Asymptotic properties of carbon based composite materials used in latent heat thermal energy storage systems

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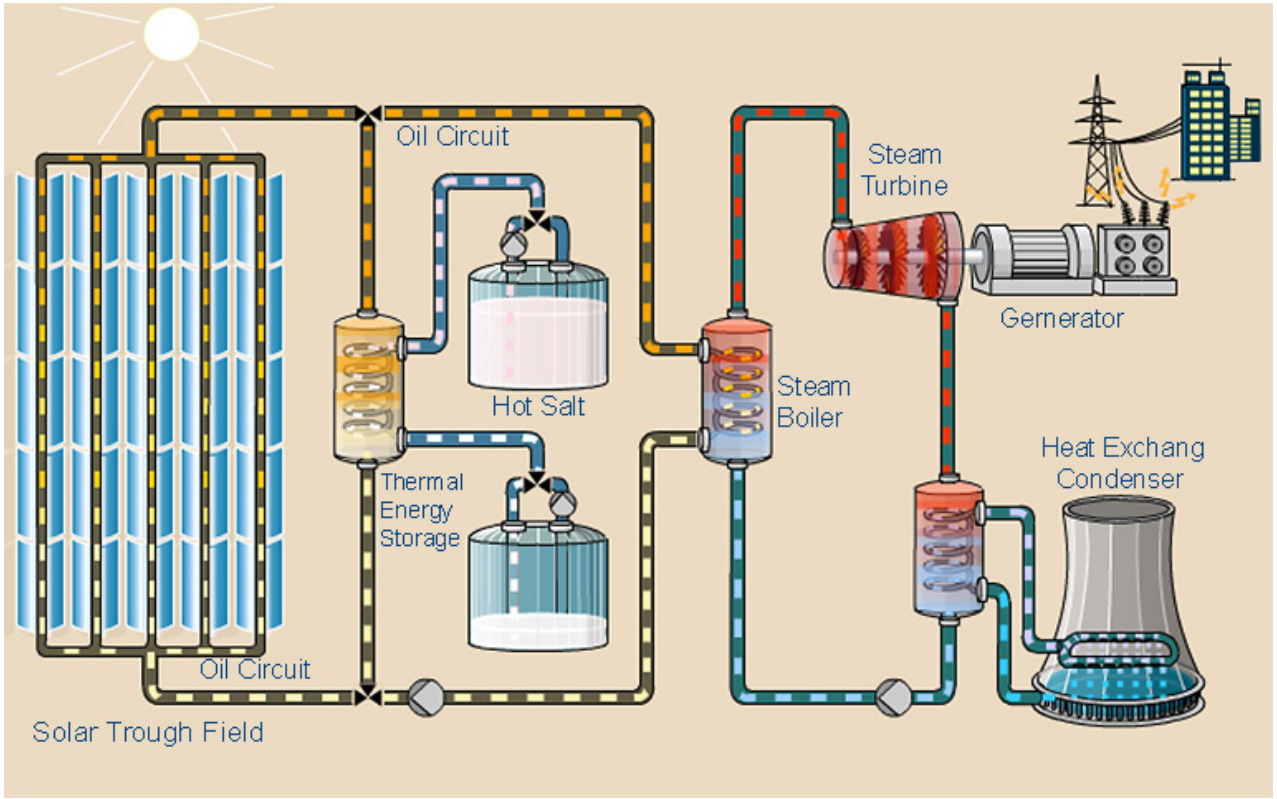
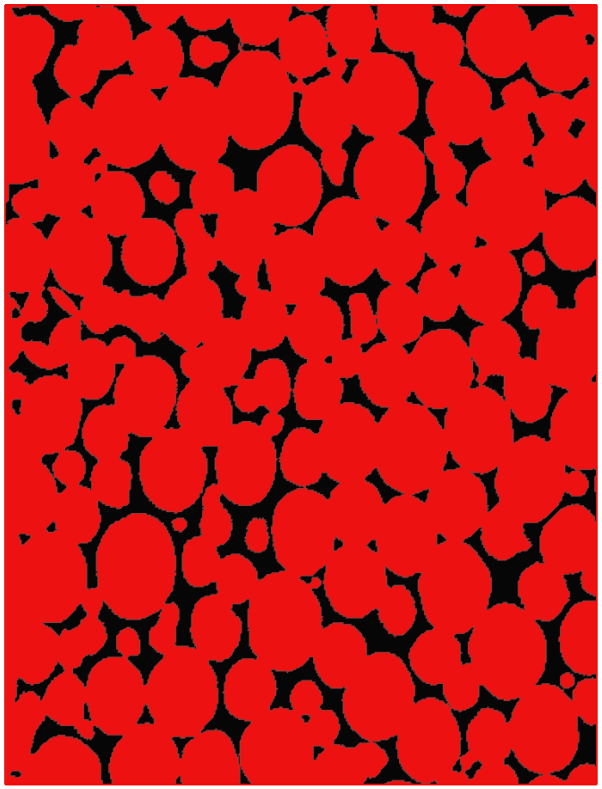
Latent heat thermal energy storage systems

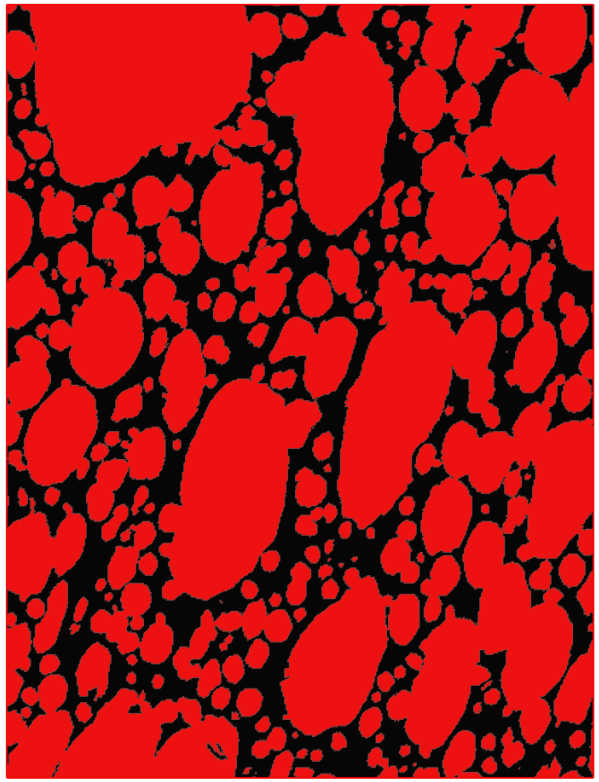


The studied composite materials

KL1_250



KD1



These composites are introduced by V. Canseco, Y. Anguy, J. J. Roa and E. Palomo in [1].

Thermo-physical properties of the composites

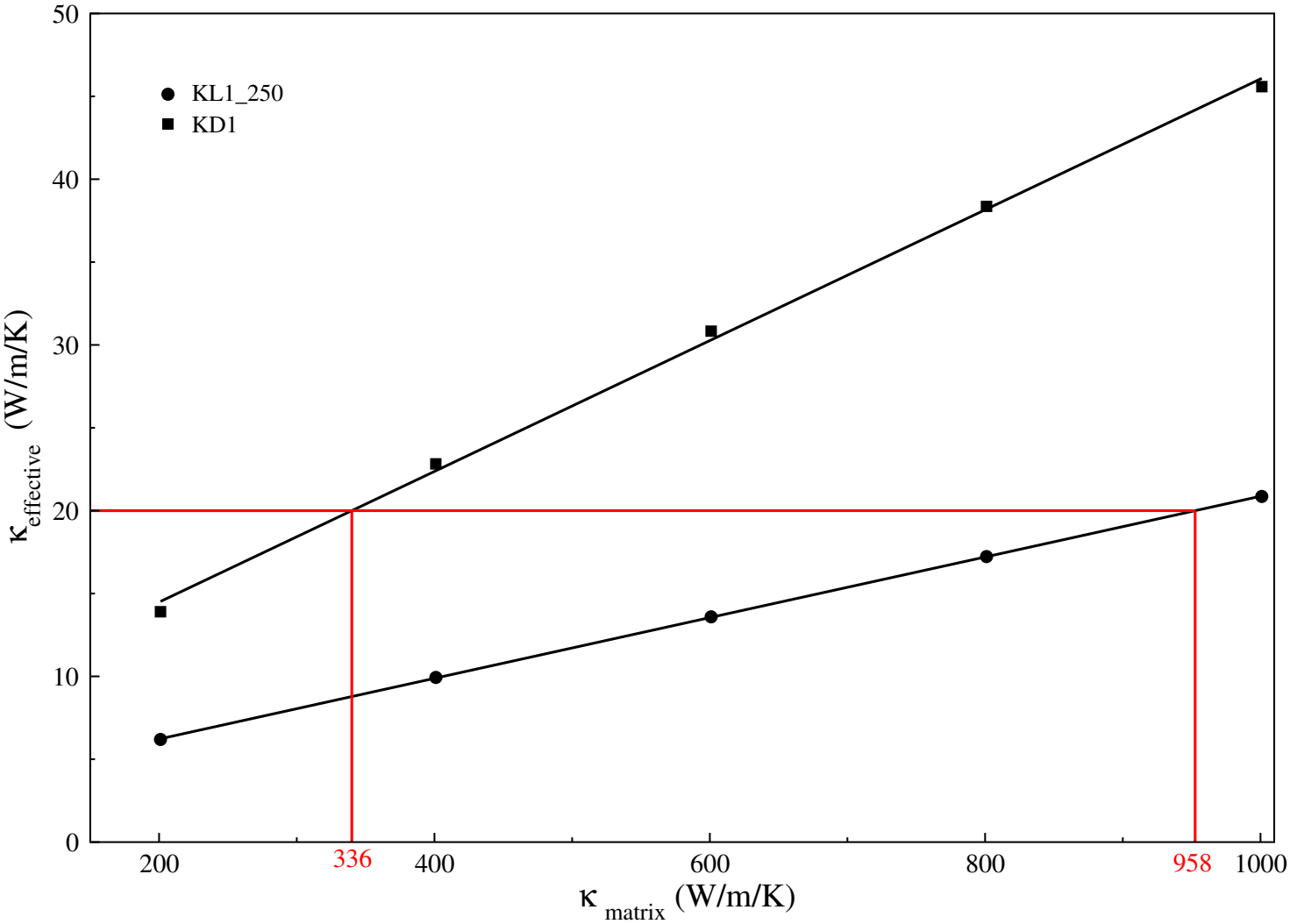
Parameter	**KL1_250**	**KD1**
porosity(%)	82	70
$\kappa_{PCM}\ (W/m/K)$	1	1
$C_{p_{PCM}}\ (J/kg/K)$	4544	4544
$C_{p_{Foam}}\ (J/kg/K)$	1402	1402
$\rho_{PCM}\ (J/kg/K)$	1634	1634
$\rho_{Foam}\ (J/kg/K)$	1318	1318
$\Delta h_m\ (KJ/kg)$	480	480

the porosity of a composite is defined as $\mathcal{E} = \frac{V_{pores}}{V_{Total}}$

We want κ_{eff} the same for both composites

We know that $\kappa_{eff} = a * \kappa_{Foam} + b$

The effective thermal conductivity of the composites



Thermal energy storage capacity

For 2 different 2D rectangular samples, of dimensions: $x_1 \times y_1$ and $x_2 \times y_2$ with porosities $\mathcal{E}_1$ and $\mathcal{E}_2$ with the same latent heat of fusion Δh_m, their thermal energy storage capacities are

$$Q_1 = \mathcal{E}_1 x_1 y_1 \Delta h_m$$

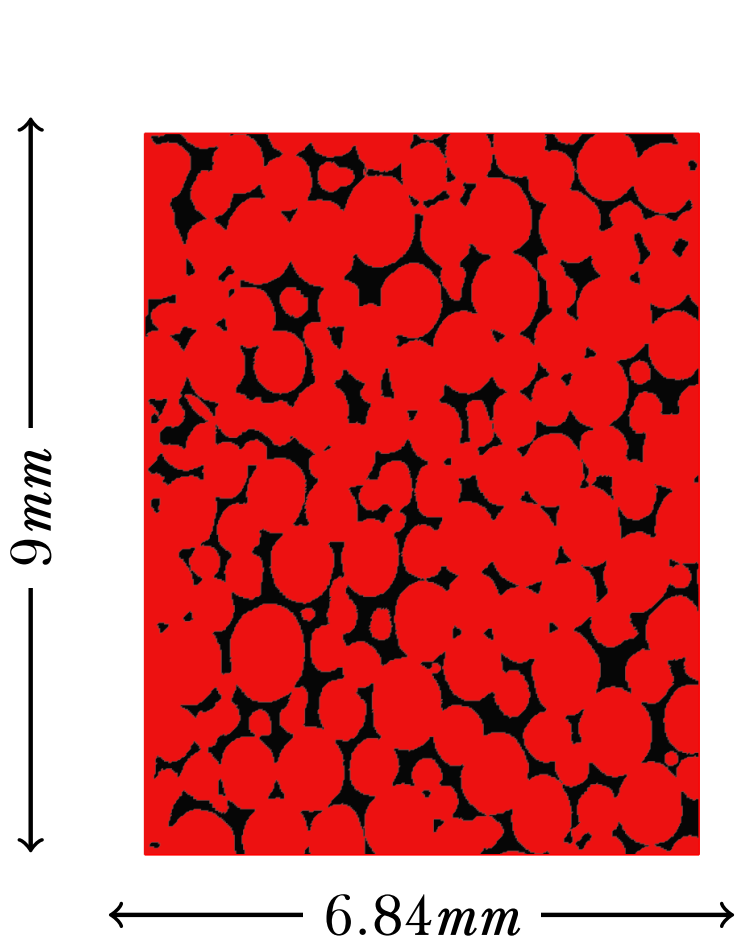
and

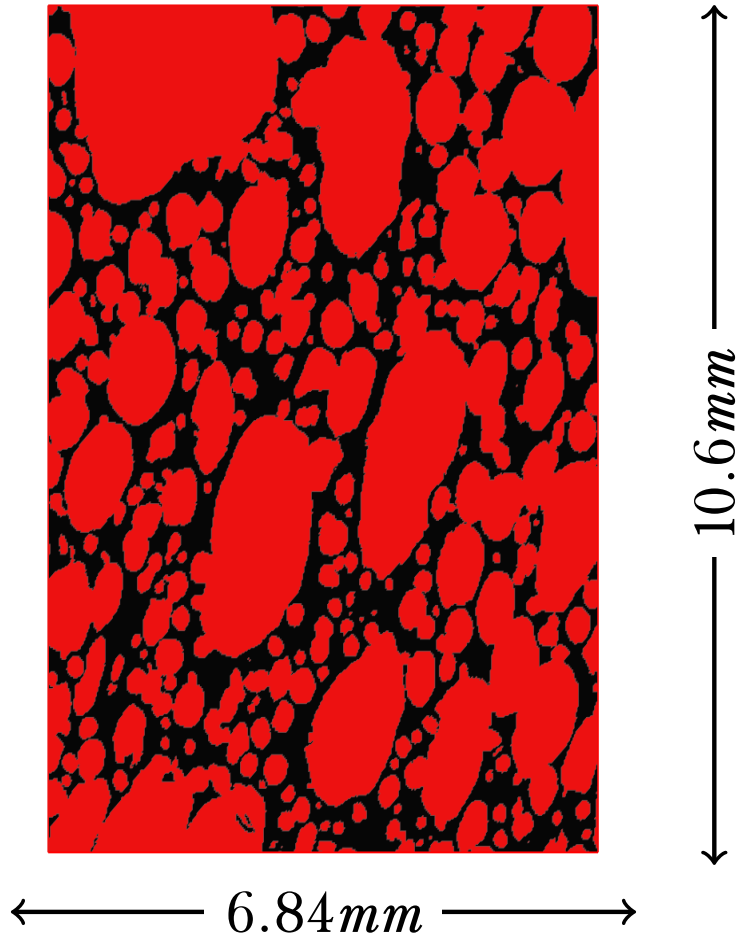
$$Q_2 = \mathcal{E}_2 x_2 y_2 \Delta h_m$$

hence to have $Q_1 = Q_2$ with $x_1 = x_2$ we need to have $\mathcal{E}_1 y_1 = \mathcal{E}_2 y_2$

