

(1)

$$\text{Eis } \text{Br}(F) = 0$$

\nwarrow alg. Luther kopp.

$$\text{Br}(F_Q) = 0$$

$$\text{Br}(R) = \frac{1}{2} \{ H \}$$

\nwarrow an \mathbb{Q}/\mathbb{Z}

$$\text{Br} \left(\begin{array}{l} \text{komplett kopp} \\ \text{mhp dvr myndlig} \\ \text{nrkopp} \end{array} \right) \cong \mathbb{Q}/\mathbb{Z}$$

$$0 \rightarrow \text{Br}(F) \rightarrow \bigoplus_v \text{Br}(F_v) \rightarrow \mathbb{Q}/\mathbb{Z} \rightarrow 0$$

Alber - Brauer-Hasse - Noether - theorem

(Max Noether)

$$\text{Witt - ring til n kopp F. } W(F) = \frac{GW(F)}{\langle N \rangle}$$

Lemma $W(F)$ giv av $\langle a \rangle$ for $a \in F^\times$.

hyperboliske planet

relasjone $\cdot \langle a \rangle \langle b \rangle = \langle ab \rangle$

$$\cdot \langle a \rangle + \langle b \rangle = \langle a+b \rangle + \langle (a+b)ab \rangle$$

$$\cdot \langle a \rangle^2 = 1$$

$$\cdot \langle a \rangle + \langle -a \rangle = 0$$

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Gleason finner

$$\text{SBil } (\mathbb{F}) \rightarrow \text{P}(\mathbb{F})$$

\rightsquigarrow \dim

$$\varepsilon: \text{GW}(\mathbb{F}) \rightarrow K_0 \mathbb{F} \cong \mathbb{Z}$$

$$\overset{1}{I} = \ker(\varepsilon) \leftarrow \text{fundamental ideal}$$

Hør indsat av

$$w(\mathbb{F}) \xrightarrow{\varepsilon_{\mathbb{Z}_2}} \mathbb{Z}_2 \quad (\text{char } h \neq 2)$$

Hør $I = \ker(\varepsilon_{\mathbb{Z}_2}) \cong \overset{n}{\underset{q}{\cap}}$

(sum av alle grupper)

Filtrasjon $\dots \subset I^{n+1} \subset I^n \subset \dots \subset I \subset w(\mathbb{F})$

\Rightarrow grønne Witt-ringer $\text{Gr}_I(w(\mathbb{F})) = \bigoplus_{n \geq 0} I/I^{n+1}$

 \curvearrowright

A

I sent av $\langle a \rangle^{-1}$ for $a \in F^\times$. Etterst etc. $\overset{x}{\curvearrowright}$ $\overset{1}{I}$ er på formen

$$q_1 - q_2 \quad q_1 = \langle a_1, \dots, a_n \rangle$$

$$q_2 = \langle b_1, \dots, b_n \rangle.$$

$$\Rightarrow x = \sum_{i=1}^n \langle a_i \rangle - \sum_{i=1}^n \langle b_i \rangle = (\sum \langle a_i \rangle - 1) - (\sum \langle b_i \rangle - 1)$$

 \blacksquare

I giv av $\langle 1, -\alpha \rangle$ för $\alpha \in F^X$: (3)

$$\begin{aligned} \langle 1, -\alpha \rangle &= \langle 1 \rangle + \langle -\alpha \rangle = \langle 1 \rangle + \langle \alpha \rangle \\ &\quad \text{P} \\ &\quad : w(F) \quad + \langle \alpha, -\alpha \rangle \\ &= \langle 1 \rangle - \langle \alpha \rangle \end{aligned}$$

ah y/ sätter I. III

Orsak kallas Pfister-former ($\langle 1, -\alpha \rangle$ för $\alpha \in F^X$)

$\ll \alpha \gg$.

$$\begin{array}{ccc} \text{Bräggs-} & & \\ \text{datorrör} & \curvearrowright & \\ \text{GW}(F) & \xrightarrow{\det} & F^X / (F^X)^2 \\ \downarrow & ? & \nearrow \text{grönmark} \\ W(F) & & \end{array} \quad \langle H \rangle = \langle 1, -1 \rangle$$

$$\det(q \oplus q') = \varepsilon \det q \cdot \det q' \quad \text{if } \varepsilon = \pm 1 \text{ where av} \\ \dim q, q' \bmod 4.$$

- {
- o Ifs $\dim q \equiv \dim q' \equiv 1 \pmod 4 \Rightarrow \varepsilon = 1$
- o Ifs $\dim q \equiv \dim q' \equiv 0 \pmod 2 \Rightarrow \varepsilon = 1$

För en gruppering α där $I \xrightarrow{\alpha} \overline{F}/(F^\times)^2$. Er suprativ (9)

Med $\det(\langle a, -1 \rangle) = \bar{a}$.

