On Approximately Cyclic Model Order Reduction for Data-Driven Systems

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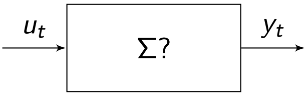
Overview

Introduction

Data-Driven Systems: Dynamical Systems and Predictive Data Analytics

We will study some of the mutually beneficial connections between:

- **Graphs**: as computational and mathematical objects
- **Matrices**: as data tables $\{x_t\}$ and **meaningful** transformations $x_t \mapsto x_{t+1} \approx \mathbf{F}_t x_t$ between them, and
- **Industrial Descriptor Systems:**



Motivation: Switched Control Systems

The motivation comes from inverse problems for industrial ***switched*** dynamical systems (***approximately***) described by ***difference equations*** of the form:

$$\hat{\Sigma}: \begin{cases} \hat{x}_1 = \hat{T}_t x_1 \\ \hat{x}_{t+1} = \hat{F}_t \hat{x}_t & , t \geq 0 \\ x_t = \hat{K}_t \hat{x}_t \end{cases}$$

where $\{x_t\}_{0 \leq t \geq T-1} \subseteq \mathbb{C}^n$ are given and $\{\hat{F}_t\}_{t \geq 0}$ are ***unknown*** or ***partially known***.

Motivation: Switched Control Systems

The previous control systems models appear in:

1. Industrial Robotics and Automation. Open problems raised by **R. Brocket, M. Chu.**
2. BIM simulation for smart and semi-smart buildings. Open problems raised by **P. Irwin.**
3. Big data predictive analytics. Open problems raised by **N. Kutz.**

Mathematical Control Engineering Approach to CFSA Computation

Given a discrete descriptor control system with an orbit determined by the **black-box** transition diagram:

$$x_t \longrightarrow \boxed{\quad \Sigma? \quad} \longrightarrow x_{t+1}$$

We aim to compute a matrix decomposition/realization:

$$\hat{\Sigma} : \begin{cases} \hat{x}_1 = \hat{T}_t x_1 \\ \hat{x}_{t+1} = \hat{F}_t \hat{x}_t & , t \geq 0 \\ x_t = \hat{K}_t \hat{x}_t \end{cases}$$

We call $\hat{\Sigma}$ a Cyclic Finite-State Approximation (**CFSA**) of $\Sigma?$.

Methods: Matrix Equation Solver Approach

One can approach the computation of a CFSA by (approximately) solving the model matrix equation:

$$\begin{cases} (Q(t)X(t) - X(t)Q(t))P(t) = \mathbf{0}_{2n} \\ Q(t)^4 = Q(t)^2 \\ Q(t)^2 = ZQ(t) = (Q(t)^2)^* \end{cases} , 0 \leq t \leq T-1$$

where $X(t)$ is to be completed preserving the structure:

$$X(t) = \begin{bmatrix} H_t & \mathbf{0}_n \\ \mathbf{0}_n & F_t \end{bmatrix}$$

An explicit computation of each $X(t)$ may be **expensive** and severely **Ill-conditioned**.

Methods: Alternative data-driven algebraic approach

Can one solve the following problem numerically?

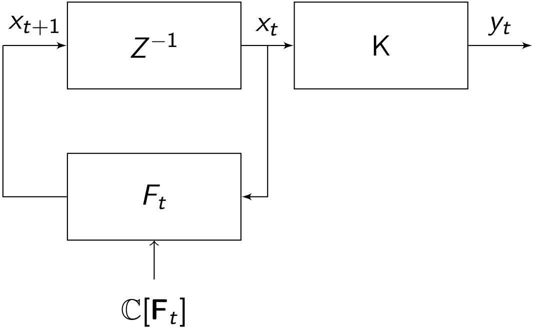
1. Given $\varepsilon > 0$, and a ε-almost S-periodic data driven system $\{x_t\}_{t\geq 1} \subset \mathbb{C}^n$,
2. Is there a matrix $\hat{\mathbf{F}} \in \mathbb{C}^{n\times n}$? such that the diagram $x_t \approx \hat{\mathbf{F}} x_1$ and $\hat{\mathbf{F}}^S \hat{\mathbf{F}}^r = \hat{\mathbf{F}}^r$ with $r, S \ll n$:

Main Result

Yes, one can solve the previous problem numerically. Proved in theorems II.1-3 in paper.

Computational Implementation

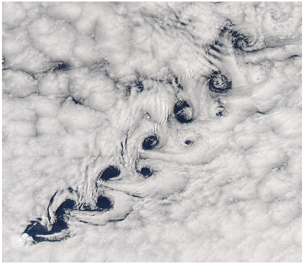
The previous computations can be implemented to derive algorithms described in general by the diagram:



Some Applications

Numerical Simulation of von Karman Vortex Streets

von Karman Vortex Streets in Nature




Numerical Simulation of von Karman Vortex Streets

CFSA for von Karman Vortex Streets Models

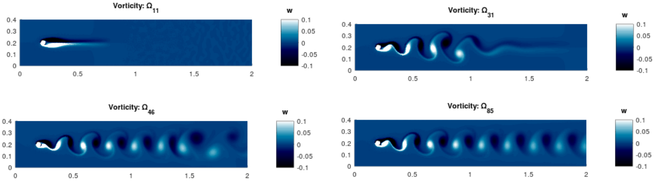


Figure: Predictive CFSA for von Kármán vortex streets based on FEM Navier-Stokes models

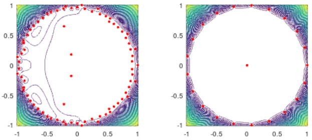


Figure: Pseudoespectral predictive scheme for CFSA of von Kármán models

Conclusion

- Every data-driven system $\Sigma : \{x_t\}_{t\geq 1}$ with an approximate state transition graph like the one shown in the figure:

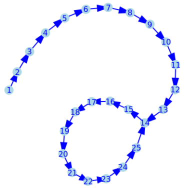


Figure: $Ind_\varepsilon(\{F_t\}) = (12, 14)$.

has a CFSA representation/realization.

Future Directions

- Combine CFSA with embedded systems techniques for BIM predictive simulations.
- Combine CFSA with embedded systems techniques for climate predictive simulations.

Questions?

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Thanks!