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MULTIVARIATE MIXED PROPORTIONAL HAZARD MODELLING OF THE JOINT RETIREMENT OF MARRIED COUPLES

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SUMMARY

We analyse the joint distribution of the durations until retirement of Danish husbands and wives. We estimate a multivariate mixed proportional hazards model that allows for interdependence in the times to retirement of spouses. We find evidence of strong complementarities in leisure times. Symmetrically for husband and wife, low own income and poor health are found to induce individual retirement (prior to spouse's retirement), whereas low spousal income is not, and neither party is found likely to substitute own for purchased care when the spouse is in poor health. Furthermore, high wealth and low income are found to spur joint (simultaneous) retirement. Copyright © 2004 John Wiley & Sons, Ltd.

1. INTRODUCTION

In this paper, we estimate models of withdrawal from the Danish labour market. As most people approaching retirement age are married and spouse's characteristics are expected to play an important role in the decision to withdraw from the labour market, we focus on married couples. We analyse the joint distribution of the durations until husband's and wife's retirement. To this end, we introduce a new and very general multivariate duration model, the semiparametric multivariate mixed proportional hazards (MMPH) model. The model allows each agent's decision to depend on the labour force status and other characteristics of the spouse, and it allows for both correlated unobserved heterogeneity factors and positive mass on simultaneous termination of individual spells.

The majority of the literature on the hazard function modelling approach has focused on univariate durations (see Kiefer, 1988a or Lancaster, 1990 for comprehensive accounts). Our MMPH model applies more generally to situations in which multiple durations share the same onset time and their endings are influenced by common factors. A key feature of many multivariate duration data sets is the positive probability of ties among durations. In our application to the retirement decisions of married couples, dependence between husband's and wife's durations may be due to complementarities in leisure times, assortative mating, or correlation in tastes (Christensen and Datta Gupta, 1994; Blau, 1998). Complementarity in leisure times introduces an incentive to retire at the same time, implying tied durations. Assortative mating and correlation in tastes imply similar behaviour of husband and wife, but not necessarily ties. Our model allows for both effects. It encompasses the multivariate exponential distribution of Marshall and Olkin (1967)

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and its extensions (Klein *et al.*, 1989; Ryu, 1993), with dependence through ties, as well as random effects or frailty models (Clayton and Cuzick, 1985; Hougaard, 1987; Lindeboom and van den Berg, 1994; Kimber, 1996), with dependence through correlated frailty factors, e.g., unobserved heterogeneity. In our data, the share of tied durations varies between 48% and 61%, much more than would be expected if the underlying model assigned zero probability to ties.

We consider statistical inference from grouped duration data. Our retirement data are available at an annual frequency only, so grouping does not require discarding information. For univariate grouped duration data, likelihood methods have been proposed by Prentice and Gloeckler (1978), Kiefer (1988b), Meyer (1990), Sueyoshi (1992) and Ryu (1994). One can specify a discrete time model directly, or specify a continuous time model and treat data as grouped. For our multivariate model, we adopt the latter approach, which has been advocated, e.g., by Heckman and Singer (1986) and Han and Hausman (1990). The resulting likelihood function depends upon the unspecified baseline hazards only through a certain finite set of parameters (integrals of baseline hazards, as in the univariate proportional hazards case; see Kiefer, 1988a), thus facilitating inference procedures.

In the next section, we describe our general modelling strategy for the bivariate case. The case of more than two durations is an obvious extension. Section 3 presents the empirical application to the retirement decisions of married couples, and Section 4 concludes.

2. BIVARIATE MIXED PROPORTIONAL HAZARDS

2.1. The BMPH Model

In this section, we introduce the bivariate mixed proportional hazards (BMPH) model. Let (T_n^1, T_n^2) be the durations until a suitably defined retirement state of the two spouses in couple n. We introduce three latent durations Y_n^k, $k = 1, 2, 3$. When Y_n^k ends, spell T_n^k is terminated, $k = 1, 2$, but if Y_n^3 ends first, then both retirement durations terminate (the couple retires jointly). Thus, $T_n^1 = \min\{Y_n^1, Y_n^3\}$ and $T_n^2 = \min\{Y_n^2, Y_n^3\}$. The exogenous covariates $W_n(t)$ may be time-varying, with realizations $w_n(t) = (x_n^1(t), x_n^2(t), z_n(t))$, where $z_n(t)$ measures the observed characteristics common to both spouses, and $x_n^k(t)$ those specific to spouse k.

We use three unobserved heterogeneity (random effects, or frailty) factors $(V_n^1, V_n^2, V_n^3) = V_n$, where V_n^k influences the kth latent duration Y_n^k. We assume that V_n follows a trivariate distribution $G(v, \kappa)$ whose Laplace transform exists, with κ an unknown parameter of dimension K. We assume that V_n and W_n are independent, and that, conditionally on (W_n, V_n), the latent durations (Y_n^1, Y_n^2, Y_n^3) are mutually independent. The key BMPH model structure is stated as follows.

Assumption 1. *Conditionally on $(W_n(t), V_n) = (w(t), v)$, Y_n^k has the proportional hazard function*

$$h^k(t|w(t), v^k) = \lambda^k(t)e^{w(t)\delta_k}v^k, \qquad k = 1, 2, 3$$

The baseline hazards λ^k are left unspecified. The novel feature is the couple hazard h^3. Write

$$H^k(t|w, v^k) \equiv \int_0^t h^k(s|w(s), v^k)ds \tag{1}$$

for the integrated hazard for Y_n^k, $k = 1, 2, 3$, i.e., the density of Y_n^k is $f^k(t|w, v^k) = h^k(t|w(t), v^k)$ $\exp\{-H^k(t|w, v^k)\}$. We now have the following.

