Assignment 6

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1 Model

Using the python implementation from `https://github.com/fskerman/vicsek_model` as a starting point, I have implemented the following Boids style model:
There are three zones; a separation zone $Z_S = (r_{s,0},\, r_{s,1}]$, an alignment zone $Z_A = (r_{a,0},\, r_{a,1}]$ and a cohesion zone $Z_C = (r_{c,0},\, r_{c,1}]$. When using intrinsic noise, a particle p_i updates the angle θ_i of its velocity vector $\vec{v}_i$, and its position $\vec{x}_i$ according to

$$\begin{aligned} \vec{x}_i(t+\Delta t) &= \vec{x}_i(t) + \Delta t\big(\cos\theta_i(t+\Delta t),\, \sin\theta_i(t+\Delta t)\big) \\ \theta_i(t+\Delta t) &= \Theta(\vec{w}_i) + \eta\,\xi_i \\ \vec{w}_i &= \vec{v}_i(t) + \langle U_{A_i}\rangle + \langle V_{C_i}\rangle - \big(\langle \hat{U_{S_i}}\rangle + \vec{u}_{m_i}\big) \end{aligned}$$

Where $\Theta(\cdot)$ is the angle $\in [-\pi,\, \pi)$ of a vector, ξ is a uniformly distributed random angle $\in [-\pi,\, \pi)$, $\langle\cdot\rangle$ denotes arithmetic mean, $\hat{\cdot}$ denotes a normalized vector, $\vec{u}_{m_i} = \min(U_{S_i})$ is the vector between p_i and its nearest neighbor, and

$$\begin{aligned} U_{A_i} &= \{\vec{u}_j : \vec{x}_j \in A_i\} \qquad & A_i &= \{\vec{x}_j : d_T(\vec{x}_i,\, \vec{x}_j) \in Z_A\} \qquad & \vec{u}_j &= \big(d_{min}[x_i,\, x_j],\, d_{min}[y_i,\, y_j]\big) \\ V_{C_i} &= \{\vec{v}_j : \vec{x}_j \in C_i\} \qquad & C_i &= \{\vec{x}_j : d_T(\vec{x}_i,\, \vec{x}_j) \in Z_C\} \end{aligned}$$

Here, $d_T(\,,)$ denotes distance on the torus and $d_E(\,,)$ denotes euclidean distance. $d_{min}(\,,)$ is the minimum of those metrics.

When using extrinsic noise, the angle is instead updated according to

$$\begin{aligned} \theta_i(t+\Delta t) &= \Theta(\vec{w}_i + \vec{r}_A + \vec{r}_C + \vec{r}_S) \\ \vec{r} &= \eta\big(\cos\xi,\, \sin\xi\big) \end{aligned}$$

That is, a unit noise vector with random angle, scaled by η is added to each external influence.

2 Polarisation

For this experiment I ran ten simulations for 100 time steps in intrinsic and extrinsic for each value of η. I calculated the mean polarisation over time for each simulation and plotted the mean over the ten simulations.

The parameters are:

$$N = 40$$
$$Z_A = (0, 0.12]$$
$$Z_C = Z_S = 0$$

Thus only the alignment zone with radius 0.12 is affecting particle behavior in these simulations.

The mean polarisation shown in fig. 1a looks more or less like expected. When $\eta = 1$, the direction that a particle takes is equally weighted between the current velocity, alignment and stochastic components, resulting in essentially random behavior. The phase transition looks fairly smooth overall, and I expect that it would look smoother given more time steps per simulation since the initial disorderly period would have less of an impact on the mean.

With extrinsic noise (fig. 1b), there does not seem to be a clear pattern. The particles are apparently much more resilient to extrinsic noise, which is also illustrated by the much less stochastic appearance of the curve in fig. 2b, compared to fig. 2a. However, I had expected to see a point where polarisation goes close to zero here as well.

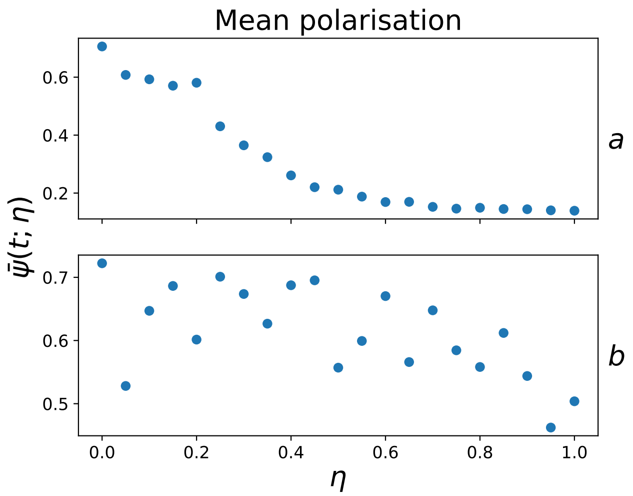

Figure 1: ***a*** *shows the plot with intrinsic noise,* ***b*** *with extrinsic noise.*

