ALVEUS

Python Application

Technical Description

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Author:
Frederik Lehn

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Nomenclatue

Table 1: Nomenclature

Variables	Description
	Scalers
S_{cum}	Cumulative scaler
S_{rate}	Rate scaler
S_{ffw}	Water fractional flow scaler
S_{ffg}	Free gas fractional flow scaler
Δ	Time delay (onset)
WCT_{ini}	Initial water-cut
	Uncertainty
	Input
b_o	Oil formation volume factor
b_g	Gas formation volume factor
b_w	Water formation volume factor
Q_o	Oil production rate at surface
Q_g	Gas production rate at surface
$Q_{g,total}$	Total gas production rate at surface
$Q_{g,lift}$	Gas lift injection rate
$Q_{g,free}$	Free gas production rate
Q_w	Water production rate at surface
Q_l	Liquid production rate at surface
$Q_{g,inj}$	Gas injection rate at surface
$Q_{w,inj}$	Water injection rate at surface
Q_α^p	Potential of phase α
Q_α^{ip}	Instantaneous potential of phase α
N_α	Cumulative of phase α
R_s	Solution gas-oil ratio
q_{hc}	In situ hydrocarbon production rate
q_{res}	In situ production rate
WHCR	Water-hydrocarbon ratio
FGOR	Free gas-oil ratio
TTGLR	Target total gas-liquid ratio
T	Production duration
T_Δ	Onset factor
FF_{WHC}	Water-hydrocarbon fractional flow
FF_{FGO}	Free gas-oil fractional flow

1. Unconstrained Forecasting

1.1. Production

Forecasting of production potentials is based on typecurves fitted to analogue data. Each typecurve consists of three continuous functions, one for liquid potential vs. time, $Q_l(t)$, one for water-cut vs. cumulative oil,

$WCT(N_o)$ and one for GOR vs. time, $GOR(t)$. Based on these functions, the well profiles are time-stepped for a provided duration. Additional to the typecurves, the potentials may be influenced by a series of scalers, namely a cumulative scaler, S_{cum}, a rate scaler, S_{rate}, a water fractional flow scaler, S_{ffw}, a gas fractional flow scaler, S_{ffg}, an onset parameters, Δ which delays the onset of fractional flow scalers, and an initial water-cut scaler, WCT_{ini}.

1.1.1. Well Spacing

Prior to time-stepping, the scalers are addjusted for well spacing differences between the analogue and the development well. The well spacing ratio is defined

$$WSR = \frac{WS_{\text{dev}}}{WS_{\text{ana}}} \tag{1}$$

where WS is the well spacing. Using this the cumulative and rate scalers are adjusted

$$\begin{aligned} S^*_{cum} &= S_{cum} \cdot WSR \\ S^*_{rate} &= S_{rate} \cdot WSR^{-0.75} \end{aligned} \tag{2}$$

This well spacing adjustment has been tested and shown to work well for water-flooded fields with horizontal wells positioned in line-drives in tight carbonate reservoirs. Potentially different methods will be required for different recovery mechanisms, development layouts and geology.

1.1.2. Time-Stepping

The time-stepping algorithm work as follows:

Calculate the liquid potential

$$Q^p_{l,t} = Q_l(t) \cdot S^*_{rate} \tag{3}$$

calculate the fractional flow of water at surface

$$WCT_t = WCT\left(\frac{N_{o,t-1}}{S^*_{cum}} + N_{o,ini}\right) \tag{4}$$

where the initial cumulative oil is found using a root-finding method with the objective-function

$$WCT_{ini} = WCT\left(N_{o,ini}\right) \Leftrightarrow 0 = WCT\left(N_{o,ini}\right) - WCT_{ini} \tag{5}$$

then calculate the fractional flow of gas at surface

$$GOR_t = GOR(t) \tag{6}$$

Now surface potentials are changed to reservoir potentials to scale the fractional flows at in situ conditions. First calculating the free gas

$$Q^p_{g,free,t} = \max(GOR_t - R_s, 0) \cdot Q^p_{l,t} \cdot (1 - WCT_t) \tag{7}$$

