# Photonic modes in colloidal / complex nematic resonators

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MARIE CURIE ACTIONS

# Outline

- **Nematic liquid crystals - properties**
- **Motivational nematic photonic systems**
- **Computational methods**
- **Example: EM eigen modes in a nematic system**
- **Possible further work**

# Nematic liquid crystals

**Liquid crystals (LC)** are an oily material:

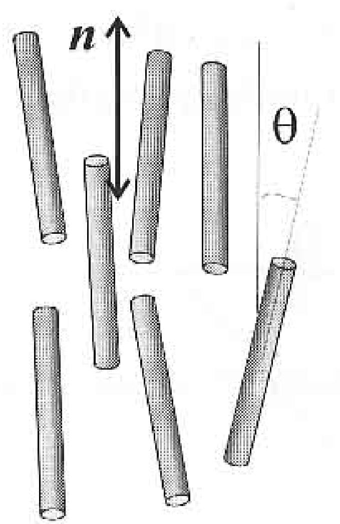
- Flow like a ***liquid***
- But are also *partially ordered* - like ***crystals***.

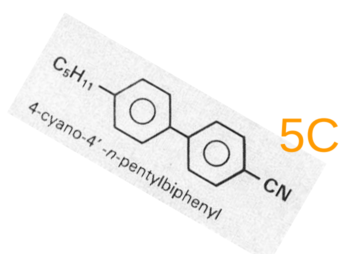
- In <u>nematic</u> LC molecules are *rodlike.*
- Tend to align in a *preferred direction*.

Isotropic liquid phase
(higher temperature)

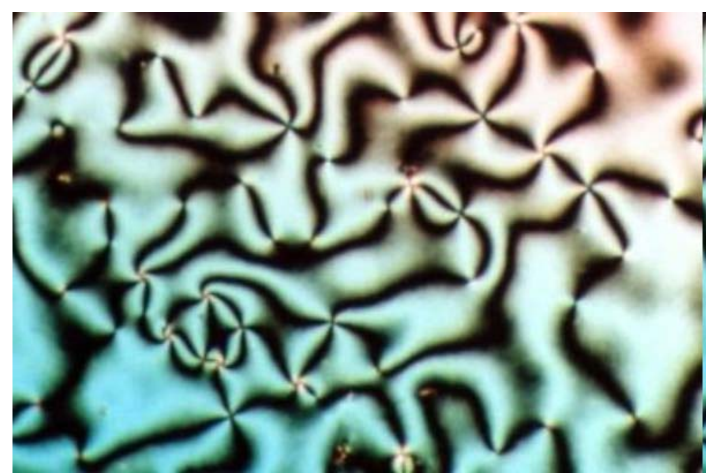
*Low enough temperature*

**Partially ordered**



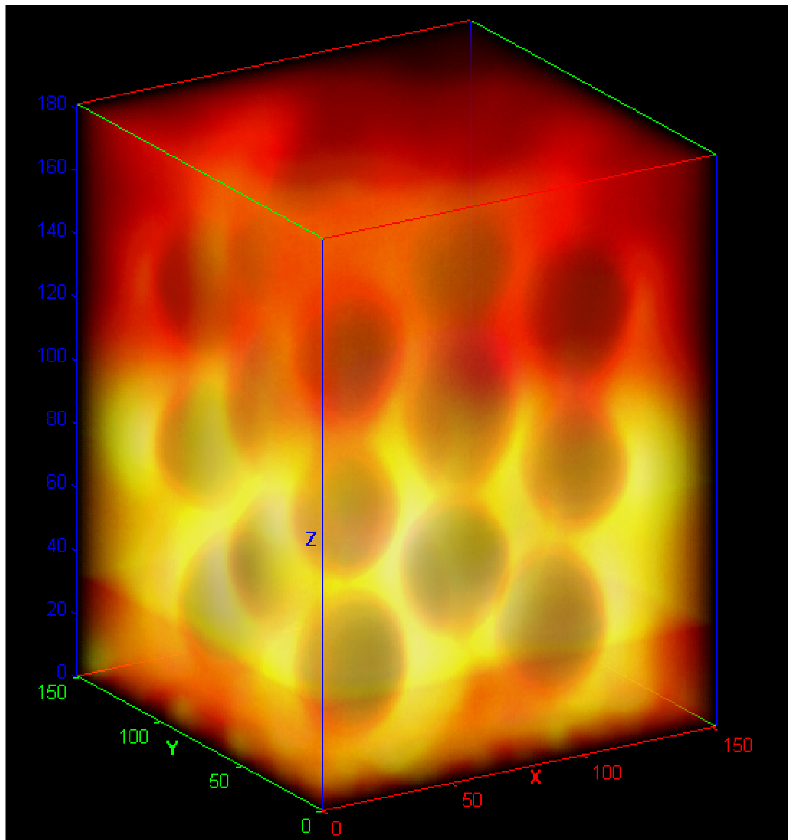

5CB

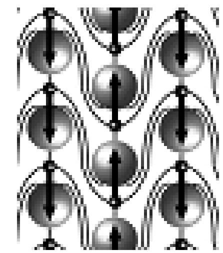


Motivational system 1:

# Nematic photonic crystals

**Larger 3D**
structures



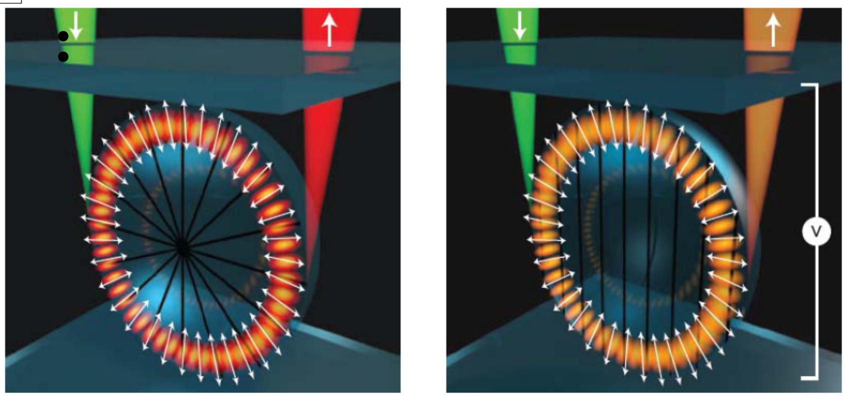
**3×3×3 dipolar crystal**. Experiment by Andriy Nych, 2012 (*submitted*).