Thm 5.6.4 (Phiser)

~~det indusert~~

$$I/\mathbb{Z} \xrightarrow{\det} \overline{F}/(F^\times)^2$$

Bru3

I generer av $\langle a \rangle = \langle 1-a \rangle$, $a \in F^\times$.

$\langle 1, -a \rangle \otimes \langle 1, -b \rangle = \langle 1, -a, -b, ab \rangle$ har föregående ab ,
som $\equiv 1$ mod $(F^\times)^2$. Så $I^2 \subseteq$ är det.

För faktorisering $\det: I/\mathbb{Z} \xrightarrow{\alpha} \overline{F}/(F^\times)^2$.

β defint% av $\beta(a) = \langle a \rangle$.

$$\begin{aligned} \beta(a) + \beta(b) - \beta(ab) &= \langle 1, -a \rangle + \langle 1, -b \rangle - \langle 1, -ab \rangle \\ &= \langle 1, -a, 1, -b, -1, ab \rangle \end{aligned}$$

$$\stackrel{\langle 1 \rangle}{=} \langle 1, -a, -b, ab \rangle = \langle a \rangle \otimes \langle b \rangle \in I.$$

$\alpha \circ \beta = \text{id}_{\overline{F}/(F^\times)^2}$. os $(\beta \circ \alpha)(\langle a \rangle) = \langle a \rangle$.

$$\therefore \beta \circ \alpha = \text{id}_{I/\mathbb{Z}}.$$

WT

Korr $W(F) \cong \mathbb{Z}/2$ \Rightarrow -1 kadrat i F . (5)

$$\text{Meth} \quad \frac{\mathbb{I}}{\mathbb{I}^2} \xrightarrow{\text{clifford-}} \mathbb{B}_r (\#)$$

injektiv

\downarrow (berigt: 82)
2-torsjon (Merhavjan)

EWD: Lemme: For en kropp F , er følgende ekvivalente.

- ① F kadratisk null (ingen grad 2-wurdele)
- ② $F^\times = (F^\times)^2$
- ③ $W(F) \cong \mathbb{Z}/2$ ring-iso
- ④ $I = 0$
- ⑤ $I/I^2 = 0$.

$W(\bar{F}) = \mathbb{Z}/2$. Envis at \bar{F} er et kadrat
 \Leftrightarrow kan skrives i-dim form: $\langle 1 \rangle$ ($\langle a \rangle \langle ab^{-1} \rangle$)

Envis 2-dim form δ desentral.

$\langle a, b \rangle \neq \langle ab^{-1} \rangle$ for $a \in F^\times$. Se øvelse
 til 2-dimensionelle former er 0 og $\langle 1 \rangle$.

(6)

$$\text{Bsp } \mathbb{R}^*/\mathbb{R}^{\pm 2} \cong \mathbb{Z}/2 = \{ \pm 1 \}$$

\Rightarrow what form has diagonalizations of $\langle 1, \dots, 1, -1, \dots, -1 \rangle$

r s

degrees on both r,s are positive. Number of $s=0$ ~~is r~~ $\Leftrightarrow r>0$

 $r=0$ or $s>0$.

$$\Rightarrow W(\mathbb{R}) \longrightarrow \mathbb{Z}$$

Signaturbildung

$$\text{sgn}: W(\mathbb{R}) \longrightarrow \mathbb{Z}$$

$$\langle 1, \dots, 1, \underbrace{-1, \dots, -1}_{s} \rangle \mapsto r-s.$$

$$\mathbb{Z}/2 = \mathbb{Z}_2. \text{ Mod } 2 \text{ (Steinberg)}$$

Bsp da $F = \mathbb{F}_q$ da $\mathbb{Z}/2$

$$\text{gib } \mathbb{Z}^2 = 0.$$

$$\mathbb{F}_q^{*} / (\mathbb{F}_q^{*})^2$$

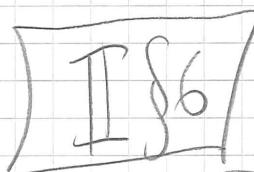
$$0 \rightarrow \mathbb{Z}/2 \rightarrow W(\mathbb{F}_q) \xrightarrow{\text{sgn}} \mathbb{Z}/2 \rightarrow 0$$

$$\Rightarrow W(\mathbb{F}_q) = \begin{cases} \mathbb{Z}/4 & q \equiv 3 \pmod{4} \\ \mathbb{Z} \oplus \mathbb{Z} & q \text{ or kladbar} \\ & q \equiv 1 \pmod{4} \end{cases}$$