Lemma 1. *The joint survivor function for the observed durations* (T_n^1, T_n^2) *is*

$$S(t_1, t_2|w, v) = \exp\{-H^1(t_1|w, v^1) - H^2(t_2|w, v^2) - H^3(\max\{t_1, t_2\}|w, v^3)\}$$

Proof: By construction,

$$\begin{aligned} P(T_n^1 > t_1, T_n^2 > t_2|w, v) &= P(\min\{Y_n^1, Y_n^3\} > t_1, \min\{Y_n^2, Y_n^3\} > t_2|w, v) \\ &= P(Y_n^1 > t_1, Y_n^2 > t_2, Y^3 > \max\{t_1, t_2\}|w, v) \end{aligned}$$

The proof now follows from the expression for H^k in (1). □

Some important features of the BMPH model are stated in Lemma 2.

Lemma 2. *The BMPH model has the following properties:*

(i) T_n^k *has the nonproportional hazard function* $h^k(t|w(t), v^k) + h^3(t|w(t), v^3)$.
(ii) Unless $h^3 \equiv 0$, *the tie probability* $P(T_n^1 = T_n^2|w, v) > 0$, *and the distribution of* (T_n^1, T_n^2) *is not absolutely continuous.*

Proof: Using Lemma 1, T_n^1 has survivor $S(t, 0|w, v) = \exp\{-H^1(t|w, v^1) - H^3(t|w, v^3)\}$, and hence hazard $-d \log S(t, 0|w, v)/dt$, proving (i). For (ii),

$$\begin{aligned} P(T_n^1 = T_n^2|w, v) &= P(Y_n^3 < \min\{Y_n^1, Y_n^2\}|w, v) \\ &= \int_0^\infty \int_{t_3}^\infty \int_{t_3}^\infty \prod_{k=1}^3 f^k(t_k|w, v^k)dt_1 dt_2 dt_3 \\ &= \int_0^\infty h^3(t|w(t), v^3) \exp\left\{-\sum_{k=1}^3 H^k(t|w, v^k)\right\} dt \end{aligned}$$

which is positive unless $h^3 \equiv 0$. □

Particular cases of the BMPH model are common frailty, $P(V^1 = V^2 = V^3) = 1$, and the (no longer mixed) bivariate proportional hazards (BPH) model where $P(V^1 = V^2 = V^3 = 1) = 1$. In the BPH model, all heterogeneity is observed, through W_n, but the probability of ties is positive, hence generating the conditional (on W_n) dependence between observed durations. The BPH model is a generalization of Marshall and Olkin's (1967) bivariate exponential distribution with three independent exponential latent durations Y^k and no regressors. Another special case of the BMPH model, obtained by imposing $h^3 \equiv 0$, is the pure heterogeneity/frailty or mixed proportional hazards (MPH) model considered by Hougaard (1987) and Lindeboom and van den Berg (1994), with all dependence between observed durations generated through correlation among unobserved heterogeneity terms, and absolutely continuous distribution.

Tied durations may be interpreted as joint retirement coordination, consistent with complementarities in leisure times of spouses. Correlation between unobserved heterogeneity terms may be interpreted as a result of assortative mating or correlation in unobservable tastes. The BMPH model accommodates both sources of dependence among durations.

2.2. Statistical Inference in the BMPH Model

We consider grouped duration data and write a_i for the fixed measurement times, $0 = a_0 < a_1 < \ldots < a_m < a_{m+1} = \infty$. Thus, the bivariate duration sample space is partitioned into $(m+1)^2$ rectangular grouping regions. Let D_n be the smallest rectangular union of grouping regions known to cover the durations for couple n, this way covering missed measurement and right-censored durations, and let the indices (i_n, j_n, k_n, l_n) define the corners of D_n,

$$D_n = \{(t_1, t_2) : a_{i_n} \le t_1 < a_{j_n}, a_{k_n} \le t_2 < a_{l_n}\}$$

The observation $(D_n; w_n)$ may then be written $(i_n, j_n, k_n, l_n; w_n)$. For a random sample of size N, the log likelihood is

$$l = \sum_{n=1}^{N} \log E_G[P(i_n, j_n, k_n, l_n | w_n, V)] \tag{2}$$

where

$$P(i, j, k, l | w, V) = S_{ik}(w, V) + S_{jl}(w, V) - S_{jk}(w, V) - S_{il}(w, V)$$

with $S_{ij}(w, v) = S(a_i, a_j | w, v)$ from Lemma 1.

We partition $w(t)$ into time-invariant and time-varying components w^1 and $w^2(t)$, assuming the latter to be constant within each grouping interval $[a_p, a_{p+1})$, and we partition δ_k into δ_k^1 and δ_k^2 of conformable dimensions. The integrated hazard H^k from (1), which enters S and hence (2), is then given by

$$H^k(a_p | w, v^k) = v^k e^{w^1 \delta_k^1} \sum_{q=1}^{p} \exp\{w^2(q)\delta_k^2 + \gamma_q^k\}$$

where γ_q^k is the log increase in the kth integrated baseline hazard,

$$\gamma_q^k = \log \int_{a_{q-1}}^{a_q} \lambda^k(s) ds$$

Thus, (2) depends on λ^k only through the $3m$ parameters $\gamma = \{\gamma_q^k : k = 1, 2, 3; q = 1, 2, \ldots, m\}$, and may be used as the full log likelihood for the $d + K + 3m$ extended parameters $\theta = (\delta, \kappa, \gamma)$, where the vector of regression coefficients $\delta = (\delta_1, \delta_2, \delta_3)$ of dimension d is the parameter of primary interest. The maximum likelihood estimator (MLE) $\hat{\theta}_N$ achieves the goal of estimating θ consistently and with an asymptotic normal distribution at the usual rate $\sqrt{N}$, without specifying the baseline hazards λ^k parametrically.

