All matematikk

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Sets

1.1 Axioms

1. There exists a set \emptyset .

Relations

Definition 2.0.1. A function between two sets A, B, written $f: A \to B$, is a subset Γ of $A \times B$ such that the restriction of the first projection, $\pi_1|_{\Gamma}: \Gamma \to A$ is injective.

[[THIS IS CIRCULAR!!]]

Definition 2.0.2. A binary operation is a function $S \times S \to S$, where S is a set.

Category theory

Groups

Definition 4.0.3. A group is a triple (G, μ, ι) where G is a set and μ is a binary operation $G \times G \to G$ and ι is a function $G \to G$, satisfying the following axioms:

• Associativity: The following diagram commutes:

$$G \times G \times G \xrightarrow{\mathrm{id} \times \mu} G \times G$$

$$\downarrow^{\mu \times \mathrm{id}} \qquad \qquad \downarrow^{\mu}$$

$$G \times G \xrightarrow{\mu} G$$

• Identity element. There exist an element $e \in G$ such that the following two diagrams commute:

$$\{e\} \times G \xrightarrow{e \times \mathrm{id}} G \times G$$

$$\downarrow^{\mu}$$

$$G$$

and

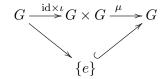
$$G \times \{e\} \xrightarrow{\mathrm{id} \times e} G \times G$$

$$\downarrow^{\mu}$$

$$G$$

where the top maps are the natural inclusions.

• Inverse element. The following diagram is commutative:

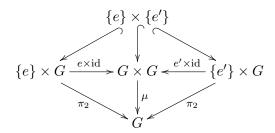


The same should also hold with id $\times \iota$ replaces with $\iota \times id$.

We will never write a group G as a triple, but only refer to the group (G, μ, ι) as just G, the maps being implicit. We will write $\iota(g)$ as g^{-1} , and $\mu(g, h)$ as gh.

Lemma 4.0.4. The identity element $e \in G$ is unique.

Bevis. The following is a commutative diagram:



Following the left arrow gives e', and following the right arrow gives e. \Box

Rings

Definition 5.0.5. A (unitary) $ring\ R$ is a set that is both an abelian group with identity element $0 \in R$ and a multiplicative group with identity element $1 \in R$.

Fields

Definition 6.0.6. A *field* is a commutative ring k with only one ideal.

6.1 Ordered fields

The real numbers