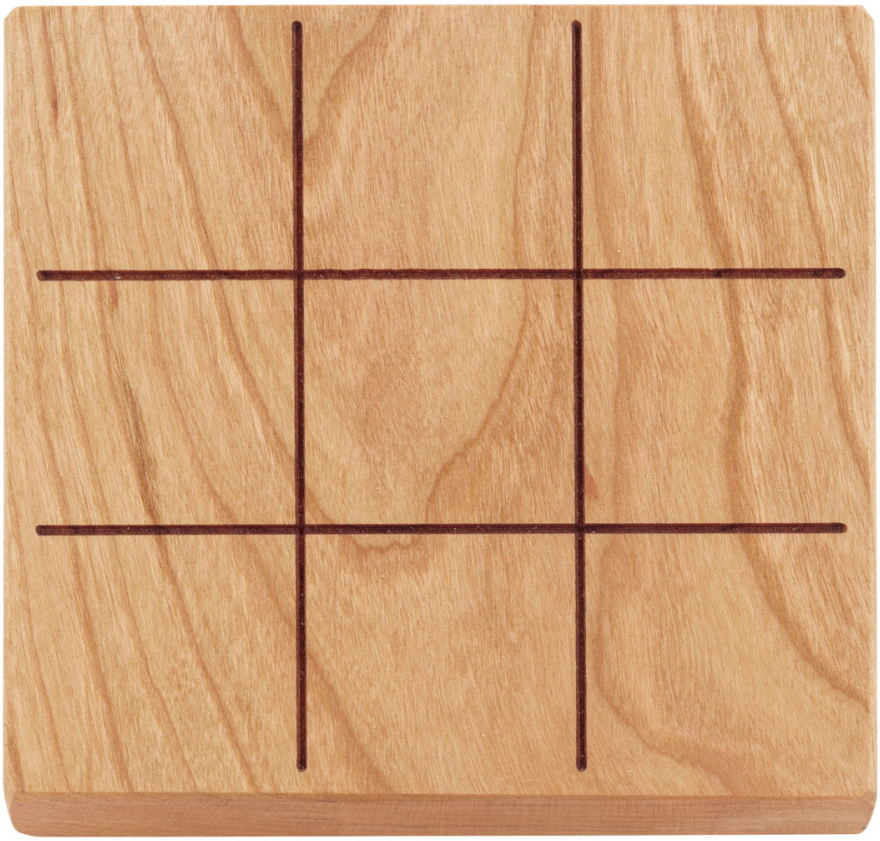
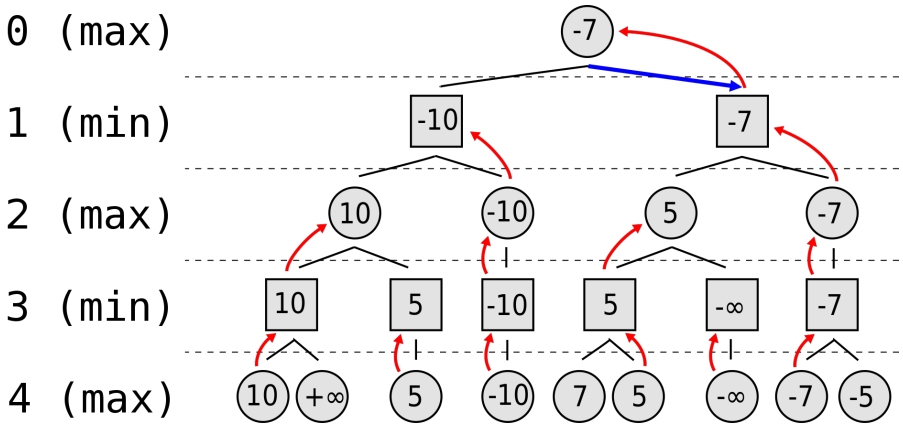


Figure 2: Minimax Tree Example



the bottom in this scenario. It would end up with a value of 10 which would be good for the maximizer. However, the minimizer needs to be taken into account, so the maximizer backtracks once to see the other move which is +infinity. It is guaranteed, that the minimizer would never pick that route, which means the score 10 can be kept in mind while the maximizer backtracks to the next point where the minimizer has a move. As is seen this route only gives both players one choice on each of their respective turns. The last move has a -10 score so it backtracks with that score once again and ends up with 10 and -10 on the 2 moves the minimizer can make. If the maximizer chooses this route the point value of the move would therefore be -10 as the minimizer always goes for the lesser score.
The algorithm then does the same thing for the other branch of moves and finds that this move will serve the maximizer better as it will yield -7 move value which is still poor for the maximizer but less so than a -10 score.