In the retirement application below, we adopt a trivariate normal distribution $N(0, \Sigma)$ for $\{\log V^k\}_{k=1}^3$ as the heterogeneity distribution $G(v, \kappa)$. The normalization to zero mean is without loss of generality, since factors of proportionality are picked up by the baseline hazards. Hence,

$\kappa = \Sigma$, involving $K = 6$ free parameters. The trivariate integral with respect to G in (2) is not given in closed form under log-normality, and we use the simulated log likelihood function

$$l(\theta, S) = \sum_{n=1}^{N} \log \left(S^{-1} \sum_{s=1}^{S} P(i_n, j_n, k_n, l_n | w_n, \widetilde{v}_n^s) \right) \tag{3}$$

with S the number of draws $\widetilde{v}_n^s$ from G per observation. The maximum simulated likelihood estimator (MSLE) of θ is the unconstrained (except that $\Sigma > 0$) maximizer $\widehat{\theta}_{N,S} = \arg\max_\theta l(\theta, S)$, which is asymptotically equivalent to the MLE as (i) $N \to \infty$ and (ii) $\sqrt{N}/S \to 0$, for any fixed number $m \geq 2$ of grouping intervals. Consistency holds even when relaxing (ii) to $S \to \infty$. Asymptotic standard errors are calculated in the usual way as the square roots of the diagonal elements of minus the inverse Hessian. In practice, a moderate value of S typically suffices (the results in Section 3 are virtually insensitive to increases in S beyond 20).

3. JOINT RETIREMENT OF MARRIED COUPLES

3.1. Data

We now turn to the application to retirement in Denmark. Our data are drawn from a random 0.5% sample of the Danish population, compiled by Statistics Denmark. The database contains annual observations for the period 1980–1990, with measurement as of November each year. Data are based exclusively on administrative registers and contain no survey element.

The official old-age pension (OAP) eligibility age during the sample period was 67. The Danish pension system in addition includes an early retirement programme labelled post-employment wage (PEW). Introduced in 1979, the PEW programme allows workers who have for at least 10 of the last 15 years contributed to a government approved unemployment insurance plan to retire any time from age 60 to 66, and to receive PEW benefits depending on previous labour market earnings until the transfer to OAP at 67. Furthermore, there is a disability retirement option available even before 60, provided specific social or disability criteria are met. We consider anyone in any of these three groups (OAP, PEW, or disability) as retired. Finally, we consider anyone working less than 5 hours per week as retired. All others are considered working.

For each married couple, we take the base year from which durations until retirement are measured to be the year in which the oldest spouse is 54 in November, which may occur prior to 1980. Our sample consists of all married couples for which this base year is 1990 or earlier, with the requirement that both husband and wife are working in the base year, and remain married to each other from the base year until either death or survey end, whichever occurs first. At this time, each ongoing spell is right-censored, i.e., the model is applied to surviving couples. Self-employed workers are eliminated. This leaves 243 couples in our sample.

Cell counts for the resulting bivariate durations (t_n^1, t_n^2) appear in Table I. We apply right-censoring to durations of 10 years and above, and 10 in the table indicates right-censoring due to this reason, death, or survey end. Censoring occurs prior to OAP eligibility, and the analysis is focused on early retirement. We return to OAP retirement in Section 3.3 below.

For durations of 7 years and up, there is notable concentration along the diagonal in Table I. Due to data grouping, some ties would be expected in any case. Our BMPH model allows testing

Table I. Observed bivariate durations

Husband's duration	Wife's duration 1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	1	0	1	0
2	0	0	0	0	0	0	1	0	0	0
3	0	0	0	0	0	0	1	0	1	0
4	0	0	0	0	0	0	1	0	1	2
5	0	0	0	0	0	0	0	1	1	0
6	0	0	0	0	0	0	1	0	1	0
7	2	2	0	1	1	0	20	3	0	7
8	3	0	1	0	5	2	1	13	1	19
9	0	1	0	1	0	0	1	2	18	10
10	1	0	0	3	0	2	5	5	3	97

Table II. Descriptive statistics

Variable	Base year only		All person-years	
	Mean	Std. dev.	Mean	Std. dev.
Owner	0.416	0.493		
Province	0.675	0.468		
Skill-husband	0.667	0.471		
Skill-wife	0.539	0.499		
Age-husband	53.51	1.795	57.72	3.258
Age-wife	49.53	4.124	53.64	4.901
PEW-husband	0.000	0.000	0.315	0.465
PEW-wife	0.000	0.000	0.097	0.296
Reform-1984	0.119	0.324	0.444	0.497
Unemp. rate	6.866	2.970	8.023	2.221

whether part of the observed pattern should be attributed to actual ties in the underlying continuous time model.[1]

Summary statistics for the regressors $w_n(t)$ appear in Table II. The first two columns report means and standard deviations for the base year only. The last two columns report statistics for all person-years for the time-varying regressors. Owner is a dummy variable indicating home ownership of either spouse. About 42% of couples own their homes. The dummy Province indicates residence outside Copenhagen (the capital), which applies to about two-thirds of the sample. Skill is a dummy indicating that the job category is manager, salaried employee (management position) or skilled employee, which is true for two-thirds of the husbands, but only 54% of the wives, and the difference is significant. Husbands are close to 54 years of age in the base year, as they tend to be the oldest in the couples. The next variable is a dummy for the PEW eligibility interval 60 to 66, which includes 32% of the person-years for husbands and 10% for wives. Reform-1984 is a dummy for the years 1984 and later, capturing the impact of a policy reform in 1984 that made

