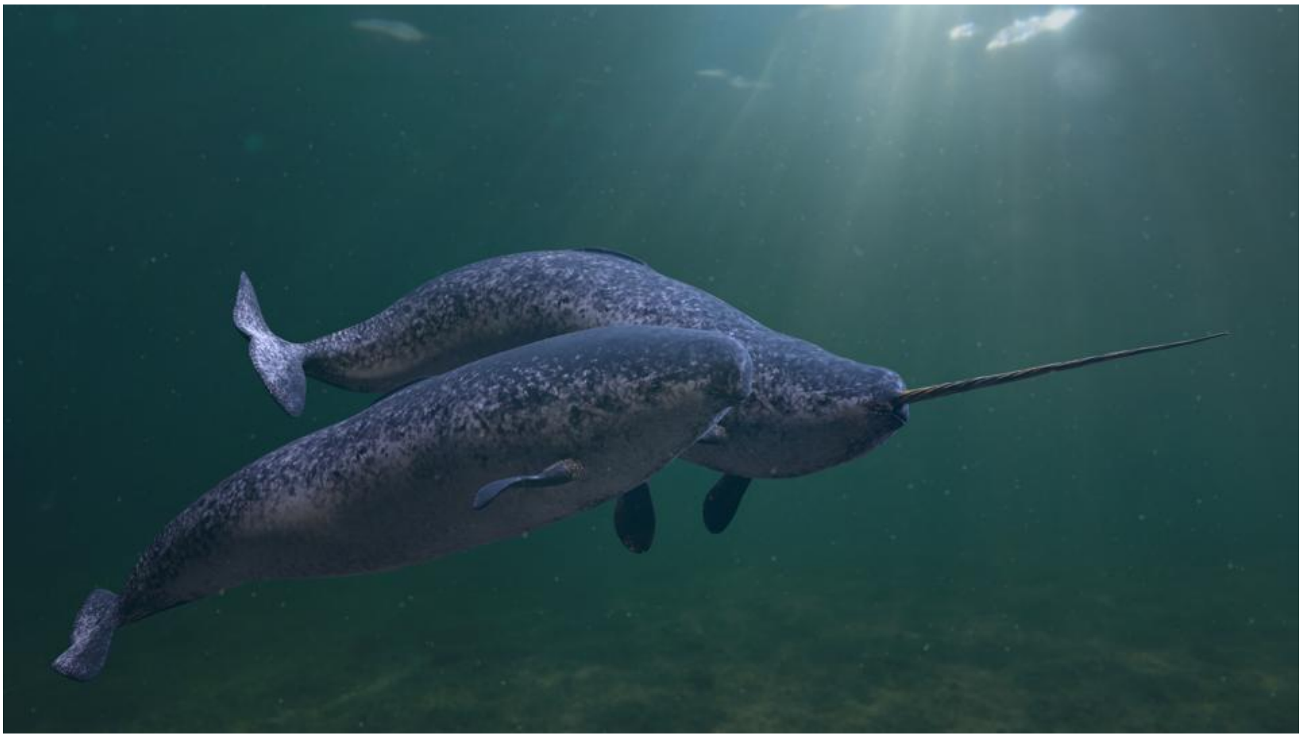
UNIVERSITY OF COPENHAGEN
FACULTY OF SCIENCE



Master's Thesis

Frederik Leander Schultz

Estimation of the effect of tagging on narwhals

Advisor: Susanne Ditlevsen, co-advisor: Outi Tervo

Handed in: December 20, 2020

Abstract

In this paper we examine the effect of tagging of Narwhals from East Greenland by examining different models that have a time since tagging component that goes towards 0. We have conducted a Kolmogorov Smirnow test, and found that all of the models suggested can be rejected, but by looking at other articles we assume that if the models included memory, not all of the models can be rejected for all of the whales. A mediation analysis was conducted, and we found that a handling time that takes more than 63 minutes results in the highest proportion, and a handling time between 55 and 62 minutes, the lowest. Lastly we have examined how long it takes before the whales' buzzing rate is back to natural, and found that this also depends on the handling time.

Contents

1 Introduction

Biologists are collecting huge amounts of behavioral data from narwhals in East Greenland by live-capturing the whales in nets and attaching small computers and measurement devices to the backs of the whales. This is called tagging. The procedure takes less than 90 minutes, and the whales are then set free. The tags collect data each second for 2-8 days on swimming depths, sound production, accelerometer data and GPS positions. The data is collected to obtain information about natural behavior of the animals. When plotting the data, it is clearly visible in the records that the whales are strongly influenced by the tagging procedure, and during some hours, differing from animal to animal, the whales dive less and produce less sound. It is therefore not convenient to include the first part of the record in the statistical analysis of natural behavior. However, it is not clear how much data should be discarded. This paper examines, using different statistical models, how much of the data can be disregarded, when looking at narwhal data.

2 Problem statement

In this thesis we wish to propose models that can help future analyses to have a stronger idea of how long after release it takes before the behavior is natural by examining different models to determine how long the silence period truly is. This paper assumes that the buzzing behavior of the whales is a good indicator of whether the whales are still effected by the tagging procedure. To examine this we wish to come up with models that estimate the buzzing patterns, using the depth and time since tagging. Furthermore we wish to include further variables such as the time the tagging procedure took, to see if this has an impact on the time it takes for the whales to act as normal. Furthermore, when looking at the diving- and buzzing patterns, it is clear to see that the whales primarily make buzzes when they are below a certain depth. Therefore we wish to examine if the tagging procedure effects the buzzing behavior directly or if the procedure only effects the depth of the dives, and thereby the buzzing.

3 Description of the data

The data have kindly been given by Greenland Institute of Natural Resources, and describes 15 different narwhales that were live captured in August 2013-2019 from a field station near the southwestern tip of Milne Land in the Scoursby Sound fjord complex in Greenland. The whales were captured by using set nets in collaboration with local Inuit hunters. Six female whales, and 9 male whales (one was captured twice and is treated as two different whales) were instrumented with Acousonde$^{\text{TM}}$ acoustic and orientation tags. The recorder provided archived data for depth, and geographical positions as well as continuous recordings of vocalisation and positions of the whales at a rate of 1 Hz. The Acousonde transmitters were attached to the whales using suction cups on the rear end of the animal and was held to the animals using materials that ensured the release of the recorder after 3-8 days of attachment. After the recorder detached it was retrieved after 1-4 days. Furthermore an additional data set has been given, with information about the length, handling time and gender of each whale. The initial data set has the following variables:

- `Ind`: A string with the name of the whale
- `Posixct`: The current date and time in the form ”YYYY-MM-DD HH:MM:SS”
- `Depth`: The current depth of the whale
- `Buzz`: A binary variable that indicates whether the whale is buzzing (1) or not (0)
- `Lat`: The latitude of the whale.
- `Lon`: The longitude of the whale
- `Area`: A string with the letters indicating the area of which the whale is currently in
- `hour`: An integer value between 0 and 23 that indicates the current hour.
- `hourminute`: A value between 0 and 23 that indicates the current time of day
- `time_since_tagging`: The time after release in seconds.

3.1 Exploration of the data

First to get an idea of what the data looks like, we start by looking at how many observations there are for each whale

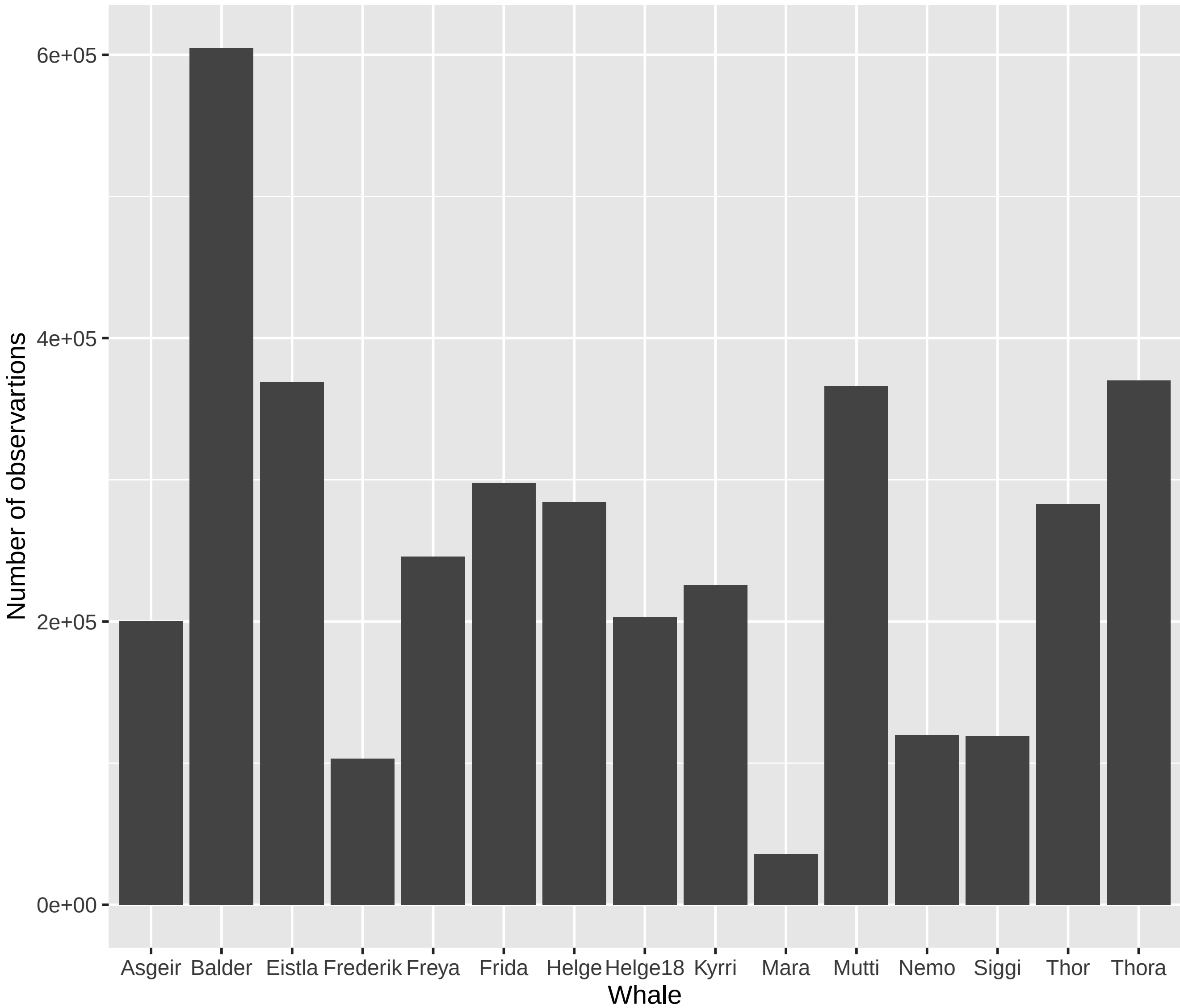


Figure 1: Number of observations the different whales

As we can see of Figure 1 it is very different how much data we have on the different whales.We see that Mara has the least amount of data and as described in Table 1 we only have 10 hours of data for her, while Balder has the most observations (168 hours).

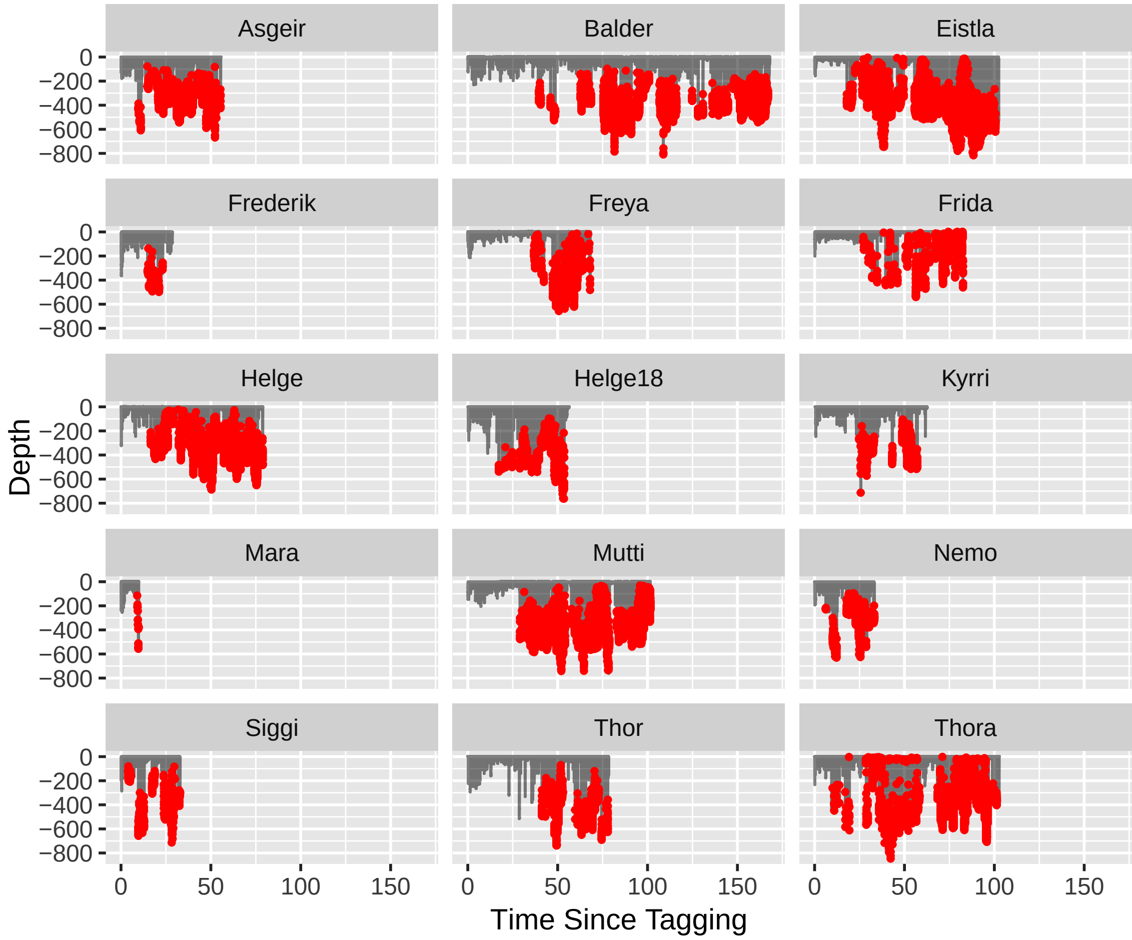


Figure 2: The diving patterns of the different whales with red dots indicating the start of a buzz

Figure 2 illustrates the diving and buzzing patterns of the different whales with the red dots being the start of a buzz. We notice that there is a clear trend that it takes a while, after the tagging procedure, during which, the whales start to dive below 200 meters. Furthermore we notice that in general there is a time frame after the tagging that the whales do not make any buzzes. This might be caused by the tagging itself, since the whales might be stressed and/or afraid of being captured again. Lastly we see that almost all of the buzzes occur when the whales are diving below a certain depth.

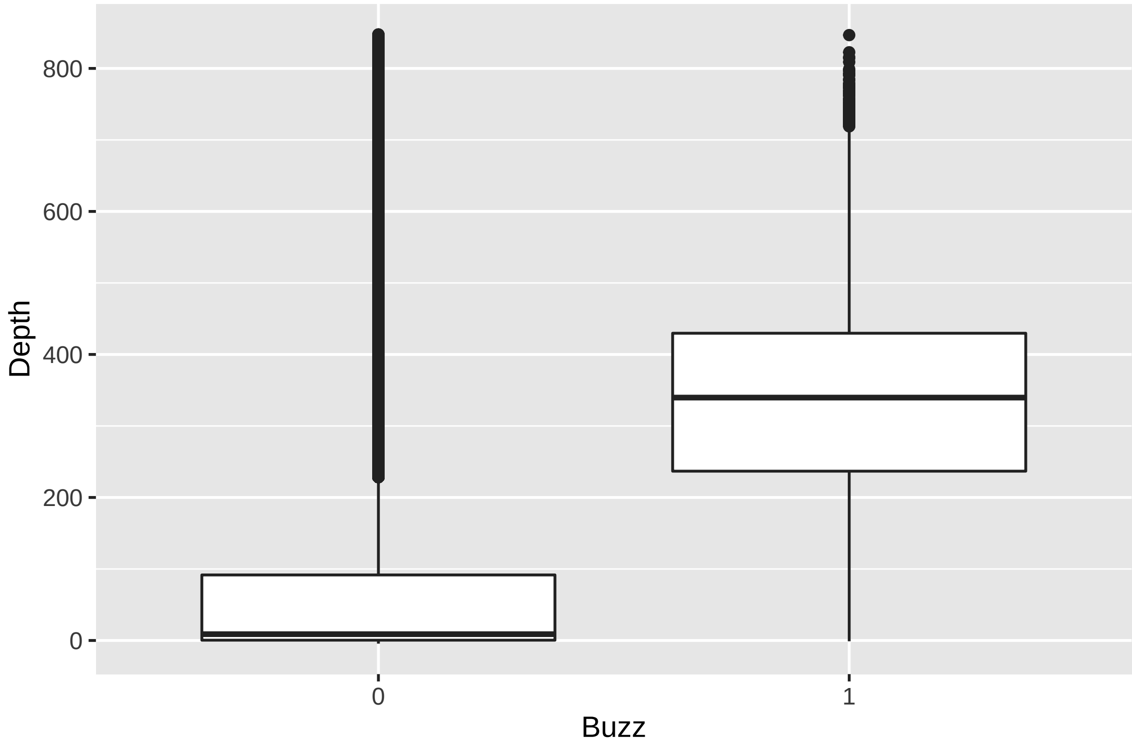


Figure 3: Boxplot of the buzzes against the depth

Looking at Figure 3 we confirm what we saw on Figure 2, in term of the buzzing primarily happening around 350 meters deep. We also notice that even though the majority of the time where the whales do not buzz is around the surface, there are times where the whales are around 200-800 meters deep and do not buzz. This might be because of the ”silence period” after the tagging, but it can also simply be that after the whales buzz, they wait some time before buzzing again.

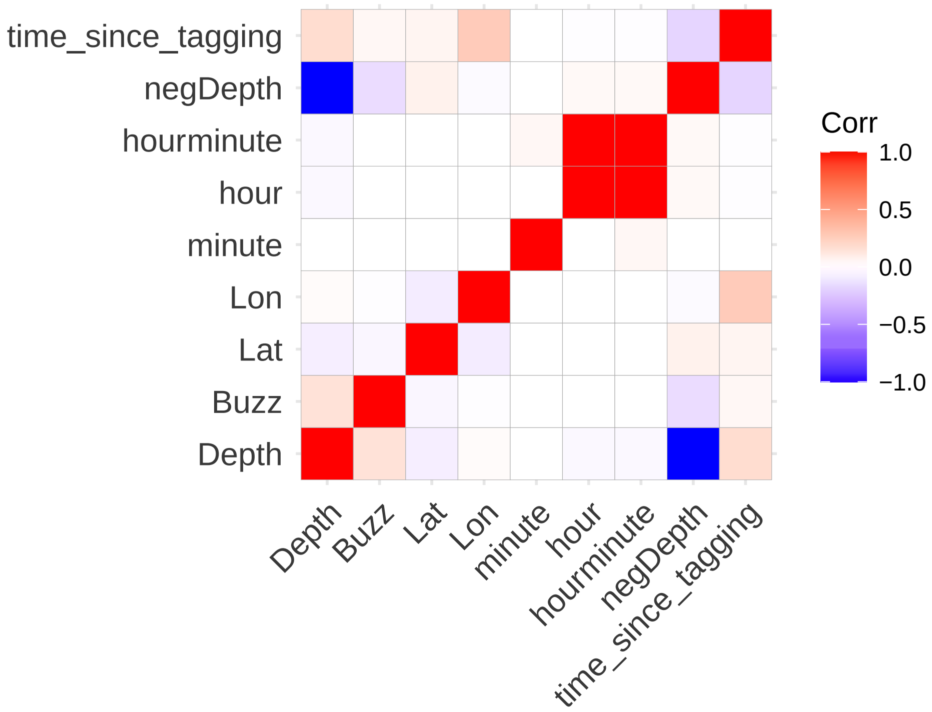


Figure 4: Correlation plot of the numeric values

Taking a look at the correlation between the different numerical values in Figure 4, we see that there is a correlation between the time since tagging and the depth, which is one thing we want to examine. This can be caused by the whales by the whales being scared/stressed of being captured again, so they do not dive that deep. Furthermore we notice that there is a correlation between the depth and the buzzes, which is to be expected since the narwhals rarely buzz when they are near the surface. If there is a negative correlation between the time since tagging and depth, the whales simply dive less because of the tagging. As we saw on Figure 3 they will not buzz as much when they are near the surface. We also notice that there is a correlation between time since tagging and the longitude, which can imply that due to the tagging procedure, the whales stay roughly around the same place the tagging was performed. This will, however, not be examined in this paper.

