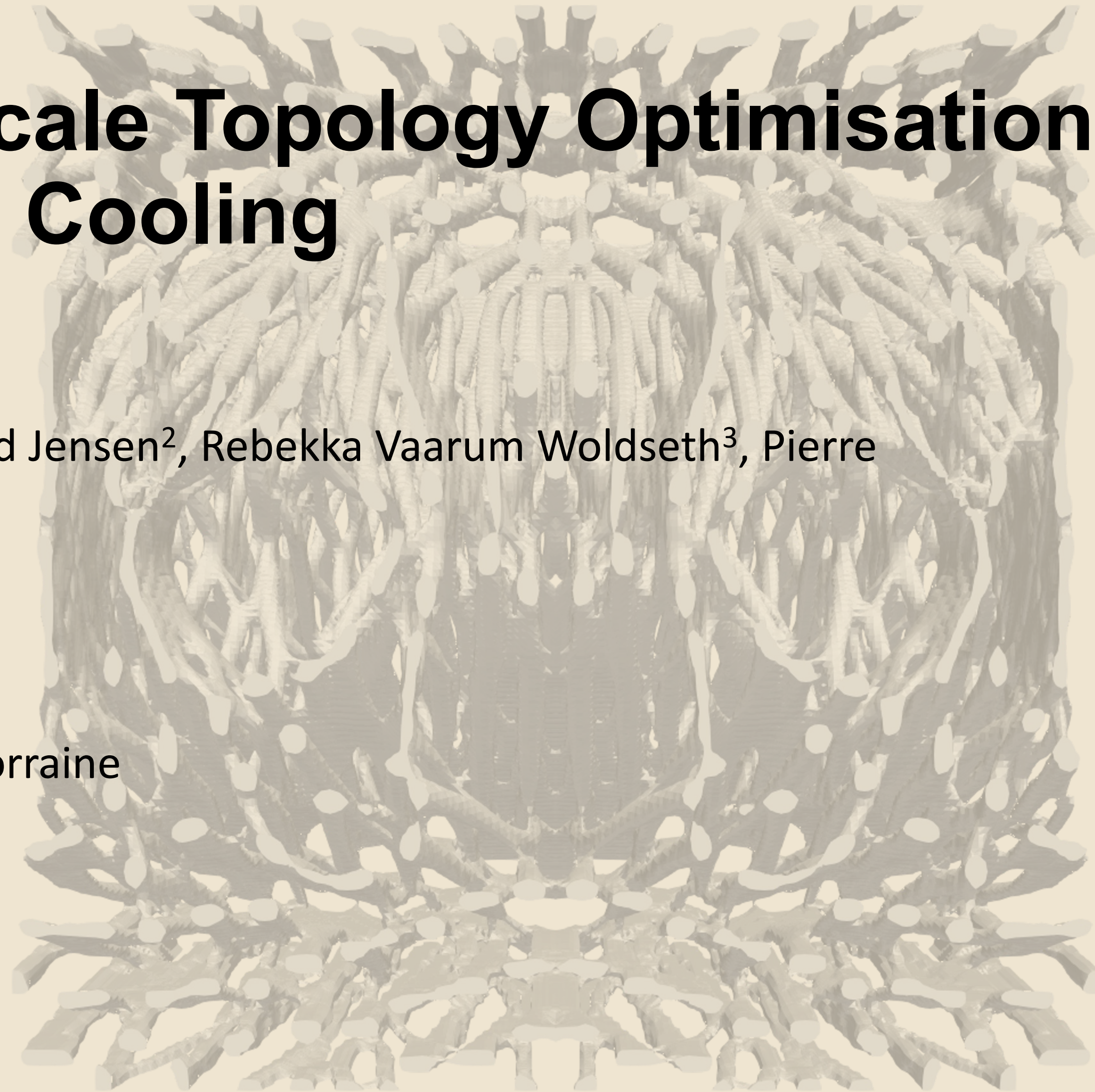
Institute of Mechanical and Electrical Engineering

The 17th FreeFEM Days

Towards Multi-Scale Topology Optimisation Of Microchannel Cooling



Hao Li[1,*], Peter Dørffler Ladegaard Jensen[2], Rebekka Vaarum Woldseth[3], Pierre Jolivet[4], Joe Alexandersen[1]

[1]University of Southern Denmark
[2]Technical University of Denmark
[3]Centre Inria de l'Université de Lorraine
[4]LIP 6, Sorbonne université

*E-mail: hli@sdu.dk

sdu.dk #sdudk

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Contents

- **The presentation is composed of**
  - Single-scale topology optimisation
    - Applications in thermal-fluid device design
    - Strengths and limitations
  - Multi-scale topology optimisation
    - Homogenisation-based design approach
    - Phasor-based de-homogenisation technique
    - Limitations and wish list

- **The material of the presentation is part of the HORIZON-MSCA-PF:**

  Advanced Design of Heat Exchangers using multiscale models and machine learning (ADeHEx), Project number: 101106842, 2023/10/1-2025/09/30.

- For any comments, suggestions or questions, do not hesitate to contact:

  hli@sdu.dk (Li, H.), joal@sdu.dk (Alexandersen, J.)

Part 1
Single-Scale TopOpt

- Physics-driven single-scale topology optimisation
- Applications in thermal-fluid device design

Physics-Driven Single-Scale TopOpt

Optimisation problem

$$\begin{aligned} \min_{\gamma \in \mathcal{D}} \quad & J(\boldsymbol{s}, \gamma) \\ \text{s.t.} \quad & \text{Governing Eqs.} \\ & g_k(\boldsymbol{s}, \gamma) \leq \bar{g}_k, \quad k = 1, 2, \ldots, m \\ & 0 \leq \gamma_i \leq 1, \quad i = 1, 2, \ldots, n \end{aligned}$$

Elemental-/nodal-wise design variable

$$\gamma = \begin{cases} 1 & \text{if } \boldsymbol{x} \in \Omega \\ 0 & \text{if } \boldsymbol{x} \in D \backslash \Omega \end{cases}$$

Lagrange function

$$\mathcal{L}(\boldsymbol{s}, \boldsymbol{\lambda}, \gamma) = f(\boldsymbol{s}, \gamma) - \boldsymbol{\lambda}^\top \boldsymbol{r}$$