(Recently built also 6×6×6)

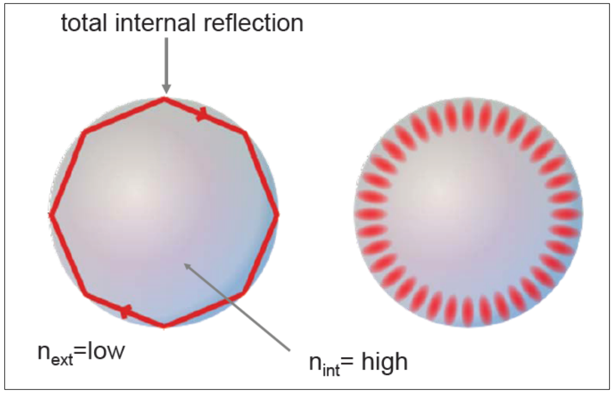
**Motivational systems 2,3:**

# **Nematic** and **chiral nematic** droplets

2




*Whispering Gallery Modes* (WGM ) in a **microresonator.**




Figures: Matjaž Humar, LC experimental group, Jožef Stefan Institute, Ljubljana.

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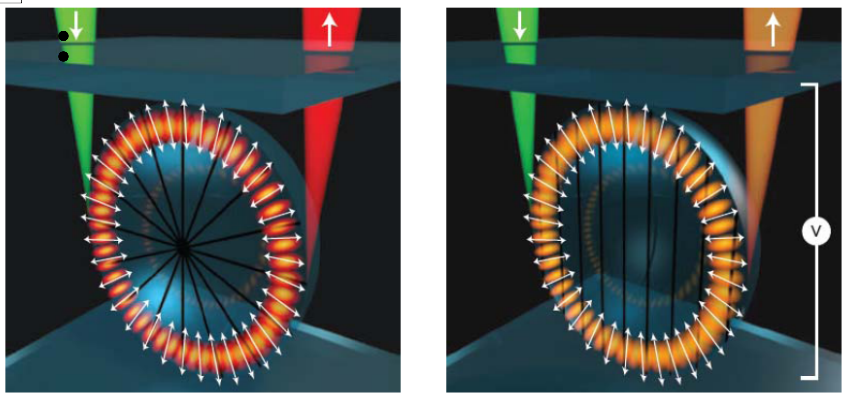

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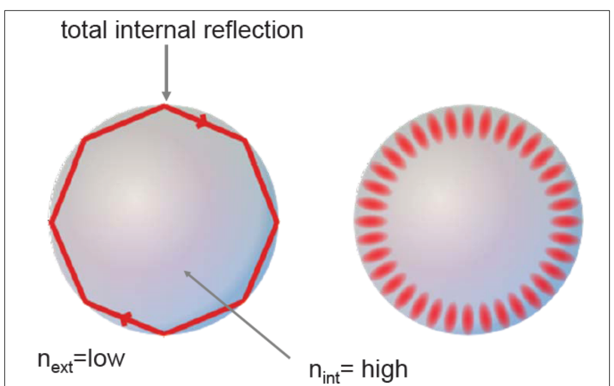
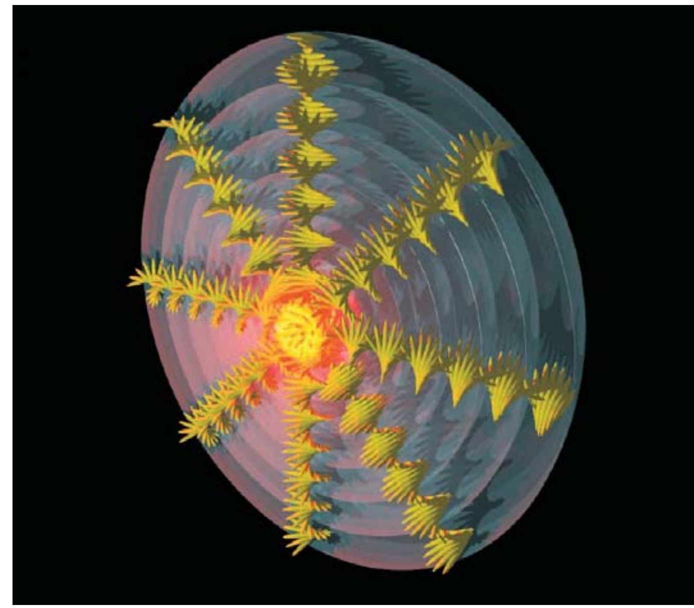



3



M. Humar, I. Muševič, *3D microlasers from self-assembled cholesteric LC*, Optics Express, 2010.

Bragg-onion optical microcavity (R ~ 15um): **stimulated light emission** (from dye molecules in the liquid crystal).

Figures: Matjaž Humar, LC experimental group, Jožef Stefan Institute, Ljubljana.

# Computational photonics

• **Detail dimensions** comparable with **wavelength**.

*Numerical solution of **full** Maxwell equations*

*Time-harmonic expansion*

1) Time-domain propagation

2) Frequency-domain response

3) Frequency-domain eigen problems

# Eigen problems in frequency-domain

**Maxwell equations:**

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{H} = 0$$

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$$\vec{\nabla}\times\underline{\underline{\varepsilon}}(\vec{r})^{-1}\vec{\nabla}\times\vec{H} = \omega^2\vec{H}$$

$$\vec{\nabla}\cdot\vec{H} = 0$$

+ Boundary conditions

(For ideal conductor:

$\overrightarrow{H}\cdot\vec{\nu} = 0$ )

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**(the *curl-curl eqn*)**

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**Fully anisotropic permittivity**

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- Example of Schroedinger eqn with periodic b.c. and localized defects: [1].

[1] E. Cances, V. Erlacher, Y. Maday, Periodic Schroedinger operators with local defects and spectral pollution, Arxiv 1111.3892.