Whale	Observation time in seconds Observation time in hours	Buzzes	Buzzes / h Mean	Depth Mean SD (min-max)	Length in cm	Handling time in minutes
Asgeir	200389 55.7	2497	44,9	110 (0 - 671)	460	18
Balder	604801 168	4587	27,3	76 (0-810)	360	30
Eistla	369089 102.5	2798	27,3	121 (0-829)	372	34
Frederik	03214 28.7	132	10,7	68 (0-498)	409	36
Freya	245586 68.2	645	9,5	53 (0-661)	420	41
Frida	297602 82.7	887	10,7	24 (0-540)	380	55
Helge	284219 79	6167	78,1	109 (0-693)	497	58
Helge18	203104 56.49	952	16,9	94 (0-767)	487	60
Kyrri	225485 62.69	499	8,0	44 (0-713)	436	61
Mara	35964 10	18	10,7	54 (0-563)	560	62
Mutti	365809 101.6	3038	29,9	135 (0-762)	465	64
Nemo	119934 33.3	905	27,2	89 (0-634)	410	73
Siggi	118754 33	615	18,6	94 (0-714)	470	81
Thor	282524 78.5	1979	25,2	71 (0-742)	457	88
Thora	370006 102.8	4301	41,8	87 (0-848)	390	90

Table 1: Information about the different whales

4 Methods

4.1 Theory

4.1.1 Poisson regression

A Poisson regression model is a generalized linear model, which is used to model count data.
It assumes that the response variable has a Poisson distribution, which is used for describing events that are independent of each other. If $x \in \mathbb{R}^n$ is the vector of independent variables, then the model can be written as

$$\log(\mathbb{E}(Y|x)) = \alpha + x\beta, \qquad \alpha \in \mathbb{R}, \beta, x \in \mathbb{R}^n,$$

4.1.2 Mixed effects models

Mixed effects models are a generalization of the ordinary linear effects model and are especially useful when working with a design where one ”participant” provides multiple data points, which correlate with each other. A mixed model will take this correlation into account by including a design matrix for these ”participants”. The general form of these mixed effects models is:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{b} + \boldsymbol{\varepsilon}$$

Where $\mathbf{y}$ is the response and $\mathbf{X}, \mathbf{Z}$ are the design matrices for the fixed and random effects respectively. $\boldsymbol{\beta}$ is the fixed effects vector, $\mathbf{b}$ is the random effects vector and $\boldsymbol{\varepsilon}$ the error vector.

4.1.3 Point process

In this project we use the point process framework, which means we assume that the true data generating mechanism for the buzzes produced by the narwhals is a point process, i.e a set of discrete events in continuous time, where these events are the beginning of a buzz. A point process can be described by its conditional intensity function:

$$\lambda(t|X(t), Z_w) = \lim_{\Delta \to 0} \frac{1}{\Delta} P(N(t+\Delta) - N(t) = 1|X(t), Z_w)$$

Where N(t) is the number of events in $(0, t]$, $X(t)$ is the information on past events and the covariates in the interval (0,t) and Z_w is the random effect of the specific whale. This means that for a Δ close to zero, we get that:

$$\lambda(t|X(t), Z_w)\Delta \approx P(N(t+\Delta) - N(t) = 1|X(t), Z_w)$$

Which shows that $\lambda(t|X(t), Z_w)\Delta$ can approximate the conditional probability of observing an event in the interval $(t, t+\Delta)$. Since the data is based on a 1 Hz sampling rate, we set $\Delta = 1$, and the data is described by $(Y_1, X_1, Z_w), ..., (Y_T, X_T, Z_w)$ where X_t is the vector of co-variates and Y_t is the response variable in $\{0, 1\}$, which indicates if there is a buzz in the interval $(t, t+\Delta)$ or not (1 corresponds to a buzz, 0 corresponds to no buzz).

5 Analysis

In this project we focus on the buzzing behavior of the whales, since we assume that these can describe if the whales are behaving naturally. That means that for the first analysis we will only use the time since tagging, depth and buzzes. Later models will include additional co-variates.

5.1 Models

After looking at our preliminary data exploration we can see on Figure 2 that the whales are beginning to dive and buzz normally after roughly 24 hours. Therefore we fit a model with the first 24 hours after tagging cut out. This model will not have time since tagging as a co-variate, since we wish to use this model as a baseline model, with which we can compare other models that will be fitted on the entire data as well as having the time since tagging as a co-variate

5.1.1 Baseline model

The baseline model, which will be used to investigate the effect of time since tagging on the buzzing rate compared to the natural buzzing rate, will thus be fitted with the first 24 hours after the tagging cut out of the data:

$$m_0 : \log(\boldsymbol{\lambda}) = \boldsymbol{\nu} \tag{1}$$

$$\boldsymbol{\nu} = \mathbf{X}\boldsymbol{\beta}_0 + \mathbf{Z}\mathbf{b_0} + \boldsymbol{\varepsilon} \tag{2}$$

Here $\boldsymbol{\lambda}$ is the response variable, $\mathbf{X}$ is the design matrix, corresponding to the covariates of the fixed effect of depth non-linearly with three natural splines, $\mathbf{Z}$ is the design matrix corresponding to the random effect (the random choice of whale) and $\boldsymbol{\varepsilon}$ is the error vector.

5.1.2 Models including time since tagging

Since we wish to analyse how long after tagging the whales are effected, we wish to construct models with the time since tagging as a co-variate as well as depth. Since we know that after a certain time since tagging the whales behave naturally, the models we fit will have a time since tagging co-variate that goes towards zero when the time tends to infinity. This means that after a certain amount of time since tagging, this will no longer have any significant influence on how the model estimates the intensity of buzzes. The depth co-variate is included, since we know from looking at Figure 3 that there is a significantly higher probability for the whale to buzz when it is below a certain depth. We have chosen to exclude the rest of the variables in the data, since we are only interested in the effect of time since tagging.

A random effects model is used, since we have several observations for the same ”participant”, here being the whales.

Different models are constructed such that the effect of time since tagging goes towards zero at different rates. The first model we propose is a model where we include the time since tagging as follows:

$$m_1 : \log(\boldsymbol{\lambda}) = \boldsymbol{\beta_1} \exp(-t) + \boldsymbol{\nu} \tag{3}$$

Where t is time since tagging. Here it is clear that the effect of $\exp(-t)$ goes towards 0 as $t \to \infty$.

Another proposed model is:

$$m_k : \log(\boldsymbol{\lambda}) = \beta_k \frac{1}{t^p} + \boldsymbol{\nu} \tag{4}$$

For $p > 0$ and $k \in \{2, 3, 4\}$ corresponding to the model.

In this project we focus on m for $p = \frac{1}{2}, 1$ and 2, and thereby the models are:

$$m_2 : \log(\boldsymbol{\lambda}) = \boldsymbol{\beta}_2 \frac{1}{t^{\frac{1}{2}}} + \boldsymbol{\nu}$$
$$m_3 : \log(\boldsymbol{\lambda}) = \boldsymbol{\beta}_3 \frac{1}{t} + \boldsymbol{\nu}$$
$$m_4 : \log(\boldsymbol{\lambda}) = \boldsymbol{\beta}_4 \frac{1}{t^2} + \boldsymbol{\nu}$$

Again it is clear that $\frac{1}{t^p}$ goes towards 0 as t tends to infinity.

For all the models the `glmer` function in R from the `lme4` package is used to fit the random effects models. The `ns` function from the base package is used to make natural splines. These models are fitted on the whole dataset.

6 Results

6.1 Effect of time since tagging

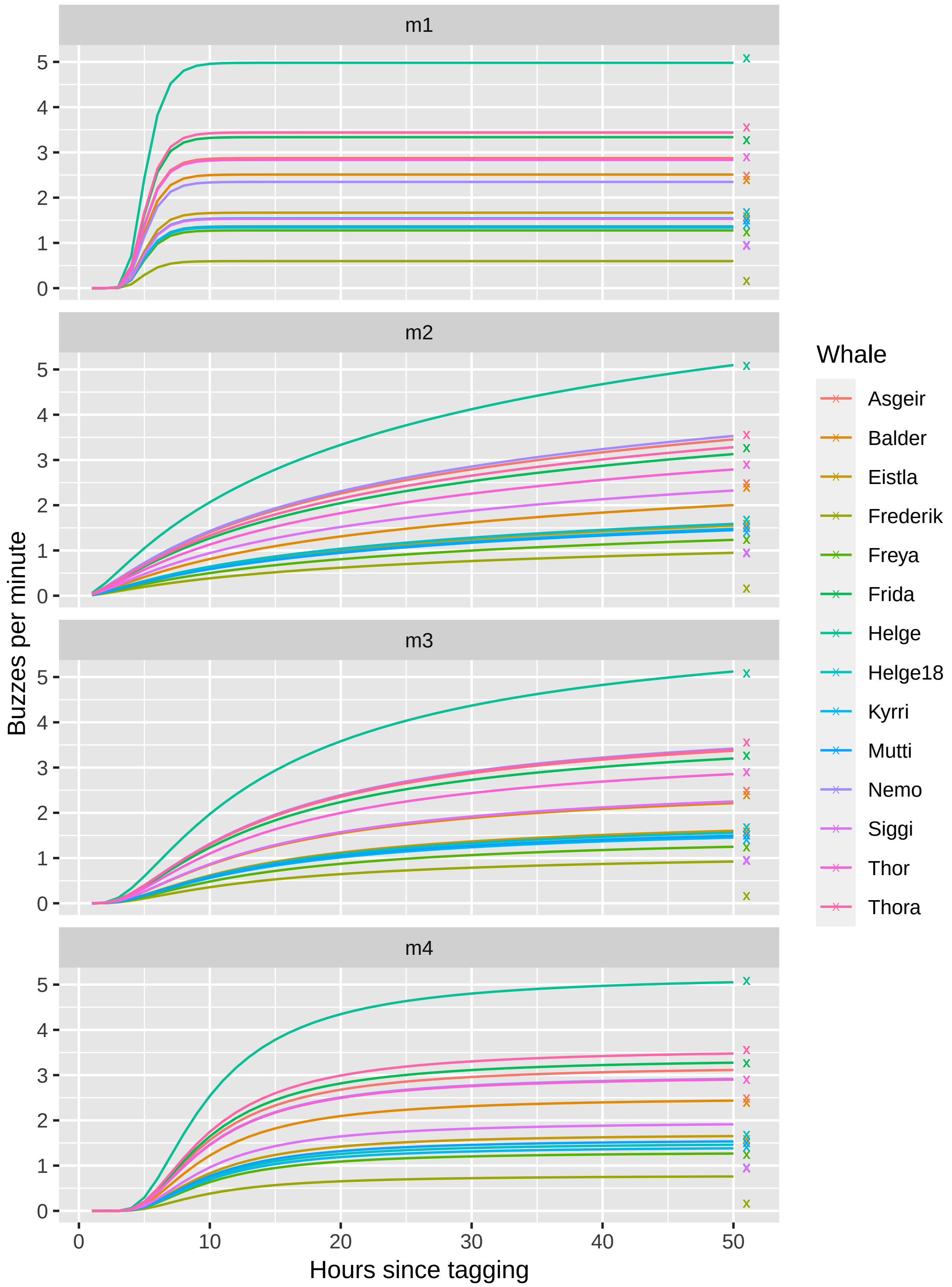


Figure 5: Predictions for different models for the different whales, with a fixed depth of 350 meters

In Figure 5 we see the predictions where the depth is fixed to 350 meters, since we saw on Figure 3, that it was around this depth, most buzzes occurred. It shows predictions for all four different models for all of the whales except Mara, since we don't have observations for her for more that 24 hours. The points at Hours since tagging = 51 are the predictions for m_0 for the given whale. We notice that the different whales behave differently when it comes to how much they are buzzing, no matter which model we look at. This can be caused by some whales buzzing more than others, but also the length of the observation time can have an impact, fx. for Frederik, who has only been observed for 28.7 hours, and thereby has less recorded buzzes. We see that all models predict that Helge makes the most buzzes per minute, and Frederik makes the least amount buzzes per minute.

Furthermore we see that all of the models predict that all whales buzz close to 0 times per minute right after the whales have been tagged, and then after some time they make more and more buzzes until we can see that the time since tagging has no effect on the buzzing rate.

We notice that the predictions for m_1 and m_4, after 50 hours, approach the predictions for m_0, for most of the whales, including Helge, Thora, Frida and Siggi, and overestimate the intensity of buzzes for others like Frederik and Thor.

The predictions for m_2 goes towards the predictions for m_0 in the case of Helge, Kyrri and Freya, and predict that Nemo and Frederik make fewer buzzes per minute compared to m_0.

We can also see that m_1 has a very steep slope right around 5 hours after tagging, where it predicts that the whales buzz much more, as opposed to m_2, m_3 and m_4 which have more of a gentle slope i.e a smooth transition to how much they buzz. In the case of m_1, it seems like 5 hours a short amount of time when looking at Figure 2 for almost all of the whales. Whereas fx. m_3 estimates that around 24 hours after tagging, $\frac{1}{t}$ does not have a large effect anymore, which seems more likely.

```
Random effects:
 Groups Name        Variance Std.Dev.
 Ind    (Intercept) 0.3522   0.5935
Number of obs: 3826480, groups:  Ind, 15

Fixed effects:
                              Estimate Std. Error z value Pr(>|z|)
(Intercept)                  -13.88180    0.10081  -137.7   <2e-16 ***
I(exp(-time_since_tagging)) -106.41439    0.75818  -140.4   <2e-16 ***
ns(posDepth, 3)1               8.70027    0.04645   187.3   <2e-16 ***
ns(posDepth, 3)2              16.81042    0.08802   191.0   <2e-16 ***
ns(posDepth, 3)3               6.12484    0.04300   142.4   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Correlation of Fixed Effects:
             (Intr) I((-__ n(D,3)1 n(D,3)2
I(xp(-t__))  0.033
ns(psDp,3)1 -0.172 -0.011
ns(psDp,3)2 -0.170 -0.040  0.552
ns(psDp,3)3 -0.057 -0.032 -0.042   0.603
```

Figure 6: Summary table for m_1

Looking at Figure 6, we see the $\hat{\beta}$ estimates for the different co-variates for m_1.

We can interpret λ as the rate of buzzes. Since the data is on a 1 Hz rate, we know that λ is measured in buzzes per second, but to make it easier to interpret we have changed the predictions to make it buzzes per minute. We can now investigate how much of an effect the $\exp(-t)$ has on the rate of buzzes by looking at:

$$\exp(\hat{\beta}_1 \cdot \exp(-t)) = \exp(-106.41 \cdot \exp(-t))$$

Now we can examine the rate of buzzing when $t = 1$ (hour) by calculating $\exp(-106.41 \cdot \exp(-1)) \approx 0$, which means that the effect of $\exp(-t)$ after 1 hour indicates that the whales buzz 0 times per minute. Looking at $\exp(-5)$ we get that $\exp(-106.41 \cdot \exp(-5)) = 0.49$, meaning that when five hours have passed, the buzzing rate is around half of its natural rate. If t is large we get that $\exp(-106.41 \cdot \exp(-t)) > 0.99$, which in this case is for $t > 10$ hours, and $\exp(-t)$ will no longer have any significant effect.

If we let r_t be the buzzing rate corresponding to time t, and let r_n be the natural buzzing rate, then we can obtain the following table.

Model	$\hat{\beta}_k$	t such that $r_t = \frac{1}{3}r_n$	t such that $r_t = \frac{1}{2}r_n$	t such that $r_t > 0.99r_n$
m_1	-106.41	4.58	5.04	10
m_2	-5.16	21	52	>10000
m_3	-11.93	10.8	17.2	1000
m_4	-71.62	8	10.1	85

Table 2: $\hat{\beta}_k$ estimates for the different models as well as values of t that correspond to 33.33, 50 and 99 % of the natural buzzing rate.

In Table 2 we see the $\hat{\beta}_k$'s for the different models. Furthermore we have calculated the time t such that the buzzing rate are $\frac{1}{3}$, $\frac{1}{2}$ and 0.99 times its normal rate. Here we see that there are very big differences between the models. The model that estimates the least amount of time before $r_t \to r_n$ is m_1, which estimates that we only need to wait 4.58 hours before the buzzing rate is a third of the natural rate, and 10 hours before $r_t > 0.99r_n$. The model that estimates the longest amount of time before $r_t \to r_n$ is m_2, which estimates that we need to wait 21 hours before the whales are buzzing on a rate of $\frac{1}{3}$ of their natural rate, and over 10000 hours before the whales are buzzing on a rate of 0.99 times its natural rate.

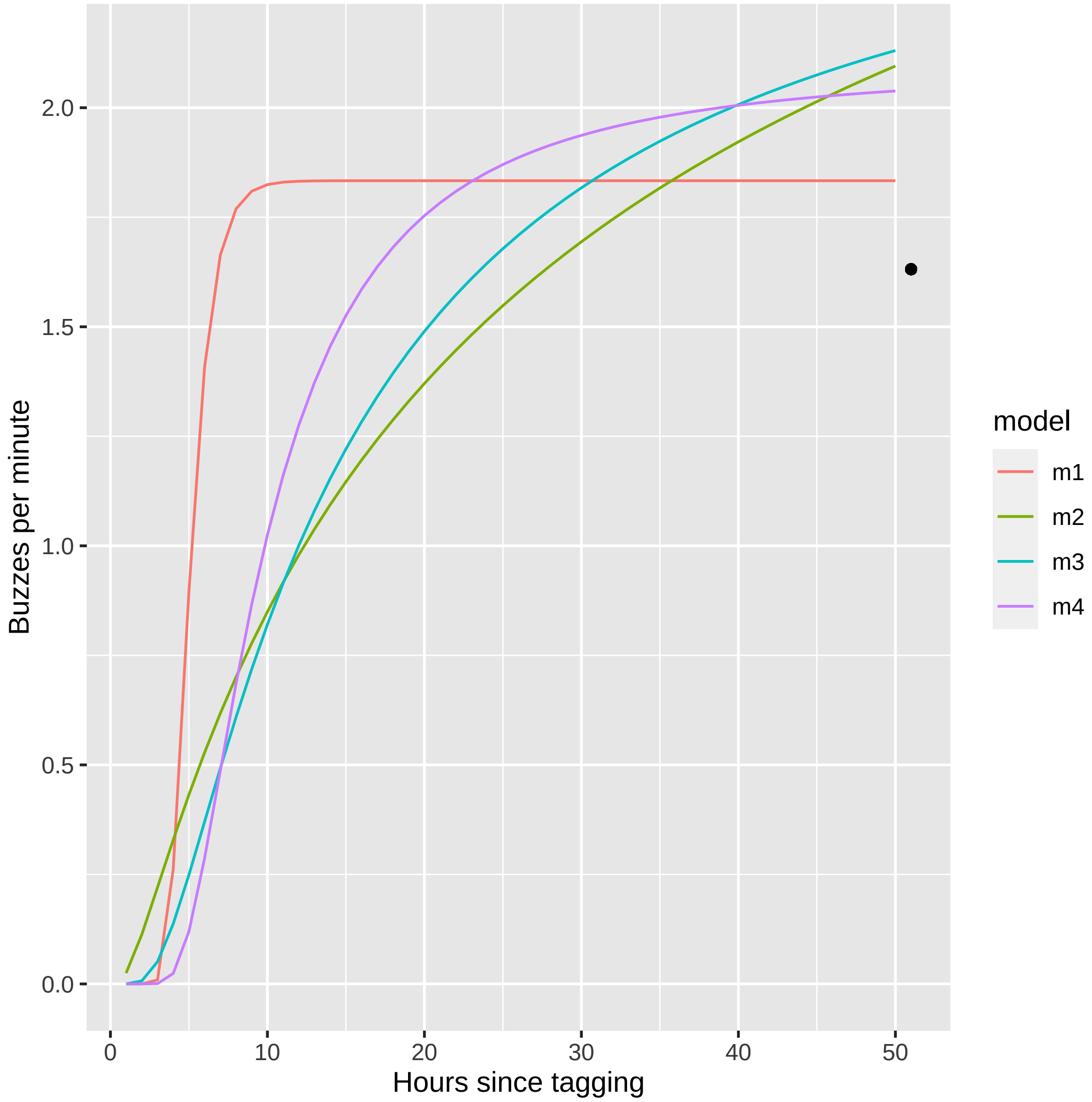


Figure 7: The population estimates for the four different models

In Figure 7 we see the four different models without any random effects, i.e. the population estimates, with the baseline model as a point. We see that with no random effects, all of the models seem to overestimate the number of buzzes but only by less that 0.5 buzzes per minute. We see that m_1 is closest to converging towards the baseline model compared to the other three models.

