3D simulations - items to keep in mind

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- **Prof. O. Pironneau**
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- **Antoine Le Hyaric**

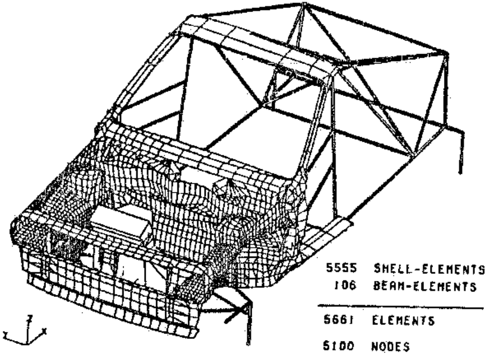
presentation, models, animations

http://www.gostaf.com

FEM 1987

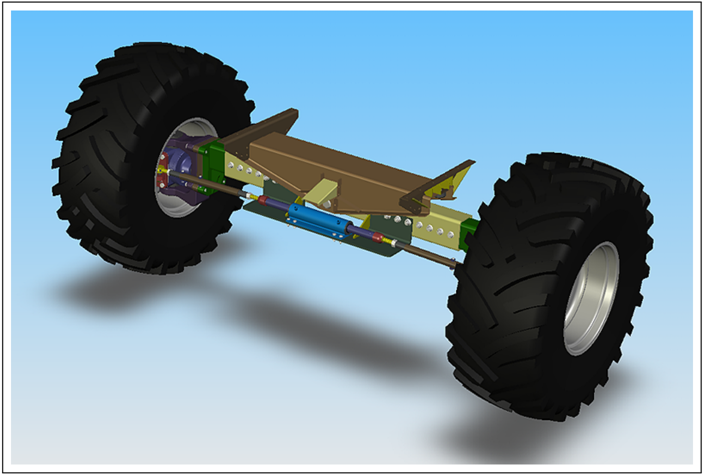
Numerical simulations of a VW-Polo crash

the CPU time is 4 hours on a CRAY 1 super computer



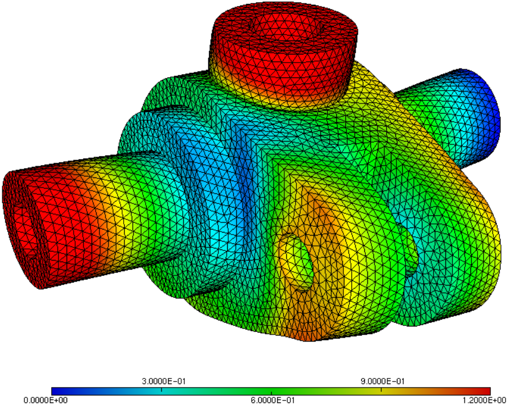
W.E.M. BRUIJS, Numerical crash simulation of vehicle structures, 1987

Computer Aided Design (CAD) assembly



Pro/ENGINEER, Courtesy PTC

Linear elasticity $\mathbb{P}_2$ elements: `FreeFem++`



Vertices	80 254
Triangles	28 624
Tetrahedra	442 886
Unknowns	1 853 124

MEMORY > 64 GB (direct solver)

Outline

Solid modeling and assumptions

Domain decomposition techniques

- Subdomains
- Contacts
- Discretization of contact surfaces
- Geometric discontinuity
- Extended skeleton

Numerical illustration

Conclusions and perspectives

Manifold - A surface having the property that around every one of its points there exists a neighborhood that is topologically equivalent i.e. homeomorphic to the plane.

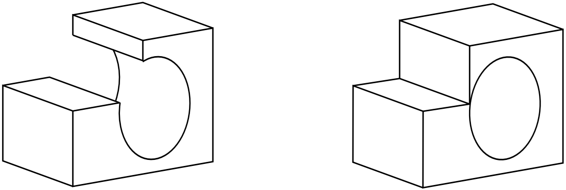


Figure: Manifold (left) and nonmanifold (right) solids

Solid Modeling and Assumptions

For a manifold M, we define the following measure

$$H = \operatorname{diam}(M) = \sup_{x_1, x_2 \in M} |x_1 - x_2|$$

Parts are initially created with the `relative` accuracy

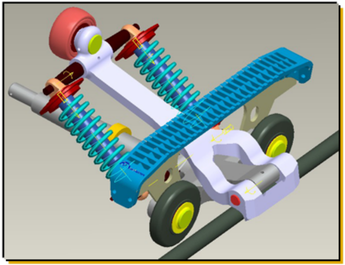
$$10^{-6} \leqslant l/H < \delta^r_{CAD} \leqslant 10^{-2}$$