calculate the hydrocarbon reservoir potential

$$q^p_{hc,t} = b_o \cdot Q^p_{l,t} \cdot (1 - WCT_t) + b_g \cdot Q^p_{g,free,t} \tag{8}$$

and the full reservoir potential

$$q^p_{r,t} = q^p_{hc,t} + b_w \cdot Q^p_{l,t} \cdot WCT_t \tag{9}$$

then the in situ water-hydrocarbon ratio may be calculated

$$WHCR_t = \frac{b_w \cdot Q^p_{l,t} \cdot WCT_t}{q^p_{hc,t}} \tag{10}$$

and the free gas-in situ oil ratio

$$FGOR_t = \frac{b_g \cdot Q^p_{g,free,t}}{b_o \cdot Q^p_{l,t} \cdot (1 - WCT_t)} \tag{11}$$

The the onset factor, T_Δ is calculated

$$T_{\Delta,t} = \begin{cases} e^{-\frac{t}{\Delta}}, & \text{if } \Delta > 0, \\ 0 \end{cases} \tag{12}$$

Then the scaled water-hydrocarbon ratio is calculated

$$WHCR^*_t = WHCR_t \cdot (S_{ffw} + (1 - S_{ffw}) \cdot T_{\Delta,t}) \tag{13}$$

and the scaled free gas-in situ oil ratio

$$FGOR^*_t = FGOR_t \cdot (S_{ffg} + (1 - S_{ffg}) \cdot T_{\Delta,t}) \tag{14}$$

which results in the following reservoir fractional flow of water

$$F_{w,t} = \frac{WHCR^*_t}{1 - WHCR^*_t} \tag{15}$$

and the reservoir fractional flow of gas

$$F_{g,t} = \frac{FGOR^*_t}{1 - FGOR^*_t} \tag{16}$$

from these values, the flow rates at surface may be calculated.

Flow of water at surface

$$Q^p_{w,t} = \frac{q^p_{r,t} \cdot F_{w,t}}{b_w} \tag{17}$$

Flow of oil at surface

$$Q^p_{o,t} = \frac{q^p_{r,t} \cdot (1 - F_{w,t}) \cdot (1 - F_{g,t})}{b_o} \tag{18}$$

Flow of gas at surface

$$Q^p_{g,t} = R_s \cdot Q^p_{o,t} + \frac{q^p_{r,t} \cdot (1 - F_{w,t}) \cdot F_{g,t}}{b_g} \tag{19}$$

The cumulative oil production is then updated in terms of a forward integration

$$N_{o,t+1} = N_{o,t} + Q_{o,t} \cdot \Delta t \tag{20}$$

and the simulation advanced to the next time-step.

1.2. Lift Gas

The required gas-lift is calculated based on a target total gas-liquid ratio, $TTGLR$. For at given time-step the gas-lift rate is calculated as

$$Q^p_{g,lift,t} = \max(TTGLR \cdot Q^p_{l,t} - Q^p_{g,t}, 0) \tag{21}$$

1.3. Injection

1.3.1. Potentials

There are available options for calculating the injection potential of a given injector. Either by applying a constant water or gas rate or by an assumption of voidage replacement. If no constant rate is supplied, the simulator automatically calculates the required injection potential.

In the latter case, injection potential is calculated on the basis of a given voidage replacement ratio

$$VR = \frac{Q^p_{w,inj} \cdot b_{w,inj} + Q^p_{g,inj} \cdot b_{g,inj}}{Q^p_o \cdot \left(b_o + (GOR - R_s) \cdot b_g\right) + Q^p_w \cdot b_w} \tag{22}$$

thus the required injected water volume is

$$Q^p_{w,inj} = VR \cdot \frac{Q^p_o \cdot \left(b_o + (GOR - R_s) \cdot b_g\right) + Q^p_w \cdot b_w}{b_{w,inj}} - \frac{Q^p_{g,inj} \cdot b_{g,inj}}{b_{w,inj}} \tag{23}$$