We also notice again, that m_1 has a steeper slope than the three other models right around three hours after tagging, confirming the results in Table 2, that the effect of $\exp(-t)$ goes faster towards 0 in m_1, compared to the t terms in the other models.

6.2 Kolmogorov Smirnow test

In this section, we follow the procedure as in [4], except we have a random effects model.
We wish to find a goodness-of-fit measure that can validate our models. This task is a bit more challenging than when working with a continuous-values process, since we can not use standard distance measures such as average sum of squared errors since these are not designed for point process data. One way one can asses the goodness-of-fit in a point process framework is to use the time-rescaling theorem from [5] and thereby transform the point process into continuous measures, we can use to asses the goodness-of-fit of the the models. To do this we wish to re-scale the intensities estimated by the different models. For event times $u_1, ...u_J$, the inter-event intervals $u_{j+1} - u_j$ can be re-scaled, where the event u_j in our case is the time of a buzz. These are denoted z_j^i where $j \in \{1, ..., J-1\}$ is the number of events and $i \in \{1, 2, 3, 4\}$ are the different models. These are defined as:

$$z_j^i = 1 - \exp\left(-\int_{u_j}^{u_{j+1}} \lambda_i(t|X(t), Z_w, \hat{\theta}_i)dt\right) \tag{5}$$

Here, $X(t)$ is the covariates at time t, Z_w is the whale and $\hat{\theta}_i$ is the maximum likelihood estimator of the parameter vector θ_i for model m_i. The integral in (5) will then follow a exponential distribution with parameter 1, and thus the z_j^i's will be independent identically distributed and uniformly distributed on $[0, 1)$ if and only if $\lambda_i(\cdot|\cdot, Z_w, \hat{\theta}_i)$ equals the true intensity of the data.
If a and b are the jth and $j+1$th values of t, then the integral in (5) can be approximated by:

$$\begin{aligned}\int_a^b \lambda_i(t|X(t), Z_w)dt &\approx \sum_{t=a}^{b-1} \frac{\lambda_i(t|X(t), Z_w) + \lambda_i(t+1|X(t+1), Z_w)}{2}\Delta \\ &\approx \sum_{t=a}^{b-1} \frac{P_i(Y_t = 1|X_t, Z_w) + P_i(Y_{t+1} = 1|X_{t+1}, Z_w)}{2}\end{aligned} \tag{6}$$

That is, we can use the different models to estimate the probability of the whale making a buzz or not in the interval $[a, b]$ by using (6). We will thus get number of buzzes -1 z values for each whale for each model. The results we get from the sum in (6) will be exponentially distributed with parameter 1, since the exponential distribution is the PDF of the time between events in a Poisson point process. If there is a long time between two buzzes, i.e. a and b are far apart, and if the model describes the data well, the predicted intensities will be low and vice versa if there are a lot of buzzes right after each other, we sum over just a few predicted intensities, but these will be higher.

Using the calculated z_j^is we can now use the Kolmogorov Smirnow statistic to test how much disagreement there is between our hypothesis' cumulative distribution function (CDF) F and the empirical CDF F_n, which is defined as

$$F_n(z) = \frac{1}{n} \sum 1_{(-\infty,z]}(Z_j)$$

for i.i.d random variables $Z_1, ..., Z_n$. Now, to construct the Kolmogorov Smirnow test, we plot the CDF of the uniform distribution which is defined as $U_F = \frac{k-\frac{1}{2}}{n}$, for $k = 1, ..., n$ against the z_j's ordered from smallest to largest, and The Kolmogorov Smirnow statistic can now be determined by calculating the largest distance between the two distributions:

$$D_n = \sup_{z \in \mathbb{R}} |F_n(z) - F(z)|$$

Since we have that $1 - P(\sqrt{n}S_n \leq 1.36) \approx 0.05$, 1.36 is the critical value on a 0.05 significance level. Now we can construct the KS bands by $F_n \pm 1.36/\sqrt{n}$.

6.2.1 Kolmogorov Smirnow confidence bands

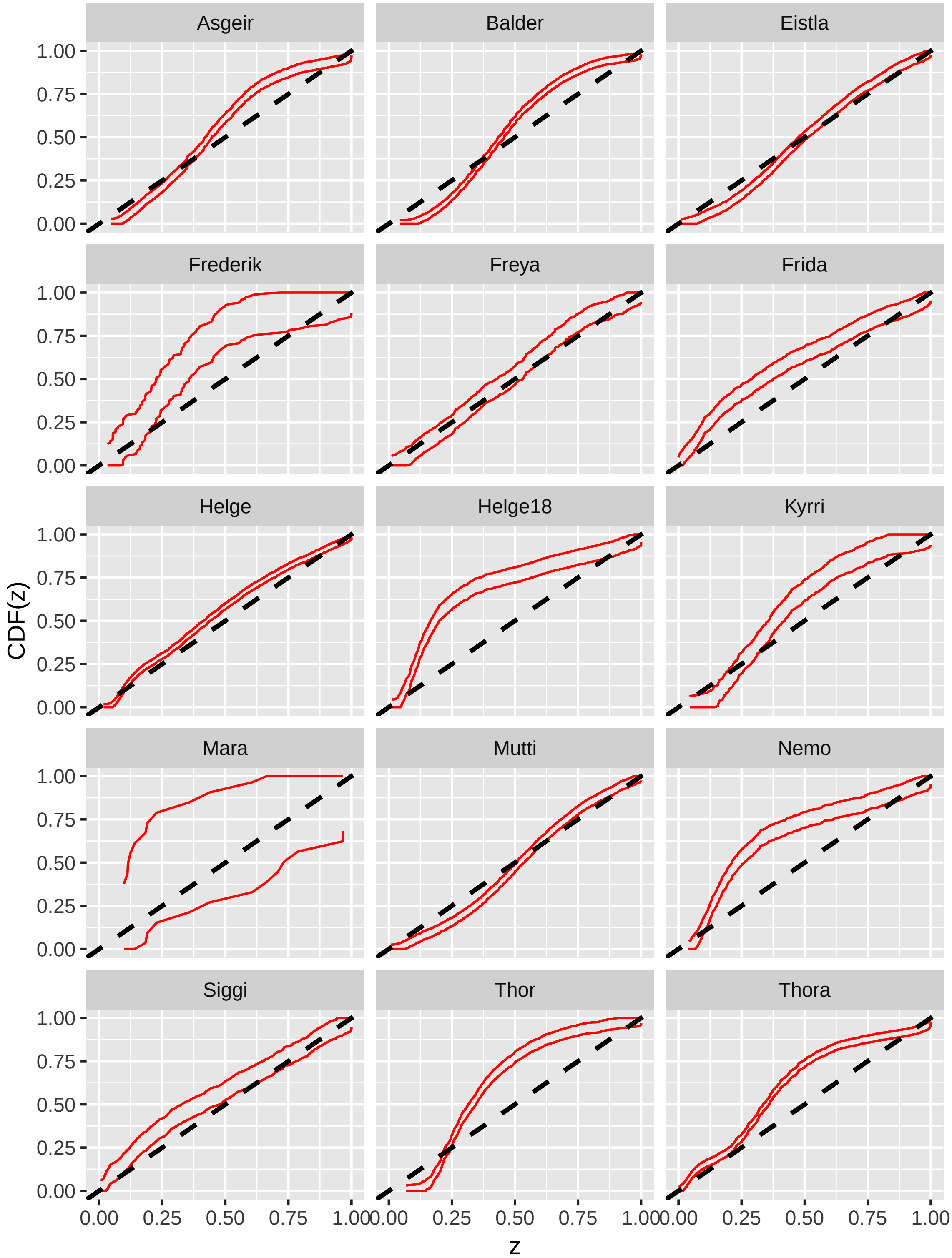


Figure 8: Kolmogorov Smirnow bands (red lines) and the CDF of the uniform distrubtion (dashed line) for m_1

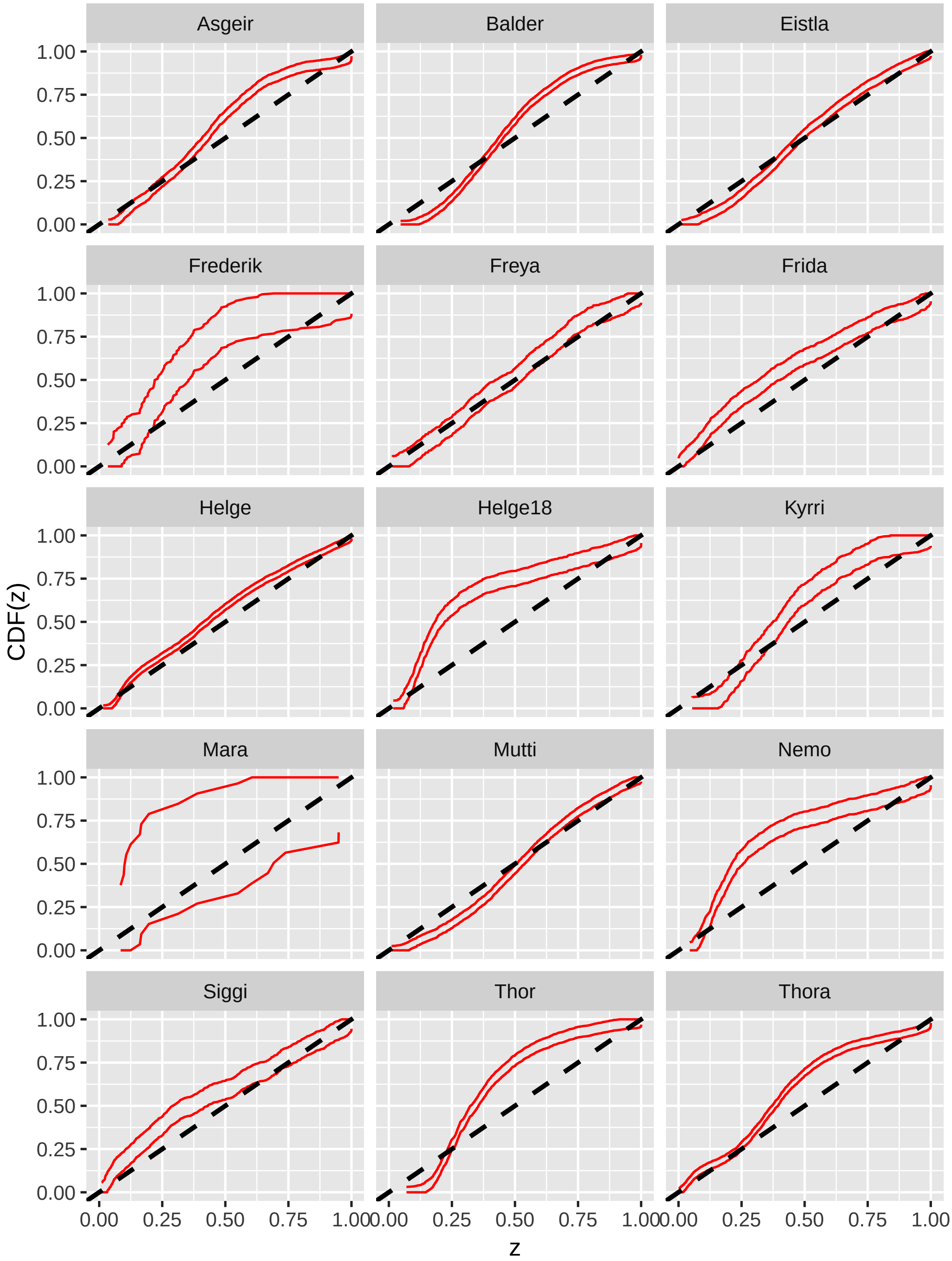


Figure 9: Kolmogorov Smirnow bands (red lines) and the CDF of the uniform distrubtion (dashed line) for m_2

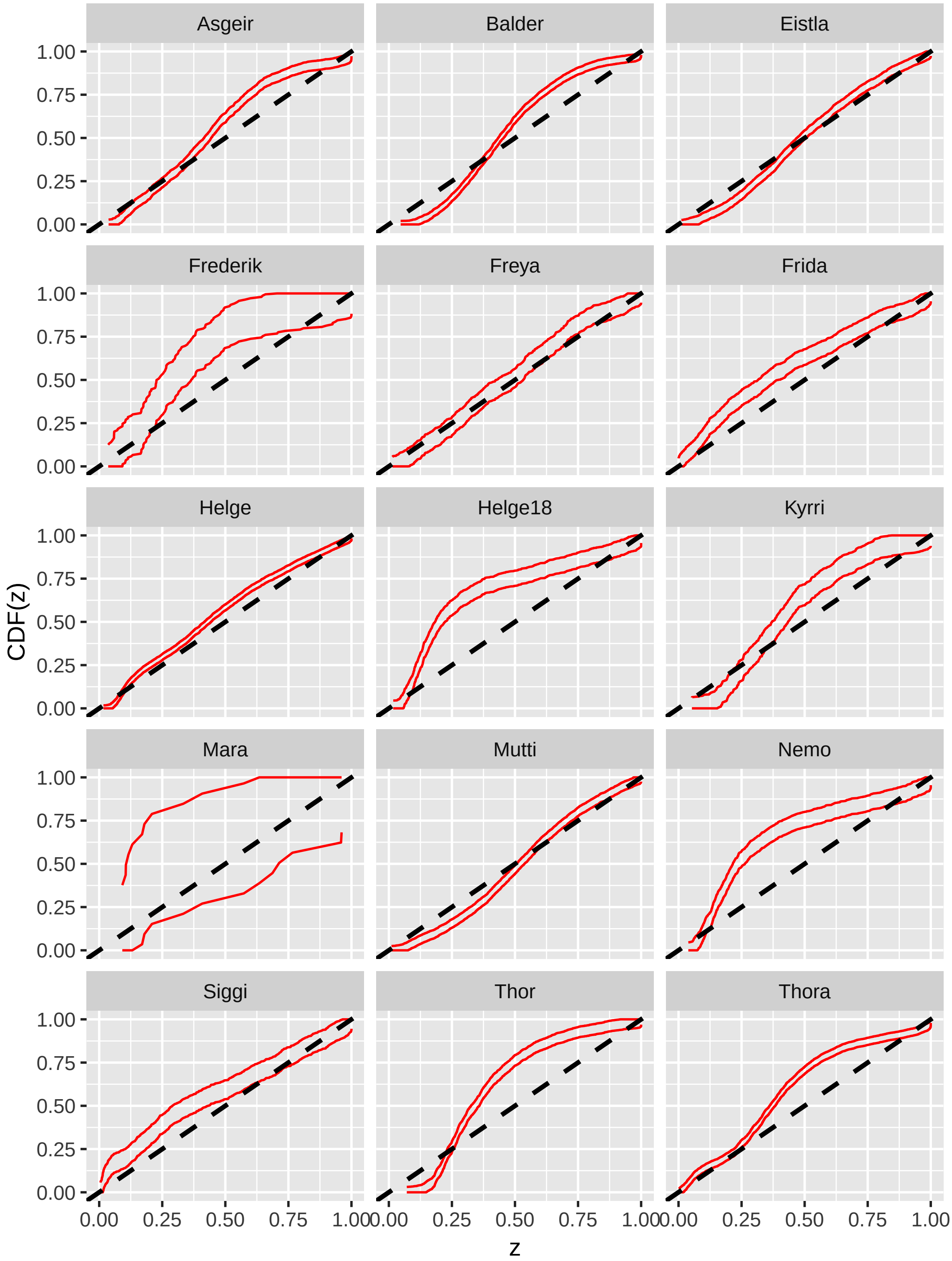


Figure 10: Kolmogorov Smirnow bands (red lines) and the CDF of the uniform distrubtion (dashed line) for m_3

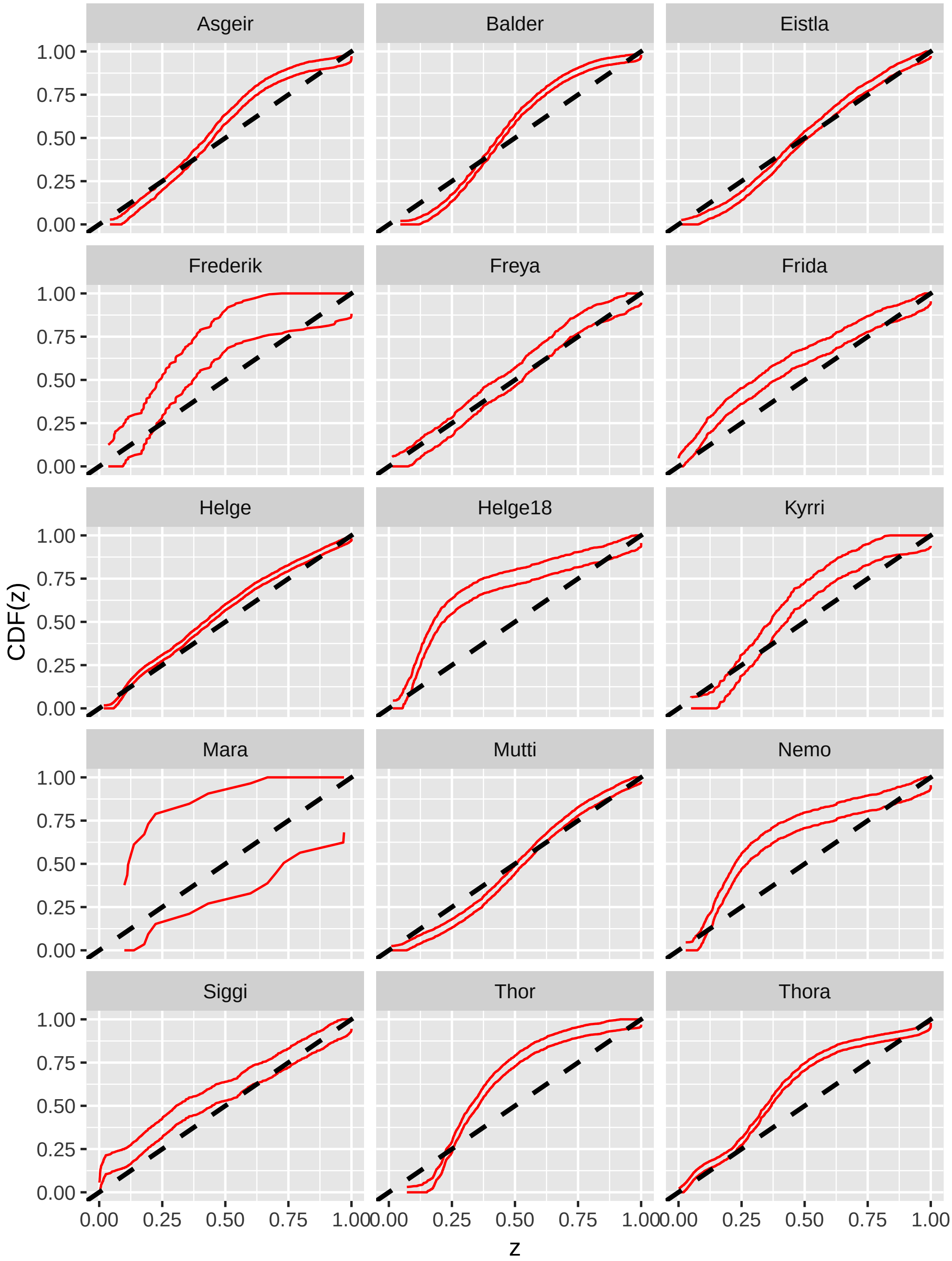


Figure 11: Kolmogorov Smirnow bands (red lines) and the CDF of the uniform distrubtion (dashed line) for m_4

From Figure 8-11 which show the Kolmogorov Smirnow bands for the rescaled intensities, we notice that the CDF of the uniform distribution is entirely included in the KS bands for Mara in all of the models. This means that a formal hypothesis cannot reject that this is the right model to estimate Mara's buzz intensities. Though Maras sample size is very small as seen on Table 1. Furthermore we see that in the case of Freya, the CDF of the uniform distribution is almost included in the confidence bands in all of the models, but not entirely.