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***Basic variational formulation of Helmholz eqn***


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***Edge (Nedelec) vector elements [1,2] used.***

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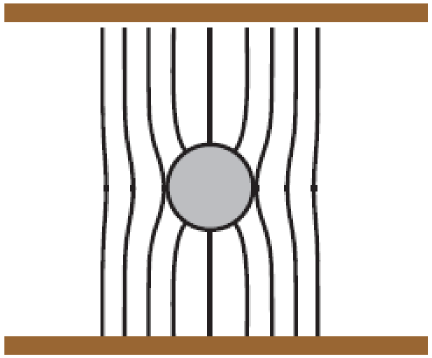
Example:

EM modes of the **nematic quadrupole** (2D)

***System geometry:***



Nematic quadrupolar configuration

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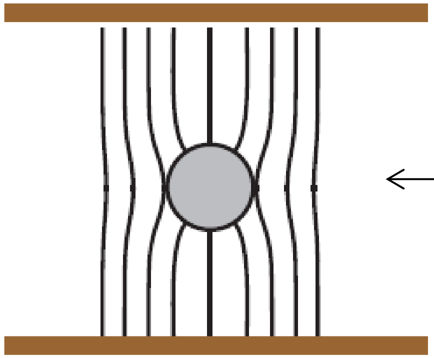

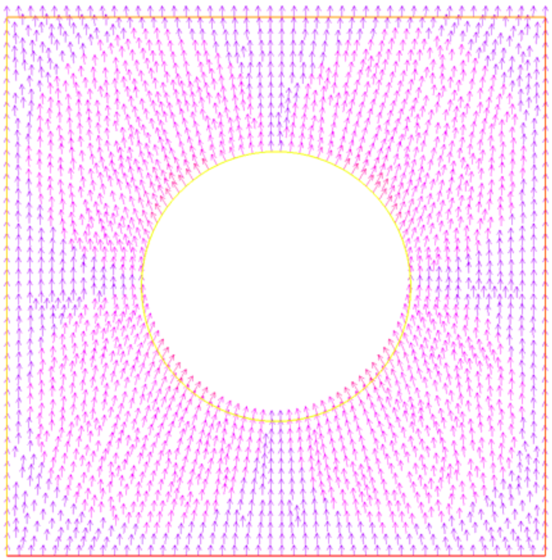

Nematic quadrupolar configuration

Nematic structure
by analytical ansatz

Induces *anisotropic dielectric tensor.*

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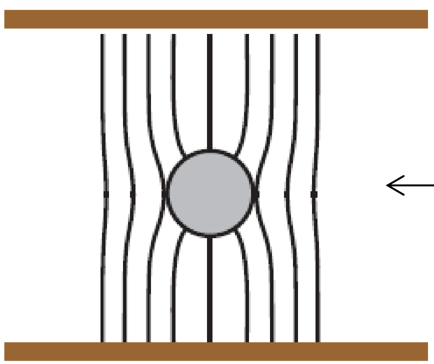

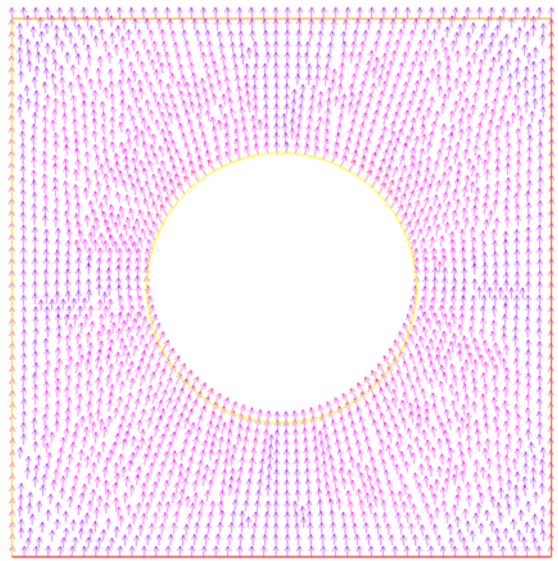

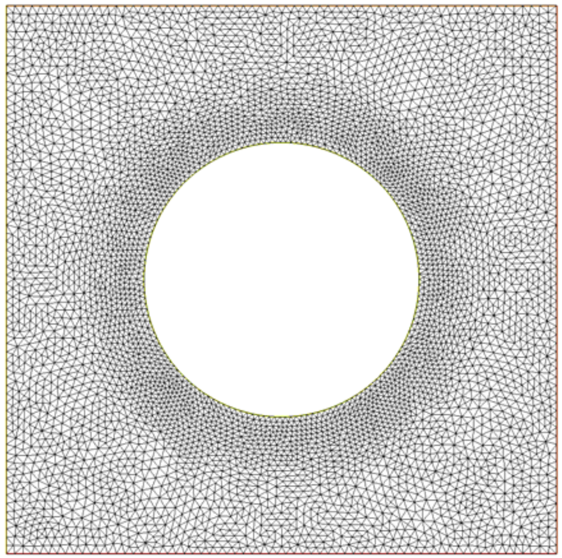

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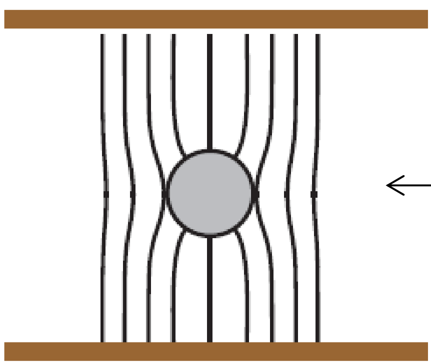

