How to Check Your Calculus Homework - Part 1

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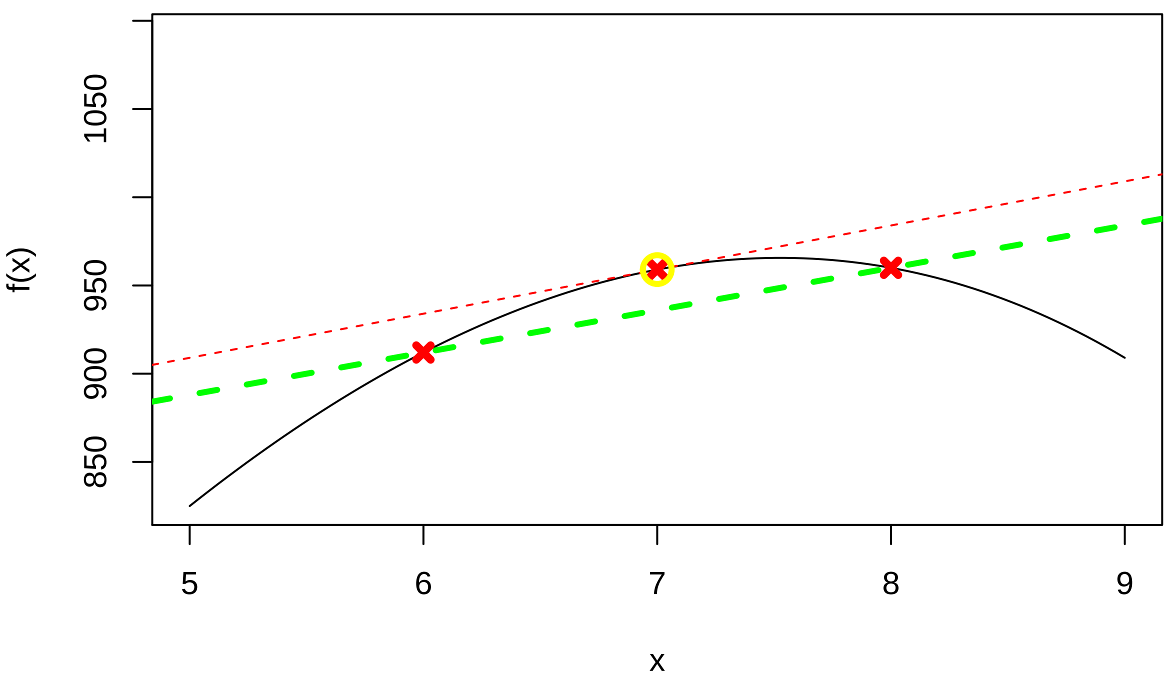
```
print_as_latex <- function(r_equation){
    r_equation <- gsub("\\*", "", r_equation)
    cat(paste("$$",r_equation,"$$"))
}
```

Taking derivatives of functions is generally a matter of applying one or more formulas from a table of derivatives. This can involve complicated algebra, so it is easy to make mistakes. Here we look at a simple way to check your work by graphing it against what the derivative should look like, based on a numerical approximation. If your analytical function for the derivative plots on top of the numerical approximation, that is good evidence that it is correct.

```
d <- function(FUN, X){
  Y <- FUN(X)
  numerical_derivative <- function(i) (Y[i+1]-Y[i-1])/(X[i+1]-X[i-1])
  plotPoints <- 2:(length(X) - 1)
  c(NA, sapply(plotPoints, numerical_derivative), NA)
}
```

How it works



$$-x^3 - 2x^2 + 200x$$

Plotting functions

```
plotFdF <- function(x, fun, dfun){
  ylim <- range(c(fun(x), d(fun,x), dfun(x)), na.rm=TRUE)
  plot(x, fun(x), type="l", ylim=ylim)  # original function for which we want derivative
  lines(x, d(fun,x), lwd=3, col="yellow")  # numerical approximation
  lines(x, dfun(x), lty=2, col="red")  # candidate analytical derivative
}
```

