

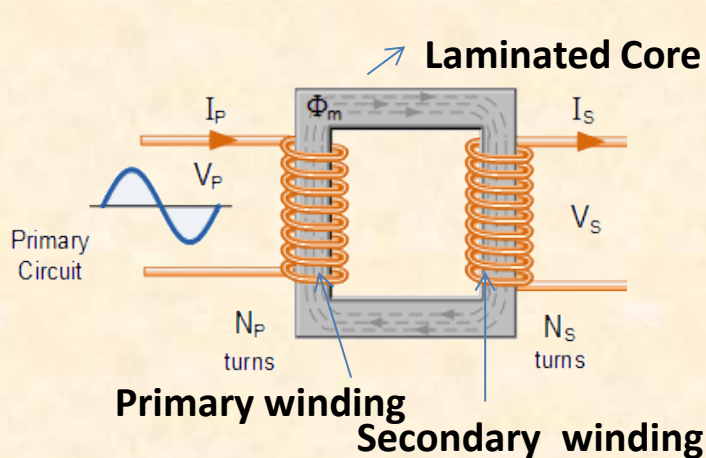
4.1 Single phase transformer

Syllabus

Construction and principle of working,
e.m.f. equation of a transformer,
different types of transformer,
losses in transformer,
voltage regulation and efficiency of
transformer (no numerical expected)

Working Principle of Transformer

- Transformer is a static(or stationary) piece of apparatus by means of which electric power in one circuit is transformed into electric power of the same frequency in another circuit.
- It can raise or lower the voltage (V) in a circuit but with corresponding decrease or increase current (I). So the product VI remains constant.
- In its simplest form, transformer consists of two inductive coils (windings) which are electrically separated but magnetically linked through a path of low reluctance i.e. by laminated core as indicated below. The two coils possess high mutual inductance.



- The primary winding is connected to a source of alternating voltage, an alternating flux is set up in laminated core, most of which is linked with secondary winding in which it produces mutually induced e.m.f. according to Faradays law of electromagnetic induction($E=Mdi/dt$).
- If the secondary winding circuit is closed, a current flows in it and so electric energy is transferred (entirely magnetically) from first coil to the second coil.

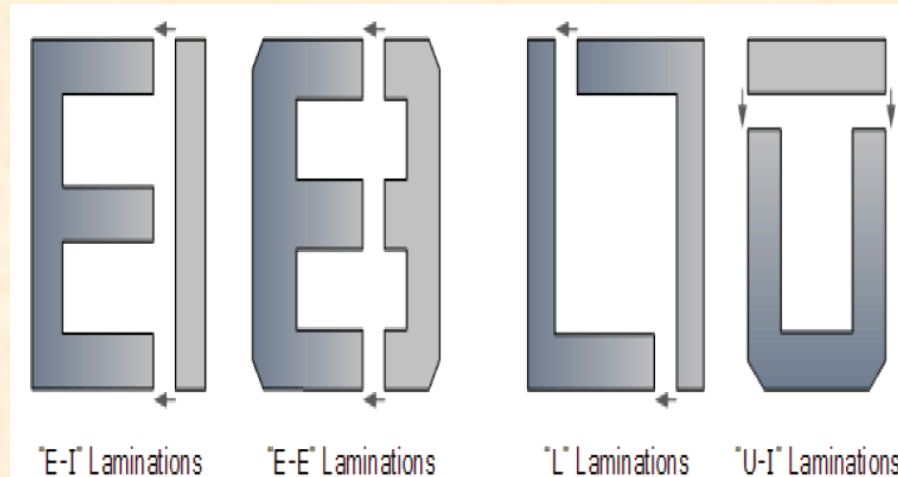
Transformer construction

➤ Two basic parts of transformer are

1. Magnetic core
2. Windings or coils

➤ **Magnetic core**

- The core of transformer either square or rectangular type in size.
- The vertical portion on which coils are wound called limb while horizontal portion is called yoke.
- Core is made of laminated type constructions to minimize eddy current losses.



- Generally high grade silicon steel laminations (0.3 to 0.5mm) are used. The high silicon content and heat treated to ensure high permeability and low hysteresis at losses at usual flux densities.

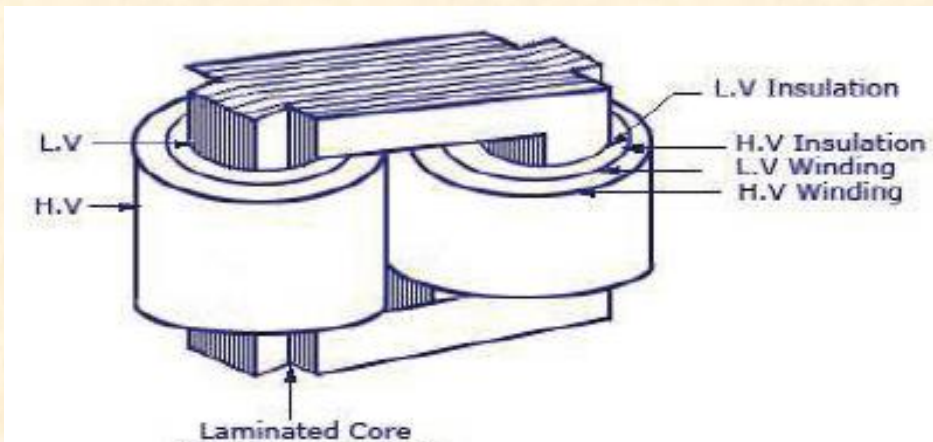
Transformer construction..

➤ Transformer winding or coils

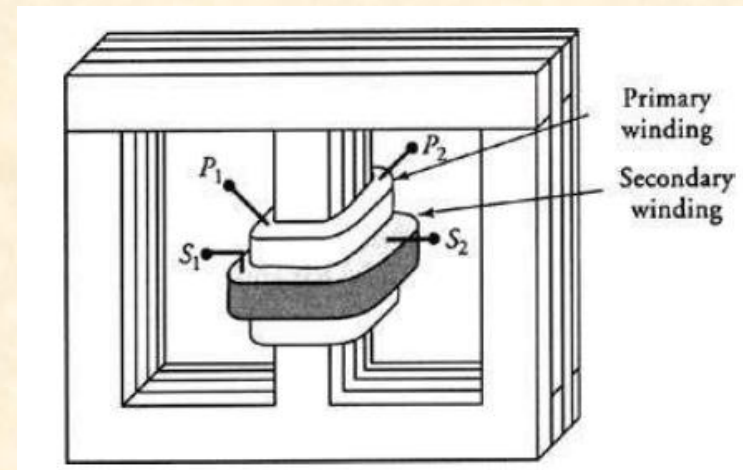
- Conducting material such as copper is used in the winding of the transformer.
- The coils are used are wound on the limbs and insulated from each other.
- The leakage flux increases which affects the performance and efficiency of transformer. To reduce the leakage flux it is necessary that the windings should be very close to each other to have high mutual induction.

