PYTHON FOR THE FINANCIAL ECONOMIST, RETAKE EXAM 2022
HEDGING A CASH FLOW AND PORTFOLIO OPTIMIZATION

Copenhagen Business School
27. February 2023

2 weeks, home assignment

The home assignment is to be answered in groups of two students (maximum of 25 A4-pages) or individually (maximum of 15 A4-pages). The students must individualize the assignment.

The take-home assignment should take the form of an academic report written in either Word or Latex converted to pdf format. It is expected that students present relevant formulas, present results using visualizations and tables, include relevant references, etc. The overall impression of how the results are presented will count in the assessment.

The analysis should be performed using Python. Please attach code / Jupyter notebooks.

If you think that you do not have all the necessary information to answer a problem, make the necessary assumptions in order to proceed and state these assumptions in the solution.

Good luck!

Hedging a cash flow

An investor is saving up for retirement in 10 years. To that end, the investor needs 100,000 USD pr. year with a 2% annual cost of living adjustment from year 11 to year 30. Thus, 100,000 USD in year 11, 102,000 in year 12, and so on. Plot the cash flows with a bar plot.

The investor seeks our advice on how to invest her wealth to ensure realizing these cash flows. Assume that the investor can trade the following bonds all with face value of 1:

- 1 year zero-coupon bond
- 5 year bullet bond with 2% coupon rate (yearly payment)
- 7 year bullet bond with 2% coupon rate (yearly payment)
- 10 year bullet bond with 3% coupon rate (yearly payment)
- 15 year bullet bond with 3% coupon rate (yearly payment)
- 20 year bullet bond with 3% coupon rate (yearly payment)
- 25 year bullet bond with 4% coupon rate (yearly payment)
- 30 year bullet bond with 4% coupon rate (yearly payment)

Assume that we are standing in the beginning of 2000 (3rd of January). FRED provides nominal yield curve data interpolated using the Nelson-Siegel-Svensson specification. Obtain and plot the yield curve. Calculate the price, duration and convexity of each bond and the retirement cash flow. Present results in tabular form and comment on your findings. Note that the price of a bond is given by

$$B_t = \sum_{T_i > t} Y_{T_i} B_t^{T_i} = \sum_{T_i > t} Y_{T_i} e^{-y_t^{T_i}(T_i - t)}$$

where Y_{T_i} is the bond payments and $y_t^{T_i}$ is the zero-coupon yield (cont. compounded). The duration is given by

$$D_t = \frac{1}{B_t} \sum_{T_i > t} Y_{T_i} (T_i - t) e^{-y_t^{T_i}(T_i - t)}$$

and the convexity is given by

$$C_t = \frac{1}{B_t} \sum_{T_i > t} Y_{T_i} (T_i - t)^2 e^{-y_t^{T_i}(T_i - t)}$$

HINT: The classes *CashFlow* and *NelsonSiegelSvensson* are available in *codelib*.

Resample the yield curve data monthly based on the first day of the month. Assume that there is exactly 1/12 year between each observations. Calculate and plot the duration and convexity for the retirement cash flow and the bonds for the next 20 years, when assuming that the one year zero coupon bond is replaced with a new one each period. Explain your findings. HINT: Note that the methods on the *CashFlow* object has an input *time_shift*.

We seek to select a hedging portfolio that matches the duration or the duration and convexity of the retirement cash flow. Basically, we need to solve the system of equations

$$\mathbf{H}^\top \mathbf{x} = \mathbf{d}$$

where $\mathbf{x}$ is the column vector of portfolio weights, $\mathbf{d}$ is the column vector of quantities to be matched (e.g. duration) and $\mathbf{H}$ is a matrix where the columns represent the quantities across all bonds (e.g. the first column may contain durations for all bonds). The system of equations will in our case not have an exact solution (if we want to use all bonds). To solve the problem, we follow Mantilla-Garcia et al. (2022) and define the minimization problem with leverage penalty

$$\min_x \frac{1}{2}\mathbf{x}^\top \boldsymbol{\Lambda} \mathbf{x} - \mathbf{x}^\top \boldsymbol{\Theta} + \lambda \left(\sum_{i=1}^{n} |x_i| \right)$$

subject to the constraint

$$\sum_{i=1}^{n} x_i = 1$$

where $\boldsymbol{\Lambda} = \mathbf{H}^\top \mathbf{H}$ and $\boldsymbol{\Theta} = \mathbf{H}\mathbf{d}$. The coefficient λ can be set such that the leverage of the portfolio is below a given leverage budget δ (set $\delta = 3$ in the provided function or in your own code). NOTE: The function *solve_hedging_problem* is provided (remember to install the package *cvxopt*). For the initial date (3rd of January 2000), find two optimal hedging portfolios when it is allowed to borrow in the 1 year zero coupon bond: One portfolio that seeks to ensure that the duration and market value of the hedging portfolio are the same as for the retirement cash flow and one portfolios that also seeks to ensure that the convexity is the same. Comment on the results.