[1] Durations below 7 occur infrequently, and never for both spouses simultaneously. Duration 7 means retirement when the oldest spouse reaches 60, the eligibility age for the popular PEW programme. Durations below 7, typically due to disability or a drop in hours, are particularly sparse for husbands, who are usually the oldest in the couples and tend to work at least until PEW eligibility. Wives' durations below 7 occur without exception in cases where the husband works through until PEW.

disability retirement more attractive and accessible. Finally, the aggregate unemployment rate is included to allow for business cycle effects.[2]

3.2. Results

In the estimation phase, right-censoring is applied to each ongoing spell at the time of retirement of the spouse, i.e., the model is applied to working (and surviving) couples. The results appear in Table III. Estimates are shown for the husband's hazard h^1 in the first panel, III(A), for the wife's hazard h^2 in III(B), and for the new couple hazard h^3 in III(C). The last lines of panel III(C) show the maximized simulated log likelihood values and numbers of parameters.

The first column shows estimates from independent univariate proportional hazard models without unobserved heterogeneity ($h^3 = 0, \upsilon^k = 1, k = 1, 2, 3$). Here, husband's hazard depends on own and couple's characteristics, x^1 and z, but not on wife's characteristics, x^2. Similarly, in panel III(B), first column, wife's hazard does not depend on husband's characteristics.

In the next column, we report estimates from the BPH model ($h^3 > 0, \upsilon^k = 1, k = 1, 2, 3$), still imposing that own hazards do not depend on spouse's characteristics. As seen in panel III(C), second column, labelled BPH, we allow the couple hazard to depend on the couple's characteristics, z, but not on the person-specific characteristics, x^1 and x^2. This way, x^k enters only own hazard k, but neither spouse's nor couple's hazard. The specification with two independent univariate proportional hazard models is rejected in a likelihood ratio (LR) test in favour of the BPH model. The χ^2-statistic takes the value 18.0, on six degrees of freedom. Thus, the new couple hazard h^3 governing joint retirement is indeed a statistically important feature when modelling retirement dates of married couples. The concentration along the diagonal in Table I is more than what could be attributed to mere data grouping.

Figure 1 compares empirical (Kaplan–Meier) retirement hazards for husbands (panel A) and wives (panel B) with the average predicted hazards from the bivariate model [using Lemma 2(i) and labelled BMPH] and the independent univariate proportional hazard models (labelled Indep. PH). A 95% confidence band is included for the bivariate model. For husbands, the two model predictions are close, and both do a reasonable job of capturing the sharp increase in retirement hazard when the oldest in the couple (typically the husband) reaches PEW eligibility around duration 7. For wives, the univariate model overestimates the hazard rate and is outside the BMPH confidence band.

Consider next the specific point estimates and their significance (asymptotic t-statistics in parentheses) in the BPH model, Table III, column 2. Starting with the couple characteristics, home ownership decreases the husband's hazard significantly, perhaps because he works longer to pay off the mortgage, whereas this variable is insignificant at conventional levels in the wife's and couple's hazards. Living in the provinces increases the retirement hazard, perhaps reflecting high prices in the capital. As expected, retirement hazards are up for both husbands and wives after the pension reform in 1984. High unemployment rates similarly increase both hazards, possibly due to increased layoff rates and difficulties in finding alternative employment,

[2] Owner and Province are measured in the base year and treated as couple-wide time-invariant regressors (type z in the model); Skill is person-specific and time-invariant (type x^k); Reform-1984 and the unemployment rate are couple-wide, time-varying ($z(t)$); and Age and PEW are individual, time-varying ($x^k(t)$).

Table III. Parameter estimates under alternative model restrictions

Panel III(A) Husband's hazard

	Ind. PH	BPH	Ext. BPH	Ext. BMPH	BMPH	MPH
Owner	−0.40	−0.52	−0.43	−0.57	−0.68	−0.56
	(−1.87)	(−2.17)	(−1.77)	(−1.79)	(−2.11)	(−1.87)
Province	0.74	0.71	0.63	0.76	0.86	0.94
	(3.49)	(3.00)	(2.54)	(2.43)	(2.70)	(3.13)
Reform-1984	0.75	0.87	0.77	0.89	0.98	0.89
	(3.27)	(3.44)	(3.00)	(3.11)	(3.23)	(3.30)
Unemp. rate	0.38	0.38	0.37	0.40	0.42	0.41
	(3.90)	(3.59)	(3.69)	(3.75)	(3.62)	(3.98)
Skill-husband	−0.75	−0.76	−0.68	−0.78	−0.93	−0.91
	(−3.46)	(−3.24)	(−2.64)	(−2.41)	(−3.10)	(−3.20)
Age-husband	0.34	0.36	0.35	0.35	0.36	0.34
	(12.23)	(2.45)	(1.62)	(1.35)	(1.45)	(1.49)
(Age-60)·PEW-husband	−0.02	−0.06	−0.09	0.07	0.10	0.15
	(−0.19)	(−0.35)	(−0.40)	(0.28)	(0.40)	(0.69)
PEW-husband	1.21	1.11	1.18	1.21	1.14	1.24
	(3.39)	(2.60)	(2.23)	(2.23)	(2.06)	(2.40)
Skill-wife			−0.23	−0.37		
			(−0.95)	(−1.18)		
Age-wife			0.02	0.03		
			(0.48)	(0.74)		
(Age-60)·PEW-wife			−0.01	0.02		
			(−0.04)	(0.07)		
PEW-wife			0.19	0.13		
			(0.49)	(0.32)		
Log α (dur. dep.)	−0.47	−0.57	−0.60	−0.59	−0.51	−0.44
	(−1.07)	(−1.06)	(−0.91)	(−0.74)	(−0.69)	(−0.61)