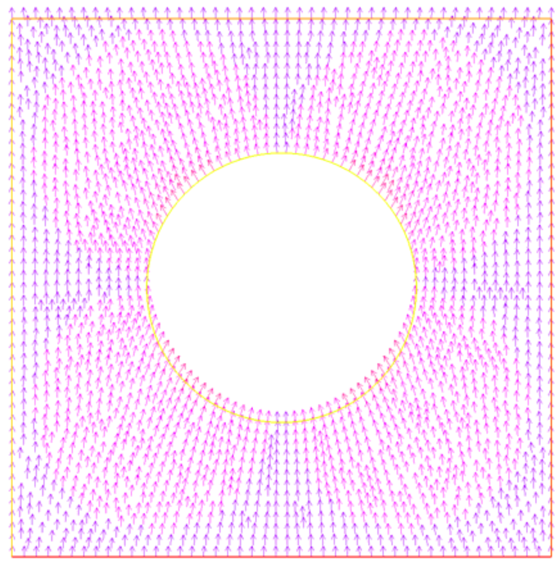

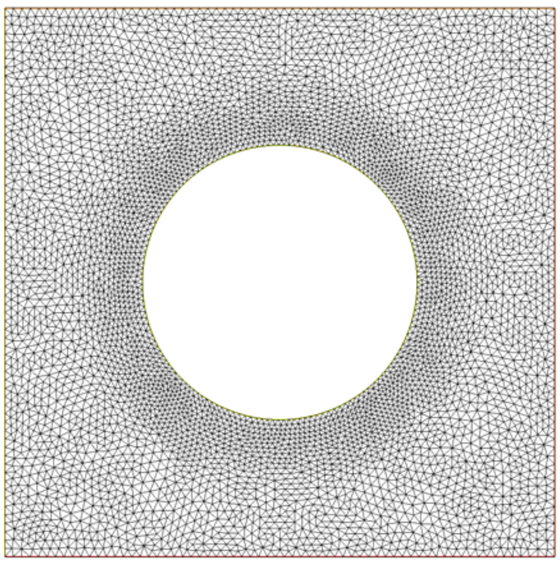

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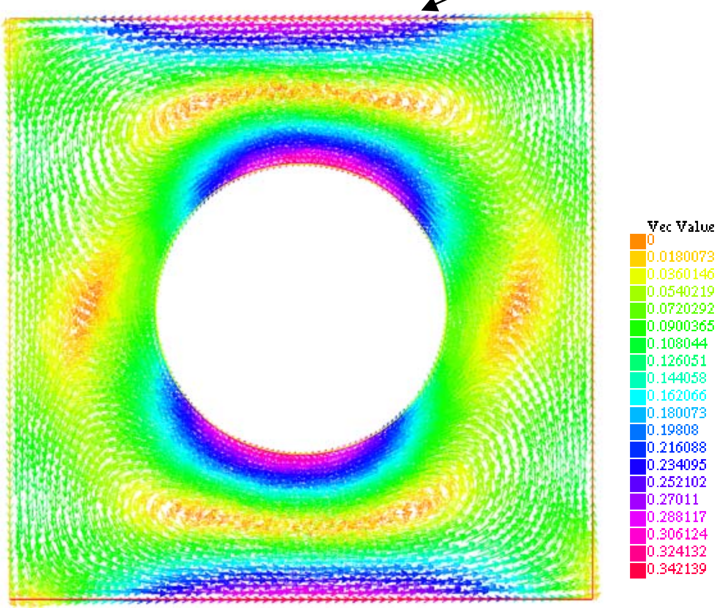
We compute EM modes
**on this nematic structure**

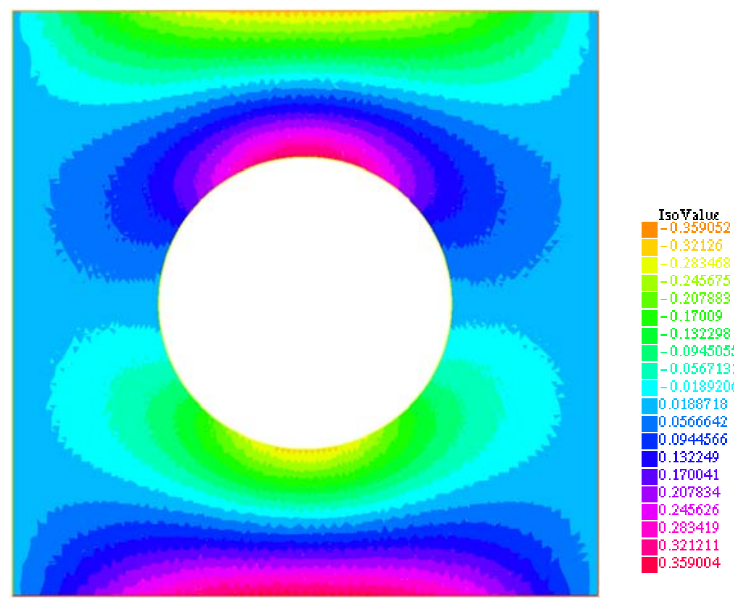
# EM modes of the **nematic quadrupole** (2D)

## 1st mode:

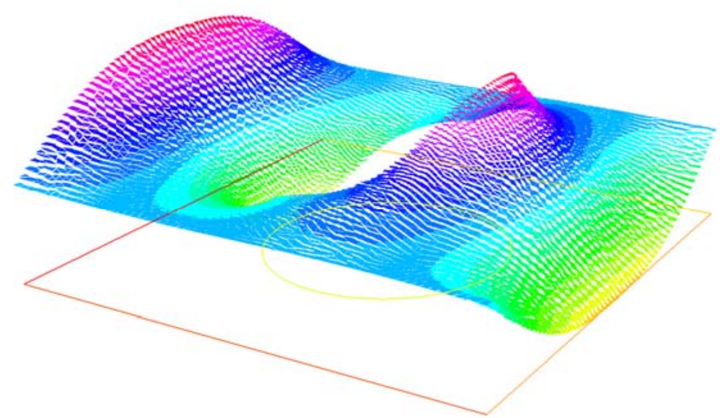
metal on surface, i.e, ideal conductor boundary conditions




***(Hx, Hy)***




***Hx***



$$\omega^2 = 0.42$$

multiplicity = 1

# EM modes of the **nematic quadrupole** (2D)

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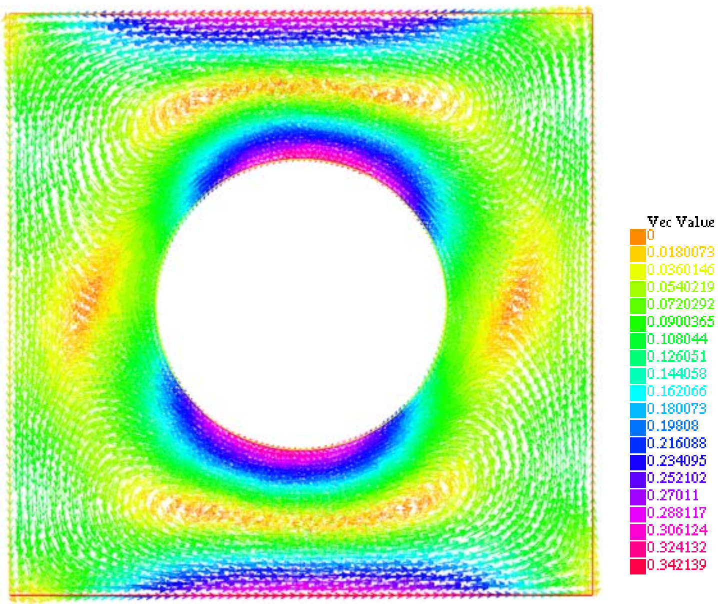