➤ Transformer classification based on construction

1. Core Type Transformer



2. Shell Type Transformer



Transformer construction..

1. Core Type

- In this type one magnetic circuit and cylindrical coils are used
- Normally L and T shaped laminations are used
- Commonly primary winding would on one limb while secondary on the other but performance will be reduce
- To get high performance it is necessary that other the two winding should be very close to each other

2. Shell Type

- In this type two magnetic circuit are used
- The winding is wound on central limbs
- For the cell type each high voltage winding lie between two voltage portion sandwiching the high voltage winding
- Sub division of windings reduces the leakage flux
- Greater the number of sub division lesser the reactance
- This type of construction is used for high voltage

Transformer classification

➤ In terms of number of windings

- Conventional transformer: two windings
- Autotransformer: one winding
- Others: more than two windings

➤ In terms of number of phases

- Single-phase transformer
- Three-phase transformer

➤ Depending on the voltage level at which the winding is operated

- Step-up transformer: primary winding is a low voltage (LV) winding
- Step-down transformer : primary is a high voltage (HV) winding

➤ Based on construction

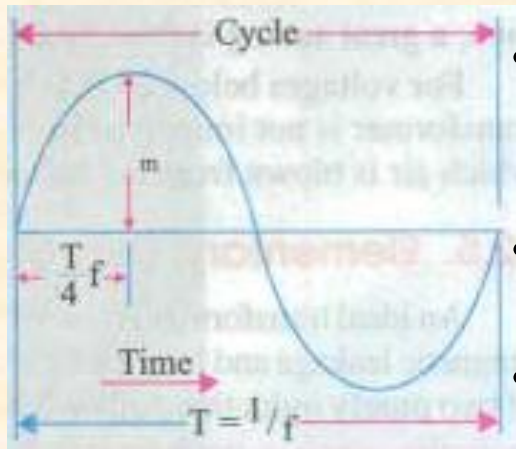
- Core Type
- Shell Type

EMF Equation of Transformer

Let N_1 = Number of turns in primary ,

N_2 = Number of turns in secondary

Φ_m = Maximum flux in core in webers = $B_m \times A$ f = Frequency of a.c. input in Hz



- Flux increases from zero value to maximum value Φ_m in $1/4f$ seconds.
- Average rate of change of flux = $\Phi_m / (1/4f) = 4f \Phi_m$ wb/s or V
- As flux varies sinusoidally then rms value of induced emf is obtained by multiplying average value with form factor.
since form factor = rms value / average value = 1.11
- rms value of emf /turn = $1.11 \times 4f \Phi_m = 4.44 f \Phi_m$ Volts
- rms value of emf in whole primary = rms value of emf /turn \times Number of turns

$$E_1 = 4.44 f \Phi_m \times N_1 = 4.44 f N_1 B_m A \dots\dots\dots(i)$$
- rms value of emf in whole secondary = rms value of emf /turn \times Number of turns

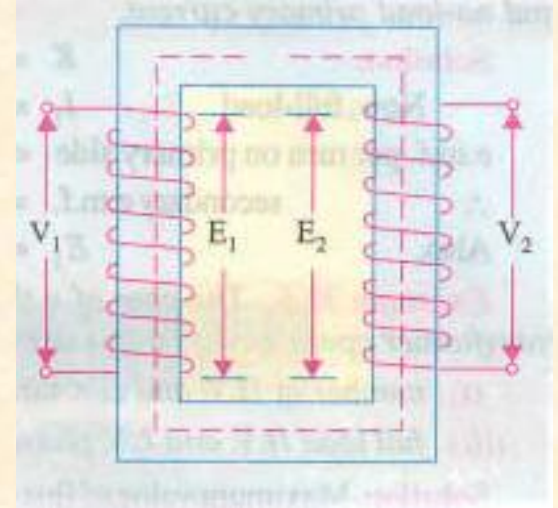
$$E_2 = 4.44 f \Phi_m \times N_2 = 4.44 f N_2 B_m A \dots\dots\dots(ii)$$

Transformation Ratio (K) of Transformer

From emf equation (i) and (ii) of the transformer

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

Where K is called transformation ratio.



- If $N_2 > N_1$ i.e. if $K > 1$, then transformer is called step up transformer.
- If $N_2 < N_1$ i.e. if $K < 1$, then transformer is called step down transformer.

For ideal transformer input VA= output VA

$$V_1 I_1 = V_2 I_2$$

$$\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$$

Hence currents are in the inverse ratio the (voltage) transformation Ratio.

Losses in the Transformer

Core losses or Iron losses

(i) Hysteresis loss (ii) Eddy current loss



Copper loss

➤ **Core losses or Iron losses:** Eddy current loss and hysteresis loss depend upon the magnetic properties of the material used for the construction of core. Hence these losses are also known as **core losses** or **iron losses**. These losses are independent of current and does not change with load. Hence are called fixed losses.

(i) Hysteresis loss in transformer: Hysteresis loss is due to reversal of magnetization in the transformer core. This loss depends upon the volume and grade of the iron, frequency of magnetic reversals and value of flux density. It can be given by, Steinmetz formula:

$$W_h = \eta B_{\max}^{1.6} f V (\text{watts})$$

where,

f = frequency of supply

η = Steinmetz hysteresis constant

V = volume of the core in m^3

Losses in the Transformer

(ii) Eddy current loss in transformer:

In transformer, AC current is supplied to the primary winding which sets up alternating magnetizing flux. When this flux links with secondary winding, it produces induced emf in it. But some part of this flux also gets linked with other conducting parts like steel core or iron body or the transformer, which will result in induced emf in those parts, causing small circulating current in them. This current is called as eddy current. Due to these eddy currents, some energy will be dissipated in the form of heat.

➤ Copper loss in transformer

Copper loss is due to ohmic resistance of the transformer windings. Copper loss for the primary winding is $I_1^2 R_1$ and for secondary winding is $I_2^2 R_2$. Where, I_1 and I_2 are current in primary and secondary winding respectively, R_1 and R_2 are the resistances of primary and secondary winding respectively. It is clear that Cu loss is proportional to square of the current, and current depends on the load. Hence copper loss in transformer varies with the load, hence called variable losses.

Rating of the Transformer

➤ kVA Rating

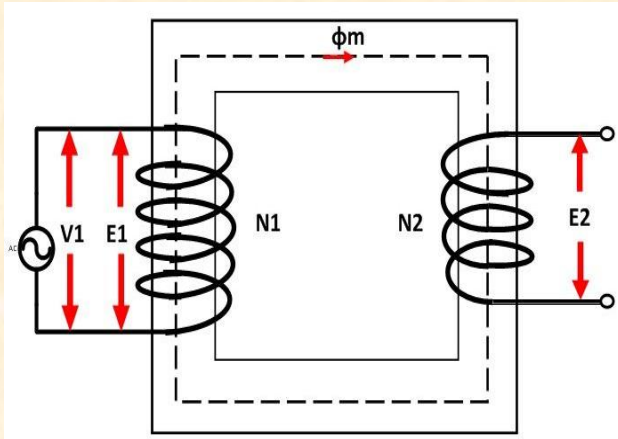
- The copper losses in the transformer depends on the current and iron loss on voltage.
- Hence the transformer total loss depends on volt-ampere (VA) and not on phase angle between voltage and current i.e losses are independent of power factor.
- Hence the rating of transformer is described in volt-ampere or kilo Volt-ampere (kVA) and not as kW.
- $V_1 I_1 = V_2 I_2$ defines the kVA rating.

