Finite element computation of two-phase flow with level set method and explicit interface detection

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A joint work with Atsushi SUZUKI (Osaka University)

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Université Pierre et Marie Curie (UPMC)

Toyama est dans la direction de nord-ouest de Tokyo.

”Toyama” veut dire qu’il y a beaucoup de montagnes en japonais. (mont Tsurugi, 3003m)

Outline

1. Numerical analysis of two-fluid flows by FreeFem++ and reinitialization procedure (K. OHMORI)
2. Finite element computation of two-phase flow with level set method and explicit interface detection (A. SUZUKI)

Model problem

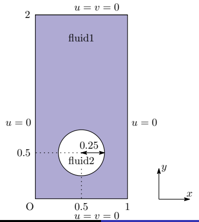
In this presentation, we present a numerical study of the immscible incompressible two-fluid flows by FreeFem++.
$\Omega \subset R^2$: a bounded domain such that
$\overline{\Omega} = \overline{\Omega_1(t)} \cup \overline{\Omega_2(t)}$ and $\Omega_1(t) \cap \Omega_2(t) = \emptyset.$

$$\Gamma(t) = \partial\Omega_1(t) \cap \partial\Omega_2(t) : \text{the interface between two fluids} \qquad (1)$$

- the density and viscosity

$$(\rho,\mu) = \begin{cases} (\rho_1,\mu_1) & \text{in } \Omega_1 \\ (\rho_2,\mu_2) & \text{in } \Omega_2 \end{cases}$$

,where $\rho_1 > \rho_2$ and $\mu_1 > \mu_2$



Mathematical Model

Immiscible incompressible two-fluid system

$$\begin{cases} \rho(\dfrac{\partial u}{\partial t}+(u\cdot\nabla)u)=-\nabla p+\nabla\cdot(2\mu D(u))+f+F_{SF} & \text{in } \Omega, \\ \nabla\cdot u=0 & \text{in } \Omega, \\ u=0 \quad \textit{(no slip condition)} & \text{on } \Gamma_{TB}, \\ u\cdot n=0 \ \textit{(free slip condition)} & \text{on } \Gamma_{LR}, \end{cases} \tag{2}$$

where $D(u)=(\nabla u+{}^t\nabla u)/2$: the viscous stress tensor,
f : the force term ($f={}^t(0 \quad -\rho g)$)
n : outer normal vector of domain Ω_1
κ : the curvature of the interface
σ : the surface tension coefficient
F_{SF} : the surface tension term.

- Transmission conditions on the interface $\Gamma(t)$:
  $[u]|_\Gamma = 0, \qquad [2\mu D(u)n - pn]|_\Gamma = \sigma\kappa n.$

Surface tension effects are taken into consideration through the following force balance at the interface $\Gamma(t)$.

The surface tension term is expressed as

$$F_{SF} = \sigma\kappa n\delta_\Gamma$$

where δ_Γ is the distribution and it holds

$$\int_\Omega \delta_\Gamma \varphi \, dx = \int_\Gamma \varphi \, d\gamma$$

for any smooth function φ.

Level set method for two-fluid flows

In level set method the interface is evolved by solving the advection equation.
The interface is define as follows

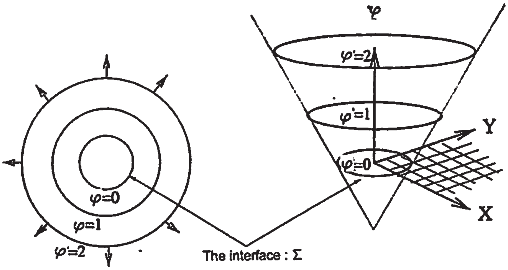
$$\Gamma(t) = \{x \in \Omega;\ \varphi(x,t) = 0\} \text{ for } 0 \leq t < T.$$

