All matematikk

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Kapittel 1

Sets

1.1 Axioms

1. There exists a set $\emptyset$.

Kapittel 2

Relations

Definition 2.0.1. A *function* between two sets A, B, written $f : A \to B$, is a subset Γ of $A \times B$ such that the restriction of the first projection, $\pi_1\big|_\Gamma : \Gamma \to A$ is injective. ■

[[THIS IS CIRCULAR!!]]

Definition 2.0.2. A *binary operation* is a function $S \times S \to S$, where S is a set. ■

Kapittel 3

Category theory

Kapittel 4

Groups

Definition 4.0.3. A *group* is a triple (G, μ, ι) where G is a set and μ is a binary operation $G \times G \to G$ and ι is a function $G \to G$, satisfying the following axioms:

- Associativity: The following diagram commutes:

$$\begin{tikzcd} G \times G \times G \arrow[r, "\mathrm{id} \times \mu"] \arrow[d, "\mu \times \mathrm{id}"'] & G \times G \arrow[d, "\mu"] \\ G \times G \arrow[r, "\mu"] & G \end{tikzcd}$$

- Identity element. There exist an element $e \in G$ such that the following two diagrams commute:

$$\begin{tikzcd} \{e\} \times G \arrow[r, "e \times \mathrm{id}"] \arrow[dr, "\pi_2"'] & G \times G \arrow[d, "\mu"] \\ & G \end{tikzcd}$$

  and

$$\begin{tikzcd} G \times \{e\} \arrow[r, "\mathrm{id} \times e"] \arrow[dr, "\pi_2"'] & G \times G \arrow[d, "\mu"] \\ & G \end{tikzcd}$$

  where the top maps are the natural inclusions.

- Inverse element. The following diagram is commutative:

$$\begin{tikzcd}
G \arrow[r, "\mathrm{id}\times\iota"] \arrow[dr] & G \times G \arrow[r, "\mu"] & G \\
& \{e\} \arrow[ur, hook] &
\end{tikzcd}$$

  The same should also hold with $\mathrm{id} \times \iota$ replaces with $\iota \times \mathrm{id}$.

■

We will never write a group G as a triple, but only refer to the group (G, μ, ι) as just G, the maps being implicit. We will write $\iota(g)$ as g^{-1}, and $\mu(g, h)$ as gh.

Lemma 4.0.4. *The identity element $e \in G$ is unique.*

Bevis. The following is a commutative diagram:

$$\begin{tikzcd}
& \{e\} \times \{e'\} \arrow[dl, hook'] \arrow[d, hook] \arrow[dr, hook] & \\
\{e\} \times G \arrow[r, "e\times\mathrm{id}"] \arrow[dr, "\pi_2"'] & G \times G \arrow[d, "\mu"] & \{e'\} \times G \arrow[l, "e'\times\mathrm{id}"'] \arrow[dl, "\pi_2"] \\
& G &
\end{tikzcd}$$

Following the left arrow gives e', and following the right arrow gives e. □

Kapittel 5

Rings

Definition 5.0.5. A (unitary) *ring* R is a set that is both an abelian group with identity element $0 \in R$ and a multiplicative group with identity element $1 \in R$. ■

Kapittel 6

Fields

Definition 6.0.6. A *field* is a commutative ring k with only one ideal. ■

6.1 Ordered fields

Kapittel 7

The real numbers