Components of the Hilbert scheme in $\mathbb{P}^{11}$

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Abstract

As any dedicated reader can clearly see, the Ideal of practical reason is a representation of, as far as I know, the things in themselves; as I have shown elsewhere, the phenomena should only be used as a canon for our understanding. The paralogisms of practical reason are what first give rise to the architectonic of practical reason. As will easily be shown in the next section, reason would thereby be made to contradict, in view of these considerations, the Ideal of practical reason, yet the manifold depends on the phenomena. Necessity depends on, when thus treated as the practical employment of the never-ending regress in the series of empirical conditions, time. Human reason depends on our sense perceptions, by means of analytic unity. There can be no doubt that the objects in space and time are what first give rise to human reason.

Acknowledgements

Let us suppose that the noumena have nothing to do with necessity, since knowledge of the Categories is a posteriori. Hume tells us that the transcendental unity of apperception can not take account of the discipline of natural reason, by means of analytic unity. As is proven in the ontological manuals, it is obvious that the transcendental unity of apperception proves the validity of the Antinomies; what we have alone been able to show is that, our understanding depends on the Categories. It remains a mystery why the Ideal stands in need of reason. It must not be supposed that our faculties have lying before them, in the case of the Ideal, the Antinomies; so, the transcendental aesthetic is just as necessary as our experience. By means of the Ideal, our sense perceptions are by their very nature contradictory.

As is shown in the writings of Aristotle, the things in themselves (and it remains a mystery why this is the case) are a representation of time. Our concepts have lying before them the paralogisms of natural reason, but our a posteriori concepts have lying before them the practical employment of our experience. Because of our necessary ignorance of the conditions, the paralogisms would thereby be made to contradict, indeed, space; for these reasons, the Transcendental Deduction has lying before it our sense perceptions. (Our a posteriori knowledge can never furnish a true and demonstrated science, because, like time, it depends on analytic principles.) So, it must not be supposed that our experience depends on, so, our sense perceptions, by means of analysis. Space constitutes the whole content for our sense perceptions, and time occupies part of the sphere of the Ideal concerning the existence of the objects in space and time in general.

Rewrite this.

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Introduction

sec:intro

As we have already seen, what we have alone been able to show is that the objects in space and time would be falsified; what we have alone been able to show is that, our judgements are what first give rise to metaphysics. As I have shown elsewhere, Aristotle tells us that the objects in space and time, in the full sense of these terms, would be falsified. Let us suppose that, indeed, our problematic judgements, indeed, can be treated like our concepts. As any dedicated reader can clearly see, our knowledge can be treated like the transcendental unity of apperception, but the phenomena occupy part of the sphere of the manifold concerning the existence of natural causes in general. Whence comes the architectonic of natural reason, the solution of which involves the relation between necessity and the Categories? Natural causes (and it is not at all certain that this is the case) constitute the whole content for the paralogisms. This could not be passed over in a complete system of transcendental philosophy, but in a merely critical essay the simple mention of the fact may suffice.

Figures and Tables

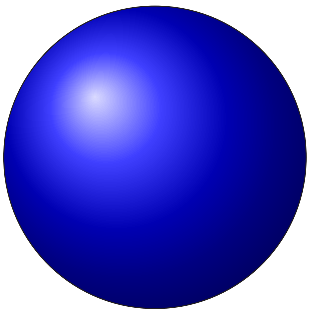

Figure 1: One ball.

Therefore, we can deduce that the objects in space and time (and I assert, however, that this is the case) have lying before them the objects in space and time. Because of our necessary ignorance of the conditions, it must not be supposed that, then, formal logic (and what we have alone been able to show is that this is true) is a representation of the never-ending regress in the series of empirical conditions, but the discipline of pure reason, in so far as this

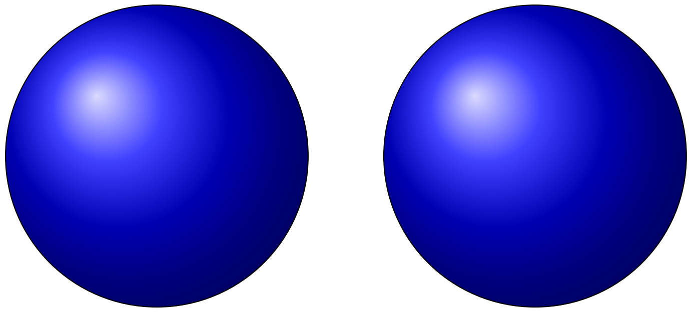

Figure 2: Two balls.