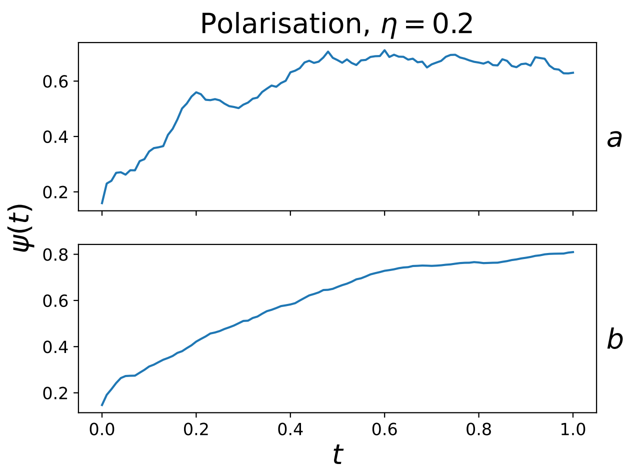

Figure 2: *Example of polarisation over time, averaged across five simulations with intrinsic (**a**) and extrinsic (**b**) noise*

3 Aggregation

I have tested two measures of aggregation; proximity, which is based on the average distance to the closest neighbor and clustering, which is based on the distribution of particles. Here I use basically the same setup as in 2, except that the simulations are run with intrinsic noise only.

Proximity is computed as

$$\frac{1}{N}\sum_i 0.5\sqrt{2} - d_T(\vec{x}_i, \vec{x}_{m_i}) ,$$

where $\vec{x}_{m_i}$ is the coordinates of the nearest neighbor of particle i. This measure is simply intended to capture how close together particles are on average.

Clustering computes the average positive deviation from the expected number of particles within radius r, given a uniform distribution of particles. It is defined as

$$\frac{1}{N}\sum_i \max(0,\, 1 + |R_i| - E_i) ,$$

where $R_i = \{x_j : d_T(x_i, x_j) \leq r\}$ is the set of neighbors that are within radius r of particle i, and $E_i = N\pi r^2$. This measure is intended to capture the degree to which particles are grouped together and the density of those groups.

Here I have used $r = 0.12$ when computing clustering, which is equal to the maximum radius of influence in these simulations. For reference, the mean proximity and clustering of 100 uniformly random samples were found to be about 0.627 and 1.158 respectively.

There is clearly a high degree of correlation between these measures in this experiment, and it is not clear from this data whether the clustering measure is actually useful.

The parameters for these simulations are:

$$\begin{aligned} N &= 40 \\ Z_C &= (0.06,\, 0.12] \\ Z_A &= (0,\, 0.06] \\ Z_S &= 0 \end{aligned}$$

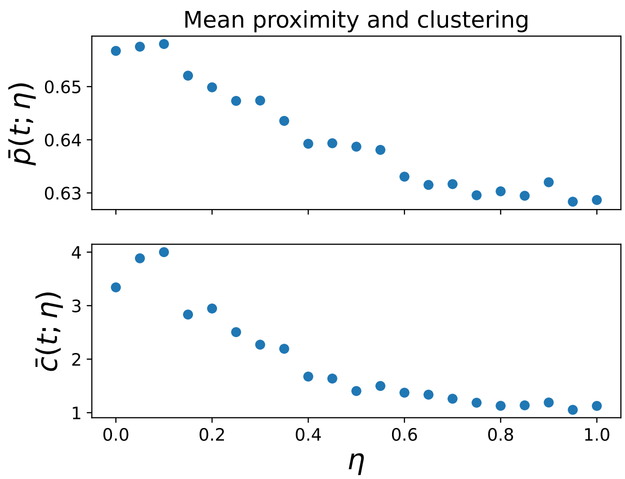

Figure 3: *Average over 10 simulations where the measure is averaged over time for each simulation.*

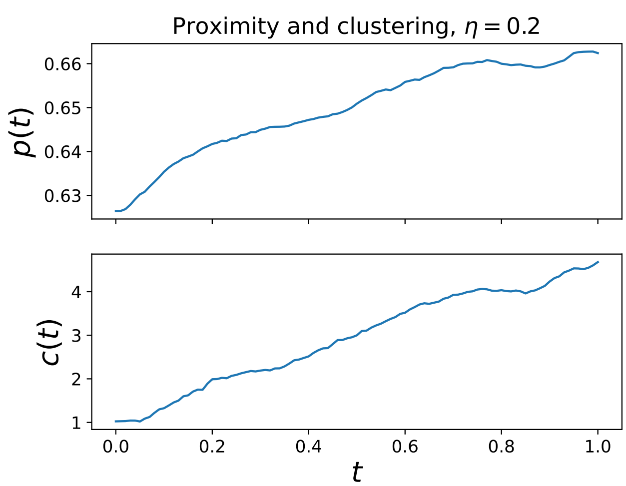

Figure 4: *Average over 10 simulations.*

A Code

All the code can be checked out at
`https://github.com/Fredrik-M/BERV-MCS/tree/master/MCS/lab_6`

B Example simulations

Some example simulations can be found in
`https://github.com/Fredrik-M/BERV-MCS/tree/master/MCS/lab_6/sim`