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①

• Goes back to Matsusaka (1952).

• Milne

Let A abelian variety / k

Then Assume k infinite.

\exists curve C/k , nonsingular, connected. Then \exists

$\gamma: J_C \rightarrow A$ def. over k .

Problem

We have $g(C) = \dim J(C) \gg \dim A$
 $\frac{g!}{g!(g-1)}$ "very, very big"

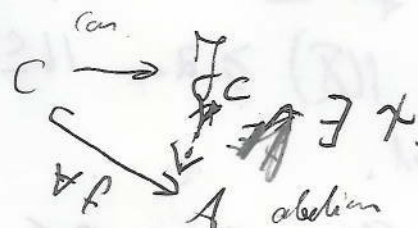
① Assume $k = \bar{k}$.

We know $A \subseteq \mathbb{P}^n$

Let $C = A \cap H_1 \cap \dots \cap H_{g-1}$. (H_i generic hyperplanes)

By Bézout $\Rightarrow C$ nonsing, connected.

Have canonical map



Have γ ① is γ surjective?

Let $A_0 = \gamma(J_C) = \langle C \rangle \subseteq A$. If $A_0 \neq A$ then $\exists A_1$
 $w/ A_1 \cap A_0 = \mu \quad w/ \# \mu < \infty$. s.t. $A_1 + A_0 = A$.

Look at $(a,b) \mapsto a+b$
 $A_1 \times A_0 \xrightarrow{\pi} A \rightarrow 0$ finite map
 $0 \rightarrow \mu$

(2)

Intersect in $\pi^{-1}(C)$.

let $a \in A_0$. $C_a = C - a = \{x-a \mid x \in C\}$

Now if $a \in A_0 \cap A_1$.

Then $C_a \times \{a\} \subseteq A_0 \times A_1$

$$\begin{array}{ccc} (x,a) & & \\ \downarrow + & \downarrow & \\ x & & C \end{array}$$

Hence $\pi^{-1}(C) = \bigsqcup_{a \in A_0 \cap A_1} C_a \times \{a\} = \pi^{-1}(C)$

so if $|A_0 \cap A_1| \geq 2 \Rightarrow \pi^{-1}(C)$ not connected!

Contradicts a connectedness theorem.

(a trick for the case $\#|A_0 \cap A_1| = 1$)

Thm $f: X \rightarrow \mathbb{P}^n$, X normal variety w/
 $\dim f(X) \geq 2$. If $H \subseteq \mathbb{P}^n$ hyperplane $\Rightarrow f^{-1}(H)$
 connected. \mathbb{Z}
 Ex. seq $0 \rightarrow \mathcal{O}_X(-n\mathbb{Z}) \rightarrow \mathcal{O}_X \rightarrow \mathcal{O}_{n\mathbb{Z}} \rightarrow 0$

+ some vanishing. + $\mathcal{O}_{n\mathbb{Z}}$ and \mathbb{Z} some
 topology

Prop If $U \subseteq \mathbb{P}_k^n$ gen dense, then $U(k) \neq \emptyset$. (3)

R Induction on n .

$$\#(P^1 \setminus U) < \infty.$$

$U(k) \neq \emptyset$ since infinite.

$\mathbb{P}^n \setminus U = \text{union of } \downarrow \text{finitely many hypersurfaces.}$

There are only finitely many hypersurfaces $\subseteq \mathbb{P}^n \setminus U$.

$\Rightarrow \exists$ as many H 's def over k .

$\Rightarrow \exists H$ s.t. $U \cap H \neq \emptyset \Rightarrow U(k) \neq \emptyset$ by induction

$$\text{let } \tilde{A} \triangleq B_0 A$$

$$\pi \downarrow A$$

ex. divisor.

If α v.a. on A , \neq

look at $\pi^* \alpha^N(-E)$ very ample.

gives $\tilde{A} \subseteq \mathbb{P}^m$ s.t. $E \subseteq \mathbb{P}^n$

\exists linear since $\mathcal{O}_{\tilde{A}}(-E)|_E \simeq \mathcal{O}_E(1)$

Then \tilde{A} is def / k . Same w/ E

look at $C = \tilde{A} \cap H_1 \cap \dots \cap H_{g-1}$

and $\tilde{A} \cap H_1 \cap \dots \cap H_{g-1} = \{pt\}$

see point on the curve.

\nexists

$\mathcal{O}(k)$

If $\#k < \infty$

Play (O. G., Poonen)

If $\#k < \infty$,

$X \subseteq \mathbb{P}_k^n$ nonsing

Then $\exists S$ hypersurface, def / k

s.t.

$\tilde{X} \cap S$ is non-singular (over \tilde{k})

(something about the proof)

(it involved ζ -functions and sieves)