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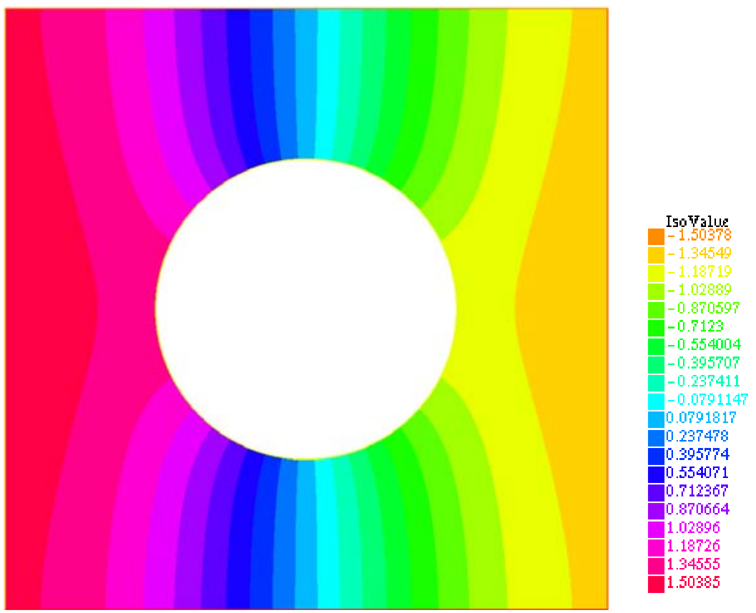
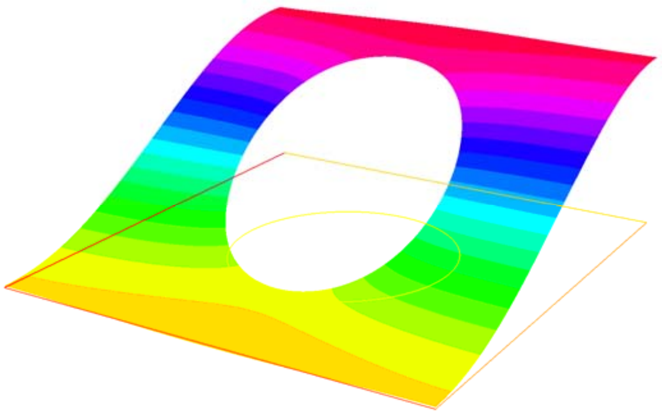
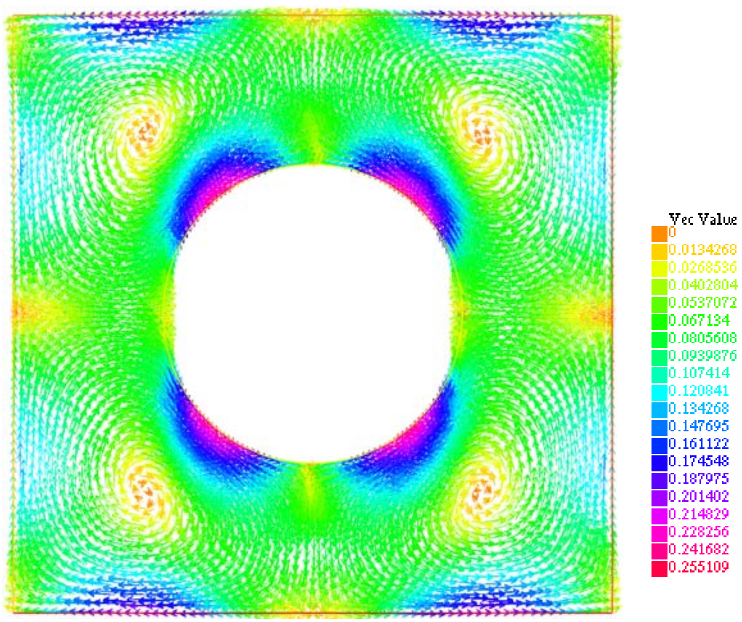



***Hz***



# EM modes of the **nematic quadrupole** (2D)

## 2nd mode:

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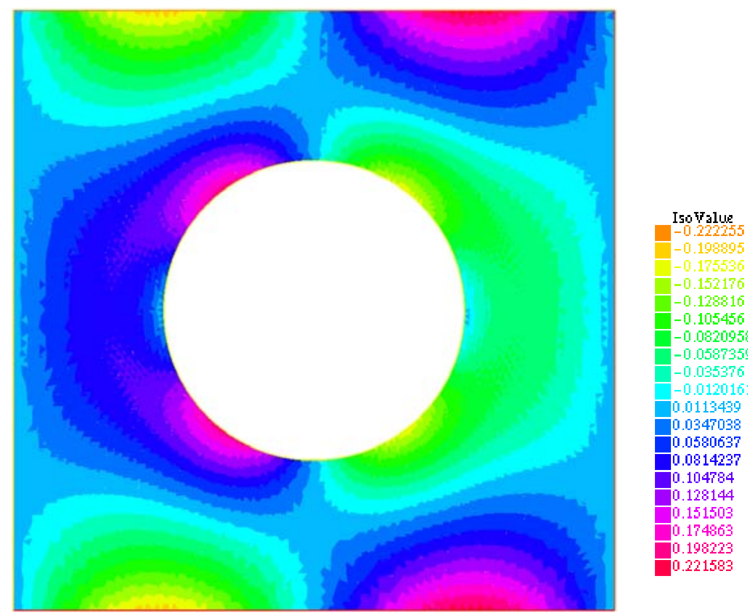