Primal solution

$$\mathbf{A}\boldsymbol{s} = \boldsymbol{b}$$

Adjoint solution

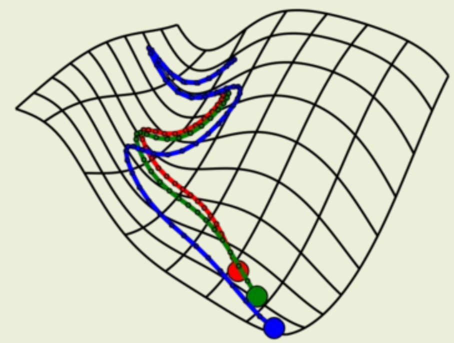
$$\mathbf{A}^\top \boldsymbol{\lambda} = \delta_{\boldsymbol{s}} f$$

Sensitivity

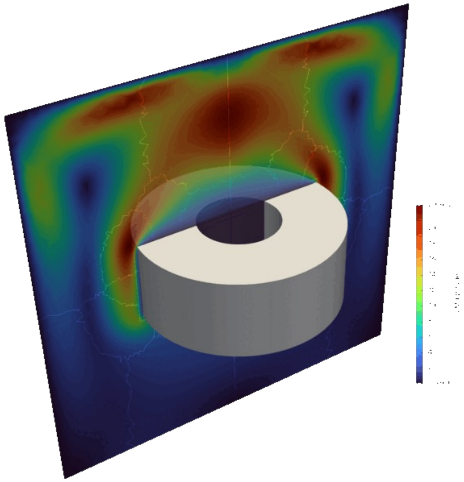
$$\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\hat{\gamma}} = \frac{\partial f}{\partial \hat{\gamma}} - \boldsymbol{\lambda}^\top \frac{\partial \boldsymbol{r}}{\partial \hat{\gamma}}$$

Gradient-based optimiser (MMA)

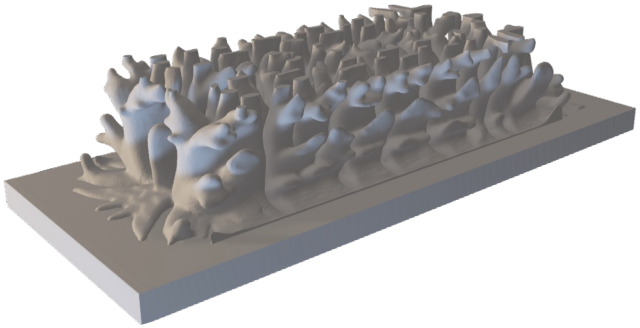
FEA +



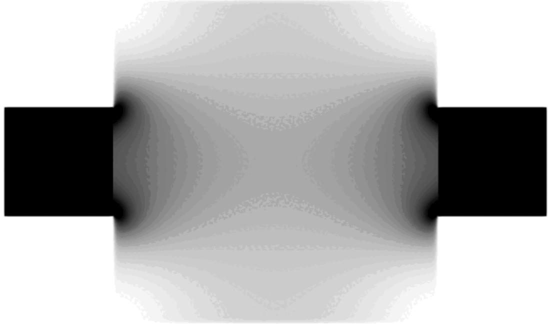
Applications in Thermal-Fluid Device Design



Passive heat sink
[Presentation]



Heat exchanger
[Click]



Liquid-cooled heat sink
[Click]

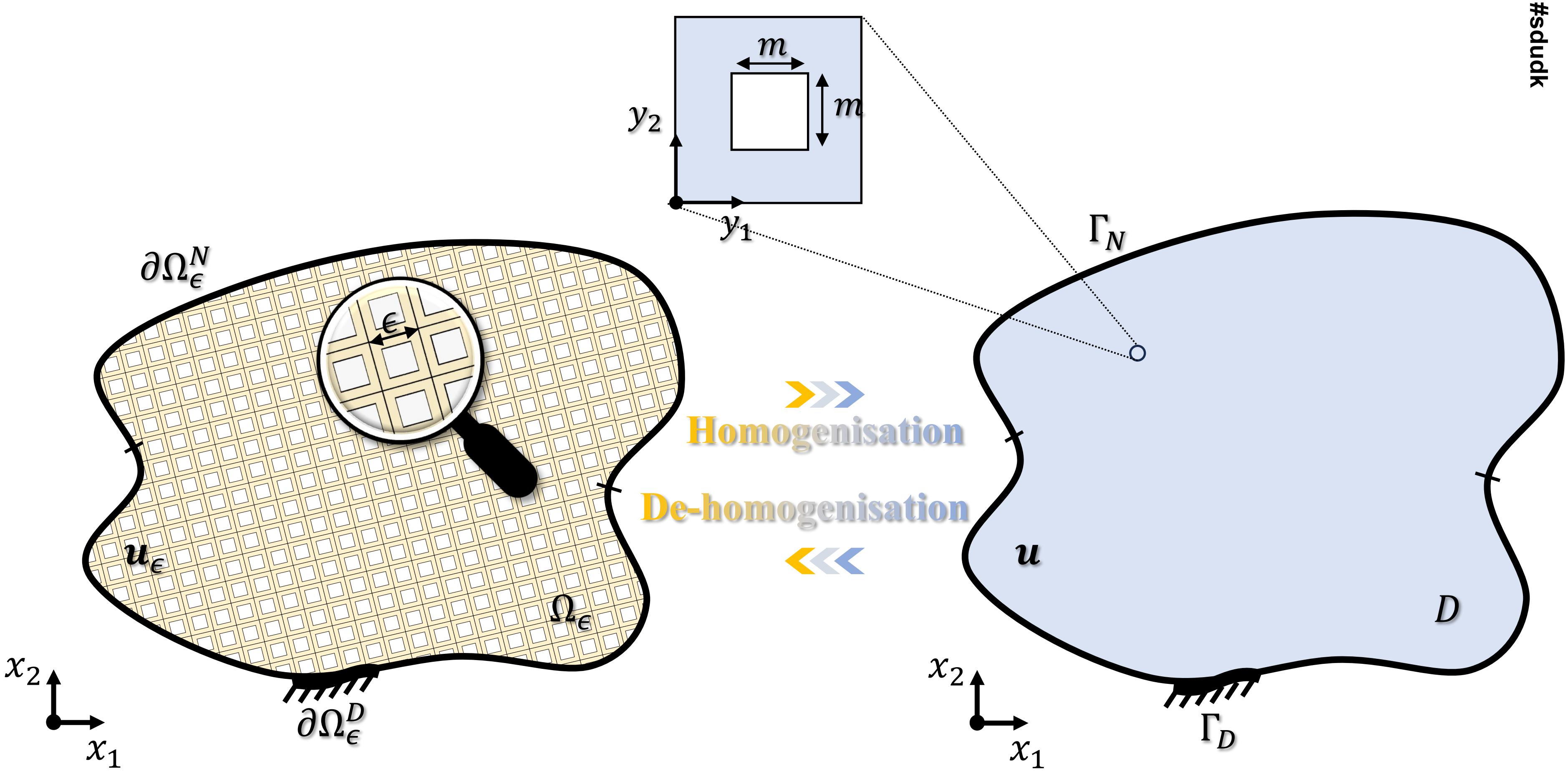
[Li et al., 2022; Li et al., 2025a]