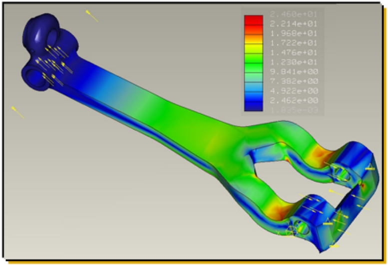
l - the smallest curve length

Default (`Pro/Engineer`)

$$\delta^r_{CAD} = 1.2 \times 10^{-3}$$

- CAD assembly modeling
- Export into neutral formats e.g. `IGES, STEP`
- Broken associativity
- Contacts -> Boundary conditions





Pro/ENGINEER Mechanism Dynamics, Courtesy PTC

Generic CAD-FEM Workflow

CAD assembly modeling

Export into neutral formats e.g. IGES, STEP

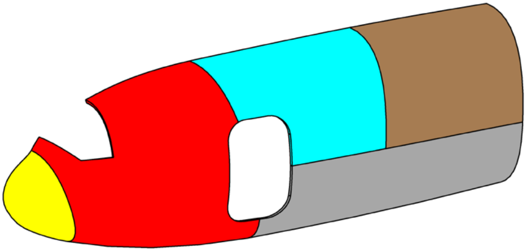
Broken associativity

Contacts -> Boundary conditions

Domain decomposition

Conforming mesh partitioning, e.g. ParMETIS, SCOTCH

Parallel solution/solvers, visualization



References

Dirichlet-Neumann method

P. Bjorstad, O. Widlund, Bramble, Furano, Marini, A. Quarteroni

Neumann-Neumann method

V. Agoshkov, V. Lebedev, J. Bourgat, R. Glowinski, P. Tallec, M. Vidrascu

FETI method

C. Farhat, F. X. Roux

Mortar method

C. Bernardi, Y. Maday, T. Patera

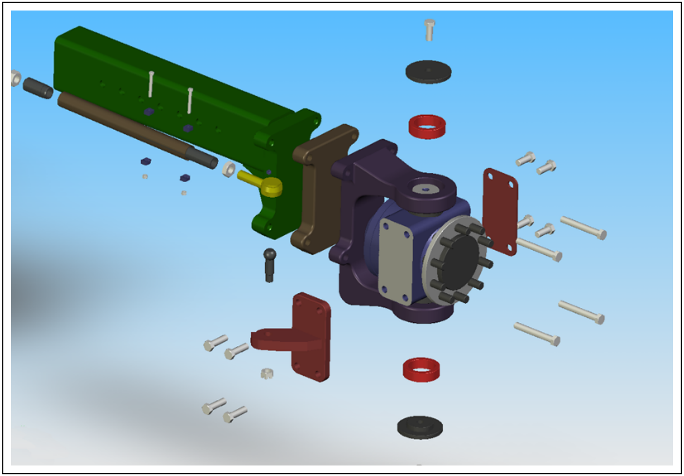
Contacts analysis

T. McDevitt, T. Laursen, B. Wohlmuth

Prolongation operators

M. Gander, T. Dickopf, R. Krause, F. Rapetti, A. Bossavit

CAD-based Domain Decomposition Method

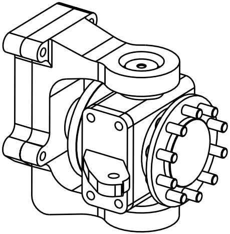


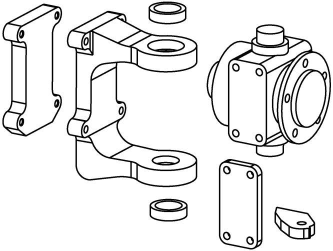
Gostaf Kirill Pichon

thesis 2012, UPMC, Paris, France.