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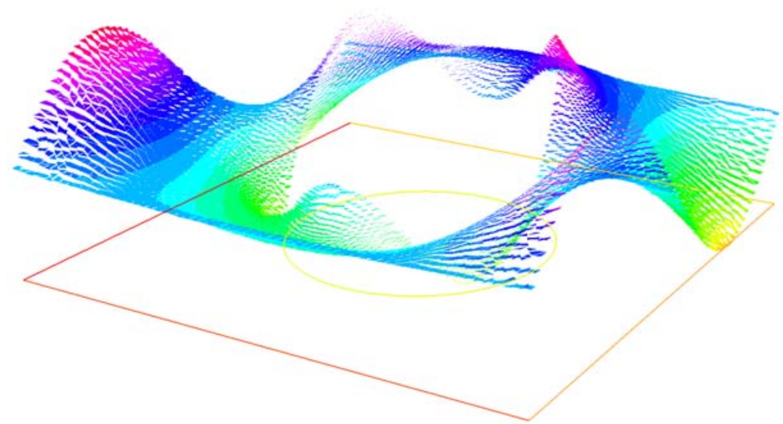

$$\omega^2 = 0.73$$

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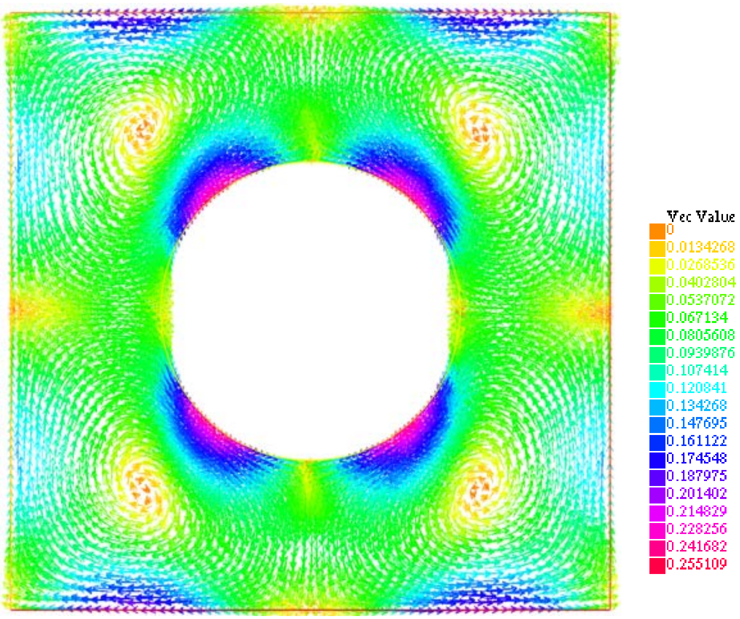


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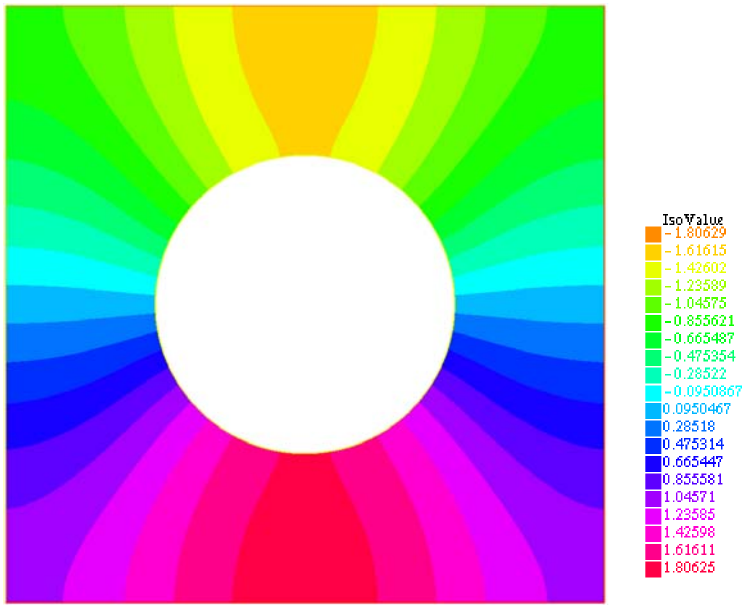
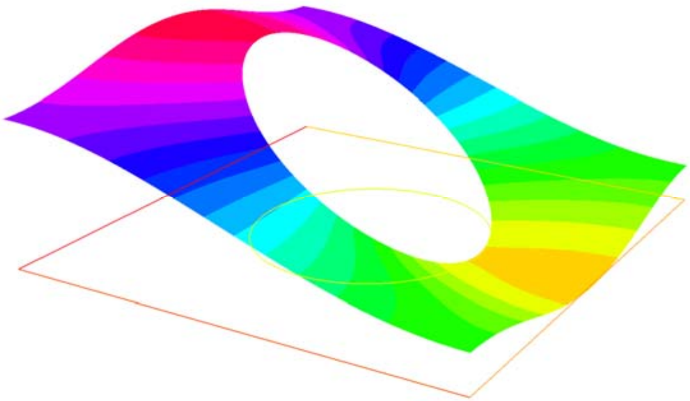



