

List of objects

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1 List of objects

1.1 Stanley-Reisner sphere X_{00}

Let E_6 be a hexagon, and let S be the join of two hexagons. This is a simplicial 3-sphere with 12 vertices. The variables are named x_1, \dots, x_6 and z_1, \dots, z_6 .

1.2 Stanley-Reisner ball Y_{00}

Let S be the Stanley-Reisner sphere from Section 1.1. Add two variables y_0 and y_1 to get a 5-dimensional variety. This is the join of two (filled) hexagons.

1.3 Toric variety Y_0

The Stanley-Reisner scheme Y_{00} deforms to the toric variety with polytope P , where P is the polytope with vertices

$$\begin{pmatrix} 2 & 2 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & -2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & -2 & -2 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 \end{pmatrix}.$$

The toric variety is a deformation of Y_{00} in Section 1.2, by a result of Sturmfels, since the simplicial complex associated to it is a triangulation of the corresponding polytope.

The polytope P is reflexive.

1.4 Singular Calabi-Yau X_0

This is a complete intersection of two anticanonical hypersurfaces in Y_0 . By general results, it is a Calabi-Yau. It has 12 singular points, each looking like $C(dP_6)$.

1.5 Cone over del Pezzo

The cone over the del Pezzo surface of degree 6 has two smoothings.

1.6 Smoother toric Y

The toric variety 1.3 deforms to a variety with one-dimensional singularities.

Remark. *I suspect that it actually smooths, but I haven't been able to prove it yet. Should be feasible.*

1.7 Smooth Calabi-Yau X

Since Y have low-dimensional singularities, two applications of Bertini shows that X_0 smooths. I have computed it to have Euler characteristic -72 .

Problem: find which deformation is induced on the singularities of X_0 .