Ideal Vs Practical Transformer

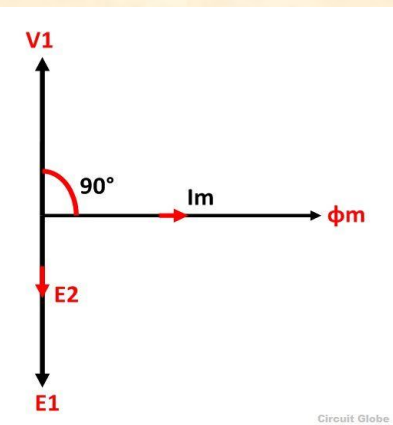
- **The transformer which is free from all types of losses is known as an ideal transformer.** It is an imaginary device which has no core loss, no ohmic resistance and no leakage flux. The ideal transformer has the following important characteristic.
 - The resistance of their primary and secondary winding is zero.
 - The core of the ideal transformer has infinite permeability. The infinite permeable means less magnetising current requires for magnetising their core.
 - The leakage flux of the transformer becomes zero, i.e. the whole of the flux induces in the core of the transformer links with their primary and secondary winding.
 - The ideal transformer has 100 percent efficiency, i.e., the transformer is free from hysteresis and eddy current loss.

While the practical transformer has finite windings resistance , some leakage flux and has all kind of losses.

Phasor Diagram of Ideal transformer



- The voltage source V_1 is applied across the primary winding of the transformer. Their secondary winding is kept open. The N_1 and N_2 are the numbers of turns of their primary and secondary winding.
- The current I_m is the magnetizing current flows through the primary winding of the transformer. The magnetizing current produces the flux ϕ_m in the core of the transformer. As the permeability of the core is infinite the flux of the core link with both the primary and secondary winding of the transformer.
- The flux link with the primary winding induces the emf E_1 because of self-induction. The direction of the induces emf is inversely proportional to the applied voltage V_1 . The emf E_2 induces in the secondary winding of the transformer because of mutual induction.



← The phasor diagram of the ideal transformer is shown in the figure below.

As the coil of the primary transformer is purely inductive the magnetising current induces in the transformer lag 90° by the input voltage V_1 . The E_1 and E_2 are the emf induced in the primary and secondary winding of the transformer. The direction of the induced emf opposite to the applied voltage as per Lenz's Law.

Theory of Practical Transformer

Practical Transformer on No Load

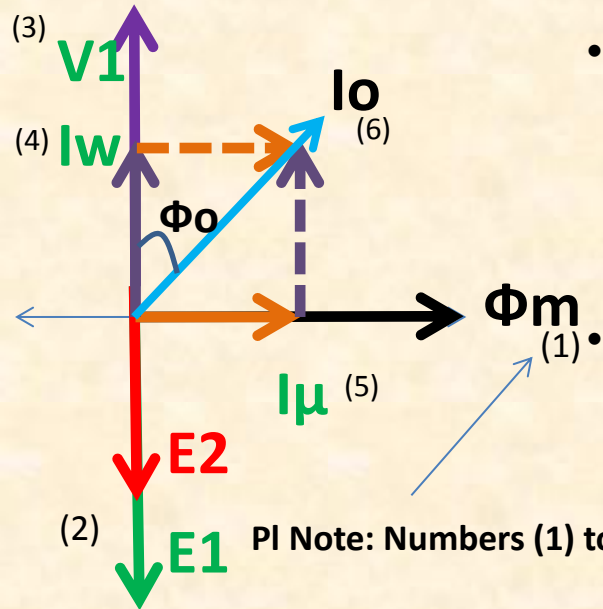
1. Consideration of Non-Zero Core and copper losses

- There is iron loss in the core and copper loss in both primary and secondary windings and these losses are not negligible.
- Even when transformer secondary is open (No Load), the primary input current is not purely reactive. The primary input current under no load condition has to supply
 - (i) iron losses (hysteresis and eddy current) in the core
 - (ii) very small copper loss in the primary (since secondary is open so no copper loss in secondary).
- Hence the no load input primary current does not lag 90° behind the applied voltage V_1 but lags behind $\Phi_0 < 90^\circ$.

Theory of Practical Transformer ..

Practical Transformer on No Load

Phasor Diagram of Practical Transformer on NO load:



- As seen from the phasor diagram, the primary current (I_0) has two components. (i) I_w which is in phase with V_1 , is known as active, working or loss component because it mainly supplies iron loss plus very small copper loss $I_w = I_0 \cos(\Phi_0)$.

(ii) The other component I_μ in quadrature with V_1 is known as magnetising component because its function is to sustain alternating flux in the core. $I_\mu = I_0 \sin(\Phi_0)$

(2) PI Note: Numbers (1) to (6) in the phasor diagram indicate steps /sequence to draw various quantities.

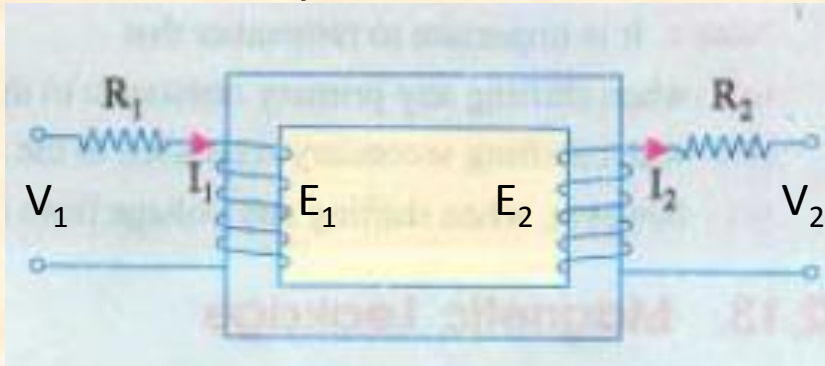
Important Points about the no load Primary current (I_0):

- The No load primary current (I_0) is very small compared to full load (I_F) current. $I_0 = 1\%$ of I_F .
- Since I_0 is very small and also the copper loss is negligible, hence the primary input current at no load is practically equal to iron loss of the transformer.
- Since the iron loss is responsible for the shift in I_0 current vector, hence the angle Φ_0 is known as hysteresis angle of advance.

Theory of Practical Transformer ..

2. Consideration of primary and secondary winding resistance

- An ideal transformer has zero winding resistance however the practical transformer has always a finite resistance of primary and secondary windings.



