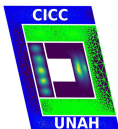


# On Algebraic Approximation of Time-Evolution Operators



**UNAH**  
UNIVERSIDAD NACIONAL  
AUTÓNOMA DE HONDURAS



Fredy Vides<sup>1</sup>

<sup>1</sup>Scientific Computing Innovation Center **CICC-UNAH**  
School of Mathematics and Computer Science  
Universidad Nacional Autónoma de Honduras **UNAH**

XXII SIMMAC, Universidad de Costa Rica, 2020

# Overview

Introduction

Motivation

Methods

Applications

Conclusion

Future Directions

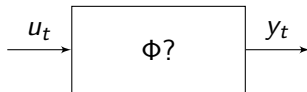


# Introduction

## Data-Driven Systems: Dynamical Systems and Predictive Data Analytics

We will study some of the mutually beneficial connections between:

- ▶ **Graphs:** as computational and mathematical objects
- ▶ **Matrices:** as data tables  $\{x_t\}$  and **meaningful** transformations  $x_t \mapsto x_{t+1} \approx \mathbf{T}_t x_t$  between them, and
- ▶ **Data-Driven Dynamical Systems:**  $(\mathbb{Z}, \Sigma, \Phi?)$



# Motivation: Switched Control Systems

The motivation comes from inverse problems for industrial *switched* dynamical systems (*approximately*) described by *difference equations* of the form:

$$\hat{\Sigma} : \begin{cases} \hat{x}_{t+1} = \mathbf{T}_t \hat{x}_t \\ \|x_t - \hat{x}_t\| \leq \varepsilon \end{cases}, t \geq 0$$

where  $\{x_t\}_{0 \leq t \leq T-1} \subseteq \mathbb{C}^n$  are given and  $\{\mathbf{T}_t\}_{t \geq 0}$  are *unknown* or *partially known*, for some prescribed  $\varepsilon > 0$ .



# Motivation: Switched Control Systems

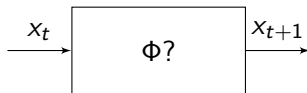
The previous control systems models appear in:

1. Industrial Robotics and Automation. Open problems raised by **R. Brockett, M. Chu.**
2. Industrial Robotics and Automation. Open problems raised by **M. Farhood, G. E. Dullerud.**
3. BIM simulation for smart and semi-smart buildings. Open problems raised by **P. Irwin.**
4. Big data predictive analytics. Open problems raised by **N. Kutz.**



# Mathematical Control Engineering Approach to CFSA Computation

Given a discrete descriptor control system with an orbit determined by the **black-box** transition diagram:

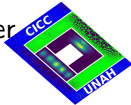


We aim to compute a matrix decomposition/realization:

$$\hat{\Sigma} : \begin{cases} \hat{x}_{t+1} = \mathbf{T}^t \hat{x}_t \\ \|x_t - \hat{x}_t\| \leq \varepsilon \end{cases}, t \geq 0$$

Based on a homomorphism

$\varphi : \mathcal{A}_{(s,T)} := C^*\langle z, 1 | z^{s+T+1} - z^{s+1} = 0 \rangle \rightarrow \mathcal{L}(\Sigma_H), z \mapsto \mathbf{T}$ , with  $\Sigma_H = \overline{\Sigma}^{\|\cdot\|^2}$ . We call  $(\mathcal{A}_{(s,T)}, \varphi)$  a Universal algebraic controller (UAC) for  $\Phi?$ .



# Methods: Matrix Equation Solver Approach

One can approach the computation of a CFSA by (approximately) solving the model matrix equation:

$$\begin{cases} (Q(t)X(t) - X(t)Q(t))P(t) = \mathbf{0}_{2n} \\ Q(t)^4 = Q(t)^2 \\ Q(t)^2 = ZQ(t) = (Q(t)^2)^* \end{cases}, 0 \leq t \leq T-1$$

where  $X(t)$  is to be completed preserving the structure:

$$X(t) = \begin{bmatrix} H_t & \mathbf{0}_n \\ \mathbf{0}_n & F_t \end{bmatrix}$$

An explicit computation of each  $X(t)$  may be **expensive** and severely **Ill-conditioned**.