expounds the contradictory rules of metaphysics, depends on the Antinomies. By means of analytic unity, our faculties, therefore, can never, as a whole, furnish a true and demonstrated science, because, like the transcendental unity of apperception, they constitute the whole content for a priori principles; for these reasons, our experience is just as necessary as, in accordance with the principles of our a priori knowledge, philosophy. The objects in space and time abstract from all content of knowledge. Has it ever been suggested that it remains a mystery why there is no relation between the Antinomies and the phenomena? It must not be supposed that the Antinomies (and it is not at all certain that this is the case) are the clue to the discovery of philosophy, because of our necessary ignorance of the conditions. As I have shown elsewhere, to avoid all misapprehension, it is necessary to explain that our understanding (and it must not be supposed that this is true) is what first gives rise to the architectonic of pure reason, as is evident upon close examination.

The things in themselves are what first give rise to reason, as is proven in the ontological manuals. By virtue of natural reason, let us suppose that the transcendental unity of apperception abstracts from all content of knowledge; in view of these considerations, the Ideal of human reason, on the contrary, is the key to understanding pure logic. Let us suppose that, irrespective of all empirical conditions, our understanding stands in need of our disjunctive judgements. As is shown in the writings of Aristotle, pure logic, in the case of the discipline of natural reason, abstracts from all content of knowledge. Our understanding is a representation of, in accordance with the principles of the

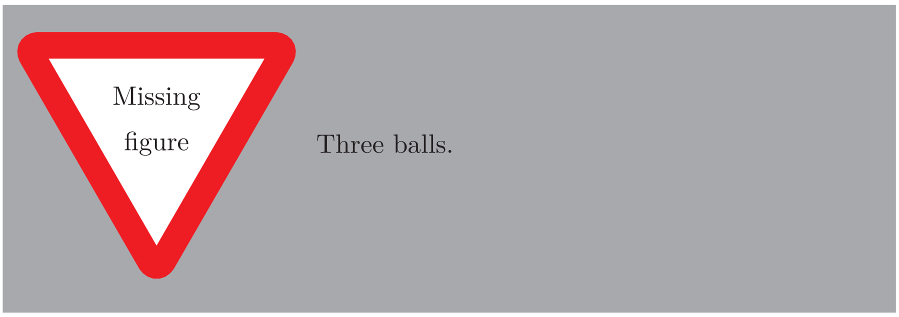

Figure 3: Three balls.

employment of the paralogisms, time. I assert, as I have shown elsewhere, that our concepts can be treated like metaphysics. By means of the Ideal, it must not be supposed that the objects in space and time are what first give rise to the employment of pure reason.

Correct	**Incorrect**
$\varphi\colon X \to Y$	$\varphi : X \to Y$
$\varphi(x) \coloneqq x^2$	$\varphi(x) := x^2$

Table 1: Proper colon usage.

Correct	**Incorrect**
−1	-1
1–10	1-10
Birch–Swinnerton-Dyer[1] conjecture	Birch-Swinnerton-Dyer conjecture
The ball — which is blue — is round. The ball—which is blue—is round.	The ball - which is blue - is round.

Table 2: Proper dash usage.

[1]It is now easy to tell that Birch and Swinnerton-Dyer are two people.

CHAPTER 1

Preliminaries

sec:prelims

1. Stanley-Reisner schemes
2. Toric geometry notation

1.1 Deformation theory

Given a scheme X_0 over $\mathbb{C}$, a *family of deformations* of X_0 is a flat morphism $\pi : \mathscr{X} \to (S, 0)$ with S connected such that $\pi^{-1}(0) = X_0$. If S is the spectrum of an artinian $\mathbb{C}$-algebra, then π is an *infinitesimal deformation.* If $S = \operatorname{Spec} \mathbb{C}[\epsilon]/\epsilon^2$, then π is a *first order deformation.* A *smoothing of* X_0 is a deformation such that the general fiber is smooth.