In our case $\mathcal{E}_{KL1_250} = 82\% > \mathcal{E}_{KD1} = 70\%$
we need to take $y_{KD1} > y_{KL1_250}$

Dimensions of the samples





The model

$$\begin{cases} \partial_t H(T) - div(\kappa \nabla T) = 0 \quad in\ \Omega_G \cup \Omega_S \\ \kappa \partial_n T = 0 \quad on \quad \Gamma_N \\ T = T_D \quad on \quad \Gamma_D \\ [\kappa \partial_n T] = 0 \quad and \quad \gamma \\ T(0,x) = T_0 \quad in \quad \Omega \end{cases}$$

The enthalpy $H(T)$ is given by:

$$H(T) = \begin{cases} [(\rho c)_S^s (1 - f(T)) + (\rho c)_S^l f(T)]T + \rho_S \Delta h_m f(T) & \text{in } \Omega_S \\ (\rho c)_G T & \text{in } \Omega_G \end{cases}$$

The time scheme

Implicit Euler : $\frac{H^{n+1}-H^n}{\Delta t} - div(\kappa \nabla T^{n+1}) = 0$

Chernoff Scheme : $H = \beta^{-1}(T)$ where $T = \beta(H) \rightarrow \quad \partial_t H = 1/\beta'(T)\partial_t T$

on définie:

$$H^{n+1} = H^n + \gamma(T^{n+1} - \beta(H^n))$$

where γ corresponds to $\frac{1}{\beta'(T)}$

$$\frac{\gamma}{\Delta t}(T^{n+1} - \beta(H^n)) - div(\kappa \nabla T^{n+1}) = 0$$

Linear Problem

$$\lambda E(\boldsymbol{u}) - div(\kappa \nabla \boldsymbol{u}) = g \quad dans \quad \Omega_G \cup \Omega_S$$

$$\kappa \partial_n \boldsymbol{u} = 0 \quad sur \quad \Gamma_N$$

$$\boldsymbol{u} = u_D \quad sur \quad \Gamma_D$$

$$[\kappa \partial_n \boldsymbol{u}] = 0 \quad et \quad R(\kappa \partial_n \boldsymbol{u})_s = [\boldsymbol{u}] \quad sur \quad \gamma$$

Sobolov spaces

$$H(div, \Omega) := \{q \in L^2(\Omega)^2 : divq \in L^2(\Omega)\}$$

$$H_{0,N}(div, \Omega) := \{q \in H(div, \Omega) : q.n = 0 \quad sur \quad \Gamma_N\}$$

Mixed Formulation

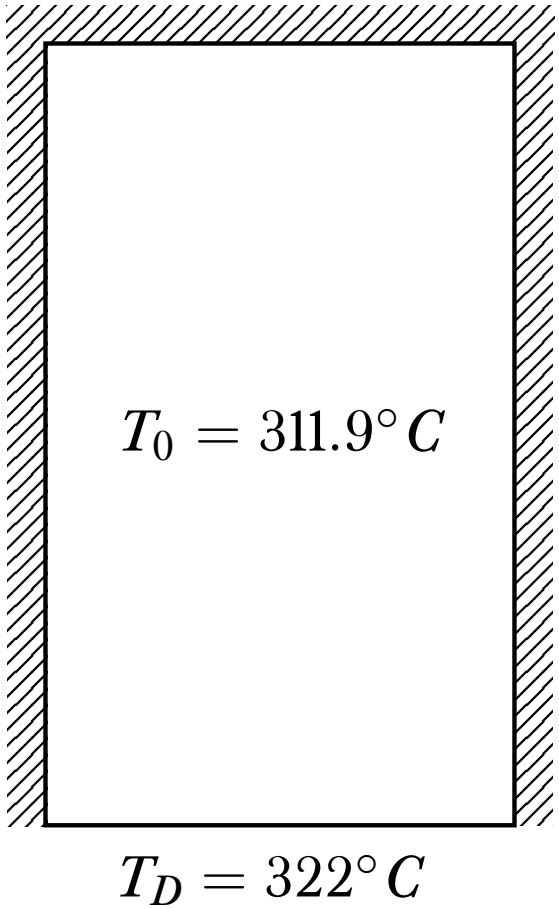
$$\boldsymbol{p} = \kappa \nabla \boldsymbol{u}$$

Find $(\boldsymbol{u}, \boldsymbol{p})$ such as

$$\int_\Omega \frac{1}{\kappa} \boldsymbol{p}.q dx + \int_\Omega \boldsymbol{u}\ div\ q\ dx - \int_\gamma R(\boldsymbol{p}.n)(q.n)\ d\gamma = \int_{\Gamma_D} u_D q.n d\gamma \quad \forall q \in H_{0,N}(div, \Omega)$$

$$\int_\Omega \lambda E(\boldsymbol{u}) v dx - \int_\Omega v\ div\ \boldsymbol{p}\ dx = \int_\Omega v g dx \quad \forall v \in L_2(\Omega)$$

Initial and boundary conditions



The analytic solution in the case of a homogeneous PCM

$$T(x,t) = T_D + (T_m - T_D)\frac{erf(\frac{x}{2\sqrt{\alpha t}})}{erf(\xi)} \quad 0 < x \leq X(t)$$

with $X(t) = 2\xi\sqrt{\alpha t}$ where ξ is solution to

$$\frac{Stefan}{erf(\xi)e^{\xi^2}} = \sqrt{\pi}\xi$$

with

$$Stefan = \frac{c(T_D - T_m)}{\Delta h_m}$$

The heat flux at surface $y = 0$ is given by

$$q(0,t) = -\frac{\kappa(T_m - T_D)}{erf(\xi)\sqrt{\pi\alpha t}}$$

The similarity variable is defined by $\mu = \frac{x}{\sqrt{\alpha t}}$

PCM with equivalent thermo-physical properties

Recall : the porosity of a composite is defied as $\mathcal{E} = \frac{V_{pores}}{V_{Total}}$

If ρ_G and ρ_S are the density of the matrix and the PCM then $\rho_{equivalent} = \mathcal{E}\rho_S + (1-\mathcal{E})\rho_G$

C_{p_G} and C_{p_S} the thermal capacities of the matrix and of the PCM,

$$C_{p_{equivalent}} = \mathcal{E}C_{p_S} + (1-\mathcal{E})C_{p_G}$$

Δh_m is the latent heat of fusion of the PCM contained in the matrix, $\Delta h_{m_{equivalent}} = \mathcal{E}\Delta h_m$

Dimensionless Variables

$$x^* = \frac{x}{x_{max}},\ t^* = \frac{\alpha t}{x^2},\ T^* = \frac{T - T_D}{T_m - T_D},\ X^* = \frac{X}{x_{max}}\ and\ q^* = \frac{x_{max} q}{\kappa(T_m - T_D)}$$

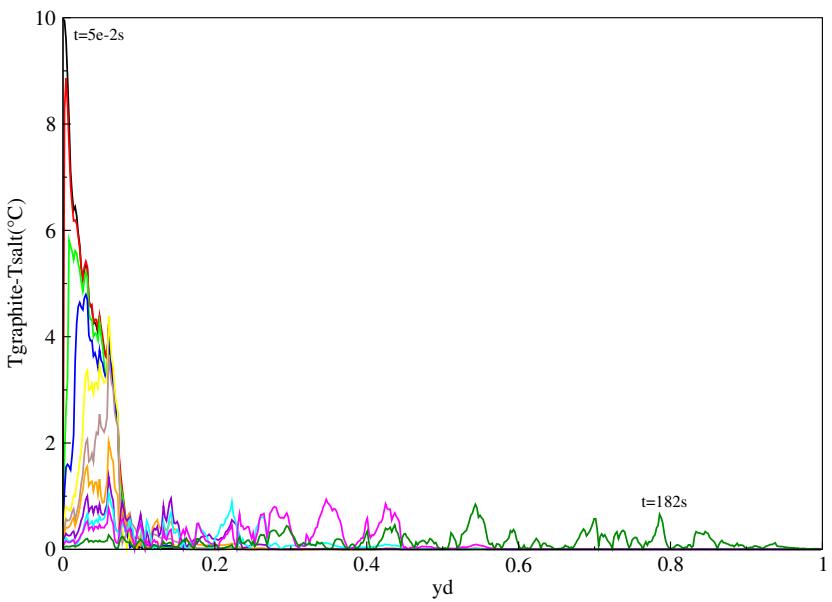
$$T^*(x^*, t) = \frac{erf(\frac{x^*}{2\sqrt{t^*}})}{erf(\xi)},\ X^*(t) = 2\xi\sqrt{t^*},\ q^*(0, t) = -\frac{1}{\sqrt{\pi} erf(\xi)\sqrt{t^*}}$$

For a composite material,we introduce the mean dimensionless temperature

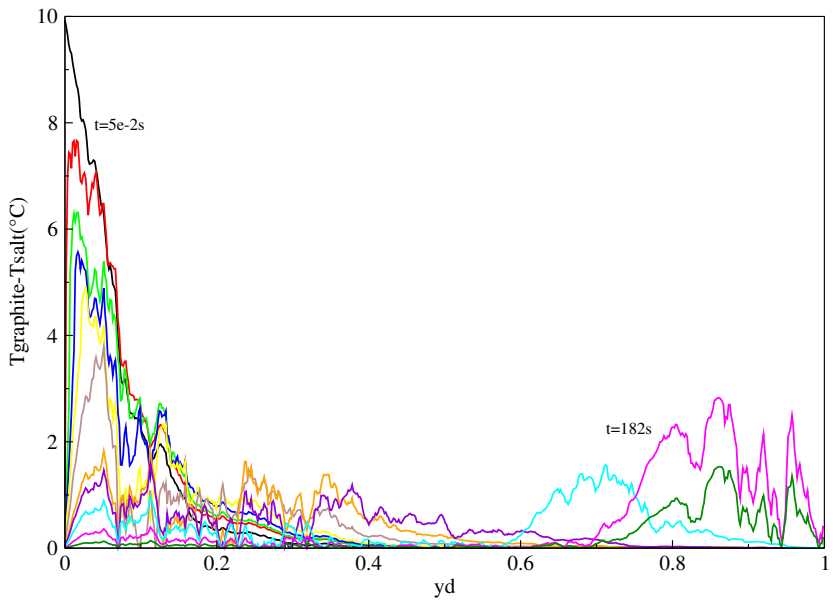
$$T^*_{Mean}(y^*, t) = \frac{1}{Nx} \sum_{n=1}^{n=Nx} T^*(x, y, t)$$