Initial level set function $\varphi(x,0)$: a signed distance from the interface $\Gamma(0)$

$$\varphi(x,0) = \begin{cases} \mathrm{dist}(x, \Gamma(t) \cup \partial\Omega) & \text{for } x \in \Omega_1(0), \\ 0 & \text{for } x \in \Gamma(0), \\ -\mathrm{dist}(x, \Gamma(t) \cup \partial\Omega) & \text{for } x \in \Omega_2(0), \end{cases} \tag{3}$$

where $\mathrm{dist}(x,\Gamma) = \min\limits_{y \in \Gamma} |x - y|$.
In the level set method, it is common to define the curvature as the divergence the normal

$$\begin{cases} n = \dfrac{\nabla\varphi}{|\nabla\varphi|}\Big|_{\varphi=0} \\ \kappa = -\nabla \cdot n = -\dfrac{\varphi_y^2 \varphi_{xx} - 2\varphi_x \varphi_y \varphi_{xy} + \varphi_x^2 \varphi_{yy}}{(\varphi_x^2 + \varphi_y^2)^{3/2}}. \end{cases} \tag{4}$$


φ
φ'=2
φ'=1
Y
φ=0
X
φ=0
φ=1
φ'=2
The interface : Σ

Variational formulation for two-fluid flows

$$
\begin{cases}
V = \{v \in H^1(\Omega)^2 \mid v = 0 \text{ on } \Gamma_1,\ v \cdot n = 0 \text{ on } \Gamma_2\}, \\
Q = \{q \in L^2(\Omega) \mid \int_\Omega q\, dx = 0\}, \\
\Psi = H^1(\Omega).
\end{cases}
\tag{5}
$$

Find $(u(t), p(t), \varphi(t)) \in V \times Q \times \Psi$ for $0 < t < T$ such that

$$
(\Pi)
\begin{cases}
\displaystyle\int_\Omega \rho(\varphi)\left\{\frac{\partial u}{\partial t} + (u\cdot\nabla)u\right\}\cdot v\, dx + 2\int_\Omega \mu(\varphi) D(u) : D(v)\, dx \\
\displaystyle -\int_\Omega p\, \nabla\cdot v\, dx - \int_\Omega q\, \nabla\cdot u\, dx \\
\displaystyle \qquad = \int_\Omega f\cdot v\, dx + \sigma \int_\Gamma \kappa n \cdot v\, d\gamma \qquad \text{for } \forall (v,q) \in V\times Q. \\
\displaystyle\int_\Omega \left\{\frac{\partial \varphi}{\partial t} + u\cdot\nabla\varphi\right\}\xi\, dx = 0 \qquad \text{for } \forall \xi \in \Psi.
\end{cases}
\tag{6}
$$

Reinitialization - Hyperbolic PDE approach

Sussman, Smereka and Osher (1994)

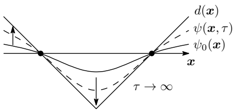
The signed distance function

$$d(x) = \begin{cases} dist(x,\Gamma) & \text{for } x \in \Omega_1, \\ 0 & \text{for } x \in \Gamma, \\ -dist(x,\Gamma) & \text{for } x \in \Omega_2. \end{cases} \quad (7)$$

has the useful property

$$|\nabla d(x)| = 1 \quad (\textit{Eikonal equation}).$$

However, after the advection step, because of error accumulation the level set function is no longer a distance function.



Hamilton-Jacobi type equation for pseudo time τ,

$$\begin{cases} \dfrac{\partial \psi}{\partial \tau} + S(\psi_0)(|\nabla \psi| - 1) = 0 \quad \text{in } \Omega, \\ \psi(x,0) = \psi_0 = \varphi(x,t), \end{cases} \tag{8}$$

where $S(\psi_0)$ is a sign function such that

$$S(\psi_0) = \begin{cases} -1 & \text{for } \psi < 0, \\ 0 & \text{for } \psi = 0, \\ 1 & \text{for } \psi > 0. \end{cases} \tag{9}$$

In practice, $S(\psi_0)$ is approximate by

$$S^*(\psi) = \frac{\psi_0}{\sqrt{\psi_0^2 + \varepsilon}}.$$

If ψ is steady state attained at $\tau = \tau_\infty$, the redistanced function ψ would be expected to satisfy

$$|\nabla \psi| = 1.$$

Subsequently, φ is recovered as $\varphi(x,t) = \psi(x,\tau_\infty)$.

Semi-discretization

$\{T_h\}_{h>0}$: a regular family of triangulation
$V_h \subset V$ and $Q_h \subset Q$: a pair of finite element spaces which satisfies uniform inf-sup condition
$\Psi_h \subset \Psi$.