- Due to this resistance there is finite voltage drop in the two windings. The results is that

(i) The secondary terminal voltage V_2 is vectorically less than secondary induced emf E_2 by an amount $I_2 R_2$ where R_2 is resistance of secondary.

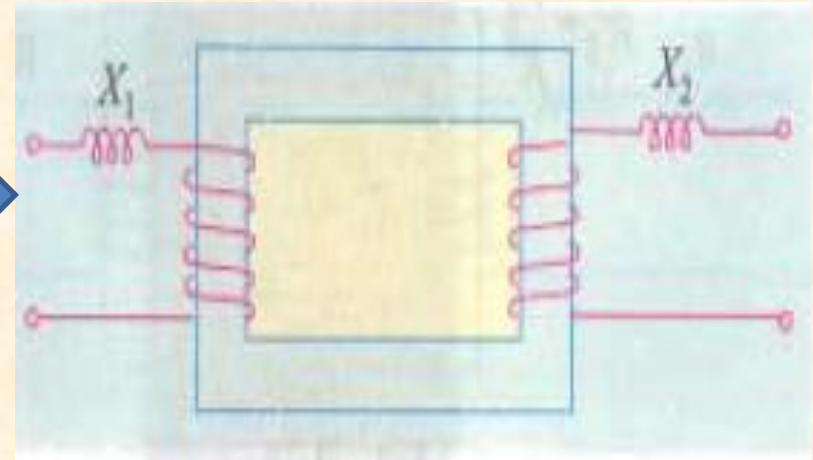
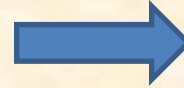
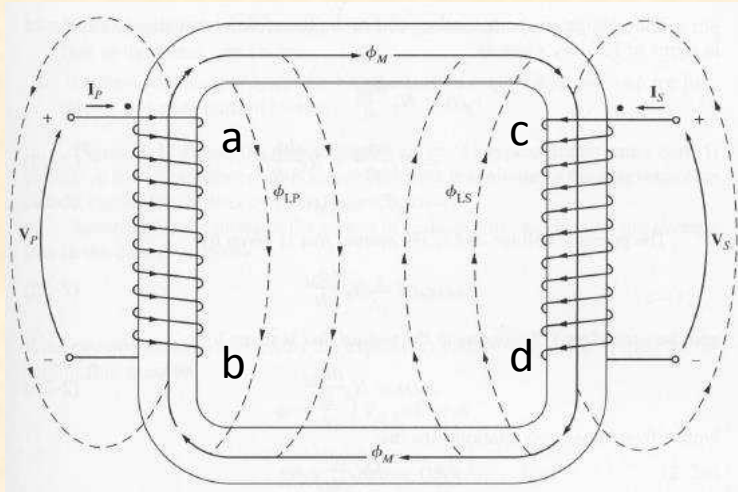
$$V_2 = E_2 - I_2 R_2 \text{ ----- (note its vector difference)}$$

(ii) Similarly, the primary induced emf E_1 is vectorically less an applied voltage V_1 by an amount $I_1 R_1$ where R_1 is resistance of primary.

$$E_1 = V_1 - I_1 R_1 \text{ ----- (note its vector difference)}$$

Theory of Practical Transformer ..

3. Consideration of magnetic Leakage at primary and secondary



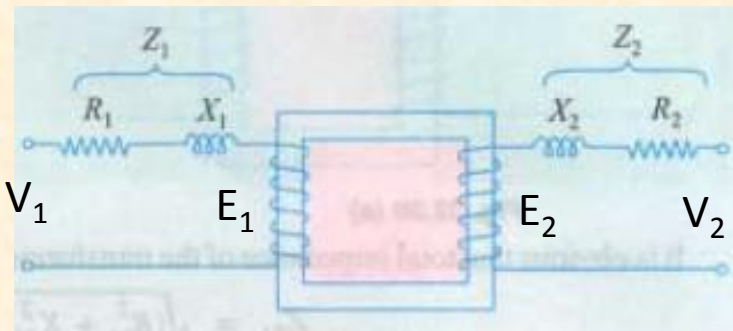
- All the flux linked with primary does not link with secondary but part of it i.e. Φ_{L1} completes its magnetic circuit by passing through air rather than through core. This leakage flux is produced when the m.m.f due to primary ampere-turns existing between points a and b acts along leakage path. Hence this flux is known as primary leakage flux and is proportional to primary ampere-turns alone and because secondary turns do not link magnetic circuit of Φ_{L1} . The flux Φ_{L1} is in phase with I_1 and it induces emf e_{L1} in Primary.
- Similarly secondary ampere-turns existing between points c and d set up flux Φ_{L2} . The flux Φ_{L2} is in phase with I_2 and it induces emf e_{L2} in secondary.
- The leakage flux induces emf in respective winding hence the leakage fluxes are represented by a small inductive coil in series with each winding.
As indicated in diagram reactances $X_1 = e_{L1}/I_1$ and $X_2 = e_{L2}/I_2$

Theory of Practical Transformer ..

3. Consideration of magnetic Leakage at primary and secondary..

Important points about leakage reactance:

- The leakage flux links one or the other winding but not both, hence it in no way contributes to the transfer of energy from primary to secondary.
- The primary voltage V_1 will have to supply reactive drop $I_1 X_1$ in addition to $I_1 R_1$. Similarly E_2 will have to supply $I_2 X_2$ in addition to $I_2 R_2$.
- The primary and secondary windings are placed close to reduce leakage flux.



- The primary and secondary impedances Z_1 and Z_2
 $Z_1 = \sqrt{R_1^2 + X_1^2}$ $Z_2 = \sqrt{R_2^2 + X_2^2}$
- The resistance and leakage reactance are responsible for voltage drops in each winding.

$$V_1 = E_1 + I_1 (R_1 + jX_1) = E_1 + I_1 Z_1$$

.....note its vector addition)

