MODELS OF COMPUTATION

Assignment 1

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Problem 1

Injective

$\forall x : A\ \exists y : B\ f(x) = y,\ \forall a, b \in A,\ a \neq b \Rightarrow f(a) \neq f(b)$

Partial

$\exists x : A\ \exists y : B\ f(x) = y$

Total

$\forall x : A\ \exists y : B\ f(x) = y$

Problem 2

One definition of something being enumerable is as follows:
A set S is enumerable if there exists a surjective $function$ $f \in \mathbb{N} \to S$
We can express the positive rational numbers as a list : $\{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ..., \frac{1}{n}\} \in S$
It is obvious that all positive rational numbers will appear somewhere in this list. Letting the index of each rational number in this list $\in \mathbb{N}$ we have for instance:
$f(0) = \frac{1}{1}$ and $f(3) = \frac{1}{4}$
From the definition of surjective, $(\forall y : B\ \exists x : A\ f(x) = y)$, it is trivial to realise that $f \in \mathbb{N} \to S$ is surjective.

Thus we can say that the set of positive rational numbers are enumerable!

Problem 3

It is obvious that a C-Program is a subset of all Programs. We know that the set Programs are enumerable thus, assuming we have a wellformed program, we can write a program such that which for an input i outputs the ith program. This follows the definition of enumerability, the surjective function $f \in \mathbb{N} \to Programs$. And since the set Programs are enumerable, this also means that its subset, all C-programs, are enumerable.

We have seen that a simple diagonalization argument shows that there are more functions in $\mathbb{N} \in \mathbb{N}$ than programs taking a natural number to a natural number. From this we can see that the set $\mathbb{N} \in \mathbb{N}$ is infinite whereas the set Programs are finite and it is trivial to realise that an infinite set cannot be a subset of a finite set. And hence it might not be possible to express some functions $\mathbb{N} \in \mathbb{N}$ in my favourite programming language.