

Lecture I-V summary

Knudsen number: $K_n = \frac{\text{mean free path}}{\text{smallest length scale}}$

Shallow water equations: Assumption $H \ll L$ and homogeneous ($\rho = \text{const}$)

- 1) Use scaling to obtain hydrostatic equilibrium and incompressibility
- 2) Obtain $\nabla_H p = \rho g \nabla_H \eta$
- 3) Use continuity and BC to obtain $\frac{Dh}{Dt} + h \nabla_H \cdot \vec{u} = 0$

Two layer model: Assumption same as before, note $\nabla_H p_0 = 0$.

- 1) Write NS equations with new continuity.
- 2) First layer same as before
- 3) The second layer uses continuous pressure. $p_2(z) = p_0 + \rho_1 g (\eta_0 - \eta_1) + \rho_2 g (\eta_1 - z)$
- 4) Obtain reduced gravity $g' = g \frac{\rho_2 - \rho_1}{\rho_1}$

Reduced gravity model: (aka 1.5) Assume $H_2 \gg H_1$.

- 1) Notice that shallow part is much faster, $\vec{u}_2 \approx 0$
- 2) Obtain relation $\nabla_H \eta_0 \approx -\frac{g'}{g} \nabla_H \eta_1$
- 3) Set $h_1 = \eta_0 - \eta_1$ which gives g' and h_1 for upper layer.

Potential vorticity: Material invariance, assume shallow. Use $\zeta = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$.

- 1) Start by $\frac{\partial(eq2)}{\partial x} - \frac{\partial(eq1)}{\partial y}$
- 2) Solve for ζ and $\frac{D}{Dt} \zeta$
- 3) Use continuity
Note: All functions of Q are material invariants.

Linear wave solution: Assume SWE, $f = f_0$ and $\vec{u} = \varepsilon \vec{\tilde{u}}$, $\vec{v} = \varepsilon \vec{\tilde{v}}$, $\eta = H + \varepsilon \tilde{\eta}$.

- 1) Neglect all ε^2 terms.
- 2) Apply Fourier's ansatz
- 3) Solve for $\omega(f_0, k, l)$
- 4) Two solutions:
 - i. $\omega = 0 \rightarrow$ Geostrophic
 - ii. $\omega \neq 0 \rightarrow$ Pointcarté waves
- 5) Use $c = \sqrt{gH}$ and Rossby radius of deformation $L_D = \frac{c}{f_0} = \frac{\sqrt{gH}}{f_0}$.
 - i. If SW: $\omega \gg f_0 \rightarrow \omega^2 = c^2(k^2 + l^2)$
 - ii. if LW: $\omega \approx f_0$ "Internal oscillations"

Geostrophic adjustment: Assume linearized waves

- 1) Use equations to derive expression for $\zeta(Q)$
- 2) Begin by doing $\frac{\partial(eq1)}{\partial x} + \frac{\partial(eq2)}{\partial y} = -g \nabla^2 \eta$
 - i. Insert ζ when possible
 - ii. Use continuity as $\frac{\partial^2 \eta}{\partial t^2} = -H \frac{\partial \eta}{\partial t} \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} \right]$
- 3) Split solution into homogeneous and particular as $\eta = \eta_w(t) + \eta_p$
- 4) Use the starting scenario to determine Q , use relation from before.
- 5) Use particular solution to derive η_p and determine geostrophic wind.