Assignment 2: ARMA Processes and Seasonal Processes

Instructions: The assignment is to be handed in via DTU Learn "FeedbackFruits" latest at March 15th at 23:59. You are allowed to hand in in groups of up to max 4 persons. You must hand in a single pdf file presenting the results using text, math, tables and plots, do not include code in the report!. Arrange the report in sections and subsections according to the questions in this document. Please indicate your student numbers on the report.

Be aware that the model parametrizations in the assignment and in your favorite software package may not be identical. You will study how the choice of coefficients (of the operator polynomials in ARMA processes) affects the structure of the process, through simulated data and empirical autocorrelation functions (ACFs). The assignment is to some extent specified in terms of R commands. Users of other software packages should find similar functions to answer the questions.

1 Stability

Let the process $\{X_t\}$ be an AR(2) given by

$$X_t + \phi_1 X_{t-1} + \phi_2 X_{t-2} = \epsilon_t$$

where $\{\epsilon_t\}$ is a white noise process with $\sigma_\varepsilon = 1$.

Answer the following:

1.1. Determine if the process is stationary for $\phi_1 = -0.7$ and $\phi_2 = -0.2$ by analysing the roots of the characteristic equation.

1.2. Is the process invertible?

1.3. Write the autocorrelation $\gamma(k)$ for the AR(2) process as function of ϕ_1 and ϕ_2.

1.4. Plot the autocorrelation $\gamma(k)$ up to $n_{\text{lag}} = 30$ for the coefficient values above.

1.5. Simulate 5 realizations of the process up to $n = 200$ observations with the coefficient values given above. Plot them in one plot. (Remember set a seed when you do simulations).

1.6. Calculate the empirical ACF of the simulations and plot them together with $\gamma(k)$, up to lag 30. Comment on the result.

1.7. Keep $\phi_2 = -0.2$ and redo plots of simulations and ACFs for the following ϕ_1-values. For each comment shortly on the outcome, especially focus on stationarity and compare the results across the simulations.
First $\phi_1 = -0.2$.

1.8. $\phi_1 = 0.7$

1.9. $\phi_1 = -0.8$

1.10. $\phi_1 = -0.85$.

1.11. Would you recommend always plotting the time series data or does it provide sufficient information just to examine the ACF?

2 Predicting monthly solar power

In renewable energy systems forecasting is really import to plan the operation of systems. The forecasting horizon can both be short, but also longer, as in the case below, where the next twelve months electricity generation on a solar PV plant is to be predicted.

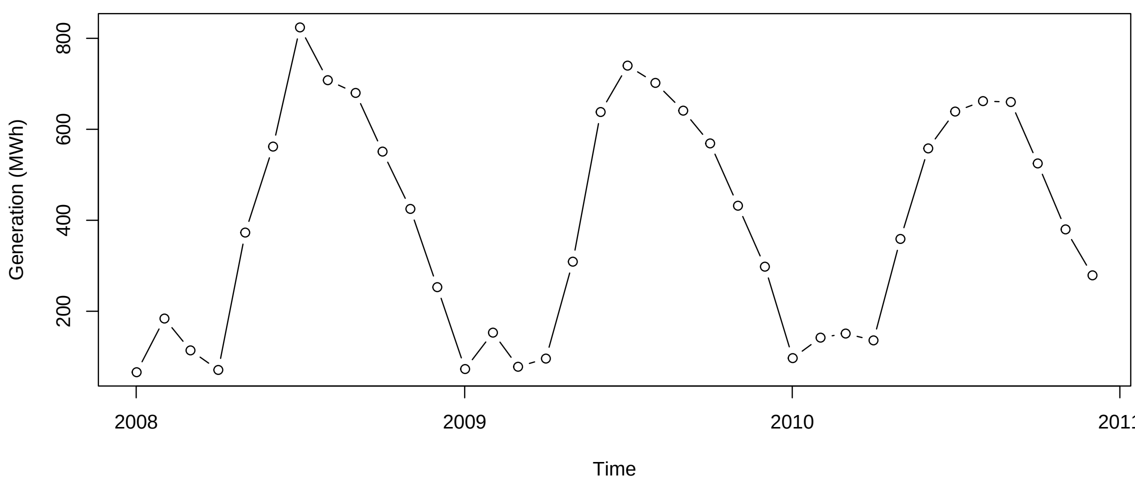
Based on historical data the following seasonal AR model has been identified a particular plant

$$(1 + \phi_1 B)(1 + \Phi_1 B^{12})(\log(Y_t) - \mu) = \varepsilon_t$$

where Y_t is the monthly energy from the plant in MWh, $\{\varepsilon_t\}$ is a white-noise process with variance σ_ε^2. The parameters $\phi_1 = -0.38$, $\Phi_1 = -0.94$ and $\mu = 5.72$ are assumed to be known. Based on 36 observations, it is found that $\sigma_\epsilon^2 = 0.22^2$.

