25.04.2021

Домашняя работа

№ 1.2.

1) $y = x^4$

$(y)' = 4x^3$

$(y)'' = 12x^2$

2) $y = 3 - 5x + x^3$

$(y)' = -5 + 3x^2$

$(y)'' = 6x$

3) $y = \frac{1}{x-1}$

$(y)' = -\frac{1}{(x-1)^2} = \frac{-1}{(x-1)^2}$

$(y)'' = (-(x-1)^{-2})' = 2(x-1)^{-3} =$

$= \frac{2}{(x-1)^3}$

4) $y = \sqrt[3]{x}$

$(y)' = \frac{1}{3\sqrt[3]{x^2}}$

$(y)'' = \frac{1}{3} \cdot (-\frac{2}{3}) \cdot x^{-\frac{5}{3}} =$

$= -\frac{2}{9x\sqrt[3]{x^2}}$

5) $y = (1-3x)^2$

$(y)' = -3 \cdot 3(1-3x)^2 =$

$= -9(1-3x)^2$

$(y)'' = -18(-3)(1-3x) =$

$= 54 - 162x$

6) $y = \cos 2x$

$(y)' = (\cos 2x)' = -2\sin 2x$

$(y)'' = -4\cos 2x$

7) $y = \sin^2 x$

$(y)' = 2\sin x \cdot \cos = \sin 2x$

$(y)'' = 2\cos 2x$

8) $y = x\cos x$

$(y)' = (x')\cos x + (\cos x)'x =$

$= \cos x - \sin x \cdot x$

$(y)'' = (-\sin x) - ((\sin x)' \cdot x +$

$+ (x)' \sin x) = -\sin x - \cos x \cdot 1 -$

$- \sin x = -2\sin x - \cos x$

№1.8.

$S_1(t) = t^3 - t^2 + 3t - 2$

$v_1(t) = 3t^2 - 2t + 3$

$3t^2 - 2t + 3 = t^2 + t + 5$

$2t^2 - 3t - 2 = 0$

$\begin{cases} t_1 = \frac{4}{2} = 2 \\ t_2 = -\frac{1}{2} \end{cases}$

$a_1(t) = 6t - 2$

$a_1(t_1) = 12 - 2 = 10 \left(\frac{м}{с^2}\right)$

$a_2(t_2) = 6\left(-\frac{1}{2}\right) - 2 = -3 - 2 = -5 \left(\frac{м}{с^2}\right)$

$S_2(t) = \frac{t^3}{3} + \frac{t^2}{2} + 5t - 8$

$v_2(t) = t^2 + t + 5$

$a_2(t) = 2t + 1$

$2 \cdot t + 1 = 0$

$t_3 = -\frac{1}{2}$

$a_2(t_3) = 0 \left(\frac{м}{с^2}\right)$

$t_4 = 2$

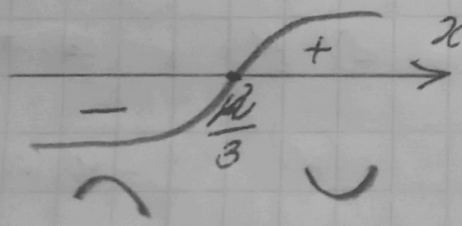
$a_2(t_4) = 5 \left(\frac{м}{с^2}\right)$

№1.11

1) $y = x^3 - 2x^2 + x - 2$

$(y)' = 3x^2 - 4x + 1$

$(y)'' = 6x - 4 = 0$



Опукла вгору $f(x)$:

$x \in (-\infty; \frac{2}{3}]$

Опукла вниз $f(x)$:

$x \in [\frac{2}{3}; +\infty)$

$x_0 = \frac{2}{3}$

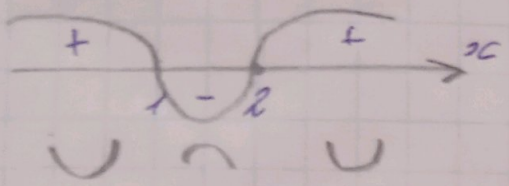
2) $y = x^4 - 6x^3 + 12x^2 - 5x + 4$

$(y)' = 4x^3 - 18x^2 + 24x - 3$

$(y)'' = 12x^2 - 36x + 24 = 0$

$x^2 - 3x + 2 = 0$

$\left[\begin{array}{l} x = 2 \\ x = 1 \end{array}\right.$



Опукла вгору $f(x)$:

$x \in [1; 2]$

Опукла вниз $f(x)$:

$x \in (-\infty; 1] \cup [2; +\infty)$

$x_0 = 1, 2$

№ 51.13

$y = 3x^5 + 10x^4 + 10x^3 - 5x - 4$

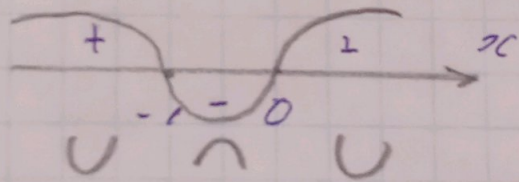
$(y)' = 15x^4 + 40x^3 + 30x^2 - 5$

$(y)'' = 60x^3 + 120x^2 + 60x = 0$

$60x(x^2 + 2x + 1) = 0$

$60x(x + 1)^2 = 0$

$\left[\begin{array}{l} x = 0 \\ x = -1 \end{array}\right.$



Опукла вниз $f(x)$:

$x \in (-\infty; -1] \cup [0; +\infty)$

Опукла вгору $f(x)$:

$x \in [-1; 0]$

$x_0 = -1; 0$

№ 51.15

$f(x) = \sin^2 x - 2x^2$

$f'(x) = 2 \sin x \cos x - 4x = \sin 2x - 4x$

$f''(x) = 2 \cos 2x - 4 = 0$

$\cos 2x = 2$ Коренів немає

$f''(x) < 0, x \in \emptyset$ Доведено