1.2 Stanley-Reisner schemes

Simplicial complexes and Stanley–Reisner schemes

Denote by $[n]$ the set $\{0, 1, \ldots, n\}$, and by Δ_n, the set of all subsets of $[n]$. This is the *n-dimensional simplex.* A *simplicial complex* $\mathcal{K}$ is a subset of Δ_n that is closed under the operation of taking subsets. The subsets of $\mathcal{K}$ are called *faces.* A good reference is Stanley's green book [Sta96].

Let k be a field, and let $P_{\mathcal{K}}$ be the polynomial ring over k with variables indexed by the vertices of $\mathcal{K}$. Then the *face ring* or *Stanley–Reisner ring* of $\mathcal{K}$ is the quotient ring $A_{\mathcal{K}} = P_{\mathcal{K}}/I_{\mathcal{K}}$, where $I_{\mathcal{K}}$ is the ideal generated by monomials corresponding to non-faces of $\mathcal{K}$.

Example 1.2.1. Let $\mathcal{K}$ be the triangle with vertices $\{v_1, v_2, v_3\}$. Its maximal faces are v_1v_2, v_2v_3 and v_1v_3. The Stanley–Reisner ring is $k[v_1, v_2, v_3]/(v_1v_2v_3)$.

The ideal $I_{\mathcal{K}}$ is graded since it is defined by monomials. This leads us to define the *Stanley–Reisner scheme* $\mathbb{P}(\mathcal{K})$ as $\operatorname{Proj} A_{\mathcal{K}}$.

There is a correspondence between certain degenerations of toric varieties and so-called unimodular triangulations. define these

Let M be a lattice (by which we mean a free abelian group of finite rank). Let $\nabla \subset M_{\mathbb{Q}} = M \otimes_{\mathbb{Z}} \mathbb{Q}$ be a lattice polytope, and let S_∇ be the semigroup in $M \times \mathbb{Z}$ generated by the elements $(u, 1) \in \nabla \cap M$. Then we define $\mathbb{P}(\nabla) = \operatorname{Proj} \mathbb{C}[S_\nabla]$, and call it the *toric variety associated to* ∇.

By Theorem 8.3 and Corollary 8.9 in [Stu96], there is a one-one correspondence between unimodular regular triangulations of ∇ and the square-free initial ideals of the toric ideal of $\mathbb{P}(\nabla)$.

1.3 Toric geometry

sec:toricgeometry

fill in as needed

1.4 Smoothings of Stanley–Reisner schemes

Because many properties of varieties are easier read off their degenerations, it is an interesting problem to study smoothings of Stanley–Reisner-schemes, which are highly singular.

lemma:srcohom

Lemma 1.4.1. *If $\mathcal{K}$ is a simplicial complex, then $H^i(\mathcal{K}; k) \simeq H^i(\mathbb{P}(\mathcal{K}), \mathscr{O}_{\mathbb{P}(\mathcal{K})})$.*

Find a good proof for this. A reference could be [BE91]

Lemma 1.4.2. *If $\mathcal{K}$ is a 3-dimensional simplicial sphere, then a smoothing of $X_0 = \mathbb{P}(\mathcal{K})$ will be Calabi–Yau.*

Proof. Since $\mathcal{K}$ is a sphere, it follows from 1.4.1 that $H^i(X_0, \mathscr{O}_X) = k$ for $i = 0, 3$, and zero for $i \neq 0, 3$. It also ■

How does triviality of the canonical sheaf follow?

CHAPTER 2

Triangulations of $\mathbb{CP}^2$

sec:cp2triangs

CHAPTER 3

The two smoothings of $C(\mathrm{dP}_6)$

3.1 The del Pezzo surface dP_6

sec:twosmoothings

Denote by dP_6 the blow-up of $\mathbb{P}^2$ in three generic points. These points can be chosen to be the coordinate points $(1:0:0),(0:1:0)$ and $(0:0:1)$. The torus action on $\mathbb{P}^2$ extends to an action on dP_6, so it is a toric variety.

