

# **Analysis of Surface Acoustic Waves Transducer Having Aperiodic Multi Electrode Cells Using a Coupled FEM/ BIE Numerical Model**

**P. Ventura\*, Pierre Dufilié\*\*, F. Hecht\*\*\***

**Laboratoire LEM3, Université Paul Verlaine, Metz,  
France**

**\*\*Phonon Corporation  
Simsbury, Connecticut, USA**

**\*\*\*Laboratoire Jacques Louis Lions, Université  
Pierre et Marie Curie, Paris, France**

# Outlines

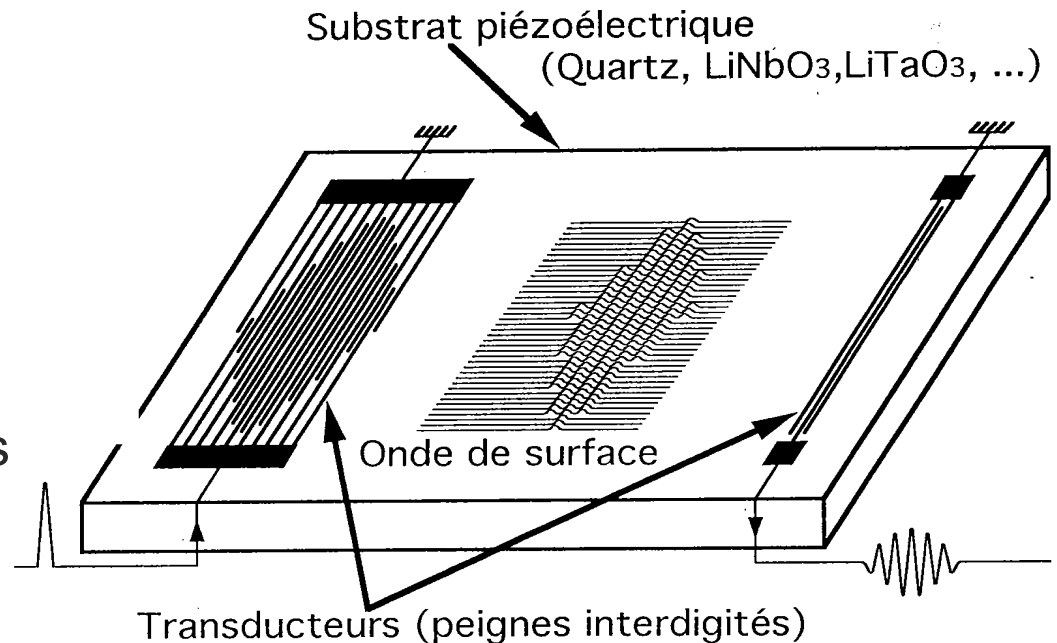
- **Introduction**
- **Physical model**
- **Weak Formulation**
- **First results - Validations**
- **Application to Multi electrodes Aperiodic Transducers**
- **Filter Frequency Response**
- **Conclusions**
- **Future works**

# Introduction

## SAW IDT components

*How is built a SAW device*

- Piezoelectric substrate
- SAW IDT transducer
- The Surface Acoustic Wave is
  - launched and detected
  - propagating