Part 2

Multi-Scale TopOpt

- Homogenisation and de-homogenisation
- 1+2 step approach
- Surrogate-modelling using Neural Networks

Homogenisation and De-homogenisation



$$-\operatorname{div}\left(\mathbb{A} : e\left(\boldsymbol{u}\right)\right) = \boldsymbol{f}$$

$$-\operatorname{div}\left(\mathbb{A}^* : e\left(\boldsymbol{u}\right)\right) = \boldsymbol{f}$$

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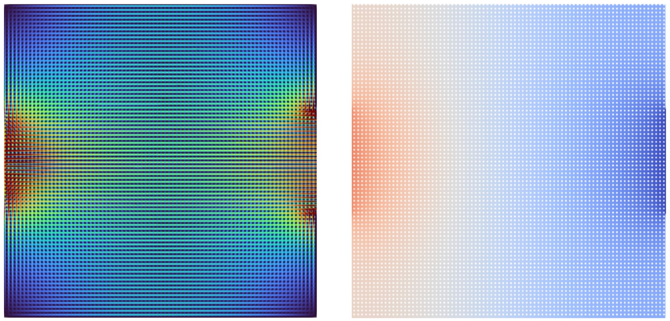
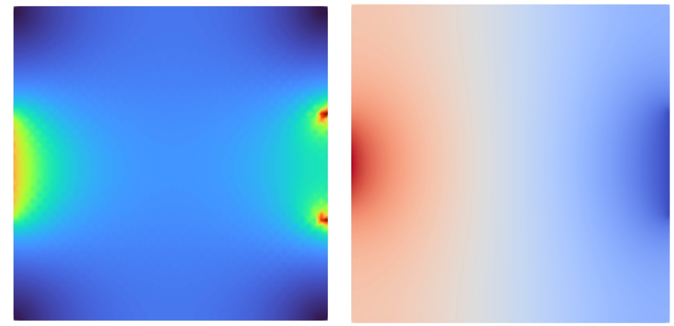
[Bendsøe & Kikuchi, 1988]

[Groen et al., 2018; Allaire et al., 2019]

Homogenisation and De-homogenisation



- Full-scale solution
    - Stokes flow



- Homogenised solution
    - Darcy's flow

$$\begin{cases} -\Delta \boldsymbol{u}_\epsilon + \nabla p_\epsilon = -\alpha_\epsilon(\boldsymbol{x})\,\boldsymbol{u}_\epsilon & \text{in } \Omega_\epsilon, \\ \operatorname{div}(\boldsymbol{u}_\epsilon) = 0 & \text{in } \Omega_\epsilon, \\ \boldsymbol{u}_\epsilon = \boldsymbol{u}_0 & \text{on } \Gamma_{\text{in}}, \\ \left(-p_\epsilon \mathbf{I} + \left(\nabla \boldsymbol{u}_\epsilon + \nabla \boldsymbol{u}_\epsilon^T\right)\right) \cdot \boldsymbol{n} = 0 & \text{on } \Gamma_{\text{out}}, \\ \boldsymbol{u}_\epsilon = 0 & \text{on } \partial\Omega_\epsilon \setminus (\Gamma_{\text{in}} \cup \Gamma_{\text{out}}), \end{cases}$$

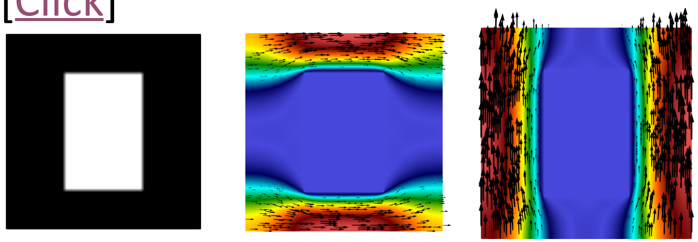
$$\begin{cases} \boldsymbol{u} = -\epsilon^2 \underbrace{\boldsymbol{\mathcal{X}}^*(m_1, m_2, \theta)}\nabla p & \text{in } D, \\ \operatorname{div}(\boldsymbol{u}) = 0 & \text{in } D, \\ \boldsymbol{u} \cdot \boldsymbol{n} = \boldsymbol{u}_0 \cdot \boldsymbol{n} & \text{on } \Gamma_{\text{in}}, \\ p = 0 & \text{on } \Gamma_{\text{out}}, \end{cases}$$

[Li et al., 2025b]

Offline library (Step #0)

- Parameterise microstructures
- Unit-cell problem
- Homogenised properties

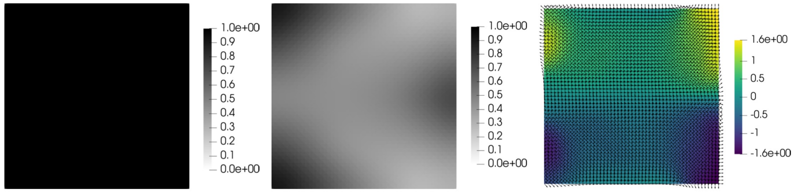
[Click]



