Title: "*Nonlinear analysis of wrinkles in film-substrate systems by Finite Element Method and Asymptotic Numerical Method using FreeFem++*"

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Abstract:
An efficient numerical model for the study of wrinkling in film/substrate systems, either planar or spherical will be presented. This numerical model is based both on the Finite Element Method (FEM) and the Asymptotic Numerical Method (ANM) [1]. Here, we propose the technical implementation of ANM for FEM in the framework of FreeFEM++ [2]. ANM is a robust continuation method for solving nonlinear problems that depend on a parameter. This technique has been applied successfully to various fields in solid and fluid mechanics, etc.
In this work, we will show that FreeFem++, because of its natural way to deal with variational formulations, can be easily used to implement ANM for instability problems in solid mechanics.
To illustrate the great efficiency of FreeFem++, we will show how to write elegant FreeFem++ scripts to compute the different steps of ANM continuation for elastic structures considering simple geometries subjected to conservative loading. Then, we will recall how ANM is used for thermo-mechanical problems involving elastic behavior and geometrical nonlinearities.
For validation purpose, we will show the case of a cantilever subjected to an applied force. The case of planar or spherical film/substrate systems [3] will be studied in great details using the new developed numerical model. More precisely, we will show that this numerical model enables the accurate following of bifurcation diagrams that involve complex buckling responses.

[1] B. Cochelin, N. Damil, M. Potier-Ferry, Méthode asymptotique numérique, Hermes, Lavoisier 2007.
[2] F. Hecht, New development in FreeFem++, Journal of Numerical Mathematics 20 (3-4) (2012) 251-265.
[3] F. Xu, S. Zhao, L. Conghua, M. Potier-Ferry, Pattern selection in core-shell spheresΩ,

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