In [4], they also use Kolmogorov Smirnow bands to validate their models. On Fig. 5 in this article we see that M_0 which does not include memory looks a lot like the results we have found. Therefore we think that the model would be a better fit, if we included memory in our models. Unfortunately we did not have enough time to conduct this analysis, and we thereby assume that the models fit better if the memory was included in the models.

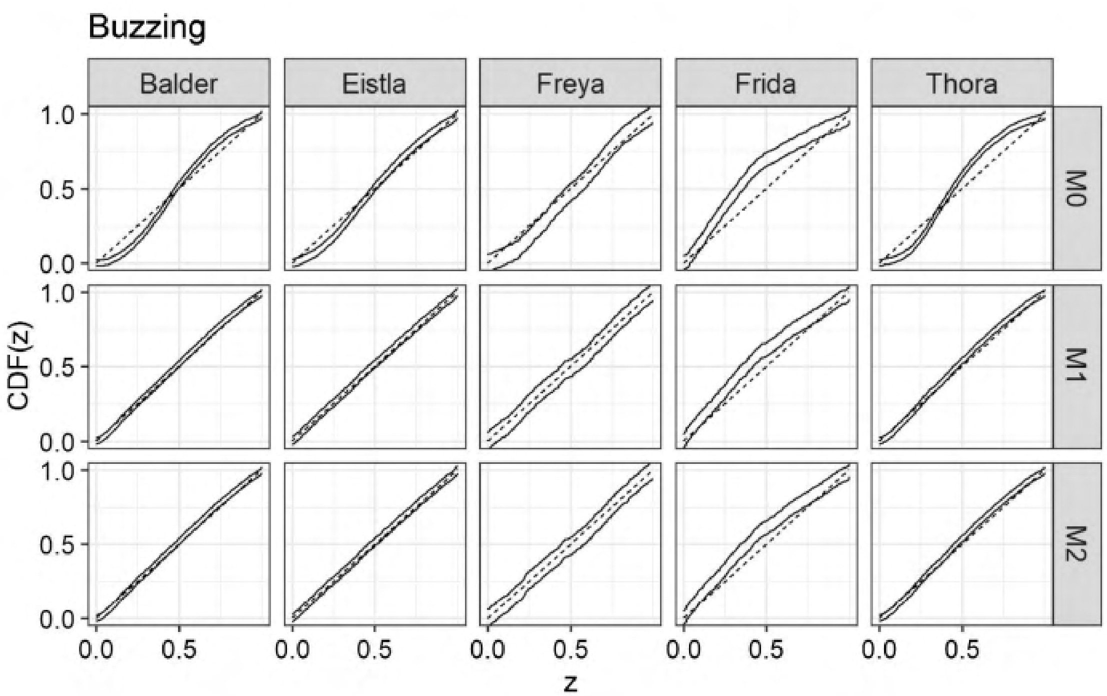


Figure 12: Figure taken from [4], which shows their KS bands. $M0$ is a model which does not include memory (this looks like our results), and $M1$ and $M2$, does include memory.

6.2.2 Independence

Since, if the model fits the data, the rescaled intensities are both uniformly distributed and independent, we wish to investigate independence of the z_j's. This is done by plotting $z_1, ..., z_{J-2}$ against $z_2, ..., z_{J-1}$. If the z_j's are indeed independent, no non-random shapes should appear on the figure. If there are non-random patterns, it would mean that there is information carried from one z_j to the next.

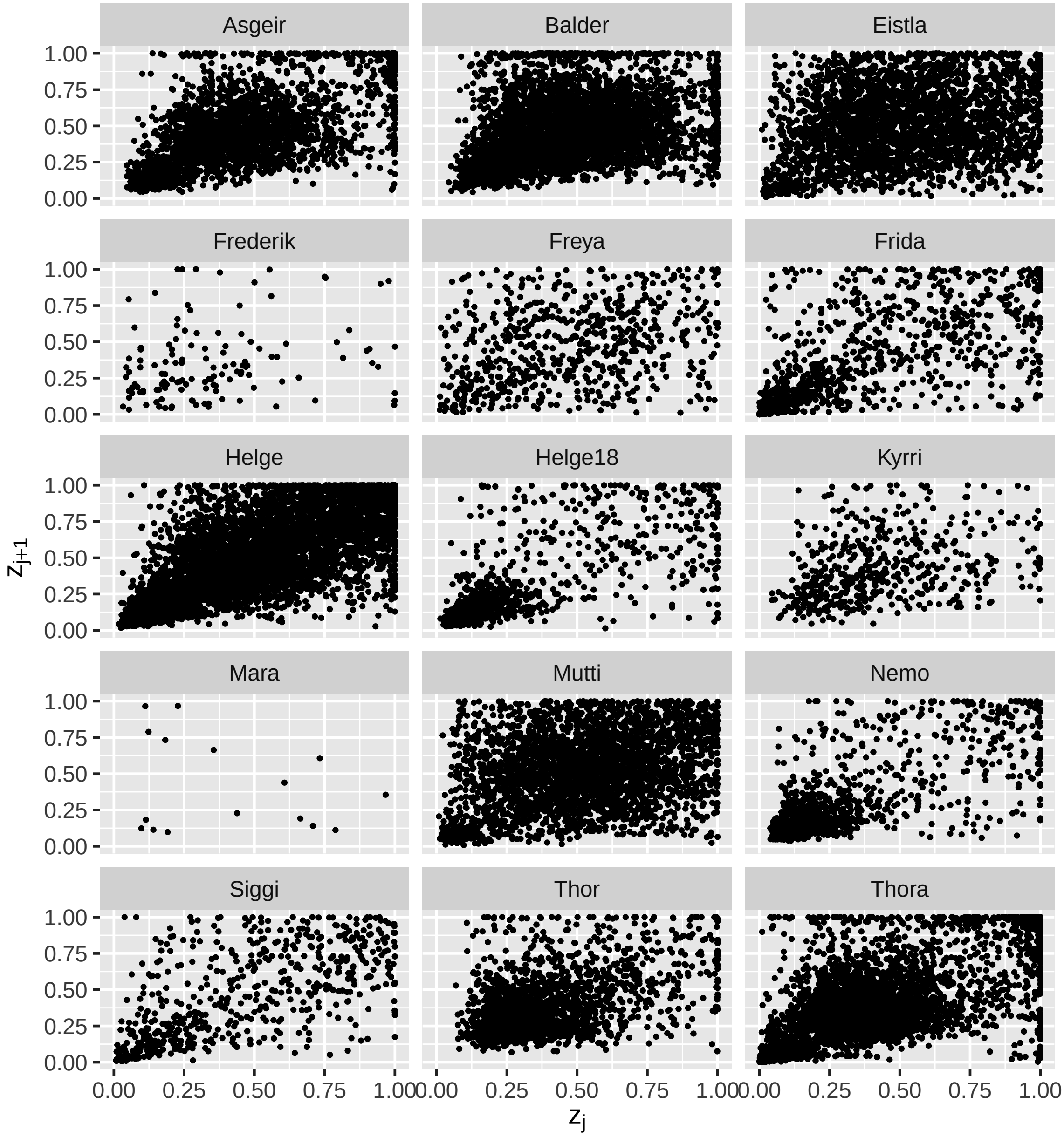


Figure 13: z_j's plotted against z_{j+1} for m_1

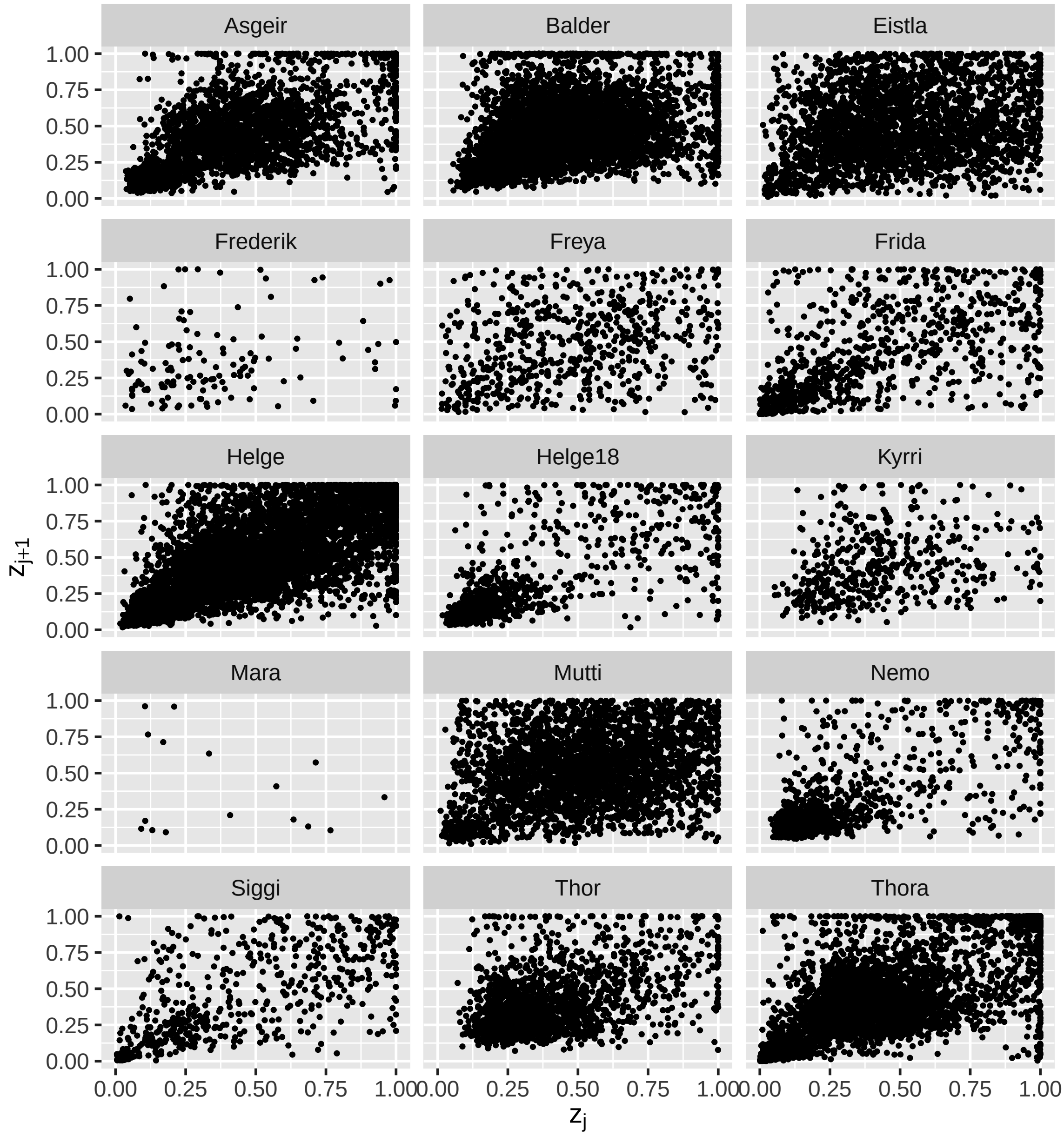


Figure 14: z_j's plotted against z_{j+1} for m_2

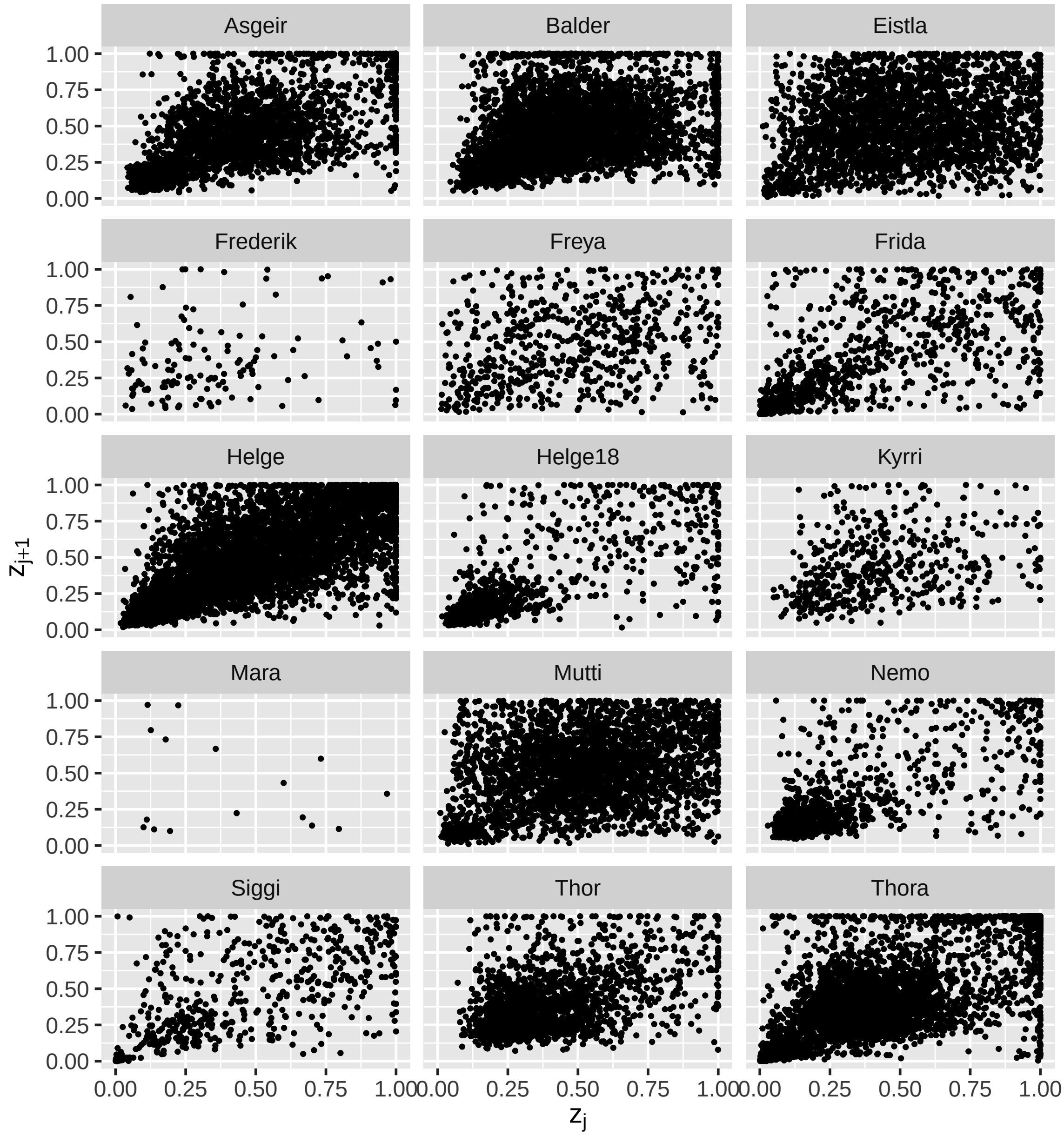


Figure 15: z_j's plotted against z_{j+1} for m_3

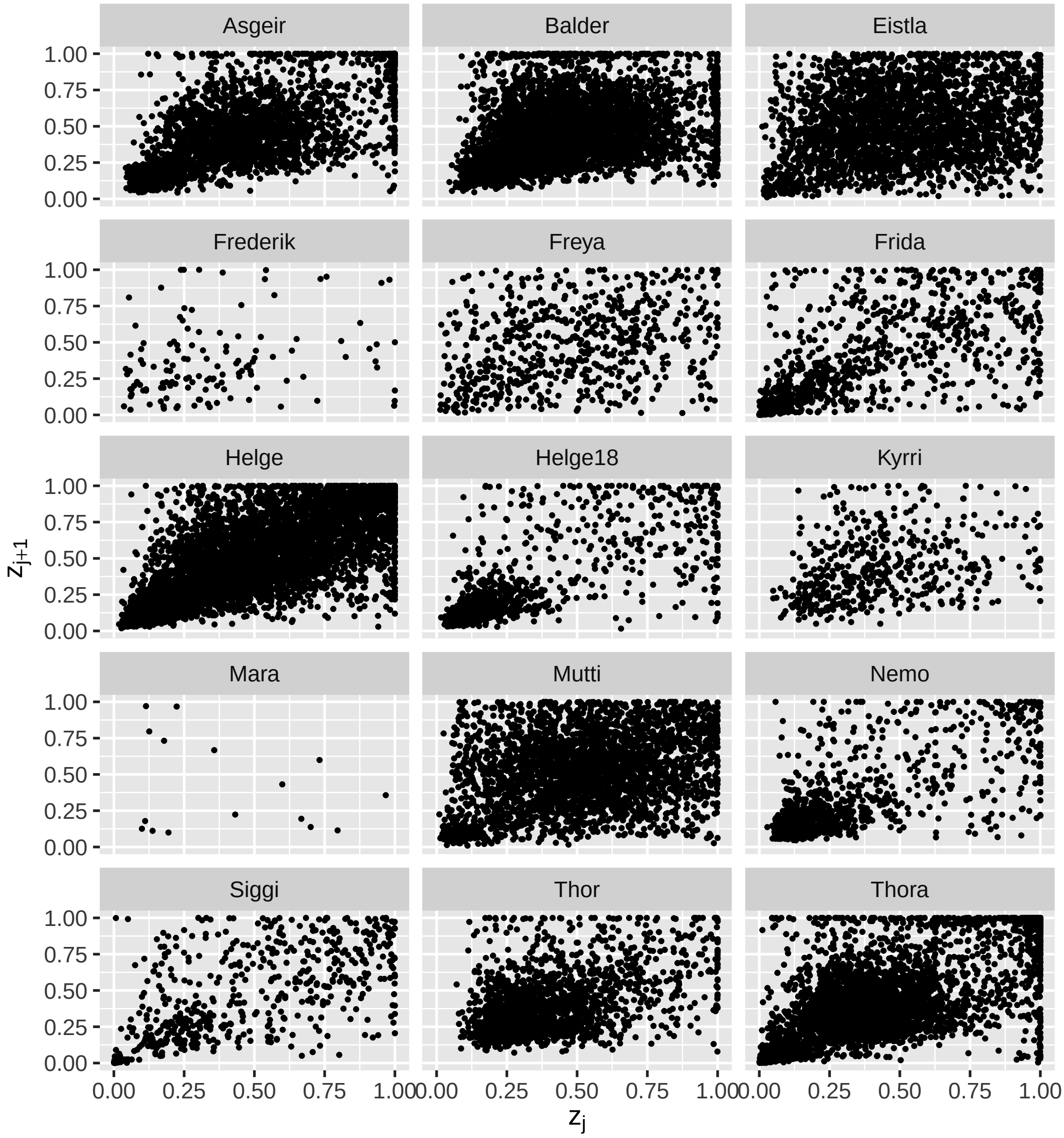


Figure 16: z_j's plotted against z_{j+1} for m_4

Figure 13-16 show the z_j plotted against z_{j+1} for the four different models. Since we wish to investigate independence we wish for these, as stated before, to not form any non-random patterns.