Comparison with a homogeneous PCM

KL1_250

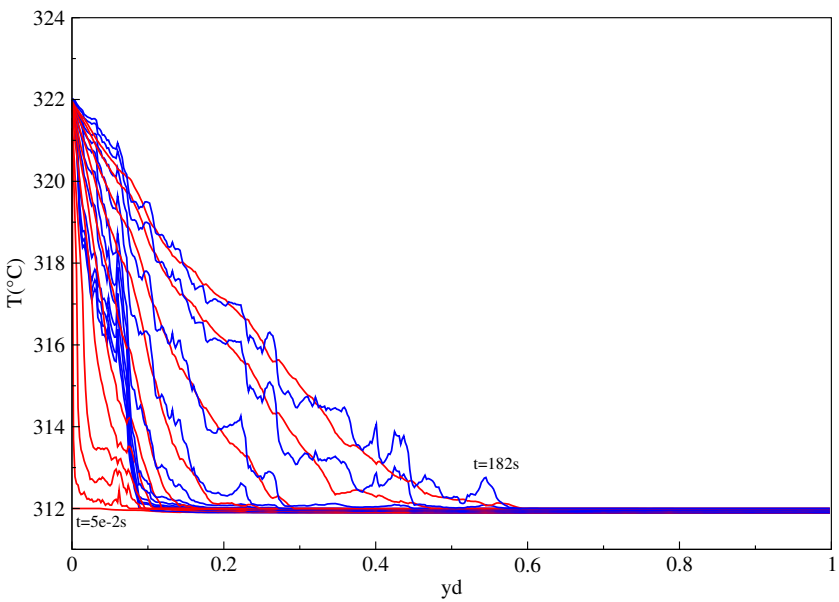


KD1

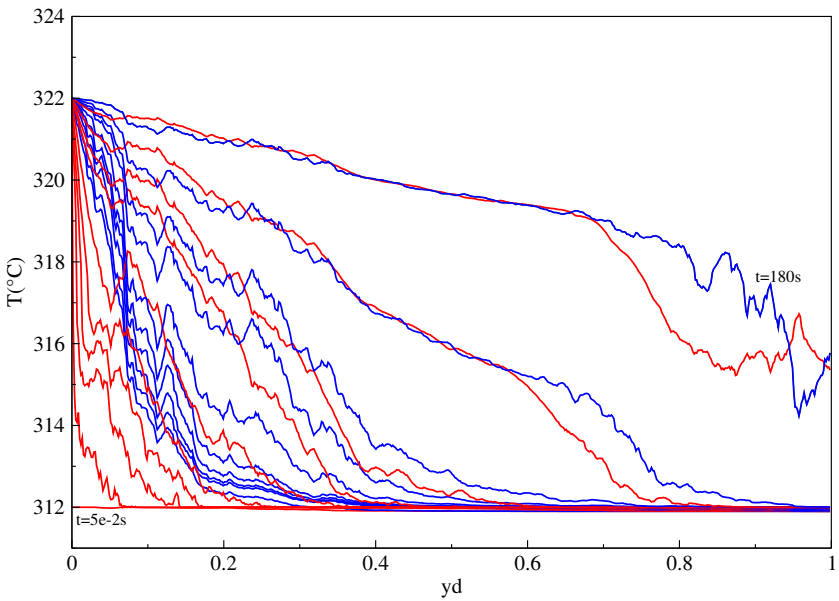


KL1_250: temperature profile

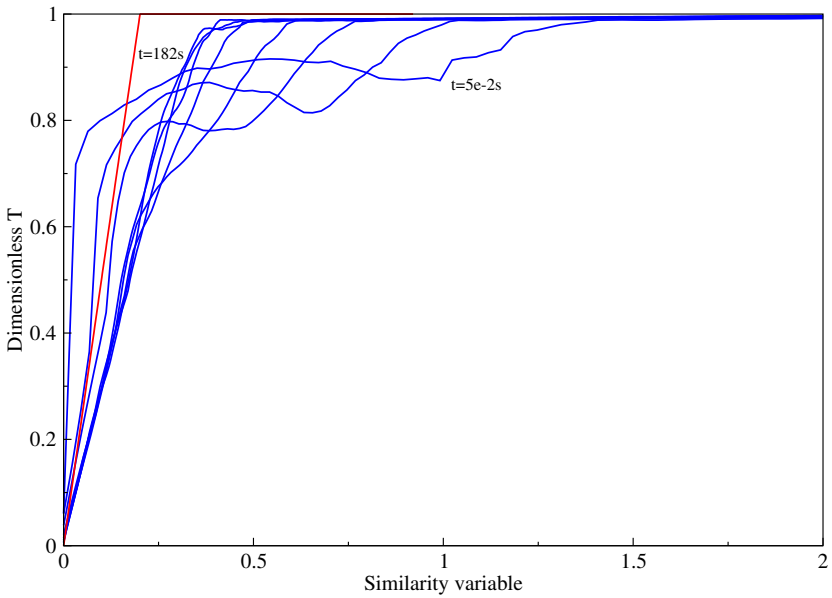
KL1_250



KD1



KL1_250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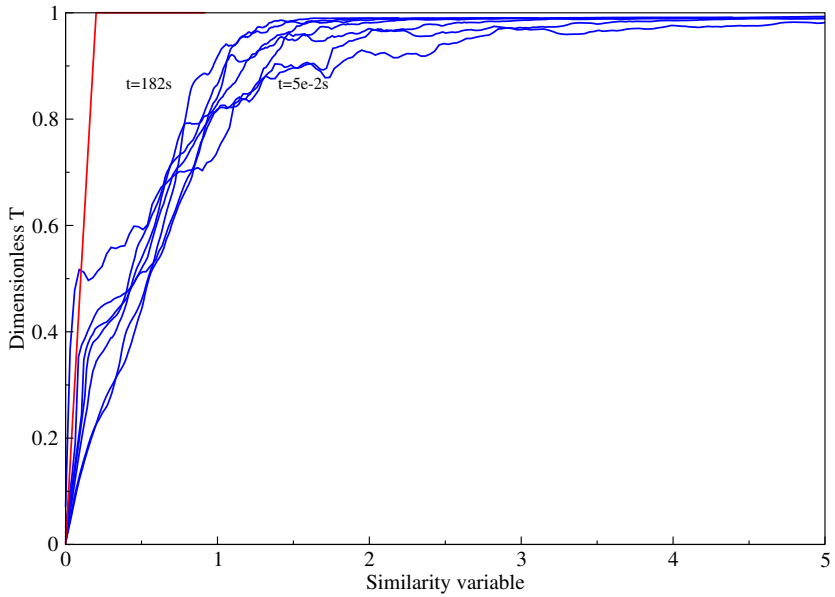
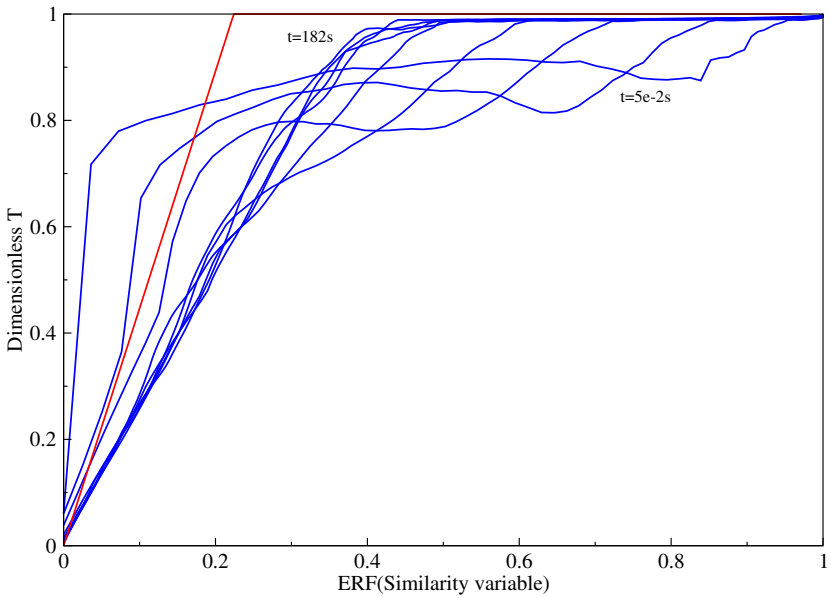
t=5e-2s
Dimensionless T
Similarity variable
0
0.2
0.4
0.6
0.8
1
0
0.5
1
1.5
2

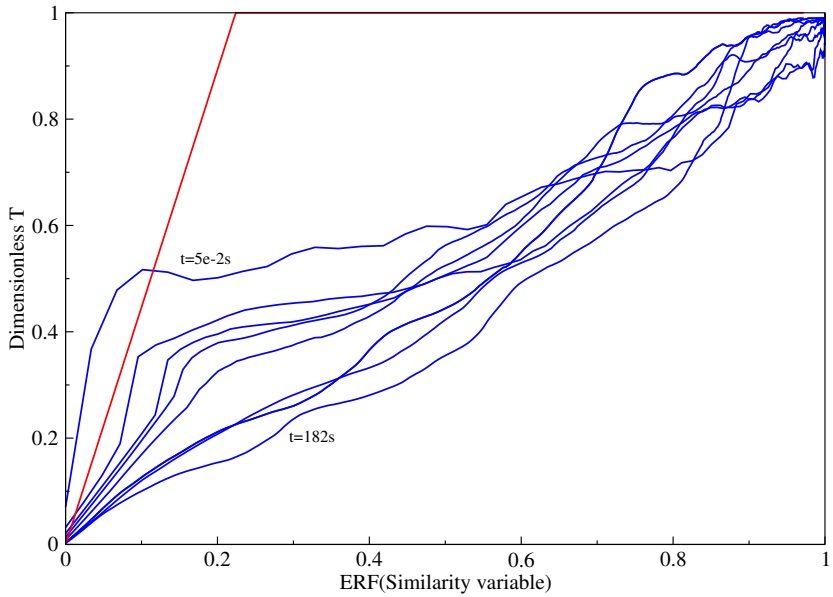
KD1

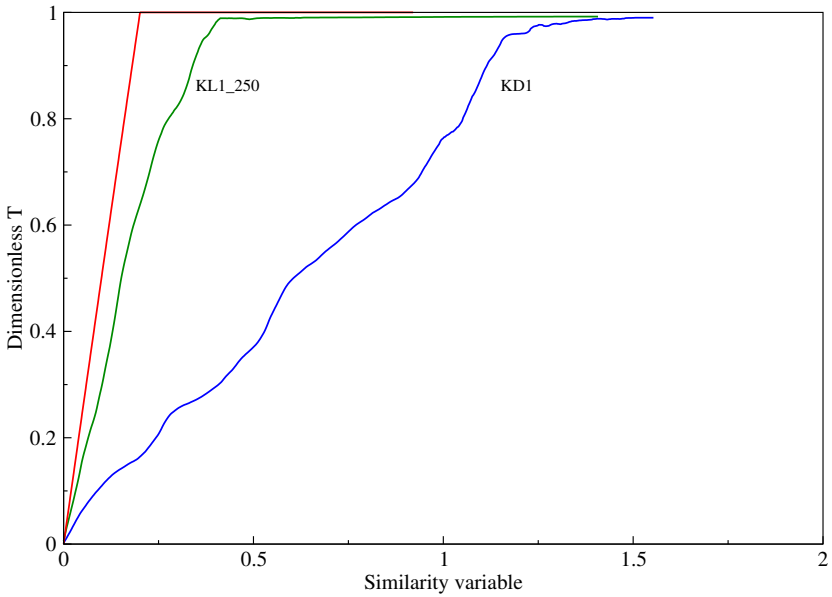

t=182s
t=5e-2s
Dimensionless T
Similarity variable
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5

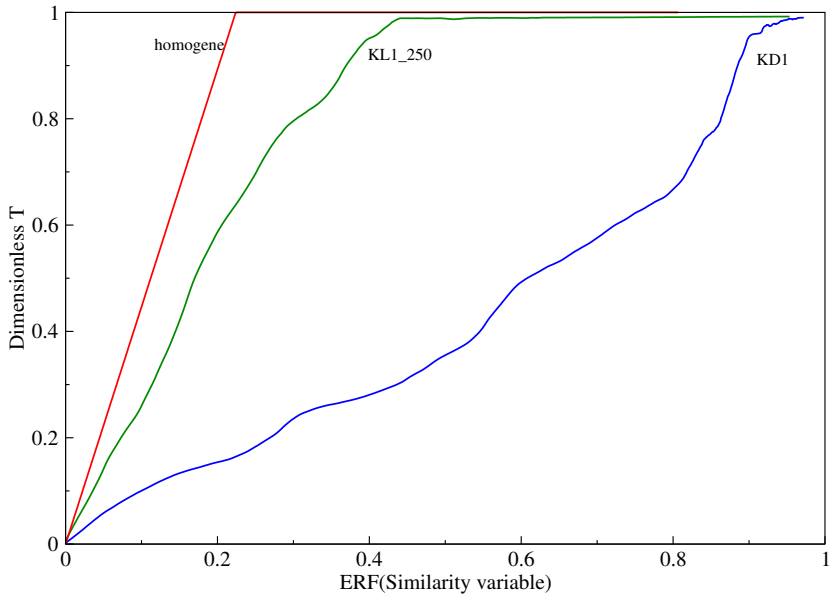
KL1_250

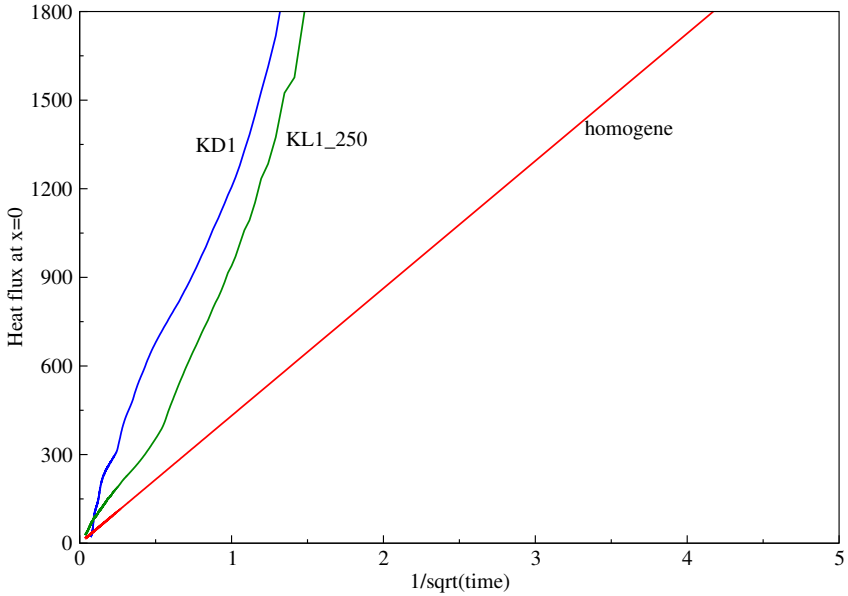

t=182s
t=5e-2s
Dimensionless T
ERF(Similarity variable)
0
0.2
0.4
0.6
0.8
1

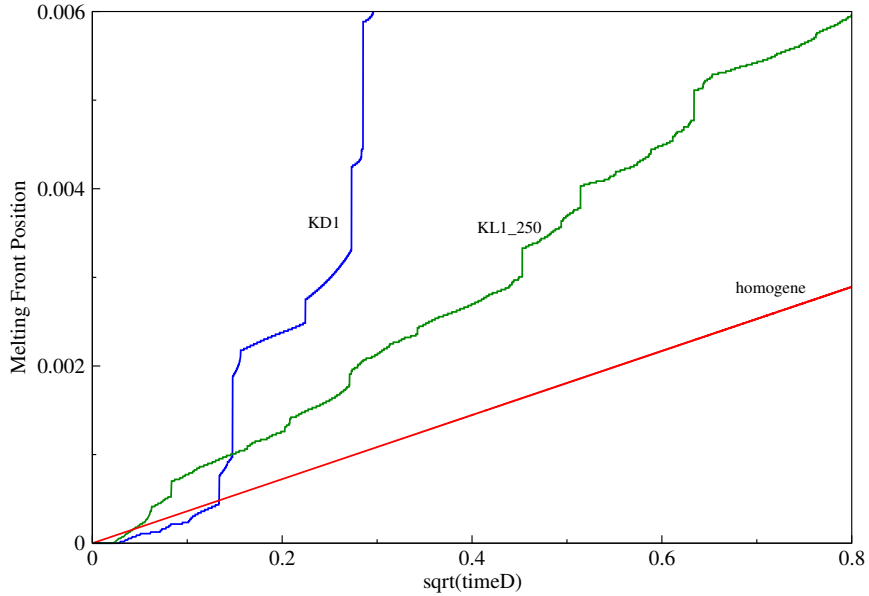
KD1


t=5e-2s
t=182s
Dimensionless T
ERF(Similarity variable)
0
0.2
0.4
0.6
0.8
1


KL1_250
KD1
Dimensionless T
Similarity variable
0
0.2
0.4
0.6
0.8
1
0
0.5
1
1.5
2


homogene
KL1_250
KD1
Dimensionless T
ERF(Similarity variable)
0
0.2
0.4
0.6
0.8
1



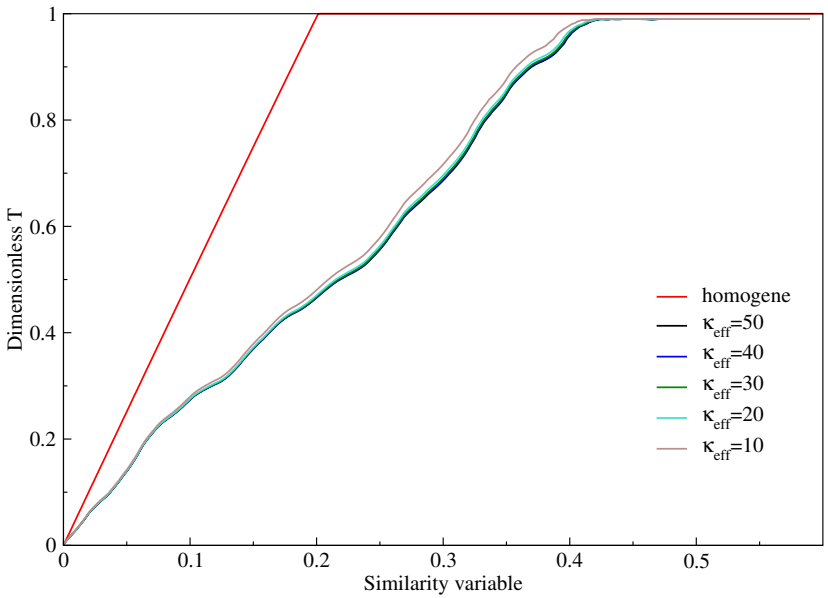


Conclusion : from $t = 182s$ KL1_250 could be assimilated to a homogeneous PCM with equivalent thermo-physical properties.

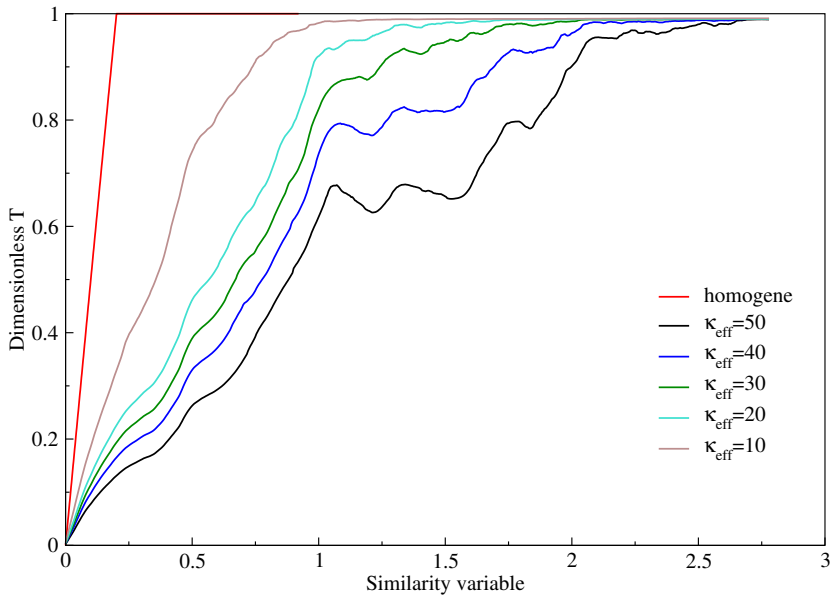
Effects of increasing the effective thermal conductivity

$\kappa_{effective}$ $(W/m/K)$	κ_{Foam} KL1_250	κ_{Foam}KD1
10	410	125
20	958	336
30	1510	580
40	2060	848
50	2615	1128

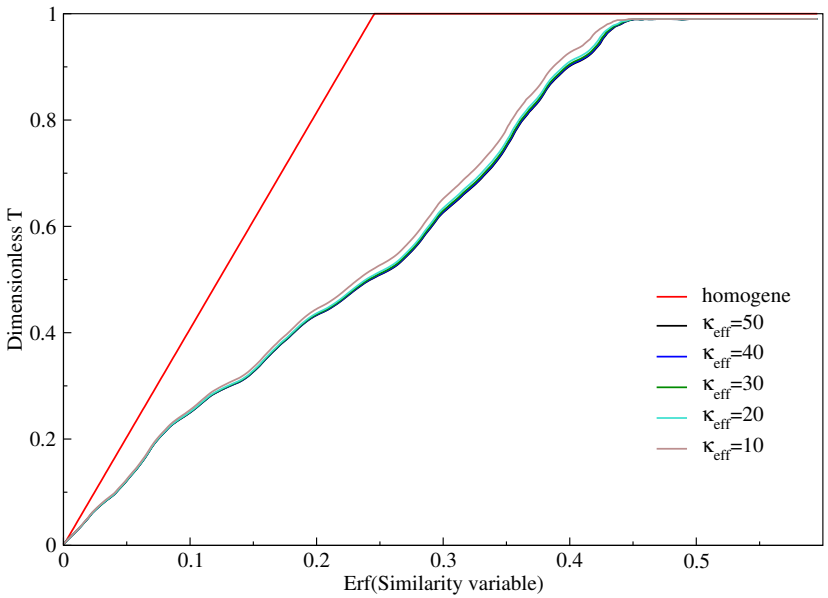
KL1_250


Dimensionless T
Similarity variable
homogene
κeff=50
κeff=40
κeff=30
κeff=20
κeff=10

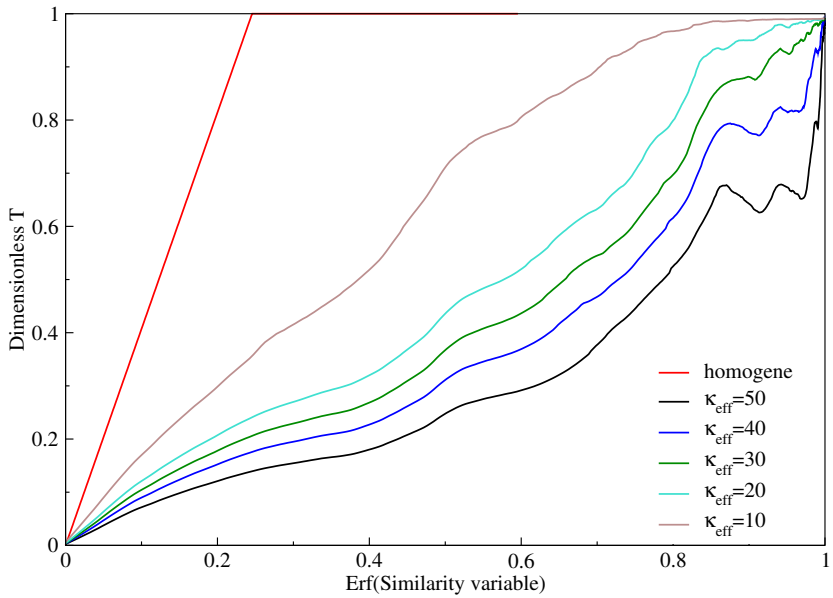
KD1


Dimensionless T
Similarity variable
homogene
κeff=50
κeff=40
κeff=30
κeff=20
κeff=10

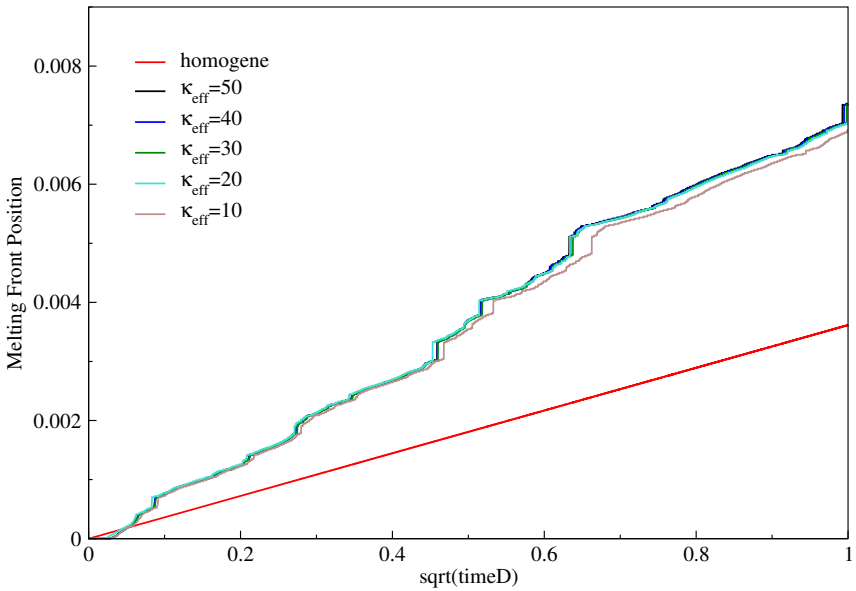
KL1_250


Dimensionless T
Erf(Similarity variable)
homogene
κeff=50
κeff=40
κeff=30
κeff=20
κeff=10

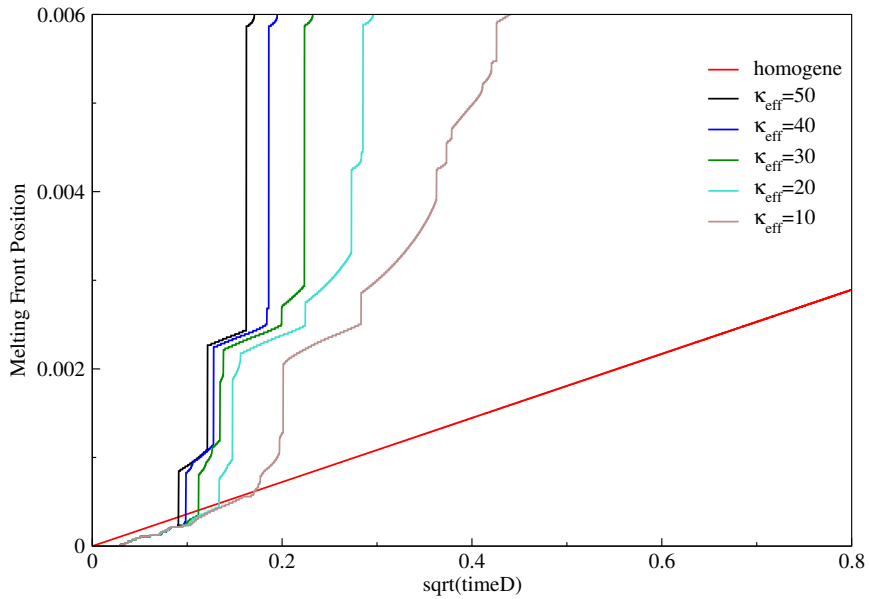
KD1


Dimensionless T
Erf(Similarity variable)
homogene
κeff=50
κeff=40
κeff=30
κeff=20
κeff=10

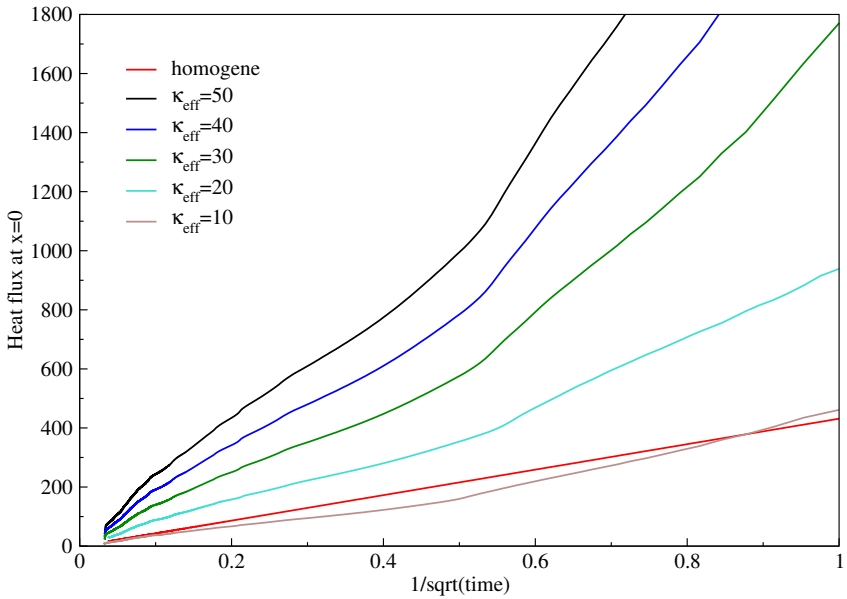
KL1_250


homogene
κeff=50
κeff=40
κeff=30
κeff=20
κeff=10
Melting Front Position
sqrt(timeD)
0
0.002
0.004
0.006
0.008
0.2
0.4
0.6
0.8
1