Find $(u_h, p_h, \varphi_h) \in V_h \times Q_h \times \Psi_h$ for $0 < t < T$ such that

$$
(\Pi') \begin{cases} \displaystyle\int_\Omega \rho(\varphi_h)\left\{\frac{\partial u_h}{\partial t} + (u_h\cdot\nabla)u_h\right\}\cdot v_h\,dx + 2\int_\Omega \mu(\varphi_h)D(u_h):D(v_h)dx \\ \displaystyle -\int_\Omega p_h\,\nabla\cdot v_h\,dx - \int_\Omega q_h\,\nabla\cdot u_h\,dx \\ \displaystyle\qquad = \int_\Omega f\cdot v_h\,dx + \sigma\int_\Omega \delta_\varepsilon(\varphi_h)\kappa n\cdot v_h\,dx \quad \text{for } \forall(v_h,q_h)\in V_h\times Q_h \\ \displaystyle\int_\Omega \left\{\frac{\partial \varphi_h}{\partial t} + u_h\cdot\nabla\varphi_h\right\}\xi_h\,dx = 0 \qquad \text{for } \forall \xi_h \in \Psi_h. \end{cases} \tag{10}
$$

The surface integration on Γ for the surface tension term is replaced by a domain integration :

$$\sigma \int_{\Gamma} \kappa n \cdot v \, d\gamma \doteq \sigma \int_{\Omega} \delta_{\varepsilon}(\varphi_h) \kappa n \cdot v_h \, dx$$

, where

$$\delta_{\varepsilon}(\varphi) = \frac{dH_{\varepsilon}}{d\varphi}(\varphi) = \begin{cases} \frac{1}{2\varepsilon}[1+\cos(\frac{\pi\varphi}{\varepsilon})] & \text{for } |\varphi| \leq \varepsilon, \\ 0 & \text{for } |\varphi| > \varepsilon. \end{cases} , \tag{11}$$

and

$$H_{\varepsilon}(\varphi) = \begin{cases} 0 & \text{for } \varphi < -\varepsilon, \\ \frac{1}{2}[1+\frac{\varphi}{\varepsilon}+\frac{1}{\pi}\sin(\frac{\pi\varphi}{\varepsilon})] & \text{for } |\varphi| \leq \varepsilon, \\ 1 & \text{for } \varphi > \varepsilon\,. \end{cases} \tag{12}$$

Full discretization

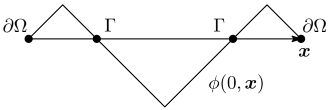
- Finite element triple (u, p, φ) - P2/P1/P2
- Stabilization for transport eq. - SUPG method

Find $(u_h^{n+1}, p_h^{n+1}, \varphi_h^{n+1}) \in V_h \times Q_h \times \Psi_h$ for $n \geq 0$ such that

$$
(\Pi_h) \left\{
\begin{aligned}
&\int_\Omega \rho^n \frac{1}{\Delta t}\left(u_h^{n+1} - u_h^n \circ \chi_h^n\right) \cdot v_h \, dx + 2\int_\Omega \mu^n D(u_h^{n+1}) : D(v_h) dx \\
&- \int_\Omega p_h^{n+1} \, \nabla \cdot v_h \, dx - \int_\Omega q_h \, \nabla \cdot u_h^{n+1} \, dx \\
&\quad = \int_\Omega f \cdot v_h \, dx + \sigma \int_\Omega \delta_\varepsilon(\widehat{\varphi_h}^n) \kappa_h^n n_h^n \cdot v_h \, dx \\
&\qquad\qquad\qquad\qquad \text{for } \forall (v_h, q_h) \in V_h \times Q_h, \\
&\int_\Omega \left(\frac{\varphi_h^{n+1} - \varphi_h^n}{\Delta t} + u_h^n \cdot \nabla \varphi_n^{n+1} \right) \left(\psi_h + \alpha u_h^n \cdot \nabla \psi_h\right) dx = 0 \\
&\qquad\qquad\qquad\qquad \text{for } \quad \forall \psi_h \in \Psi_h.
\end{aligned}
\right. \tag{13}
$$

where $\widehat{\varphi_h}^n$ is removed from φ_h^n by the following way

$$\delta_\varepsilon(\widehat{\varphi_h}^n(x)) = \begin{cases} \delta_\varepsilon(\varphi_h^n(x)) & \text{if } dist(x,\Gamma) > 2\varepsilon, \\ 0 & \text{otherwise.} \end{cases}$$



