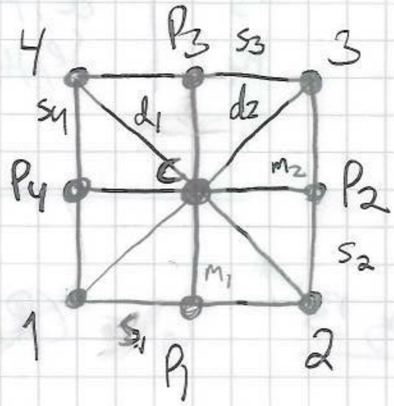
Sect 16

$$X = \{1, 2, 3, 4, s_1, s_2, s_3, s_4, m_1, m_2, d_1, d_2, C, P_1, P_2, P_3, P_4\}$$

D_4 acts on diagonals, vertices, etc ...



(1) Find the fixed sets X_σ for each $\sigma \in D_4$.

σ	X_σ
e	X
ρ_1	$\{C\}$
ρ_2	$\{C, d_1, d_2, m_1, m_2\}$
ρ_3	$\{C\}$
μ_1	$\{C, P_1, P_3, m_1, m_2, s_1, s_3\}$
μ_2	$\{C, P_2, P_4, m_1, m_2, s_2, s_4\}$
δ_1	$\{C, 2, 4, d_1, d_2\}$
δ_2	$\{C, 1, 3, d_1, d_2\}$

Recall $\mu_1 = \{\text{mirror through } m_1\}$

$\mu_2 = \{\text{mirror through } m_2\}$

$\delta_1 = \{\text{mirror through } d_1\}$

$\delta_2 = \{\text{— " — } d_2\}$

(2) Find the isotropy subgroups G_x for each $x \in X$.

$x \in X$	G_x	$x \in X$	G_x	$x \in X$	
1	$\{e, \delta_2\} \simeq \mathbb{Z}_2$	s_1	$\{e, \mu_1\}$	m_1	$\{e, \mu_1, \rho_2, \mu_2\} \simeq \mathbb{Z}_2 \times \mathbb{Z}_2$
2	$\{e, \delta_1\} \simeq \mathbb{Z}_2$	s_2	$\{e, \mu_2\}$ $\simeq \mathbb{Z}_2$	m_2	— " —
3	$\{e, \delta_2\} \simeq \mathbb{Z}_2$	s_3	$\{e, \mu_1\}$	d_1	$\{e, \delta_1, \delta_2, \rho_2\} \simeq \mathbb{Z}_2 \times \mathbb{Z}_2$
4	$\{e, \delta_1\} \simeq \mathbb{Z}_2$	s_4	$\{e, \mu_2\}$	d_2	— " —
				C	D_4

$x \in X$	G_x	
P_1	$\{e, \mu_1\}$	
P_2	$\{e, \mu_2\}$	$\mathbb{Z}_2$
P_3	$\{e, \mu_3\}$	
P_4	$\{e, \mu_2\}$	

(ops)

Opps 13 $\quad G = (\mathbb{R}, +)$

$$G \curvearrowright \mathbb{R}^2$$

$$r \longmapsto \left(P \longmapsto \begin{pmatrix} \cos r & -\sin r \\ \sin r & \cos r \end{pmatrix} P \right)$$

a) Obvious.

b) The orbit of P = circle of radius $|P|$ around 0.

c) $G_P = \{ r \mid r \cdot P = P \}$

$= \{$ rotations that are multiples of $2\pi \}$

$= \{ \dots, -2\pi, 0, 2\pi, 4\pi, \dots \}$

$= 2\pi \mathbb{Z}$

$\simeq \mathbb{Z}$.

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Section 17

1. Find the # of orbits in $\{1,2,3,4,5,6,7,8\}$ under $\langle(1,3,5,6)\rangle \leq S_8$.

Orbit of 1 $= \{1,3,5,6\}$

All other elements are fixed so the other orbits are $\{2\}, \{4\}, \{7\}, \{8\}$.

Hence there are 5 orbits.

$\backsim$

We now use Corollary 17.2 to do the problem:

$$\# \text{orbits} = \frac{1}{4}(8+4+4+4)$$

$$= \frac{20}{4} = \underline{\underline{5}}.$$

2. Same, but now w/ $G = \langle (13), (247) \rangle$.

Note that $G = \{e, (13), (247), (274)\}$.

Then

$$\# \text{orbits} = \frac{1}{4}(8+6+5+5) = \frac{24}{4} = \underline{\underline{6}}$$

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Section 11

11.1 List the elements of $\mathbb{Z}_2 \times \mathbb{Z}_4$. Find the order of each element. Is the group cyclic?

Order 8.

$g \in G$	order
(0,0)	1
(0,1)	4
(0,2)	2
(0,3)	4
(1,0)	2
(1,1)	4
(1,2)	2
(1,3)	4

No, its not cyclic, b.c. there are no elements of order 8.

(2) No, Do it yourself (it is cyclic)

(9) Find all proper non-trivial subgroups of $\mathbb{Z}_2 \times \mathbb{Z}_2$.

$$\begin{tikzcd}
 & \mathbb{Z}_2 \times \mathbb{Z}_2 \arrow[dl, dash] \arrow[dr, dash] & \\
\{0\} \times \mathbb{Z}_2 \arrow[dr, dash] & & \mathbb{Z}_2 \times \{0\} \arrow[dl, dash] \\
 & \{0\} \times \{0\} &
\end{tikzcd}$$

(10) — // ——— $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$

$$\begin{tikzcd}[column sep=small]
 & & \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \arrow[dll, dash] \arrow[d, dash] \arrow[drr, dash] & & & \\
\{0\} \times \mathbb{Z}_2 \times \mathbb{Z}_2 \arrow[d, dash] \arrow[dr, dash] & & \mathbb{Z}_2 \times \{0\} \times \mathbb{Z}_2 \arrow[d, dash] \arrow[dr, dash] & & \mathbb{Z}_2 \times \mathbb{Z}_2 \times \{0\} \arrow[d, dash] \arrow[dr, dash] & \\
\{0\} \times \{0\} \times \mathbb{Z}_2 & \{0\} \times \mathbb{Z}_2 \times \{0\} & \{0\} \times \{0\} \times \mathbb{Z}_2 & \mathbb{Z}_2 \times \{0\} \times \{0\} & \{0\} \times \mathbb{Z}_2 \times \{0\} & \mathbb{Z}_2 \times \{0\} \times \{0\} \\
 & & \{0\} \times \{0\} \times \{0\} \arrow[ull, dash] \arrow[ul, dash] \arrow[u, dash] \arrow[ur, dash] \arrow[urr, dash] \arrow[urrr, dash] & & &
\end{tikzcd}$$

(11) Find all subgroups of order 4 of $\mathbb{Z}_2 \times \mathbb{Z}_4$.

Answer $\{0\} \times \mathbb{Z}_4$ and $\mathbb{Z}_2 \times 2\mathbb{Z}_4$