Summary

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1 Dimension of some cohomology groups

Group	0	1	2	3	4	5	Euler-characteristic
$H^i(X, \Omega_{\mathbb{P}^{12}} \otimes \mathscr{O}_X)$	0	1	0	167	0	0	-168
$H^i(Y, I_Y/I_Y^2)$	0	36	0	12	2	0	-46
$H^i(Y, \Omega_Y)$	0	1	12	2	0	0	-9

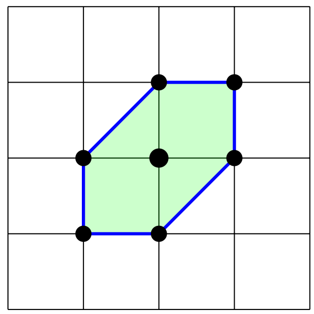
2 The singular locus of Y

By computing in each chart and taking closures, it can be computed that the singular locus of Y is of dimension 1, and consists of the union of projective lines:

3 Deformations of dP_6

We will need some results about deformations of the cone over dP_6.

Recall that dP_6 is the toric variety whose associated polytope is the hexagon:



This induces an embedding into $\mathbb{P}^6$. Let $\mathbb{P}^6$ have coordinates $y_0, x_1..x_6$ (corresponding to the center and the vertices, respectively). Then the ideal of dP_6 inside $\mathbb{P}^6$ is given by the 2×2-minors of the matrix

$$\begin{vmatrix} x_1 & y_0 & x_6 \\ x_2 & x_3 & y_0 \\ y_0 & x_4 & x_5 \end{vmatrix} \leq 1. \tag{1}$$

The $\mathbb{Z}_6$-symmetry is visible by permuting columns and rows. Since dP_6 is smooth, the only singularity of its affine cone, $C(dP_6)$, is the origin. One can compute that $T^1(C(dP_6)) = 3$, and that the versal base space splits into two components. Up to a change of variable, the two deformations of the affine cone is given by

$$\begin{vmatrix} x_1 & y_0 + t_3 & x_6 \\ x_2 & x_3 & y_0 + t_3 \\ y_0 + t_3 & x_4 & x_5 \end{vmatrix} \leq 1.$$

and

$$\begin{vmatrix} x_1 & y_0 - t_2 + t_1 & x_6 \\ x_2 & x_3 & y_0 + t_1 + t_2 \\ y_0 - t_2 - t_1 & x_4 & x_5 \end{vmatrix} \leq 1.$$

Note that by the form of the equations (1), there is a factorization.

$$\begin{tikzcd} dP_6 \arrow[r, hook] \arrow[dr, hook] & \mathbb{P}^2 \times \mathbb{P}^2 \arrow[d, hook] \\ & \mathbb{P}^8 \end{tikzcd}$$

The two deformations behave differently with resepct to this factorization. The first deformation is induced by a deformation of $C(\mathbb{P}^2 \times \mathbb{P}^2)$, leaving the ideal fixed. The other deformation is obtained by also deforming the ideal.