# Introduction

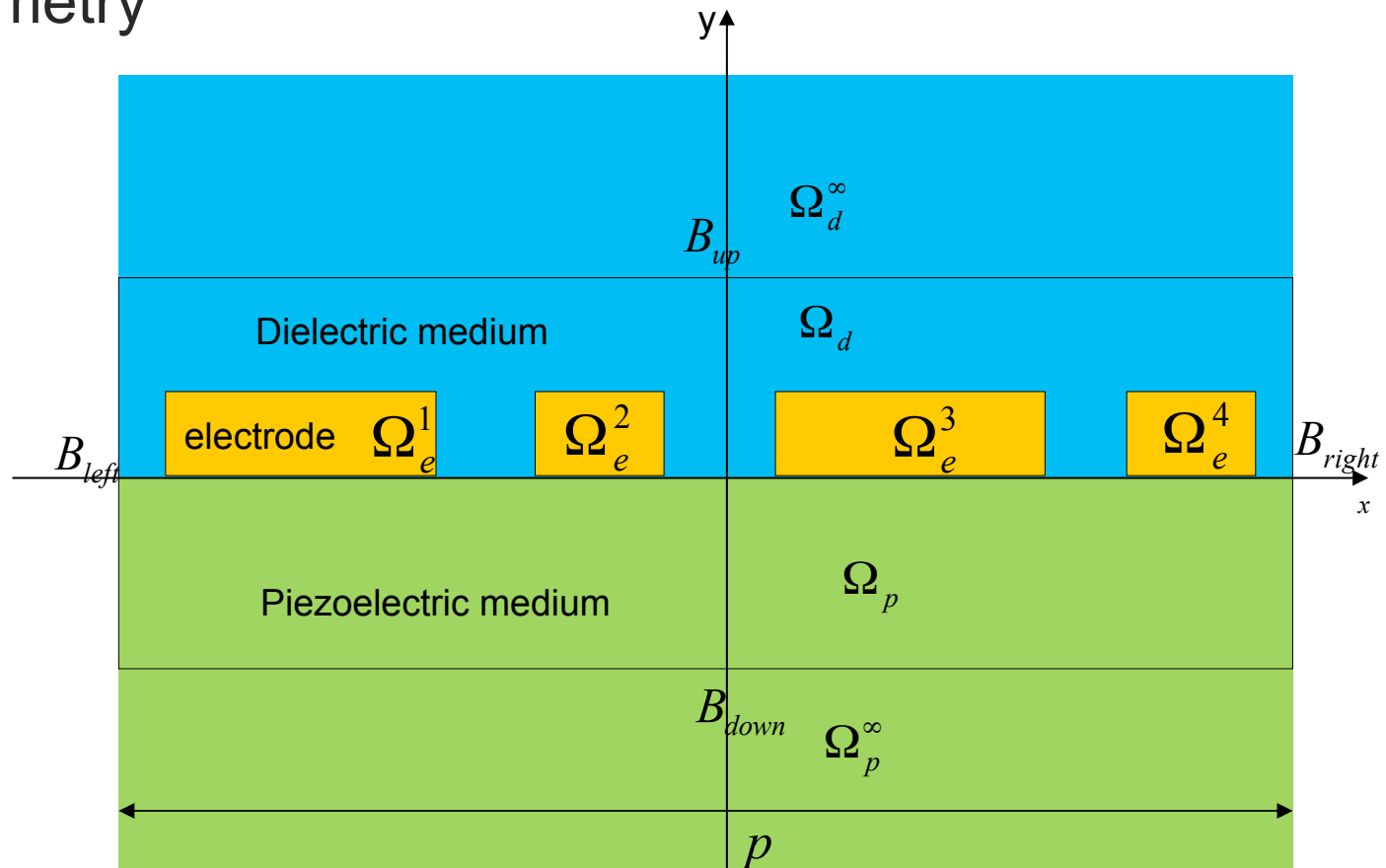
- For many years, SAW devices were designed with periodic (bandpass filters, resonators) or locally periodic (dispersive devices) electrode structures.
- Analysis of these structures were accurately made using models for single electrodes in a periodic structure.
- Current devices incorporate more complex electrode pattern structures such as Hanma-Hunsinger cells and EWC Cells.
- The design of high performance SAW devices requires an accurate model of the electro-acoustic parameters of each cell.
- A coupled FEM/BIE numerical model able to deal with aperiodic multi-electrode cell has been developed.

# Introduction

- published in the Journal of Computational Physics, 246 (2013), pp. 265-274  
“Original FEM/BIE numerical model for analyzing infinite periodic surface acoustic wave transducers”, F. Hecht, P. Ventura, P. Dufilié
- To be published in the proceedings of the IEEE Ultrasonics Symposium 2013,  
“Analysis of SAW Transducers Having Aperiodic Multi-Electrode Cells Using a Coupled FEM/BIE Numerical Model”, P. Ventura, P. Dufilié, F. Hecht

# Physical model

## ■ Geometry



# Physical model

## ■ Assumptions :

- 2D analysis (very long electrode) : plain strain approximation
- p periodic along the x axis
- harmonic electrical excitation of the electrodes:  $V_n(\gamma) = V_0 e^{-j2\pi n\gamma}$   
(allow to apply Bloch-Floquet's theorem and take into account only a single period of the array)
- electrical assumption: no dielectric losses in the electrode
- mechanical assumption : the metallic electrode are homogeneous isotropic, elastic materials