Homogenisation-based TO (Step #1)

- Homogenised problem
- Gradient-based optimiser

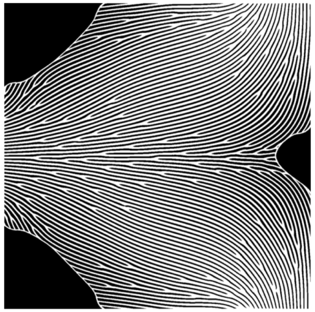
[Click]



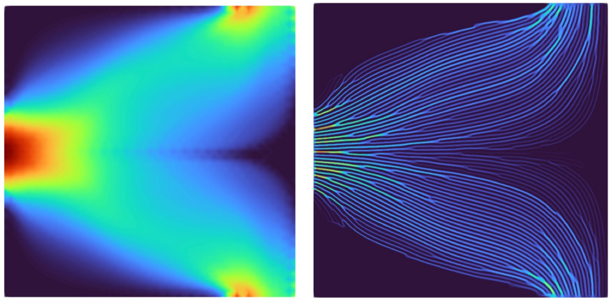
(a) m_1. (b) m_2. (c) θ.

De-homogenisation (Step #2)

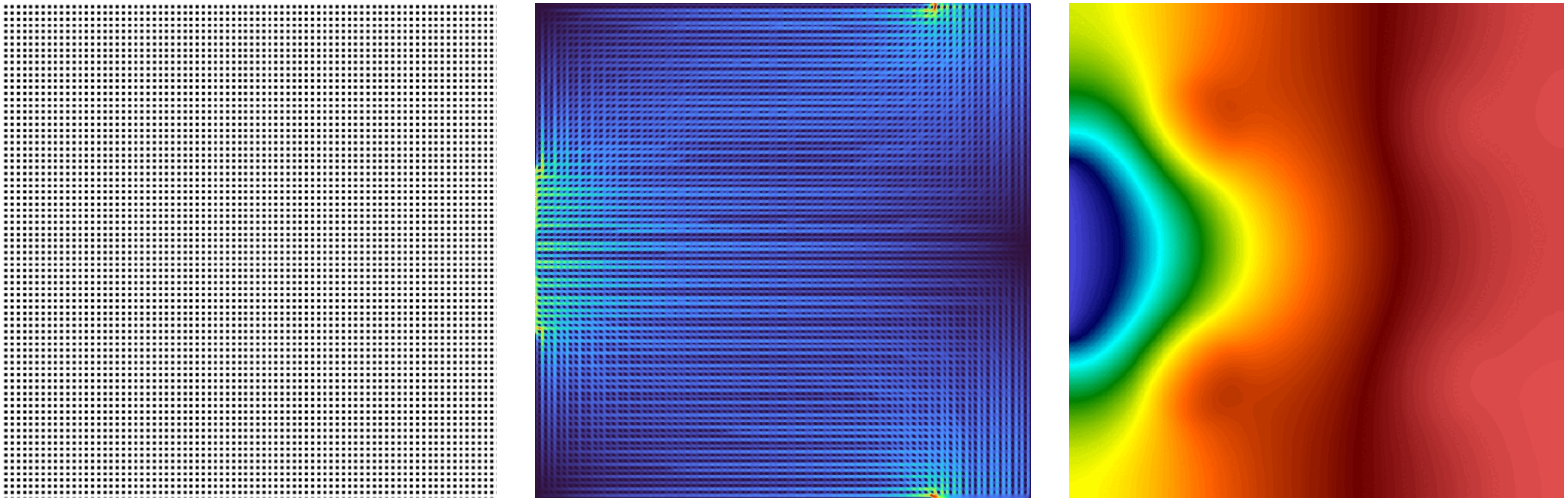
- Phasor-based approach [Click]



Post-analysis



On-the-fly dehomogenisation & full-scale simulation



- Upsample fine-scale microchannels on a **3213×3213 fine mesh**
- The objective value achieves a **35.3% improvement** w.r.t. initial design
- Our method VS hypothetical brute-force TO
    - Brute-force TO: 10 min 34 $sec/iter$ using 10 MPI processes -> 21,334 min for 1,000 iterations
    - Our method: 0.64 sec ->**10 min for $1,000$ iterations**

Three orders-of-magnitude reduction in computational cost in terms of total runtime!

- **Wish list:**
    - Navier-Stokes
    - ”2.5D” design
    - …

[Li et al., 2025c] [Woldseth et al., 2024a] [Woldseth et al., 2024b]

Numerical Tools

PyFreeFEM[1]

PETSc

python™

FREEFEM

upgrade your meshes

PyTorch

[Feppon, 2024; Presentation]

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Thank you for your kind attention!

Hao Li[1,*], Peter Dørffler Ladegaard Jensen[2], Rebekka Vaarum Woldseth[3], Pierre Jolivet[4], Joe Alexandersen[1]