and equivalently the required injected gas volume

$$Q^p_{g,inj} = VR \cdot \frac{Q^p_o \cdot \left(b_o + (GOR - R_s) \cdot b_g\right) + Q^p_w \cdot b_w}{b_{g,inj}} - \frac{Q^p_{w,inj} \cdot b_{w,inj}}{b_{g,inj}} \tag{24}$$

assuming an injector can only inject one fluid at a time, this reduces to

$$Q^p_{w,inj} = VR \cdot \frac{Q^p_o \cdot \left(b_o + (GOR - R_s) \cdot b_g\right) + Q^p_w \cdot b_w}{b_{w,inj}} \tag{25}$$

and

$$Q^p_{g,inj} = VR \cdot \frac{Q^p_o \cdot \left(b_o + (GOR - R_s) \cdot b_g\right) + Q^p_w \cdot b_w}{b_{g,inj}} \tag{26}$$

respectively. One injector may support a set of producers $\mathcal{P} = \{P_1, ..., P_n\}$, with a certain proportion, p_P of the support going to producer P. Thus for any single injector the required injection becomes

$$Q^p_{w,inj} = VR \cdot \sum_{P \in \mathcal{P}} p_P \cdot \frac{Q^p_{o,P} \cdot \left(b_{o,P} + (GOR_P - R_{s,P}) \cdot b_{g,P}\right) + Q^p_{w,P} \cdot b_{w,P}}{b_{w,inj}} \tag{27}$$

and

$$Q^p_{g,inj} = VR \cdot \sum_{P \in \mathcal{P}} p_P \cdot \frac{Q^p_{o,P} \cdot \left(b_{o,P} + (GOR_P - R_{s,P}) \cdot b_{g,P}\right) + Q^p_{w,P} \cdot b_{w,P}}{b_{g,inj}} \tag{28}$$

A correction is applied to the injectors, such that the cumulative voidage replacement equals 1, i.e.

$$Q^p_{w,inj} = Q^p_{w,inj} \cdot \left(VR \cdot \sum_{p \in \mathcal{P}} p_P\right)^{-1}, \quad Q^p_{g,inj} = Q^p_{g,inj} \cdot \left(VR \cdot \sum_{p \in \mathcal{P}} p_P\right)^{-1} \tag{29}$$

Imagine a line-drive flanked by an injector, which only provides $p_P = 0.5$ support to the only adjacent well. If no correction is applied to (27) and (28) the calculated injection will be lower than what will realistically be injected, considering not all of the injected phase will go directly into the producer. There has to be a correction for the fluid which is not going to a well, but just into the reservoir, in order to have appropriate values for the constrained forecasting. In a thermodynamic model this would of course not be a linear scaling, as the pressure gradient would be substantially different for the various directions the water and gas could go in, but (29) serves as an approximation.

1.3.2. WAG

If the well is a WAG injector the required water and gas potential is calculated as outlined in 1.3.1 followed by a time-stepping to get the correct amount of WAG-cycles. An assumption is made that a WAG well starts with gas injection and ends with water injection after all the cycles have finished. Given the parameters T_{cycle} (change-over) and n_{cycles} (number of cycles) a time-stepping is initiated, determining the injected phase at any given time.

Algorithm 1: WAG cycling

$fluid = gas$;
$cycle = 0$;
$t_{change} = 0$;
for t *in* T **do**
    if *fluid* == *gas* **then**
        calculate $Q^{p}_{g,inj,t}$ with (28);
        Set $Q^{p}_{w,inj,t} = 0$;
    else
        calculate $Q^{p}_{w,inj,t}$ with (28);
        Set $Q^{p}_{g,inj,t} = 0$;
    end
    if $(t - t_{change}) \geq T_{cycle}$ **then**
        if *fluid* == *gas* **then**
            fluid = water
        else
            fluid = gas
        end
        $cycle$+=1;
        $t_{change} = t$;
        if $cycle$== n_{cycle} **then**
            break
        end
    end
end

2. Constrained Forecasting

The constrained forecasting model is a choke-model.