Fever (Mihal)

⑥

$$W(\mathbb{Q}) = \mathbb{Z} \oplus \bigoplus_{p \text{ primtal}} W(\mathbb{F}_p)$$



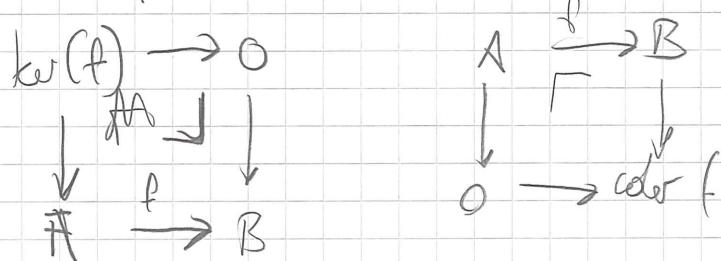
kan av en delst haengen

"Niels Henrik Abel
er også med haengen
oppfattet er"

① Additiv haengen

② Mestha haengen: additiv + hypot / hokjøre

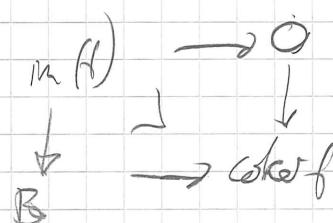
① Enhver mesti har hypot og hokjøre
def



② Enhver monist maf er en hypot, og enhver opimaf
er en hokjøre.

$\text{im}(f) \neq$

$$\text{im}(f: A \rightarrow B) = \ker(B \rightarrow \text{ker } f).$$



(8)

~~Def~~ K_0 of

↙ $\text{Hom}_R(\text{longer})$

$$\langle [A] \mid A \in \mathcal{O}(cd) \text{ my } \rangle$$

$$0 \xrightarrow{\text{on}} \hookrightarrow A \xrightarrow{\sim} B \xrightarrow{\sim} 0$$

$$[A] = [B] + [\mathbb{K}]$$

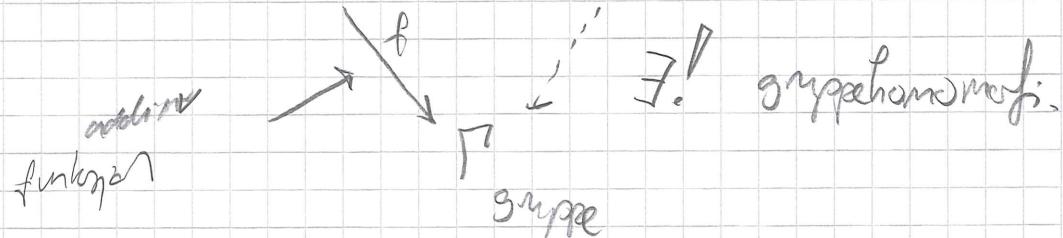
$$\text{Sonder } [A \oplus A'] = [A] + [A'].$$

Für ab.

$$K_0^{\oplus}(A) \rightarrow K_0(A)$$

Universal epimorp (6,1.2).

$$\mathcal{O}(cd) \longrightarrow K_0(A)$$



$$F: cd \rightarrow B \quad \text{additiv} \quad \text{faktor linz}$$

$$\text{Hom}_{\mathcal{O}}(A, B) \longrightarrow \text{Hom}_B(F(A), F(B)) \text{ e en}$$

gruppenhomomorf für alle $A, B \in \mathcal{O}(cd)$.

ebett an den Bogen k. e. s.

$$\begin{array}{c} \text{induziert} \\ \not\cong \\ K_0(A) \longrightarrow K_0(B) \\ [A] \longrightarrow [F(A)] \end{array}$$

(9)

$\text{et} \subseteq \mathbb{B}$ fall unterhängt.

hins i er abhängt.