Panel III(B) Wife's hazard

	Ind. PH	BPH	Ext. BPH	Ext. BMPH	BMPH	MPH
Owner	−0.25	−0.38	−0.29	−0.27	−0.33	−0.18
	(−0.93)	(−1.32)	(−1.02)	(−0.73)	(−0.90)	(−0.54)
Province	0.34	0.20	0.12	0.13	0.21	0.43
	(1.40)	(0.73)	(0.41)	(0.37)	(0.59)	(1.33)
Reform-1984	0.62	0.73	0.67	0.77	0.81	0.74
	(2.59)	(2.55)	(2.45)	(2.45)	(2.49)	(2.65)
Unemp. rate	0.33	0.36	0.33	0.33	0.36	0.34
	(3.30)	(3.28)	(3.08)	(2.92)	(3.02)	(3.29)
Skill-husband			0.07	0.19		
			(0.26)	(0.53)		
Age-husband			0.07	0.12		
			(1.10)	(1.07)		
(Age-60)·PEW-husband			−0.33	−0.39		
			(−1.75)	(−1.83)		
PEW-husband			0.21	0.18		
			(0.55)	(0.41)		
Skill-wife	−0.30	−0.38	−0.53	−0.66	−0.47	−0.33
	(−1.28)	(−1.40)	(−1.82)	(−1.82)	(−1.36)	(−1.10)
Age-wife	0.13	0.13	0.16	0.17	0.14	0.13
	(2.37)	(2.65)	(2.87)	(2.66)	(2.14)	(2.28)
(Age-60)·PEW-wife	0.12	0.08	0.14	0.30	0.22	0.30
	(0.71)	(0.40)	(0.69)	(1.29)	(0.95)	(1.45)
PEW-wife	1.02	1.15	1.09	1.19	1.25	1.12
	(2.65)	(2.73)	(2.69)	(2.70)	(2.79)	(2.87)
Log α (dur. dep.)	−0.25	−0.45	−0.67	−0.73	−0.39	−0.18
	(−1.06)	(−2.06)	(−2.04)	(−1.69)	(−1.45)	(−0.75)

Table III. (*Continued*)
Panel III(C) Couple hazard

	Ind. PH	BPH	Ext. BPH	Ext. BMPH	BMPH	MPH
Owner		17.80	28.01	128.33	22.78	
		(0.88)	(0.78)	(1.08)	(0.07)	
Province		1.85	43.81	21.62	2.11	
		(1.31)	(0.73)	(1.24)	(0.99)	
Reform-1984		−0.39	0.12	0.23	−0.12	
		(−0.52)	(0.13)	(0.20)	(−0.13)	
Unemp. rate		0.20	0.41	0.46	0.16	
		(0.75)	(1.06)	(0.93)	(0.52)	
Skill-husband			−0.99	−0.99		
			(−0.94)	(−0.89)		
Age-husband			−2.14	−5.13		
			(−0.75)	(−1.14)		
(Age-60)·PEW-husband			3.52	6.75		
			(1.10)	(1.56)		
PEW-husband			23.39	34.42		
			(0.64)	(1.03)		
Skill-wife			36.22	127.53		
			(0.77)	(1.16)		
Age-wife			−0.13	−0.11		
			(−1.27)	(−1.01)		
(Age-60)·PEW-wife			0.027	0.08		
			(0.05)	(0.12)		
PEW-wife			−0.03	−0.10		
			(−0.02)	(−0.07)		
Log α (dur. dep.)		1.02	0.43	0.32	1.03	
		(3.05)	(0.26)	(0.16)	(3.16)	
Log likelihood	−557.2	−548.2	−543.5	−541.1	−546.4	−555.2
No. of parameters	20	26	42	48	32	23

Note: Asymptotic t-statistics in parentheses.

combined with the government's attempts to create jobs for the young by providing incentives for the older to retire. This variable has the highest asymptotic t-statistic for both husbands and wives.

Turning to the person-specific characteristics, skilled husbands work longer than unskilled. Both husbands and wives exhibit significantly higher retirement hazards when older, and particularly when PEW eligible. Including (Age-60)·PEW allows for a structural change in the age coefficient when reaching PEW eligibility, but this is insignificant. Thus, retirement hazards increase with age, exhibiting a parallel shift up at the PEW eligibility age.

The reported results are based on the parametric Weibull baseline hazard, $\lambda^k(t) = \alpha_k t^{\alpha_k - 1}$, where $\alpha_k > 0, k = 1, 2, 3$. We fail to reject this specification in an LR test against the general model with nonparametric baselines. The nonlinear restrictions are $\gamma_j^k = \log[(a_j)^{\alpha_k} - (a_{j-1})^{\alpha_k}]$, and the test is asymptotically $\chi^2_{3(m-1)}$, with $m = 9$ in the application. For convenience, Table III reports $\log\alpha$, whose sign indicates that of duration dependence. Thus, individual duration dependence is negative, and the effect is insignificant for husbands (consistent with exponential baseline) and barely significant for wives. Couple baseline duration dependence is positive and strongly significant, with a t-statistic in excess of 3, demonstrating a significant propensity to retire together.

Figure 1. Predicted versus actual hazards

The third column of each panel shows the results from an extended (saturated) BPH model, allowing all three hazards to depend on all characteristics. By entering x^k into both spouse's and couple's hazard, we give it two different chances to matter outside own hazard h^k. Still, none of the additional parameters is significant. Based on the LR test, the preferred model is that in column 2, where retirement is coordinated among spouses, but individual characteristics only enter own hazards.