# Physical model

- The Piezoelectric domain  $\Omega_p$  and the Elastic domain  $\Omega_E$  obeys Newton's second law:

$$\nabla \cdot \mathbf{T} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}$$

- The Piezoelectric domain  $\Omega_p$ , the Elastic domain  $\Omega_E$  and the Dielectric domain  $\Omega_D$  obeys the quasistatic Maxwell's equation:

$$\nabla \cdot \mathbf{D} = 0$$



# Physical model

## ■ The constitutives equations:

For  $\Omega_P$

$$\begin{cases} \mathbf{T}_{ij} = \mathbf{C}_{ijkl}^{\mathbf{E}} \mathbf{S}_{kl} - \mathbf{e}_{ijk} \mathbf{E}_k \\ \mathbf{D}_i = \mathbf{e}_{kli} \mathbf{S}_{kl} + \varepsilon_{ik}^{\mathbf{S}} \mathbf{E}_k \end{cases}$$

For  $\Omega_E$

$$\mathbf{T}_{ij} = (\lambda + \mu) \delta_{ij} \mathbf{S}_{kk} - 2\mu \mathbf{S}_{ij}$$

For  $\Omega_D$

$$\mathbf{D}_i = \varepsilon_{ik} \mathbf{E}_k$$

# Physical model

## ■ $\gamma$ periodic boundary conditions:

- For the interfaces  $B_u^l$  and  $B_u^r$  :  $\Phi(+p/2, y) = e^{-j2\pi\gamma} \Phi(-p/2, y)$
- For the interfaces  $B_d^l$  and  $B_d^r$  : 
$$\begin{cases} \mathbf{u}(+p/2, y) = e^{-j2\pi\gamma} \mathbf{u}(-p/2, y) \\ \Phi(+p/2, y) = e^{-j2\pi\gamma} \Phi(-p/2, y) \end{cases}$$

# Weak Formulation


Finds  $(\mathbf{u}, \phi)$  in  $V_\gamma^3(\Omega_p \cup \Omega_e) \times V_\gamma^3(\Omega)$  (satisfying  $\phi = 1$  in the electrode) such that for all  $(\mathbf{v}, \psi)$  in  $V_\gamma^3(\Omega_p \cup \Omega_e) \times V_\gamma^3(\Omega)$  (satisfying  $\phi = 0$  in the electrode)

$$\begin{aligned} & \int_{\Omega_p \cup \Omega_e} \bar{\mathbf{S}}(\mathbf{v}) \cdot \mathbf{T}(\mathbf{u}) d\Omega - \rho \omega^2 \int_{\Omega_p \cup \Omega_e} \bar{\mathbf{v}} \cdot \mathbf{u} d\Omega \\ & - \int_{\Omega_p \cup \Omega_e \cup \Omega_d} \bar{\mathbf{E}}(\psi) \cdot (\mathbf{eS}(u) + \varepsilon \mathbf{E}(\phi)) d\Omega \\ & = \int_{B_d} \bar{\mathbf{v}} \cdot (\mathbf{T} \cdot \mathbf{n}) d\Gamma + \int_{B_u \cup B_d} \bar{\psi} (\mathbf{D}(\phi) \cdot \mathbf{n}) d\Gamma \end{aligned}$$

$V_\gamma(\Omega)$  is the mathematical space of  $L^2(\Omega)$  satisfying  $\gamma$ -harmonic periodic boundary conditions

# Weak Formulation

- Incorporates  $\gamma$  periodic boundary conditions in the variational formulation:
- The idea is to transform a  $\gamma$  periodic problem into a periodic problem using the relationship:

$$u(x, y) = \varphi_\gamma(x) \cancel{u_0(x, y)} \quad \varphi_\gamma(x) = e^{-j2\pi\gamma \frac{x}{p}}$$


Periodic function

- Modify the variational formulation which allows to use only the periodic option in FreeFem++

