

Lecture I-V summary

Knudsen number: $K_n = \text{Mean free path} / \text{smallest length scale}$

Shallow water equations: Assumption $H \ll L$ and homogeneous ($\rho = \text{const.}$)

- 1) Use scaling to obtain hydrostatic equilibrium and incompressibility
- 2) Obtain $\nabla_H p = \rho g \nabla_H \eta$
- 3) Use continuity and BC to obtain $\frac{Dh}{Dt} + h \nabla_{Hu} = 0$

Two layer model: Assumption same as before, note $\nabla_H p_0 = 0$.

- 1) Write NS equations with new continuity.
- 2) First layer same as before
- 3) The second layer uses continuous pressure. $p_2(z) = p_0 + \rho_1 g(\eta_0 - \eta_1) + \rho_2 g(\eta_1 - z)$
- 4) Obtain reduced gravity $g' = g \frac{\rho_2 - \rho_1}{\rho_1}$

Reduced gravity model: (aka 1.5) Assumption $H_2 \gg H_1$,

- 1) Notice that shallow part is much faster, $u_2 \approx 0$
- 2) Obtain relation $\nabla_H \eta_0 \approx -\frac{g'}{g} \nabla_H \eta_1$
- 3) Set $h_1 = \eta_0 - \eta_1$ which gives g' and h_1 for upper layer.

Potential vorticity: Material invariance, assume shallow. Use $\zeta = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$.

Note: All functions of Q are material invariants.

Linear wave solution: Assume SWE, $f = f_0$ and $u = \varepsilon u'$, $v = \varepsilon v'$, $\eta = H + \varepsilon \eta'$.

- 1) Neglect all ε^2 terms.
- 2) Apply Fourier's ansatz
- 3) Solve for $\omega(f, k, l)$
- 4) Two solutions:
 - a) $\omega = 0 \rightarrow \text{Geostrophic}$
 - b) $\omega \neq 0 \rightarrow \text{Pointcarté waves}$
- 5) Use $c = \sqrt{gH}$ and Rossby number of deformation $L_D = \frac{c}{f_0} = \frac{\sqrt{gH}}{f_0}$.
 - a) If SW: $\omega \gg f_0 \rightarrow \omega^2 = c^2(k^2 + l^2)$
 - b) if LW: $\omega \approx f_0$ "Internal oscillations"

Geostrophic adjustment: Assume linearized waves

- 1) Use equations to derive expression for $\zeta(Q)$
- 2) Begin by doing $\frac{\partial(eq1)}{\partial x} + \frac{\partial(eq2)}{\partial y} = -g \nabla H^2 \eta$
 - a) Insert ζ when possible
 - b) Use continuity as $\frac{\partial^2 \eta}{\partial t^2} = -H \frac{\partial \eta}{\partial t} \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} \right]$
- 3) Split solution into homogeneous and particular as $\eta = \eta_w(t) + \eta_p$
- 4) Use the starting scenario to determine Q , use relation from before.
- 5) Use particular solution to derive η_p and determine geostrophic wind.