The last three columns show the MSL results obtained after adding unobserved heterogeneity [v^k in (2)]. None of the qualitative results changes when moving from the third to the fourth column. Log likelihood only improves very slightly, and insignificantly so. Similarly, the change in log likelihood is insignificant when moving from the fourth to the fifth column, labelled BMPH, corresponding to the preferred model in the second column, but adding the six parameters in Σ. We fail to reject the exclusion restrictions that individual characteristics only enter in own hazards (x^k in h^k), whether we allow for unobserved heterogeneity or not. In results not shown, we also fail to reject this reduction when carried out separately for husbands and wives. Thus, there is symmetry by gender, and no support for the male-chauvinist hypothesis (Killingsworth, 1983) that wife's hazard depends on husband's variables, but not the reverse.

The straight BPH model (column 2) is not rejected against the BMPH model (column 5). This suggests that the dependence between retirement dates is due to coordination, as captured by h^3, consistent with complementarities in leisure times. Of course, the possibility remains that all dependence is due to correlation between unobserved heterogeneity terms, consistent with assortative mating/matching on unobservables. Thus, in the BMPH model, the durations T_n^1 and T_n^2 are conditionally independent given (W_n, V_n) if and only if $h^3 \equiv 0$, which is the pure MPH hypothesis, column 6. From the LR test, the MPH model is rejected in favour of column 5, BMPH, although only barely.[3] This indicates that heterogeneity is not the sole source of dependence. As column 2 is clearly not rejected against column 5, there is better agreement with the notion that all dependence between retirement dates is due to coordination, $h^3 > 0$.

The overall BMPH specification test should be carried out for the initial unrestricted model with nonparametric baseline hazards. An (2000) proposes a new split-sample test (S-test). The sample is randomly split into two halves, yielding independent estimates of δ, say $\hat{\delta}^1$ and $\hat{\delta}^2$. Under correct specification, both are consistent and asymptotically normal, and the test statistic is

$$U = N\widehat{\Delta}'(\hat{V}_1 + \hat{V}_2)^{-1}\widehat{\Delta} \tag{4}$$

where $\widehat{\Delta} = \hat{\delta}^2 - \hat{\delta}^1$ and $\widehat{V}_i$ is the asymptotic variance of $\hat{\delta}^i$, which is estimated consistently using the foregoing. Under the null, U is asymptotically χ^2_d. The S-test (4) fails to reject the general BMPH model, as well as the version corresponding to the preferred model in column 2, both at the 5% level.

3.3. Financial and Health Variables

We conduct a separate analysis of all couples for which financial and health data are available. By redefining the base year as the year where the oldest in the couple is 60 in November, and requiring this to be 1980 or later, all spells are contained within the 1980–1990 observation interval. This allows access to financial and health data, but at the cost of a smaller sample size, 144 couples. The focus is now on later retirement, and OAP eligibility may be reached before survey end. OAP eligibility, financial and health variables are expected to be important for the retirement decision. Table IV shows the observed bivariate duration data for this senior sample. As in Table I, there is notable concentration along the diagonal. Furthermore, wife's duration tends to exceed husband's.

The new explanatory variables include Income for each spouse and Wealth for the couple, as well as an indicator of whether each person's current job is in the public sector. In addition, an indicator for poor health is included for each spouse. Income refers to all disposable income, including labour market earnings, transfers, etc. Wealth refers to the value of all real and financial assets, net of any debts. Health is considered poor in a given year if sick leave benefits are received during the year, which in turn occurs if a sickness spell lasts for more than 13 weeks. To retain parsimony, Owner, Province, Skill, the unemployment rate and the reform variable are eliminated. Furthermore, in the senior sample, the husband is the oldest in every couple, so increases in duration move in lockstep with husband's age. Since the baseline hazard allows duration dependence, age is eliminated as a regressor for the husband.

[3] The χ^2-statistic takes the value 17.6, on nine degrees of freedom (the critical value at the 5% level is 16.9). The parameters saved are the five regression coefficients in δ_3 (including an intercept not reported), the couple duration dependence α_3, the correlations of v^1 and v^2 with v^3, and the variance of v^3.

Table IV. Observed bivariate durations for senior sample

Husband's duration	Wife's duration									
	1	2	3	4	5	6	7	8	9	10
1	14	2	0	3	0	0	2	0	2	0
2	1	12	1	5	1	4	4	2	0	3
3	1	0	17	1	1	3	0	1	1	2
4	0	1	0	8	2	1	2	1	1	0
5	1	0	0	2	3	2	0	1	3	0
6	2	0	1	1	1	9	2	1	1	0
7	0	0	0	0	2	0	4	1	1	1
8	0	0	0	0	0	0	0	1	0	0
9	0	0	0	0	1	0	0	1	0	2
10	0	0	0	0	0	0	0	0	0	1

Table V. Descriptive statistics for senior sample

Variable	Base year only		All person-years	
	Mean	Std. dev.	Mean	Std. dev.
Wealth	66,287	58,537	73,259	63,447
Income-husband	23,621	7,819	20,523	8,634
Income-wife	13,437	4,380	13,232	4,899
Public-husband	0.299	0.458	0.198	0.399
Public-wife	0.493	0.500	0.462	0.499
Health-husband	0.111	0.314	0.092	0.288
Health-wife	0.104	0.306	0.098	0.298
OAP-husband	0.000	0.000	0.082	0.274
PEW-wife	0.042	0.200	0.242	0.428
Age-wife	54.444	3.869	56.684	4.254

Descriptive statistics appear in Table V. On average, each couple's wealth roughly amounts to twice the sum of husband's and wife's annual income, but is far more variable across couples. Half the wives and two-thirds of the husbands work in the private sector. Initially, 11% of husbands and 10% of wives are in poor health. These fractions decrease in later person-years due to attrition, and those husbands who keep working end up in better health than wives. An OAP eligibility indicator (ages 67 and above) is included for husbands, 27 of whom reach eligibility during the sample period, and PEW is retained for wives.

