All matematikk

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Sets

1.1 Axioms

Relations

Definition 2.0.1. A binary relation is a map $S \times S \rightarrow$

Groups

Definition 3.0.2. A group is a triple (G, μ, ι) where G is a set and μ is a binary operation $G \times G \to G$ and ι is a function $G \to G$, satisfying the following axioms:

• Associativity: The following diagram commutes:

$$G \times G \times G \xrightarrow{\operatorname{id} \times \mu} G \times G$$

$$\downarrow^{\mu \times \operatorname{id}} \qquad \qquad \downarrow^{\mu}$$

$$G \times G \xrightarrow{\mu} G$$

• Identity element. There exist an element $e \in G$ such that the following two diagrams commute:

$$\{e\} \times G \xrightarrow{e \times \mathrm{id}} G \times G$$

$$\downarrow^{\mu}$$

$$G$$

and

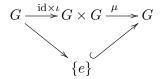
$$G \times \{e\} \xrightarrow{\mathrm{id} \times e} G \times G$$

$$\downarrow^{\mu}$$

$$G$$

where the top maps are the natural inclusions.

• Inverse element. The following diagram is commutative:

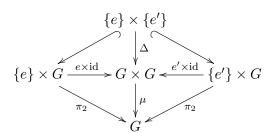


The same should also hold with id $\times \iota$ replaces with $\iota \times id$.

We will never write a group G as a triple, but only refer to the group (G, μ, ι) as just G, the maps being implicit. We will write $\iota(g)$ as g^{-1} , and $\mu(g, h)$ as gh.

Lemma 3.0.3. The identity element $e \in G$ is unique.

Bevis. Suppose e' is another identity element. Then ee'=e' by the first diagram. But also ee'=e by the second diagram. Then e=e'. Alternatively, the following is a commutative diagram:



Rings

Fields

Definition 5.0.4. A *field* is a commutative ring k with only one ideal.

5.1 Ordered fields

The real numbers