All matematikk

Fredrik Meyer

24. september 2014

Kapittel 1

Sets

1.1 Axioms

Kapittel 2

Relations

Definition 2.0.1. A *binary relation* is a map $S \times S \rightarrow$ ■

Kapittel 3

Groups

Definition 3.0.2. A *group* is a triple (G, μ, ι) where G is a set and μ is a binary operation $G \times G \to G$ and ι is a function $G \to G$, satisfying the following axioms:

- Associativity: The following diagram commutes:

$$\begin{tikzcd} G \times G \times G \arrow[r, "\mathrm{id}\times\mu"] \arrow[d, "\mu\times\mathrm{id}"'] & G \times G \arrow[d, "\mu"] \\ G \times G \arrow[r, "\mu"] & G \end{tikzcd}$$

- Identity element. There exist an element $e \in G$ such that the following two diagrams commute:

$$\begin{tikzcd} \{e\} \times G \arrow[r, "e\times\mathrm{id}"] \arrow[dr, "\pi_2"'] & G \times G \arrow[d, "\mu"] \\ & G \end{tikzcd}$$

  and

$$\begin{tikzcd} G \times \{e\} \arrow[r, "\mathrm{id}\times e"] \arrow[dr, "\pi_2"'] & G \times G \arrow[d, "\mu"] \\ & G \end{tikzcd}$$

  where the top maps are the natural inclusions.

- Inverse element. The following diagram is commutative:

$$\begin{tikzcd} G \arrow[r, "\mathrm{id}\times\iota"] \arrow[dr] & G \times G \arrow[r, "\mu"] & G \\ & \{e\} \arrow[ur, hook] & \end{tikzcd}$$

  The same should also hold with $\mathrm{id} \times \iota$ replaces with $\iota \times \mathrm{id}$.

■

We will never write a group G as a triple, but only refer to the group (G, μ, ι) as just G, the maps being implicit. We will write $\iota(g)$ as g^{-1}, and $\mu(g, h)$ as gh.

Lemma 3.0.3. *The identity element $e \in G$ is unique.*

Bevis. Suppose e' is another identity element. Then $ee' = e'$ by the first diagram. But also $ee' = e$ by the second diagram. Then $e = e'$. Alternatively, the following is a commutative diagram:

$$\begin{tikzcd} & \{e\} \times \{e'\} \arrow[dl, hook'] \arrow[d, "\Delta"] \arrow[dr, hook] & \\ \{e\} \times G \arrow[r, "e\times\mathrm{id}"] \arrow[dr, "\pi_2"'] & G \times G \arrow[d, "\mu"] & \{e'\} \times G \arrow[l, "e'\times\mathrm{id}"'] \arrow[dl, "\pi_2"] \\ & G & \end{tikzcd}$$

□

Kapittel 4

Rings

Kapittel 5

Fields

Definition 5.0.4. A *field* is a commutative ring k with only one ideal. ■

5.1 Ordered fields

Kapittel 6

The real numbers