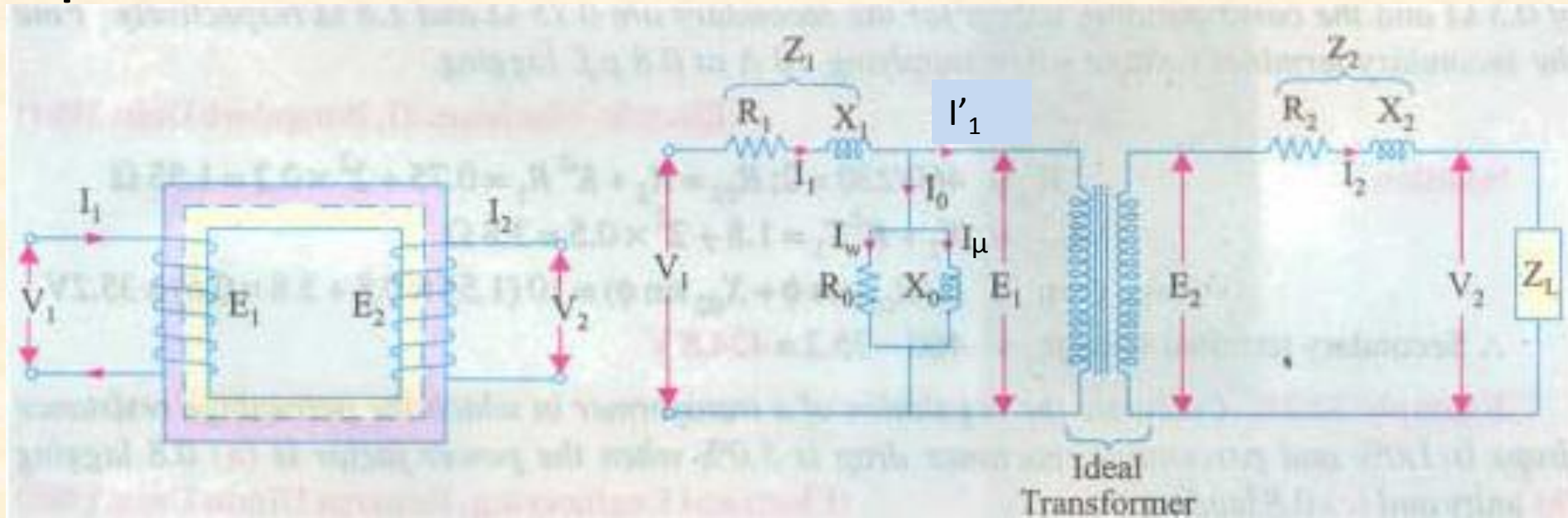
Similarly, there are $I_2 R_2$ and $I_2 X_2$ drops in secondary which combine with V_2 to give E_2 .

$$E_2 = V_2 + I_2 (R_2 + jX_2) = V_2 + I_2 Z_2$$

.....note its vector addition)

Theory of Practical Transformer ..

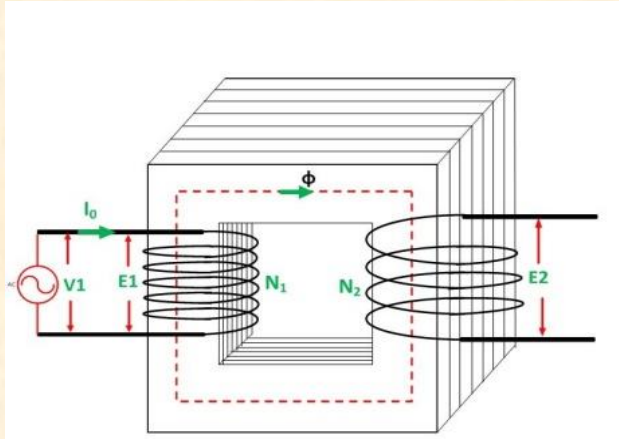
Equivalent Circuit of the Transformer



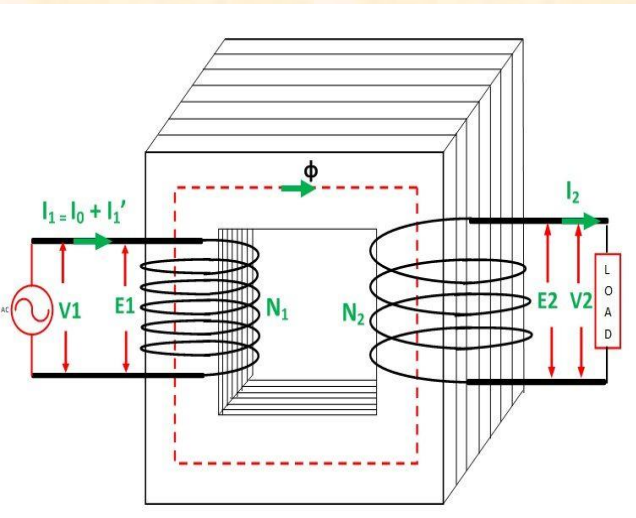
- The exact equivalent circuit is shown above where the winding resistance and leakage reactance of the winding are drawn in series with respective winding and are imagined to be external to the winding.
- The no load primary current I_0 is simulated by a pure inductance X_0 taking the magnetising component I_μ and non inductive resistance R_0 taking working component I_w , connected in parallel across primary circuit.
- The value of E_1 and V_2 are obtained by vector subtractions as $E_1 = V_1 - I_1 R_1$ and $V_2 = E_2 - I_2 R_2$
i.e. $\vec{E}_1 = \vec{V}_1 - \vec{I}_1 \vec{R}_1$ and $\vec{V}_2 = \vec{E}_2 - \vec{I}_2 \vec{R}_2$
- E_1 and E_2 are related as $E_2/E_1 = N_2/N_1 = K$

Theory of Practical Transformer ..

➤ Operation of the Transformer on Load Condition



- When secondary of the transformer is kept open, it draws the no-load current from the main supply. The no-load current induces the magnetomotive force $N_0 I_0$ and this force set up the flux Φ in the core of the transformer. The circuit of the transformer at no load condition is shown in the figure below.



- When the load is connected to the secondary of the transformer, the I_2 current flows through the secondary winding. The secondary current induces the magnetomotive force $N_2 I_2$ on the secondary winding of the transformer. This force set up the flux Φ_2 in the transformer core. The flux Φ_2 oppose the flux Φ , according to Lenz's law

Theory of Practical Transformer ..

➤ Operation of the Transformer on Load Condition

- As the flux ϕ_2 opposes the flux ϕ , the resultant flux of the transformer decreases and this flux reduces the induced EMF E_1 . Thus, the strength of the V_1 is more than E_1 and an additional primary current I'_1 drawn from the main supply. The additional current is used for restoring the original value of the flux in the core of the transformer so that the $V_1 = E_1$. The primary current I'_1 is in phase opposition with the secondary current I_2 . Thus, it is called the primary counter balancing current.
- The additional current I'_1 induces the magnetomotive force $N_1 I'_1$. And this force set up the flux ϕ'_1 . The direction of the flux is same as that of the ϕ and it cancels the flux ϕ_2 which induces because of the MMF $N_2 I_2$.

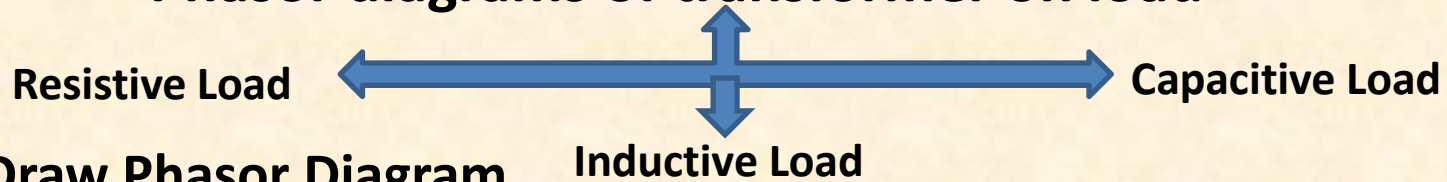
Now, $N_1 I'_1 = N_2 I_2$ Therefore,
$$I'_1 = \left(\frac{N_2}{N_1} \right) I_2 = K I_2$$

- The phasor difference between V_1 and I_1 gives the power factor angle ϕ_1 of the primary side of the transformer.
- The power factor of the secondary side depends upon the type of load connected to the transformer. if the load is inductive as shown in the above phasor diagram, the power factor will be lagging, and if the load is capacitive, the power factor will be leading. The total primary current I_1 is the vector sum of the current I_0 and I'_1 . i.e

$$\overline{I_1} = \overline{I_0} + \overline{I'_1}$$

Theory of Practical Transformer ..