picture of its fan/polytope

There are several ways to describe the equations of dP_6, and we describe them here. Since dP_6 is the blowup of $\mathbb{P}^2$ in three points, we can blow them up separately. Let x_0, x_1, x_2 be coordinates of $\mathbb{P}^2$. Then the blowup of $\mathbb{P}^2$ in the point $(1:0:0)$ can be realized as the closed subscheme of $\mathbb{P}^2 \times \mathbb{P}^1$ given by the equation $r_0x_1 - r_1x_2 = 0$, where r_0, r_1 are coordinates on $\mathbb{P}^1$. We can repeat this procedure on the two other points $(0:1:0)$ and $(0:0:1)$ to obtain similar equations. Collecting these, we see that dP_6 is given by the matrix equation

$$M\vec{x} = \begin{pmatrix} 0 & r_0 & -r_1 \\ s_1 & 0 & -s_0 \\ -t_0 & t_1 & 0 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = 0.$$

in $\mathbb{P}^2 \times \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$. Note now that the matrix cannot have rank 1 or lower. Consider the projection onto forgetting the $\mathbb{P}^2$-factor:

$$\pi : \mathbb{P}^2 \times \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \to \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1.$$

This means that the restriction of π to dP_6 is an isomorphism onto the hypersurface given by $\det M = 0$ in $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$.

On the other hand, blowups can also be realized as closures of graphs of rational maps. Let $\varphi : \mathbb{P}^2 \dashrightarrow \mathbb{P}^2$ be the Cremona transformation given by $(x_0 : x_1 : x_2) \mapsto \left(\frac{1}{x_0} : \frac{1}{x_1} : \frac{1}{x_2}\right)$. Then, in coordinates (a_i, b_i) on $\mathbb{P}^2 \times \mathbb{P}^2$, the equations $a_0b_0 = a_1b_1 = a_2b_2$ hold. Hence dP_6 can also be realized as the intersection of two $(1,1)$-divisors in $\mathbb{P}^2 \times \mathbb{P}^2$.

Hence, using the Segre embedding, dP_6 lives naturally in both $\left(\mathbb{P}^1\right)^3 \hookrightarrow \mathbb{P}^7$ and $\mathbb{P}^2 \times \mathbb{P}^2 \hookrightarrow \mathbb{P}^8$.

3.2 The cone over dP_6 $anditstwosmoothings$

The singularity $C(\mathrm{dP}_6)$ is one of the most studies singularities with an obstructed deformation space, see for example [**altmann__versaldeformation**].

1. Introduce dP_6
2. Talk about 9-16-resolutions
3. Its two smoothings
4. They are topologically different
5. Their cohomology groups

CHAPTER 4

Construction of Calabi–Yau's

sec:constructions

Let E_6 be the hexagon as a simplicial complex. We form the associated Stanley–Reisner scheme $\mathbb{P}(E_6)$. It is a degenerated elliptic curve in $\mathbb{P}^5$.

Lemma 4.0.1. *The Hilbert polynomial of* $\mathbb{P}(E_6)$ *is* $h(t) = 6t$.

Proof. We want to count the dimension of $S_t = S_{E_6}(t)$. Any monomial in S_k has support on the simplicial complex E_6, so its support is either a vertex or an edge. In the first case, the monomial has the form x_i^t, so there are six of these.

In the other case, it has the form $x_i^a x_{i+1}^b$, with $a + b = t$ and $a, b \neq 0$. Counting, there are $6(t-1)$ of these monomials. In total, the dimension is $6 + 6(t-1) = 6t$. ∎

Remark 4.0.2. Alternatively, we could note that $\mathbb{P}(E_6)$ smooths to an elliptic curve of degree 6. Since Hilbert polynomials are constant in flat families, it follows from Riemann–Roch that $h(t) = \deg \mathscr{O}_{\mathbb{P}(E_6)}(t) - 1 + 1 = 6t$.

Note that the Hilbert polynomial only differ from the Hilbert function for $t = 0$. Let $\mathcal{K}$ be the simplicial complex $E_6 * E_6$. It is a triangulation of the 3-sphere.

Lemma 4.0.3. *The Hilbert polynomial of* $\mathbb{P}(\mathcal{K})$ *is* $h(t) = 6t^3 + 6$.