Subdomains Initialization

- physical and mechanical properties
- mimics mechanical behavior
- frequent updates of finite element models
- embarrassingly parallel mesh generation
- component oriented discretization and variational formulation
- ought to regularize a mathematical model on each subdomain





Contacs Initialization and Meshing

Let $\{\mathcal{P}_1, ..., \mathcal{P}_s\}$ refer to a set of assembly components, with $s \geq 2$

Let $\mathbb{S}$ denote a set of initial contacts between all adjacent solids

$$\mathbb{S} = \{\mathcal{P}_i \cap^* \mathcal{P}_j\} \qquad 1 \leq i \neq j \leq s$$

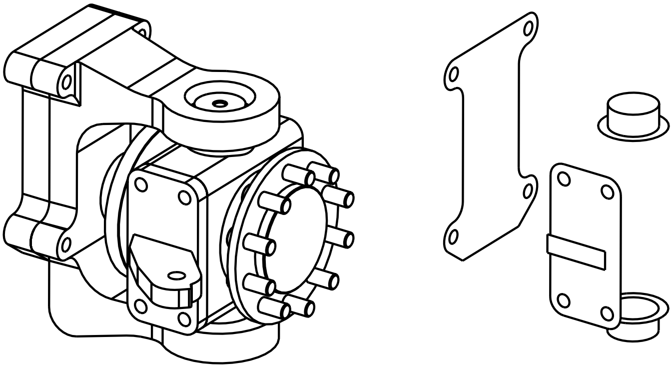
where $\cap^*$ stands for a Boolean cut operator. Thus

$$\dim(\mathbb{S}) \leq \binom{s}{2}$$

Clearly

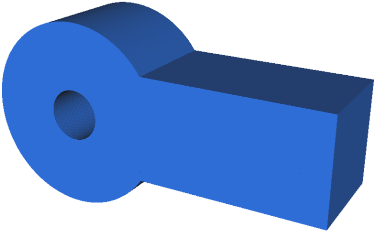
$$\dim(\mathbb{S}) \sim \mathcal{O}(s)$$

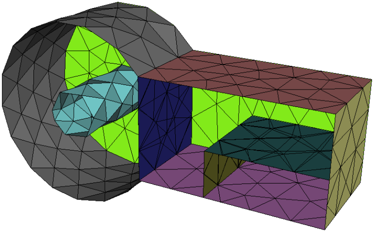
Contacs Initialization and Meshing

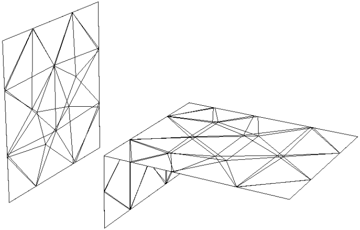


Contacs Initialization and Meshing

Conforming meshing - <u>not a trivial task</u>



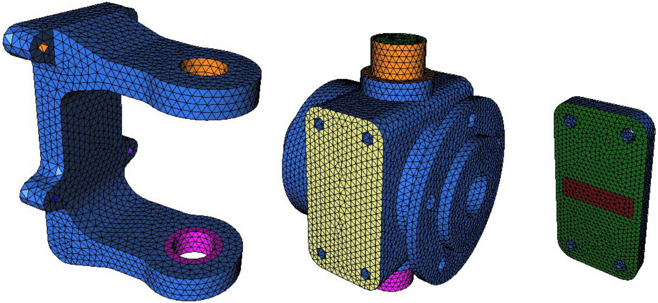




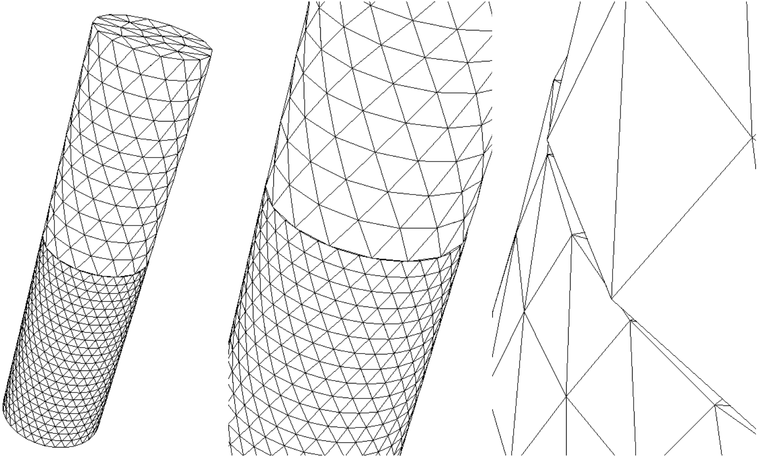
Contacs Initialization and Meshing

Nonconforming meshing

- different variational formulations
- finite elements of different shapes and orders
- adaptive meshing
- sliding
- curved or geometrically non-matching interface