Estimation results appear in Table VI, which is laid out as Table III. In the senior sample, it turns out that the log-normal unobserved heterogeneity distribution does not lead to a well-behaved and tractable simulated likelihood function (3). Instead, a discrete heterogeneity distribution is adopted, where V^k may take either of two values, say A_l^k, $l = 1, 2$, i.e., there is a total of $2^3 = 8$ support points. We use the parametrization

$$P(V^k = A_{l_k}^k, k = 1, 2, 3) = \frac{\exp(\tau_{l_1 l_2 l_3})}{\sum_{i_1, i_2, i_3 = 1,2} \exp(\tau_{i_1 i_2 i_3})}$$

Table VI. Financial and health variables

Panel VI(A) Husband's hazard

	Ind. PH	BPH	Ext. BPH	Ext. BMPH	BMPH	MPH
Income-husband	−0.31	−0.31	−0.32	−0.61	−0.51	−0.51
	(−8.89)	(−7.99)	(−8.28)	(−6.54)	(−6.54)	(−7.46)
Public-husband	−2.61	−2.58	−2.84	−5.23	−4.19	−4.25
	(−4.38)	(−4.32)	(−4.48)	(−5.54)	(−5.06)	(−5.10)
Health-husband	0.94	0.95	1.05	2.91	2.07	1.81
	(2.85)	(2.58)	(2.91)	(3.80)	(2.86)	(2.91)
OAP-husband	−1.32	1.15	0.75	0.90	1.85	−1.95
	(−1.79)	(1.46)	(0.71)	(0.80)	(2.09)	(−2.21)
PEW-wife			0.43	1.35		
			(1.10)	(2.39)		
Age-wife			−0.03	−0.14		
			(−0.63)	(−2.34)		
Income-wife			0.02	−0.01		
			(0.81)	(−0.04)		
Public-wife			0.12	0.58		
			(0.45)	(1.64)		
Health-wife			−0.68	−0.62		
			(−1.75)	(−1.15)		
Wealth	0.04	0.03	0.02	0.01	0.03	0.05
	(1.98)	(1.25)	(0.76)	(0.16)	(1.13)	(1.64)
Log α (dur. dep.)	0.44	0.42	0.41	0.93	0.88	0.81
	(4.36)	(3.80)	(3.40)	(5.07)	(4.75)	(4.74)

Panel VI(B) Wife's hazard

	Ind. PH	BPH	Ext. BPH	Ext. BMPH	BMPH	MPH
Income-husband			−0.01	−0.10		
			(−0.04)	(−1.83)		
Public-husband			−0.51	0.63		
			(−0.94)	(0.79)		
Health-husband			−1.27	−1.54		
			(−2.35)	(−2.10)		
OAP-husband			1.17	2.64		
			(2.25)	(2.38)		
PEW-wife	1.24	1.54	1.89	3.91	2.58	2.14
	(2.59)	(2.75)	(3.08)	(3.62)	(3.83)	(3.05)
Age-wife	−0.04	−0.09	−0.11	−0.22	−0.17	−0.08
	(−0.53)	(−1.18)	(−1.38)	(−2.38)	(−2.20)	(−0.79)
Income-wife	−0.27	−0.25	−0.33	−0.55	−0.39	−0.40
	(−5.95)	(−5.03)	(−5.48)	(−4.29)	(−5.59)	(−5.12)
Public-wife	−2.41	−2.33	−2.45	−4.43	−2.87	−3.08
	(−3.31)	(−3.22)	(−3.37)	(−3.76)	(−3.80)	(−3.46)
Health-wife	1.51	1.55	1.85	2.89	2.46	1.92
	(4.14)	(4.07)	(4.46)	(3.73)	(4.48)	(3.67)
Wealth	0.04	0.02	0.04	−0.08	−0.01	0.01
	(1.51)	(0.55)	(1.01)	(−1.56)	(−0.31)	(0.10)
Log α (dur. dep.)	0.43	0.41	0.28	0.27	0.56	0.52
	(2.38)	(2.29)	(1.25)	(1.04)	(3.01)	(2.37)

(*continued overleaf*)

Table VI. (*Continued*)
Panel VI(C) Couple hazard

	Ind. PH	BPH	Ext. BPH	Ext. BMPH	BMPH	MPH
Income-husband		−1.16	−0.92	−3.45	−1.00	
		(−1.65)	(−2.28)	(−2.36)	(−2.08)	
Income-wife		−1.16	−0.97	−3.08	−1.12	
		(−1.87)	(−2.63)	(−2.77)	(−2.31)	
Wealth		0.11	0.08	0.68	0.61	
		(1.70)	(2.27)	(2.31)	(2.12)	
Log α (dur. dep.)		−0.56	−1.31	−2.59	1.30	
		(−0.24)	(−0.61)	(−2.14)	(0.02)	
Log likelihood	−239.7	−234.0	−225.4	−203.2	−213.5	−223.2
No. of parameters	15	20	29	39	30	20

Note: Asymptotic t-statistics in parentheses.

normalizing $\tau_{222} = 0$, without loss of generality, so that there are seven intensity rates $\tau_{l_1 l_2 l_3}$. The zero mean normalization implies that

$$A_1^1 = -A_2^1 \frac{\sum\limits_{l_2,l_3=1,2} \exp(\tau_{2l_2l_3})}{\sum\limits_{i_2,i_3=1,2} \exp(\tau_{1i_2i_3})}$$

and similarly A_1^k is a function of A_2^k and the intensity rates, $k = 2, 3$. There are three free support points A_2^k, so the dimension of $\kappa = (A, \tau)$ is $K = 3 + 7 = 10$, to be compared with $K = 6$ in the log-normal specification. In the pure MPH model with $h^3 = 0$, we have $K = 5$, compared to $K = 3$ in the log-normal case. Clearly, the log-normal specification is preferred on grounds of parsimony, but the discrete alternative is evidently sometimes required empirically. Furthermore, the trivariate integral with respect to G in (2) is given analytically in the discrete case, thus avoiding simulation.

