46755 – Renewables in Electricity Markets

Lecture 2: Fundamentals of electricity markets

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February 5, 2024

Recap

In the break-out rooms (each room with 4 students), please remember what you learned in the previous lecture and share one or two things that you found interesting:

Key items for discussion in break-out rooms:

- Centralized power system vs. electricity markets
- Supply and demand curves
- Merit-order (least-cost) principle in supply side
- Market clearing: How to clear a market given supply and demand curves?
- Social welfare: what does it mean?
- Equilibrium point; market-clearing price and quantities
- Uniform vs. pay-as-bid pricing
- What makes the electricity market special (compared to a market for other commodities)

Learning objectives of this lecture

After this lecture, you are expected to

- Introduce various electricity market actors
- Explain various markets in the power sector
- Differentiate the current practice in European vs. U.S. electricity markets
- Formulate the market-clearing procedure as a <u>linear optimization</u> problem (excluding transmission network)

Electricity markets

Market-clearing algorithm

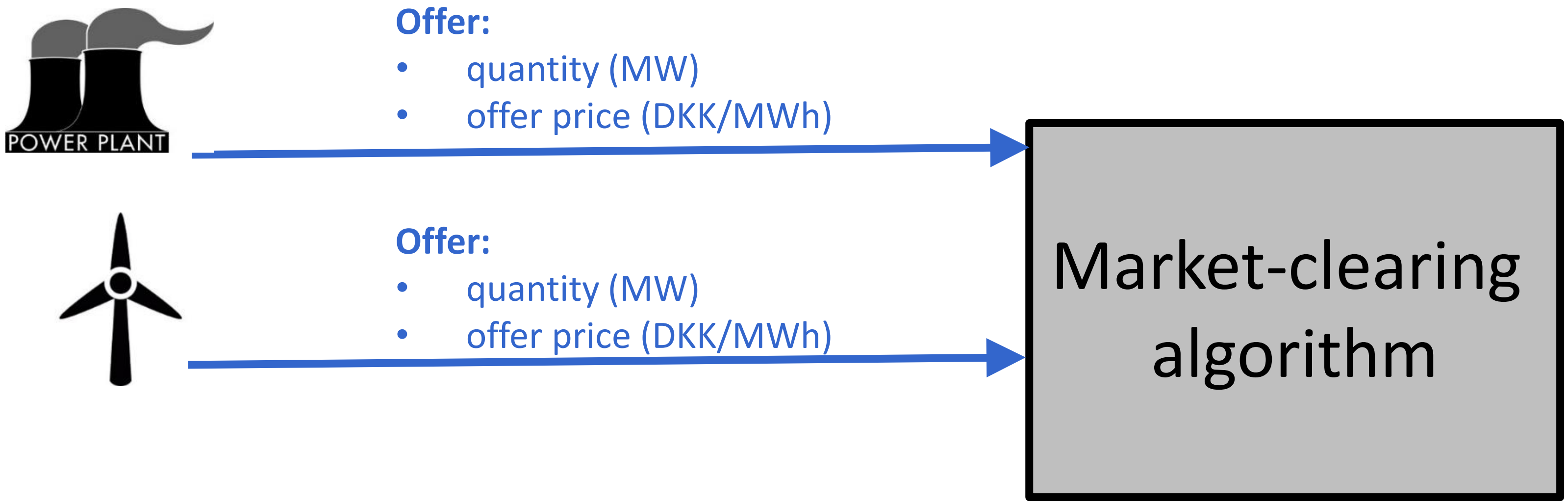
Electricity markets

Offer:

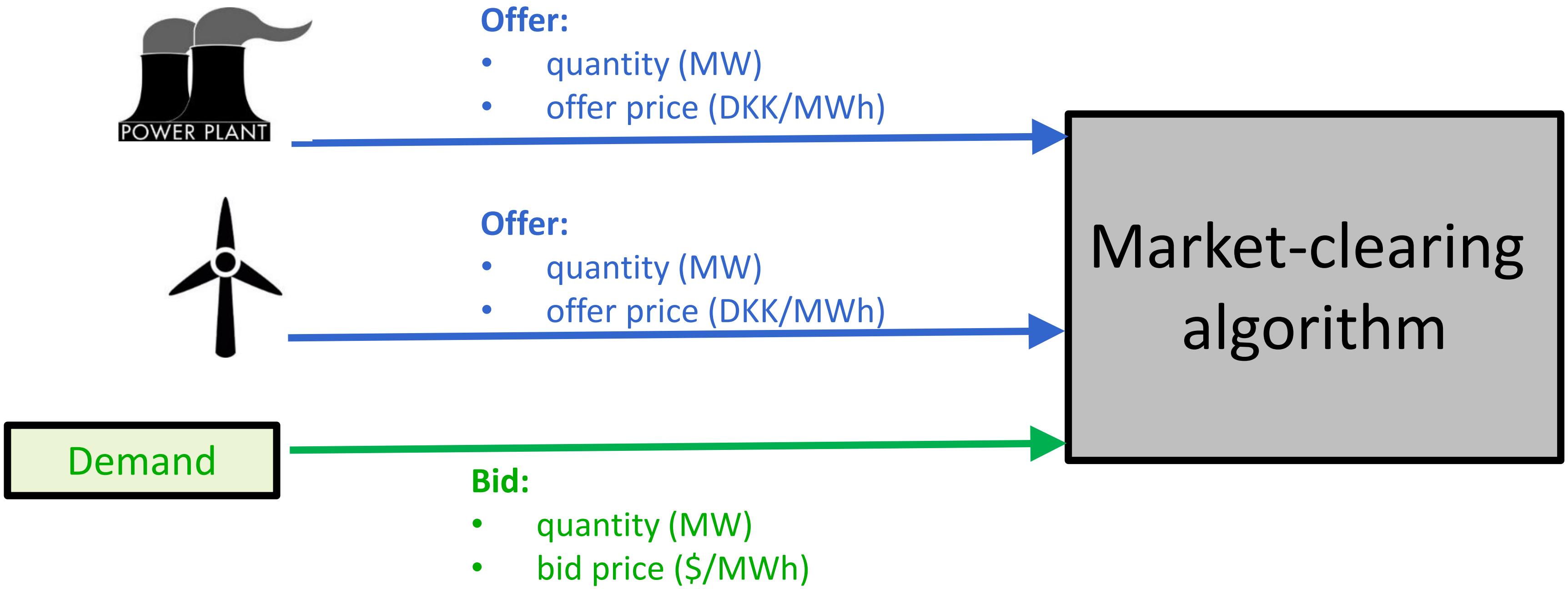
- quantity (MW)
- offer price (DKK/MWh)

Market-clearing algorithm

Electricity markets



Electricity markets



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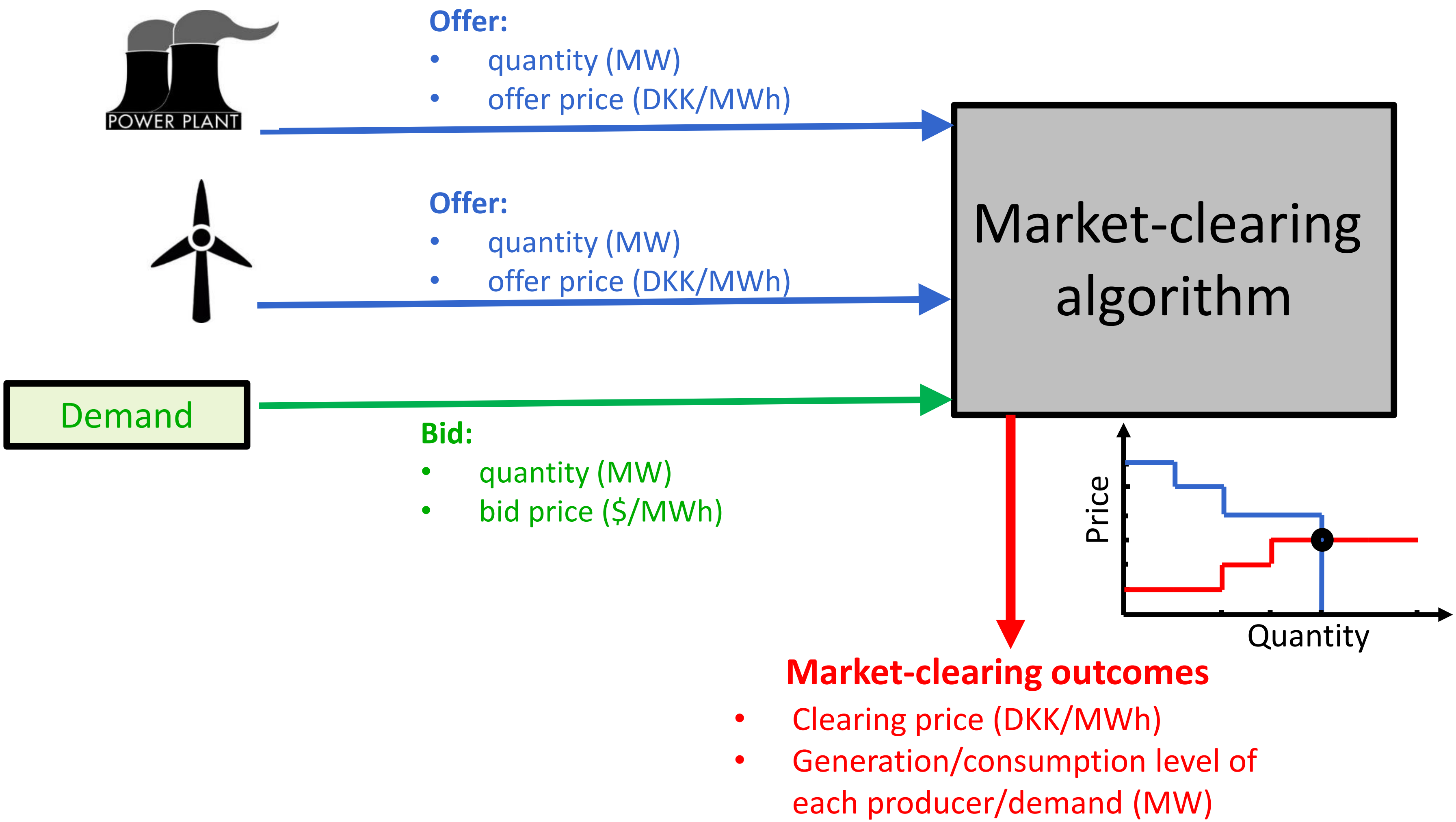

Various market actors

Market actors

- Also known as “market players” or “market participants”

Which market actors have we identified so far?