2.1. Performance Tables

Production potentials are calculated using a time-stepping scheme and thus defined as a function of time. When calculating the rates, a certain reservoir volume is extracted from the reservoir at each time-step, $q_{res,t} \leq q^{ip}_{res,t}$. When a well is choked a lower volume of reservoir fluids are extracted from the reservoir, and thus the instantaneous potential in the next time-step is different from the initially forecasted potential. For this reason, a Performance Table is defined for each well, which is tracked as a function of the extracted reservoir volume. Because there is only one choke position (i.e. oil, gas and water are extracted by the same fraction) and fluid properties remain constant throughout the simulation, the cumulative oil can be used as a proxy for the cumulative reservoir volume.

After each time-step the Performance Table is progressed by updating the cumulative oil

$$N_{o,t+1} = N_{o,t} + Q^{ip}_{oil,t} \cdot x \cdot \Delta t \tag{30}$$

where $x \in [0, 1]$ is the choke-position. The simulation is then progressed to the next time-step, $t + 1$ at which time the instantaneous oil, gas and water potential can be calculated by interpolation of the potential vs. the cumulative oil extracted so far, at time t

$$Q^{ip}_{\alpha,t} = Q^{p}_{\alpha}\left(N_{o,t-1}\right) + \frac{Q^{p}_{\alpha}\left(N_{o,t}\right) - Q^{p}_{\alpha}\left(N_{o,t-1}\right)}{N_{o,t} - N_{o,t-1}}, \quad \alpha \in \{o, w, g\} \tag{31}$$

In an unconstrained system with full availability $Q^{ip}_{\alpha,t} = Q^{p}_{\alpha,t}$, but if $u < 1$ or any part of the system is choked for even one time-step they diverge in the time domain, $Q^{ip}_{\alpha,t} \neq Q^{p}_{\alpha,t}$. The reason for tracking them against the cumulative oil is that they remain consistent in the cumulative domain independent of choked production.

Similarly the injectors have a Performance Table which is tracked against the cumulative injected volume.

2.2. Linear System

The linear programming problem is a primal-dual problem with the primal formulation

$$\begin{aligned} \min_{x} \quad & g^T x \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \end{aligned} \tag{32}$$

and the dual formulation

$$\begin{aligned} \max_{y} \quad & b^T y \\ \text{s.t.} \quad & A^T y + s = g \\ & s \geq 0 \end{aligned} \tag{33}$$

The constrained solver is built around the assumption of maintaining voidage replacement. Thus the linear system is a linear algebra formulation of the voidage replacement ratio equation

$$VR = \frac{Q_{w,inj} \cdot b_{w,inj} + Q_{g,inj} \cdot b_{g,inj}}{Q_o \cdot \left(b_o + (GOR - R_s) \cdot b_g\right) + Q_w \cdot b_w} = \frac{q_{inj}}{q_{res}} \Leftrightarrow q_{res} - \frac{1}{VR} \cdot q_{inj} = 0 \tag{34}$$

i.e. the voidage replacement ratio is split into two terms, one for the reservoir volume leaving the reservoir and the other for injected volume entering the reservoir

$$\begin{aligned} q_{res} &= Q_o \cdot \left(b_o + (GOR - R_s) \cdot b_g\right) + Q_w \cdot b_w \\ q_{inj} &= Q_{w,inj} \cdot b_{w,inj} + Q_{g,inj} \cdot b_{g,inj} \end{aligned} \tag{35}$$

assuming an injector can support multiple producers, the injected fluid from an injector will be going by some proportion to each producer it supports. Hence for a given producer, P which are supported by the list of injectors, $\mathcal{I}_P$ the voidage replacement requirement will be

$$q_{res,P} - \sum_{I \in \mathcal{I}_P} \frac{p_{I_P}}{VR_I} \cdot q_{inj,I} = 0 \tag{36}$$

Where p_I is the proportion of support from producer I. Additionally to this the production stream and the injection streams have a choke which can be varied if constraints are violated, thus the rate can be split into two terms, namely the instantaneous potential multiplied by a choke.