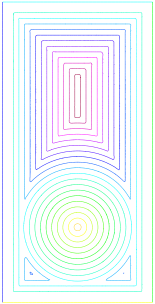


Figure: Modified contour line of level set function

Numerical examples

	ρ_1	ρ_2	μ_1	μ_2	g	σ	Re	E_o
Skirted bubble	1000	1	10	0.1	0.98	1.96	35	125

Table: Physical parameters and dimensionless numbers

The signed distance function : $\varphi_0 = \sqrt{(x-0.5)^2 + (y-0.5)^2} - 0.25,$

	$n=40$	$n=80$	$n=160$
$(\Delta t, \Delta\tau)$	(0.025, 1.0e-5)	(0.0125, 1.0e-5)	(0.00625, 1.0e-5)

Table: Time steps for computation and reinitialization

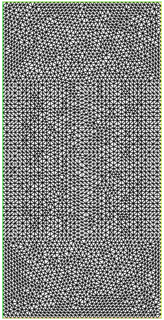


Figure: Finite element mesh (n=40)

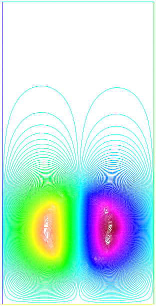


Figure: Streamline (t=0.0)

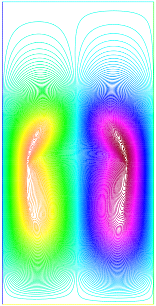


Figure: Streamline (n=80, t=3.0)

	$n = 40$	$n = 80$	$n = 160$
No. of iteration = 10	1.06535	1.07677	1.07162
No. of iteration = 20	1.06710	1.06587	1.06584
No reinitialization	1.53014	1.51965	1.57794

Table: L^1-norm of level set function for reinitialization

	$n = 40$	$n = 80$	$n = 160$
No. of iteration = 10	1.10606	1.08379	1.14104
No. of iteration = 20	1.10451	1.08257	1.15037
No reinitialization	1.10299	1.08873	1.14063

Table: Center of mass $y_c(t = 3)$, where Ω_2 is the region that the bubble occupies,

- The center of mass

$$y_c = \frac{\int_{\Omega_2} y \, d\Omega}{\int_{\Omega_2} 1 \, d\Omega}.$$

Time	$n = 40$	$n = 80$	$n = 160$
t=3.0	85/120	154/240	314/480
t=3.0	82/120	154/240	314/480

Table: Number of reinitialization

Time	$n = 40$	$n = 80$
t=3.0	0.200631	0.201418

Table: Mass conservation of the bubble

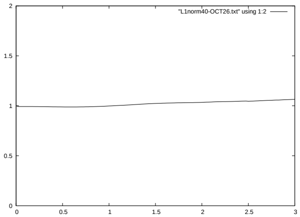


Figure: L^1-norm (n=40)

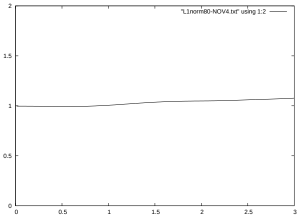
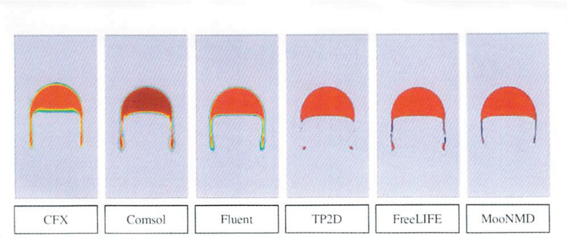
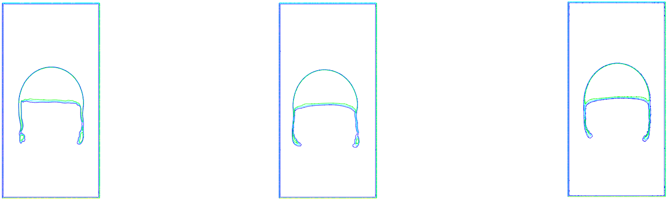


Figure: L^1-norm (n=80)



- TP2D – TU Dortmund, Inst. Applied Math.
- FreeLIFE – EPFL Lausanne, Inst. of Analysis and Sci. Comp.
- MooNMD – Uni. Magdeburg, Inst. of Analysis and Num. Math.