A simple example

A more complicated example

$$f(x) = (2sin(x) - sin(2x))/(x - sin(x))$$

Here we apply the quotient rule; if

$$f(x) = g(x)/h(x)$$

then

$$f'(x) = (g'(x) * h(x) - g(x) * h'(x))/(h(x))^2$$

Set

$$g(x) = (2sin(x) - sin(2x))$$

and

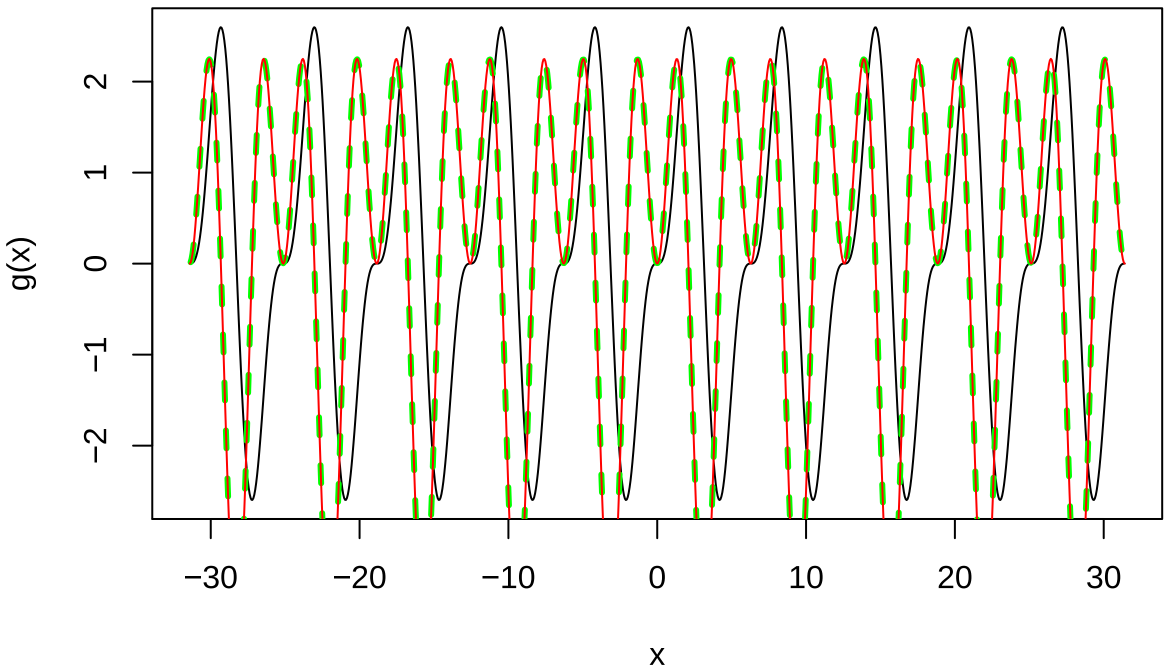
$$h(x) = (x - sin(x))$$

So we can break the problem down into three parts:

1. find the derivative of g(x)
2. find the derivative of h(x)
3. use these together with the quotient rule to find the derivative of f(x)

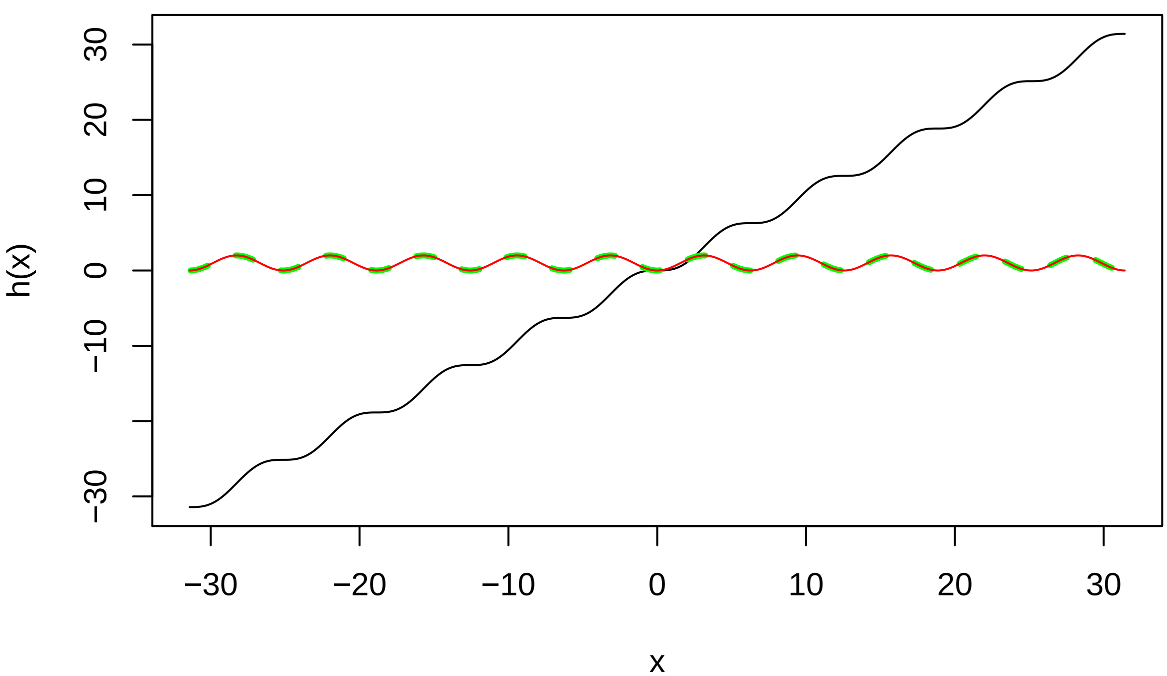
1. derivative of g(x)

```
x <- seq(-10 * pi, 10 * pi, length=1000)
g <- function(x) 2 * sin(x) - sin(2*x)
dg <- function(x) 2*(cos(x) - cos(2*x))
plot(x, g(x), type="l")
lines(x, d(g,x), lwd=3, lty=2, col="green")  # numerical approximation
lines(x, dg(x), col="red")  # candidate analytical solution
```