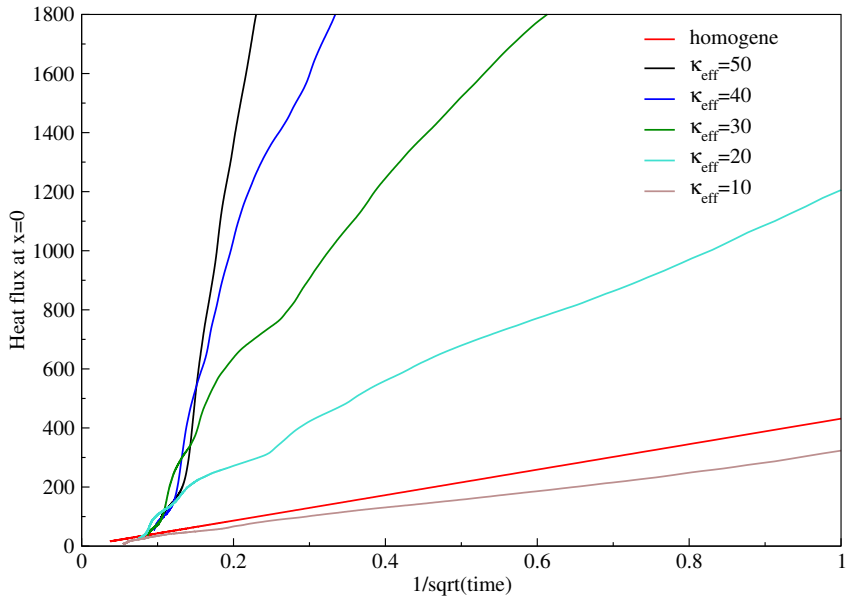
KD1

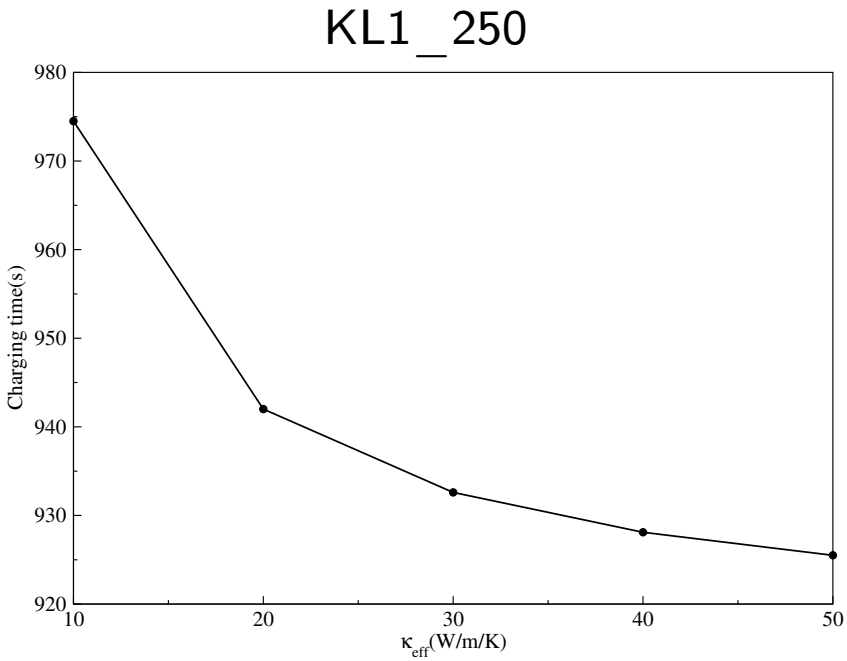

homogene
κeff=50
κeff=40
κeff=30
κeff=20
κeff=10
Melting Front Position
sqrt(timeD)
0
0.002
0.004
0.006
0.2
0.4
0.6
0.8

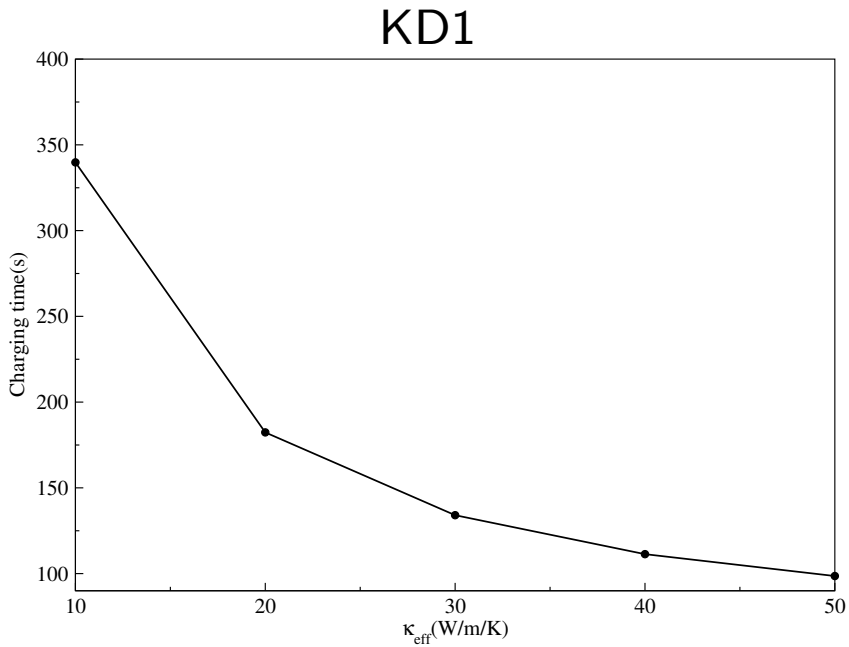
KL1_250


Heat flux at x=0
1/sqrt(time)
homogene
κeff=50
κeff=40
κeff=30
κeff=20
κeff=10
0
200
400
600
800
1000
1200
1400
1600
1800
0.2
0.4
0.6
0.8
1

KD1


Heat flux at x=0
1/sqrt(time)
homogene
κeff=50
κeff=40
κeff=30
κeff=20
κeff=10
0
200
400
600
800
1000
1200
1400
1600
1800
0.2
0.4
0.6
0.8
1





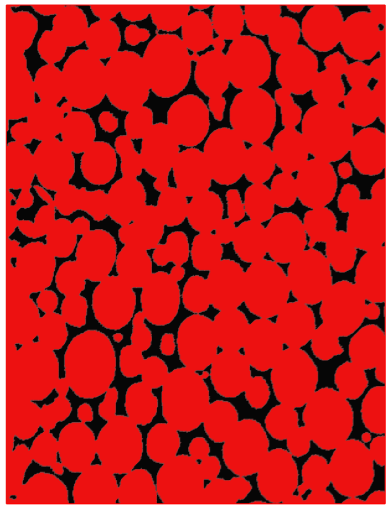
Conclusions :

- Changing the values of the effective thermal conductivity has more effects in the case of KD1 because the volume occupied by the matrix represents 30% of the total volume where for KL1_250 it represents only 18% of the total volume.
- Increasing the value of the effective thermal conductivity allows to accelerate the charging/discharging cycles.

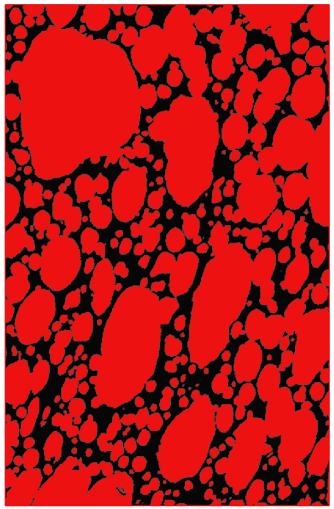
Influence of the pores size

- Dilatation in both directions :
  $f_{new}(x, y) = f(\alpha x, \alpha y)$ with $\alpha \in \{0.6, 0.7, 0.8, 0.9, 1\}$

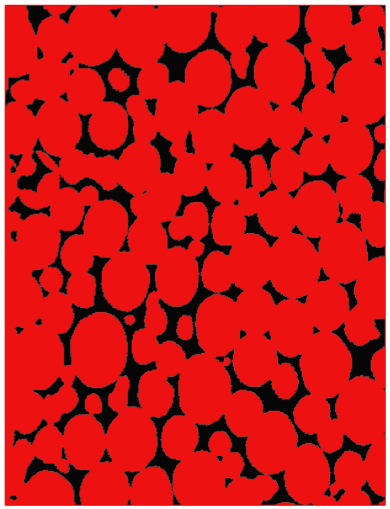
- $\kappa_{eff} = 20 \ W/m/K, \forall \alpha$



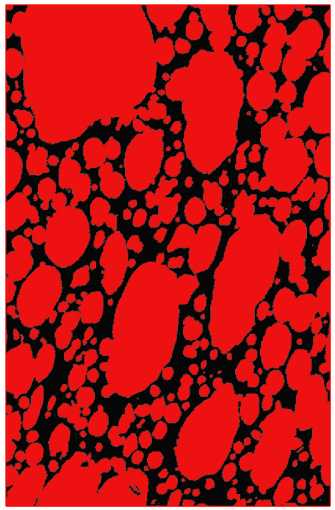
initial



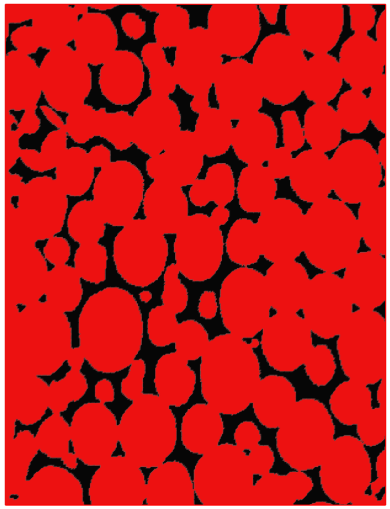
initial



$\alpha = 0.9$



$\alpha = 0.9$



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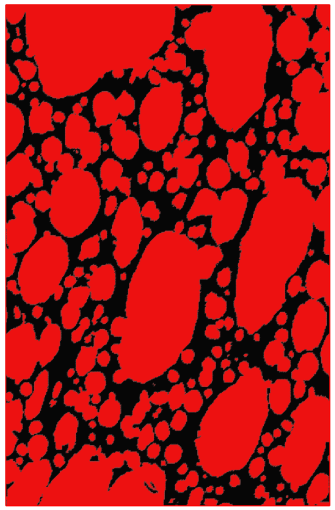

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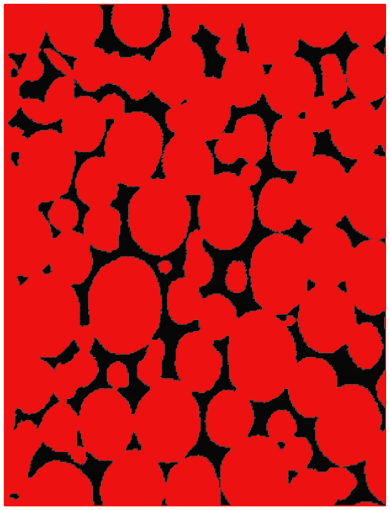

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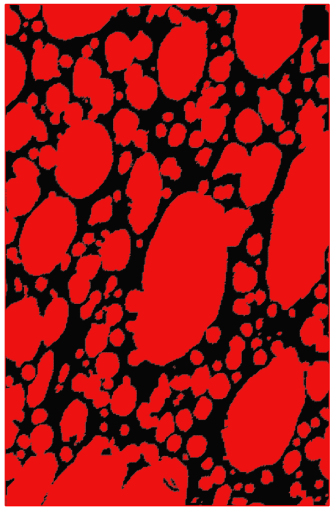

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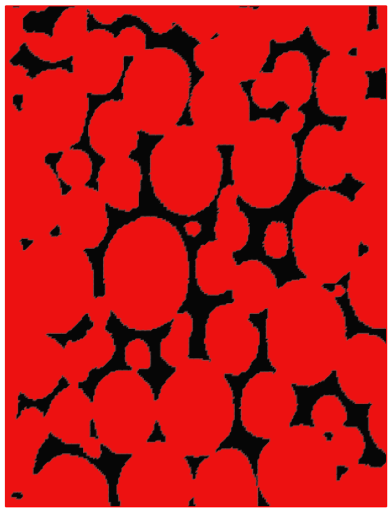

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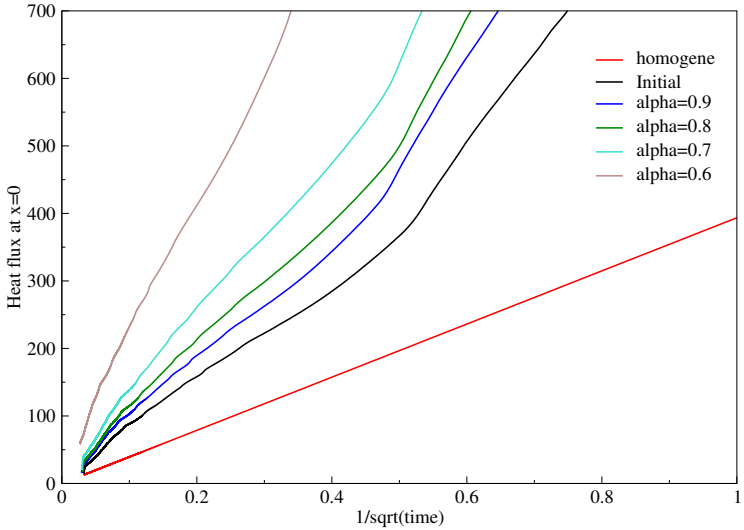
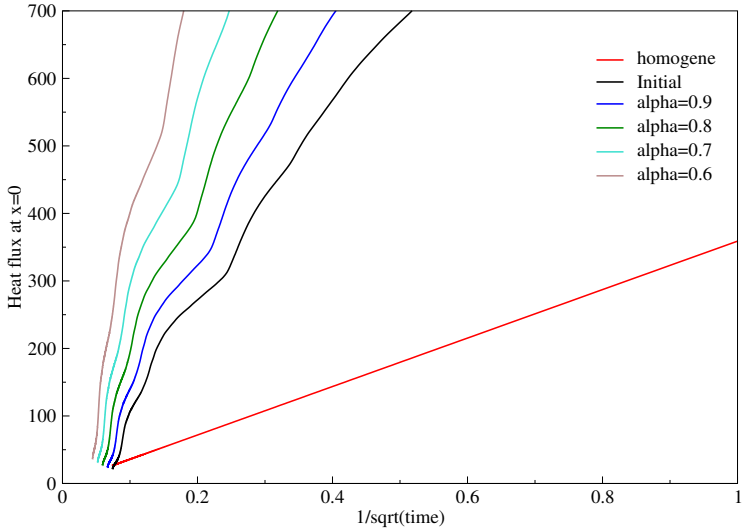
$\alpha = 0.6$


Heat flux at x=0
1/sqrt(time)
homogene
Initial
alpha=0.9
alpha=0.8
alpha=0.7
alpha=0.6