$n = 40$ $n = 80$ $n = 160$

Conclusion

J. Cahouet and J.-P. Chabard, Some fast 3D finite element solver for the generalized Stokes, problem, Int. J. Numer. Meth. Fluids, Vol. 8, pp.869–895(1988)

1. Robustness of the algorithm (Good)
2. Simplicity of the algorithm (Excellent)
3. No parameter to tune (No good)
4. Mass balance (Good)
5. Accuracy (Good)

So far, so good !

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Je vous remercie de votre attention.

weak formulation

$$
\begin{aligned}
V &= \{v \in H^1(\Omega)^2\,;\, v\cdot n = 0 \text{ on } \Gamma_{\mathsf{RL}},\, v = 0 \text{ on } \Gamma_{\mathsf{TB}}\},\\
Q &= \{q \in L^2(\Omega)^2\,;\, (q,1) = 0\},\\
\Psi &= H^1(\Omega)\,.
\end{aligned}
$$

initial condition : $u(0,x)$, $\varphi(0,x)$
find $\{u(t), p(t), \varphi(t)\} \in V \times Q \times \Psi$

$$
\begin{aligned}
\int_\Omega \rho(\varphi)(\partial_t u + u\cdot\nabla u)\cdot v + 2\int_\Omega \mu(\varphi) D(u) : D(v) - \int_\Omega p\nabla\cdot v - \int_\Omega q\nabla\cdot u = \\
-\int_\Omega \rho(\varphi) g\, e_2 \cdot v + \sigma \int_\Gamma \kappa\, n\cdot v \qquad \forall (v,q) \in V \times Q\,,\\
\int_\Omega \partial_t\varphi\psi + u\cdot\nabla\varphi\psi = 0 \qquad \forall \psi \in \Psi\,.
\end{aligned}
$$

$$
\rho(\varphi)(t,x) = \begin{cases} \rho_1 & \varphi(t,x) > 0 \\ \rho_2 & \varphi(t,x) < 0. \end{cases}
$$
, $\mu(\varphi)$ as same as $\rho(\varphi)$.

$e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, g : gravity acceleration coefficient

discretization scheme

Δt : time step, $\mathcal{T}_h$ mesh decomposition with triangle K.

$$V_h = \{u|_K \in (P_1(K))^2; v\cdot n = 0 \text{ on } \Gamma_{\mathsf{RL}}, v = 0 \text{ on } \Gamma_{\mathsf{TB}}\} \subset H^1(\Omega)^2,$$
$$Q_h = \{p|_K \in P_1(K); \textstyle\int_\Omega p = 0\} \subset L^2_0(\Omega),$$
$$\Psi_h = \{\phi|_K \in P_1(K)\} \subset H^1(\Omega).$$

characteristic finite element method + penalty stabilization / SUPG
(u^m, φ^m) : known, find $(u^{m+1}, p^{m+1}, \varphi^{m+1}) \in V_h \times Q_h \times \Psi_h$,

$$\int_\Omega \rho_h^m \frac{1}{\Delta t}(u_h^{m+1} - u_h^m \circ X_h^m)\cdot v_h + 2\int_\Omega \mu_h^m D(u_h^{m+1}) : D(v_h)$$
$$-\int_\Omega p_h^{m+1}\nabla\cdot v_h - \int_\Omega q_h \nabla\cdot u_h^{m+1} - \delta\sum_K h_K^2 \int_K \nabla p_h^{m+1}\cdot\nabla q_h =$$
$$-\int_\Omega \rho_h^m g\, e_2\cdot v_h + \sigma\int_{\Gamma_h^m} \kappa^m\, n^m\cdot v_h \quad \forall (v_h, q_h) \in V_h\times Q_h\,,$$
$$\int_\Omega \left(\frac{\widetilde{\varphi_h}^{m+1} - \varphi_h^m}{\Delta t} + u_h^m\cdot\nabla\varphi_h^m\right)(\psi_h + \alpha u_h^m\cdot\nabla\psi_h) = 0 \quad \forall \psi_h \in \Psi_h\,.$$

$\delta > 0$: stability parameter, $h_K = \mathrm{diam}(K)$, $\alpha > 0$

computation of the surface tension effect

how to compute surface integration? $\sigma \int_\Gamma \kappa\, n \cdot v$

$\kappa = -\nabla_S \cdot n = -\nabla \cdot n, \quad n = \frac{\nabla \varphi}{|\nabla \varphi|}|_{\varphi=0}, \quad \nabla_S$: tangential gradient