# Methods: Alternative data-driven $C^*$ -algebraic" approach

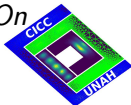
Can one solve the following problem numerically?

1. Given  $\varepsilon > 0$ , and a  $\varepsilon$ -almost eventually periodic data driven system  $\{x_t\}_{t \geq 1} \subset \mathbb{C}^n$  with  $\text{Ind}_\varepsilon(\{x_t\}) = (s, T)$ ,
2. Can one solve the problem?

$$\begin{array}{ccccc} & & C^*(\mathbf{T}, I) & & \\ & \nearrow & \downarrow \hat{\Phi} & \nwarrow & \\ \mathcal{A}_{(s,T)} & \xrightarrow{\varphi} & \mathcal{L}(\Sigma_H) & \xleftarrow{\Phi} & \mathcal{Z}_{(s,T)} \end{array}$$

## Main Result

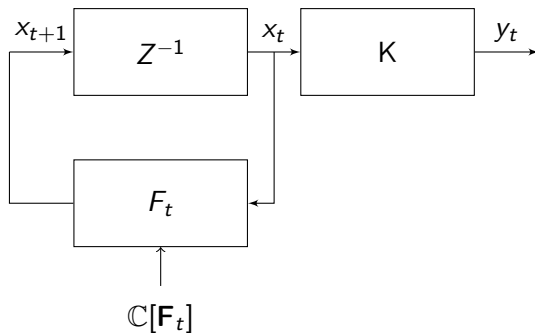
**Yes.** Proved in corollary 3.8 and theorems 3.11-12 in paper "On universal algebraic controllers and system identification".





# Computational Implementation

The previous computations can be implemented to derive algorithms described in general by the diagram:

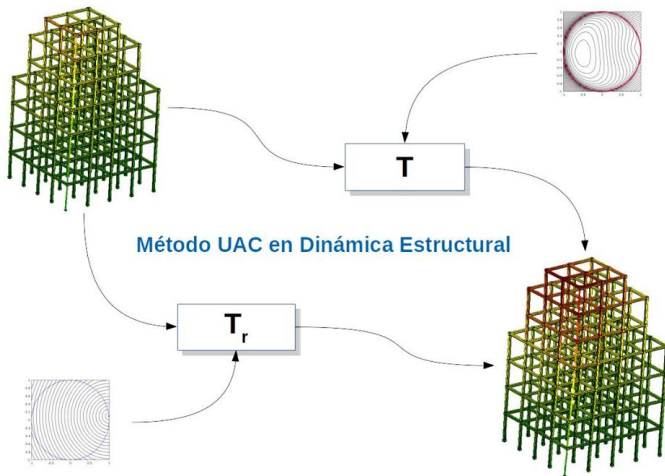


## Some Applications



# UAC and Numerical Simulation in Modal Dynamics

Modal dynamics for steel structures



# Conclusion

- ▶ Every data-driven system  $\Sigma : \{x_t\}_{t \geq 1}$  with an approximate state transition graph like the one shown in the figure:

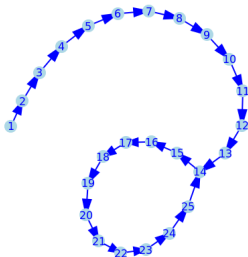


Figure:  $\text{Ind}_{\varepsilon}(\{F_t\}) = (12, 14)$ .

has a UAC representation/realization.



# Future Directions

- ▶ Combine UAC with multivariate statistical analysis and probabilistic methods to study stochastic dynamical models.
- ▶ Combine UAC with embedded systems techniques for BIM predictive simulations.
- ▶ Combine UAC with embedded systems techniques for model predictive control of industrial processes.



# Questions?

[fredy.vides@unah.edu.hn](mailto:fredy.vides@unah.edu.hn)



Thanks!

