Introduction to Type 1 Chebyshev filters

Alessandro Marcelli

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0.1 Low-pass Type 1 Chebyshev filters

Type 1 Chebishev low pass filters are characterized by a very small transient band, at the price of oscillations in the passing band. The higher the filter's oreder, the smaller the transient band is and the more oscillations in passing band we have.

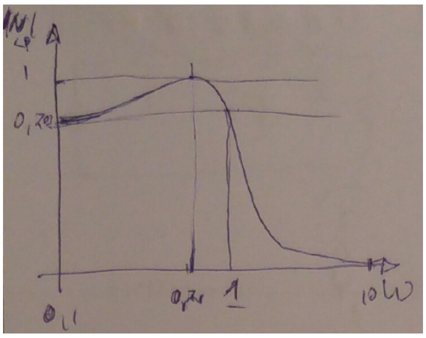

Figure 1: Example of a Low-pass Chebyshev filter

A filter of the n-th order is characterized by the following transfer function

$$|H_{LP}^{(n)}(j\omega)|^2 = \frac{H_0^2}{1+\epsilon^2 \cdot C_n^2(\omega)} \tag{1}$$

where

1. $\omega = 2\pi f$
2. H_0 is the **maximum amplification in the filter's passing band**
3. ϵ is the **ripple factor**, whose contribute to the max amplitude in the passing band is given by
$$k_p = 20 \cdot \log_{10}(\sqrt{1+\epsilon^2}) \qquad [dB] \tag{2}$$
4. $C_n(\omega)$ is the **n-th order Chebyshev polynomial** (which can be found tabulated [2]), where **n** corresponds to the filter order, which are defined as such

$$C_n(\omega) = cos(n \cdot arccos(\omega)) \qquad \omega \in [0,1] \tag{3}$$
$$C_n(\omega) = cos(n \cdot arccosh(\omega)) \qquad \omega \geq 1 \tag{4}$$

More than $H_{LP}^{(n)}(j\omega)$ it's usually preferred the **normalized transfer function**, defined as

$$|N_{LP}^{(n)}(j\omega)|^2 = \frac{|H_{LP}^{(n)}(j\omega)|^2}{H_0^2} = \frac{1}{1+\epsilon^2 \cdot C_n^2(\omega)} \tag{5}$$

0.2 Pole calculation

Poles for Type 1 Chebyshev lay along the left side of an ellpise, with one real for odd values of n. Studying the tranfer function the following expression for pole calculation can be derived:

$$p_k = \sigma_k + j\omega_k \tag{6}$$
$$\sigma_k = -sin(u_k) \cdot sinh(v) \tag{7}$$
$$\omega_k = +cos(u_k) \cdot cosh(v) \tag{8}$$
$$u_k = \frac{2k-1}{2n} \cdot \pi \tag{9}$$
$$v = \frac{1}{n} \cdot arcsinh\left(\frac{1}{\epsilon}\right) \tag{10}$$

0.2.1 Example: 7th order low-pass filter with $\epsilon = 1$

For a 7th order low-pass filter with $\epsilon = 1$ the normalized transfer function becomes

$$|N_{LP}^{(7)}(j\omega)|^2 = \frac{1}{1 + (64\omega^7 - 112\omega^5 + 56\omega^3 - 7\omega)^2} \tag{11}$$

Using the given formulas we get

$$u_1 = \frac{1}{14} \cdot \pi \tag{12}$$
$$u_2 = \frac{3}{14} \cdot \pi \tag{13}$$
$$u_3 = \frac{5}{14} \cdot \pi \tag{14}$$
$$u_4 = \frac{1}{2} \cdot \pi \tag{15}$$
$$u_5 = \frac{9}{14} \cdot \pi \tag{16}$$
$$u_6 = \frac{11}{14} \cdot \pi \tag{17}$$
$$u_7 = \frac{13}{14} \cdot \pi \tag{18}$$
$$v = 0.1259 \tag{19}$$

This leads to the following 7 poles

$$p_1 = -0.0281 + j \cdot 0.9827 \tag{20}$$
$$p_2 = -0.0787 + j \cdot 0.7880 \tag{21}$$
$$p_3 = -0.1137 + j \cdot 0.4373 \tag{22}$$
$$p_4 = -0.1262 \tag{23}$$
$$p_5 = -0.1137 - j \cdot 0.4373 \tag{24}$$
$$p_6 = -0.0787 - j \cdot 0.7880 \tag{25}$$
$$p_7 = -0.0281 - j \cdot 0.9827 \tag{26}$$

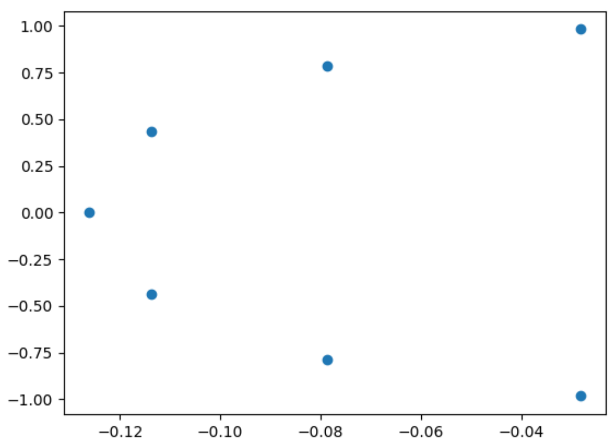

Figure 2: Poles on the plane

0.3 Chebyshev tables

For the calculation of normalized impedences, one usually consults the Chebyshev tables [1], characterized by the filter's order and desired **ripple factor** ϵ.

