

# Lecture I-V summary

Knudsen number:  $K_n$  = Mean free path / smallest length scale

Shallow water equations: Assumption  $H \ll L$  and homogeneous ( $\rho = \text{const}$ )

- 1) Use scaling to obtain hydrostatic equilibrium and incompressibility
- 2) Obtain  $\nabla_H p = \rho g \nabla_H \eta$
- 3) Use continuity and BC to obtain  $\frac{Dh}{Dt} + h \nabla_H u = 0$

Two layer model: Assumption same as before, note  $\nabla_H p_0 = 0$ .

- 1) Write NS equations with new continuity.
- 2) First layer same as before
- 3) The second layer uses continuous pressure.  $p_2(z) = p_0 + \rho_1 g (\eta_0 - \eta_1) + \rho_2 g (\eta_1 - z)$
- 4) Obtain reduced gravity  $g' = g \frac{\rho_2 - \rho_1}{\rho_1}$

Reduced gravity model: (aka 1.5) Assumption  $H_2 \gg H_1$ .

- 1) Notice that shallow part is much faster,  $u_2 \approx 0$
- 2) Obtain relation  $\nabla_H \eta_0 \approx -\frac{g'}{g} \nabla_H \eta_1$
- 3) Set  $h_1 = \eta_0 - \eta_1$  which gives  $g'$  and  $h_1$  for upper layer.

Potential vorticity: Material invariance, assume shallow. Use  $\zeta = \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$ .

Note: All functions of  $Q$  are material invariants.

Linear wave solution: Assume SWE,  $f = f_0$  and  $u = \varepsilon \tilde{u}$ ,  $v = \varepsilon \tilde{v}$ ,  $\eta = H + \varepsilon \tilde{\eta}$ .

- 1) Neglect all  $\varepsilon^2$  terms.
- 2) Apply Fourier's ansatz
- 3) Solve for  $\omega(f, k, l)$
- 4) Two solutions:
  - i.  $\omega = 0 \rightarrow$  Geostrophic
  - ii.  $\omega \neq 0 \rightarrow$  Pointcarté waves
- 5) Use  $c = \sqrt{gH}$  and Rossby number of deformation  $L_D = \frac{c}{f_0} = \frac{\sqrt{gH}}{f_0}$ .
  - i. If SW:  $\omega \gg f_0 \rightarrow \omega^2 = c^2(k^2 + l^2)$
  - ii. if LW:  $\omega \approx f_0$  "Internal oscillations"

Geostrophic adjustment: Assume linearized waves

- 1) Use equations to derive expression for  $\zeta(Q)$
- 2) Begin by doing  $\frac{\partial(eq1)}{\partial x} + \frac{\partial(eq2)}{\partial y} = -g \nabla H^2 \eta$ 
  - i. Insert  $\zeta$  when possible
  - ii. Use continuity as  $\frac{\partial^2 \eta}{\partial t^2} = -H \frac{\partial \eta}{\partial t} \left[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} \right]$
- 3) Split solution into homogeneous and particular as  $\eta = \eta_w(t) + \eta_p$
- 4) Use the starting scenario to determine  $Q$ , use relation from before.
- 5) Use particular solution to derive  $\eta_p$  and determine geostrophic wind.