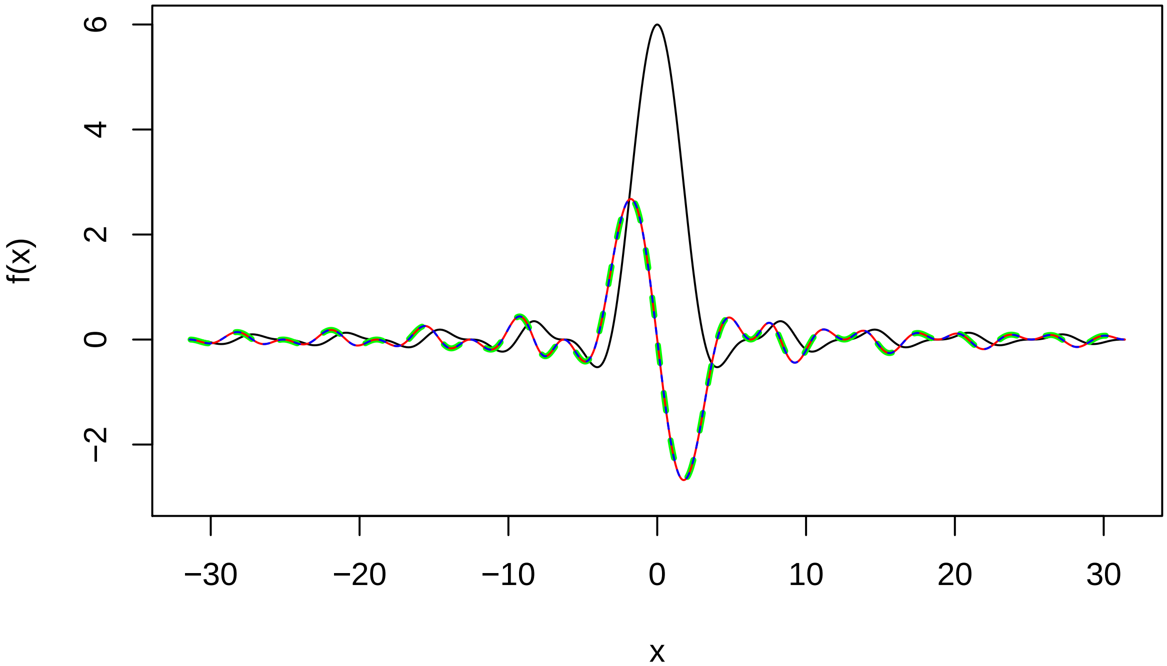
2. derivative of h(x)

```
h <- function(x) x - sin(x)
dh <- function(x) 1 - cos(x)
plot(x, h(x), type="l")
lines(x, d(h,x), lwd=3, lty=2, col="green")  # numerical
lines(x, dh(x), col="red")  # analytical
```



3. Apply the quotient rule

```
f <- function(x) (2 * sin(x) - sin(2*x)) / (x - sin(x))
plot(x, f(x), type="l", ylim=c(-3,6))
lines(x, d(f,x), lwd=3, lty=2, col="green")  # numerical
df1 = function(x) (dg(x)*h(x) - g(x)*dh(x)) / (h(x))^2
df2 = function(x) ((2*(cos(x) - cos(2*x)) * (x - sin(x))) - (2 * sin(x) - sin(2*x)) * (1 - cos(x)) )/ (
lines(x, df1(x), col="red")
lines(x, df2(x), col="blue", lty=2)
```



$$f'(x) = ((2 * (cos(x) - cos(2 * x)) * (x - sin(x))) - (2 * sin(x) - sin(2 * x)) * (1 - cos(x)))/(x - sin(x))^2$$