# List of objects

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October 12, 2016

## Contents



## 1 Notation

We will use the following notation.

- If $X_0$ is a degeneration of $X$, we write $X \rightsquigarrow X_0$.

## 2 List of objects

### 2.1 Stanley-Reisner sphere $X_{00}$

Let $E_6$ be a hexagon, and let $S$ be the join of two hexagons. This is a simplicial 3-sphere with 12 vertices. The variables are named $x_1, \ldots, x_6$ and $z_1, \ldots, z_6$.

## 2.2 Stanley-Reisner ball $Y_{00}$

Let $S$ be the Stanley-Reisner sphere from Section 2.1. Add two variables $y_0$ and $y_1$ to get a 5-dimensional variety. This is the join of two (filled) hexagons.

## 2.3 Toric variety $Y_0$

The Stanley-Reisner scheme $Y_{00}$ deforms to the toric variety with polytope $P$, where $P$ is the polytope with vertices

$$\begin{pmatrix} 2 & 2 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & -2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & -2 & -2 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 \end{pmatrix}.$$

The toric variety is a deformation of $Y_{00}$ in Section 2.2, by a result of Sturmfels, since the simplicial complex associated to it is a triangulation of the corresponding polytope.

The polytope $P$ is reflexive.

## 2.4 Singular Calabi-Yau $X_0$

This is a complete intersection of two anticanonical hypersurfaces in $Y_0$. By general results, it is a Calabi-Yau. It has 12 singular points, each looking like $C(dP_6)$.

## 2.5 Cone over del Pezzo

The cone over the del Pezzo surface of degree 6 has two smoothings.

## 2.6 Smoother toric $Y$

The toric variety 2.3 deforms to a variety with one-dimensional singularities.

**Remark.** *I suspect that it actually smooths, but I haven't been able to prove it yet. Should be feasible.*

### 2.7 Smooth Calabi-Yau $X$

Since $Y$ have low-dimensional singularities, two applications of Bertini shows that $X_0$ smooths. I have computed it to have Euler characteristic $-72$.

Problem: find which deformation is induced on the singularities of $X_0$.