Find the optimal hedging portfolio for the next 20 years when seeking to hedge both duration and convexity. How does the optimal hedge portfolio change over time? Visualize the duration and convexity of the retirement cash flow and the hedge portfolio in the same figure to check that you have a reasonable hedge.

Optimal portfolios

An investor can invest in 10 different assets: Government bonds, Investment-grade bonds, High-yield bonds, Emerging markets gov. bonds, Equities (developed markets), Equities (Emerging markets), Private equity, Infrastructure, Real Estate, and Hedgefunds. The continuously compounded / log-returns are assumed to be i.i.d. multivariate normally distributed

$$\mathbf{r}_{t+1,1} \sim MVN(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

with correlation matrix

$$\mathbf{C} = \begin{bmatrix} 1.0 & 0.6 & 0.1 & 0.3 & -0.1 & -0.1 & -0.2 & -0.1 & -0.1 & -0.1 \\ 0.6 & 1.0 & 0.6 & 0.6 & 0.2 & 0.2 & 0.2 & 0.1 & 0.1 & 0.3 \\ 0.1 & 0.6 & 1.0 & 0.7 & 0.7 & 0.6 & 0.6 & 0.4 & 0.3 & 0.7 \\ 0.3 & 0.6 & 0.7 & 1.0 & 0.5 & 0.6 & 0.4 & 0.2 & 0.2 & 0.5 \\ -0.1 & 0.2 & 0.7 & 0.5 & 1.0 & 0.7 & 0.8 & 0.4 & 0.4 & 0.8 \\ -0.1 & 0.2 & 0.6 & 0.6 & 0.7 & 1.0 & 0.7 & 0.4 & 0.4 & 0.7 \\ -0.2 & 0.2 & 0.6 & 0.4 & 0.8 & 0.7 & 1.0 & 0.4 & 0.4 & 0.7 \\ -0.1 & 0.1 & 0.4 & 0.2 & 0.4 & 0.4 & 0.4 & 1.0 & 0.3 & 0.4 \\ -0.1 & 0.1 & 0.3 & 0.2 & 0.4 & 0.4 & 0.4 & 0.3 & 1.0 & 0.4 \\ -0.1 & 0.3 & 0.7 & 0.5 & 0.8 & 0.7 & 0.7 & 0.4 & 0.4 & 1.0 \end{bmatrix},$$

vector of volatilities

$$\mathbf{v} = \left[0.037, 0.055, 0.119, 0.107, 0.153, 0.217, 0.204, 0.14, 0.108, 0.094\right]^\top,$$

and expected log return

$$\boldsymbol{\mu} = \left[0.019, 0.022, 0.049, 0.043, 0.061, 0.083, 0.102, 0.056, 0.041, 0.038\right]^\top - \frac{1}{2}\mathrm{Diag}(\boldsymbol{\Sigma}),$$

Describe how to obtain the mean-variance efficient frontier based on a 1 year investment horizon with a long-only constraint, e.g. what expected return vector and covariance matrix to use and how is the optimization problem defined? Find at least 50 portfolios on the efficient frontier. Additionally, calculate the 1 year minimum variance portfolio and tangency portfolio (assuming a risk free rate of 0%). Simulate at least 100 random portfolios. Visualize the results.

Explain how to calculate the relative risk contribution when using standard deviation as risk measure. Calculate and plot the relative risk contribution for the minimum-variance and tangency portfolio.

The investor wants to examine another optimal portfolio, namely the equal risk contribution (ERC) portfolio. Explain how to calculate the ERC portfolio. Calculate the portfolio and add it to your previous visualization (see e.g. Maillard et al. (2010)).

Present expected return, standard deviation and Sharpe ratio (assuming a risk free rate of 0%) for the minimum-variance, tangency portfolio and ERC portfolio in tabular form. Comment on the results.

The investor wants to understand the distribution of realized Sharpe ratios over the next 50 years. Simulate yearly assets returns for the next 50 years, calculate the yearly returns of the minimum-variance, tangency and ERC portfolio when rebalancing yearly to the initial weights, and estimate the average return, standard deviation and Sharpe ratio based on the yearly portfolio returns. Repeat this 10,000 times. Visualize the distribution of realized average return, standard deviation and Sharpe ratio (kernel density plot or histogram of the 10,000 simulated average returns for the different portfolios, etc.). Comment on your results.

For each of the 10,000 simulations, calculate the minimum-variance, tangency and ERC portfolio weights based on estimated expected return and covariances. Visualize the dispersion of the optimal portfolios, e.g. using boxplots. Calculate and visualize the yearly expected return, standard deviation and Sharpe ratio for the optimal portfolios based on the true distributional parameters. Comment on your findings.

References

Maillard, S., T. Roncalli, and J. Teïletche (2010): "The Properties of Equally Weighted Risk Contribution Portfolios," *The Journal of Portfolio Management*, 36, 60–70.

Mantilla-Garcia, D., L. Martellini, V. Milhau, and H. E. Ramirez-Garrido (2022): "Improving Interest Rate Risk Hedging Strategies through Regularization," *Financial Analysts Journal*, 78, 18–36.