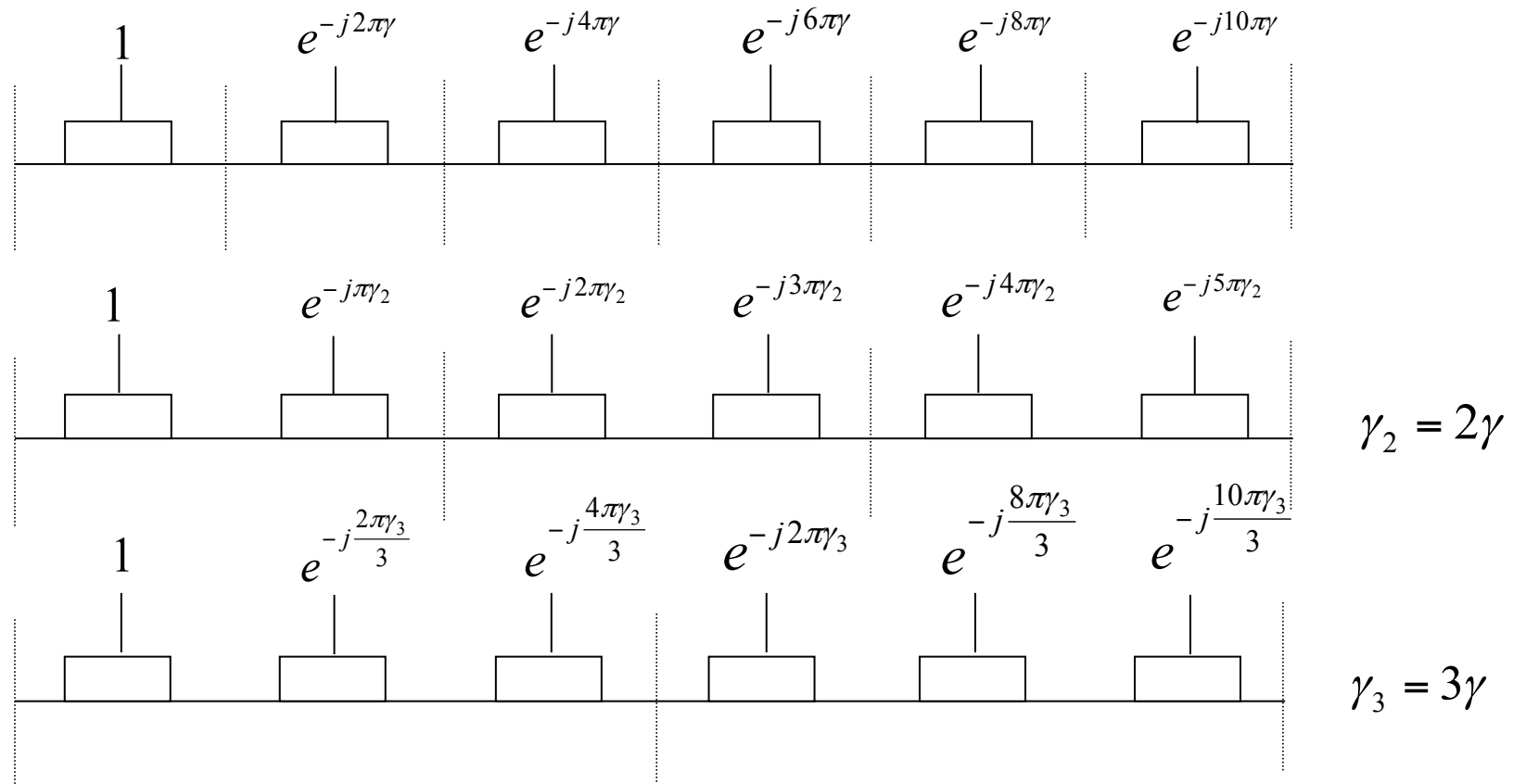
# Weak Formulation

$$\int_{B_d} \bar{\mathbf{v}} \cdot (\mathbf{T} \cdot \mathbf{n}) d\Gamma + \int_{B_u \cup B_d} \bar{\psi} (\mathbf{D}(\phi) \cdot \mathbf{n}) d\Gamma$$

BIE terms take into account periodic harmonic Green's function at the dielectric and at the piezoelectric boundaries, lead to full matrix coupling degree of freedom belonging to the upper and lower boundaries.