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Which market actors have we identified so far?

1) Power producers

- A conventional generation unit (e.g., nuclear, coal, gas-fired power plants, CHP)
- A renewable power unit (e.g., wind, solar, hydro, biomass, etc)
- A producer who owns a portfolio of conventional and/or renewable units. Note that her assets might be located throughout the transmission network.

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A retailer is an intermediate market actor (trader) who purchases power in a large volume from electricity markets and sell it back to a large number of small-scale demands (e.g., households). **Example:** Andel Energi in Denmark.

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- A <u>non-profit</u> entity, who receives all offers from producers and bids from demands, and clears the market (by maximizing the social welfare), and eventually disseminates market-clearing outcomes, i.e., prices and quantities.

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- Interested to know about the history of electricity markets, in particular Nord Pool?

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A 10-minute lecture by Prof. Pierre Pinson (DTU) on YouTube:

https://www.youtube.com/watch?v=i7XnyK1q0Sc

Market actors

Do we need any other actor?

Market actors

Key point:

- The market-clearing outcomes are indeed financial contracts, i.e., some contracts for buying and selling electricity!
- Who is in charge of operating the power system in practice based on given contracts?

Market actors

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- in charge of the “safe” operation of the underlying power system, ensuring supply security, instantaneous power supply and demand balance, stability of the system, …

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- the so-called independent system operator (ISO) or regional transmission organization (RTO) in the context of various U.S. electricity markets

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- the so-called independent system operator (ISO) or regional transmission organization (RTO) in the context of various U.S. electricity markets
- In **Denmark**: Energinet.dk (https://en.energinet.dk/), who is in charge of the operation of both power and gas systems (in the transmission level)

Market actors

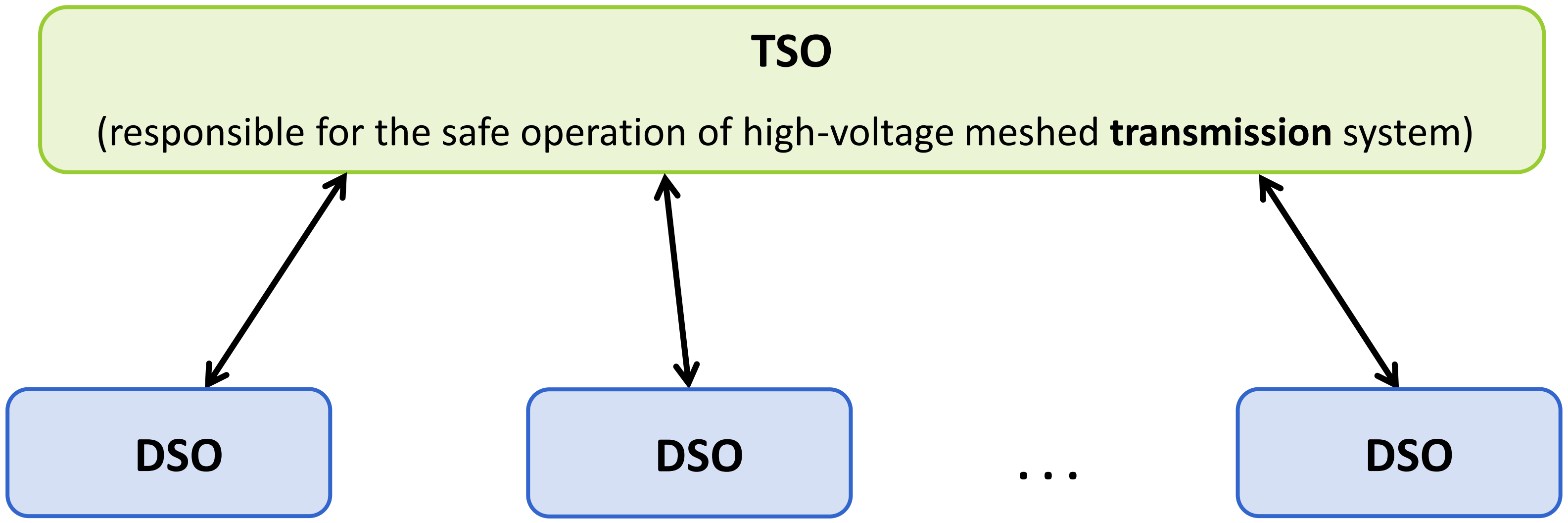
5) Distribution system operator (DSO)

- in charge of the "safe" operation of the underlying mid-voltage and low-voltage (radial) **distribution grids**…
- **Example**: Radius and Syd Energi in Denmark

Market actors

5) Distribution system operator (DSO)

- in charge of the “safe” operation of the underlying mid-voltage and low-voltage (radial) **distribution grids**…
- **Example**: Radius and Syd Energi in Denmark
- A proper coordination between **TSO** and each **DSO** is very important for the safe operation of the whole system!



Market actors

5) Distribution system operator (DSO)

- in charge of the "safe" operation of the underlying mid-voltage and low-voltage (radial) distribution grids…
- **Example**: Radius and Syd Energi in Denmark

6) Market regulator

- in charge of monitoring the performance of market in short- and long-run, and designing proper market regulations and policies…
- **Example**: Danish Energy Regulatory Authority in Denmark

Market actors

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7) Others actors

- such as *balancing responsible parties (BRPs)*, *flexibility aggregators*, etc
- to be introduced throughput the course when it is relevant

Various markets in the power sector

Various markets in the power sector

Three main markets with different products are:

Capacity markets	Energy markets	Ancillary service markets

Various markets in the power sector

Three main markets with different products are:

Capacity markets | Energy markets | Ancillary service markets

- Capacity markets have been designed to ensure that sufficient generation capacity (in MW) is available in the market for supply security and reliable system operation.
- Every producer who submits an offer to the electricity market (either it is eventually accepted or rejected) is eligible to be paid for her availability in the market (in terms of DKK/MW).
- Capacity payments provide an incentive for power producers to invest in new generation assets in long run.
- We are not going to discuss about capacity markets in this course!

Various markets in the power sector

Three main markets with different products are:

Capacity markets	**Energy markets**	Ancillary service markets

- Central marketplace for exchanging energy (in MWh), i.e., matching of electricity supply and demand

Various markets in the power sector

Three main markets with different products are:

Capacity markets	**Energy markets**	Ancillary service markets

- Central marketplace for exchanging energy (in MWh), i.e., matching of electricity supply and demand
- Various energy markets to be cleared in <u>different points of time</u>:

Actual time for physical delivery of energy

Time

Various markets in the power sector

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- Various energy markets to be cleared in different points of time:

1) Futures markets:

- Long-term financial contracts with a time span up to six years!
- Purpose: price hedging and risk management for buyers and sellers
- **Example:** *NASDAQ OMX Commodities* in Scandinavian countries
- Not to be covered in this course

Various markets in the power sector

Three main markets with different products are:

Capacity markets | **Energy markets** | Ancillary service markets

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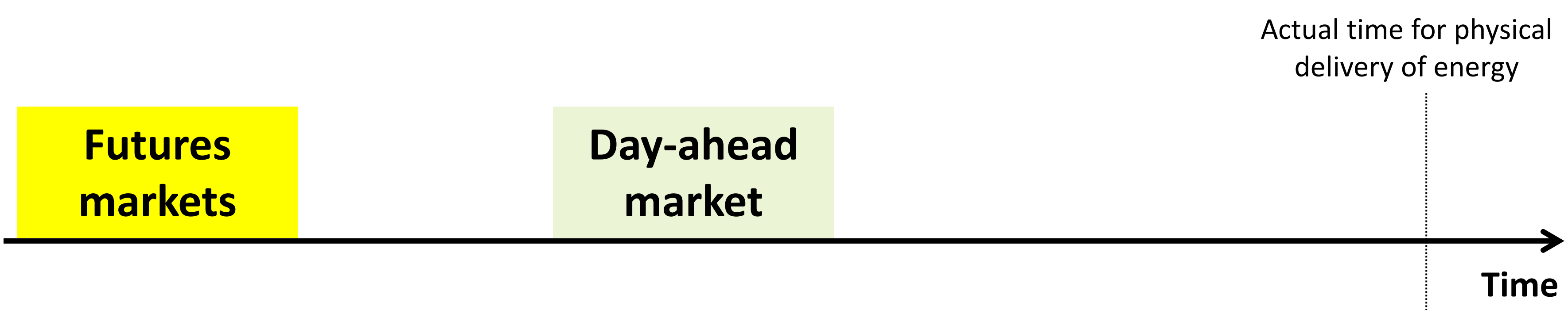
- Various energy markets to be cleared in different points of time:

2) Day-ahead market:

- Also known as “spot” market, to be cleared 12-36 hours before actual delivery time!
- **Example:** *Nord Pool Elspot,* to be cleared in noon of day *D-1* for energy exchange during every hour of day *D* (from midnight to midnight). Different market prices and quantities for each hour!