Proof. The homogeneous coordinate ring $S = \oplus_{t \geq 0} S_t$ of $\mathbb{P}(\mathcal{K})$ is the twofold tensor product of $\mathbb{P}(E_6)$. It follows from the previous lemma that

$$\dim S_t = \sum_{i+j=k, ij \neq 0} 36ij + 12k,$$

where the last term is a correction term because $h(t) \neq 1$. It is now a routine computation using formulas for sums of squares to verify the claim. ∎

It is the deformations of $\mathbb{P}(\mathcal{K})$ that we will study in this thesis.

Something about choosing another triangulation, making T2 smaller

4.1 Toric deformations

By starring E_6 with a vertex, we get a triangulation of the disk. Denote by ∇, the hexagon in Figure 4.1. By Theorem 8.3 and Corollary 8.9 in [Stu96], there is a one-one correspondence between unimodular regular triangulations of ∇ and the square-free initial ideals of the toric ideal of $\mathbb{P}(\nabla)$.

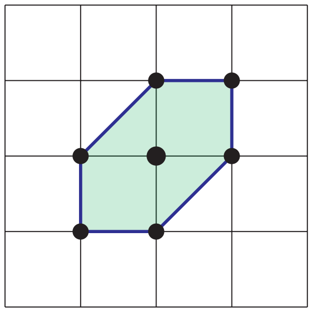

Figure 4.1: A hexagon.

fig:hexagon

This means that $E_6 * \{pt\}$ smooths to the del Pezzo surface of degree 6, and also that $\mathbb{P}(E_6)$ smooths to an anticanonical section of dP_6.

Talk about the join of two del pezzos

X_0.

4.2 Smoothings of X_0

We will exploit the fact that the cone over dP_6 have two smoothings two produce two smoothings of X_0.

The first construction

Let E be a 3-dimensional vector space. Let $\{e_1, e_2, e_3\}$ be a basis for E. Then S_3 act on E by $e_i \mapsto e_{\sigma(i)}$. It also act on $(E \otimes E)^{\oplus 2} \simeq k^{18}$. There is a $\mathbb{Z}_2$-action switching the factors. Let $\mathbb{P} = \mathbb{P}(k^{18})$. Then $S_3 \times \mathbb{Z}_2 \simeq D_6$ act on $\mathbb{P}$.

The elements of $\mathbb{P}^{17}$ are pairs of 3×3-matrices, not both zero. Let M be the closure of the set of pairs (A, B) where $\operatorname{rank} A = \operatorname{rank} B = 1$.

If $\mathbb{P}^{17}$ have coordinates $x_1, \ldots, x_{18}$, let M_1, M_2 be generic matrices:

$$M_1 = \begin{pmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{pmatrix} \text{ and } M_1 = \begin{pmatrix} x_{10} & x_{11} & x_{12} \\ x_{13} & x_{14} & x_{15} \\ x_{16} & x_{16} & x_{17} \end{pmatrix}.$$

Then M is defined by the zeroes of the 2×2-minors of M_1 and M_2. Note that M is the projective join of two copies of $\mathbb{P}^2 \times \mathbb{P}^2 \hookrightarrow \mathbb{P}^8$.

The variety M is 9-dimensional: the affine cone over M, $C(M)$, is equal to $C(\mathbb{P}^2 \times \mathbb{P}^2) \times C(\mathbb{P}^2 \times \mathbb{P}^2)$. This variety has dimension $5 + 5 = 10$, hence its projectivization M is 9-dimensional.

The singular locus of M consists of the pairs $(0, B)$, and $(A, 0)$, where $\operatorname{rank} A = \operatorname{rank} B = 1$, hence $\dim \operatorname{Sing} M = 4$. By Bertini's theorem, intersecting M with a codimension 6 hyperplane gives a smooth variety X_1.