Question ? φ : approximated by P1 $\Rightarrow \kappa = 0$?

surface divergence formula with integration by part

$$\int_\Gamma \kappa v \cdot n = -\int_\Gamma \nabla_S \cdot n\, v \cdot n = -\int_\Gamma \nabla_S \cdot v\, .$$

cf. J.-F. Gerbeau, T. Lelièvre, C. Le Bris, *JCP*, 184 (2003)

Γ_h^m : P1-interpolation of zero-level set $\{\varphi_h^m(x) = 0\}$,

$\Gamma_h = \bigcup_l \Gamma_h \cap K_l = \bigcup_l \gamma_l$. time step m : fixed.

in 2-D case, surface gradient is easily obtained as

$\nabla = \nabla_N + \nabla_S = n(n \cdot \nabla) + t(t \cdot \nabla). \quad n = (n_1, n_2)^T, t = (n_2, -n_1)^T$

$$\begin{aligned} -\int_{\Gamma_h} \nabla_S \cdot v_h &= -\sum_l \int_{\gamma_l} t_l(t_l \cdot \nabla) \cdot v_h \\ &= -\sum_l \int_{\gamma_l} \begin{bmatrix} n_2 \\ -n_1 \end{bmatrix} (n_2 \partial_1 - n_1 \partial_2) \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \end{aligned}$$

reinitialization of the level set?

Eikonal eq. $|\nabla \varphi| = 1$ from signed distance function
$|\nabla \varphi^{m+1}| = 1$ is not satisfied in vicinity of $\Gamma(t)$
$\Rightarrow$ reinitialization procedure by solving PDE
cf. M. Sussman, P. Smereka, S. Osher, JCP, 114 (1994)

mathematical consideration by Dr. N. Hamamuki
Reinitialization process with hyperbolic PDE by Sussman et al. does not change zero-level set of the signed distance function itself.

reconstruction of signed distance function $\widetilde{\varphi_h}^{m+1} \Rightarrow \varphi_h{}^{m+1}$
from zero-level set in finite elements, $\bigcup_l \Gamma_h^{m+1} \cap K_l$

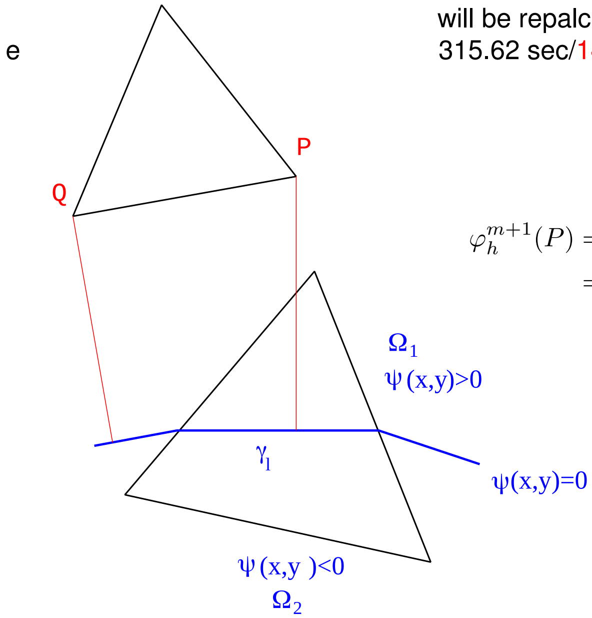
cf. M. K. Touré, A. Soulaïmani, doi:10.1016/j.camwa.2016.02.028

$$\varphi_h^{m+1}(P) = \mathrm{dist}(P, \Gamma_h^{m+1}) = \min_l \mathrm{dist}(P, \gamma_l^{m+1})$$

with reduction of computational cost by selecting FE nodes P in neighborhood of Γ_h^m