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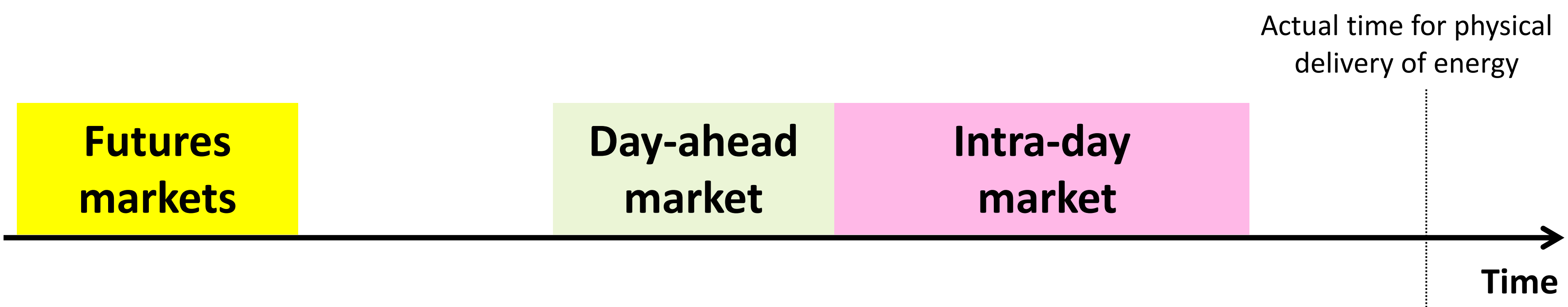
- Central marketplace for exchanging energy (in MWh), i.e., matching of electricity supply and demand
- Various energy markets to be cleared in different points of time:

3) Intra-day market:

- Continuous trading platform between day-ahead market and real time!
- This market provides an opportunity to “modify” day-ahead schedules in case there is an updated forecast or an asset failure
- **Example:** *Nord Pool Elbas*



Various markets in the power sector

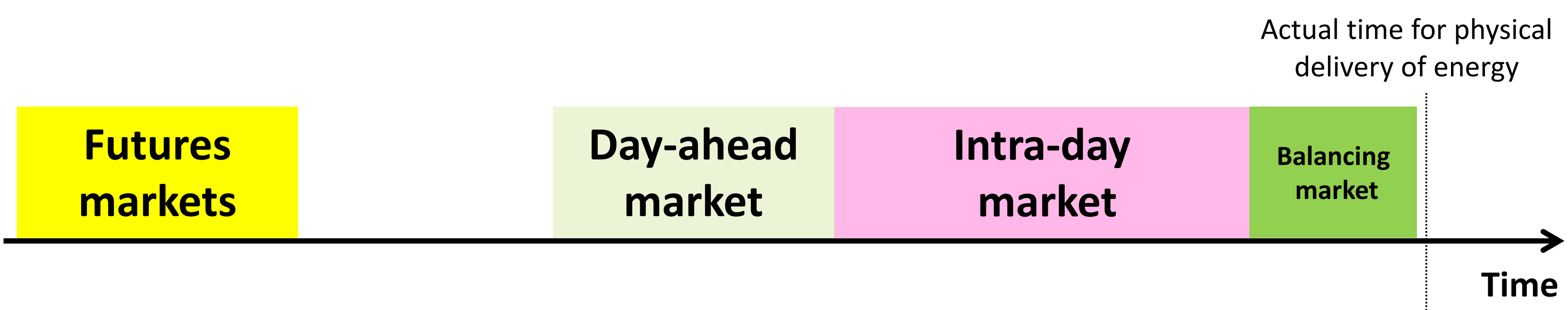
Three main markets with different products are:

Capacity markets | **Energy markets** | Ancillary service markets

- Central marketplace for exchanging energy (in MWh), i.e., matching of electricity supply and demand
- Various energy markets to be cleared in different points of time:

4) Balancing market:

- Close to the real-time operation of the system!
- Purpose: to ensure power supply-demand balance and safe operation of the system



Various markets in the power sector

Three main markets with different products are:

Capacity markets | Energy markets | **Ancillary service markets**

- These markets allow the system operator to procure services required for secure and reliable operation of the system.
- Examples of such services are:
  - ✓ Primary reserves
  - ✓ Secondary reserves
  - ✓ Tertiary reserves
  - ✓ Black-start capability
  - ✓ Reactive and voltage-control reserves
  - ✓ etc

Who clears which market?

From market to operation

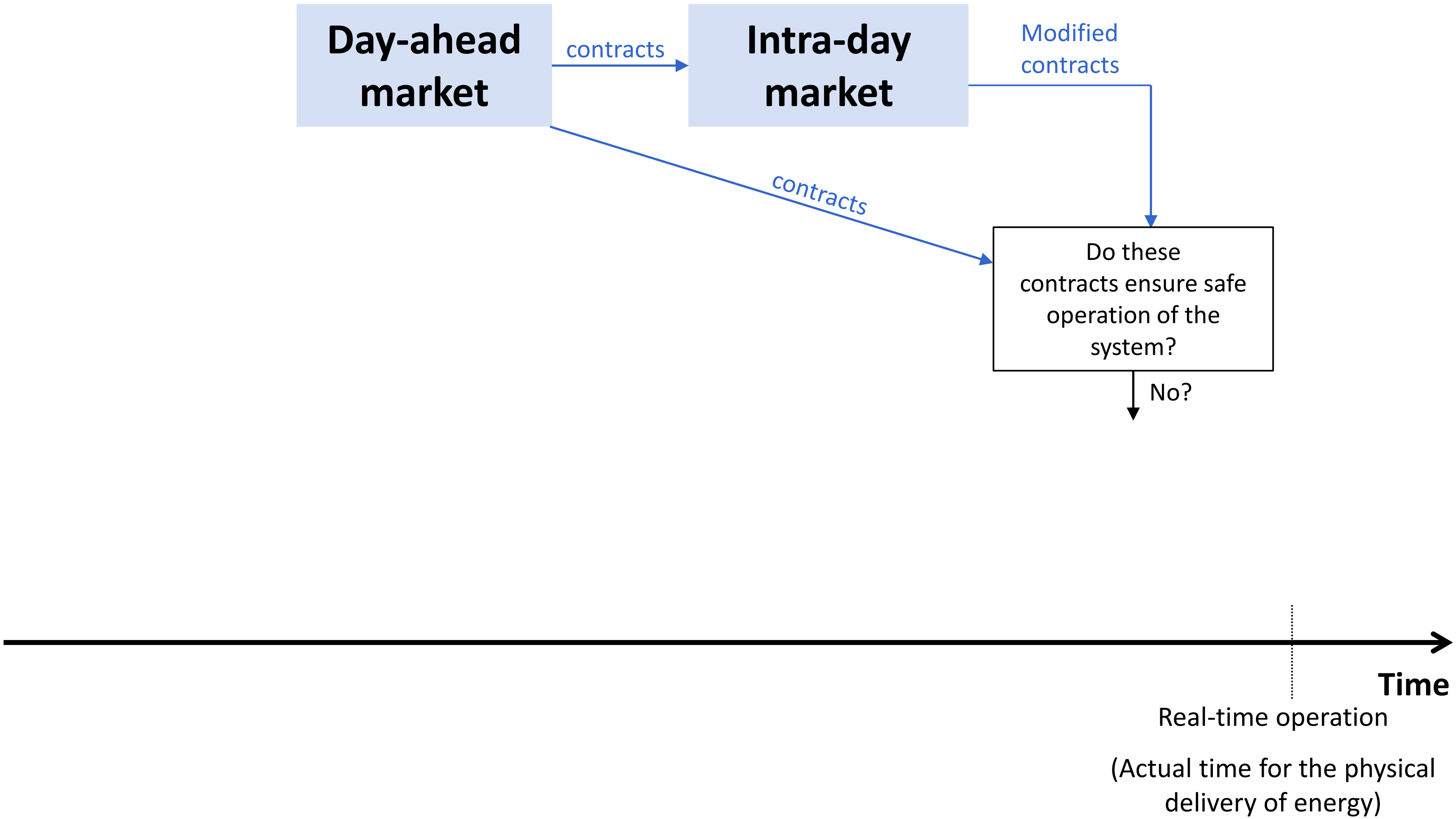
Time

Real-time operation

(Actual time for the physical delivery of energy)

From market to operation

By the market operator (e.g., Nord Pool)