Def $G_0 R$: R ring, noethersch (höhere-noethersch)

$M(R)$ Kategorien an end. generale R -moduln
 $\hookrightarrow \subseteq \text{mod-}R$
 absth kategor.

$$G_0 R \stackrel{\circ}{=} K_0 M(R).$$

Ring-abbildung $R \rightarrow S$ (Noethersch), hins S ad-gn R -modul,

$$\rightsquigarrow f_* : M(S) \rightarrow M(R)$$

$$\rightsquigarrow f_* : G_0 S \Rightarrow G_0 R$$

an S er flach R -modul, sa

$$f^* : M(R) \rightarrow M(S)$$

$$\overset{\circ}{M} \mapsto M \underset{R}{\otimes} S. \text{ abhängt } \not\propto$$

$$\Leftarrow \overset{\circ}{f} : G_0 R \rightarrow G_0 S.$$

hins S hat endlich resolution an flache R -moduln, sichtbar etc

$$f^*([M]) = \sum_{i \geq 0} [\text{Tor}_i^R(M, S)].$$

$$F \text{ kropp. } \Rightarrow G_F = R_F = \mathbb{Z}$$

On \mathbb{R} integrable wrt μ \rightarrow $L^1(\mathbb{R})$

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$$\underline{\text{Ex}}) \quad G_0(\mathbb{Z}) \cong \mathbb{Z}\{[Z]\}.$$

$$0 \rightarrow \mathbb{Z} \xrightarrow{\cdot n} \mathbb{Z} \rightarrow \mathbb{Z}_n \rightarrow 0$$

$$\Rightarrow [z_h] = 0 \in G_0(\mathbb{Z})$$

So not a set of $\mathbb{Z}^n \neq \mathbb{Z}^m$ for $n \neq m$.

↳ Bush at \mathbb{Q} -ell for \mathbb{Z} -model. as ab.

Samme argument for R et handikablaande $\Rightarrow G_R \cong \mathbb{Z}$

Es p-grupp. $N_p = \text{absl. p-grupp}$

$$k_0(\Lambda^6_p) \cong \mathbb{Z}[\sqrt[p]{\zeta_p}].$$

Kompositionsräume $H_1 \triangleleft \dots \triangleleft H_m = M$ abelsche Gruppen.

String insertions are stable p -groups. $l(\alpha) = n$

$$\text{Jordan-Hölder} \Rightarrow l: K_0(\mathrm{Ab}_p) \rightarrow \mathbb{Z} / p \quad [\mathbb{Z}/p] \mapsto 1$$

(11)

Påsid: \mathbb{Z}/p er generator. (induktion på lengde)

$$\text{Eks 6.3.2} \quad K_0(\mathrm{Ab}_{\text{endek}}) = \bigoplus_{p \text{ prim}} K_0(\mathrm{Ab}_p)$$

$$\text{Eks 6.2.4} \quad M(\mathbb{Z}/p^n) \xrightarrow{\text{echter}} \mathrm{Ab}_p \quad \text{is } \mathbb{Z} \text{ på } K_0.$$

$$G_0(\mathbb{Z}/p) = \mathbb{Z} \{ [\mathbb{Z}/p] \}.$$

$$K_0(\mathbb{Z}/p^n) = \mathbb{Z} \{ [\mathbb{Z}/p^n] \}$$

center-kompleks med $[\mathbb{Z}/p^n] \mapsto n[\mathbb{Z}/p]$
(spesielt surjektiv)



For X noethersk spaen. Kategorier av finne Cl_X -moduler.

$M(X) =$ kategorier av holone Cl_X -moduler. ($\mathrm{Coh}(X)$).
Er abelisk.

$$G_0 X = K_0 X \stackrel{\triangle}{=} K_0 M(X)$$

$$X \xrightarrow{f} Y \rightsquigarrow f^*: M(Y) \rightarrow M(X)$$

(2)

$$\text{def } \forall F \mapsto f^*F = F \otimes Q_X.$$

Hvis f flat, så er $f^*(-)$ mots. sif för ab. på G_b :

$$f^*: G_0 Y \rightarrow G_0 X.$$

"kontravariant funktor för fler av"

Hvis f endlig, så bewar f^* kohesive ~~co~~moduler.

$$\Leftrightarrow f^*: H(X) \rightarrow H(Y) \text{ är eksakt}$$

$$\text{För også } f^*G(X) \rightarrow f^*G(Y).$$



Hvis f proper, så er direktebilder f_*F kohesivt hvis F kohesivt og
bildmålet $R^i f_* F$. (höge direktaffäder)

Lemma f , proper ger robust ab. $f_* G_0 X \rightarrow G_0 Y$

$$[Z] \mapsto \sum (-1)^i [R^i f_* F].$$

Gör \oplus faktoriell för proper abbildningar.