$$W^m(\zeta) = \{x \in \Omega; |\varphi^m(x)| < \zeta\}, \quad \zeta > 0$$

reconstruction of signed distance function

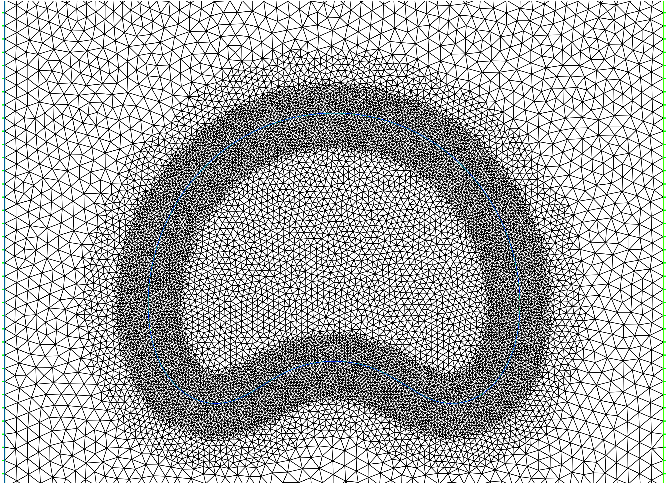


will be repalced by `distance()`
315.62 sec/140.67 sec @ t=0.25

$$\varphi_h^{m+1}(P) = \mathrm{dist}(P, \Gamma_h^{m+1}) = \min_l \mathrm{dist}(P, \gamma_l^{m+1})$$

isotropic mesh refinement to save DOF for linear solver

mesh around the bubble by `FreeFem++`

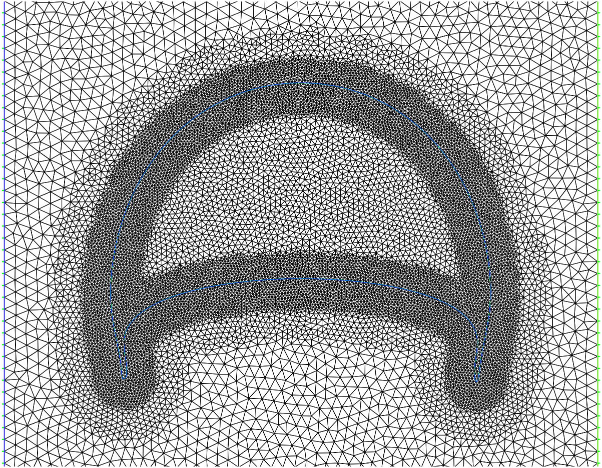


layers are defined by $W^m(\zeta_k)$, $\zeta_1 < \zeta_2 < \zeta_3$.
The signed distance function is totally reconstructed and re-meshing when center of mass of the bubble moves a lot, e.g. 0.01

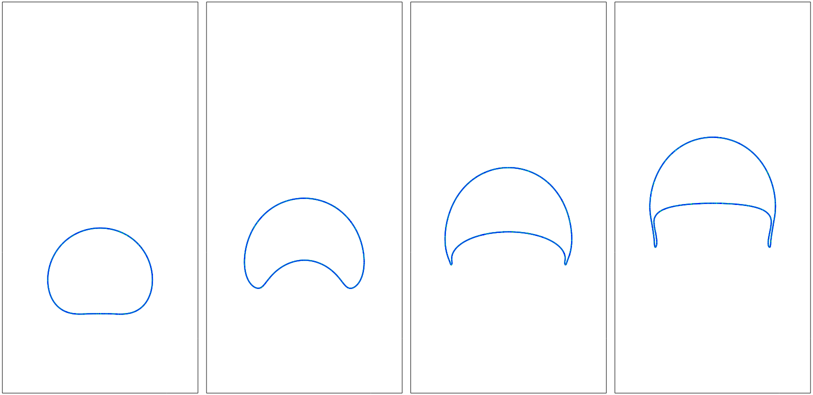
numerical setting of a test problem

density ρ_k		viscosity μ_k		σ	# of K	# of vertex	h_{min}	h_{max}	Δt
1,000	1.0	10	0.1	1.96	32,656	16,479	0.00356	0.0279	0.005