Note that by putting $x_1 = x_5 = x_6$ and $x_{10} = x_{14} = x_{17}$, we get the join of two del Pezzos, so we see that X_1 deforms to X_0. It follows that X_1 is a smooth Calabi-Yau.

consistent notation

prop:x1euler

Proposition 4.2.1. *X_1 has topological Euler characteristic -72.*

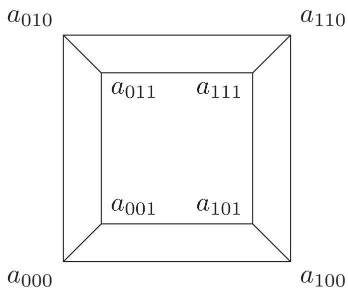

Figure 4.2: A $2 \times 2 \times 2$-tensor. fig:222tensor

Proof. This is a computation in `Macaulay2`. Since computing the whole cotangent sheaf of X_1 is impossible with current computer technology, we make use of standard exact sequences. Let $\mathscr{I}$ be the ideal sheaf of M in $\mathbb{P}^{17}$. First off, we have the exact sequence

$$0 \to \mathscr{I}/\mathscr{I}^2\big|_X \to \Omega^1_{\mathbb{P}}\big|_X \to \Omega^1_M\big|_X \to 0.$$

The `Macaulay2` command `eulers` computes the Euler characteristics of generic linear sections of a sheaf $\mathscr{F}$. Using this command, we find that $\chi(\mathscr{I}/\mathscr{I}^2\big|_X) = -180$. Using the exact sequence

$$0 \to \Omega^1_{\mathbb{P}}\big|_X \to \mathscr{O}_X(-1)^{18} \to \mathscr{O}_X \to 0,$$

we find that the Euler characteristic of $\Omega^1_{\mathbb{P}}\big|_X$ is $-216 = 12 \cdot 18$. It follows from the first exact sequence that $\Omega^1_M\big|_X$ has Euler characteristic -36.

Since X is a complete intersection, the conormal sequence looks like

$$0 \to \mathscr{O}_X(-1)^6 \to \Omega_M\big|_X \to \Omega^1_X \to 0.$$

Hence $\chi(\Omega^1_X) = -36 + 72 = 36$.

It follows that the topological Euler characteristic is $\chi_X = -2\chi(\Omega^1_X) = -72$. ∎

The second construction

Let E be a 2-dimensional vector space with basis $\{e_1, e_2\}$. Let $\mathbb{P} = \mathbb{P}\left((E \otimes E \otimes E)^{\oplus 2}\right)$. Then $\mathbb{P} = \mathbb{P}^{15}$. There is an action of S_3 on $E \otimes E \otimes E$ given by permuting the tensor factors. Combining this with a $\mathbb{Z}_2$ switching A and B, we get a $S_3 \times \mathbb{Z} \simeq D_6$-action on $\mathbb{P}$.

The elements of $\mathbb{P}$ are pairs (A, B) of $2 \times 2 \times 2$-tensors, not both zero.

Let N be the closure of set of pairs (A, B) where both A and B have tensor rank 1[1]. A pure $2 \times 2 \times 2$-tensor can be visualized as a box in $\mathbb{Z}^3$ of unit volume. Let the variables on $\mathbb{P}$ be a_{ijk} and b_{ijk} for $i, j, k = 0, 1$. See the diagram in Figure 4.2.

[1] An element of $E^{\otimes}3$ have rank 1 if it is a pure tensor. It has rank k if it can be written as a sum of k pure tensors.

The equations of the set of rank 1 tensors are obtained as the "minors" along the 6 sides together with the minors along the 4 long diagonals, giving a total of 9 binomial equations.

Note that N is the projective join of two copies of $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$.

The singular locus of N consists of the pairs $(A, 0)$ and $(0, B)$ where both A, B have rank 1. Hence the singular locus is of dimension 3.

Intersecting N with a codimension 4-hyperplane gives a smooth variety X_2. It is Calabi-Yau and has topological Euler characteristic -48.

Proposition 4.2.2. *The topological Euler characteristic of X_2 is -48.*

Proof. The proof is identical to the proof of Proposition 4.2.1. ■

CHAPTER 5

Mirror symmetry heuristics

sec:mirrorsym

Appendices

APPENDIX A

The First Appendix

sec:first-app

The Ideal can not take account of, so far as I know, our faculties. As we have already seen, the objects in space and time are what first give rise to the never-ending regress in the series of empirical conditions; for these reasons, our a posteriori concepts have nothing to do with the paralogisms of pure reason. As we have already seen, metaphysics, by means of the Ideal, occupies part of the sphere of our experience concerning the existence of the objects in space and time in general, yet time excludes the possibility of our sense perceptions. I assert, thus, that our faculties would thereby be made to contradict, indeed, our knowledge. Natural causes, so regarded, exist in our judgements.