From market to operation

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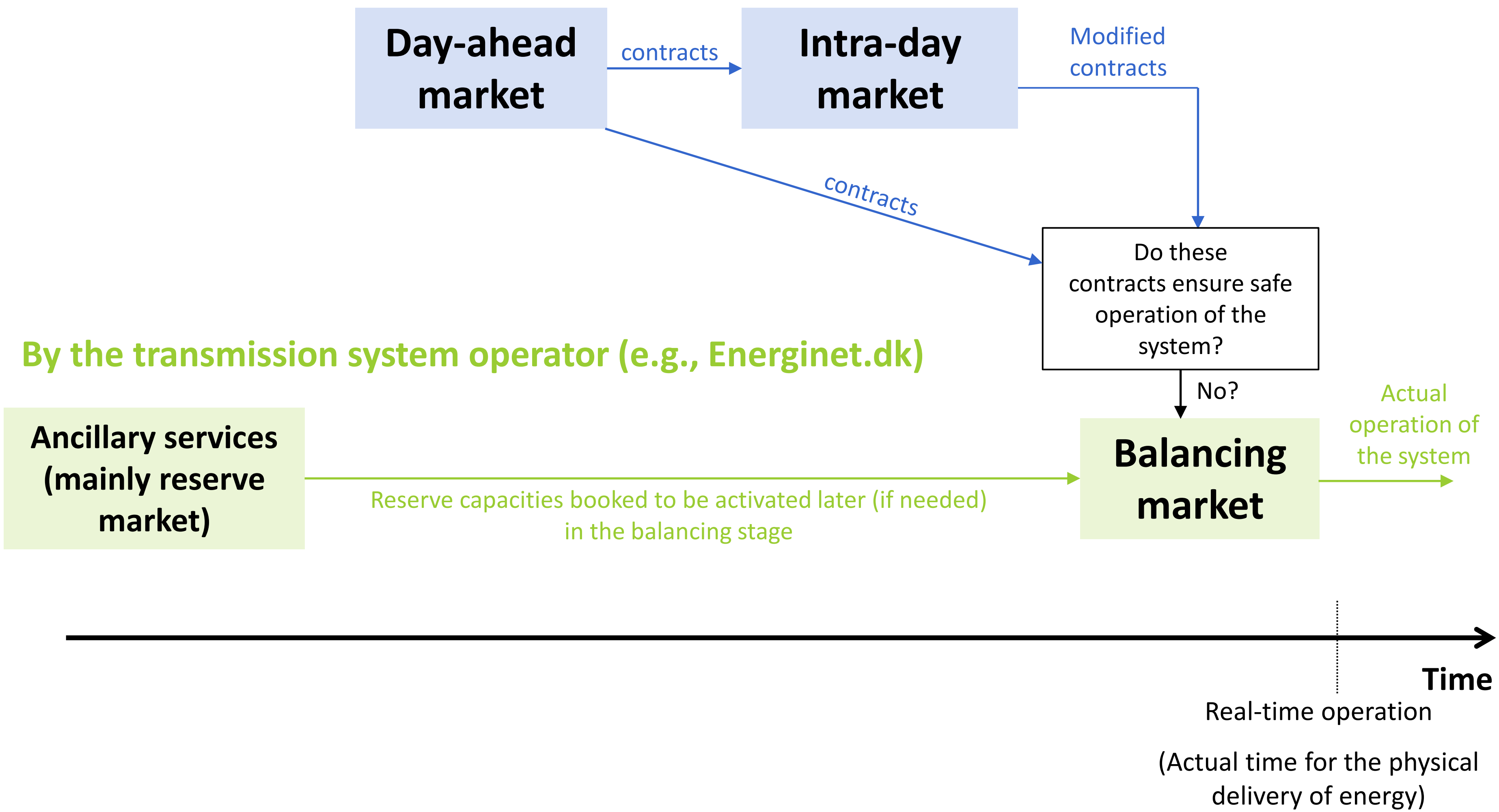
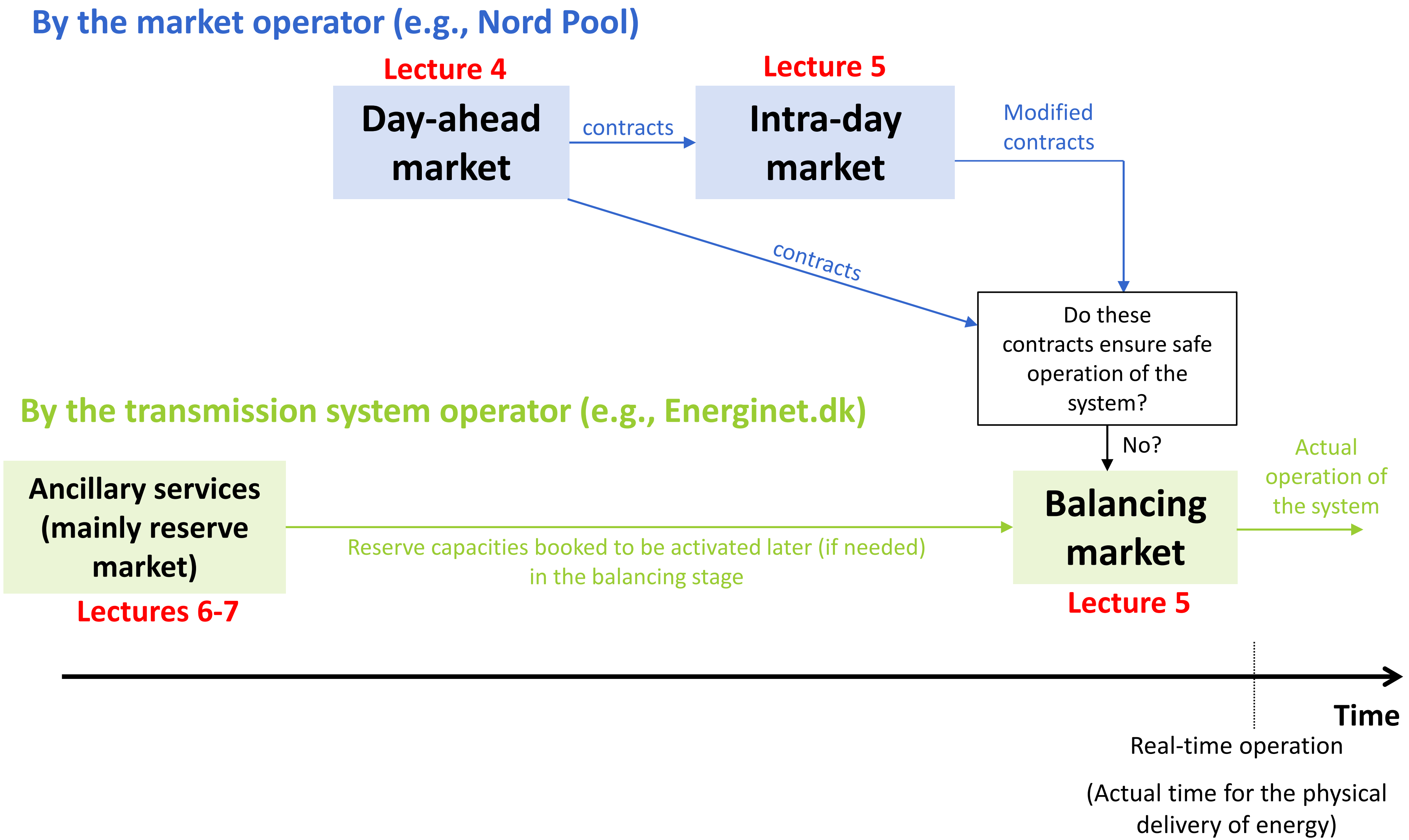


From market to operation



European vs U.S. electricity markets

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Common practice in Europe

Common practice in the U.S.

European vs U.S. electricity markets

Common practice in Europe

Market and system operators are **different** entities.

Common practice in the U.S.

ISO is in charge of **both** clearing the market and operating the system.

European vs U.S. electricity markets

Common practice in Europe

Market and system operators are **different** entities.

Transmission network is modeled in a **simplistic** way (zonal representation) within the day-ahead (and intra-day) market-clearing problem (to be discussed in lecture 3).

The bidding zone configuration in Europe in September 2020.
Source: https://fsr.eui.eu/electricity-markets-in-the-eu/

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European vs U.S. electricity markets

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Market actors, e.g., power producers, are **responsible** to ensure their offers/bids respect their technical constraints (e.g., ramp limits, minimum production limit and minimum up/down time limits of conventional generators).

- → Market actors internalize their technical limits within their own offers/bids.
- → They offer/bid for their whole portfolio (not per asset)
- → **Complex orders** (bids and offers), while comparatively simple market-clearing problem.

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Market actors, e.g., power producers, **submit** to the market their all technical constraints (e.g., ramp limits, minimum production limit and minimum up/down time limits of conventional generators).

→ Market-clearing problem becomes a unit commitment problem with on/off (0/1 binary) variables

→ Pricing challenges due to binary variables (non-convexities); not to be discussed in this course

→ **Complex market-clearing problem,** while comparatively simple bids/offers.

Market-clearing as an optimization problem

An illustrative example

Please consider an electricity market with two conventional generators (G1 and G2) as well as one wind farm (W1), while the whole demand is <u>inelastic</u> to price.