$$q^{ip}_{res,P} \cdot x_P - \sum_{I \in \mathcal{I}_P} p_{I_P} \cdot VR_I^{-1} \cdot q^{ip}_{inj,I} \cdot x_I = 0 \tag{37}$$

where $x_P \in [0, 1]$ is the production choke and $x_I \in [0, 1]$ is the injection choke. The system also has to satisfy a series of other inequality constraints, which are the mechanical constraints of the equipment such as wells, manifolds, compressors, separators and pipelines. Hence the system to be solved is the so called homogeneous model

$$\begin{aligned} Ax - b\tau &= 0 \\ A^T y + s - g\tau &= 0 \\ -g^T x + b^T y - \kappa &= 0 \\ x, s, \tau, \kappa &\geq 0 \end{aligned} \tag{38}$$

where x is the variables, namely the choke position of each well. A is the constrained system matrix and b is the constrained system results. g determines the weighting of the objective function. y is the solution to the dual problem, s is a slack variable and τ and κ are scalars used in the solving of the system and has no physical meaning.

Even though the problem, is stated with just one system of equations, A and b for equality and inequality constraints, practical algorithms are implemented taking both A_{eq} and b_{eq} for the equality constraints and A_{iq} and b_{iq} for the inequality constraints. It is also possible to apply lower, lb, and upper, ub, bounds on the solution vectors. Thus that is how the problem will be written going forward.

The size of the linear system is

- x: $n_{well} \times 1$
- A_{eq}: $n_{producer} \times n_{well}$
- b_{eq}: $n_{producer} \times 1$
- A_{iq}: $n_{constraint} \times n_{well}$
- b_{iq}: $n_{constraint} \times 1$
- lb: $n_{well} \times 1$
- ub: $n_{well} \times 1$

where the number of constraints are given by

$$n_{constraint} = \sum_{E \in \mathcal{E}} n_{constraint,E} \tag{39}$$

where $\mathcal{E}$ is the set of equipment, which is comprised of producers, injectors and the remaining surface network. Each equipment in the set will have a number of constraints $n_{constraint,E} \in [0, 7]$ where the 7 possible constraints are for oil, total gas, water, liquid, lift gas, gas injection and water injection. If the equipment is unconstrained the constraint is inactive and will be reduced from the linear system of equations.

For a simple system with 3 producers and 2 injectors in a line-drive, such that two bounding producers are 1-side supported, i.e. with a proportion of $p = 0.5$ and the middle producer is supported by both injectors, each with a proportion of $p = 0.5$.

The resulting linear system for the equality constraints is

$$A_{eq}x = b_{eq} \Leftrightarrow \begin{bmatrix} a_{P_1} & 0 & 0 & -a_{I_1,P_1} & 0 \\ 0 & a_{P_2} & 0 & -a_{I_1,P_2} & -a_{I_2,P_2} \\ 0 & 0 & a_{P_3} & 0 & -a_{I_2,P_3} \end{bmatrix} \cdot \begin{bmatrix} x_{P_1} \\ x_{P_2} \\ x_{P_3} \\ x_{I_1} \\ x_{I_2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{40}$$

where the coefficients, a, are given by

$$\begin{aligned} a_{P_j} &= q^{ip}_{res,P_j} \\ a_{I_k,P_j} &= p_{I_k,P_j} \cdot VR^{-1}_{I_k} \cdot q^{ip}_{inj,I_k} \end{aligned} \tag{41}$$

as seen in (37). Assume the wells are tied into a processing facility with the following setup

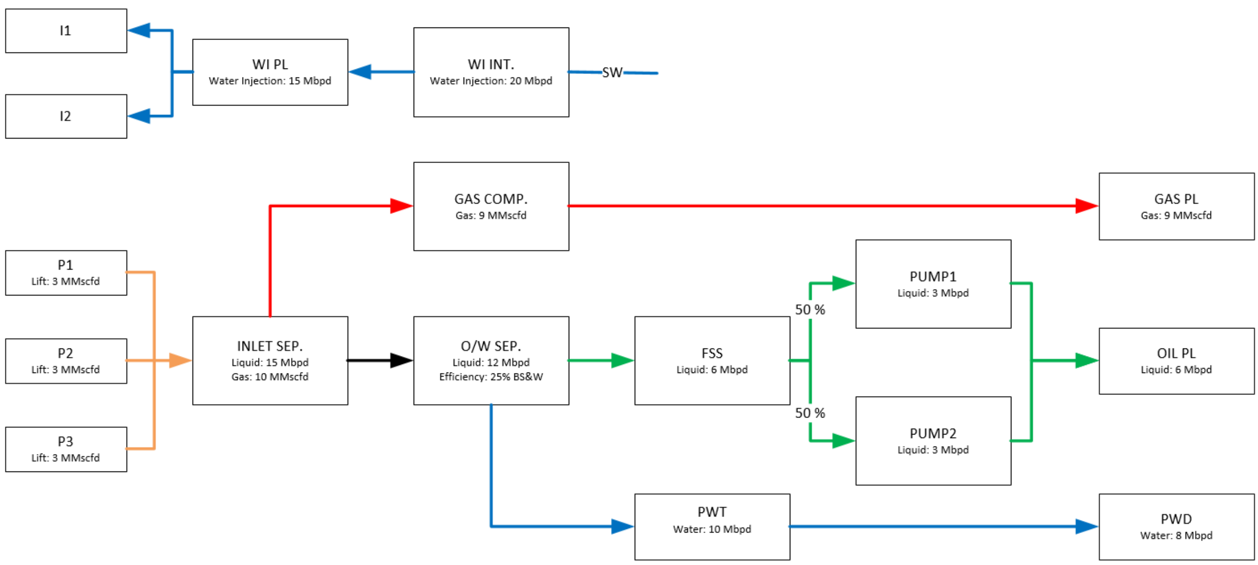

Figure 1: Network setup used as example.

I.e. the produced fluids from the wells go into an Inlet Separator which splits it into a liquid phase which is routed to an O/W Separator and a gas phase which is routed to a gas compressor and subsequently to an gas export pipeline. The water separated out in the O/W separator is routed to a Produced Water Treatment (PWT) equipment which then routes it to a Produced Water Disposal (PWD) equipment for disposal. The oil from the O/W separator is routed to a Final Stage Separator (FSS) and then into one of two pumps in parallel which then routes it into a liquid export pipeline. Injection water is extracted from the sea via a Water Injection Intake which routes it to a Water Injection Pipeline that transfers it to the water injection wells.

This system will be inequality constrained by the mechanical constraints of the processing equipment. Assume that each of the producers are constrained on gas lift by, $c_{lg,P}$, the inlet separator is constrained on the liquid and the total gas by $c_{l,IS}$ and $c_{tg,IS}$ respectively, the gas compressor is constrained on the total gas by $c_{tg,GC}$, the O/W-separator is constrained on the liquid by $c_{l,OW}$ and has an efficiency of 25 %, the PWT is constrained on the water by $c_{w,PWT}$, and the PWD is constrained on water by $c_{w,PWD}$. The FFS is constrained on the liquid by $c_{l,FFS}$, the pumps prior to the oil pipeline are both constrained on the liquid by $c_{l,PU}$, the oil export pipeline is constrained by $c_{l,OEX}$ and the gas export pipeline is constrained by

$c_{tg,GEX}$. The injection intake is constrained on the injection water by $c_{w,inj,INT}$ and the injection pipeline is constrained on the injection water by $c_{w,inj,IPL}$.

The potential flow through a piece of equipment is given by the stream that has passed through all the previous pieces of equipment in the chain. In order to arrive at the potentials, a piece of equipment has to be recursively mapped down to the base entity in the chain, namely the wells.

$$\sum_{W \in \mathcal{W}_E} \left(x_W \cdot \sum_{\alpha \in \beta} Q^{ip}_{\alpha,W} \cdot s^{nw}_{\alpha,E_W} \right) \leq u \cdot c_{\beta,E} \tag{42}$$

where $\mathcal{W}_E$ is the set of wells connected by a surface network to equipment E, $Q^{ip}_{\alpha,W}$ is the instantaneous potential of phase α for well W, s^{nw}_{α,E_W} is the stream of phase α going through the surface network to equipment E from the path starting at well W and lastly $c_{\beta,E}$ is the constraint on equipment E for the sum of phases in β.

The stream, $s^{nw}_{\alpha,E_W} \in [0, 1]$ is the product of the fraction of the stream going through the preceding pieces of equipment in the network, i.e.

$$s^{nw}_{\alpha,E_W} = \prod_{E \in \mathcal{E}_{E_W}} s_{\alpha,E} \tag{43}$$

The six basic phases are oil, gas, water, lift gas, gas injection and water injection. These may be combined in a set of phases, e.g. liquid would be $\alpha \in \beta = \{o, w\}$.

Then the inequality constrained system looks as follows

$$A_{iq}x \leq b_{iq} \Leftrightarrow \begin{bmatrix} a_{lg,P_1} & 0 & 0 & 0 & 0 \\ 0 & a_{lg,P_2} & 0 & 0 & 0 \\ 0 & 0 & a_{lg,P_3} & 0 & 0 \\ a_{l,IS_{P_1}} & a_{l,IS_{P_2}} & a_{l,IS_{P_3}} & 0 & 0 \\ a_{tg,IS_{P_1}} & a_{tg,IS_{P_2}} & a_{tg,IS_{P_3}} & 0 & 0 \\ a_{tg,GC_{P_1}} & a_{tg,GC_{P_2}} & a_{tg,GC_{P_3}} & 0 & 0 \\ a_{l,OW_{P_1}} & a_{l,OW_{P_2}} & a_{l,OW_{P_3}} & 0 & 0 \\ a_{l,FSS_{P_1}} & a_{l,FSS_{P_2}} & a_{l,FSS_{P_3}} & 0 & 0 \\ a_{l,PU1_{P_1}} & a_{l,PU1_{P_2}} & a_{l,PU1_{P_3}} & 0 & 0 \\ a_{l,PU2_{P_1}} & a_{l,PU2_{P_2}} & a_{l,PU2_{P_3}} & 0 & 0 \\ a_{w,PWT_{P_1}} & a_{w,PWT_{P_2}} & a_{w,PWT_{P_3}} & 0 & 0 \\ a_{w,PWD_{P_1}} & a_{w,PWD_{P_2}} & a_{w,PWD_{P_3}} & 0 & 0 \\ a_{tg,GEX_{P_1}} & a_{tg,GEX_{P_2}} & a_{tg,GEX_{P_3}} & 0 & 0 \\ a_{l,OEX_{P_1}} & a_{l,OEX_{P_2}} & a_{l,OEX_{P_3}} & 0 & 0 \\ 0 & 0 & 0 & a_{w,inj,INT_{I_1}} & a_{w,inj,INT_{I_2}} \\ 0 & 0 & 0 & a_{w,inj,IPL_{I_1}} & a_{w,inj,IPL_{I_2}} \end{bmatrix} \cdot \begin{bmatrix} x_{P_1} \\ x_{P_2} \\ x_{P_3} \\ x_{I_1} \\ x_{I_2} \end{bmatrix} \leq \begin{bmatrix} u \cdot c_{lg,P} \\ u \cdot c_{lg,P} \\ u \cdot c_{lg,P} \\ u \cdot c_{l,IS} \\ u \cdot c_{tg,IS} \\ u \cdot c_{tg,GC} \\ u \cdot c_{l,OW} \\ u \cdot c_{l,FSS} \\ u \cdot c_{l,PU} \\ u \cdot c_{l,PU} \\ u \cdot c_{w,PWT} \\ u \cdot c_{w,PWD} \\ u \cdot c_{tg,GEX} \\ u \cdot c_{l,OEX} \\ u \cdot c_{w,inj,INT} \\ u \cdot c_{w,inj,IPL} \end{bmatrix} \tag{44}$$

where $u \in [0, 1]$ is the overall network availability (or uptime-fraction) and the coefficients, a, are given by

$$a_{\beta,E_W} = \sum_{\alpha \in \beta} Q^{ip}_{\alpha,W} \cdot s^{nw}_{\alpha,E_W} \tag{45}$$