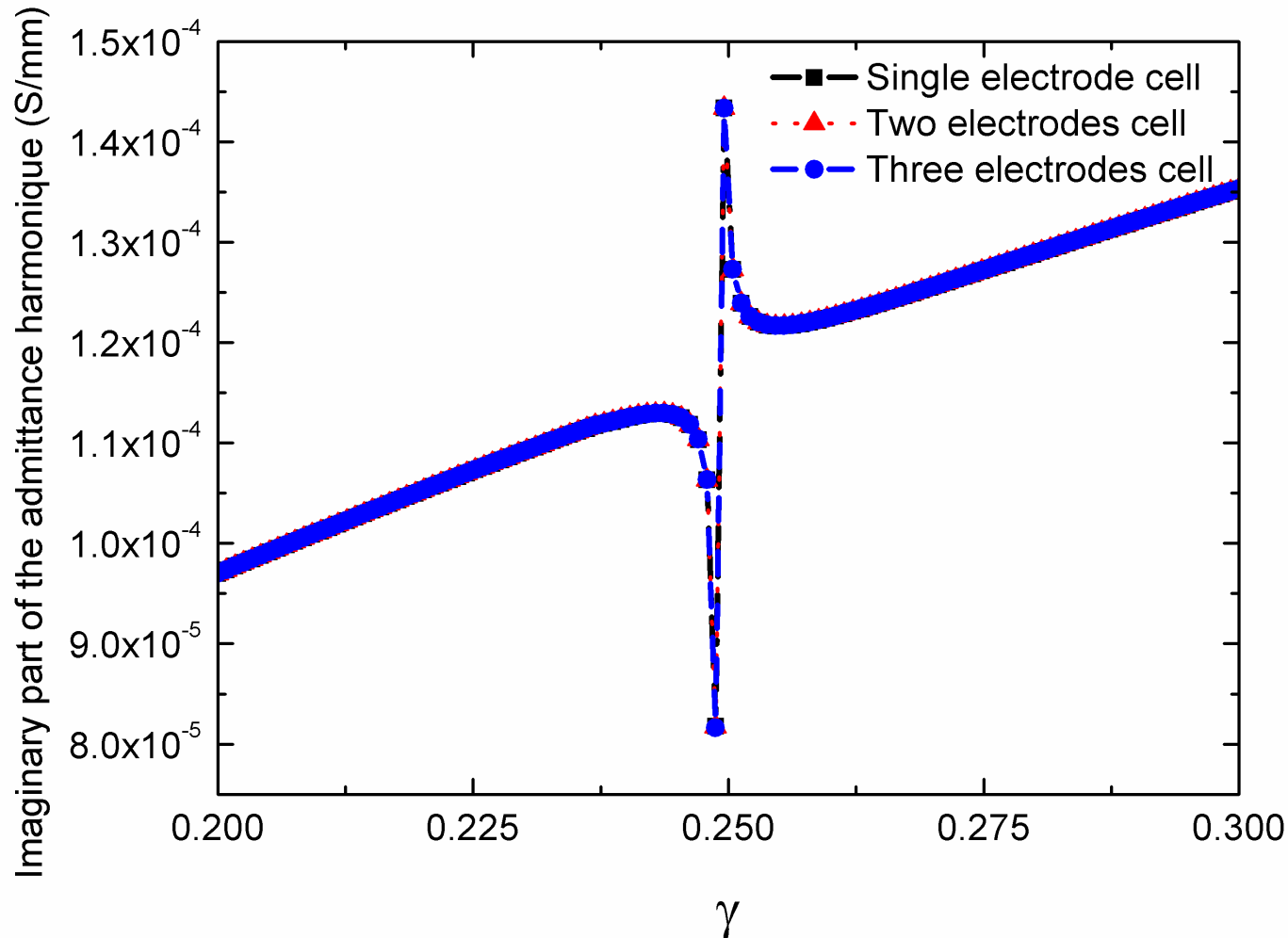
# First Results - Validation

- Purpose : compare the computation of the harmonic admittance of a single electrode periodic array with the harmonic admittance of aperiodic multi electrode cells having two, and three electrodes



# First Results - Validation

Plots of the imaginary part of the harmonic admittance



# Application to Aperiodic Multi-Electrode Transducers

- In the neighborhood of the pole, the surface acoustic wave contribution to the harmonic admittance is assumed to be:

$$Y(\gamma, f) \approx Y_r + \frac{F}{\sin^2(\pi\gamma) - \sin^2(\pi\gamma_{sc})}$$

