Summary

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1 Dimension of some cohomology groups

Group	0	1	2	3	4	5	Euler-characteristic
$H^i(X, \Omega_{\mathbb{P}^{12}} \otimes \mathscr{O}_X)$	0	1	0	167	0	0	-168
$H^i(Y, I_Y/I_Y^2)$	0	36	0	12	2	0	-46
$H^i(Y, \Omega_Y)$	0	1	12	2	0	0	-9

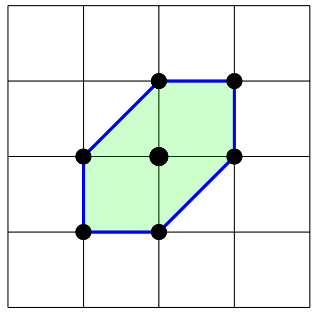
2 The singular locus of Y

By computing in each chart and taking closures, it can be computed that the singular locus of Y is of dimension 1, and consists of the union of projective lines:

3 Deformations of dP_6

We will need some results about deformations of the cone over dP_6.

Recall that dP_6 is the toric variety whose associated polytope is the hexagon:



This induces an embedding into $\mathbb{P}^6$. Let $\mathbb{P}^6$ have coordinates $y_0, x_1..x_6$ (corresponding to the center and the vertices, respectively). Then the ideal of dP_6 inside $\mathbb{P}^6$ is given by the 2×2-minors of the matrix

$$\begin{vmatrix} x_1 & y_0 & x_6 \\ x_2 & x_3 & y_0 \\ y_0 & x_4 & x_5 \end{vmatrix} \leq 1. \tag{1}$$

The $\mathbb{Z}_6$-symmetry is visible by permuting columns and rows. Since dP_6 is smooth, the only singularity of its affine cone, $C(dP_6)$, is the origin. One can compute that $T^1(C(dP_6)) = 3$, and that the versal base space splits into two components. Then one of the components is given by:

$$\begin{vmatrix} x_1 & y_0 & x_6 \\ x_2 & x_3 & y_0 - t_1 \\ y_0 - t_2 & x_4 & x_5 \end{vmatrix} \leq 1.$$

There is another way to write the equations. See Figure 1. One obtains the equations for this "$2\times 2\times 2$-tensor" by taking 2×2-minors along the faces and along long diagonals.

The other component of the deformation space of $C(dP_6)$ is given by Figure 2.

It is clear that the one-dimensional component is a smoothing of $C(dP_6)$, since it can be obtained as a generic hyperplane in $C(\mathbb{P}^1\times\mathbb{P}^1\times\mathbb{P}^1)$.

Computing the discrimant locus of the two-dimensional family, we find that as long as (t_1, t_2) lies outside lines $t_1 = t_2$, $t_1 = 0$ and $t_2 = 0$, the deformation is smooth.

Remark. *The union of these lines constitute the 1-dimensional rays of the fan of dP_6. Can this be a coincidence?*

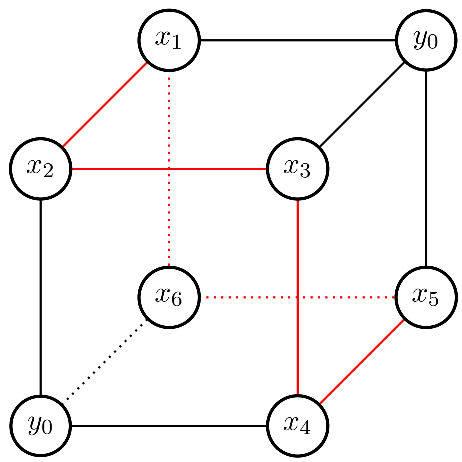


Figure 1: Equations of $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$.

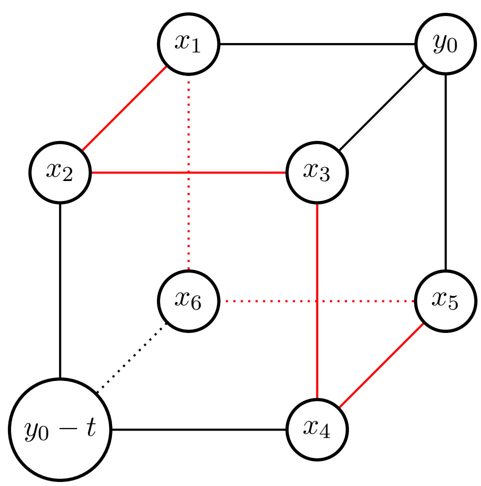


Figure 2: Deforming $C(dP_6)$.