As an example, here are some table values for $\epsilon = 1.0$

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	...
⋮									
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000			
6	3.5045	0.7684	4.6061	0.7929	4.4641	0.6033	5.8095		
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000	
⋮									

Let's see how the examples above can be interpreted using a LC ladder configuration.

1. 5th order filter

N	C_2	L_2	C_3	L_3	C_1	Z_{out}
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000

```
--------------L_2-------L_3------------------
|         |         |         |         |
Gen       C_1       C_2       C_3       R_out
|         |         |         |         |
---------------------------------------------
```

2. 6th order filter

N	C_1	L_1	C_2	L_2	C_3	L_3	Z_{out}
6	3.5045	0.7684	4.6061	0.7929	4.4641	0.6033	5.8095

```
----L_1-------L_2-------L_3------------------
|         |         |         |         |
Gen       C_1       C_2       C_3       R_out
|         |         |         |         |
---------------------------------------------
```

3. 7th order filter

N	C_2	L_2	C_3	L_3	C_4	L_4	C_1	Z_{out}
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000

```
--------------L_2-------L_3-------L_4------------------
|         |         |         |         |         |
Gen       C_1       C_2       C_3       C_4       R_out
|         |         |         |         |         |
--------------------------------------------------------
```

We can notice two thing:

1. For odd order filters the second-to-last impedence in the table actually corresponds to the first net of the filter

2. For even order filters one can't have the same input and output loads, so a scaling factor is mandatory

0.4 Components calculation

The values for a n-th order filter with a specific **ripple factor** ϵ and a specific **output load** Z_{out} are calculated denormalizing the values found in Chebyshev tables via the following equations:

$$C_i = \frac{C_{i_{norm}}}{2\pi f_c \cdot Z_{out}} \tag{27}$$

$$L_i = \frac{L_{i_{norm}} \cdot Z_{out}}{2\pi f_c} \tag{28}$$

$$R_{out} = g_{n+1} \cdot Z_{out} \tag{29}$$

The term g_{n+1} is the last impedence value on the n-th row of the table.
Let's take a look once again at the $\epsilon = 1$ case, and choose

$$R_{out} = 50\ [\Omega] \tag{30}$$

$$f_c = 1\ [MHz] \tag{31}$$

1. 5th order filter

C_2 $[nF]$	L_2 $[\mu H]$	C_3 $[nF]$	L_3 $[\mu H]$	C_1 $[nF]$	Z_{out} $[\Omega]$
11.083	6.0622	14.445	6.0622	11.083	50.000

2. 6th order filter

C_1 $[nF]$	L_1 $[\mu H]$	C_2 $[nF]$	L_2 $[\mu H]$	C_3 $[nF]$	L_3 $[\mu H]$	Z_{out} $[\Omega]$
11.155	6.1147	14.662	6.3097	14.210	4.8009	290.48

3. 7th order filter

C_2 $[nF]$	L_2 $[\mu H]$	C_3 $[nF]$	L_3 $[\mu H]$	C_4 $[nF]$	L_4 $[\mu H]$	C_1 $[nF]$	Z_{out} $[\Omega]$
11.199	6.1458	14.765	6.3972	14.765	6.1458	11.199	50.000

0.5 Low-Pass to Band-pass conversion

Low-pass and Band-pass filters are bound together by the following variable change

$$s = p + \frac{1}{p}\ ;\ p \in \mathbb{C} \tag{32}$$

This relation brings us two consequences:

1. Studying the inductors' impedances one finds that

$$Z_{LP}(s) = ks \rightarrow Z_{BP}(p) = kp + \frac{k}{p}\ ;\ p \in \mathbb{C} \tag{33}$$

This means that we have to add a capacitor in series to every inductor. We can calculate the values using the following relation

$$C_{s_k} = \frac{\Delta f_{rel}}{2\pi f_{mid} \cdot L_{k_{norm}} \cdot Z_{out}} \tag{34}$$

2. Conversely, studying the capacitors' conductances one finds that

$$Y_{LP}(s) = ks \rightarrow Y_{BP}(p) = kp + \frac{k}{p}\ ;\ p \in \mathbb{C} \tag{35}$$

This means we have to put an inductor in parallel to each capacitor, which we can calculate with the following relation

$$L_{p_k} = \frac{\Delta f_{rel} \cdot Z_{out}}{2\pi f_{mid} \cdot L_{k_{norm}}} \tag{36}$$

Following, as an example, the schematic for a 5th order Band-pass filter

```
-------------------------L_2---C_s_2----------L_3---C_s_3------------------------------
|                 |   |                   |   |                   |   |         |
Gen               C_1 L_p_1               C_2 L_p_2               C_3 L_p_3     R_out
|                 |   |                   |   |                   |   |         |
---------------------------------------------------------------------------------------
```

1st net components	
$C_1 = 11.083\ [nF]$	$L_{1_{par}} = 31.352\ [nH]$
2st net components	
$L_2 = 6.0622\ [\mu H]$	$C_{2_{ser}} = 57.317\ [pF]$
$C_2 = 11.083\ [nF]$	$L_{2_{par}} = 31.352\ [nH]$
3st net components	
$L_3 = 6.0622\ [\mu H]$	$C_{3_{ser}} = 57.317\ [pF]$
$C_3 = 14.445\ [nF]$	$L_{3_{par}} = 24.054\ [nH]$
Output load	
$R_{out} = 50.000\ [\Omega]$	

Bibliography

[1] Kirt Blattenberger. Chebyshev Filter Lowpass Prototype Element Values . https://www.rfcafe.com/references/electrical/cheby-proto-values.htm. [Online].

[2] Hobart Pao Daniel Liu, Swapnil Das. Chebyshev Polynomials - Definition and Properties . https://brilliant.org/wiki/chebyshev-polynomials-definition-and-properties/. [Online].