The explanition written here is based on this video from 0:00 to 2:30.

2.3 Implementation of Minimax

Listing 1: python example

```
# AI makes a move based on minmax algorithmic search for the most rational move to make
def ai_move(board):
    best_move = board
    best_value = float('-inf')
    for move in all_possible_moves(board, AI):
        #print(move)
        move_val = minimax(move, 0, False)
        #print(move_val)
        if move_val > best_value or best_value == None:
            best_move = move
            best_value = move_val
    return best_move
```

The function ai_move gets called when its the AI's turn to play. The function generates a list of all possible moves from the current board, which the function recieves from the arguments. This list is then iterated over to run the minimax recursion on each of these possible moves. When the result move_val is received for the current move, this value is then checked to see if it is higher or equal to the stored best_value. If it is, that stored value is then overridden with the new best value.

Listing 2: python example

```
def minimax(board, depth, isMax):
score = evaluate(board)

if score == 10:
    return score
if score == -10:
    return score
if board.count(EMPTY) == 0:
    return 0

if isMax:
    best = float('-inf')
    for move in all_possible_moves(board, AI):
        best = max(best, minimax(move, depth + 1, not isMax))
    return best
else:
    best = float('inf')
    for move in all_possible_moves(board, HUMAN):
        best = min(best, minimax(move, depth + 1, not isMax))
    return best
```

This is the implementation made in the Original Jupyter Notebook. The function always starts by checking the board with another function called evaluate. This function checks for win conditions and returns either 10 if the AI has a winning board and likewise -10 for the Human player. If no one has won yet it returns 0.
Next the received score is checked to find out if the recursion for this move should end. This is done if the score is -10, 10 or 0, and as expected the move will not be considered as best if the score is -10 as the AI does not want the human player to win.
The next part of the implementation is the interesting part. First the current simulated move is checked to see if it is the maximizer's (Human) or the minimizer's (AI) turn. Depending on the result of this the recursion is set up oppositely. A "best" variable is started at the worst value for the player being -infinity for the maximizer and infinity for the minimizer. This is done to ensure that there is a value to overwrite on the very first run of the algorithm. A list of possible moves is then created for this current simulated board and iterated over. This is where the recursion happens. The minimax function is called within it self on the current considered move of the list. This ensures that all possible moves are evaluated for this new current board as well, because every time the function returns "best" it backtracks to check the next move. Remember the tree example explained in figure 2.

The implementation described here is based on the pseudo code explained in this video from 2:30 to 5:20.

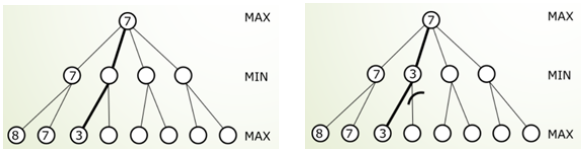


Figure 3: An example of Alpha Beta Pruning.

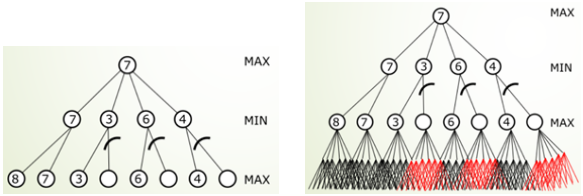


Figure 4: An example of many branches being pruned.

3 Alpha/Beta Pruning

3.1 Introduction to Alpha/Beta Pruning

Alpha/Beta pruning is a way by which the Minimax Algorithm is made more efficient. This happens by stopping consideration of possible moves, that will never be better than previously considered moves.