For brevity, going straight to the log likelihood values [panel VI(C)], we note two main differences from the previous analysis. First, unobserved heterogeneity now leads to a significant improvement (last three columns against first three). The relevant test statistics are all in excess of 30, yielding p-values below 0.05%. Second, the extensions in column 3 relative to column 2, and in column 4 relative to column 5, are no longer redundant. Thus, each individual hazard is found to depend on spouse's characteristics, whether or not unobserved heterogeneity is controlled for. The final model is the most general BMPH specification in the fourth column. There is still symmetry by gender, and no support for the Killingsworth (1983) hypothesis. The test of the MPH model against the BMPH model (column 6 against column 5) also rejects, so not all dependence is due to unobserved heterogeneity, and part of the concentration along the diagonal in Table IV is attributed to ties in the underlying model.

Comparing the findings from Tables III and VI, the new couple hazard h^3 suffices for explaining the dependence among early retirement dates, where spouse's characteristics have little role to play (at least so long as financial and health variables are unavailable). On the other hand, mixing through unobservables and conditioning on spouse's financial, health and other characteristics are required in addition to h^3 for explaining the observed dependence in later retirement.

Figure 2 is organized as Figure 1 and compares actual and predicted retirement hazards for the senior sample. Predictions from the univariate and bivariate (BMPH, Table VI, column 4) models are again similar and close to the sample hazard for husbands (panel A). The bivariate prediction captures the shape better than the univariate around duration 7, when the husband reaches OAP eligibility, but the confidence band is wide. For wives, the univariate model prediction underestimates the hazard towards the end of the sample period, where it moves outside the BMPH confidence band.

Several financial and health variables are highly significant in Table VI. Higher own income reduces both husband's and wife's hazard significantly, i.e., the substitution effect dominates the income effect for both, in agreement with the results of Blau (1998) from US data. As a new result, both individual incomes enter significantly in the couple's hazard, panel VI(C). If either party's income is lower, the spouses are more likely to retire together. The extended models also include the cross-earnings effects [columns 3 and 4 in panels VI(A) and VI(B)]. Wife's income is insignificant in the husband's hazard, and not really treated as a household variable. Husband's income does lower the wife's retirement hazard [panel VI(B)], but the effect is barely significant in the most general BMPH model (column 4).

Figure 2. Predicted versus actual hazards (senior sample)

Poor own health significantly increases both husband's and wife's retirement hazard, as expected. The extended models also include the cross-effects of health [columns 3 and 4 in panels VI(A) and VI(B)]. Both husband and wife tend to defer retirement if the spouse is in poor health. The effect is barely significant for the husband [at the 5% level, one-sided test, panel VI(A), column 3], but strongly significant for the wife [panel VI(B)]. These findings differ from those in earlier studies on US data by Anderson *et al.* (1980), Clark *et al.* (1980) and Henretta and O'Rand (1983), who found wives more likely than husbands to substitute own for purchased care when the spouse was ill. Our results for Denmark are more in line with Pozzebon and Mitchell (1989), who later found evidence of symmetry by gender in the cross-effects of health in US data, and with Blau and Riphahn's (1998) results from German data.

Higher wealth significantly increases the couple hazard [panel VI(C)]. It is interesting that the explanatory power of this couple-wide regressor comes through via the new hazard h^3. Sufficient wealth allows couples to retire to spend leisure time together.

Both spouses work longest if employed in the public sector, perhaps due to lower physical requirements or more flexibility in these jobs. Spouse's sector affiliation does not enter significantly in either party's individual hazard. Both hazards increase when the wife reaches PEW eligibility, but otherwise decline with wife's age. Duration dependence is strongly positive for the husband, whose age is left out as a regressor, and insignificant for the wife in the extended models. Interestingly, the main effect of husband's reaching OAP eligibility is to increase wife's retirement hazard. Such retirement will typically be under the PEW programme, undertaken to share leisure time with the older husband.

4. CONCLUSION

We have introduced a multivariate mixed proportional hazard model. The model incorporates both correlated unobserved heterogeneity terms and a positive tie probability as sources of dependence among durations. In our application to the joint retirement decisions of Danish married couples, we find that the new couple hazard consistent with complementarities in leisure times is important when modelling early retirement, and that a mixing unobserved heterogeneity distribution consistent with assortative mating/matching on unobservables is required in addition when modelling later retirement, controlling for financial and health variables. Thus, when spouses retire early, they frequently do so to spend leisure time together, and retirement dates are actively coordinated. If they retire later, spouses' decisions still resemble each other, and more so than what can be explained by financial, health and other observed heterogeneity. This is at least in part because spouses are driven by similar motives and tastes, and not only because they wish to spend as much time together as possible, which could have been achieved through early retirement. In addition, we find symmetry by gender in the cross-earnings effects, and in the cross-effects of health. Neither husband nor wife substitutes own for purchased care when the spouse is in poor health. Both income and wealth are found to be important determinants of couple-wide retirement coordination.

Overall, our results confirm that retirement is a household decision which is not made by the individual alone, independently of the spouse (Mitchell and Fields, 1982; Hurd, 1990). We reject the Killingsworth (1983) male-chauvinist hypothesis that the husband determines his retirement timing individually, while the wife takes the husband's timing as given in determining her retirement timing. This hypothesis is rejected in favour of a model where both husband and wife jointly determine the timing of retirement. The jointness is not just caused by correlation of unobservables, but appears to be inherent in some underlying decision process.

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