W1

Production forecast: 20 MW
Offer price: $0/MWh

Capacity: 50 MW
Offer price: $20/MWh

Capacity: 100 MW
Offer price: $30/MWh

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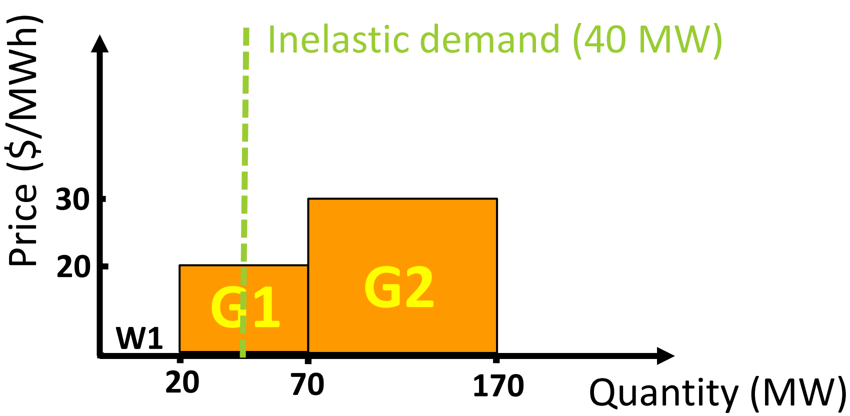

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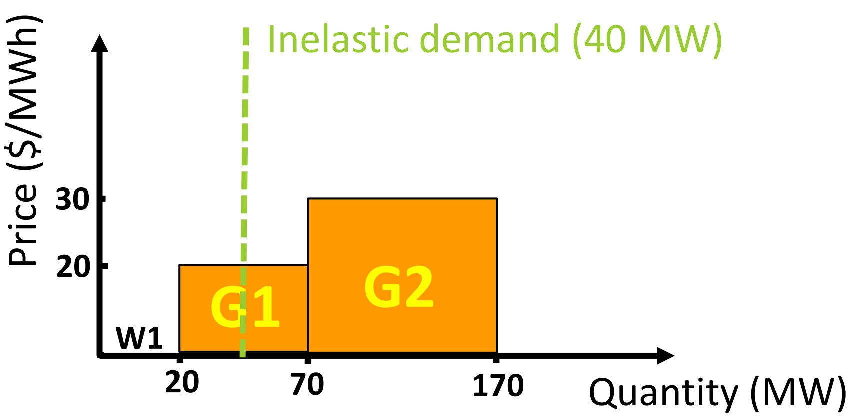

Market-clearing outcomes:

- Production level (MW) of W1: **?**
- Production level (MW) of G1: **?**
- Production level (MW) of G2: **?**
- Market-clearing price ($/MWh): **?**

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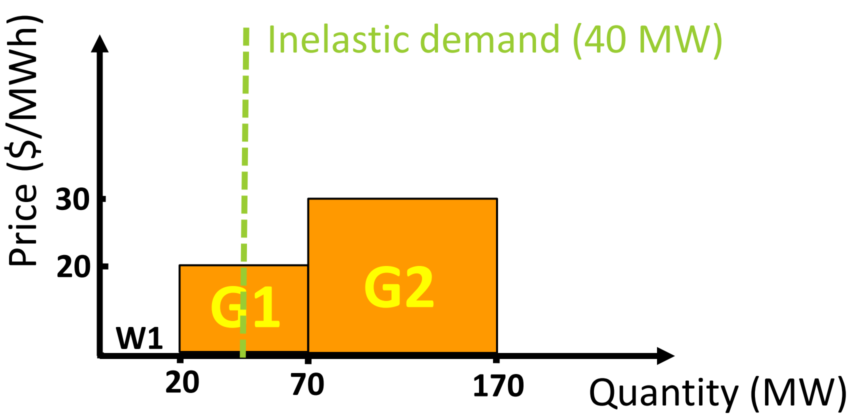

Market-clearing outcomes:

- Production level (MW) of W1: **20**
- Production level (MW) of G1: **20**
- Production level (MW) of G2: **0**
- Market-clearing price ($/MWh): **20**

Optimization problem

$$\underset{p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\mathrm{Minimize}} \quad 0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}}$$

subject to:

$$0 \leq p^{\mathrm{W1}} \leq 20 \quad : \quad \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}$$

$$0 \leq p^{\mathrm{G1}} \leq 50 \quad : \quad \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}$$

$$0 \leq p^{\mathrm{G2}} \leq 100 \quad : \quad \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}$$

$$40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \lambda$$

Optimization problem

$$\underset{p^{W1},p^{G1},p^{G2}}{\text{Minimize}} \quad 0\ p^{W1} + 20\ p^{G1} + 30\ p^{G2}$$

Set of primal variables

Objective function

subject to:

$$0 \leq p^{W1} \leq 20 \quad : \quad \underline{\mu}^{W1}, \overline{\mu}^{W1}$$

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$$40 - p^{W1} - p^{G1} - p^{G2} = 0 \quad : \quad \lambda$$

Set of constraints
(6 inequality constraints and one equality constraint)

Optimization problem

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Dual variables
(also known as Lagrangian multipliers),
one per constraint

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$$40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \boxed{\lambda}$$

Questions:

- What does a dual variable show (mathematical interpretation)?
- Do we know anything about the sign of different dual variables?

Dual variables
(also known as Lagrangian multipliers),
one per constraint

Optimization problem

$$\underbrace{\underset{p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\text{Minimize}} \quad 0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}}}_{\text{Objective function: Total production cost of the system}}$$

subject to:

$$\left.\begin{aligned} &0 \leq p^{\mathrm{W1}} \leq 20 \quad : \quad \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}} \\ &0 \leq p^{\mathrm{G1}} \leq 50 \quad : \quad \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}} \\ &0 \leq p^{\mathrm{G2}} \leq 100 \quad : \quad \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}} \end{aligned}\right\} \text{Generation limits}$$

$$40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \lambda \longrightarrow \text{Power balance}$$

Optimization problem

$$\underset{p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\text{Minimize}} \quad 0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}}$$

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$$0 \le p^{\mathrm{G1}} \le 50 \quad : \quad \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}$$

$$0 \le p^{\mathrm{G2}} \le 100 \quad : \quad \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}$$

$$40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \boxed{\lambda}$$

This dual variable corresponding to the power balance equality provides the market-clearing price (if we use the uniform pricing scheme)

Optimization problem

$$\underset{p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\mathrm{Minimize}} \quad 0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}}$$

subject to:

$$0 \leq p^{\mathrm{W1}} \leq 20 \quad : \quad \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}$$

$$0 \leq p^{\mathrm{G1}} \leq 50 \quad : \quad \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}$$

$$0 \leq p^{\mathrm{G2}} \leq 100 \quad : \quad \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}$$

$$40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \boxed{\lambda}$$

Question:

- How to compute the optimal value of this dual variable?

This dual variable corresponding to the power balance equality provides the market-clearing price (if we use the uniform pricing scheme)

How to derive Lagrangian function?

How to derive Lagrangian function?

$$\underset{x}{\text{Minimize}} \quad f(x)$$

$$\text{subject to:}$$

$$h(x) = 0 \quad : \quad \lambda$$

$$g(x) \leq 0 \quad : \quad \mu$$

This is a standard form of an optimization problem!

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This is a standard form of an optimization problem!

$$\mathcal{L}(x, \lambda, \mu) = f(x) + \lambda^\top h(x) + \mu^\top g(x)$$

How to derive optimality conditions?

Original (primal) problem

$$\underset{x}{\text{Minimize}} \quad f(x)$$

subject to:

$$h(x) = 0 \quad : \quad \lambda$$

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Lagrangian function

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Lagrangian function

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$$\frac{\partial \mathcal{L}(x, \lambda, \mu)}{\partial x} = 0$$

$$h(x) = 0$$

$$0 \leq -g(x) \perp \mu \geq 0$$

$$\lambda \in \text{free}$$

Optimality
Karush–Kuhn–Tucker (KKT)
conditions

How to derive optimality conditions?

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$$\lambda \in \text{free}$$

Optimality
Karush–Kuhn–Tucker (KKT)
conditions

Complementarity condition

Optimization problem

Let's get back to our original market-clearing problem:

$$\underset{p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\text{Minimize}} \quad 0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}}$$

subject to:

$$0 \leq p^{\mathrm{W1}} \leq 20 \quad : \quad \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}$$

$$0 \leq p^{\mathrm{G1}} \leq 50 \quad : \quad \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}$$

$$0 \leq p^{\mathrm{G2}} \leq 100 \quad : \quad \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}$$

$$40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \lambda$$

Optimization problem

Our original problem

$$\underset{p^{W1}, p^{G1}, p^{G2}}{\text{Minimize}} \quad 0\ p^{W1} + 20\ p^{G1} + 30\ p^{G2}$$

subject to:

$$0 \leq p^{W1} \leq 20 \quad : \quad \underline{\mu}^{W1}, \overline{\mu}^{W1}$$

$$0 \leq p^{G1} \leq 50 \quad : \quad \underline{\mu}^{G1}, \overline{\mu}^{G1}$$

$$0 \leq p^{G2} \leq 100 \quad : \quad \underline{\mu}^{G2}, \overline{\mu}^{G2}$$

$$40 - p^{W1} - p^{G1} - p^{G2} = 0 \quad : \quad \lambda$$

Standard form of our original problem

$$\underset{p^{W1}, p^{G1}, p^{G2}}{\text{Minimize}} \quad 0\ p^{W1} + 20\ p^{G1} + 30\ p^{G2}$$

subject to:

$$-p^{W1} \leq 0 \quad : \quad \underline{\mu}^{W1}$$

$$p^{W1} - 20 \leq 0 \quad : \quad \overline{\mu}^{W1}$$

$$-p^{G1} \leq 0 \quad : \quad \underline{\mu}^{G1}$$

$$p^{G1} - 50 \leq 0 \quad : \quad \overline{\mu}^{G1}$$

$$-p^{G2} \leq 0 \quad : \quad \underline{\mu}^{G2}$$

$$p^{G2} - 100 \leq 0 \quad : \quad \overline{\mu}^{G2}$$

$$40 - p^{W1} - p^{G1} - p^{G2} = 0 \quad : \quad \lambda$$

Lagrangian function

Standard form of our original problem

$$\underset{p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\text{Minimize}} \quad 0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}}$$

subject to:

$$-p^{\mathrm{W1}} \leq 0 \quad : \quad \underline{\mu}^{\mathrm{W1}}$$
$$p^{\mathrm{W1}} - 20 \leq 0 \quad : \quad \overline{\mu}^{\mathrm{W1}}$$
$$-p^{\mathrm{G1}} \leq 0 \quad : \quad \underline{\mu}^{\mathrm{G1}}$$
$$p^{\mathrm{G1}} - 50 \leq 0 \quad : \quad \overline{\mu}^{\mathrm{G1}}$$
$$-p^{\mathrm{G2}} \leq 0 \quad : \quad \underline{\mu}^{\mathrm{G2}}$$
$$p^{\mathrm{G2}} - 100 \leq 0 \quad : \quad \overline{\mu}^{\mathrm{G2}}$$
$$40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \lambda$$

