Resolution of joins

Fredrik Meyer

December 8, 2016

1 Blowing up $\mathbb{P}^{17}$ in two disjoint linear subspaces

Let $\mathbb{P}^{17}$ have coordinates $x_0, \ldots, x_{17}$. Let H_1 be the $\mathbb{P}^8$ defined by $x_9 = x_{10} = \ldots = x_{17} = 0$. Let H_2 be the complementary linear subspace defined by $x_0 = x_1 = \ldots = x_8 = 0$.

Let N be the blowup of $\mathbb{P}^{17}$ in the union of H_1 and H_2. Then N is the subset of $\mathbb{P}^{17} \times \mathbb{P}^8_{y_i} \times \mathbb{P}^8_{z_i}$ defined by the 2×2-minors of

$$\begin{pmatrix} x_0 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ y_0 & y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & y_7 & y_8 \end{pmatrix}$$

and of

$$\begin{pmatrix} x_0 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ z_0 & z_1 & z_2 & z_3 & z_4 & z_5 & z_6 & z_7 & z_8 \end{pmatrix}.$$

We have a projection $Y \to \mathbb{P}^8 \times \mathbb{P}^8$. Let $\mathbb{P}^{8,8}$ denote $\mathbb{P}^8 \times \mathbb{P}^8$. Then Y is a $\mathbb{P}^1$-bundle over $\mathbb{P}^{8,8}$. It is not hard to see that the corresponding locally free rank 2 vector bundle is $\mathscr{E} = \mathscr{O}_{\mathbb{P}^{8,8}}(1,0) \oplus \mathscr{O}_{\mathbb{P}^{8,8}}(0,1)$. Hence $Y = \mathbb{P}(\mathscr{E})$.