Heat flux at x=0
1/sqrt(time)
homogene
Initial
alpha=0.9
alpha=0.8
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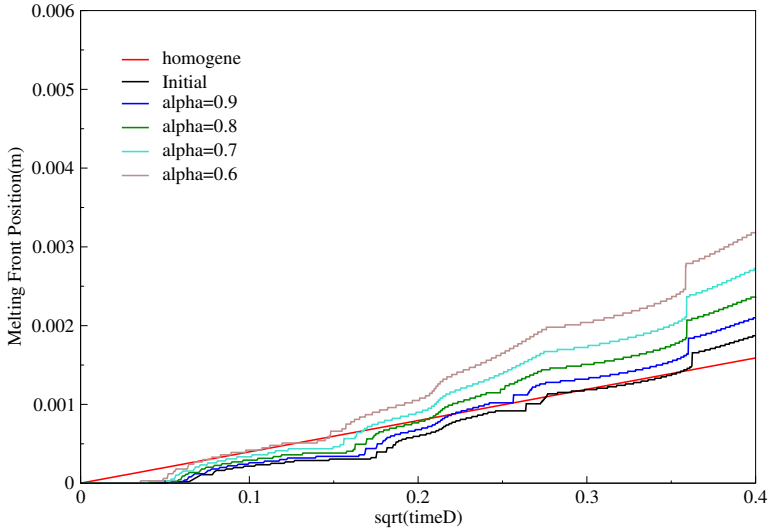
alpha=0.6


Melting Front Position(m)
sqrt(timeD)
homogene
Initial
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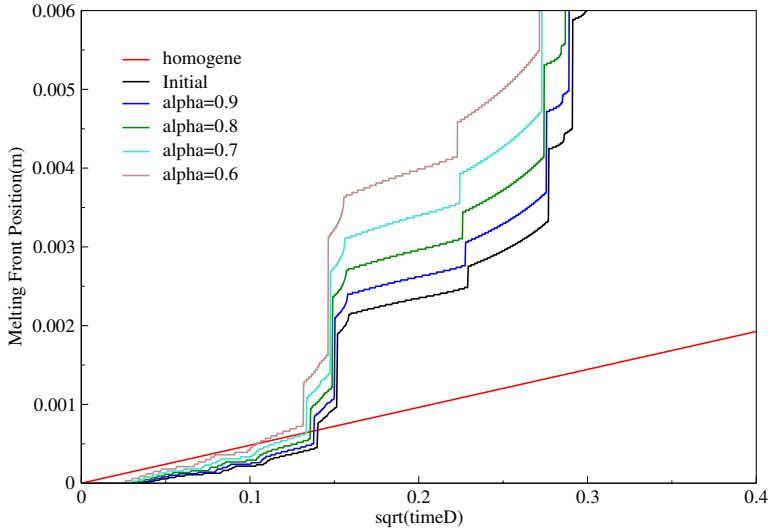
alpha=0.8
alpha=0.7
alpha=0.6


Melting Front Position(m)
sqrt(timeD)
homogene
Initial
alpha=0.9
alpha=0.8
alpha=0.7
alpha=0.6

Conclusions

Enlarging the pores allow to

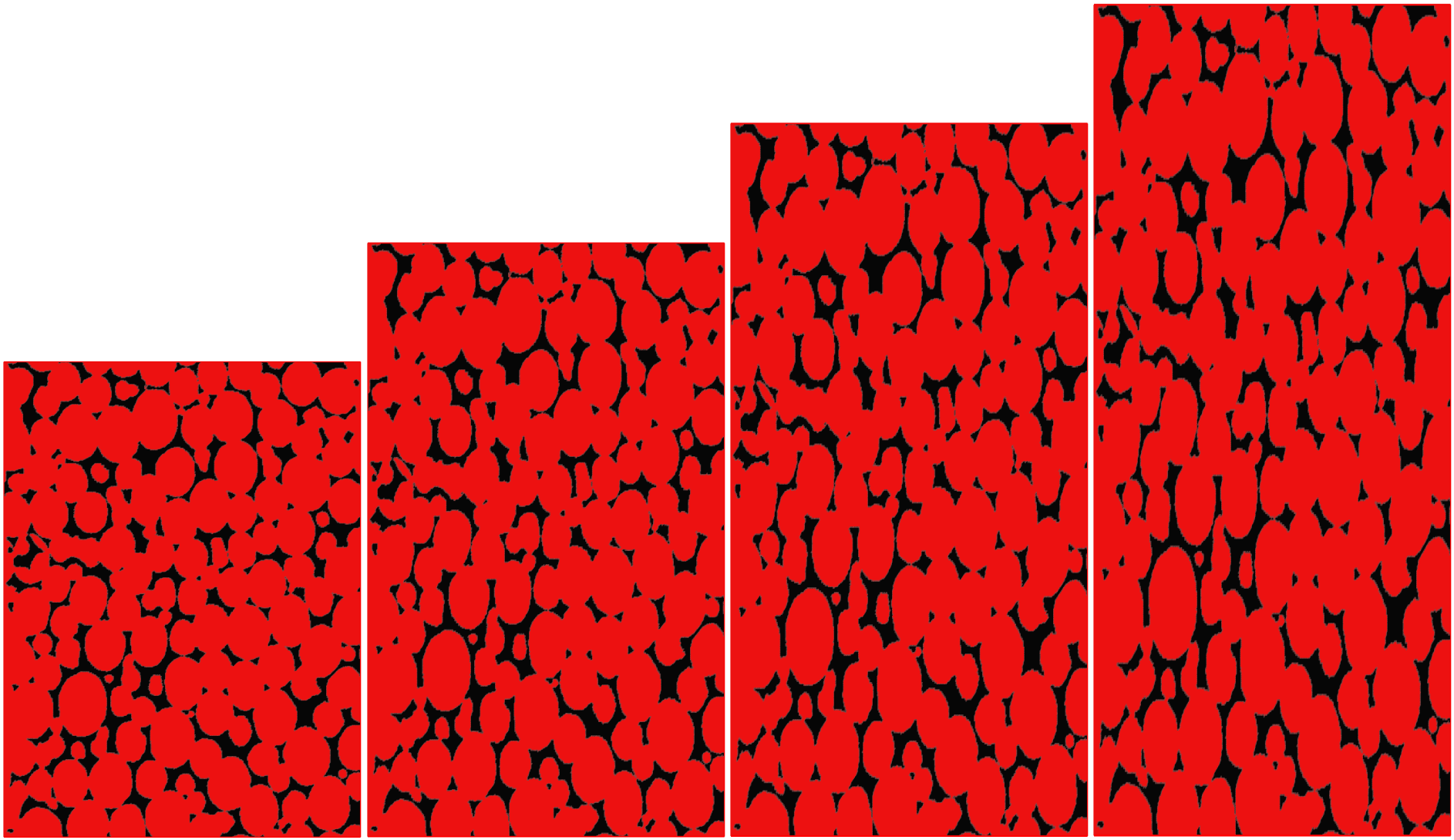
- accelerate the melting process inside the ports.
- increasing the heat flux while keeping a constant thermal conductivity.

Influence of the pores shape

- Dilatations in both directions :
  $f_{nouveau}(x, y) = f(\alpha x, \beta y)$ avec $\alpha, \beta \in \{0.6, 0.7, 0.8, 0.9, 1\}$

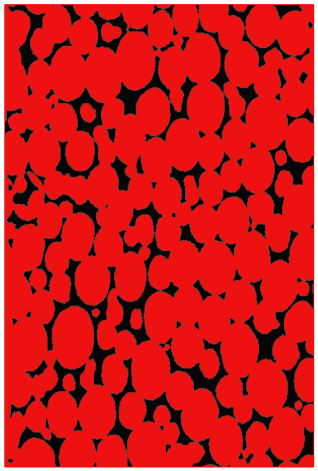
- $\kappa_{eff} = 20 \ W/m/K, \forall \alpha, \beta$

Dilatations de KL1_250 dans la direction y



$\beta = 0.9$ $\beta = 0.8$ $\beta = 0.7$ $\beta = 0.6$

Dilatations de KL1_250 dans la direction x



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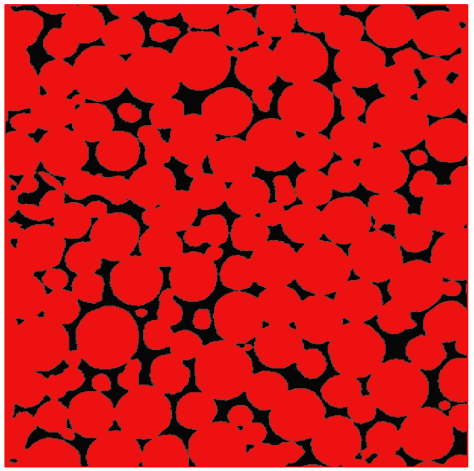

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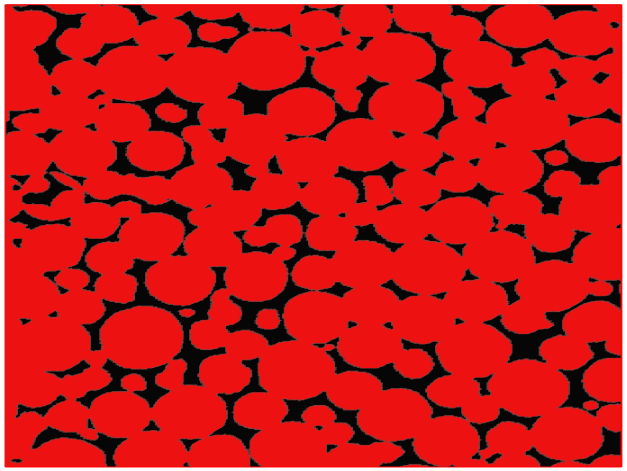

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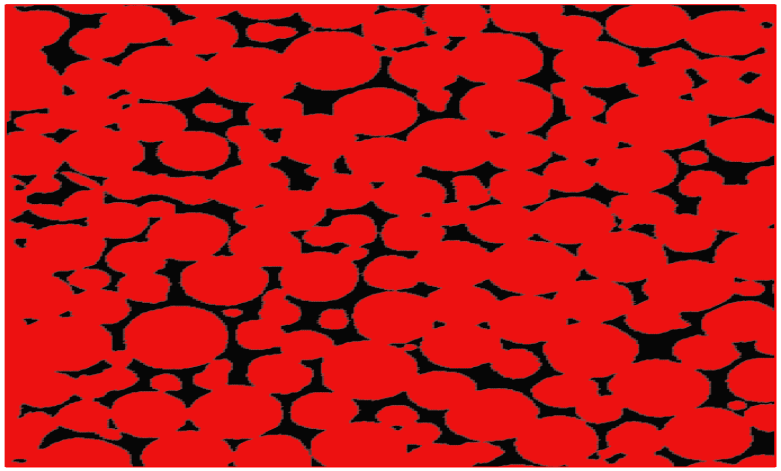

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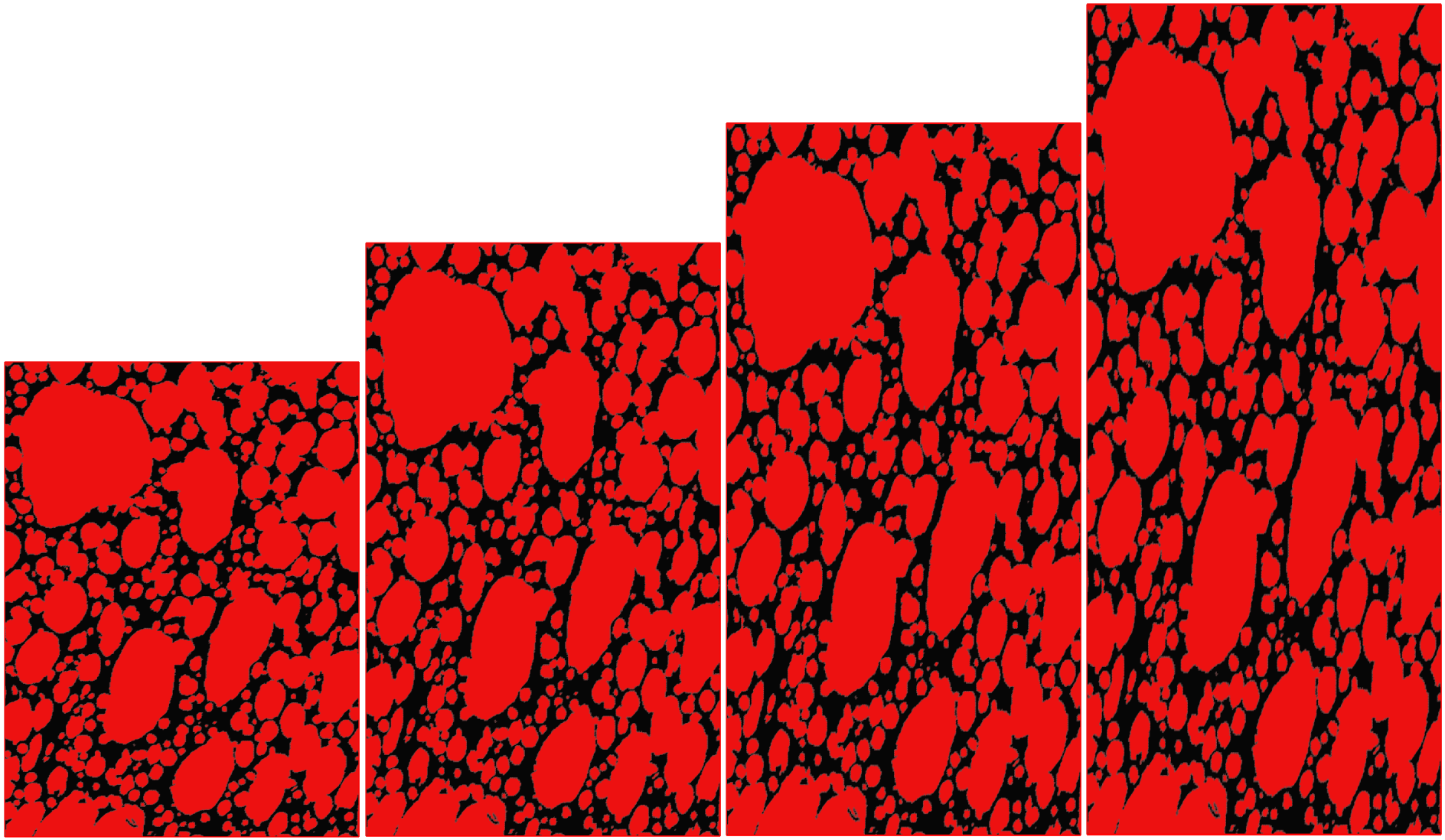
Dilatations de KD1 dans la direction *y*