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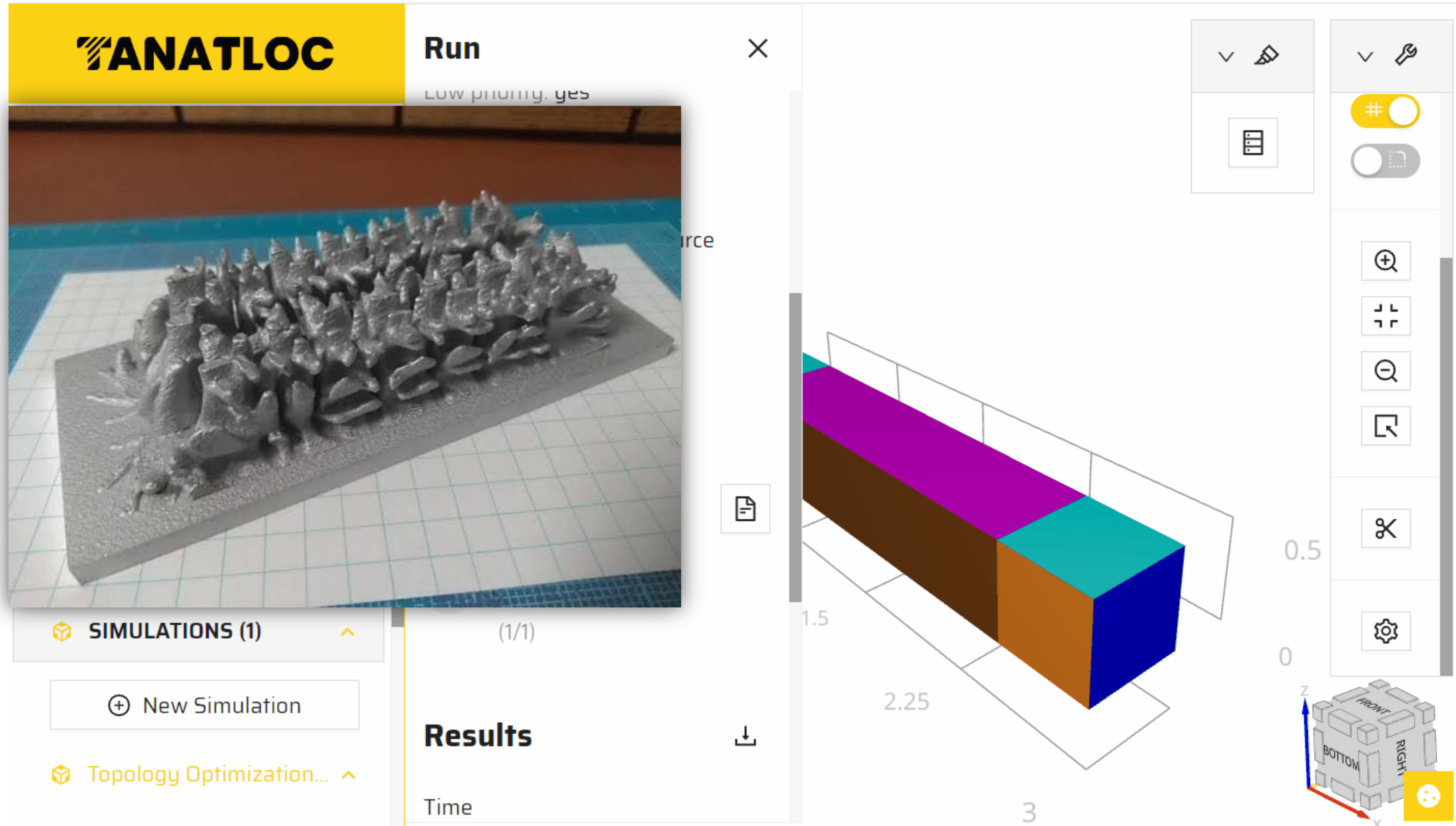
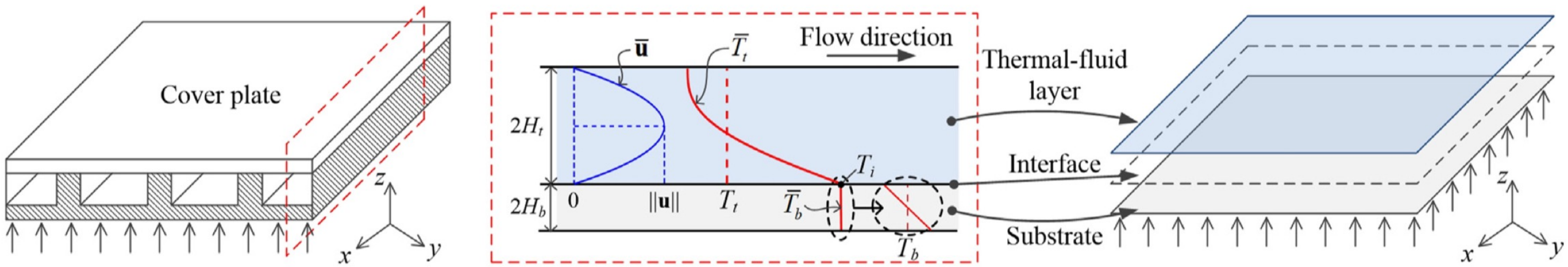
*E-mail: hli@sdu.dk

Appendix A: Prototyping & TANATLOC GUI Interface



Thanks to Hiroshi Ogawa (DENSO), Simon Garnotel (AIRTHIUM)

Appendix B: Multi-layer Modelling of Heat Sink



$$\begin{bmatrix} \mathbf{A}_{00} & \mathbf{A}_{01} \\ \mathbf{A}_{10} & \mathbf{A}_{11} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{Tt} \\ \boldsymbol{x}_{Tb} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_{T_t} \\ \boldsymbol{f}_{T_b} \end{bmatrix}$$

The linear system with the transpose of the matrix associated with the KSP object can be efficiently solved using **KSPSolveTranspose**

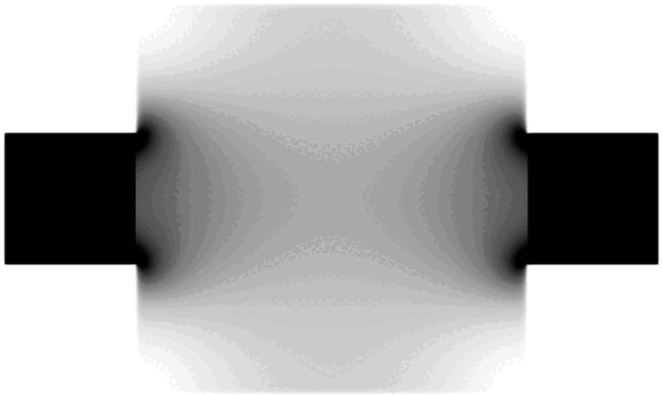
$$\mathbf{A}^{\top}\boldsymbol{\lambda} = \delta_{\boldsymbol{s}} f$$

[Yan et al., 2019]