Phasor diagrams of transformer on load

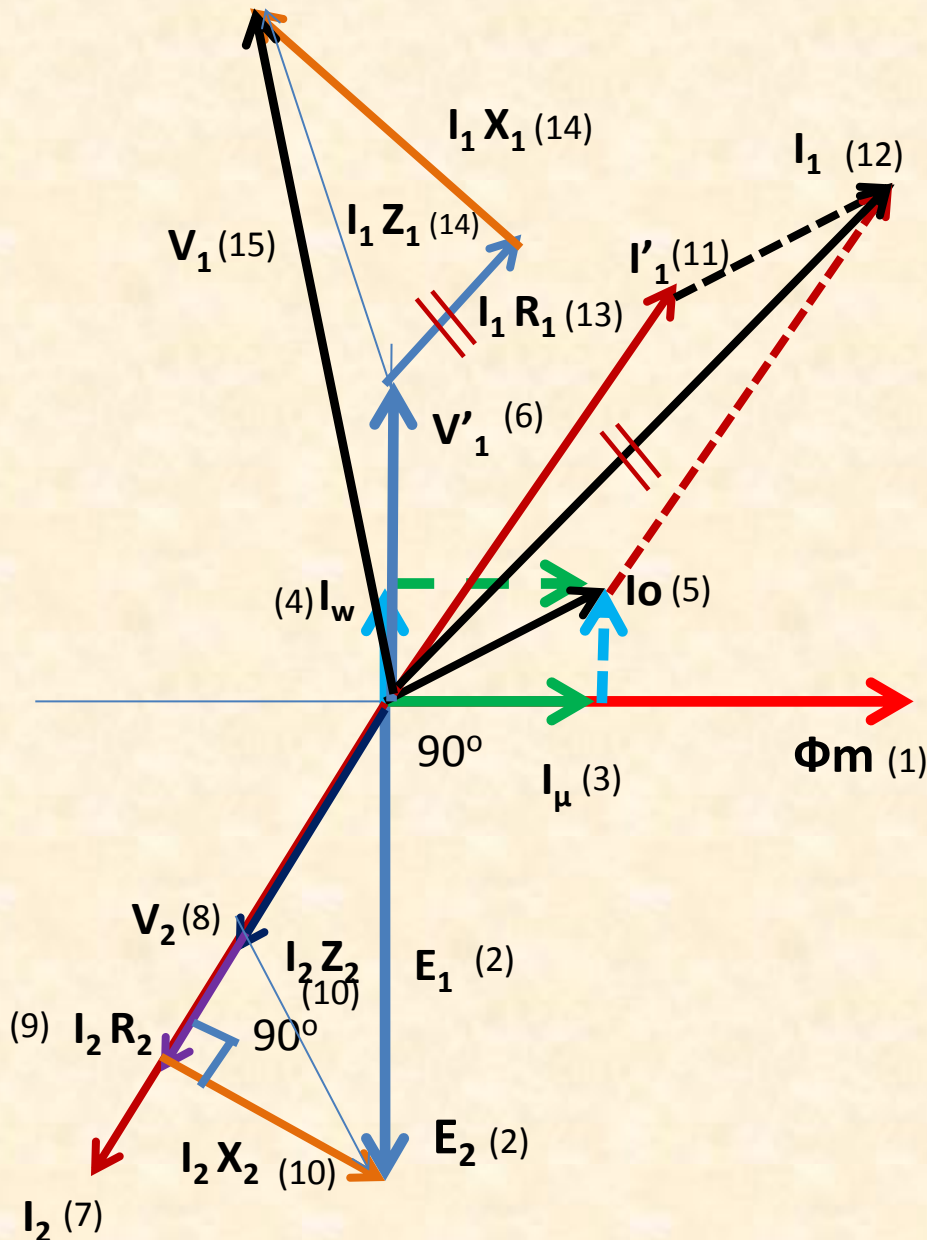


Steps to Draw Phasor Diagram

- Take flux ϕ a reference \rightarrow Draw I_0 as vector sum of I_w and I_μ i.e. $(\vec{I_0} = \vec{I_w} + \vec{I_\mu})$
- Induces emf E_1 and E_2 lags the flux by 90 degrees.
- The component of the applied voltage to the primary equal and opposite to induced emf in the primary winding. E_1 is represented by V_1' .
- Current I_0 lags the voltage V_1' by Φ_0 degrees.
- Draw current I_2 is drawn w.r.t E_2 based on type of load (resistive, inductive or capacitive)
- The resistance and the leakage reactance of the windings result in a voltage drop, and hence secondary terminal voltage V_2 is the phasor difference of E_2 and voltage drop. $\vec{E_2} = \vec{V_2} + \vec{I_2 R_2}$. Here $I_2 R_2$ is in phase with I_2 and $I_2 X_2$ is in quadrature with I_2 .
- The total current flowing in the primary winding is the phasor sum of I_1' and I_0 . where Current I_1' is drawn equal and opposite to the current I_2
- Primary applied voltage V_1 is the phasor sum of V_1' and the voltage drop in the primary winding.
- $V_1 = V_1' +$ voltage drop $I_1 R_1$ is in phase with I_1 and $I_1 X_1$ is in quadrature with I_1 .
- The phasor difference between V_1 and I_1 gives the power factor angle ϕ_1 of the primary side of the transformer.

Theory of Practical Transformer ..

