Toolbox

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1 Techniques

1.1 Compute the ideal of an affine toric variety

Suppose given an affine toric X_σ defined by a full-dimensional rational polyhedral convex cone in $N_\mathbb{R} \simeq \mathbb{R}^d$. Then the coordinate ring of X_σ is given by the semigroup algebra $k[S_\sigma]$, where $S_\sigma = \sigma^\vee \cap M$.

Here $\sigma^\vee$ is the dual cone and M is the dual lattice $N^\vee$. There is a canonical k-algebra basis for $k[S_\sigma]$ given by the *Hilbert basis* of the semigroup $\sigma^\vee \cap M$. This gives us a presentation $k[\mathbf{x}] \to k[S_\sigma]$.

Thus there are three steps in computing the toric ideal:

1. First compute the dual cone $\sigma^\vee$.
2. Compute a Hilbert basis $\{m_1, \cdots, m_r\}$ of $\sigma^\vee \cap M$.
3. Compute the kernel of the map

$$\begin{aligned} k[x_1, \cdots, x_r] &\to k[S_\sigma] \\ x_i &\mapsto m_i. \end{aligned}$$

Here is a `Macaulay2` session that starts with a cone $\sigma \subseteq \mathbb{N}_\mathbb{R}$, and prints the corresponding toric ideal.

```
M = matrix{{3,1},{1,2}}
C = posHull M
Cd = dualCone C
hB = transpose matrix apply(hilbertBasis Cd, a -> entries a_0)
I  =  toricGroebner(hB, QQ[vars(0..#hB-1)])
```