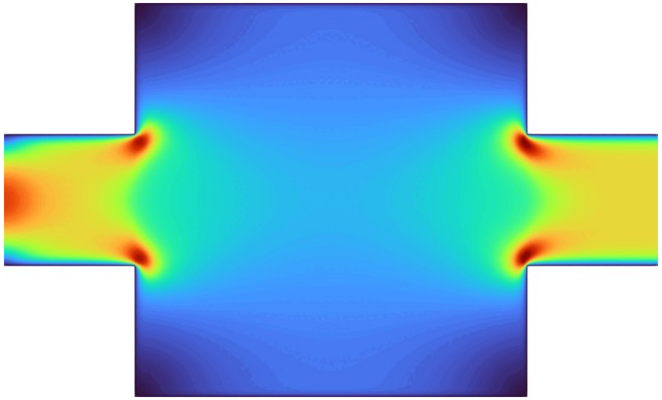
https://petsc.org/release/manualpages/KSP/KSPSolveTranspose/

[Back]

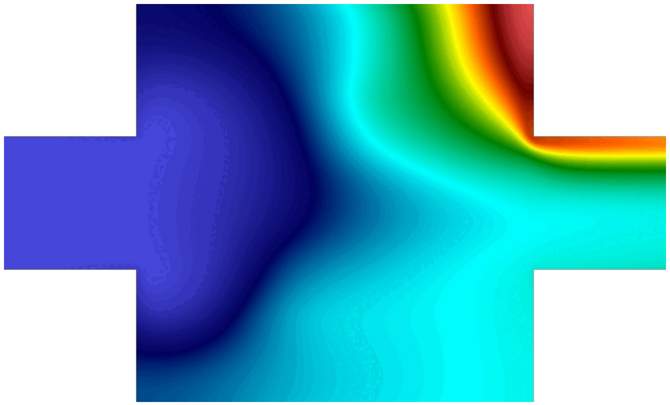
Appendix B: Multi-layer Modelling of Heat Sink



Topology



Velocity



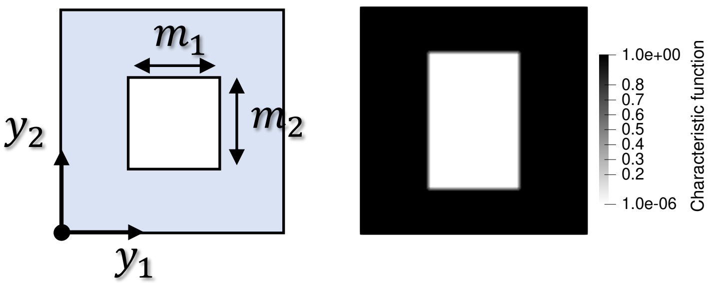
Temperature on the base plate

[Under preparation]

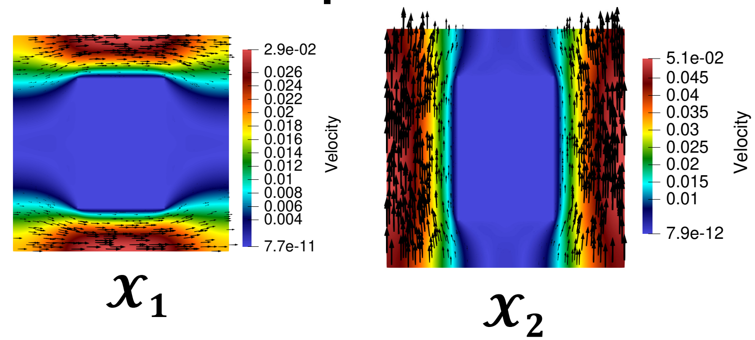
[Back]

Appendix C: Construction of Offline Library

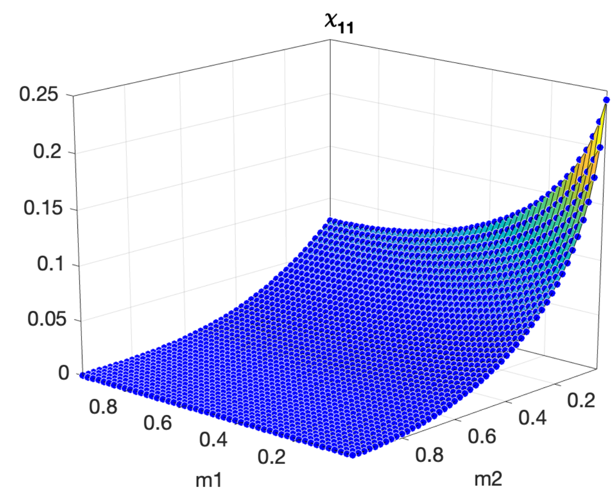
Parameterised microstructure



Bi-linear interpolation



$$\mathcal{X}_{ij}^H = \int_{Y(m_1,m_2)} \mathcal{X}_j(y) \boldsymbol{e}_i \mathrm{d}y, \quad 1 \leq i, j \leq 2,$$



Unit-cell problem

$$\begin{cases} \Delta \boldsymbol{\mathcal{X}}_i(\boldsymbol{y}) + \nabla \psi_i(\boldsymbol{y}) = \boldsymbol{e}_i - \alpha(\boldsymbol{y}) \boldsymbol{\mathcal{X}}_i(\boldsymbol{y}) & \text{in } Y\left(m_1, m_2\right), \\ \operatorname{div}\left(\boldsymbol{\mathcal{X}}_i(\boldsymbol{y})\right) = 0 & \text{in } Y\left(m_1, m_2\right), \\ \displaystyle\int_{Y(m_1,m_2)} \psi_i(\boldsymbol{y}) \, \mathrm{d}y = 0, & \\ y \rightarrow \boldsymbol{\mathcal{X}}_i(\boldsymbol{y}) & Y\text{-periodic.} \end{cases}$$

Block matrix:

$$\boldsymbol{A}_{\text{Stokes}} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} & \boldsymbol{0} \\ \boldsymbol{B}^T & \boldsymbol{S} & \boldsymbol{h} \\ \boldsymbol{0} & \boldsymbol{h}^T & \boldsymbol{0} \end{bmatrix},$$