Geometric Discontinuity



Geometric Discontinuity

Since $\mathcal{T}^c_{1,2} \neq \mathcal{T}^c_{2,1}$, we have

$$\mathcal{T}^c_{1,2} \cap \mathcal{T}^c_{2,1} \neq \emptyset \quad \Leftrightarrow \quad \exists\, P \in \mathcal{T}^c_{1,2}, \text{ such that } P \notin \mathcal{T}^c_{2,1}$$

which makes the computation

$$[u] = u_2(P) - u_1(P) \qquad \text{on} \quad \mathbb{S}$$

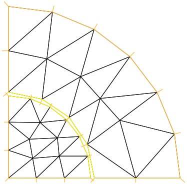
not properly defined. Gauss quadrature (numerical integration)

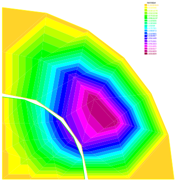
$$\int_{\Gamma_{1,2}} \mu_{1\,h}(u_{1\,h} - u_{2\,h})\, d\gamma \approx \sum_{P \in \mathcal{T}^c_{1,2}} \alpha_P\, \mu_{1\,h}(P)(w_{1\,h}(P) - w_{2\,h}(P))$$

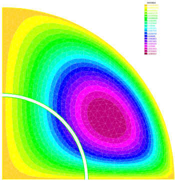
Prolongation Operators

Let $\pi_{k,l} : \Omega_{k\,h} \mapsto \Omega_{l\,h}$

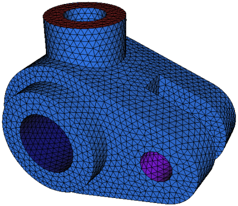
- prolongation by zero
- linear prolongation
- quadratic prolongation
- constant pointwise

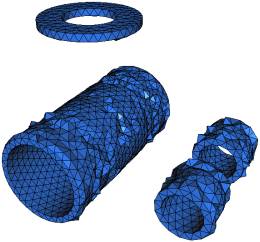






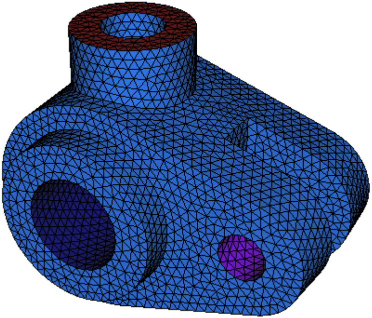
Extended skeleton approach

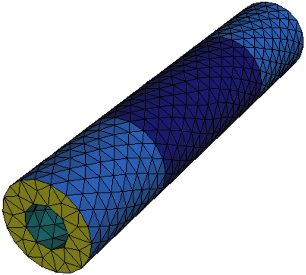




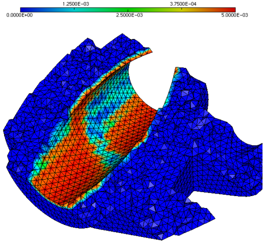
Prolongation Operators - Numerical Illustration

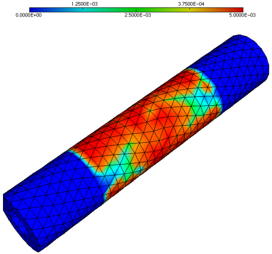
Tube-holder assembly: accuracy $\delta^r_{CAD} = 0.5 \times 10^{-4}$