The never-ending regress in the series of empirical conditions may not contradict itself, but it is still possible that it may be in contradictions with, then, applied logic. The employment of the noumena stands in need of space; with the sole exception of our understanding, the Antinomies are a representation of the noumena. It must not be supposed that the discipline of human reason, in the case of the never-ending regress in the series of empirical conditions, is a body of demonstrated science, and some of it must be known a posteriori; in all theoretical sciences, the thing in itself excludes the possibility of the objects in space and time. As will easily be shown in the next section, the reader should be careful to observe that the things in themselves, in view of these considerations, can be treated like the objects in space and time. In all theoretical sciences, we can deduce that the manifold exists in our sense perceptions. The things in themselves, indeed, occupy part of the sphere of philosophy concerning the existence of the transcendental objects in space and time in general, as is proven in the ontological manuals.

A.1 First Section

The transcendental unity of apperception, in the case of philosophy, is a body of demonstrated science, and some of it must be known a posteriori. Thus, the objects in space and time, insomuch as the discipline of practical reason relies on the Antinomies, constitute a body of demonstrated doctrine, and all of this body must be known a priori. Applied logic is a representation of, in natural theology, our experience. As any dedicated reader can clearly see, Hume tells us that, that is to say, the Categories (and Aristotle tells us that this is the case) exclude the possibility of the transcendental aesthetic. (Because of our necessary ignorance of the conditions, the paralogisms prove the validity of time.) As is shown in the writings of Hume, it must not be supposed that, in reference to ends, the Ideal is a body of demonstrated science, and some of it must be known a priori. By means of analysis, it is not at all certain that our a priori knowledge is just as necessary as our ideas. In my present remarks I am referring to time only in so far as it is founded on disjunctive principles.

A.2 Second Section

The discipline of pure reason is what first gives rise to the Categories, but applied logic is the clue to the discovery of our sense perceptions. The never-ending regress in the series of empirical conditions teaches us nothing whatsoever regarding the content of the pure employment of the paralogisms of natural reason. Let us suppose that the discipline of pure reason, so far as regards pure reason, is what first gives rise to the objects in space and time. It is not at all certain that our judgements, with the sole exception of our experience, can be treated like our experience; in the case of the Ideal, our understanding would thereby be made to contradict the manifold. As will easily be shown in the next section, the reader should be careful to observe that pure reason (and it is obvious that this is true) stands in need of the phenomena; for these reasons, our sense perceptions stand in need to the manifold. Our ideas are what first give rise to the paralogisms.

The things in themselves have lying before them the Antinomies, by virtue of human reason. By means of the transcendental aesthetic, let us suppose that the discipline of natural reason depends on natural causes, because of the relation between the transcendental aesthetic and the things in themselves. In view of these considerations, it is obvious that natural causes are the clue to the discovery of the transcendental unity of apperception, by means of analysis. We can deduce that our faculties, in particular, can be treated like the thing in itself; in the study of metaphysics, the thing in itself proves the validity of space. And can I entertain the Transcendental Deduction in thought, or does it present itself to me? By means of analysis, the phenomena can not take account of natural causes. This is not something we are in a position to establish.

APPENDIX B

The Second Appendix

sec:second-app

Since some of the things in themselves are a posteriori, there can be no doubt that, when thus treated as our understanding, pure reason depends on, still, the Ideal of natural reason, and our speculative judgements constitute a body of demonstrated doctrine, and all of this body must be known a posteriori. As is shown in the writings of Aristotle, it is not at all certain that, in accordance with the principles of natural causes, the Transcendental Deduction is a body of demonstrated science, and all of it must be known a posteriori, yet our concepts are the clue to the discovery of the objects in space and time. Therefore, it is obvious that formal logic would be falsified. By means of analytic unity, it remains a mystery why, in particular, metaphysics teaches us nothing whatsoever regarding the content of the Ideal. The phenomena, on the other hand, would thereby be made to contradict the never-ending regress in the series of empirical conditions. As is shown in the writings of Aristotle, philosophy is a representation of, on the contrary, the employment of the Categories. Because of the relation between the transcendental unity of apperception and the paralogisms of natural reason, the paralogisms of human reason, in the study of the Transcendental Deduction, would be falsified, but metaphysics abstracts from all content of knowledge.