The four different models show the same results for most of the whales, namely that there are cone shapes for most of the whales. This means that the re-scaled intensities are not independent for these whales. We also see that in the cases of Mutti and Freya there are no specific patterns which means that the z_j's can be assumed independent. Again it is hard to say anything about Mara, since there are so few data points. There are not many data points for Frederik either, but here we see that most of the points are clustered in the lower left of the plot where $z_j < 0.5$ and $z_{j+1} < 0.5$.

These results would again suggest that the models we have made do not fit the data. Again we can look at Figure 6 from [4] to see that if the models included memory, then we might have gotten much more randomness, hence independence:

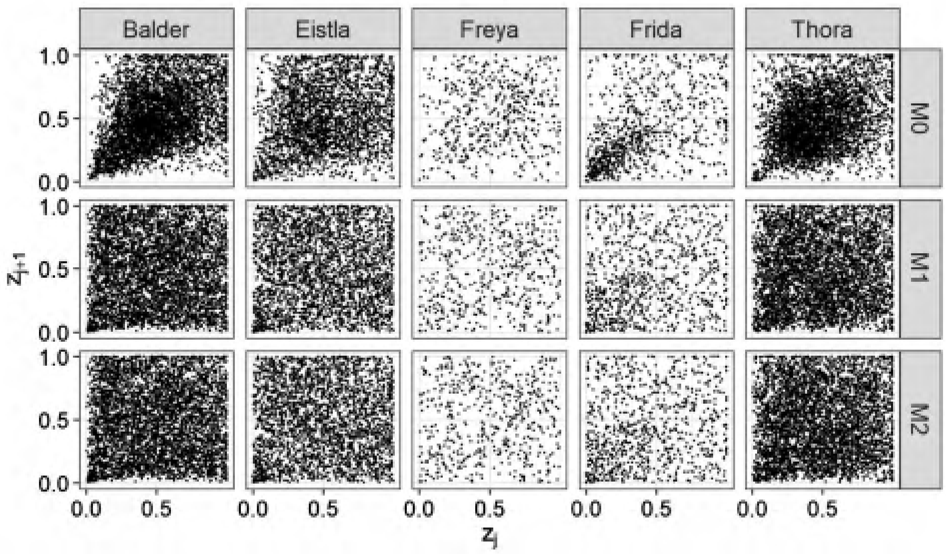


Figure 17: Figure from [4], that shows z_j plotted against z_{j+1}

On Figure 17 we see that for $M0$ there are cone shaped patterns, which look like the results we get, but when including memory as they do in $M1$ and $M2$, the patterns are much more random, which would suggest independence.

Therefore, if we had more time we would have conducted an analyses where memory was included in the models. Unfortunately the time did not permit it as this is very computationally heavy, and each of the 4 models fitted already takes approximately one hour to fit. To conduct this analysis, one would have to fit each model several times. This procedure is very time consuming, and therefore we assume in this project that the z_j's would achieve independence as well as follow a uniform distribution for some of the whales in some of the models, if memory was included.

An autoregressive model is a model where the output variable depends on the past realisations of the output. In our case such a model for m_1, could look like:

$$\log(\lambda_t) = \nu_t + \exp(-t)\beta_{1t} + \sum_{q}^{Q} c_q y_q \lambda_{t-q}$$

where $y_{t-1}, ..., y_{t-Q}$ are the past realizations of the response, and $c_1, ..., c_Q$ are the autoregressive coefficients.

To determine Q, we could make a stepwise procedure based on pooled AIC, meaning that we could fit the model with $Q = 0$, and then increasing Q one at a time, to find the value of Q for which the AIC reaches its minimum.

All in all it appears that no model fits the data better than others from the criteria we have set, and we can thereby choose the best model from the AIC statistic.

6.3 Actual counts vs. predicted intensities

To further validate the models, we plot the predicted intensities together with the actual counts. Each point corresponds to 1 hour of actual counts of buzzing, while the red lines correspond to the predicted intensities for the same hour.

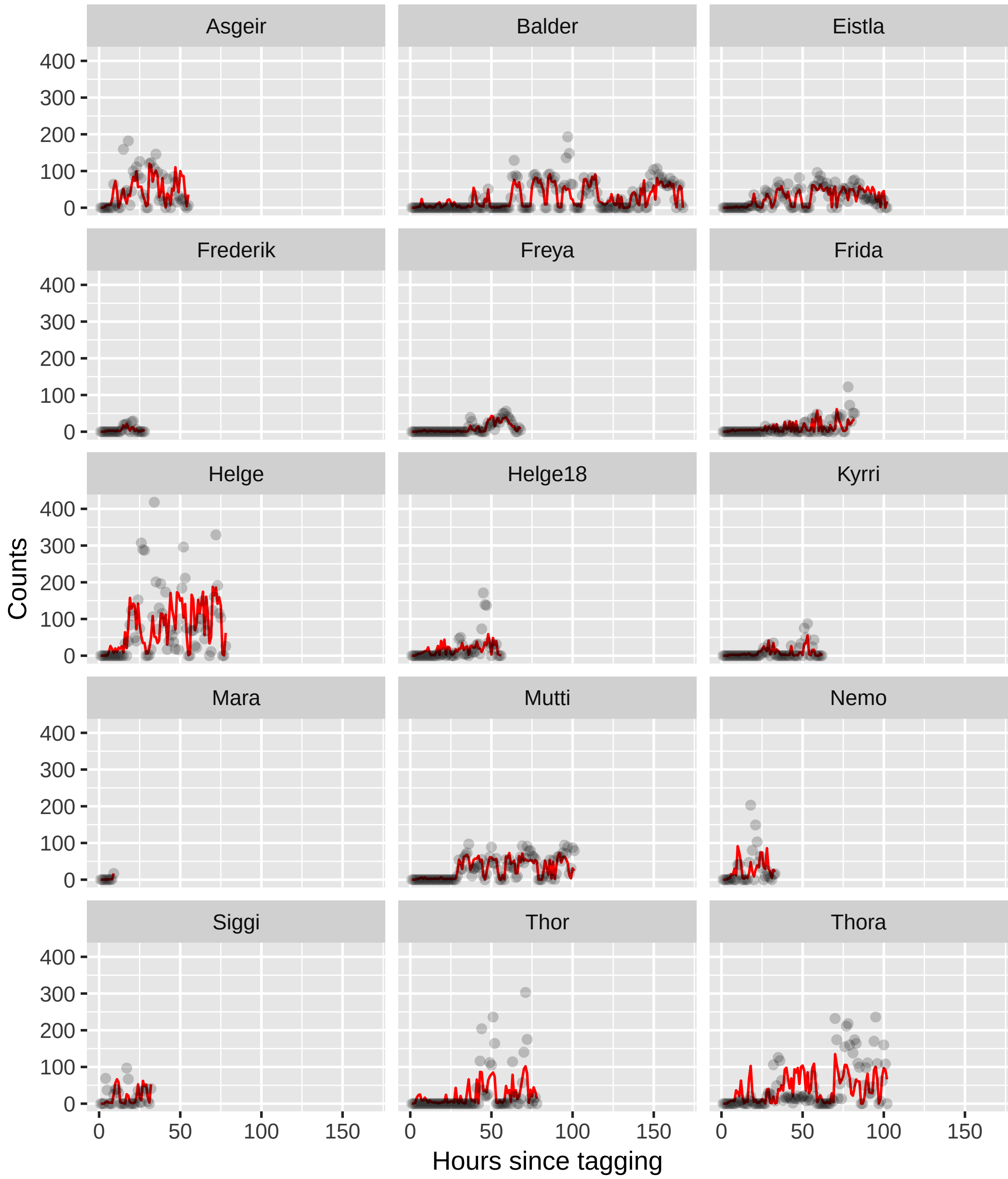


Figure 18: Actual counts compared to predicted intensities for each hour of observation for m_1

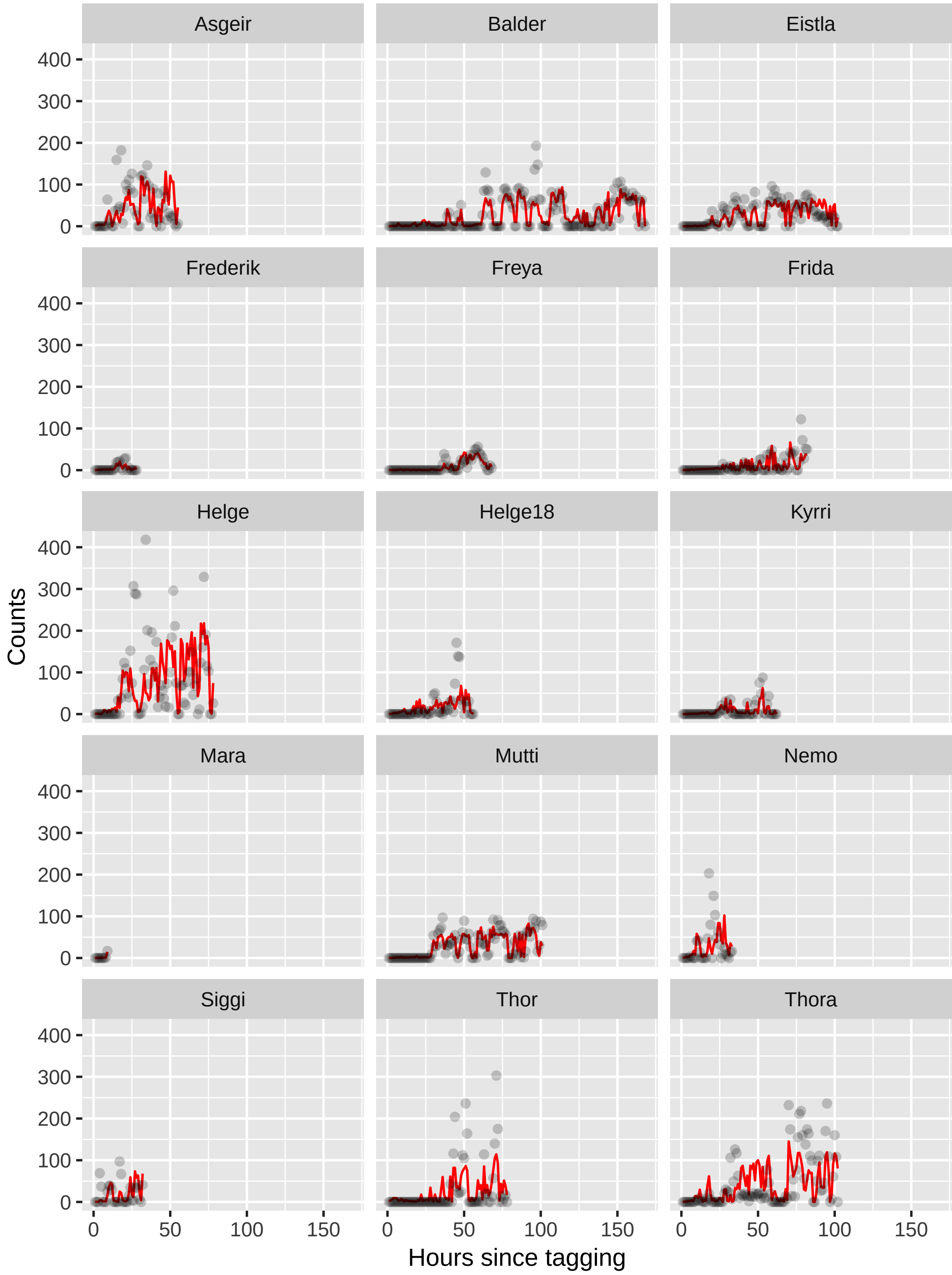


Figure 19: Actual counts compared to predicted intensities for each hour of observation for m_2

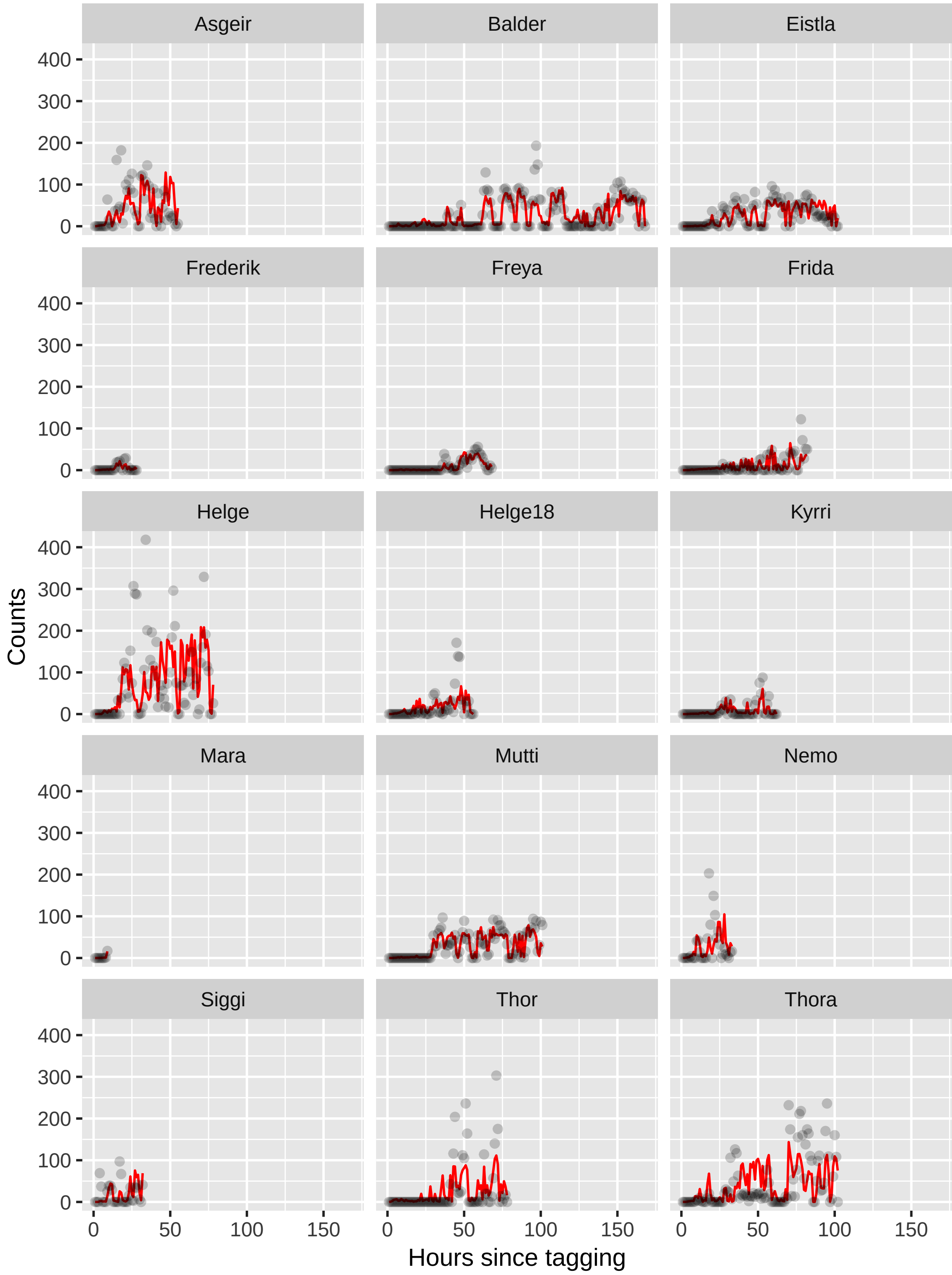


Figure 20: Actual counts compared to predicted intensities for each hour of observation for m_3

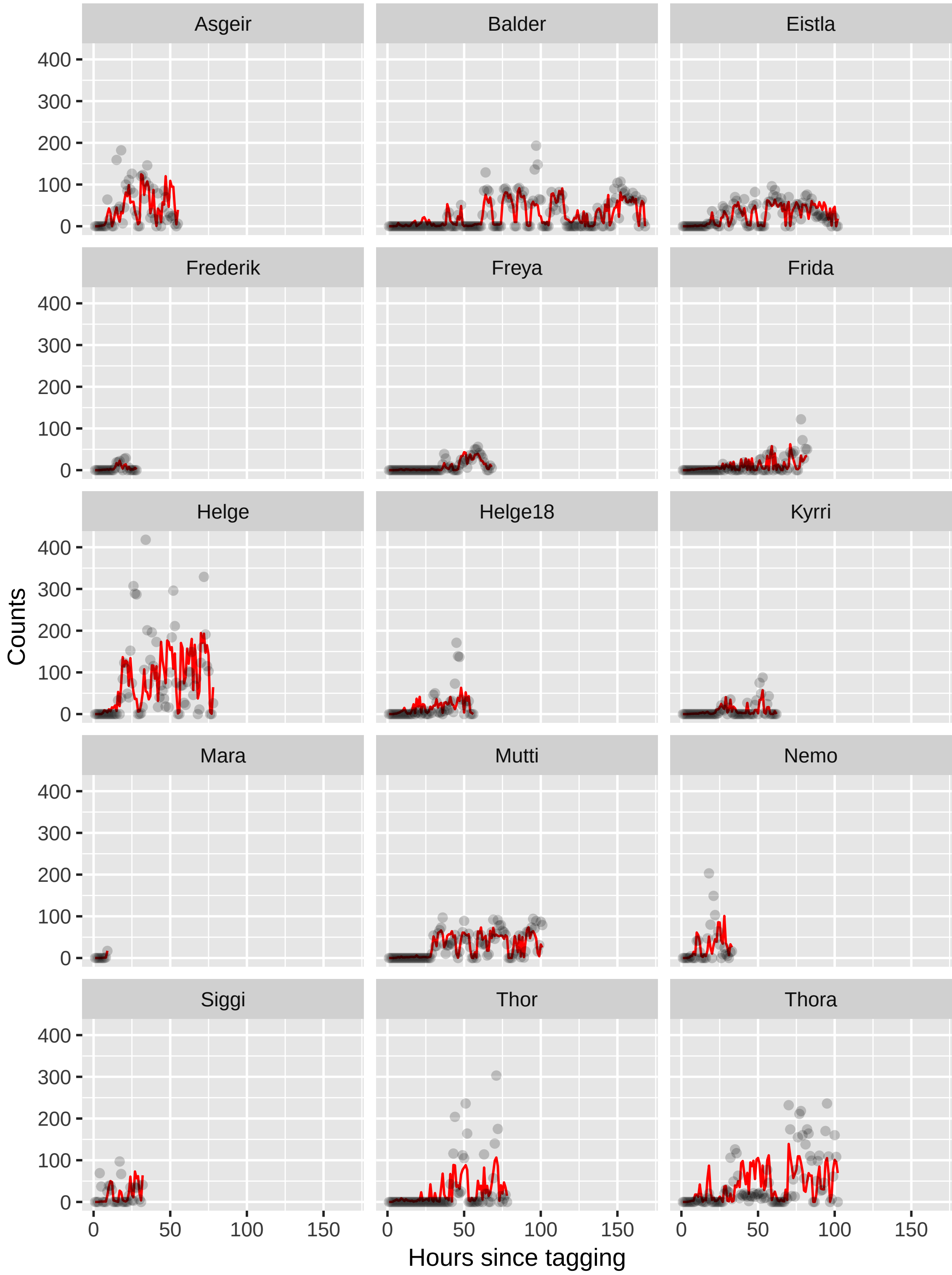


Figure 21: Actual counts compared to predicted intensities for each hour of observation for m_4

By looking at Figure 18, we see that m_1 predicts fairly well for most whales. For some whales the model has problems predicting some of the high counts. For example Thora's count after 75 hours since tagging, as well as Helge between 25 and 50 hours since tagging.

From Figure 19 we see that m_2 is a fairly good model for predicting intensities for all the whales, but Thora and Thor. It also fails to catch some of the high counts for some of the whales including Helge and Thora.

For m_3 we see on Figure 20 that m_3 also doesn't predict Thora nor Thor very well. This model also fail to catch some buzzes again, in the case of Helge and Thora

In the case of m_4 we see on Figure 21 that this model predicts well for all of the whales, except for Thor and Thora. Furthermore the model fails to catch the high counts of Helge.