***Hz***

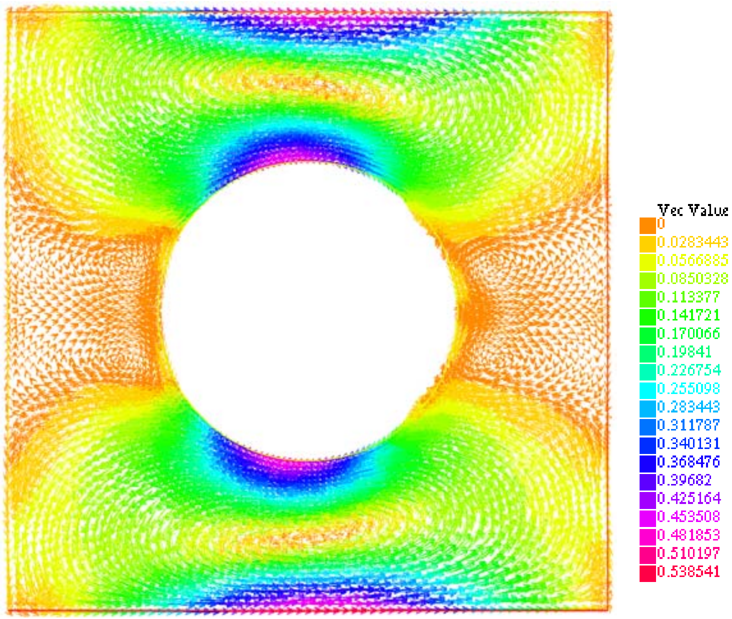


$$\omega^2 = 0.73$$

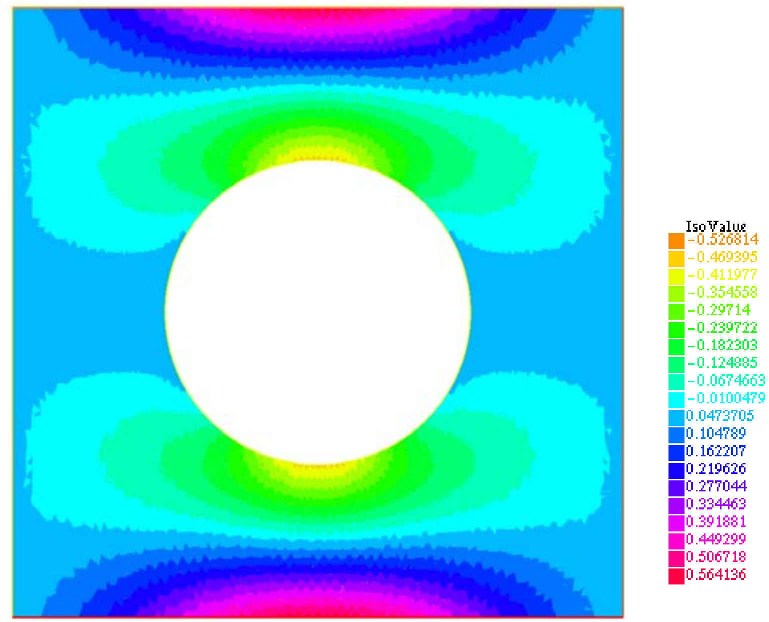
multiplicity = 1

# EM modes of the **nematic quadrupole** (2D)

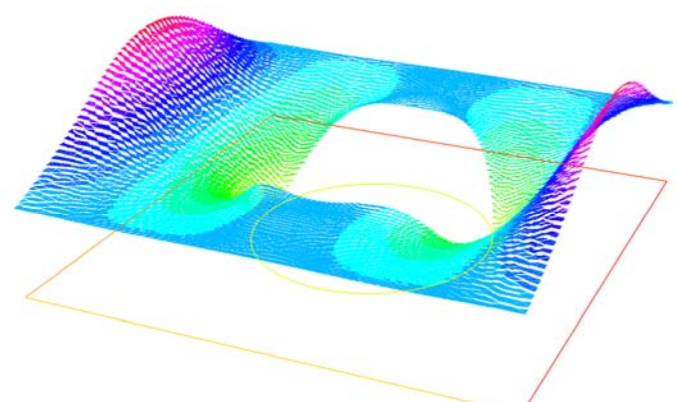
## 3rd mode:




***(Hx, Hy)***




***Hx***

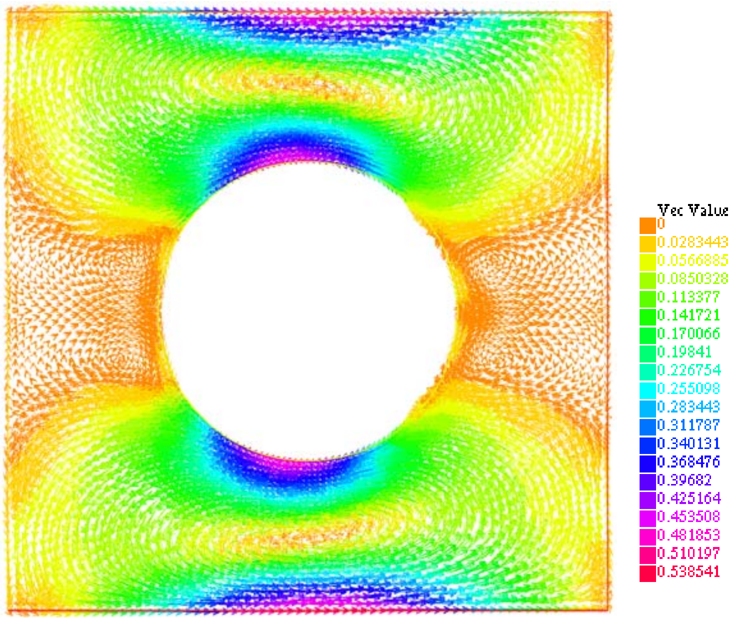


$$\omega^2 = 1.79$$

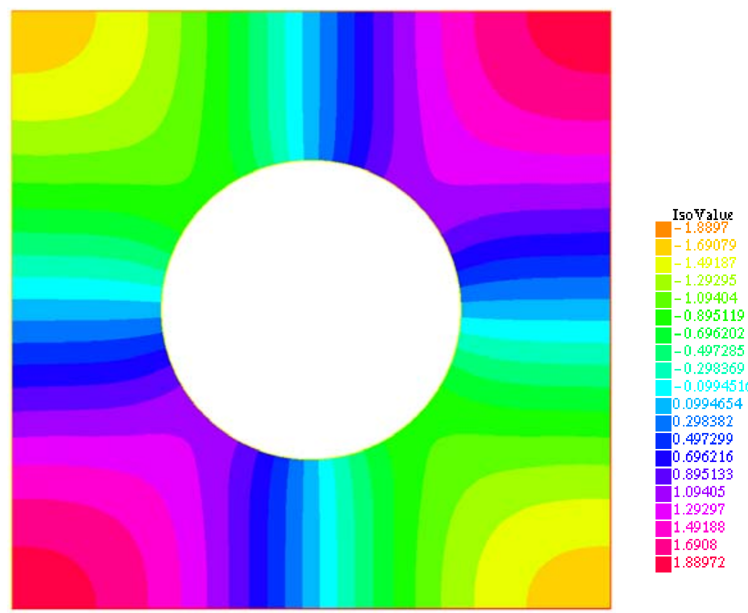
multiplicity = 1

# EM modes of the **nematic quadrupole** (2D)

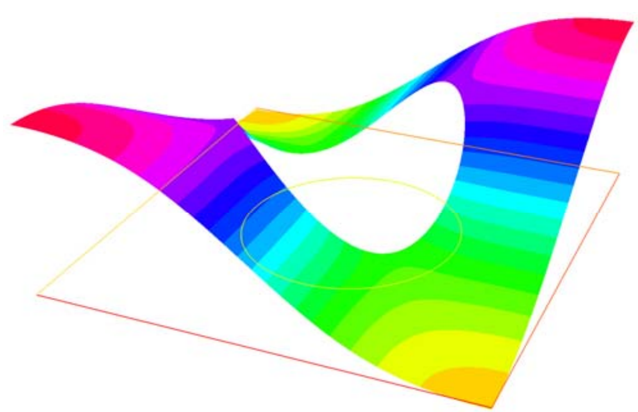
## 3rd mode:




***(Hx, Hy)***




***Hz***

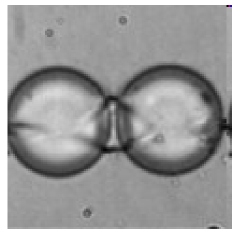


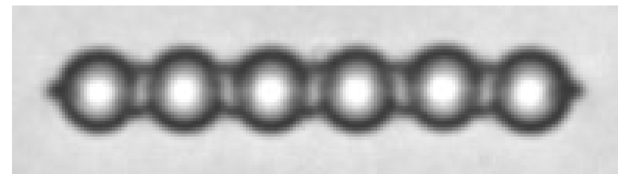
$$\omega^2 = 1.79$$

multiplicity = 1

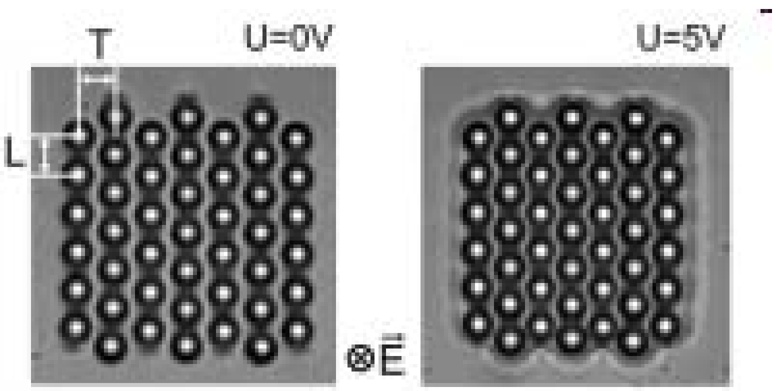
# EM modes for more complex colloidal nematic structures?

For **1D** structures?



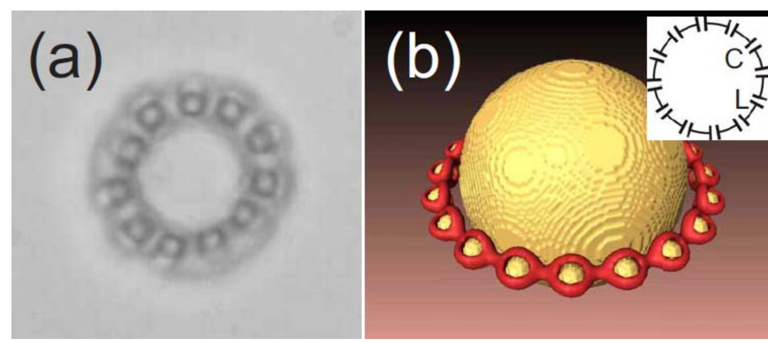


**EM waveguide**?




For **2D** structures – crystals?

**Large and small particles**?



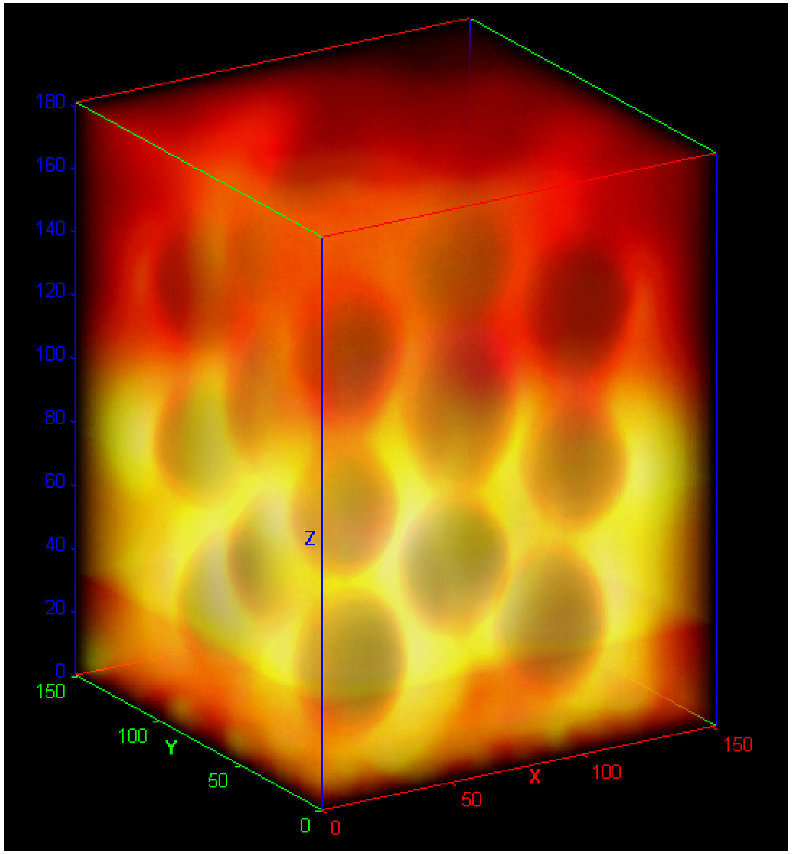

**Ring-split resonator**?

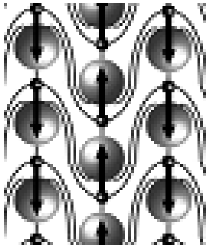
(Škarabot, Ravnik et al., PRE, 2008.)

Photos: I. Muševič group, PRE, PRL, Science, 2006-2010.

# Nematic photonic crystals?

**Large 3D** structures



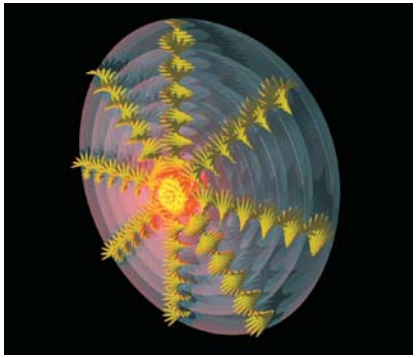
**3×3×3 dipolar crystal**. Experiment by Andriy Nych, 2010 (*to be published*).

Recently built also 6×6×6

# Future work

## Computational:

– Computation of 3D modes in a **chiral nematic droplet** with at least 5-6 layers, on one processor.



## Theoretical / mathematical:

– **Statistical** behaviour and **coherence** phenomena of EM resonant modes in chiral nematic droplet.

## Further:

– Going to Schroedinger equation and computation of a (periodical) **quantum system**?