1. Resistive Load: The numbers in bracket indicates steps to draw phasor diagram .

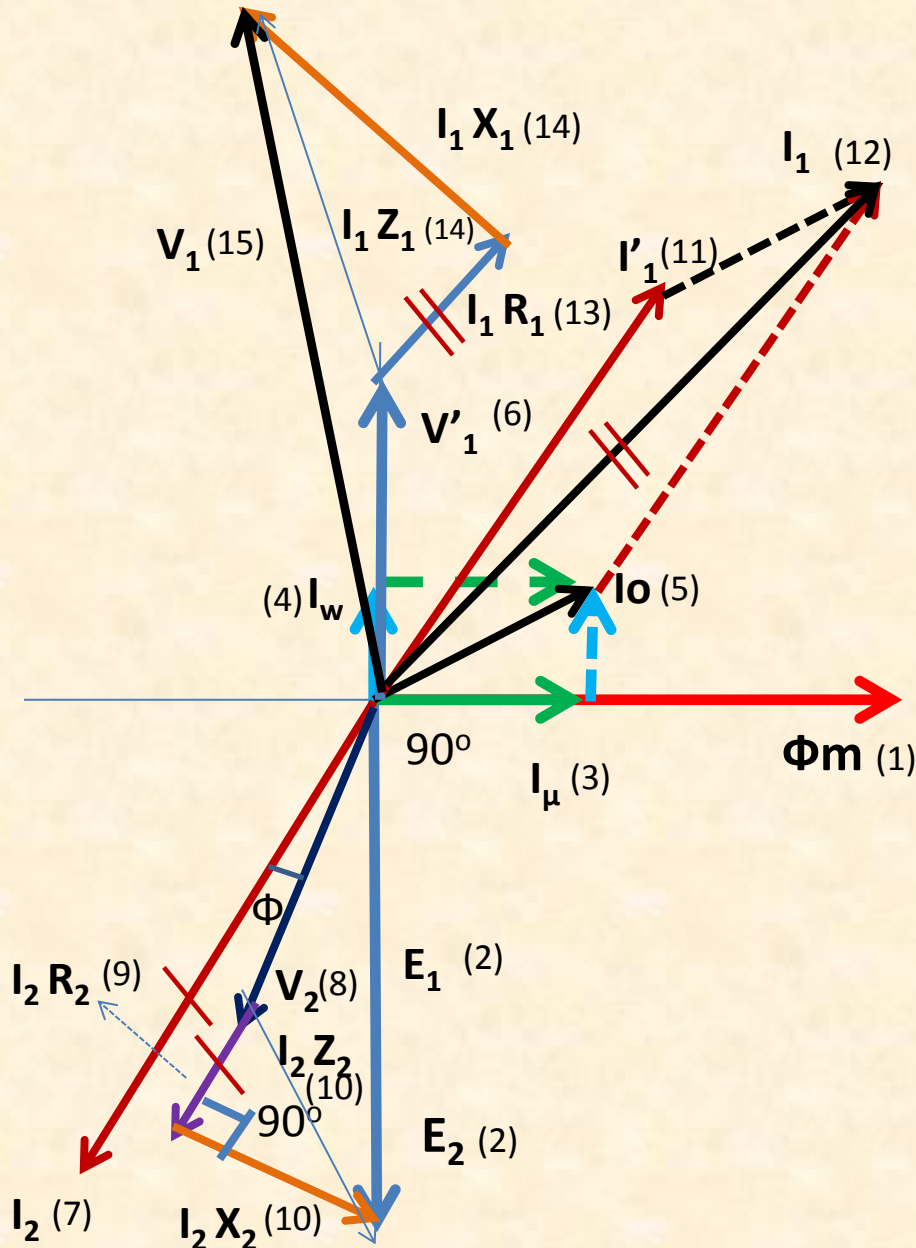


Steps

- flux ϕ_m a reference (1)
- E_1 and E_2 lags the flux by 90 degrees (2)
- I_w and I_μ and I_0 ($\vec{I_0} = \vec{I_w} + \vec{I_\mu}$) (3,4,5)
- E_1 is represented by V'_1 . (6)
- Draw current I_2 (7)
- V_2 in phase with I_2 (resistive Load) (8)
- $I_2 R_2$ in phase with I_2 (9)
- $I_2 X_2$ in 90° w.r.t I_2 (10)
- $\vec{E_2} = \vec{V_2} + \vec{I_2 Z_2}$. (10)
- Current I'_1 is drawn equal and opposite to the current I_2 (11)
- ($\vec{I_1} = \vec{I_0} + \vec{I'_1}$) (12)
- voltage drop $I_1 R_1$ is in phase with I_1 (13)
- $I_1 X_1$ is in quadrature with I_1 (14)
- $\vec{I_1 Z_1} = \vec{I_1 R_1} + \vec{I_1 X_1}$. (14)
- $\vec{V_2} = \vec{V'_1} + \vec{I_1 Z_1}$ (15)

Theory of Practical Transformer ..