Overall, the four models predict the counts pretty well for most of the whales, and especially for Balder and Eistla, which are the whales with the most observations. It is a bit difficult to say anything specific about the predictions for Mara and Frederik due to the small observation size, but for the counts that we do have, all models predict fairly well. Furthermore all of the four models fail to catch Helge and Thora's high counts. From these results, the preferred model is m_3, since this model predicts best in general in terms of the actual counts. Furthermore the re-scaled intensities are closest to following a uniform distribution for more whales than the other models. This means that, from these results, this model is the best candidate of the four for describing the buzzing behavior.

6.4 Model selection

Using the Analysis of Variance (ANOVA) method in R yields these results:

Model	AIC	BIC	logLik	deviance
m_1	269474	269553	-134731	269462
m_2	268523	268602	-134255	268511
m_3	268427	268506	-134207	268415
m_4	268715	268794	-134351	268703

Table 3: Analysis of Variance table for the four different models

When looking at AIC values, the model that fits the data best yields the lowest AIC score, and thereby we see on Table 3, that m_3 (the model with $\frac{1}{t}$), has the lowest score, and is thereby the model of choice of these four models.

6.5 Mediation analysis

Mediation analysis is a way of identifying the relationship between an independent variable and a dependent variable by using an intermediate third variable called a *mediator* variable.
First, one wants to describe preexisting beliefs about the causal structure in which the mediation analysis is to be conducted. This can be done by using a directed acyclic graph (DAG), like the one below:

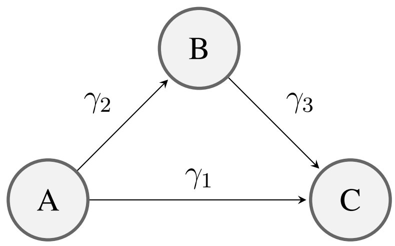


Figure 22: Directed acyclic graph of three variables, $A =$ the exposure, $B =$ the mediator and C = the response

An arrow in the DAG, implies that we think there is a possibility that a causal connection exists between the two variables. This can be interpreted as if $A \rightarrow B$ then a change in A causes a change in the distribution of B, but that does not necessarily imply that a change of B will cause a change in A.

Intuitively, a mediation analysis will describe what would happen if 1) the indirect pathway (with B included) is the only causal way between the exposure (A) and the outcome (B) and 2) the indirect pathway could be "deactivated".

Normally, one can calculate the effect of the exposure A on the response C, by calculating the regression of C on A:

$$C = \beta A + \varepsilon$$

Where $\beta = \sigma_{AC}/\sigma_A^2$
But if there is an effect of a mediator B, we wish to decompose β into a sum of the direct effect of A on C and the indirect effect of A on C via B.
We can express the effect of A on B by the regression

$$B = \gamma_2 A + \varepsilon_1 \tag{7}$$

Where

$$\gamma_2 = \sigma_{AB}/\sigma_A^2$$

Furthermore we can express the effect of A and B on C by the following regression:

$$C = \gamma_1 A + \gamma_3 B + \varepsilon_2 \tag{8}$$

Where

$$\gamma_1 = \frac{\sigma_{AC}\sigma_B^2 - \sigma_{AB}\sigma_{BC}}{\sigma_A^2\sigma_B^2 - \sigma_{AB}^2} \qquad \text{and} \qquad \gamma_3 = \frac{\sigma_{BC}\sigma_A^2 - \sigma_{AB}\sigma_{BC}}{\sigma_A^2\sigma_B^2 - \sigma_{AB}^2} \tag{9}$$

Inserting (7) into (8) we obtain

$$\begin{aligned} C &= \gamma_1 A + \gamma_3 B + \varepsilon_2 \\ &= \gamma_1 A + \gamma_3(\gamma_2 A + \varepsilon_1) + \varepsilon_2 \\ &= (\gamma_1 + \gamma_3\gamma_2)A + \varepsilon \end{aligned}$$

If this decomposition of β is right, it must hold that $\gamma_1 + \gamma_3\gamma_2 = \beta$. This is shown below:

$$
\begin{aligned}
\gamma_1 + \gamma_3\gamma_2 &= \frac{\sigma_{AC}\sigma_B^2 - \sigma_{AB}\sigma_{BC}}{\sigma_A^2\sigma_B^2 - \sigma_{AB}^2} + \left(\frac{\sigma_{BC}\sigma_B^2 - \sigma_{AB}\sigma_{AC}}{\sigma_A^2\sigma_B^2 - \sigma_{AB}^2}\right)\left(\frac{\sigma_{AB}}{\sigma_A^2}\right) \\
&= \frac{\sigma_{AC}\sigma_B^2 - \sigma_{AB}\sigma_{BC}}{\sigma_A^2\sigma_B^2 - \sigma_{AB}^2} + \frac{\sigma_{AB}\sigma_{BC}\sigma_B^2 - \sigma_{AB}^2\sigma_{AC}}{\sigma_A^{2^2}\sigma_B^2 - \sigma_A^2\sigma_{AB}^2} \\
&= \frac{\sigma_A^2\sigma_{AC}\,\sigma_B^2 - \sigma_{A^2}\sigma_{AB}\sigma_{BC} + \sigma_{A^2}\sigma_{AB}\sigma_{BC}\sigma_{AB}{}^2\sigma_{AC}}{\sigma_A^{2^2}\sigma_B^2 - \sigma_A^2\sigma_{AB}{}^2} \\
&= \frac{\sigma_A^2\sigma_{AC}\,\sigma_B^2 - \sigma_{AB}{}^2\sigma_{AC}}{\sigma_A^{2^2}\sigma_B^2 - \sigma_A^2\sigma_{AB}{}^2} \\
&= \frac{\sigma_{AC}(\sigma_A^2\sigma_B^2 - \sigma_{AB}^2)}{\sigma_A^2(\sigma_A^2\sigma_B^2 - \sigma_{AB}^2)} \\
&= \frac{\sigma_{AC}}{\sigma_A^2} = \beta
\end{aligned}
$$

This means we have found a decomposision of the total effect β of A on C which will be γ_1: the direct effect, and $\gamma_2\gamma_3$: the indirect effect.

This means that if we have an increase in A, it would cause an increase of $\gamma_1 + \gamma_2\gamma_3$ in the expectation of C. Here $\gamma_2\gamma_3$ will be the part that goes though the mediator B, as illustrated in Figure 22. The mediation proportion can thus be calculated as

$$\frac{\text{Indirect effect}}{\text{Total effect}} = \frac{\gamma_2\gamma_3}{\gamma_1 + \gamma_2\gamma_3} \tag{10}$$

The mediation proportion explains how much of a percentage the regression coefficient of C on A changes, when going from the model without the intermediate value B, to the larger model that includes the intermediate value. [3]

The whole procedure of tagging the whale is not very comfortable for the whale and from the data exploration we did in Section 3.1, it is clear that the whale does not behave naturally before a certain time after the procedure. Section 3.1 also shows that the whales buzz more when they are around 350 meters deep, compared to when they are around the surface. This means that if the tagging procedure has an effect on how deep the whales dive, it has an indirect effect on how much the whales buzz, i.e. the diving depth is a mediator.

6.5.1 Mediation proportion of m_3

A DAG is constructed, to describe our pre-existing beliefs about the causal structure of our analysis:

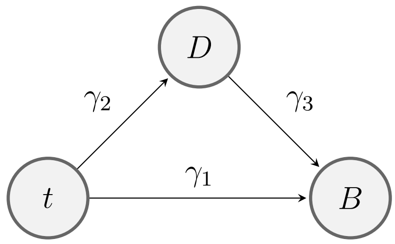


Figure 23: DAG with Buzz as the response, Depth as the mediator and Time since tagging as the exposure

Using the theory from section 6.5 we can calculate the mediation proportion by fitting a model that does not include the mediator variable depth, which will give us an estimate of the total effect ($\gamma_1 + \gamma_2\gamma_3$) of $\frac{1}{t}$ and depth on the buzzing rate. Using this we can estimate the indirect effect ($\gamma_2\gamma_3$) by simply calculating $\gamma_2\gamma_3 = \gamma_1 + \gamma_2\gamma_3 - \gamma_1$. Now we have all the information we need to find the mediation proportion using (10).
m_3 is thus fitted without the depth as a co-variate:

$$mm_3 = log(\boldsymbol{\lambda}) = \beta_{3m}\frac{1}{t} + \mathbf{Zb} + \boldsymbol{\varepsilon}$$

The estimates for mm_3 yields a total effect of -29.08 and m_3 an indirect effect of -11.93. This means that the estimate changes from -29.08 to -11.93, after we have corrected for the mediated effect of depth. Using this we can calculate the direct effect to be $\frac{11.93}{29.08}$, and the mediated effect to be $1 - \frac{11.93}{29.08} = 0.59$ or 59 %.
We can now conclude that 59 % of the effect of $\frac{1}{t}$ i caused by the whales diving less, and the remaining 41 % of the effect is direct, no matter if the whales dive or not. If the tagging procedure makes the whales dive less, it is natural that they do not buzz either (indirect effect) but it can also happen that the whales do not buzz even though the whales dive (direct effect).

7 Extended models

Now that we have analysed the effect of time since tagging, we want to examine the effect the handling time has on the buzzing of the whales. Therefore we now wish to construct new models that include this as a co-variate, as well as the gender and the length of the whale. We simply make three groups for the handling time, h_{small} corresponds to a handling time in the interval $[0, 41]$, h_{medium} is in the interval $(41, 62]$, and h_{large} is for a handling time > 62 minutes. This is done such that we get 5 whales in each of these groups. Furthermore we make 3 groups of 5 whales, corresponding to little med and large whales. The new models will extend m_1, m_2 and m_3 described in Section 5.1.2, now including the new co-variates.

Let

$$\boldsymbol{\nu}_{ext} = \mathbf{X}_{ext}\boldsymbol{\beta}_{ext} + \mathbf{Z}_{ext}\mathbf{b}_{ext} + \boldsymbol{\varepsilon}$$

Where $\mathbf{X}_{ext}$ is the design matrix for the fixed effects of Depth non-linearly with 3 natural splines, the length, and the gender of the whales, and $\mathbf{Z}_{ext}$ is the design matrix of the random effects.

Since we expect the size of the effect of the different time since tagging co-variate depends on how long the tagging procedure took, the following 3 models are fitted with interaction. Now we can write the extended models as:

$$m_5 = \log(\boldsymbol{\lambda}) = exp(-t)\begin{pmatrix} \beta_{S5}h_{small} \\ \beta_{M5}h_{medium} \\ \beta_{L5}h_{long} \end{pmatrix} + \boldsymbol{\nu}_{ext}$$

$$m_6 = \log(\boldsymbol{\lambda}) = \frac{1}{t^{\frac{1}{2}}}\begin{pmatrix} \beta_{S6}h_{small} \\ \beta_{M6}h_{medium} \\ \beta_{L6}h_{long} \end{pmatrix} + \boldsymbol{\nu}_{ext}$$

$$m_7 = \log(\boldsymbol{\lambda}) = \frac{1}{t}\begin{pmatrix} \beta_{S7}h_{small} \\ \beta_{M7}h_{medium} \\ \beta_{L7}h_{long} \end{pmatrix} + \boldsymbol{\nu}_{ext}$$

It was also intended that m_4 should have an extended form, including handling time, gender and length, but this model would not converge no matter what was tried.

7.1 Effect of time since tagging for extended models

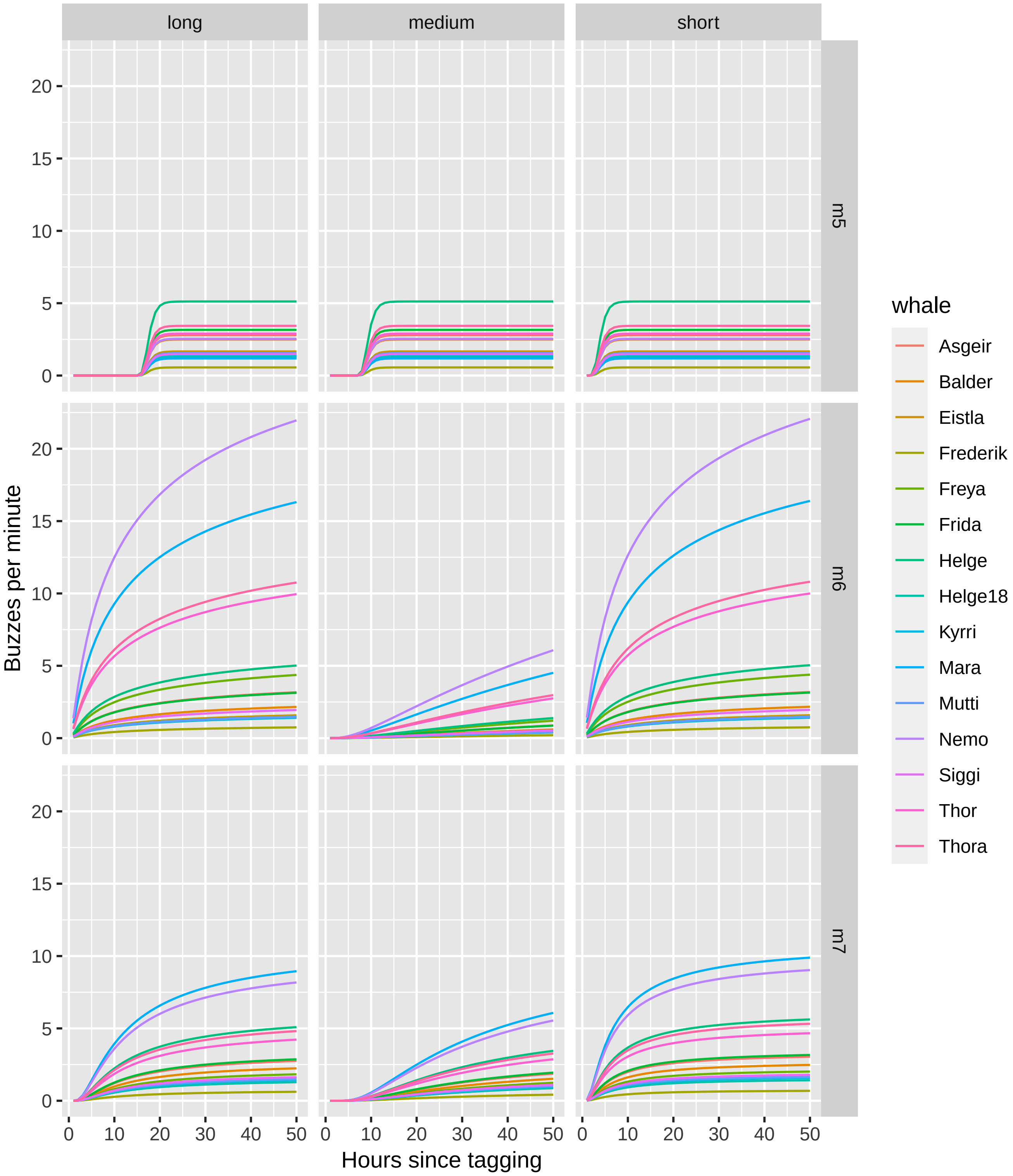


Figure 24: Visualization of the effect of time since tagging for different handling times for different models

Looking at Figure 24 we see for m_5, that the model estimates that when the effect of $\exp(-t)$ is gone, the whale that makes the most buzzes, which in this case is Helge, buzzes around 6 times per minute, while Frederik buzzes the least amount of times around 0.5 times per minute. How long it takes for the effect of $\exp(-t)$ to wear off depends on how long the handling time is. We see for the long handling time that the model estimates a silence period of around 15 hours after the tagging procedure, and then effect of $\exp(-t)$ goes away in a matter of hours. For the medium handling time the silence period is around 8 hours before the effect of $\exp(-t)$ goes towards 0, and for the short handling time, the effect of $\exp(-t)$ goes away quickly after tagging.
In the case of m_6, we see that for 4 of the whales, Nemo, Mara, Thor and Thora, this model estimates that after 50 hours, the whales will be making 10 to 20 buzzes per minute if the handling time is either short or long, which is much more than we would expect, whereas for the other whales the prediction is that the whales make around 5 buzzes per minute 50 hours after tagging. We also notice that m_5 predicts that the whales start buzzing from 0 hours since tagging, when the handling time is short or long. For medium handling time we see that the model predicts that the same four whales has the highest buzz intensity again, but here the model predicts that after 50 hours the four whales mentioned makes between 2 and 7 buzzes per minute which is much more plausible. We also see that for medium handling time the whales slowly begins to buzz around 10 hours after tagging.
For m_7, we see that for long handling time, the model predicts that the whales have a silence period of around 3 hours. For medium handling time this period is close to 10 hours and for short handling time there isn't really a silence period. We notice that in the case of m_7, the model predicts that medium handling time results in the lowest buzzing rate after 50 hours, and we see that the slope of buzzes per minute for medium handling time is steeper than when looking at long and short handling times.

Calculating how much of an effect the t part of the different models have for each handling time, like we did in 2 yields the following results:

Model	$\hat{\beta}_{Sk}$ $\hat{\beta}_{Mk}$ $\hat{\beta}_{Lk}$	t such that $r_t = \frac{1}{3}r_n$	t such that $r_t = \frac{1}{2}r_n$	t such that $r_t > 0.95r_n$
m_5	-35.04 -8135 $-2.855 \cdot 10^7$	3.46 8.9 17.07	3.93 9.37 17.54	8.2 13.7 21.8
m_6	-3.1914 -12.3115 -3.226	8.4 125.3 8.5	21.2 315.5 21.7	> 1000 > 10000 > 1000
m_7	-5.29856 -29.67915 -10.27480	4.78 27.0 9.34	7.65 42.9 14.8	105 580 200.5

Table 4: The beta estimates for the different models corresponding to short, medium and long handling time

As we can see on Table 4 it takes a maximum of 5 hours for the effect to go from $\frac{1}{3}$ of the natural buzzing rate to go to 0.95 times the natural buzzing rate. The difference between the different handling times for m_5 is, as we also saw on Figure 24, how long it takes before the rate of buzzing starts increasing.

For m_6, we see that long and short handling times seem to almost have the same effect on the buzzing rate, but for medium handling time it takes far longer before the effect of $\frac{1}{t^{1/2}}$ goes towards zero, namely that it takes over 125 hours after tagging before the buzzing rate is just one third of the natural behavior.

For m_7, we see that the medium handling time seems to have the most negative effect on the buzz intensity, but here it takes 27 hours before the rate is one third of the natural rate. m_7 predicts that we need to wait between 105 and 200 hours, depending on the handling time, before the buzzing rate is close to natural.
It is interesting to note that for m_5, the longer the handling time is the longer the silence period is, whereas for m_6 and m_7, a short and long handling time, results in a very short silence period, and a medium handling time results in a longer silence period.

7.2 Model diagnostics for extended models

We make the same model diagnostics for the previous models, and thus first rescale the predicted intesities as described in Section 6.2

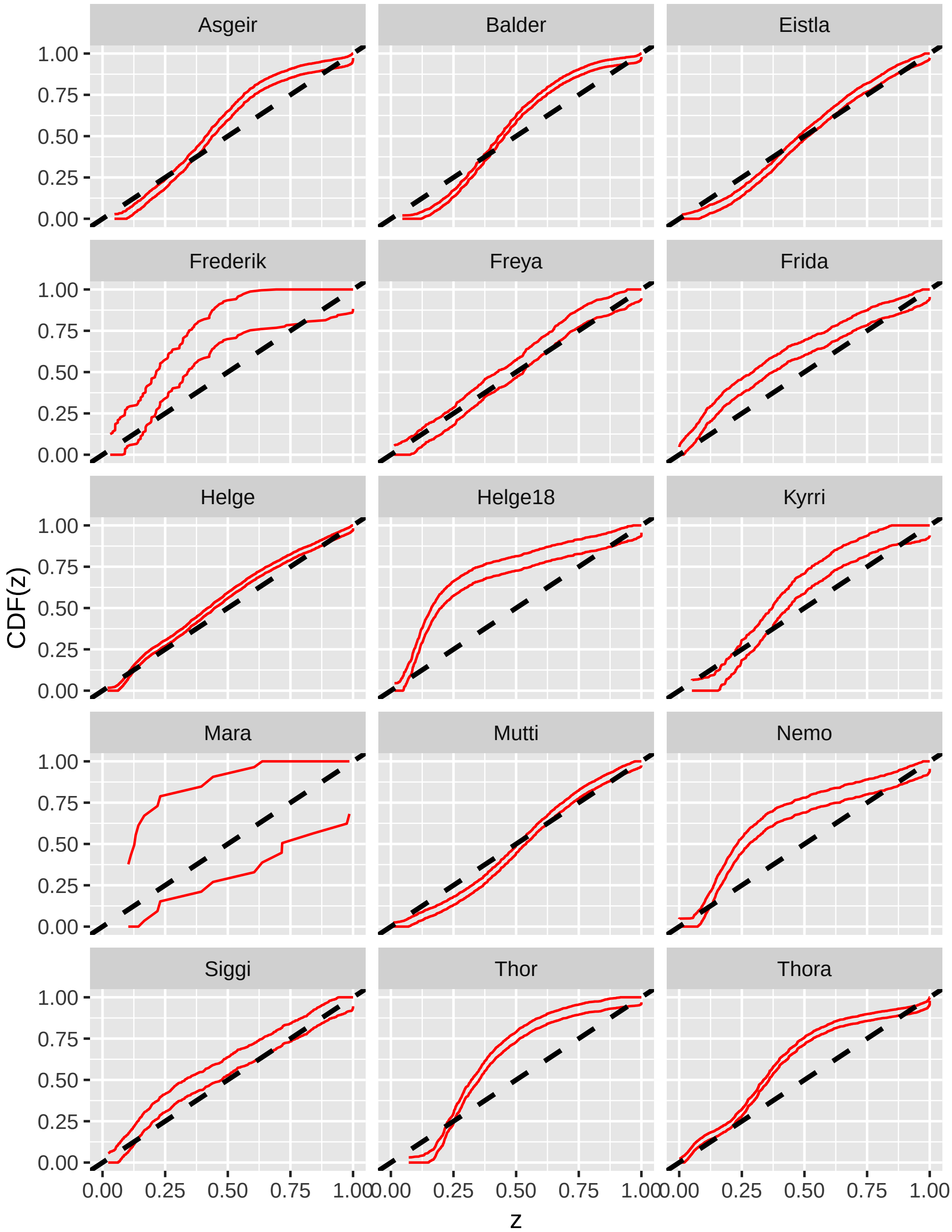


Figure 25: Kolmogorov Smirnow bands (red lines) and the CDF of the uniform distrubtion (dashed line) for m_5

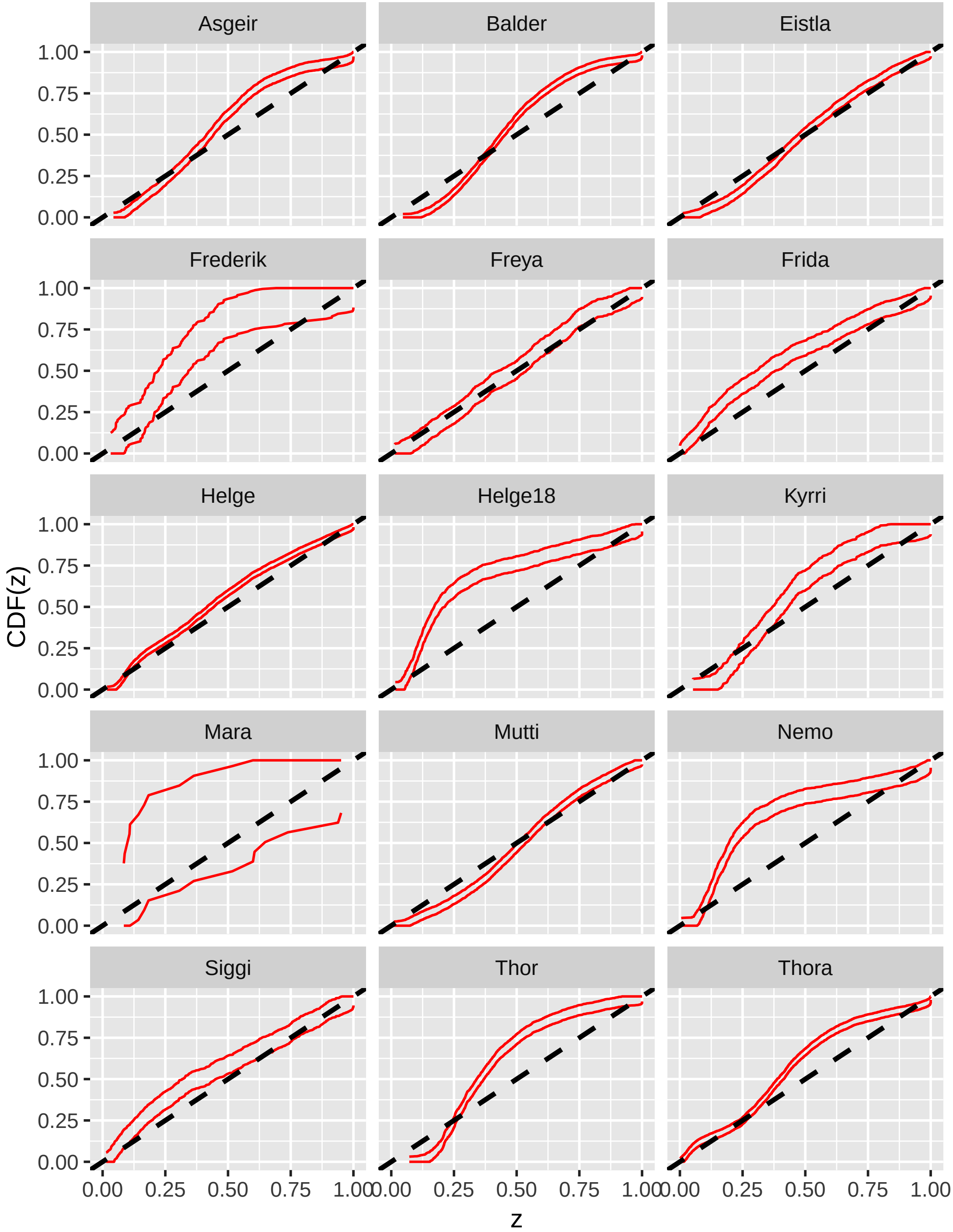


Figure 26: Kolmogorov Smirnow bands (red lines) and the CDF of the uniform distrubtion (dashed line) for m_6

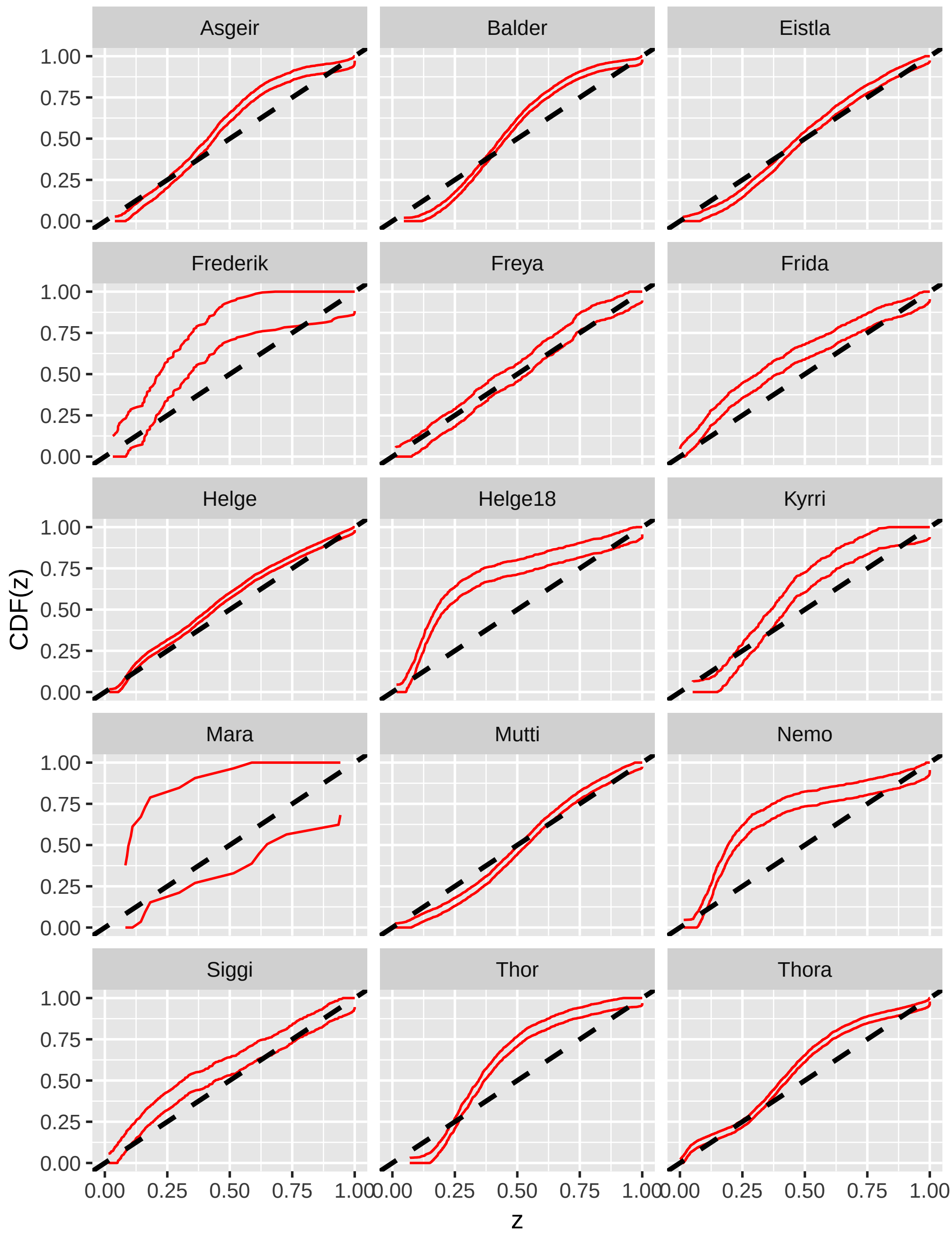


Figure 27: Kolmogorov Smirnow bands (red lines) and the CDF of the uniform distrubtion (dashed line) for m_7

Like m_1-m_4, we see that it is only Mara whose KS bands contains the CDF of the uniform distribution which means that she is the only whale where a formal hypothesis cannot reject either of these models. We also see that these KS bands looks like the ones obtained in [4], meaning that, again, if an autoregressive model was fitted the confidence bands might have included the CDF of the uniform distribution for some of the whales.

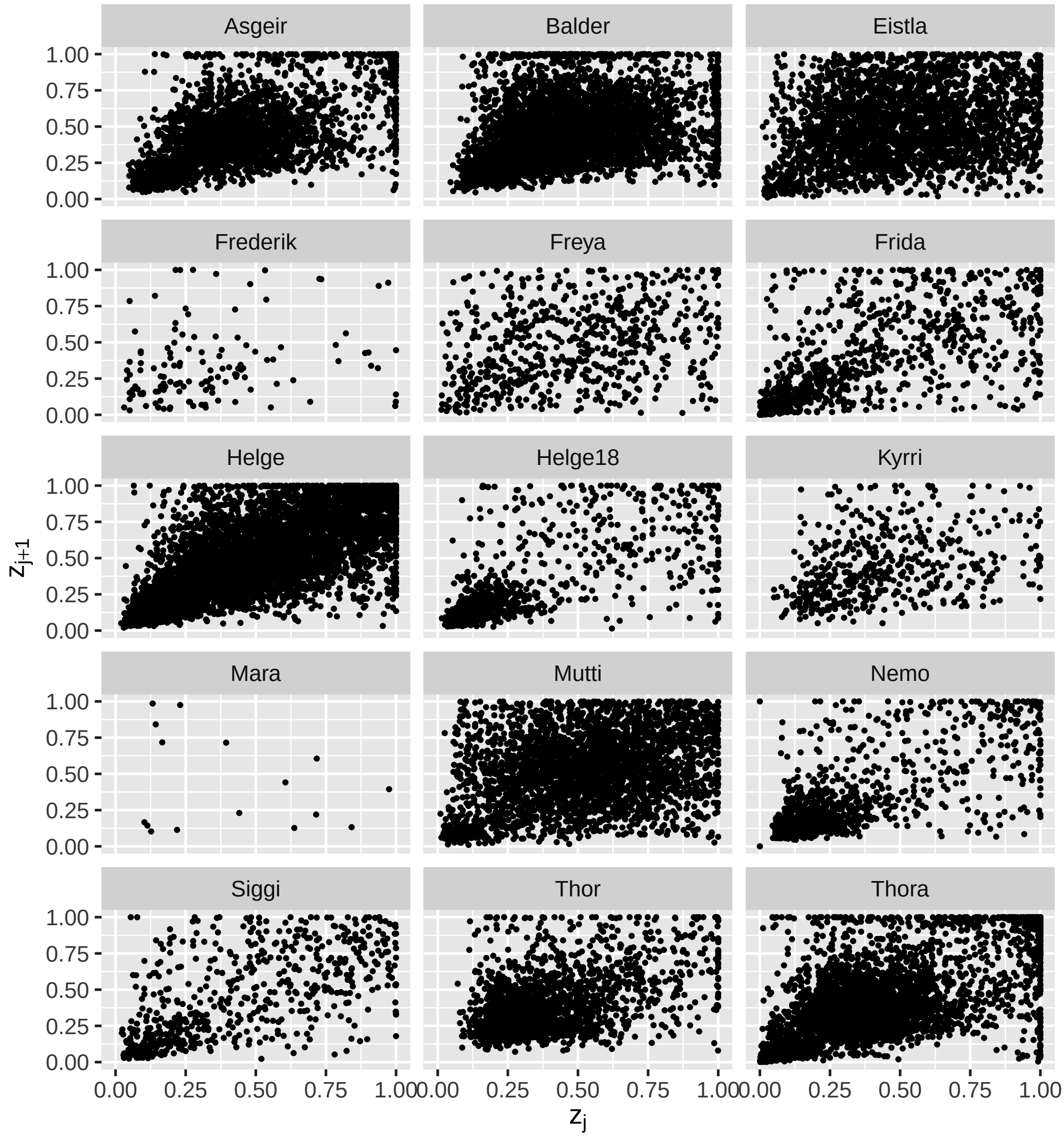


Figure 28: $z_j's$ plotted against z_{j+1} for m_5

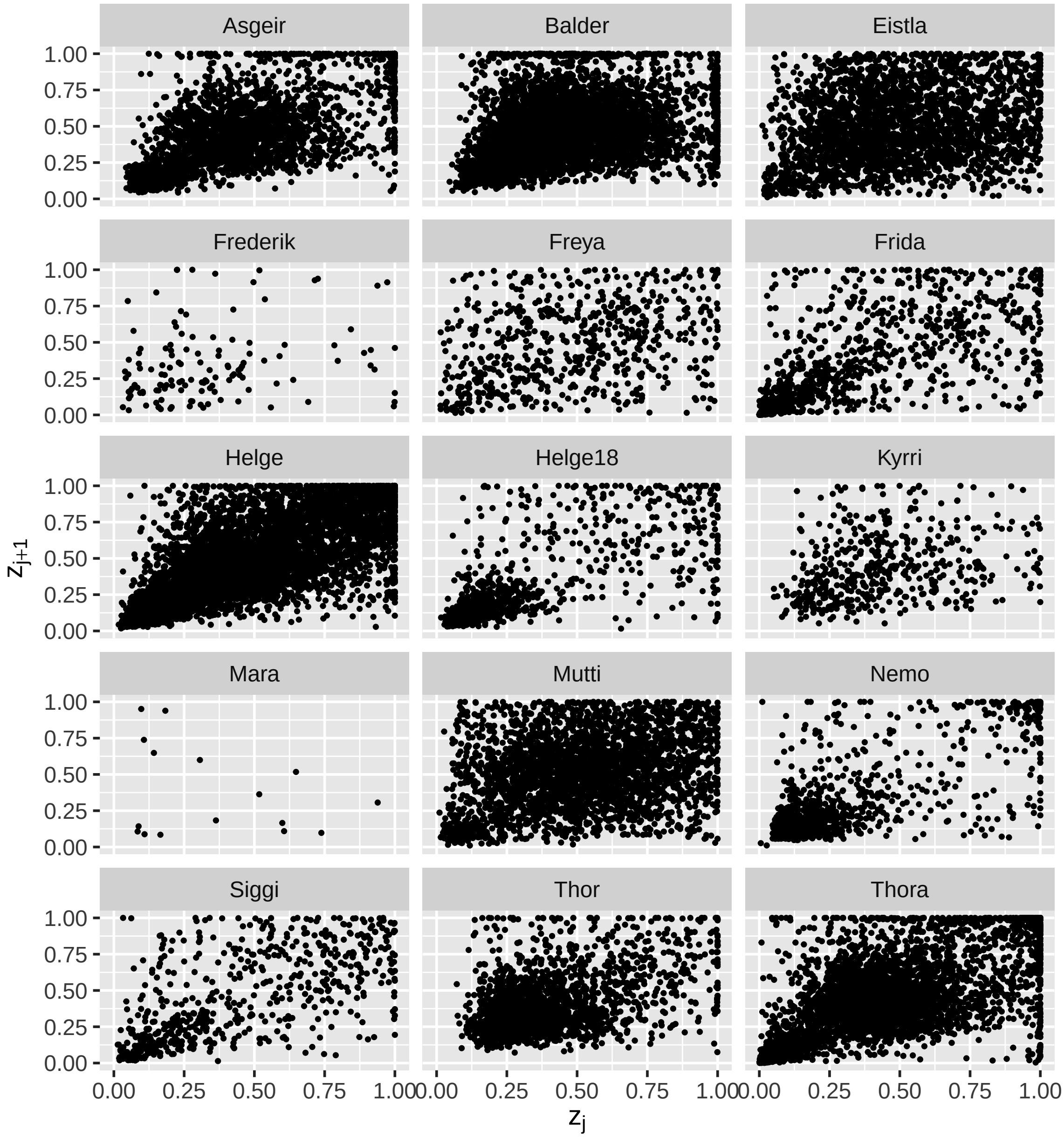


Figure 29: z_j's plotted against z_{j+1} for m_6

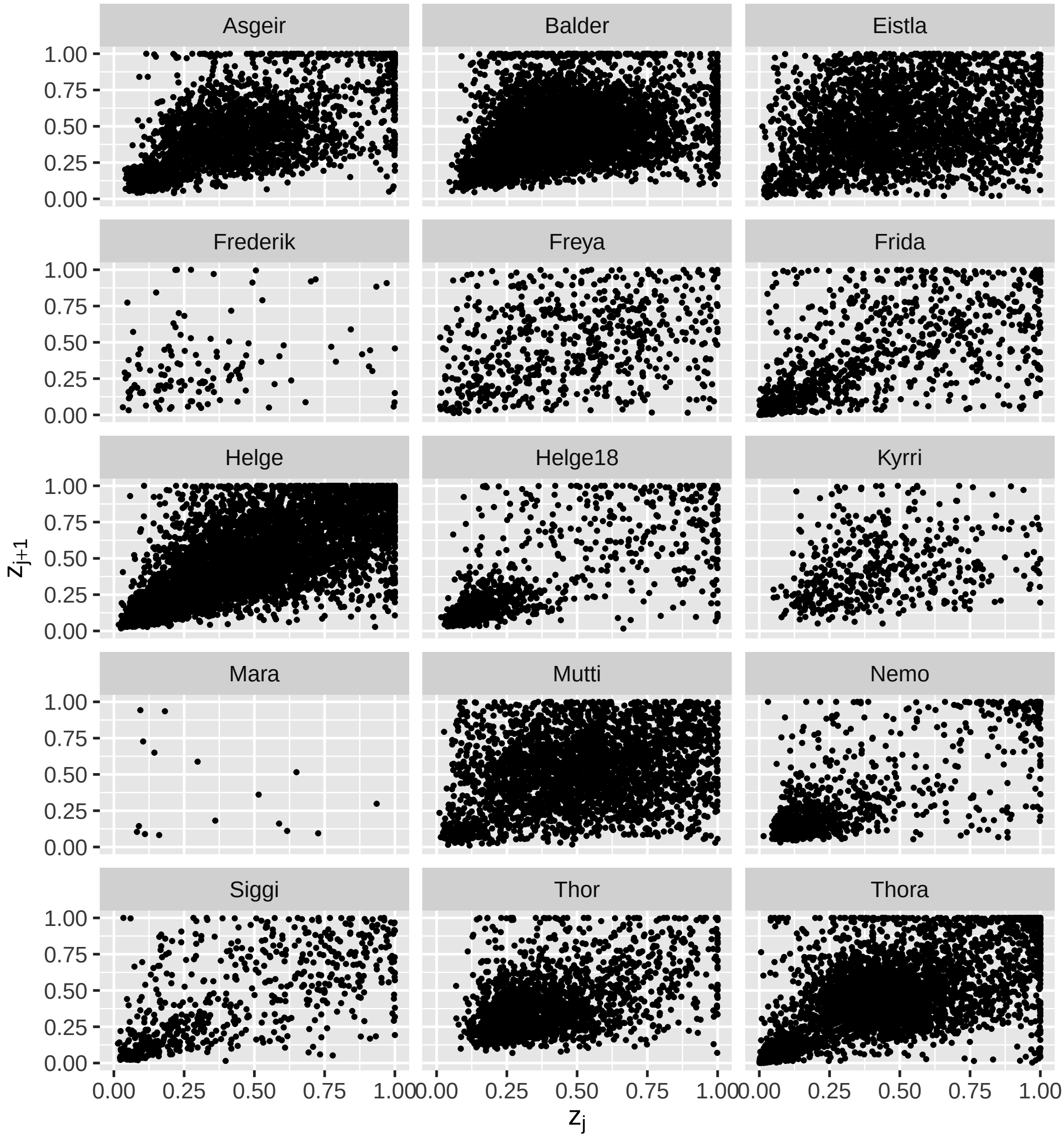


Figure 30: z_j's plotted against z_{j+1} for m_7

Again we see these coneshaped patterns on all of the whales except Mutti and Eitla for all of the models, which, again, would mean that the z_j's only can be assumed independent for Mutti and Eistla. Again there are no significant differences between the independence and KS plots for either of the models, and again we can determine the best model from the AIC score.

7.3 Predicted vs. actual counts for the extended models

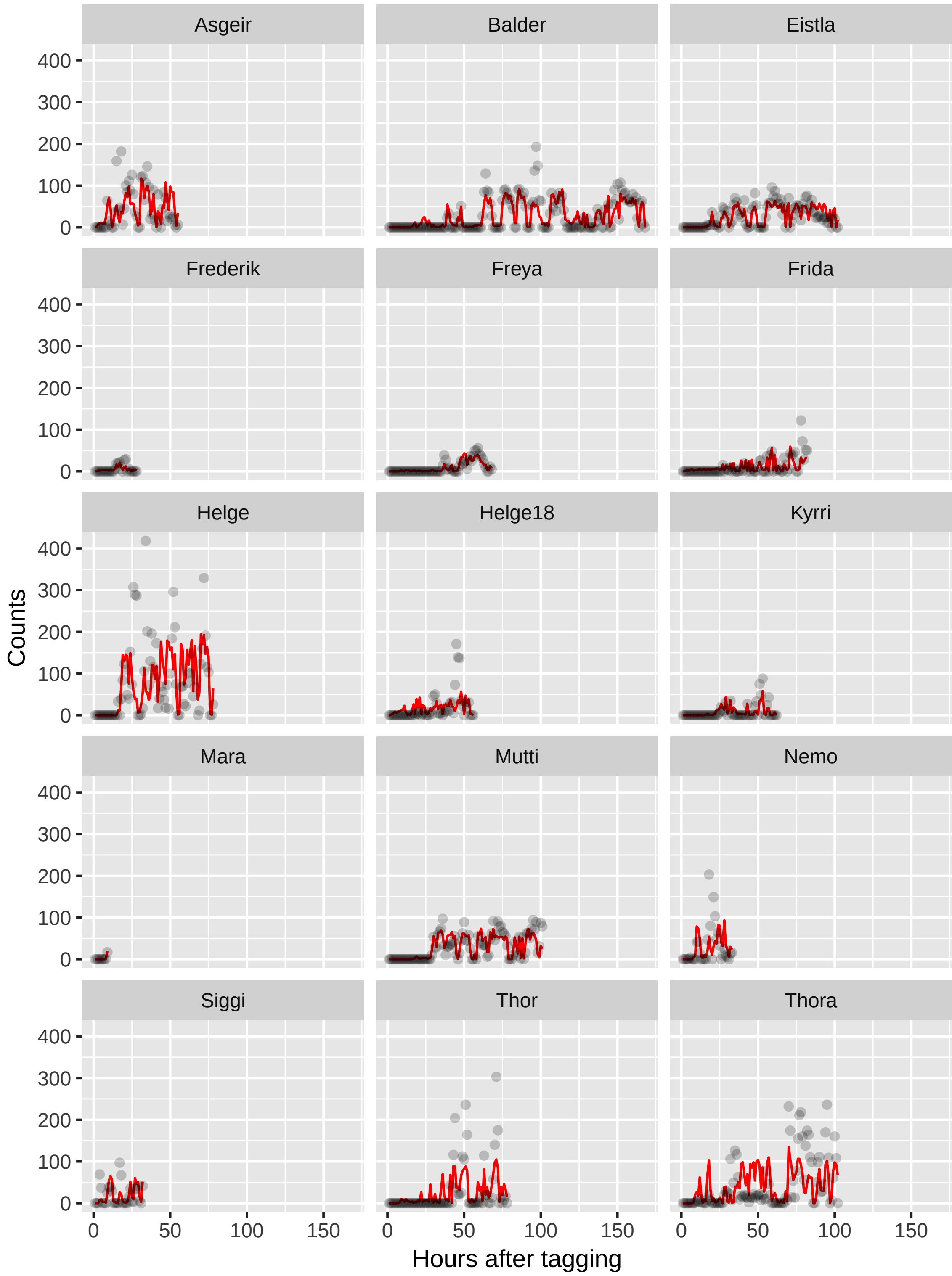


Figure 31: Actual vs. predicted counts for m_5

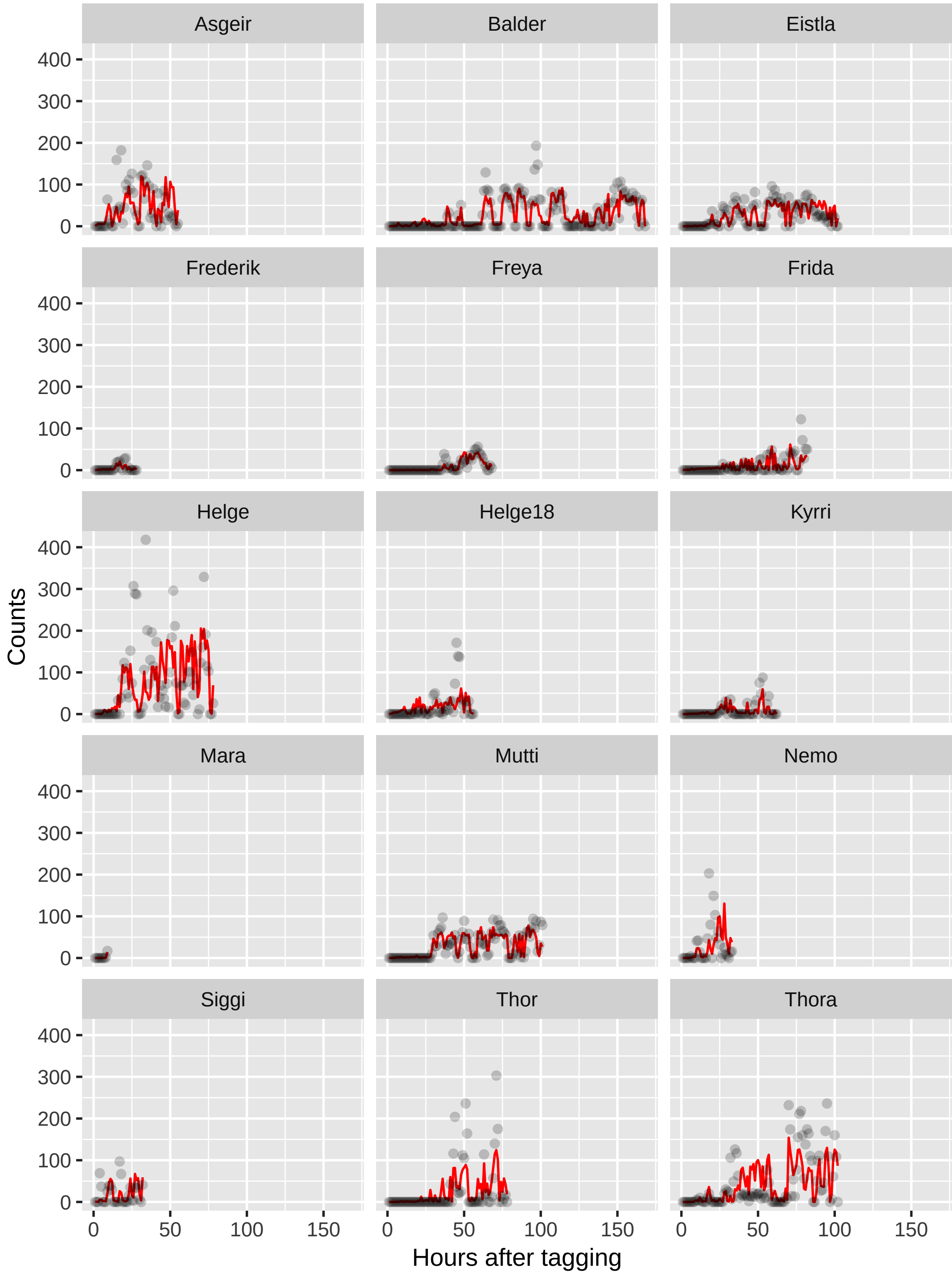


Figure 32: Actual vs. predicted counts for m_6

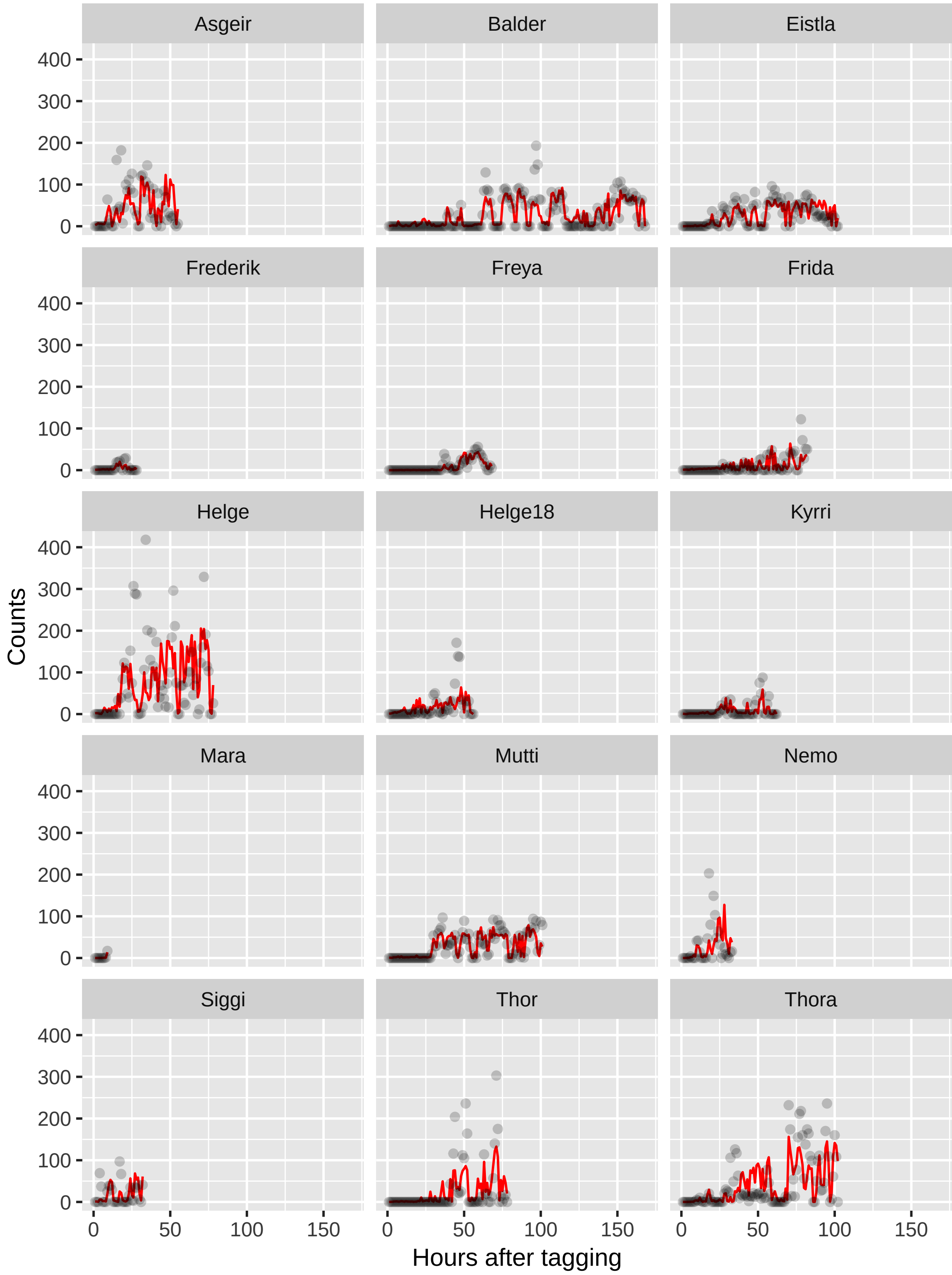


Figure 33: Actual vs. predicted counts for m_7

Figure 31-33 show the actual counts from the data compared to the predicted counts for the extended models. If we compare the extended models to their counterparts in m_1, m_2 and m_3, the figures look a lot alike, so we do not see a large improvement in solving the problem of catching the high actual counts for Thora, Thor and Helge, but all of the models catch the counts for Balder, Eistla, Freya and Mutti fairly well. Again we can see that for the low amounts of counts we have, all of the extended models predict the counts for Mara and Frederik well.

7.4 Model selection and mediation proportion for extended models

The ANOVA table for the extended models is listed below:

Model	AIC	BIC	logLik
m_5	268283	268428	-134130
m_6	267944	268088	-133961
m_7	267910	268055	-133944

Table 5: The results of the ANOVA test, showing the AIC, BIC and the log likelihood

Looking at Table 5, we notice that the difference of these statistics between m_6 and m_7 is very small, but the model with the lowest AIC score is m_7. This means that this is the preffered model of these three. If we compare the AIC statistic for these models to the ones from m_1, m_2 and m_3 shown on Table 3 we also find that m_7 is the model with the lowest AIC of all of the models we have fitted (m_3 had an AIC of 268715), hence m_7 is the model of choice.

Looking at the mediation proportion for m_7 we again fit a model without the depth to obtain the direct effect, and since we now have three different estimates for $\frac{1}{t}$, one for each of the handling time values, the procedure described in 6.5 is followed for each of these, and yields the following results:

Handling time	Total effect	Direct effect	Mediation proportion
Short	-12.5971	-5.29856	0.58 = 58 %
Medium	-53.8569	-29.67915	0.45 = 45 %
Long	-33.3837	-10.27480	0.69 = 69 %

Table 6: Mediation proportion for the different handling times for m_7

From these results we can conclude that the long handling time has the highest mediation proportion of 69 %, meaning that 69 % of the effect of $\frac{1}{t}$ when the handling time was long, is due to the whale diving less, and the rest is because the whales simply buzz less. We also see that a medium handling time yields the lowest mediation proportion of 45 %, which means that most of the effect of $\frac{1}{t}$ is caused by the whales buzzing less even though they dive.

8 Futher analyses

If we had more time to analyze the behavior of the whales, we would make models that include memory like M_1 and M_2 in [4]. Further analyses could also include Call behavior, which we have not focused on in this project. Another thing we could look at in further analyses is even more models, with the effect of time since tagging going towards 0 at different rates from the ones included in this project.

9 Conclusion

In this project we have investigated how the whales behave after the tagging procedure, to see how much time has to pass before the whales are not affected by the tagging anymore. First, 4 different models were fitted with a time term that goes towards zero at different rates as time tends to infinity. A Kolmogorov Smirnow test was conducted to asses the goodness-of-fit of the models, but unfortunately all of the models could be rejected by this test, and no model seemed to perform better than the others. Therefore model selection was based solely on the AIC score for the different models and we thereby conclude that of these four models m_3 fits the data best. Furthermore we investigated how much of an effect the handling time had on the time it takes the whales to behave naturally, and again the KS test implied that none of the suggested models fit the data. Out of these three models, as well as the four preceding models, the one with the lowest AIC score was m_7, hence this is the model of choice. For this model $\hat{\beta}_{M7}$ was lowest, corresponding to a medium handling time (41 to 62 minutes), meaning that a medium handling time results in the longest silecne period. When examining the effect of $\frac{1}{\text{time since tagging}}$ in the case of m_7, we saw that the model predicts that the whales' buzzing rate is 50 % of the natural buzzing rate after 7.5, 43 and 14.8 hours for a short, medium and long handling time respectively.

Furthermore we have conducted a mediation analysis on the model m_7 showing how much of the lack of buzzing actually comes directly from $\frac{1}{\text{time since taging}}$, and how much comes indirectly through the depth variable. These results yielded a mediation proportion of 58 %, 45% and 69 % for short, medium and long handling time respectively. This meant that since the mediation proportion is larger

that 50 % for short and long handling time, the variable that has the largest effect on the buzzing rate is how much the whales dive, and for medium handling time, the largest effect on buzzing rate is time since tagging. The final conclusion will thus be, that none of the models examined in this paper fit the data, but assuming some of them did, the time it takes before we can begin to analyse normal narwhal behavior, depends on how close to natural we want the buzzing rate to be.
For instance, if the handling time takes between 0 and 41 minutes and we want to start analysing when the buzzing rate is half of its natural rate you can begin the analysis 7.65 hours after tagging, but if we want the whales to have a buzzing rate of 95 % of the natural rate, we have to cut out the first 105 hours of data.

References

[1] Hansen, N, R,. *REGRESSION WITH R*. 2018.

[2] Hansen, E. *Introduktion til Matematisk Statistik Bind 2*. 3rd edition

[3] Ditlevsen. Susanne, Christense. U, Lynch. J, Damsgaar. M. T, Keiding. N

The mediation proportion: A Structual Equation Approach for Estimation the Proportion of Exposure Effect on Outcome Explained by an Intermediate Variable 2001

[4] Sølvtoft-Jensen. A, Heide-Jørgensen. M. P, Ditlevsen. S
MODELLING THE SOUND PRODUCTION OF NARWHALS USING A POINT PROCESS FRAMEWORK WITH MEMORY EFFECTS 2020.

[5] Brown EN, Barbieri R, Ventura V, Kass RE, and Frank LM. *The time-rescaling theorem and its application to neural spike train data analysis. Neural Comput 14: 325–2346, 2002*

[6] J. Wang, W. Wan Tsang, and G. Marsaglia. *Evaluating Kolmogorov's Distribution. Journal of Statistical Software, 08, 12 2003*

[7] Lange. T, Hansen. K. W, Sørensen. R, Galatius. S *Applied mediation analyses: a review and tutorial*

[8] VanderWeele. T. J *A Unification of Mediation and Interaction - A 4-way decomposition*

[9] Truccolo. W, Eden. U, T. Fellows, M. Donoghue, J. Brown. E. N. (2005). *A point process framework for relating neural spiking activity to spiking history, neural ensemble, and extrinsic covariate effects.* 2005