①

~~Noen løsninger~~ (some solutions)

Kap 9

⑦ $(145)(78)(257) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 1 & 3 & 5 & 8 & 6 & 2 & 7 \end{pmatrix}$

„read from the right"

⑧ $(1327)(486) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 7 & 2 & 8 & 5 & 4 & 1 & 6 \end{pmatrix}$

⑨ $(12)(478)(21)(72815) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 4 & 3 & 7 & 8 & 6 & 2 & 1 \end{pmatrix}$

⑬ ⓐ $|\overset{\sigma}{\overset{\shortparallel}{(1457)}}| = 4$ = length of the cycle,

because $1 \overset{\sigma}{\mapsto} 4 \overset{\sigma^2}{\mapsto} 5 \overset{\sigma^3}{\mapsto} 7 \overset{\sigma^4}{\mapsto} 1$, etc.

ⓑ The order of a cycle is equal to its length.

ⓒ $|\sigma| = |(45)(237)| = 6$

$|\tau| = |(14)(3578)| = 4$

ⓓ F.ex ⑩ $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 6 & 3 & 7 & 4 & 5 & 1 \end{pmatrix} = (18)(364)(57)$

By computation $\Rightarrow$ order is 6.

e) The order of σ is the least common multiple, lcm, of the orders of the factors in its decomposition into disjoint cycles. (2)

(prove this)

$\boxed{\text{Chapter 10}}$

Exc 10.1 – 5

I do only no. 4.

Find the cosets of $\langle 4 \rangle$ in $\mathbb{Z}_{12}$.

$$\langle 4 \rangle = \{0, 4, 8\}$$

$$\langle 4 \rangle + 1 = \{1, 5, 9\}$$

$$\langle 4 \rangle + 2 = \{2, 6, 10\}$$

$$\langle 4 \rangle + 3 = \{3, 7, 11\}$$

We have exhausted all elements of $\mathbb{Z}_{12}$, so these are all cosets.

9 12

Chapter 13 ③

① yes

② $\phi: \mathbb{R} \to \mathbb{Z}$
$x \mapsto \lfloor x \rfloor$ = greatest integer $\leq x$

NO!

- The kernel consists of all numbers in the interval $[0,1)$ - but this is not a subgroup of $\mathbb{R}$! (in fact, $\mathbb{R}$ have only <u>one</u> proper subgroup, namely zero)

- <u>OR</u> Note that

$$\phi\left(\frac{1}{2}+\frac{1}{2}\right) = \phi(1) = 1$$

while $\phi\left(\frac{1}{2}\right) + \phi\left(\frac{1}{2}\right) = 0 + 0 = 0.$!

③ Yes.

④ $\phi: \mathbb{Z}_6 \to \mathbb{Z}_2$ (yes)
$x \mapsto x \bmod 2$

⑤ $\phi: \mathbb{Z}_9 \to \mathbb{Z}_2$
$x \mapsto x \bmod 2$

Not well defined!
$9 \equiv 18$ in $\mathbb{Z}_9$ but
$\phi(9)=1$ and $\phi(18) = 0$!

⑥ $\phi : (\mathbb{R}, +) \to (\mathbb{R}^*, \cdot)$

$x \mapsto 2^x$

YES!

In fact this is a <u>isomorphism</u>