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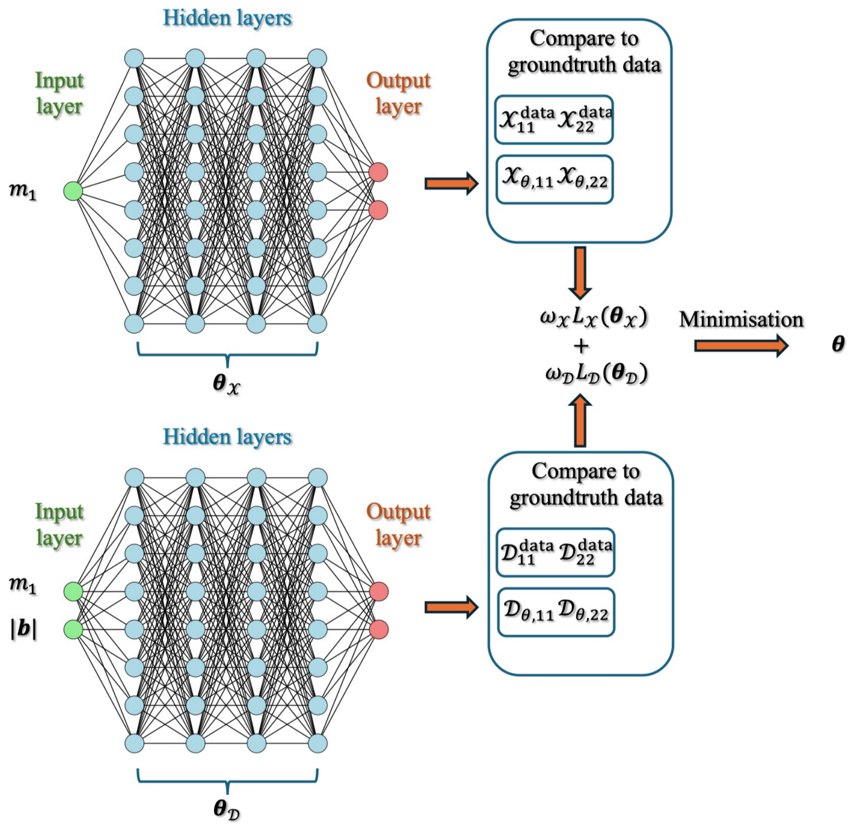
[Li et al., 2025b]

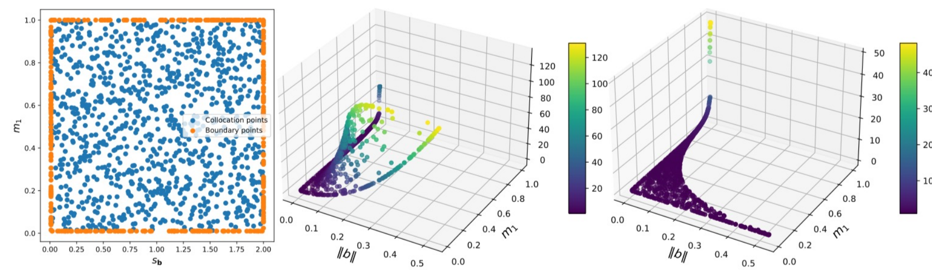
[Back]

Appendix C: Construction of Offline Library

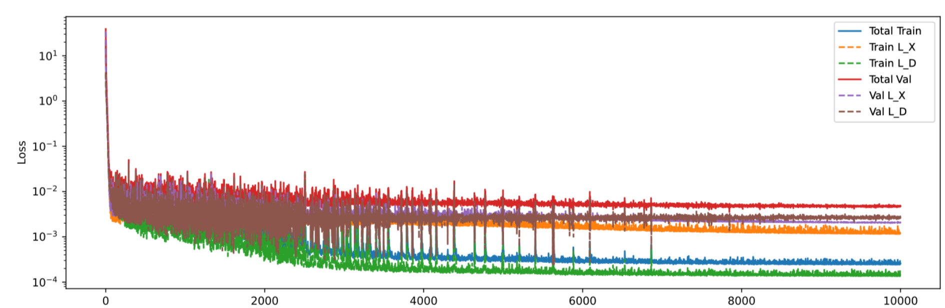
Neural Networks learning nonlinear operators

$$\begin{cases} \mathrm{Pe}_{\mathrm{loc}}\, \epsilon^{-1} \boldsymbol{R}(\theta)\bar{\boldsymbol{b}}_1 \cdot \nabla T - \operatorname{div}\left(\underbrace{\boldsymbol{\mathcal{D}}^{*}\left(m_1, \left|\boldsymbol{b}_1\right|, \theta\right)}\nabla T\right) = Q\left(\boldsymbol{x}\right) & \text{in } D, \\ T = T_0 & \text{on } \Gamma_{\mathrm{in}}, \\ \boldsymbol{\mathcal{D}}^{*}\left(m_1, \left|\boldsymbol{b}_1\right|, \theta\right) \nabla T \cdot \boldsymbol{n} = 0 & \text{on } \partial D \backslash \Gamma_{\mathrm{in}}, \end{cases}$$

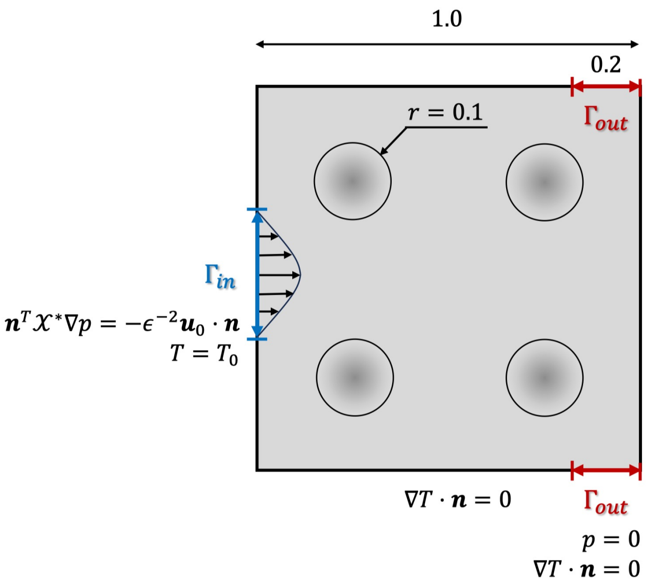




(a) Collocation points and boundary points. (b) $\mathcal{D}_{11}^{\mathrm{data}}$. (c) $\mathcal{D}_{22}^{\mathrm{data}}$.