2. Inductive Load: The numbers in bracket indicates steps to draw phasor diagram .

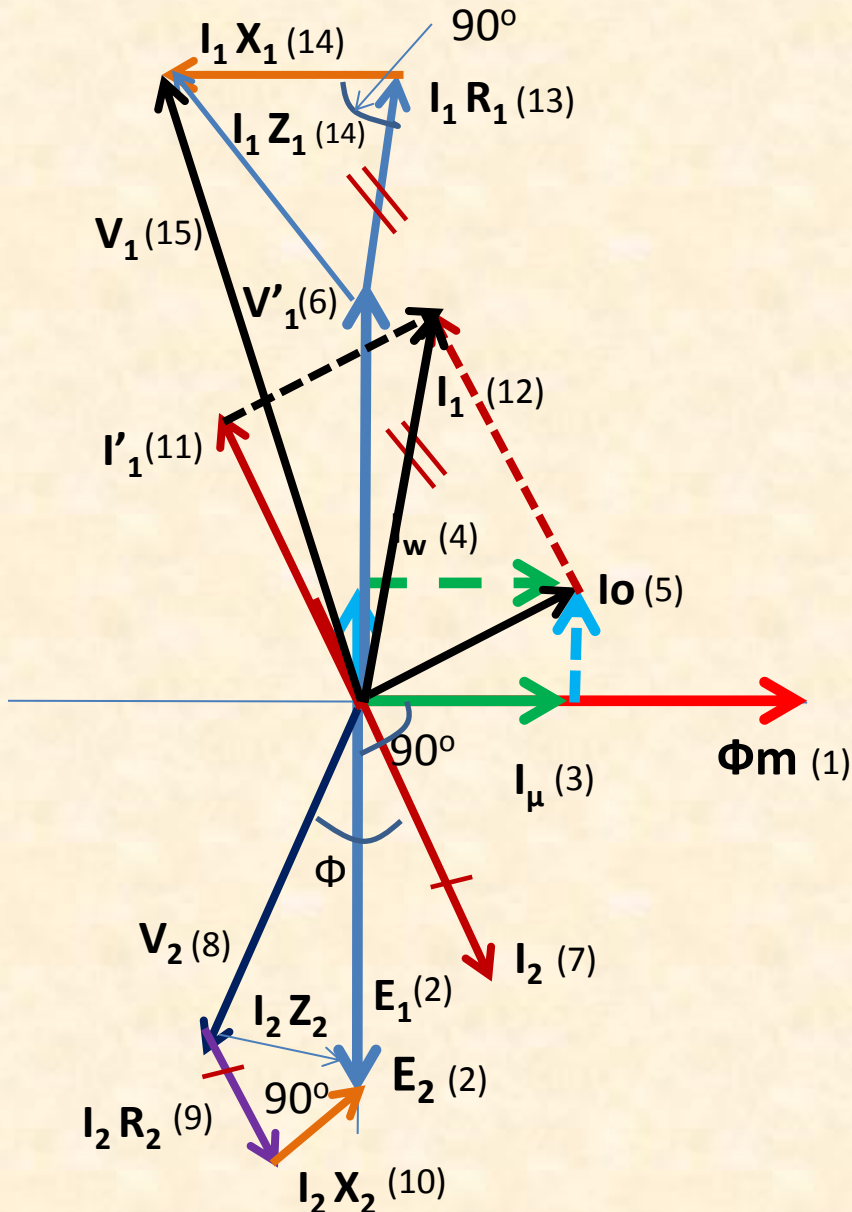


Steps

- flux ϕ_m a reference (1)
- E_1 and E_2 lags the flux by 90° (2)
- I_w and I_μ and I_0 ($\vec{I_0} = \vec{I_w} + \vec{I_\mu}$) (3,4,5)
- E_1 is represented by V_1' . (6)
- Draw current I_2 (7)
- V_2 leading with I_2 (inductive Load) (8)
- I_2R_2 in phase with I_2 (9)
- I_2X_2 in 90° w.r.t I_2 (10)
- $\vec{E_2} = \vec{V_2} + \vec{I_2Z_2}$. (10)
- Current I_1' is drawn equal and opposite to the current I_2 (11)
- ($\vec{I_1} = \vec{I_0} + \vec{I_1'}$) (12)
- voltage drop I_1R_1 is in phase with I_1 (13)
- I_1X_1 is in quadrature with I_1 (14)
- $\vec{I_1Z_1} = \vec{I_1R_1} + \vec{I_1X_1}$. (14)
- $\vec{V_2} = \vec{V_1'} + \vec{I_1Z_1}$ (15)

Theory of Practical Transformer ..

3. Capacitive Load: The numbers in bracket indicates steps to draw phasor diagram .



Steps

- flux ϕ_m a reference (1)
- E_1 and E_2 lags the flux by 90 degrees (2)
- I_w and I_μ and I_o ($\vec{I_o} = \vec{I_w} + \vec{I_\mu}$) (3,4,5)
- E_1 is represented by V_1' . (6)
- Draw current I_2 (7)
- I_2 leading with V_2 (Capacitive Load) (8)
- $I_2 R_2$ in phase with I_2 (9)
- $I_2 X_2$ in 90° w.r.t I_2 (10)
- $\vec{E_2} = \vec{V_2} + \vec{I_2 Z_2}$. (10)
- Current I_1' is drawn equal and opposite to the current I_2 (11)
- ($\vec{I_1} = \vec{I_o} + \vec{I_1'}$) (12)
- voltage drop $I_1 R_1$ is in phase with I_1 (13)
- $I_1 X_1$ is in quadrature with I_1 (14)
- $\vec{I_1 Z_1} = \vec{I_1 R_1} + \vec{I_1 X_1}$. (14)
- $\vec{V_2} = \vec{V_1'} + \vec{I_1 Z_1}$ (15)

Theory of Practical Transformer ..

Efficiency of Transformer:

efficiency of a transformer can be defined as the output power divided by the input power.

$$\text{efficiency, } \eta = \frac{\text{Output Power}}{\text{Input Power}} \times 100\%$$

$$= \frac{\text{Input Power} - \text{Losses}}{\text{Input Power}} \times 100\%$$

$$= 1 - \frac{\text{Losses}}{\text{Input Power}} \times 100\%$$

Condition for maximum efficiency:

Copper loss = $I_1^2 R_1$ and Iron loss = W_i

$$\text{efficiency} = 1 - \frac{\text{losses}}{\text{input}} = 1 - \frac{I_1^2 R_1 + W_i}{V_1 I_1 \cos \Phi_1}$$

$$\eta = 1 - \frac{I_1 R_1}{V_1 \cos \Phi_1} - \frac{W_i}{V_1 I_1 \cos \Phi_1}$$

differentiating above equation with respect to I_1

$$\frac{d\eta}{dI_1} = 0 - \frac{R_1}{V_1 \cos \Phi_1} + \frac{W_i}{V_1 I_1^2 \cos \Phi_1}$$

$$\eta \text{ will be maximum at } \frac{d\eta}{dI_1} = 0$$

Hence efficiency η will be maximum at

$$\frac{R_1}{V_1 \cos \Phi_1} = \frac{W_i}{V_1 I_1^2 \cos \Phi_1}$$

$$\frac{I_1^2 R_1}{V_1 I_1^2 \cos \Phi_1} = \frac{W_i}{V_1 I_1^2 \cos \Phi_1}$$

$$I_1^2 R_1 = W_i$$

When Copper loss = iron loss the efficiency of the transformer is maximum.

Theory of Practical Transformer ..

ALL-DAY EFFICIENCY:

It is defined as the ratio of the energy (kilowatt-hours) delivered by the transformer in a 24-hour period to the energy input in the same period of time. To determine the all-day efficiency, it is necessary to know how the load varies from hour to hour during the day.

$$\text{All day efficiency} = \frac{\text{output (in kWh)}}{\text{input (in kWh)}} \quad (\text{for 24 hours})$$

All day efficiency of a transformer is always less than ordinary efficiency of it.

Theory of Practical Transformer ..

Voltage Regulation of the Transformer

- The voltage regulation of a transformer is defined as the change in secondary terminal voltage when the transformer loading is at its maximum, i.e. full-load applied while the primary supply voltage is held constant. Regulation determines the voltage drop (or increase) that occurs inside the transformer as the load voltage becomes too low as a result of the transformers loading being too high which therefore affects its performance and efficiency.
- Voltage regulation is expressed as a percentage (or per unit) of the no-load voltage. Then if E represents the no-load secondary voltage and V represents the full-load secondary voltage, the percentage regulation of a transformer is given as:

$$\text{Voltage Regulation} = \frac{E_2 - V_2}{E_2}$$

$$\% \text{ Voltage Regulation} = \frac{E_2 - V_2}{E_2} \times 100$$

Theory of Practical Transformer ..

Transformer Voltage Regulation...

$$\text{Voltage regulation} = \frac{E_2 - V_2}{\text{Secondary rated voltage}} \times 100$$

the secondary rated voltage of a transformer is equal to the secondary voltage at no load i.e. E_2

$$\begin{aligned} \text{So percentage regulation} &= \frac{E_2 - V_2}{E_2} \times 100 = \frac{\text{Voltage drop in transformer at full load}}{\text{No-load rated secondary voltage, } E_2} \times 100 \\ &= \frac{I_2 R_{02} \cos \phi + I_2 X_{02} \sin \phi}{E_2} \times 100 \end{aligned}$$

When the power factor is leading, the percentage regulation is given as

$$\text{Percentage regulation} = \frac{I_2 R_{02} \cos \phi - I_2 X_{02} \sin \phi}{E_2} \times 100$$

The percentage regulation can be given as

$$\% \text{ regulation} = \frac{I_2 R_{02} \cos \phi \pm I_2 X_{02} \sin \phi}{E_2} \times 100$$

+ ve sign for lagging power factor and
- ve sign for leading power factor

$$= \left(\frac{I_2 R_{02}}{E_2} \cos \phi \pm \frac{I_2 X_{02}}{E_2} \sin \phi \right) \times 100$$