

Modulære funksjoner og former

I.4 Milne

$\Gamma(1)$

Har sett på undergrupper Γ av $SL_2(\mathbb{Z})$. Virker på $H \subseteq \mathbb{C}$.
 endelig indeks

Kvotienten $\Gamma \backslash H = Y(\Gamma) \subset X(\Gamma)$

kompatiblisering. Lesse til tussen.

Om $\Gamma = \Gamma(1)$, $X(\Gamma(1)) = \mathbb{P}^1_{\mathbb{C}}$.

Gitte $\Lambda \subseteq \mathbb{C} \leadsto \mathbb{C}/\Lambda$ elliptisk kurve. $SL_2(\mathbb{Z})$ virker på mengden av gitte.

(H. stiller spørsmål som folk gjør når en :-)

En meromorf

Def $\forall f: H \rightarrow \mathbb{C}$ er en modular funksjon for Γ dersom den er invariant. i.e. $f(\gamma(z)) = f(z) \quad \forall \gamma \in \Gamma$. og har av endelig orden i kuspene.

(Kussten står litt i definisjonen)

Kusp ~~Punkt~~ p ? (se Milne)

Def En Fuchsian funksjon $f: H \rightarrow \mathbb{C}$ for Γ er en modular form av vekt $2k$ dersom $f\left(\frac{az+b}{cz+d}\right) = (cz+d)^{2k} f(z)$ for $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma$ + holomorf i kuspene.

Vi har funktioner

$$\mathbb{C}(X(\Gamma)) = \mathbb{C}\{f \mid f \text{ modular funktion for } \Gamma\} \quad (2)$$

$$\Gamma_0(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid c \equiv 0 \pmod{N} \right\} \subseteq SL_2(\mathbb{Z})$$

$$\mathbb{C}(X(\Gamma_0(N))) \ni j(z)$$

Kan også se på $j(Nz)$

$$\begin{array}{c} X(\Gamma_0(N)) \\ \mu \downarrow \\ X(\Gamma_0(1)) = \mathbb{P}_\mathbb{C}^1 \end{array}$$

se på $\mathbb{C}(j)[Y]$, da er minimalpolynomiet til $j(Nz)$ over denne givet $\mu = N \cdot \prod_{p|N} (1 + \frac{1}{p})$

$$F(x, y) = x^{p+1} + y^{p+1} - x^p y^p - xy \pmod{p} \quad N=p$$

$j, j(Nz)$

Ta de tilsv. elliptiske kurver:

$$\left\{ \begin{array}{l} \text{orden } N \\ \text{under } 3 \end{array} \right\} \subseteq E_{j(z)} \xrightarrow{N:1} E_{j(Nz)}$$

„moduli problem“

$$\boxed{N=2}$$

$$\begin{aligned} F(x, y) = & x^3 + y^3 - x^2 y^2 + 1488 xy(x+y) \\ & - 162000(x^2 + y^2) + 40773375 xy \\ & + 8748000000(x+y) - 157464 \cdot 10^9 = 0 \end{aligned}$$

(PAUSE)

Modulare form \rightarrow k-fold diff. form

(3)

$$w = f(z) dz \quad f(z) \text{ meromorph}$$

Wie ist diese Γ -invariant?

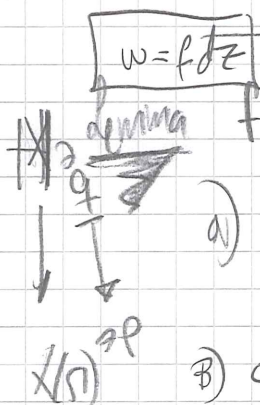
$$\begin{aligned} \gamma^* w &= f(\gamma(z)) \cdot d(\gamma(z)) = f(\gamma(z)) \frac{a(cz+d) - (cz+d)}{(cz+d)^2} dz \\ &= f(\gamma(z)) (cz+d)^{-2} dz \end{aligned}$$

$$= w \quad \text{deshalb} \quad f(\gamma(z)) = (cz+d)^2 f(z)$$

Summe richtig sein $f(\gamma(z)) = (cz+d)^{2k} f(z)$ für
dann an Wert k.

$X(\Gamma)$ hat in der annen genus g_Γ . = nur holomorphe
diff.-form.

$$\dim \{k\text{-fold diff. form}\}^{\text{RR}} = 1 - g + k(2g - 2)$$



f modular form, vgl. 2k, w k-fold

$$a) \text{ord}_q f = \text{ord}(w) + k(z) \quad \text{diff. form}$$

$$b) \text{ord}_p f = \text{ord}_p w + k$$

$$c) \text{ord}_q f = \text{ord}_p w \quad \text{ellipt.}$$

$$k(2g-2) + 1 - g > 0$$

$$k > \frac{g-1}{2g-2} = \frac{g-1}{2(g-1)} = \frac{1}{2}$$

$$\dim M_{2k} = (2k+1)(g-1) + \sum_p k(1 - \frac{1}{e_p})$$

$$n = r(1)$$

$$g=0$$

$$v_{\infty}=1$$

$$p, i$$

(4)

$$\dim M_{2k} = 1 - k + \left\lfloor \frac{k}{2} \right\rfloor + \left\lfloor \frac{2k}{3} \right\rfloor \quad k \geq 1$$

$$= 0, \underset{G_2}{1}, \underset{G_3}{1}, 1, 1, 2, 1, 2$$

Ringen an module f\u00fcr $M_{2k} = \mathbb{C}[G_2, G_3]$

$$y \in \mathbb{R} \frac{G_2^3}{\Delta} \quad \text{funktion}$$

$$j(z) = \frac{1}{q} + \dots$$