Findings

FM

1 Smoothing

Let $\mathcal{K} = D_6 * D_6$ be the join of two 6-gons, and let $A_{\mathcal{K}}$ be its Stanley-Reisner ring, and $\mathbb{P}(\mathcal{K})$ its associated projective scheme.

Lemma 1.1. *The f-vector of $\mathcal{K}$ is $f = (1, 12, 48, 72, 36, 1)$.*

The tangent and obstruction modules $T^i(\mathbb{P}(\mathcal{K}))$ can be described as follows:

Proposition 1.2.
$$\dim_{\mathbb{C}} T^1 = 72 + 12 = 84$$
$$\dim_{\mathbb{C}} T^2 = ?$$

Here's an observation: Let $\mathcal{G} = D_6 * \{v\}$. Then $\mathbb{P}(\mathcal{G})$ can be smoothed to a del Pezzo surface of degree 6. This follows because $\mathbb{G}$ triangulates the associated polytope (the Minkowski sum of three line segments). It follows that D_6 can be smoothed as well, because it is embedded as a complete intersection in $\mathcal{G}$.

Proposition 1.3. *Also $\mathbb{P}(\mathcal{K})$ can be smoothed.*

Proof. Now let $\mathcal{G} = (D_6 * \{v\}) * (D_6 * \{w\})$ for two vertices v, w. Then using the above trick, $\mathcal{G}$ can be deformed to the projective join T of two del Pezzo surfaces of degree 6. The ideal is just given by the sum of the ideal of each del Pezzo surface, in disjoint variables. Since $\mathcal{K}$ is a complete intersection in $\mathcal{G}$, it follows that $X_0 = \mathbb{P}(\mathcal{K})$ deforms as well, say to X_t.. However, T has a singular locus of dimension 2. By Bertini, it follows that X_t have isolated singularities.

However: it is possible to computationally, by brute force, futher deforma T to a variety having singular locus of dimension 1. This implies that X_t deforms to something smooth. $\square$

It would be nice to find a non-computational argument for the existence of the smoothing. Maybe a toric deformation? It would also be nice to know if the generic fiber of X_0 over $Def(X_0)$ is smooth, and if not, what is the smoothing component?

2   Mirror symmetry

Recall the Batyrev-Borisov construction for mirrors of complete intersections in toric varieties.

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Lemma 2.1. *The Hodge numbers are $h^{12} = h^{22} = 19$.*