Residual Harmonic Admittance  $\nearrow Y_r$

$F$   $\nwarrow$  Amplitude of the pole

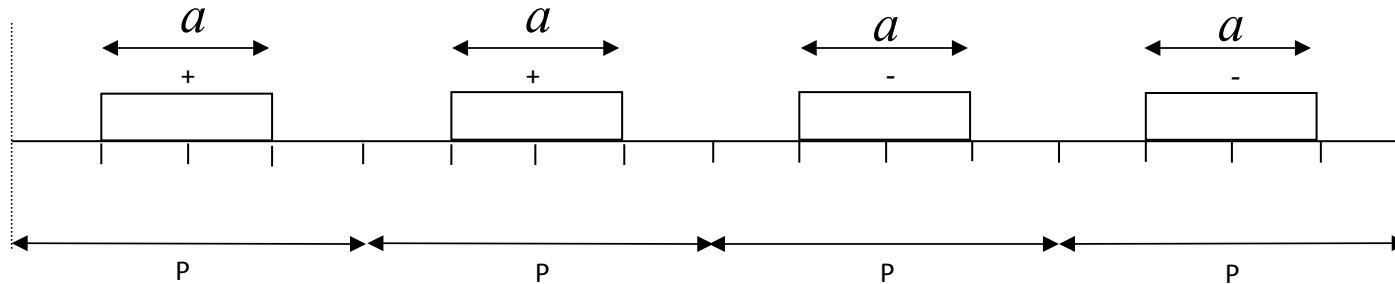
$\nwarrow \pi\gamma_{sc}$  Position of the pole

- From the linear interpolation of the amplitude  $F$  and the quadratic interpolation of the phase  $\sin^2(\pi\gamma_{sc})$ , it is possible to derive the phase velocity  $V$ , the reflectivity  $r$ , the piezoelectric coupling  $G$ , and the directivity  $\delta$  within the cell

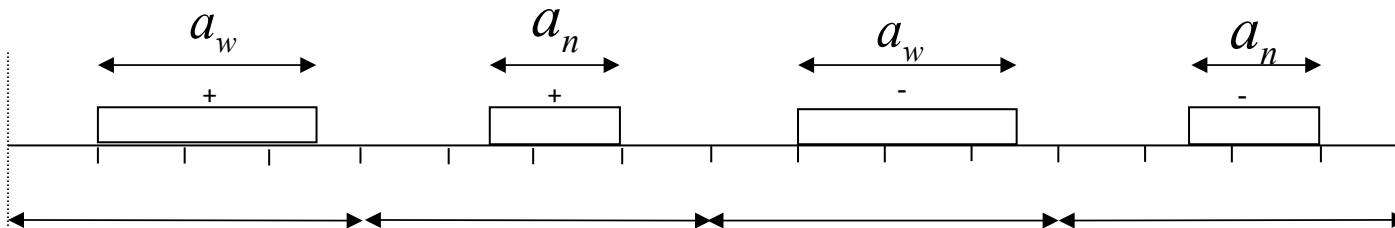


# Application to Aperiodic Multi-Electrode Transducers

## ■ Classical Split Finger (SF) Periodic Cell



## ■ Modified Hanma-Hunsinger (HH) Cell



# Application to Aperiodic Multi-Electrode Transducers

- First computation : the piezoelectric substrate is Y+38° X propagation Quartz, the electrode are not buried, and the metal thickness is 1600 Å. For the SF cell,  $a/p = 0.55$ , for HH cell  $a_o/p = 0.65$   
 $a_n/p = 0.45$
- Second computation : the piezoelectric substrate is Y+36° X propagation Quartz, the electrode are not buried, and the metal thickness is 3200 Å. Same a/p ratio than for the first computation

