Embedding of X

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1 Embedding in $\left(\mathbb{P}^2\right)^4$

Let $H = \mathbb{P}^{11}$ be spanned by twelve generic block matrices of the form

$$\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}, \tag{1}$$

where A, B er 3×3-matrices. This is a linear subspace in $\mathbb{P}^{17} = \mathbb{P}(E \otimes E \oplus E \otimes E)$, where E is a 3-dimensional vector space.

Let X be as before [[add explanation]]

We have a map $\pi_1 : X \to \mathbb{P}^2 \times \mathbb{P}^2$ defined by sending a block matrix to its first block.

Proposition 1.1. *The map $\pi_1 : X \to \mathbb{P}^2 \times \mathbb{P}^2$ is a morphism.*

Proof. X can be desribed as $H \cap M$, where M is the join of two disjoint copies of $\mathbb{P}^2 \times \mathbb{P}^2$. The singular locus is the set of block matrices where A or B is zero in (1). This is a set of dimension 4. Intersecting with a general $\mathbb{P}^{11}$ (which is of codimension 6) kills off the singular locus, and also the indeterminacy locus of the map $(A, B) \mapsto A$, so that we indeed get a morphism. $\square$

By eliminating variables, the matrices A, B can be written as

$$A = \begin{pmatrix} l_1 & x_1 & x_2 \\ x_3 & l_2 & x_5 \\ x_6 & x_7 & l_3 \end{pmatrix} \text{ and } B = \begin{pmatrix} l_4 & x_{10} & x_{11} \\ x_{12} & l_5 & x_{14} \\ x_{15} & x_{16} & l_6 \end{pmatrix},$$

where the l_i are general linear forms in the x_i appearing in the matrices. Thus the fiber $\pi^{-1}(A)$ is