Summary

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1 Dimension of some cohomology groups

Group	0	1	2	3	4	5	Euler-characteristic
$H^i(X, \Omega_{\mathbb{P}^{12}} \otimes \mathscr{O}_X)$	0	1	0	167	0	0	-168
$H^i(Y, I_Y/I_Y^2)$	0	36	0	12	2	0	-46
$H^i(Y, \Omega_Y)$	0	1	12	2	0	0	-9

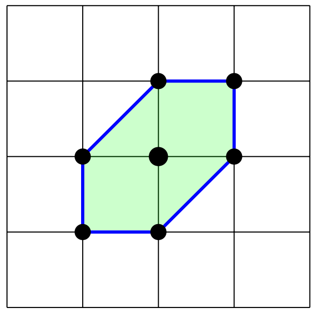
2 The singular locus of Y

By computing in each chart and taking closures, it can be computed that the singular locus of Y is of dimension 1, and consists of the union of projective lines:

3 Deformations of dP_6

We will need some results about deformations of the cone over dP_6.

Recall that dP_6 is the toric variety whose associated polytope is the hexagon:



This induces an embedding into $\mathbb{P}^6$. Let $\mathbb{P}^6$ have coordinates $y_0, x_1..x_6$ (corresponding to the center and the vertices, respectively). Then the ideal of dP_6 inside $\mathbb{P}^6$ is given by the 2×2-minors of the matrix

$$\begin{vmatrix} x_1 & y_0 & x_6 \\ x_2 & x_3 & y_0 \\ y_0 & x_4 & x_5 \end{vmatrix} \leq 1. \tag{1}$$

The $\mathbb{Z}_6$-symmetry is visible by permuting columns and rows. Since dP_6 is smooth, the only singularity of its affine cone, $C(dP_6)$, is the origin. One can compute that $T^1(C(dP_6)) = 3$, and that the versal base space splits into two components: a line and a plane intersecting transversely.

Note that this representation of the ideal gives us an embedding of dP_6 into $\mathbb{P}^2 \times \mathbb{P}^2$ as a section of $\mathscr{O}_{\mathbb{P}^2 \times \mathbb{P}^2}(1,1) \oplus \mathscr{O}_{\mathbb{P}^2 \times \mathbb{P}^2}(1,1)$ (namely as the zeros of $t_{12} - t_{23} = t_{23} - t_{31}$ (where t_{ij} are the natural coordinates on the product).

3.1 The first smoothing of the affine cone

We attempt to give explicit descriptions of the two affine smoothing components of $C(dP_6)$.

One of the components is given by:

$$\begin{vmatrix} x_1 & y_0 & x_6 \\ x_2 & x_3 & y_0 - t_1 \\ y_0 - t_2 & x_4 & x_5 \end{vmatrix} \leq 1.$$

That is, as the 2×2-minors of the above matrix. This time we see that the affine cone $C(dP_6)$ embeds naturally in the affine cone over $C(\mathbb{P}^2 \times \mathbb{P}^2)$, again as the intersection of two hyperplanes, but with some coefficients added.

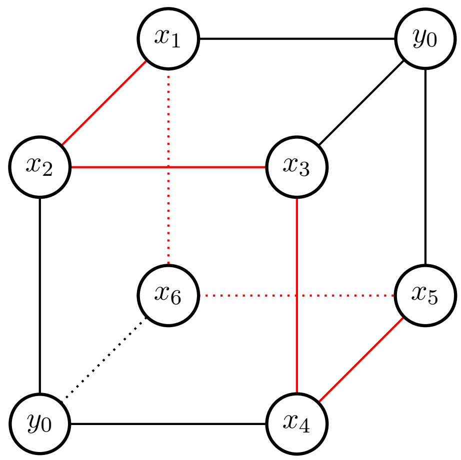


Figure 1: Equations of $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$.

It can be computed that the locus of points in $\mathbb{A}^2$ with singular fibers have ideal generated by $st(s+t) = s^2t + t^2s$, namely the union of the axes and a line.

3.2 The other smoothing of the affine cone

The other smoothing is derived from another way of writing the equations of dP_6. See Figure 1. One obtains the equations for this "$2 \times 2 \times 2$-tensor" by taking 2×2-minors along the faces and along long diagonals.

It is clear that the one-dimensional component is a smoothing of $C(dP_6)$, since it can be obtained as a generic hyperplane in $C(\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1)$.

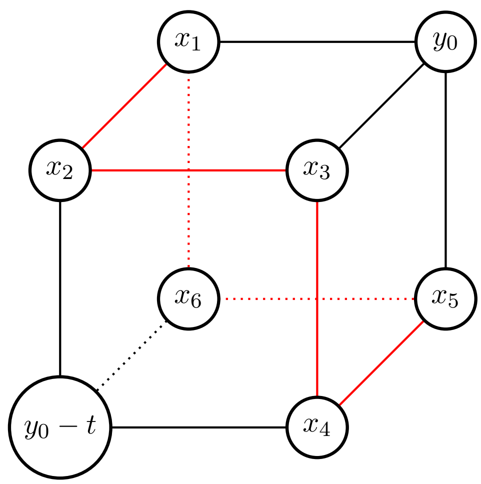


Figure 2: Deforming $C(dP_6)$.