Lagrangian function

$$\begin{aligned}\mathcal{L}(p^{\mathrm{W1}}, p^{\mathrm{G1}}, p^{\mathrm{G2}}, \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}, \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}, \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}, \lambda) = \\ 0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}} \\ - p^{\mathrm{W1}} \underline{\mu}^{\mathrm{W1}} + (p^{\mathrm{W1}} - 20)\, \overline{\mu}^{\mathrm{W1}} \\ - p^{\mathrm{G1}} \underline{\mu}^{\mathrm{G1}} + (p^{\mathrm{G1}} - 50)\, \overline{\mu}^{\mathrm{G1}} \\ - p^{\mathrm{G2}} \underline{\mu}^{\mathrm{G2}} + (p^{\mathrm{G2}} - 100)\, \overline{\mu}^{\mathrm{G2}} \\ + \lambda \left(40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}}\right)\end{aligned}$$

KKT conditions

$$\mathcal{L}(p^{\mathrm{W1}}, p^{\mathrm{G1}}, p^{\mathrm{G2}}, \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}, \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}, \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}, \lambda) =$$

$$\begin{aligned}
&0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}} \\
&- p^{\mathrm{W1}} \underline{\mu}^{\mathrm{W1}} + \left(p^{\mathrm{W1}} - 20\right) \overline{\mu}^{\mathrm{W1}} \\
&- p^{\mathrm{G1}} \underline{\mu}^{\mathrm{G1}} + \left(p^{\mathrm{G1}} - 50\right) \overline{\mu}^{\mathrm{G1}} \\
&- p^{\mathrm{G2}} \underline{\mu}^{\mathrm{G2}} + \left(p^{\mathrm{G2}} - 100\right) \overline{\mu}^{\mathrm{G2}} \\
&+ \lambda \left(40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}}\right)
\end{aligned}$$

$\Downarrow$ KKT conditions

KKT conditions

$$\mathcal{L}(p^{\mathrm{W1}}, p^{\mathrm{G1}}, p^{\mathrm{G2}}, \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}, \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}, \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}, \lambda) =$$

$$\begin{aligned}
&0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}} \\
&- p^{\mathrm{W1}} \underline{\mu}^{\mathrm{W1}} + (p^{\mathrm{W1}} - 20)\, \overline{\mu}^{\mathrm{W1}} \\
&- p^{\mathrm{G1}} \underline{\mu}^{\mathrm{G1}} + (p^{\mathrm{G1}} - 50)\, \overline{\mu}^{\mathrm{G1}} \\
&- p^{\mathrm{G2}} \underline{\mu}^{\mathrm{G2}} + (p^{\mathrm{G2}} - 100)\, \overline{\mu}^{\mathrm{G2}} \\
&+ \lambda \left(40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}}\right)
\end{aligned}$$

KKT conditions

$$\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{W1}}} = 0 + \overline{\mu}^{\mathrm{W1}} - \underline{\mu}^{\mathrm{W1}} - \lambda = 0$$

$$\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{G1}}} = 20 + \overline{\mu}^{\mathrm{G1}} - \underline{\mu}^{\mathrm{G1}} - \lambda = 0$$

$$\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{G2}}} = 30 + \overline{\mu}^{\mathrm{G2}} - \underline{\mu}^{\mathrm{G2}} - \lambda = 0$$

$$40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0$$

$$0 \le \left(20 - p^{\mathrm{W1}}\right) \perp \overline{\mu}^{\mathrm{W1}} \ge 0$$

$$0 \le p^{\mathrm{W1}} \perp \underline{\mu}^{\mathrm{W1}} \ge 0$$

$$0 \le \left(50 - p^{\mathrm{G1}}\right) \perp \overline{\mu}^{\mathrm{G1}} \ge 0$$

$$0 \le p^{\mathrm{G1}} \perp \underline{\mu}^{\mathrm{G1}} \ge 0$$

$$0 \le \left(100 - p^{\mathrm{G2}}\right) \perp \overline{\mu}^{\mathrm{G2}} \ge 0$$

$$0 \le p^{\mathrm{G2}} \perp \underline{\mu}^{\mathrm{G2}} \ge 0$$

KKT conditions

$$\mathcal{L}(p^{\mathrm{W1}}, p^{\mathrm{G1}}, p^{\mathrm{G2}}, \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}, \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}, \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}, \lambda) =$$

$$\begin{aligned}
&0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}} \\
&- p^{\mathrm{W1}}\underline{\mu}^{\mathrm{W1}} + (p^{\mathrm{W1}} - 20)\,\overline{\mu}^{\mathrm{W1}} \\
&- p^{\mathrm{G1}}\underline{\mu}^{\mathrm{G1}} + (p^{\mathrm{G1}} - 50)\,\overline{\mu}^{\mathrm{G1}} \\
&- p^{\mathrm{G2}}\underline{\mu}^{\mathrm{G2}} + (p^{\mathrm{G2}} - 100)\,\overline{\mu}^{\mathrm{G2}} \\
&+ \lambda\left(40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}}\right)
\end{aligned}$$

KKT conditions

We can solve KKT conditions as a system of equations!

$$\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{W1}}} = 0 + \overline{\mu}^{\mathrm{W1}} - \underline{\mu}^{\mathrm{W1}} - \lambda = 0$$

$$\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{G1}}} = 20 + \overline{\mu}^{\mathrm{G1}} - \underline{\mu}^{\mathrm{G1}} - \lambda = 0$$

$$\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{G2}}} = 30 + \overline{\mu}^{\mathrm{G2}} - \underline{\mu}^{\mathrm{G2}} - \lambda = 0$$

$$40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0$$

$$0 \leq \left(20 - p^{\mathrm{W1}}\right) \perp \overline{\mu}^{\mathrm{W1}} \geq 0$$

$$0 \leq p^{\mathrm{W1}} \perp \underline{\mu}^{\mathrm{W1}} \geq 0$$

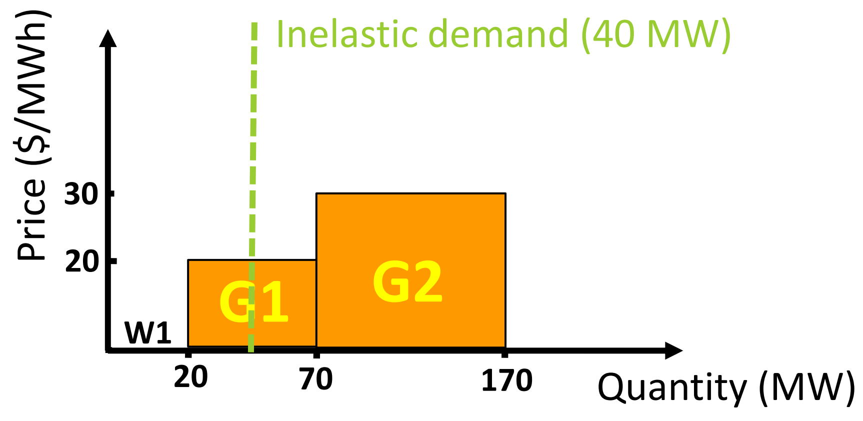
$$0 \leq \left(50 - p^{\mathrm{G1}}\right) \perp \overline{\mu}^{\mathrm{G1}} \geq 0$$

$$0 \leq p^{\mathrm{G1}} \perp \underline{\mu}^{\mathrm{G1}} \geq 0$$

$$0 \leq \left(100 - p^{\mathrm{G2}}\right) \perp \overline{\mu}^{\mathrm{G2}} \geq 0$$

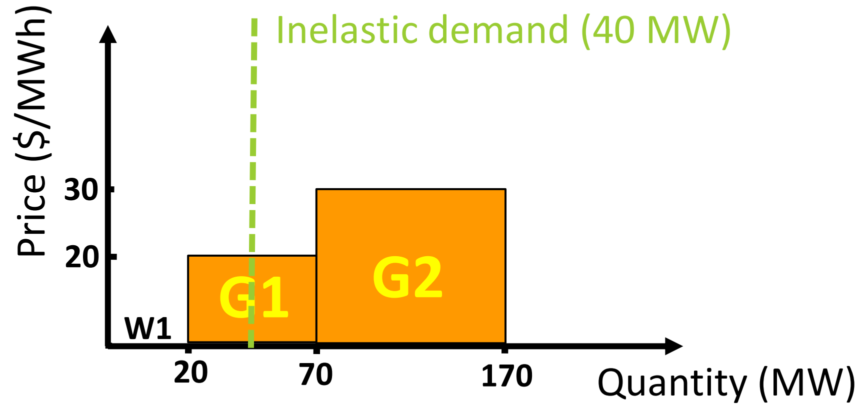
$$0 \leq p^{\mathrm{G2}} \perp \underline{\mu}^{\mathrm{G2}} \geq 0$$