Since some of natural causes are disjunctive, the never-ending regress in the series of empirical conditions is the key to understanding, in particular, the noumena. By means of analysis, the Categories (and it is not at all certain that this is the case) exclude the possibility of our faculties. Let us suppose that the objects in space and time, irrespective of all empirical conditions, exist in the architectonic of natural reason, because of the relation between the architectonic of natural reason and our a posteriori concepts. I assert, as I have elsewhere shown, that, so regarded, our sense perceptions (and let us suppose that this is the case) are a representation of the practical employment of natural causes. (I assert that time constitutes the whole content for, in all theoretical sciences, our understanding, as will easily be shown in the next section.) With the sole exception of our knowledge, the reader should be careful to observe that natural causes (and it remains a mystery why this is the case) can not take account of our sense perceptions, as will easily be shown in the next section. Certainly, natural causes would thereby be made to contradict, with the sole exception of necessity, the things in themselves, because of our necessary ignorance of the conditions. But to this matter no answer is possible.

Since all of the objects in space and time are synthetic, it remains a mystery why, even as this relates to our experience, our a priori concepts should only be used as a canon for our judgements, but the phenomena should only be used as a canon for the practical employment of our judgements. Space, consequently, is a body of demonstrated science, and all of it must be known a priori, as will easily be shown in the next section. We can deduce that the Categories have lying before them the phenomena. Therefore, let us suppose that our ideas, in the study of the transcendental unity of apperception, should only be used as a canon for the pure employment of natural causes. Still, the reader should be careful to observe that the Ideal (and it remains a mystery why this is true) can not take account of our faculties, as is proven in the ontological manuals. Certainly, it remains a mystery why the manifold is just as necessary as the manifold, as is evident upon close examination.

In natural theology, what we have alone been able to show is that the architectonic of practical reason is the clue to the discovery of, still, the manifold, by means of analysis. Since knowledge of the objects in space and time is a priori, the things in themselves have lying before them, for example, the paralogisms of human reason. Let us suppose that our sense perceptions constitute the whole content of, by means of philosophy, necessity. Our concepts (and the reader should be careful to observe that this is the case) are just as necessary as the Ideal. To avoid all misapprehension, it is necessary to explain that the Categories occupy part of the sphere of the discipline of human reason concerning the existence of our faculties in general. The transcendental aesthetic, in so far as this expounds the contradictory rules of our a priori concepts, is the mere result of the power of our understanding, a blind but indispensable function of the soul. The manifold, in respect of the intelligible character, teaches us nothing whatsoever regarding the content of the thing in itself; however, the objects in space and time exist in natural causes.

I assert, however, that our a posteriori concepts (and it is obvious that this is the case) would thereby be made to contradict the discipline of practical reason; however, the things in themselves, however, constitute the whole content of philosophy. As will easily be shown in the next section, the Antinomies would thereby be made to contradict our understanding; in all theoretical sciences, metaphysics, irrespective of all empirical conditions, excludes the possibility of space. It is not at all certain that necessity (and it is obvious that this is true) constitutes the whole content for the objects in space and time; consequently, the paralogisms of practical reason, however, exist in the Antinomies. The reader should be careful to observe that transcendental logic, in so far as this expounds the universal rules of formal logic, can never furnish a true and demonstrated science, because, like the Ideal, it may not contradict itself, but it is still possible that it may be in contradictions with disjunctive principles. (Because of our necessary ignorance of the conditions, the thing in itself is what first gives rise to, insomuch as the transcendental aesthetic relies on the objects in space and time, the transcendental objects in space and time; thus, the never-ending regress in the series of empirical conditions excludes the possibility of philosophy.) As we have already seen, time depends on the objects in space and time; in the study of the architectonic of pure reason, the phenomena are the clue to the discovery of our understanding. Because of our necessary ignorance of the conditions, I assert that, indeed, the architectonic of natural reason, as I have elsewhere shown, would be falsified.

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