- detta beräknas

Raissege-teoremet 6.3

(B)

da $B \subset A$ mhtn av at alle hengere. hvor

① $B \subset A$ er det mhtn av A

② Punkt under undergr. og kriterier.

③ evt. objekt A i A har en endelig filtrasjon $A = A_0 \supseteq A_1 \supseteq \dots \supseteq A_n = 0$

hvor alle A_i / A_{i+1} er my i B .

Da er $B \subset A$ dvs og mhtn iso $K_0 B \xrightarrow{\cong} K_0 A$.

Beweis $\ell : K_0 B \rightarrow K_0 A$

surjektiv Fra filtrasjonen

$$[A] = \sum [A_i / A_{i+1}] \in K_0 A$$

influenser: litt mer jobb se selv.

for 6.3 $I \subset R$ noethersh. Da er $G_0 R \cong G_0 (R/I)$.

Beg M $\not\cong$ Raissege. Mf $M(R)$.

$$M \supset M_I \supset M_{I^2} \supset \dots \supset 0$$

my M_I / M_{I^2} adm R_I -modul.

(M)

Ker \times mørkhed. sammensætning af reduktion

$$X_{\text{red}} \cdot \text{ker } G_0 X \approx G_0 X_{\text{red}}.$$

Indsteg 6.3.3 $I \subset R$, ideale $M_I(R) \subset M(R)$.

Aberth underhænger: består af M my fibrasjoner

$$M \supseteq MI \supseteq \dots \supseteq M I^n = 0$$

$$\text{Ansæt} \not\Rightarrow \text{ker } M_I(R) = K_0(M(I)) = G_0(R/I).$$

Samme for sløjfespor? $i: \mathbb{Z} \hookrightarrow X$

$$M_Z(X) \subseteq M(X) \quad \text{abert underh. my}$$

grader my større på \mathbb{Z}

$$\text{Lå } Q_X/J \cong Q_Z, J \text{ idealringet i } C_X.$$

x på $M(\mathbb{Z})$ som mørkh. av alle Q_X -moduler M ; $M_J(x)$

som opfylder $ME=0$

Alle M i $M_Z(J)$ har en endelig fiktasjon.

$$M \supseteq MI \supseteq M I^2 \supseteq \dots \supseteq M I^n = 0 \quad \text{my konstru.}$$

$$n(\mathbb{Z}) \quad K_0 M_Z(J) = G_Z.$$

Abhängigkeitsoperator 6.4 (Alex Heller)

(8)

A lion which hangs off B in Sur-überhangen
of. Da er folende Schaus erhat:
(S 119)

$$K_0 B \rightarrow K_0 A \rightarrow K_0(\mathcal{A}/B)$$

Oppositely: Sins aufwärts $\mathbb{H}_2 S^*$. B Sur-überhängt:

Abh. überhangen. fully indep. , hängt es
abhangt.

$$\text{If } S \quad 0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0, \text{ sa } B + C \rightleftharpoons A, C \in \mathcal{B}.$$

Reflexiv \mathcal{A}/B . En mat. $f: A \rightarrow B$ iof er en B -iso om ~~ter~~ ~~oder~~ ~~der~~ \mathcal{B}

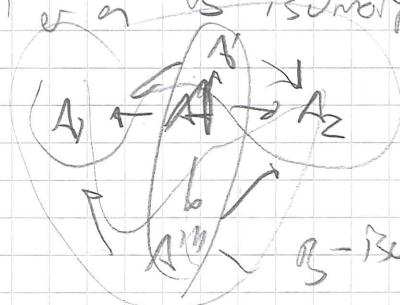
($\text{Coker } f \in \mathcal{O}(B)$).

$$\mathcal{O}(A/B) \stackrel{\circ}{=} \mathcal{O}A$$

$\mathcal{O}(A/B)$ = ekivalensklasser av diagrammer i A

$$A_1 \xrightarrow{f} A_2 \quad A_1 \xleftarrow{f} A'_1 \xrightarrow{g} A_2$$

for f er en B -isomorf. os to slite diagrammer
er isomofe om



B -isomorf.

$$\begin{array}{|c|} \hline \text{Sammlung} \\ (A_1 \xleftarrow{f} A'_1 \xrightarrow{g} A_2) \\ \hline \end{array} \quad \begin{array}{|c|} \hline (A_1 \xleftarrow{f} A_2) \\ (\text{se } b) \\ \hline \end{array}$$