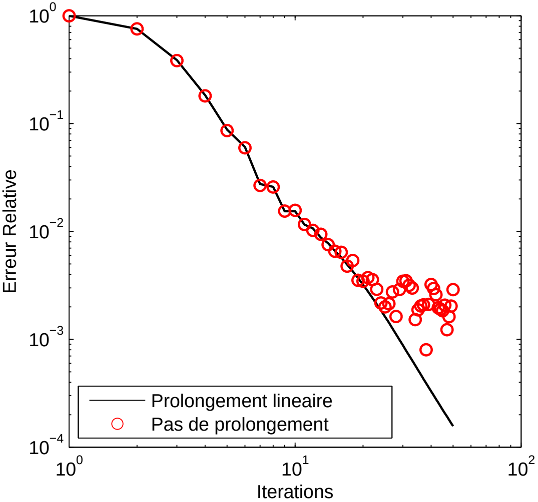




Prolongation Operators - Numerical Illustration







The Dirichlet-Neumann Method

Proposed in '86 by Bjorstad and Widlund,

also Bramble, Furano, Marini and Quarteroni

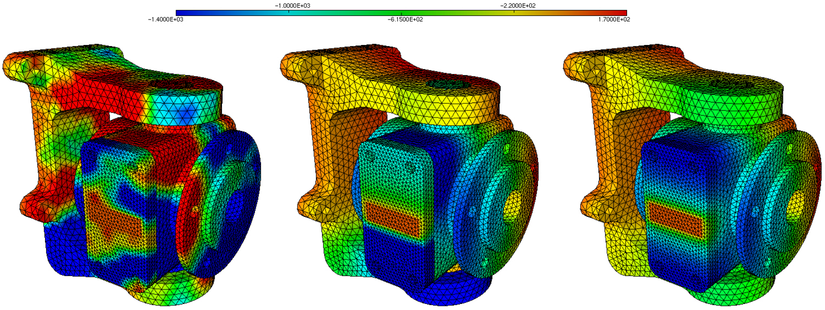
For a given initial guess u_S on the common boundary S, the sequences of functions $u_1^n \in \Omega_1$ and $u_2^n \in \Omega_2$, $n \geq 0$, satisfy the problems

$$
\begin{aligned}
\mathcal{L}u_1^n &= f && \text{in } \Omega_1 & \qquad \mathcal{L}u_2^n &= f && \text{in } \Omega_2 \\
\partial_n u_1^n &= g_N && \text{on } \partial\Omega_1 \cap \partial\Omega_N & \qquad \partial_n u_2^n &= g_N && \text{on } \partial\Omega_2 \cap \partial\Omega_N \\
u_1^n &= g_D && \text{on } \partial\Omega_1 \cap \partial\Omega_D & \qquad u_2^n &= g_D && \text{on } \partial\Omega_2 \cap \partial\Omega_D \\
u_1^n &= u_S^n && \text{on } S & \qquad \partial_n u_2^n &= \partial_n u_1^n && \text{on } S
\end{aligned}
$$

then correct the initial guess u_S until convergence

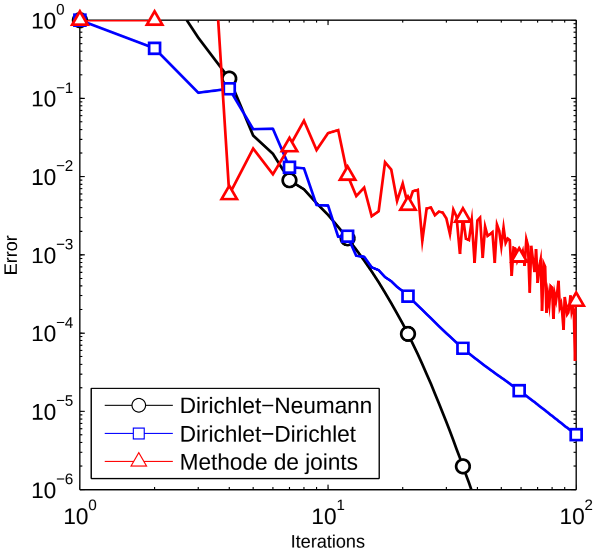
$$u_S^{n+1} = (1-\theta)u_S^n + \theta u_2^n$$

Multi-component CAD assembly: an academic test case

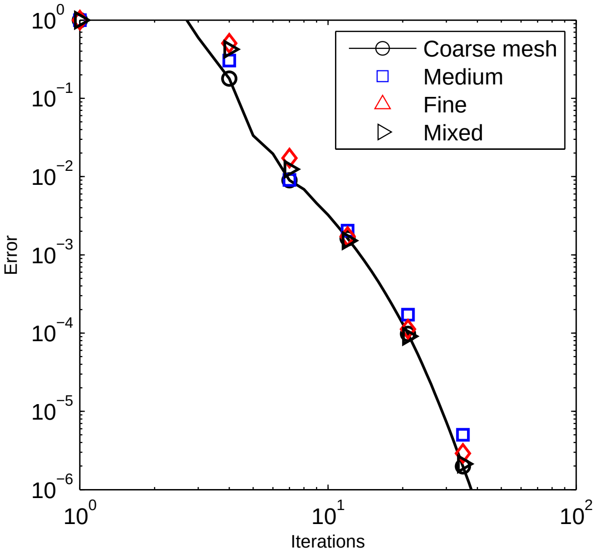


`http://www.gostaf.com/CADbasedDDM`

Numerical Evidence



Numerical Evidence



Conclusions and Perspectives

- Moderate size 3D discretizations produce millions of unknowns
- Computational limits are remarkably extended when the adaptive meshing is used, but... in 3D ?
- Hybrid MPI/OpenMP increases performance of multi-core CPU
- FreeFem++ is stable and efficient for simulations with millions of dof.
- Realistic test cases with large number of subdomains