On Algebraic Approximation of Time-Evolution Operators

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Overview

Introduction

Data-Driven Systems: Dynamical Systems and Predictive Data Analytics

We will study some of the mutually beneficial connections between:

- **Graphs**: as computational and mathematical objects
- **Matrices**: as data tables $\{x_t\}$ and **meaningful** transformations $x_t \mapsto x_{t+1} \approx \mathbf{T}_t x_t$ between them, and
- **Data-Driven Dynamical Systems:** $(\mathbb{Z}, \Sigma, \Phi?)$

$$\xrightarrow{u_t} \boxed{\quad \Phi? \quad} \xrightarrow{y_t}$$

Motivation: Switched Control Systems

The motivation comes from inverse problems for industrial ***switched*** dynamical systems (***approximately***) described by ***difference equations*** of the form:

$$\hat{\Sigma}:\begin{cases}\hat{x}_{t+1}=\mathbf{T}_t\hat{x}_t\\ \|x_t-\hat{x}_t\|\leq\varepsilon\end{cases},t\geq 0$$

where $\{x_t\}_{0\leq t\geq T-1}\subseteq\mathbb{C}^n$ are given and $\{\mathbf{T}_t\}_{t\geq 0}$ are ***unknown*** or ***partially known***, for some prescribed $\varepsilon>0$.

Motivation: Switched Control Systems

The previous control systems models appear in:

1. Industrial Robotics and Automation. Open problems raised by **R. Brocket, M. Chu.**
2. Industrial Robotics and Automation. Open problems raised by **M. Farhood, G. E. Dullerud.**
3. BIM simulation for smart and semi-smart buildings. Open problems raised by **P. Irwin.**
4. Big data predictive analytics. Open problems raised by **N. Kutz.**

Mathematical Control Engineering Approach to CFSA Computation

Given a discrete descriptor control system with an orbit determined by the **black-box** transition diagram:

$$x_t \longrightarrow \boxed{\quad \Phi? \quad} \longrightarrow x_{t+1}$$

We aim to compute a matrix decomposition/realization:

$$\hat{\Sigma} : \begin{cases} \hat{x}_{t+1} = \mathbf{T}^t \hat{x}_t \\ \|x_t - \hat{x}_t\| \leq \varepsilon \end{cases}, t \geq 0$$

Based on a homomorphism $\varphi : \mathscr{A}_{(s,T)} := C^*\langle z, 1 | z^{s+T+1} - z^{s+1} = 0 \rangle \to \mathscr{L}(\Sigma_H)$, $z \mapsto \mathbf{T}$, with $\Sigma_H = \overline{\Sigma}^{\|\cdot\|_2}$. We call $(\mathscr{A}_{(s,T)}, \varphi)$ a Universal algebraic controller (**UAC**) for $\Phi?$.

Methods: Matrix Equation Solver Approach

One can approach the computation of a CFSA by (approximately) solving the model matrix equation:

$$\left\{ \begin{array}{l} (Q(t)X(t) - X(t)Q(t))P(t) = \mathbf{0}_{2n} \\ Q(t)^4 = Q(t)^2 \\ Q(t)^2 = ZQ(t) = (Q(t)^2)^* \end{array} \right. , 0 \leq t \leq T-1$$

where $X(t)$ is to be completed preserving the structure:

$$X(t) = \begin{bmatrix} H_t & \mathbf{0}_n \\ \mathbf{0}_n & F_t \end{bmatrix}$$

An explicit computation of each $X(t)$ may be **expensive** and severely **Ill-conditioned**.

Methods: Alternative data-driven C*-algebraic" approach

Can one solve the following problem numerically?

1. Given $\varepsilon > 0$, and a ε-almost eventually periodic data driven system $\{x_t\}_{t\geq 1} \subset \mathbb{C}^n$ with $\mathrm{Ind}_\varepsilon(\{x_t\}) = (s, T)$,
2. Can one solve the problem?

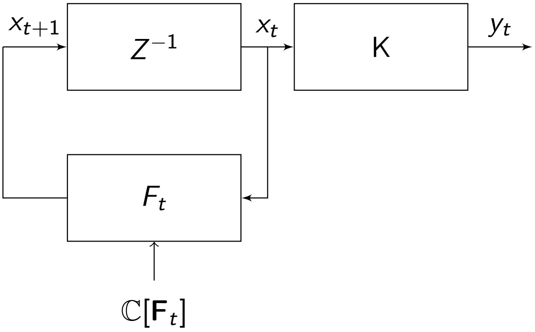
$$
\begin{tikzcd}
 & C^*(\mathbf{T}, I) \arrow[d, "\hat{\Phi}"'] & \\
\mathscr{A}_{(s,T)} \arrow[ur, two heads] \arrow[r, "\varphi"'] & \mathscr{L}(\Sigma_H) & \mathscr{Z}_{(s,T)} \arrow[l, dashed, "\Phi"'] \arrow[ul, hook]
\end{tikzcd}
$$

Main Result

Yes. Proved in corollary 3.8 and theorems 3.11-12 in paper *"On universal algebraic controllers and system identification"*.

Computational Implementation

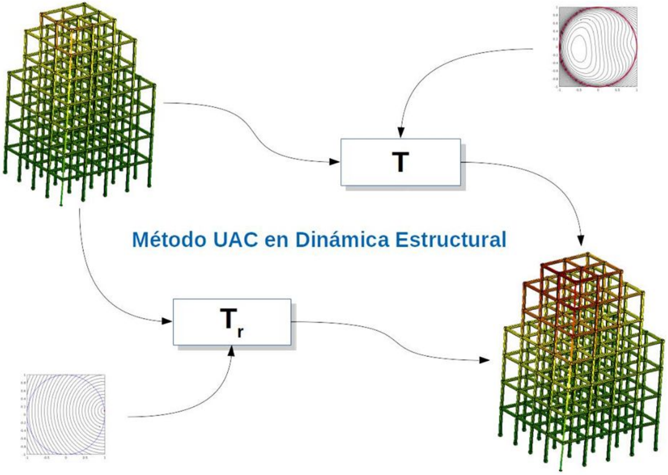
The previous computations can be implemented to derive algorithms described in general by the diagram:



Some Applications

UAC and Numerical Simulation in Modal Dynamics

Modal dynamics for steel structures



Conclusion

- Every data-driven system $\Sigma : \{x_t\}_{t\geq 1}$ with an approximate state transition graph like the one shown in the figure:

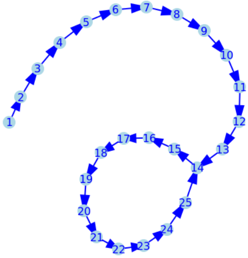


Figure: $Ind_{\varepsilon}(\{F_t\}) = (12, 14)$.

has a UAC representation/realization.

Future Directions

- Combine UAC with multivariate statistical analysis and probabilistic methods to study stochastic dynamical models.
- Combine UAC with embedded systems techniques for BIM predictive simulations.
- Combine UAC with embedded systems techniques for model predictive control of industrial processes.

Questions?

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Thanks!