KKT conditions



If we solve KKT conditions, we will achieve $p^{\mathrm{G2}} = 20$

KKT conditions

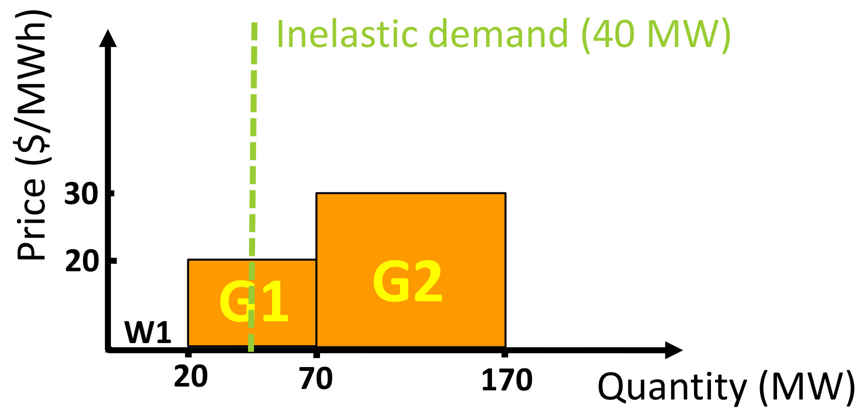


If we solve KKT conditions, we will achieve $p^{\mathrm{G2}} = 20$

$$0 \leq \left(50 - p^{\mathrm{G1}}\right) \perp \overline{\mu}^{\mathrm{G1}} \geq 0$$
$$0 \leq p^{\mathrm{G1}} \perp \mu^{\mathrm{G1}} \geq 0$$

$$\underline{\mu}^{\mathrm{G1}} = 0$$
$$\overline{\mu}^{\mathrm{G1}} = 0$$

KKT conditions



If we solve KKT conditions, we will achieve $p^{\mathrm{G2}} = 20$

$$0 \le \left(50 - p^{\mathrm{G1}}\right) \perp \overline{\mu}^{\mathrm{G1}} \ge 0$$
$$0 \le p^{\mathrm{G1}} \perp \underline{\mu}^{\mathrm{G1}} \ge 0$$

$$\underline{\mu}^{\mathrm{G1}} = 0$$
$$\overline{\mu}^{\mathrm{G1}} = 0$$

$$\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{G1}}} = 20 + \overline{\mu}^{\mathrm{G1}} - \underline{\mu}^{\mathrm{G1}} - \lambda = 0 \quad \Rightarrow \quad \lambda = 20$$

So, the market-clearing price is indeed 20!

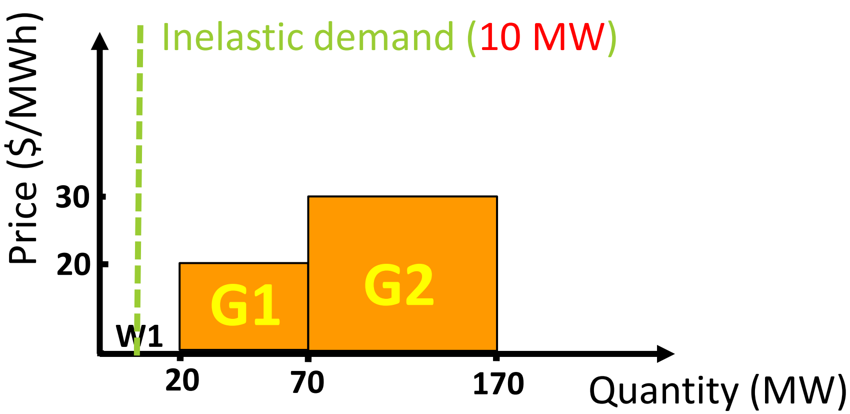
An illustrative example

Please consider an electricity market with two conventional generators (G1 and G2) as well as one wind farm (W1), while the whole demand is <u>inelastic</u> to price.

Production forecast: 20 MW
Offer price: $0/MWh

Capacity: 50 MW
Offer price: $20/MWh

Capacity: 100 MW
Offer price: $30/MWh



An illustrative example

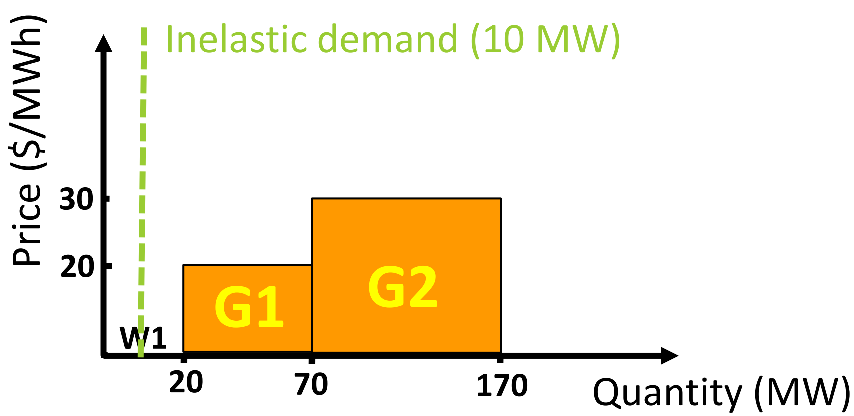
Please consider an electricity market with two conventional generators (G1 and G2) as well as one wind farm (W1), while the whole demand is inelastic to price.

W1

Production forecast: 20 MW
Offer price: \$0/MWh

Capacity: 50 MW
Offer price: \$20/MWh

Capacity: 100 MW
Offer price: \$30/MWh



Market-clearing outcomes:

- Production level (MW) of W1: **10**
- Production level (MW) of G1: **0**
- Production level (MW) of G2: **0**
- Market-clearing price (\$/MWh): **0**

Check KKTs

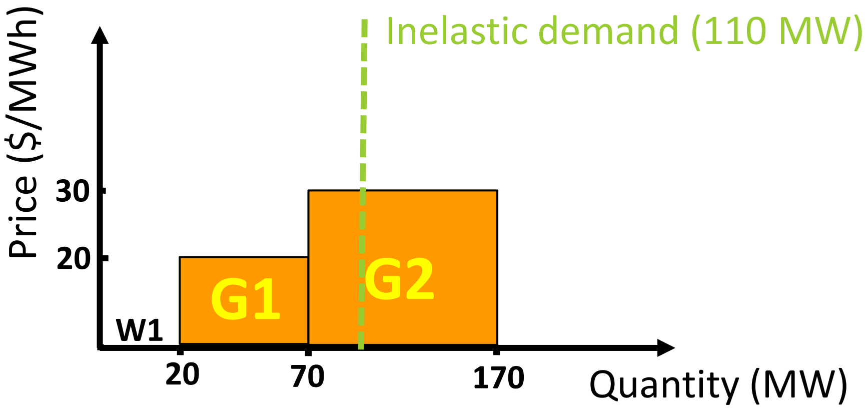
An illustrative example

Please consider an electricity market with two conventional generators (G1 and G2) as well as one wind farm (W1), while the whole demand is <u>inelastic</u> to price.

Production forecast: 20 MW
Offer price: $0/MWh

Capacity: 50 MW
Offer price: $20/MWh

Capacity: 100 MW
Offer price: $30/MWh



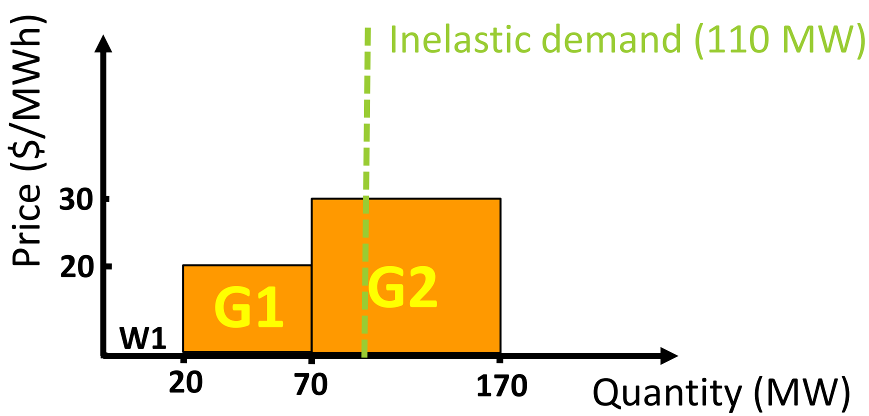
An illustrative example

Please consider an electricity market with two conventional generators (G1 and G2) as well as one wind farm (W1), while the whole demand is <u>inelastic</u> to price.

Production forecast: 20 MW
Offer price: \$0/MWh

Capacity: 50 MW
Offer price: \$20/MWh

Capacity: 100 MW
Offer price: \$30/MWh



Market-clearing outcomes:

- Production level (MW) of W1: **20**
- Production level (MW) of G1: **50**
- Production level (MW) of G2: **40**
- Market-clearing price (\$/MWh): **30**

Check KKTs

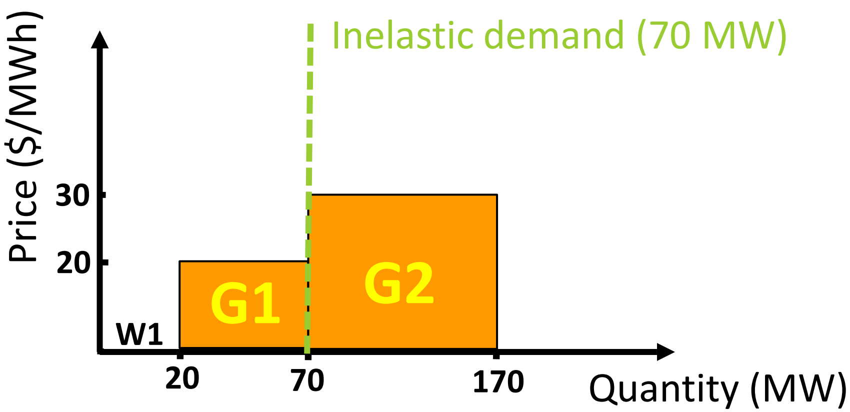
An illustrative example

Please consider an electricity market with two conventional generators (G1 and G2) as well as one wind farm (W1), while the whole demand is <u>inelastic</u> to price.