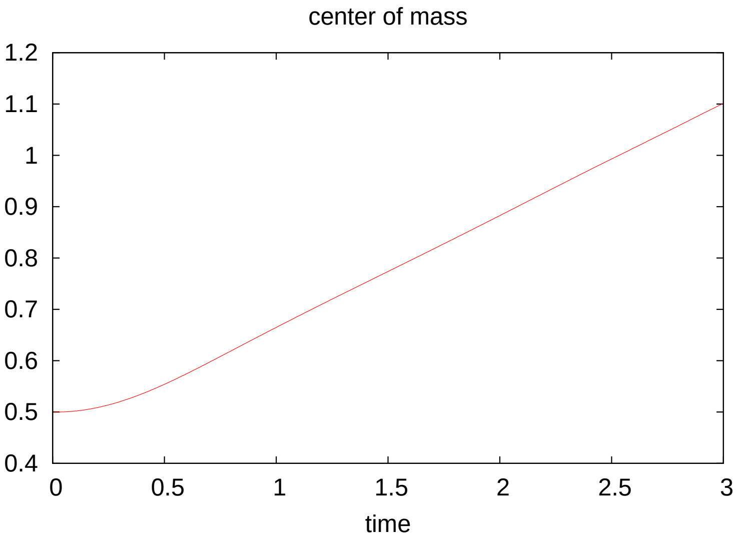
mesh around the bubble at time $t = 2.5$. $\Omega = (0,1) \times (0,2)$.



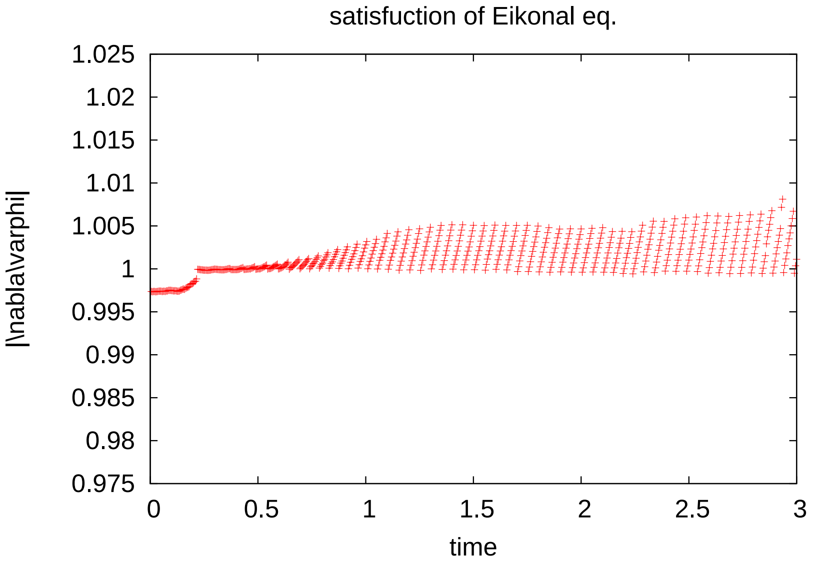
time evolution of a test problem



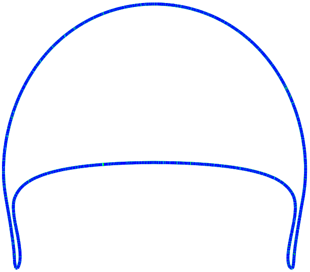
preliminary results : 1/3

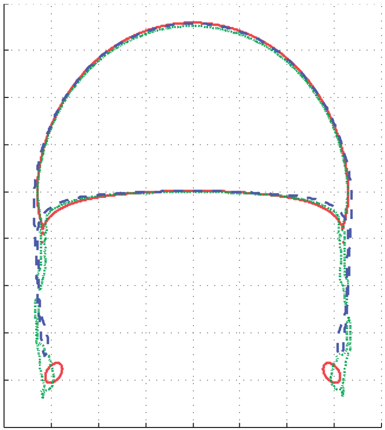


preliminary results : 2/3



preliminary results : 3/3





red	TP2D	S. Turek, D. Kuzmin, S. Hysing	level set
green	FreeLIFE	E. Burman, N. Parolini	level set
blue	MooNMD	L. Tobiska, S. Ganesan	ALE

S. Hysing, S. Turek, D. Kuzmin, N. Parolini, E. Burman,
S. Ganesan, L. Tobiska, *Int. J. Numer. Meth. Fluids. 60 (2009)*

conclusion

- P1/P1/P1 finite element for velocity/pressure/levelset unknowns
- surface tension is treated by a surface divergence formula
- reconstruction of level set function from geometrical information
- isotropic mesh refinement around the zero-level set boundary
- Dissection direct solver is used to solve generalized Stokes eqs., which indefinite and singular

ongoing

- qualitative comparison with other schemes in the benchmark paper

future work

- P2 approximation to level set
  iso-parametric approximation to zero-level set curve in each finite element