The file `datasolar.csv` holds the measurements of monthly electricity generation from the plant available.

A plot of the time series is



2.1. Introduce $X_t = \log(Y_t) - \mu$ and re-write the model to calculate the residuals $\hat{\varepsilon}_{t+1|t}$. Do a model validation by checking the assumptions of i.i.d. errors.

2.2. With the specified model calculate $\hat{Y}_{t+k|t}$ for $t = 36$ and $k = 1, \ldots, 12$, i.e. predict the power for the following twelve months. Provide the values in a table and plot them extending the observed time series (note, remember to transform back to power).

2.3. Calculate 95% prediction intervals for the twelve months ahead and add them to the plot. Use Eq. (5.149)-(5.151) and note that you can do the calculation it without including the seasonal part! So just use the AR(1) part of the model.

2.4. Comment: would you trust the forecast? Do you think the prediction intervals have correct width (all the time)?

3 Simulating seasonal processes

A process $\{Y_t\}$ is said to follow a multiplicative $(p, d, q) \times (P, D, Q)_s$ seasonal model if

$$\phi(B)\Phi(B^s)\nabla^d\nabla_s^D Y_t = \theta(B)\Theta(B^s)\epsilon_t$$

where $\{\epsilon_t\}$ is a white noise process, and $\phi(B)$ and $\theta(B)$ are polynomials of order p and q, respectively. Furthermore, $\Phi(B^s)$ and $\Theta(B^s)$ are polynomials in B^s. All according to Definition 5.22 in the textbook.

Note: `arima.sim` does not have a seasonal module, so model formulations as standard ARIMA processes have to be made when using that function.

Simulate the following models. Plot the simulations and the associated autocorrelation functions (ACF and PACF). Comment on each result:

3.1. A $(1,0,0) \times (0,0,0)_{12}$ model with the parameter $\phi_1 = 0.6$.

3.2. A $(0,0,0) \times (1,0,0)_{12}$ model with the parameter $\Phi_1 = -0.9$.

3.3. A $(1,0,0) \times (0,0,1)_{12}$ model with the parameters $\phi_1 = 0.9$ and $\Theta_1 = -0.7$.

3.4. A $(1,0,0) \times (1,0,0)_{12}$ model with the parameters $\phi_1 = -0.6$ and $\Phi_1 = -0.8$.

3.5. A $(0,0,1) \times (0,0,1)_{12}$ model with the parameters $\theta_1 = 0.4$ and $\Theta_1 = -0.8$.

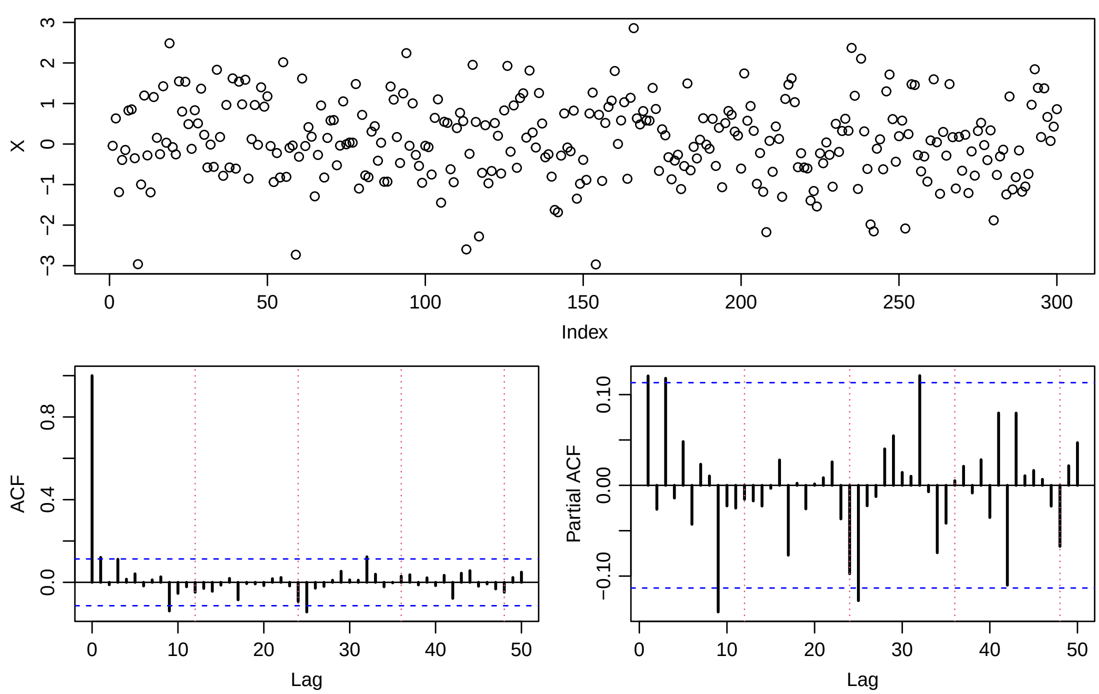
3.6. A $(0,0,1) \times (1,0,0)_{12}$ model with the parameters $\theta_1 = -0.4$ and $\Phi_1 = 0.7$.

3.7. Summarize your observations on the processes and the autocorrelation functions. Which conclusions can you draw on the general behavior of the autocorrelation function for seasonal processes?

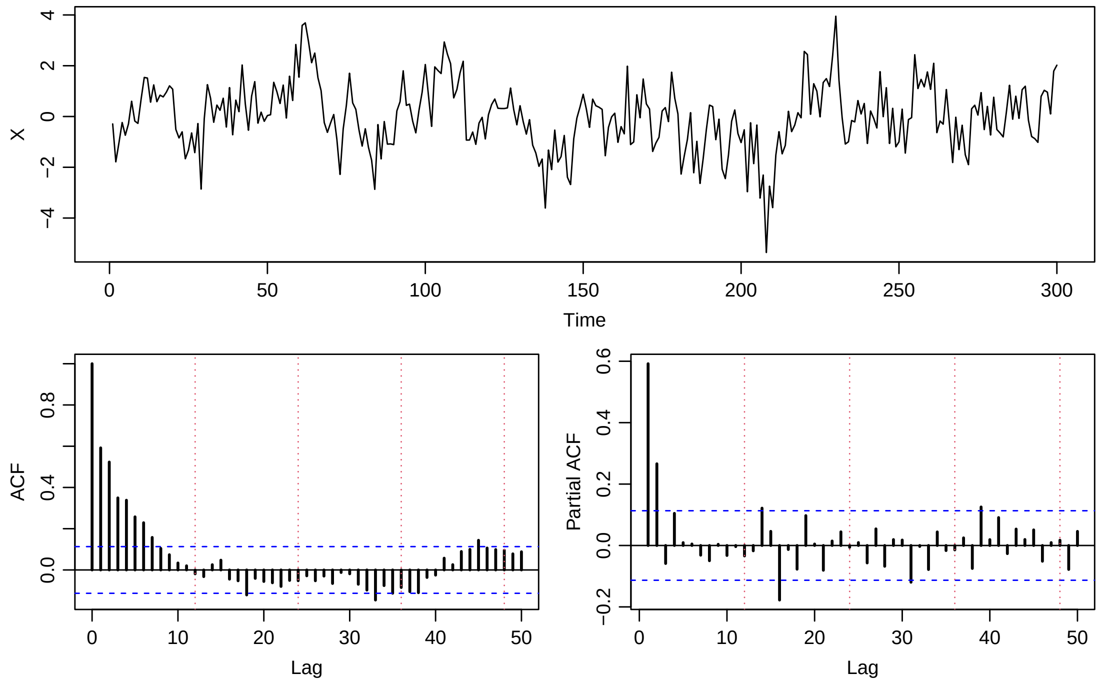
4 Identifying arma models

Below are plots of three simulated ARMA processes: time series plot, ACF and PACF. Guess the ARMA model structure for each of them, and give a short reasoning of your guess.

4.1. Process 1:



4.2. Process 2:



4.3. Process 3: