

# Lab 6 23302010034 实验报告

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## 实验内容

实现了一个图类，支持矩阵或者链表表示方法 实现了对应的Vertex、Edge 在此基础上，基于拓扑排序，实现了lab要求的最长路径算法

## 最长路径算法实现方式

```
void dfsHelper(VertexType* v,vector<bool>& visited,vector<int>& result){
    visited[v->getId()]=true;
    for(const pair<VertexType *, int>&target:adjList[v->getId()]){
        VertexType * neighbor=target.first;
        int weight=target.second;
        if(!visited[neighbor->getId()]){
            dfsHelper(neighbor,visited,result);
        }
    }
    result.push_back(v->getId());
}

std::vector<int> dfs(VertexType* start){
    if (!hasAdjList) {
        throw std::runtime_error("Adjacency list representation is required
for DFS.");
    }

    std::vector<bool> visited(verticesNum+1, false);
    std::vector<int> result;

    dfsHelper(start, visited, result);

    return result;
}

int longestPath(VertexType* start){
    if (!hasAdjList) {
        throw std::runtime_error("Adjacency list representation is required
for longest path computation.");
    }

    vector<int> topOrder=dfs(start);
    std::reverse(topOrder.begin(),topOrder.end());

    vector<int> distance(verticesNum,INT_MIN);
    distance[start->getId()]=start->getWeight();

    for(int id:topOrder){
```

```

        if(distance[id]!=INT_MIN){
            for (const auto& [neighbor, weight] : adjList[id]) {
                int vId = neighbor->getId();
                int vertexWeight=neighbor->getWeight();
                distance[vId] = std::max(distance[vId], distance[id] +
vertexWeight);
            }
        }
    }
    int longestPath = INT_MIN;
    for (int d : distance) {
        if (d != INT_MIN) {
            longestPath = std::max(longestPath, d);
        }
    }

    return longestPath;
}

int longestPath(){
    vector<int> longestPaths;
    for(VertexType&vertex:vertexVec){
        int longest_path=longestPath(&vertex);
        longestPaths.push_back(longest_path);
    }
    int longestPath = INT_MIN;
    for (int d : longestPaths) {
        if (d != INT_MIN) {
            longestPath = std::max(longestPath, d);
        }
    }

    return longestPath;
}

```

## 算法描述

首先给定了一个以链表储存的图，这个图没有环且权重在vertex上我们要获得其最长路径

### 步骤

- 1 进行拓扑排序，通过深度优先遍历并记录遍历顺序，然后reverse得到拓扑排序结果
- 2 对拓扑排序好的结果进行动态规划的遍历（第一个就是start），维护一个距离数组表示当前到对应点的最大距离，然后应用方程 $distance[vId] = \max(distance[vId], distance[id] + vertexWeight)$ ，注意到这个方程一定能得到最长路径，当遍历结束时，也就得到了start到各个点的最大距离
- 3 题目没有指定开始点，则可以默认遍历所有的节点，找到其中距离最远的那个点，最终得到长度

### 复杂度分析

对于每个起点开始，dfs时间复杂度是 $O(V+E)$ ，在此基础上，更新状态方程的时间复杂度是，访问了每个顶点每个邻居一次，复杂度为 $O(V+E)$ ，故单个顶点复杂度为 $O(V+E)$ ，为线性时间 对于所有顶点的遍历，复杂度为

$O(V(V+E))$

正确性验证

以图1为例

```
void test_1(){
    vector<int> weight={4,5,4,6,4,5,6,7,3,4,2,2,3,2,5};
    vector<Vertex<>> vertex=Graph<>::convertFromWeight(weight);
    vector<vector<int>> matrix=
    {{0,0,0,1,1,0,0,0,0,0,0,0,0,0,0},
    {0,0,0,0,0,1,1,0,0,0,0,0,0,0,0},
    {0,0,0,0,0,0,0,1,0,0,0,0,0,0,0},
    {0,0,0,0,0,0,0,0,0,0,0,0,1,0,0},
    {0,0,0,0,0,0,0,0,1,1,0,0,0,0,0},
    {0,0,0,0,0,0,0,0,1,1,0,0,0,0,0},
    {0,0,0,0,0,0,0,0,0,0,1,1,0,0,0},
    {0,0,0,0,0,0,0,0,0,0,1,1,0,0,0},
    {0,0,0,0,0,0,0,0,0,0,0,0,1,1,0},
    {0,0,0,0,0,0,0,0,0,0,0,0,0,1,0},
    {0,0,0,0,0,0,0,0,0,0,0,0,0,0,1},
    {0,0,0,0,0,0,0,0,0,0,0,0,0,0,0},
    {0,0,0,0,0,0,0,0,0,0,0,0,0,0,0},
    {0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}};

    vector<Edge<>> edges=Graph<>::convertFromMatrix(vertex,matrix);

    Graph<> graph=Graph<>
    (true,false,GraphRepresentationType::LIST_FORM,edges,vertex);
    int longest=graph.longestPath();
    cout<<"Longest for graph 1: "<<longest<<endl;
}
```

根据给定的四个示例进行验证，发现输出符合预期

```
Longest for graph 1: 18
Longest for graph 2: 8
Longest for graph 3: 27
Longest for graph 4: 16
PS D:\Code\C++\2024 Fall Data Structure Code\lab_6> █
```

说明正确实现了这个lab的要求