3.2 Explanation of Alpha/Beta Pruning

Alpha Beta Pruning is an optimization method for the Minimax algorithm that eliminates branches or entire subtrees. It is so named because it introduces the two variables or values Alpha and Beta into the Minimax algorithm. According to Akshay L Aradhya who wrote this article: "Alpha is the best value that the maximizer currently can guarantee at that level or above. Beta is the best value that the minimizer currently can guarantee at that level or above." Figure 3 shows how first the leftmost branches are explored and the minimizer picks "7" which is stored in the variable Alpha. After that the second leftmost branch is explored and the Minimizer is presented with a choice that is smaller than 7. Given that there is a choice smaller than Alpha present already, and it is the Minimizer's turn to choose, meaning that it will undoubtedly have a chance to pick a value smaller than Alpha, it would not be worth it for the Maximizer to explore the rest of the branch and the branch is pruned as seen in figure 3. Alpha Beta Pruning can potentially prune a lot of branches and thus make the examination of choices or moves through Minimax considerably faster than if it simply went through every single branch as seen in figure 4.

3.3 Implementation of Alpha/Beta Pruning

This section shows the changes to the minimax function and more, which was added to version 3 of the notebook in the code folder. This was done to better see the results of the pruning. First thing that was done was to add a counter to the minimax function in both version 3 and 4 of the code. This counter increments every time a new future move is considered. The idea was to show a count of how many moves the AI had considered before making the most rational choice. This way the number of considered moves on both versions of the code could be compared: one before implementing pruning and one after.

Listing 3: python example

```
1     def minimax(board, depth, alpha, beta, isMax):
2         global count
3         score = evaluate(board)
4         count += 1
5         if score == 10:
6             return score
7         if score == -10:
8             return score
9         if board.count(EMPTY) == 0:
10            return 0
11
12        if isMax:
13            maxEval = float('-inf')
14            for move in all_possible_moves(board, AI):
15                evaluation = minimax(move, depth + 1,
                      alpha, beta, not isMax)
16                maxEval = max(maxEval, evaluation)
17                alpha = max(alpha, evaluation)
18                if beta <= alpha:
19                    break
20            return maxEval
21        else:
22            minEval = float('inf')
23            for move in all_possible_moves(board, HUMAN):
24                evaluation = minimax(move, depth + 1,
                      alpha, beta, not isMax)
25                minEval = min(minEval, evaluation)
26                beta = min(beta, evaluation)
27                if beta <= alpha:
28                    break
29            return minEval
```

To implement Alpha Beta Pruning 2 more parameters had to be added to the function: alpha and beta. These start with the values -infinity and infinity. The recursive call was then updated to have these arguments included. For the maximizer the alpha value was set to whichever is greater between the current alpha value and the evaluation. Likewise it is done for the minimizer between

which is lesser of beta's current value and the evaluation. This means that every time the minimax function is run, the alpha value is updated if it is the maximizing players turn. Likewise the beta value is updated on the minimizing players turn. No matter whose turn it is beta is checked to see if is less than or equal to alpha, and if this is true, the rest of the moves are pruned.

4 Discussion

4.1 Compare results

This subsection will be used to show the results of the pruning.
In the notebook without pruning (V3) on turn 1 the AI considered 549945 different moves. In the notebook with pruning (V4) on turn 1 the AI considered 30709 different moves.
This means that on turn one V4 managed to prune away around 93% of the moves that would be considered without pruning.
On turn 3 this went from 7331 down to 1519 which is again a significant decrease of around 80%. These prunings of course become smaller in number as the game had fewer possible moves, and at turn 7 it would no longer prune anything.
These findings can be seen better by running files MinimaxV3.ipynb and MinimaxV4.ipynb in the code folder of this project.

4.2 Conclusion

The Jupyter Notebook TicTacToe_MinimaxV4.ipynb is the resulting implementation of Alpha Beta Pruning that was compared with the original notebook. Version 4 has significantly fewer considered moves, presumably resulting in a slightly faster game for this implementation of Tic-Tac-Toe. If implemented on a game like chess, which requires the algorithm to check many more moves per depth, pruning would likely save a considerable amount of time compared to the bare minimax function.
Future implementations, that would be interesting to include, could be: input validation, Multiplayer (humans), AI versus AI games, allowing for picking X or O, a choice of board size and potentially different games. Of those it would be relevant to test Alpha Beta Pruning on future versions with allowing for picking X or O and on different board sizes.
For a small project like this, simply comparing the amount of considered moves between notebook versions has worked well. On the other hand, more precise tools could have been used to compare and to test efficiency and impact of Alpha Beta Pruning.