Production forecast: 20 MW
Offer price: $0/MWh

Capacity: 50 MW
Offer price: $20/MWh

Capacity: 100 MW
Offer price: $30/MWh



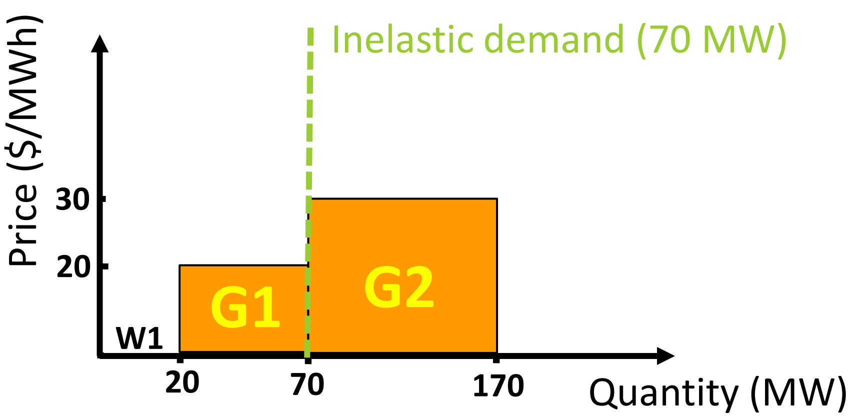
An illustrative example

Please consider an electricity market with two conventional generators (G1 and G2) as well as one wind farm (W1), while the whole demand is <u>inelastic</u> to price.

Production forecast: 20 MW
Offer price: $0/MWh

Capacity: 50 MW
Offer price: $20/MWh

Capacity: 100 MW
Offer price: $30/MWh



Market-clearing outcomes:

- Production level (MW) of W1: **20**
- Production level (MW) of G1: **50**
- Production level (MW) of G2: **0**
- Market-clearing price ($/MWh): **[20,30]**

Price multiplicity: any price between 20 and 30 (including both) can clear the market (check KKTs)!

An illustrative example

Please consider an electricity market with two conventional generators (G1 and G2) as well as one wind farm (W1), while there is a single demand (D1) which is elastic to price.

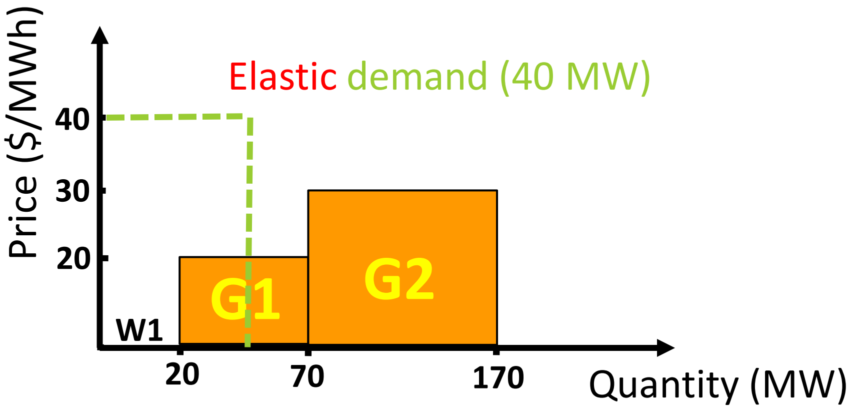
Production forecast: 20 MW
Offer price: $0/MWh

Capacity: 50 MW
Offer price: $20/MWh

Capacity: 100 MW
Offer price: $30/MWh

Demand D1

Maximum load: 40 MW
Bid price: $40/MWh



An illustrative example

Please consider an electricity market with two conventional generators (G1 and G2) as well as one wind farm (W1), while there is a single demand (D1) which is elastic to price.

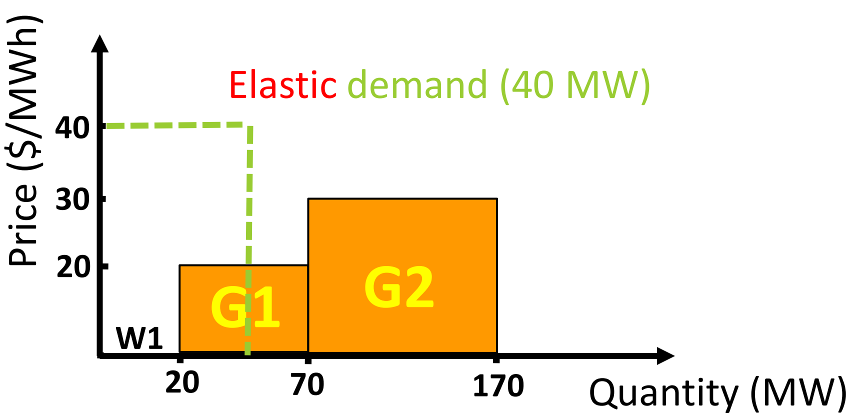
Production forecast: 20 MW
Offer price: $0/MWh

Capacity: 50 MW
Offer price: $20/MWh

Capacity: 100 MW
Offer price: $30/MWh

Demand D1

Maximum load: 40 MW
Bid price: $40/MWh



Market-clearing outcomes:

- Production level (MW) of W1: **20**
- Production level (MW) of G1: **20**
- Production level (MW) of G2: **0**
- Consumption level (MW) of D1: **40**
- Market-clearing price ($/MWh): **20**

Check KKTs

Optimization problem with elastic demand

$$\underset{\boxed{p^{\mathrm{D1}}},p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\text{Maximize}} \quad 40 \boxed{p^{\mathrm{D1}}} - 0\ p^{\mathrm{W1}} - 20\ p^{\mathrm{G1}} - 30\ p^{\mathrm{G2}}$$

Newly added primal variable: Consumption level of elastic demand D1

subject to:

$$0 \le p^{\mathrm{W1}} \le 20 \quad : \quad \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}$$

$$0 \le p^{\mathrm{G1}} \le 50 \quad : \quad \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}$$

$$0 \le p^{\mathrm{G2}} \le 100 \quad : \quad \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}$$

$$0 \le \boxed{p^{\mathrm{D1}}} \le 40 \quad : \quad \underline{\mu}^{\mathrm{D1}}, \overline{\mu}^{\mathrm{D1}}$$

$$\boxed{p^{\mathrm{D1}}} - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \lambda$$

Optimization problem with elastic demand

$$\underset{p^{\mathrm{D1}},p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\text{Maximize}} \quad 40\ p^{\mathrm{D1}} - 0\ p^{\mathrm{W1}} - 20\ p^{\mathrm{G1}} - 30\ p^{\mathrm{G2}}$$

subject to:

$$0 \leq p^{\mathrm{W1}} \leq 20 \quad : \quad \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}$$

$$0 \leq p^{\mathrm{G1}} \leq 50 \quad : \quad \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}$$

$$0 \leq p^{\mathrm{G2}} \leq 100 \quad : \quad \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}$$

$$0 \leq p^{\mathrm{D1}} \leq 40 \quad : \quad \boxed{\underline{\mu}^{\mathrm{D1}}, \overline{\mu}^{\mathrm{D1}}}$$ Newly added dual variables

$$p^{\mathrm{D1}} - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \lambda$$

Optimization problem with elastic demand

$$\underset{p^{\mathrm{D1}},p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\text{Maximize}} \quad \underbrace{40\ p^{\mathrm{D1}} - 0\ p^{\mathrm{W1}} - 20\ p^{\mathrm{G1}} - 30\ p^{\mathrm{G2}}}_{\text{Objective function: Social welfare}}$$

subject to:

$$0 \le p^{\mathrm{W1}} \le 20 \quad : \quad \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}$$

$$0 \le p^{\mathrm{G1}} \le 50 \quad : \quad \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}$$

$$0 \le p^{\mathrm{G2}} \le 100 \quad : \quad \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}$$

$$0 \le p^{\mathrm{D1}} \le 40 \quad : \quad \underline{\mu}^{\mathrm{D1}}, \overline{\mu}^{\mathrm{D1}}$$

$$p^{\mathrm{D1}} - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \lambda$$

Market clearing as an optimization problem

Compact form:

Market clearing as an optimization problem

Compact form:

$$\underset{p_g^{\mathrm{G}},p_d^{\mathrm{D}}}{\text{Maximize}} \quad SW = \sum_d U_d \; p_d^{\mathrm{D}} - \sum_g C_g \; p_g^{\mathrm{G}} \qquad (1a)$$

subject to:

$$0 \leq p_d^{\mathrm{D}} \leq \overline{P}_d^{\mathrm{D}} \quad \forall d \qquad (1b)$$

$$0 \leq p_g^{\mathrm{G}} \leq \overline{P}_g^{\mathrm{G}} \quad \forall g \qquad (1c)$$

$$\sum_d p_d^{\mathrm{D}} - \sum_g p_g^{\mathrm{G}} = 0 \qquad : \lambda \qquad (1d)$$

U_d : bid price of demand d

C_g : offer price of generator g

$\overline{P}_d^{\mathrm{D}}$: maximum load of demand d

$\overline{P}_g^{\mathrm{G}}$: capacity of generator g

Thanks for your attention!