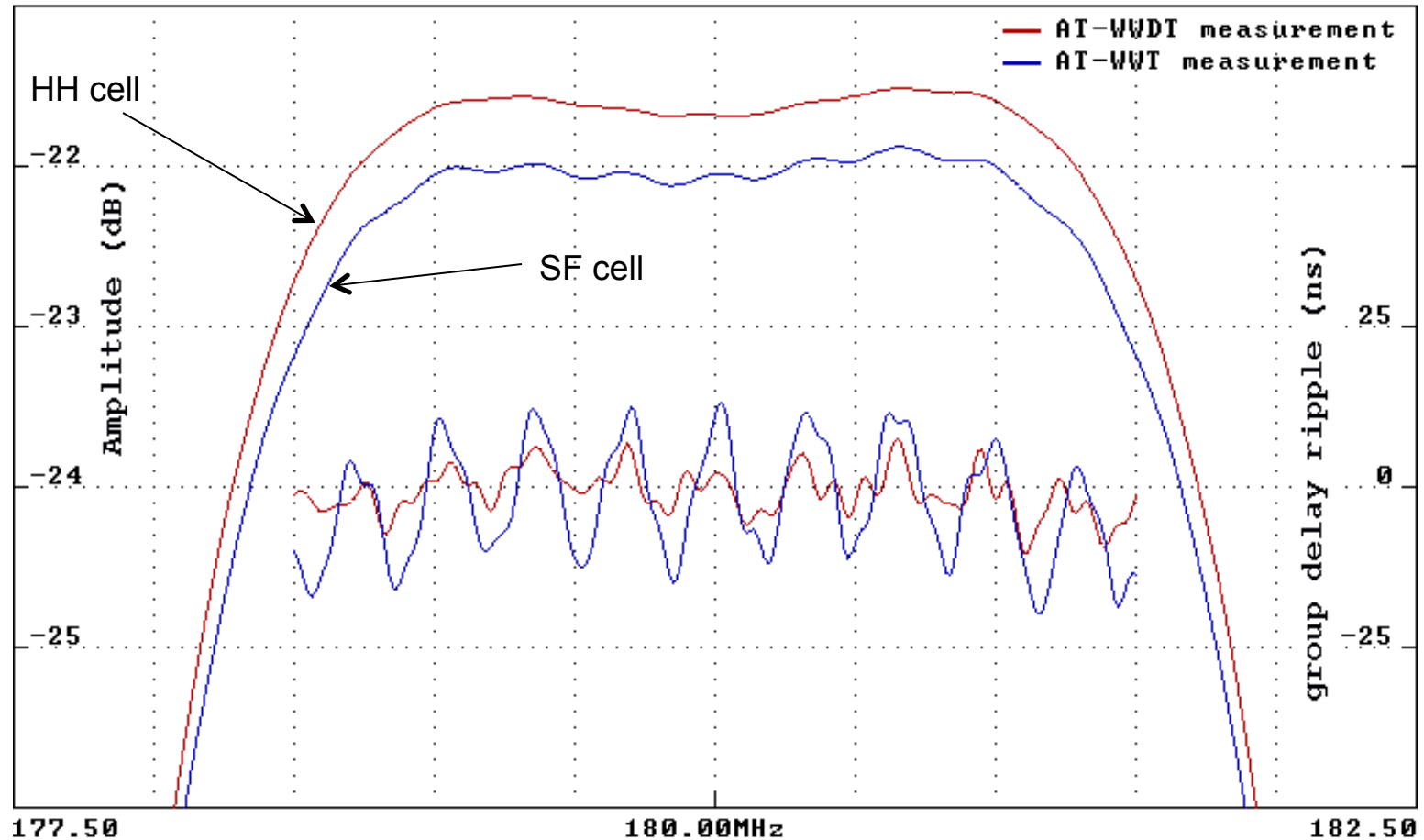
# Application to Aperiodic Multi-Electrode Transducers

	FEM/BIE	Analytical model
split finger (1)		
V (m/s)	3146.15	3143.56
G (S/mm)	1.25E-7	1.14E-7
Split finger (2)		
V	3129.64	3121.92
G (S/mm)	1.30E-7	1.13E-7

# Application to Aperiodic Multi-Electrode Transducers

	FEM/BIE	Analytical model
HH (1)		
V	3146.193	3143.98
r (%)	0.352	0.349
$\delta$ (degree)	54	
G (S/mm)	1.26E-7	1.13E-7
HH (2)		
V	3129.841	3123.40
r (%)	0.774	0.768
$\delta$ (degree)	52	
G (S/mm)	1.31E-7	1.12E-7

# Filter Frequency Response



Lower Insertion Loss for HH cell

# Conclusions

- The coupled Finite Element Model / Boundary Integral Equation (FEM/BIE) for a single periodic strip has been successfully extended to include aperiodic cells in a periodically repeating cell structure.
- An efficient validation method has been used, giving a very good degree of confidence in the software.
- Comparison for a modified Hanma-Hunsinger cell on Quartz have been presented.

# Future works

- Develop a 3D periodic model using FreeFem++ 3D to take into account transverse wave guiding effects
- Take into account non linear acoustical effects
- code parallelization