Lastly, since a choke position is limited to between fully closed (0) and fully open (1), the chokes have to be constrained by a lower and upper bound. Even if there are no constraints, the production still has to honor the system availability, so the upper bound is given by the availability, u

$$lb \leq x \leq ub \Leftrightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq \begin{bmatrix} x_{P_1} \\ x_{P_2} \\ x_{P_3} \\ x_{I_1} \\ x_{I_2} \end{bmatrix} \leq \begin{bmatrix} u \\ u \\ u \\ u \\ u \end{bmatrix} \tag{46}$$

Below will follow an instruction on how the network-streams are calculated.

Network-Stream

A network-stream is defined by how a single phase moves through the surface setup. Processors are allowed to split streams by a certain fraction, $s_{\alpha,E} \in [0, 1]$ to two different pieces of proceeding equipment. For instance the inlet separator will split the streams such that all gas goes to the gas compressor and oil and water goes to the O/W separator.

The split of fluids going into the inlet separator from the producers are simply 1 as all produced fluids are routed to the inlet separator

$$s^{nw}_{\alpha,IS_{P_i}} = 1, \quad i = 1, 2, 3, \quad \alpha \in \{o, g, w\} \tag{47}$$

the split into the gas compressor via the inlet separator is

$$s^{nw}_{\alpha,GC_{P_i}} = s^{nw}_{\alpha,IS_{P_i}} \cdot \begin{cases} 1, & \text{if } \alpha = g, \\ 0 \end{cases}, \quad i = 1, 2, 3, \quad \alpha \in \{o, g, w\} \tag{48}$$

with the liquids going to the O/W separator

$$s^{nw}_{\alpha,OW_{P_i}} = s^{nw}_{\alpha,IS_{P_i}} \cdot \begin{cases} 1, & \text{if } \alpha \in \{o, w\}, \\ 0 \end{cases}, \quad i = 1, 2, 3, \quad \alpha \in \{o, g, w\} \tag{49}$$

The streams going to the PWT are defined by the efficiency of the O/W separator

$$s^{nw}_{\alpha,PWT_{P_i}} = s^{nw}_{\alpha,OW_{P_i}} \cdot \begin{cases} 0.75, & \text{if } \alpha = w, \\ 0 \end{cases}, \quad i = 1, 2, 3, \quad \alpha \in \{o, g, w\} \tag{50}$$

and the streams going into the PWD is

$$s^{nw}_{\alpha,PWD_{P_i}} = s^{nw}_{\alpha,PWT_{P_i}} \cdot \begin{cases} 1, & \text{if } \alpha = w, \\ 0 \end{cases}, \quad i = 1, 2, 3, \quad \alpha \in \{o, g, w\} \tag{51}$$

the other stream from the O/W separator going to the FSS is again defined by the efficiency of the O/W separator

$$s^{nw}_{\alpha,FSS_{P_i}} = s^{nw}_{\alpha,OW_{P_i}} \cdot \begin{cases} 1, & \text{if } \alpha = o, \\ 0.25, & \text{if } \alpha = w,, \\ 0 \end{cases} \quad i = 1, 2, 3, \quad \alpha \in \{o, g, w\} \tag{52}$$

As seen so far Alveus allows for splitting the various streams by a fixed fraction. In practice other more advanced options are available such as spill-over, but this will require exporting the case and running it in Phaser. So in this example it will be assumed that the liquid streams going to the pumps are split equally (but any fraction is allowed), i.e.

$$s^{nw}_{\alpha,PU_{j,P_i}} = s^{nw}_{\alpha,FSS_{P_i}} \cdot \begin{cases} 0.5, & \text{if } \alpha \in \{o, w\}, \\ 0 \end{cases}, \quad i = 1, 2, 3, \quad j = 1, 2, \quad \alpha \in \{o, g, w\} \tag{53}$$