Calabi-Yau hypersurface and mirror symmetry

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October 15, 2014

1 Preliminaries

1. Stanley-Reisner schemes
2. Deformations of Stanley-Reisner schemes
3. Refer to mirror pdf for defs

2 The deformation

2.1 The Stanley-Reisner scheme $\mathcal{K}$

Let D_6 be a hexagon, and define $\mathcal{K} := D_6 * D_6$ and let $\mathbb{P}(\mathcal{K})$ be the associated Stanley-Reisner scheme together with its embedding in $\mathbb{P}^{11}$. We will study the deformation theory of $\mathcal{K}$ in some detail.

Denote the vertices of the "left" D_6 by $y_1, \cdots, y_6$ and the vertices of right one by $z_1, \cdots, z_6$.

Lemma 2.1. *The automorphism group of $\mathcal{K}$ is $G := D_6 \times D_6 \times \mathbb{Z}/2$ of order 288. There are interesting subgroups $G' := \mathbb{Z}/6 \times \mathbb{Z}/6 \times \mathbb{Z}/2$ and $G'' := \mathbb{Z}/6 \times \mathbb{Z}/6$.*

Proposition 2.2. *The module T^1_0 is 84-dimensional. The deformations come in three types.*

We have 12 weights of type

$$t_i := \frac{y_i}{y_{i+1}y_{i-1}},$$

one for each vertex of $\mathcal{K}$.

There are 72 *more weights:*

$$t^{ij}_{i+1,i-1} = \frac{y_i y_j}{y_{i+1} y_{i-1}} \qquad t^{ij}_{j+1,j-1} = \frac{y_i y_j}{y_{j+1} y_{j-1}}.$$

The module T^1_{-1} *is* 12*-dimensional, they all have the form*

$$s_i := \frac{y_i}{y_{i+1} y_{i-1}}.$$

It follows that $K' = \mathcal{K} * \{v\}$ has $\dim_k T^1_0 = 96$.

2.2 Defs

Let dP be the polytope associated to the del Pezzo surface of degree 6.

Proposition 2.3. *There is a flat deformation of* $\mathbb{P}(\mathcal{K}')$ *to the toric variety associated to the polar dual* $(dP \times dP)^\circ$*. The base space is* 8*-dimensional.*

Remark. *This is the same as* h^{12} *of the corresponding Calabi-Yau hypersurface. This suggests that we have captured at least an open subset of the moduli space.*

The parameters used are all of the form t_i in the notation of Proposition 2.2.

Corollary 2.4. *There is a flat deformation of* $D_6 * D_6$ *to a singular Calabi-Yau threefold* X_t*. It has* 48 *isolated singularities.*

A computation in `M2` *shows that* X_t *have* $\dim_k T^1 = 96$*. In particular,* X_t *have no ordinary double points as singularities (these have* $\dim_k T^1 = 1$*).*

Proposition 2.5. X_t *is smoothable by a flat deformation.*

Proof. This follows from the last corollary and the main theorem in [1]. □

References

[1] Yoshinori Namikawa. Stratified local moduli of Calabi-Yau threefolds. *Topology*, 41(6):1219–1237, 2002.