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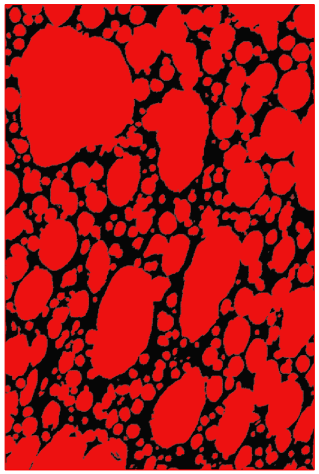
Dilatations de KD1 dans la direction x



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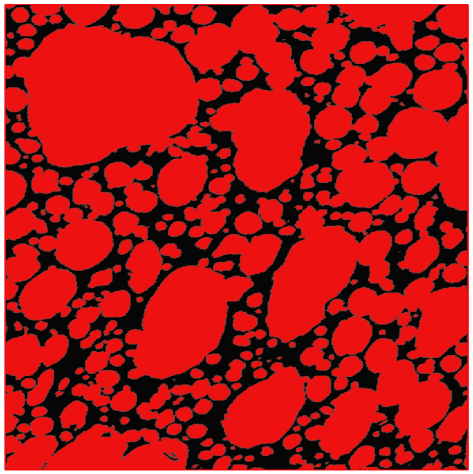

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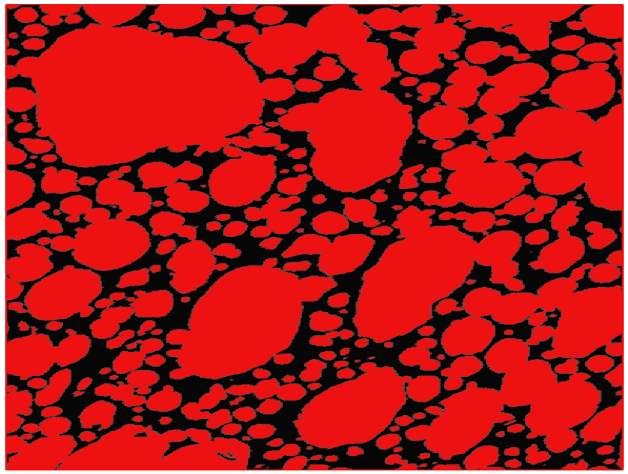

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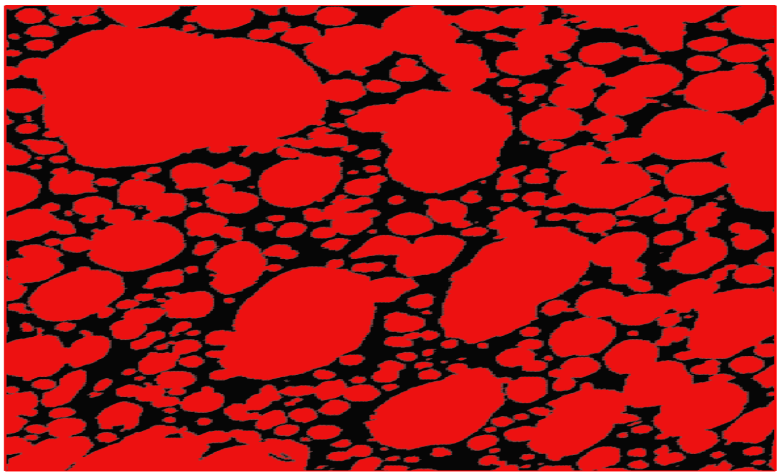

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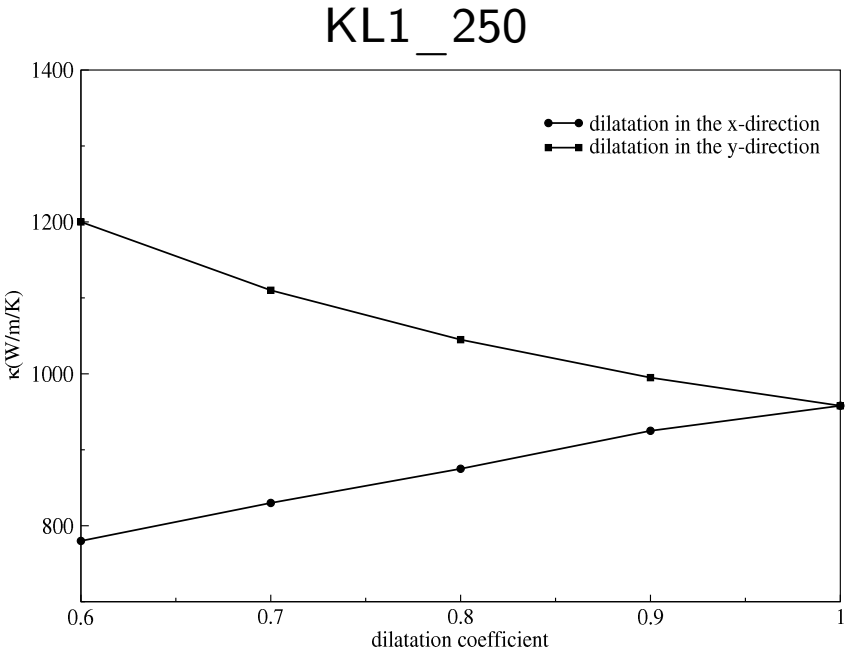
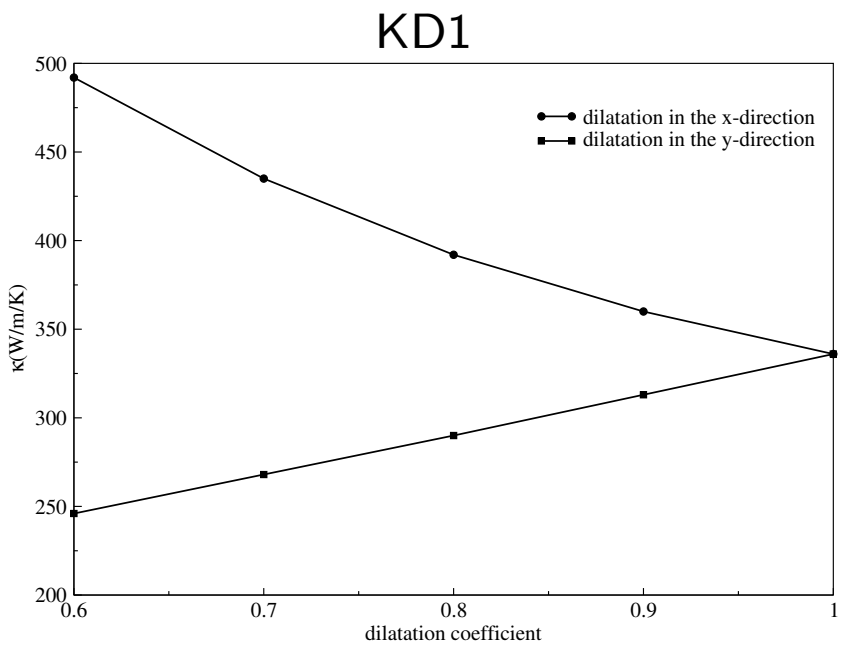
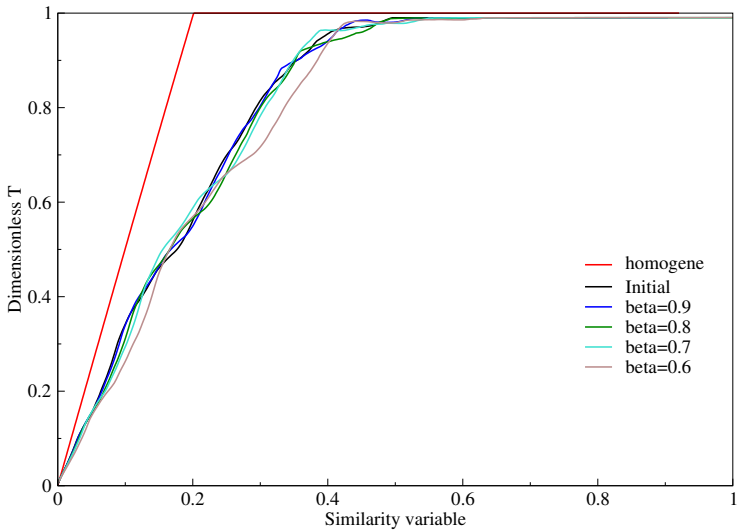


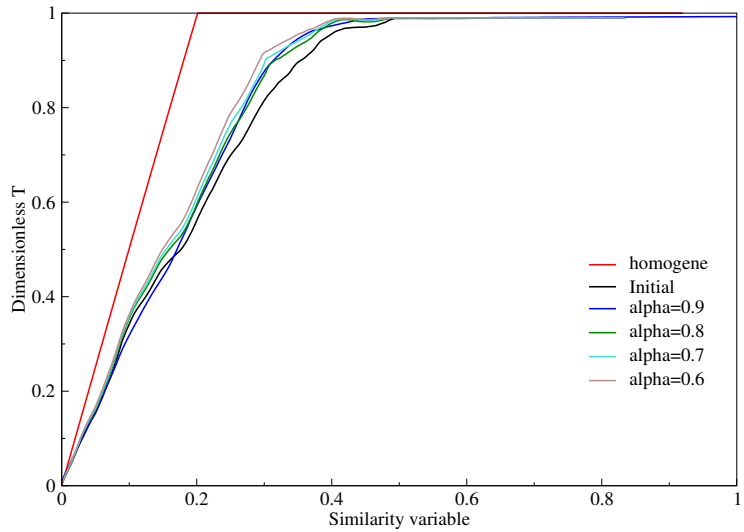


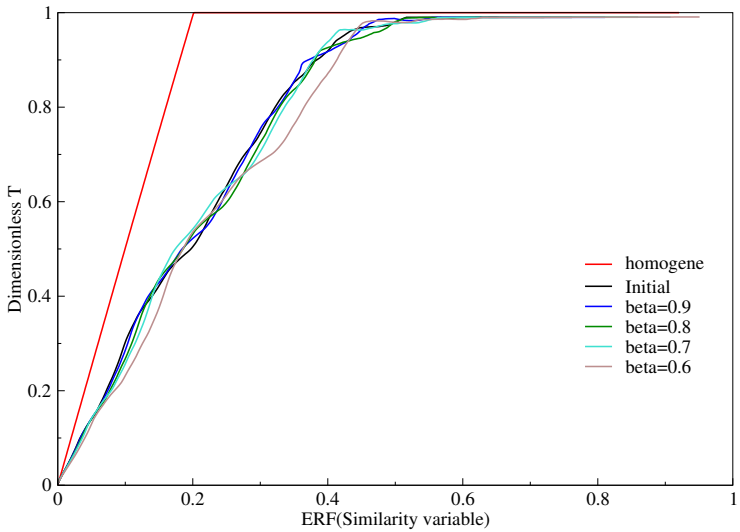
- For KL1_250 we can see that the more we poll in the ***y*** direction the more we need to increase the thermal conductivity of the matrix and the more we poll in the ***x*** direction the more we increase the thermal conductivity of the matrix.
- On note the opposite for KD1.

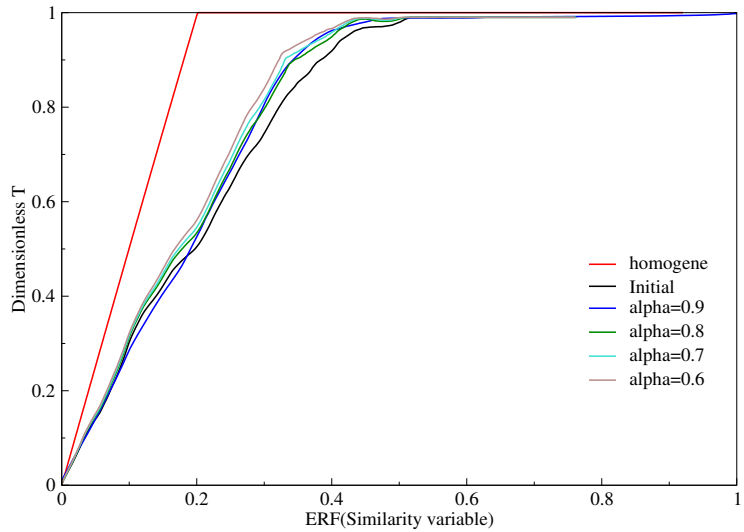
Explication : for KL1_250 les pores s'étalent dans la direction ***y*** et on sollicite dans cette direction $\rightarrow$ on doit augmenter $\kappa_{matrice}$.

Résultats pour KL1_250

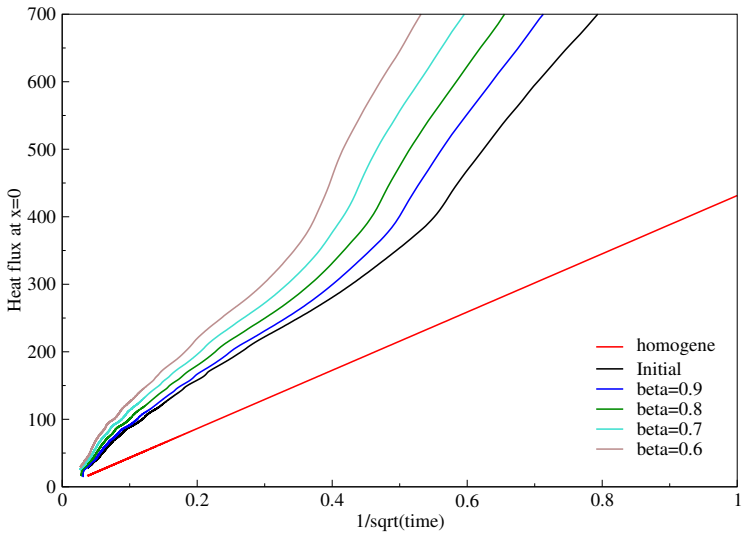


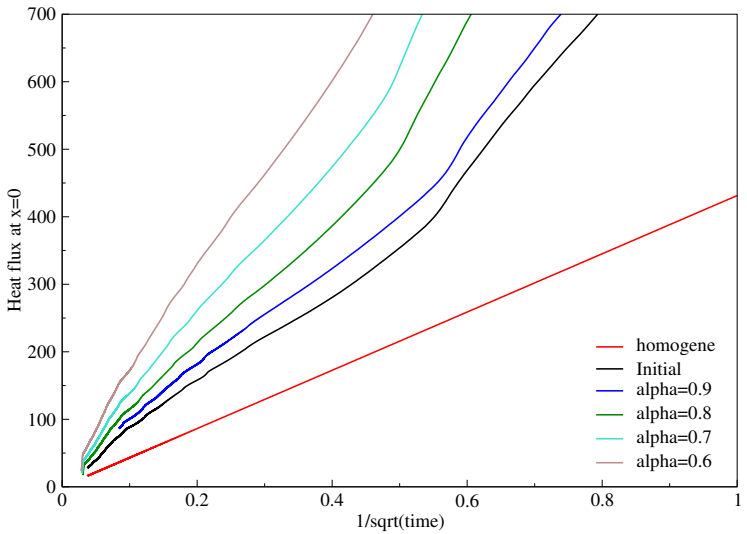


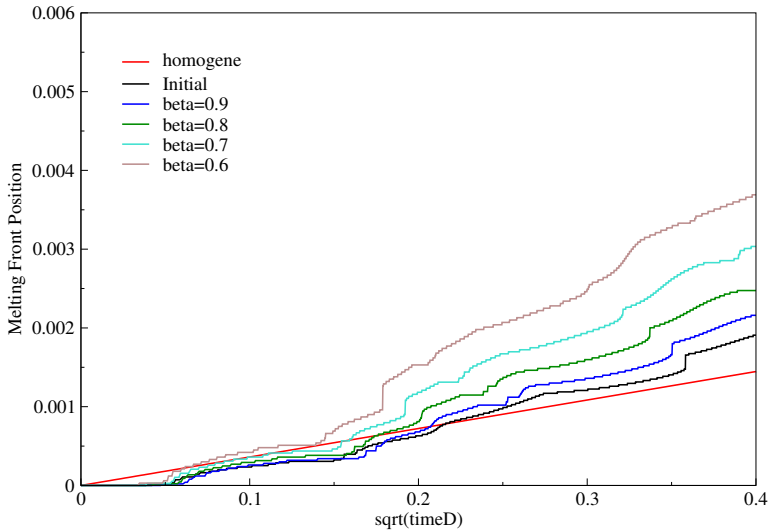


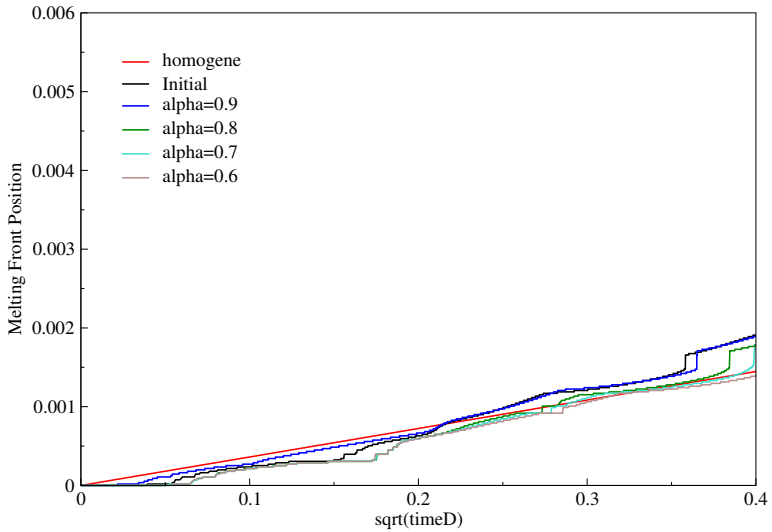


Résultats pour KL1_250

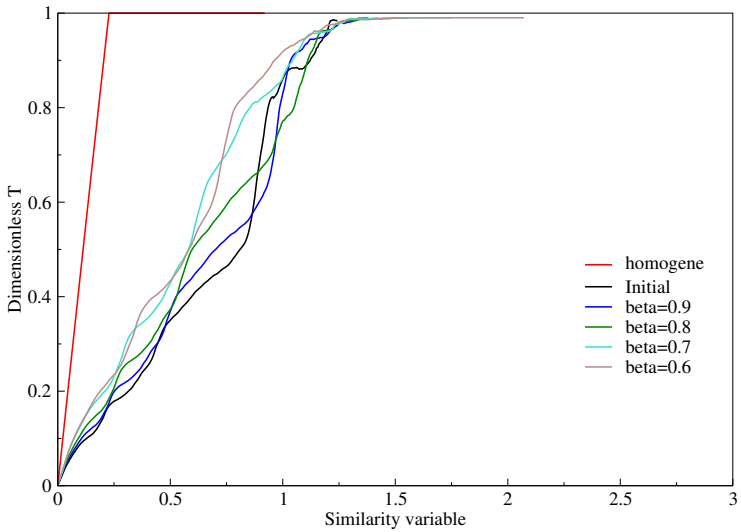


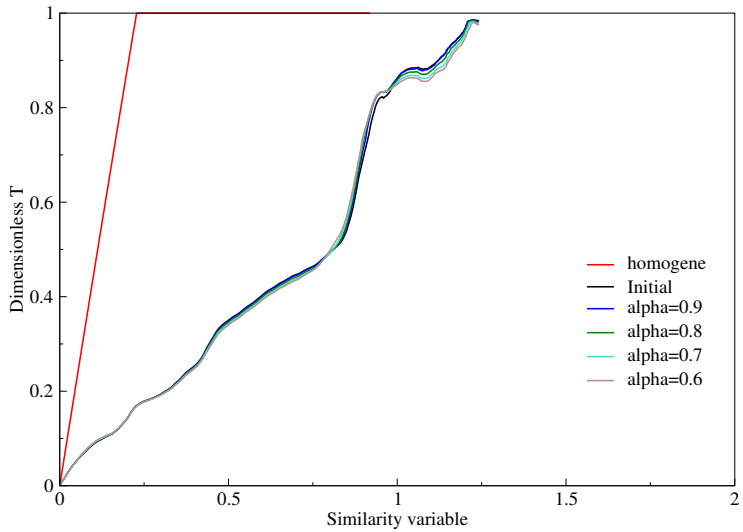


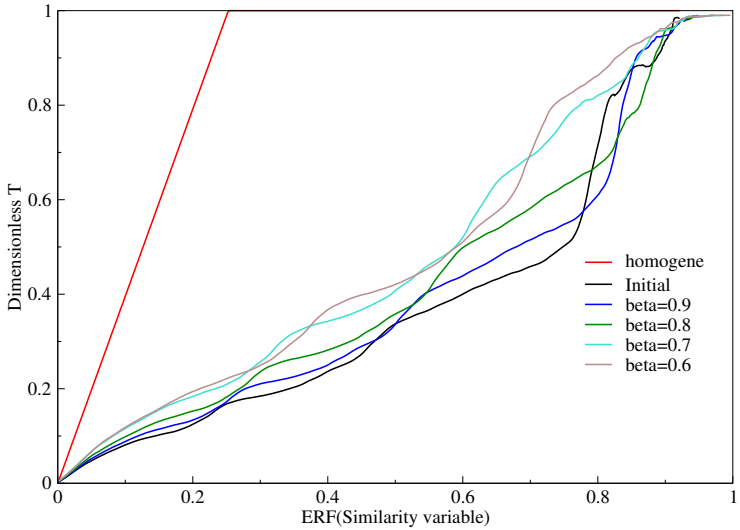


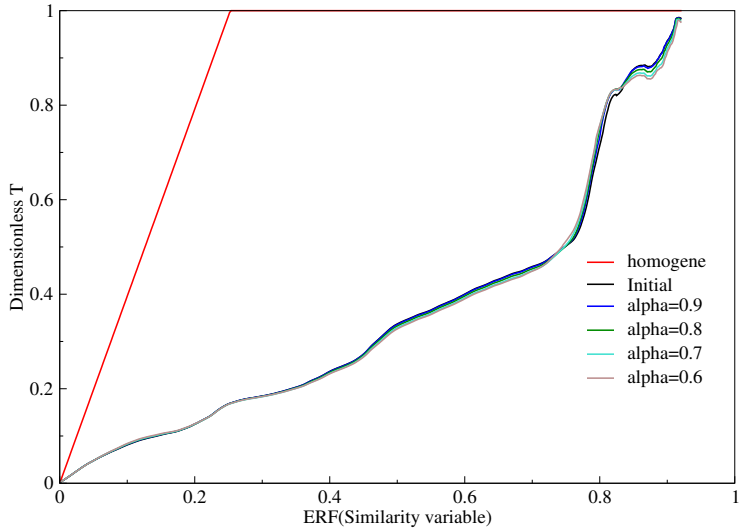


Résultats pour KD1

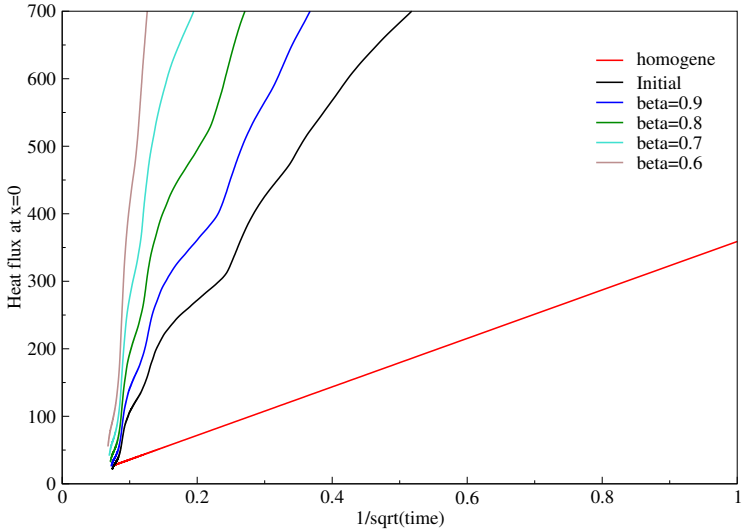


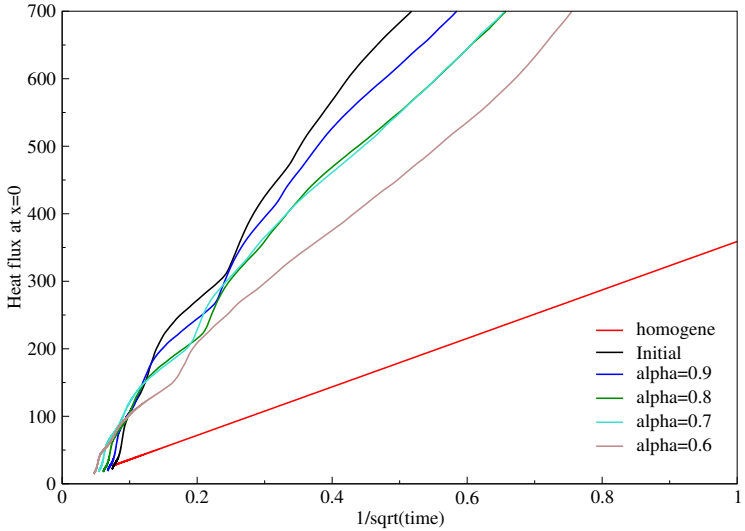


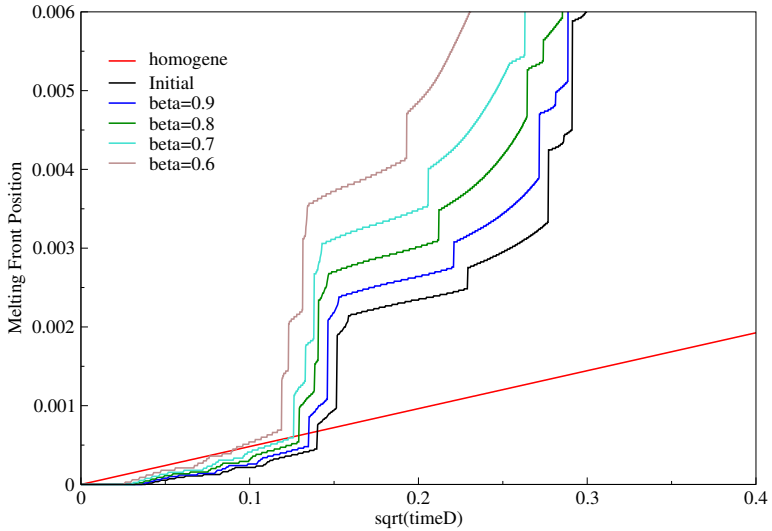


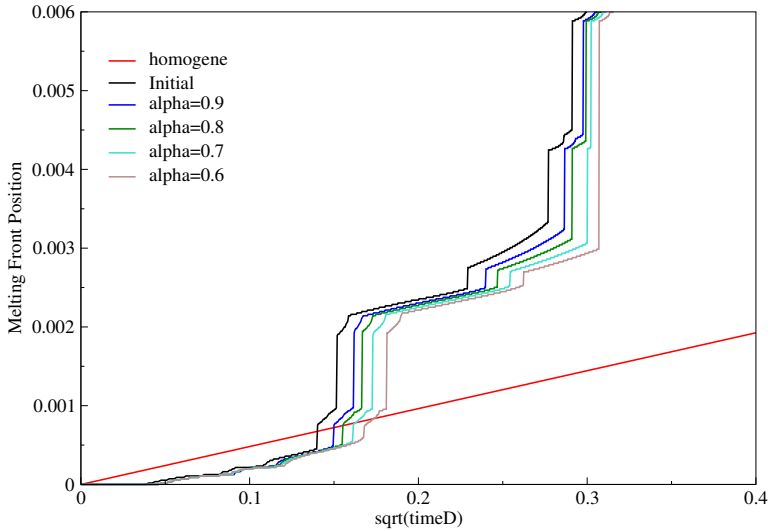


Résultats pour KD1









Conclusions

En dilatant dans la direction ***x***

- Pour KL1_250 on remarque des variations dans les profiles de températures ainsi qu'aux flux surfaciques puisqu'on augmente la valeur de $\kappa_{matrice}$.
- Pour KD1 on a presque le même comportement pour les différents α puisqu'on ne sollicite pas dans cette direction.

En dilatant dans la direction ***y*** on constate que dans les deux cas les fronts de fusions vont plus vites puisque les pores se sont allongés dans le sens où les matériaux sont "chauffés".
Donc pour accélérer les cycles de charges/décharges il faudrait dilater les pores dans le sens du gradient de température.

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