Appendix D: Design Problem



$$\min_{m_1, m_2, \theta \in \mathcal{D}} J(\Omega) = \int_D Q(\boldsymbol{x}) T \, \mathrm{d}\Omega,$$

Thermal compliance

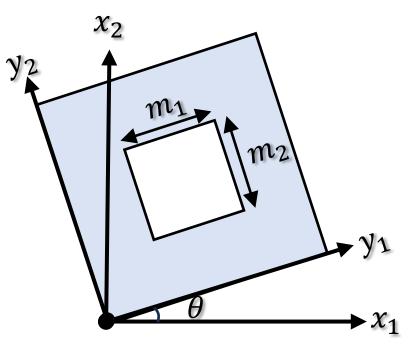
$$G_1 = \epsilon^2 \frac{1/\mathrm{DP}_0}{|\Gamma_{\mathrm{in}}|} \int_{\Gamma_{\mathrm{in}}} p \, \mathrm{d}\Gamma - C_{\mathrm{DP}} \leq 0,$$

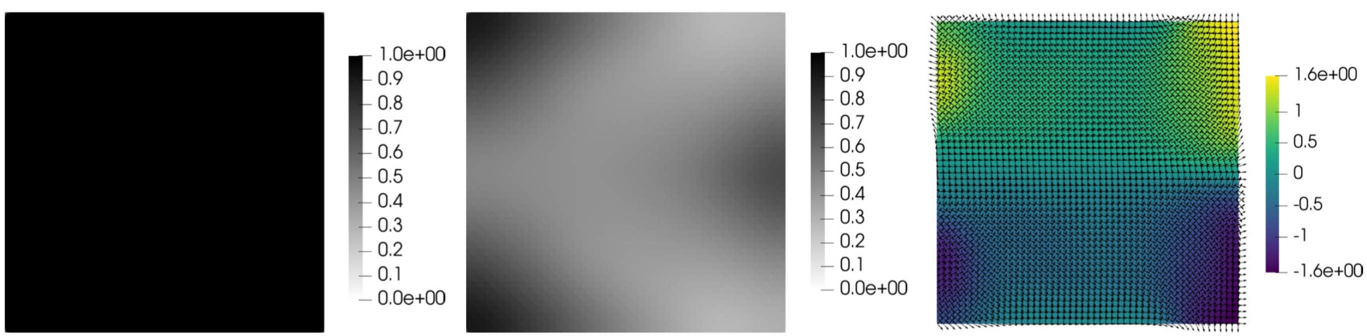
Pressure drop constraint

$$G_2 = \frac{\int_D 1 - \hat{\tilde{m}}_1 \hat{\tilde{m}}_2 \, \mathrm{d}\Omega}{\int_D \mathrm{d}\Omega} - V_{\max} \leq 0,$$

Volume fraction constraint

$$0 \leq m_1(\boldsymbol{x}), m_2(\boldsymbol{x}), \theta(\boldsymbol{x}) \leq 1 \quad \forall \boldsymbol{x} \in D,$$

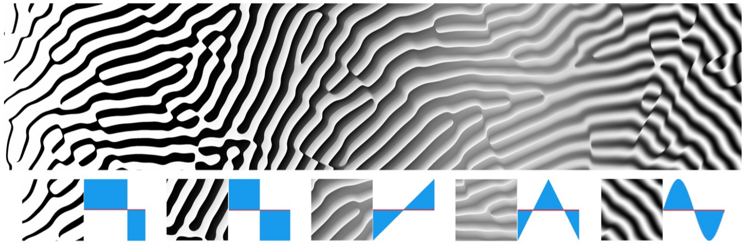




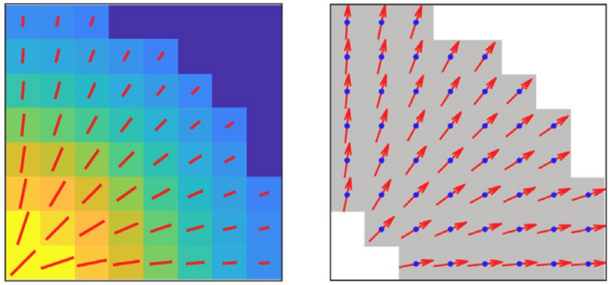
(a) m_1. (b) m_2. (c) θ.

Appendix E: Phasor-based De-homogenisation

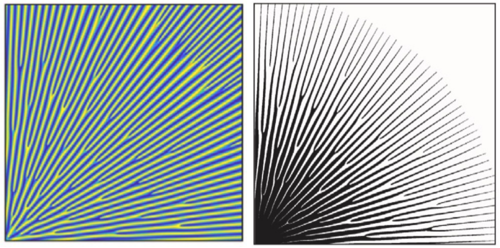
- **Phasor noise**
  - Smoothly varying local control of orientation and periodicity



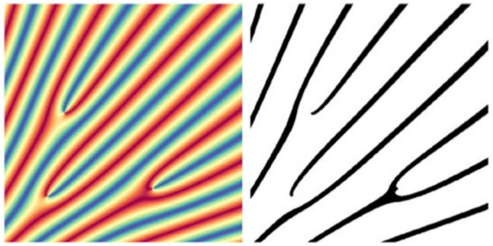
- **Layer-wise translation from multiscale to set of phasor kernels**



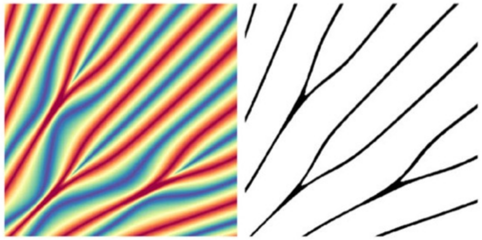
- **Wave-field translations and thickness projection**



- **Branch point locations and phasor singularities**



- **Post-processing connects and pinches the bifurcations**

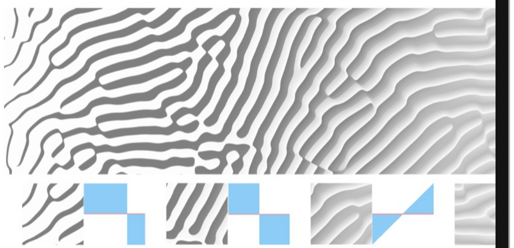


[Tricard et al., 2019; Woldseth et al., 2024a; Woldseth et al., 2024b]

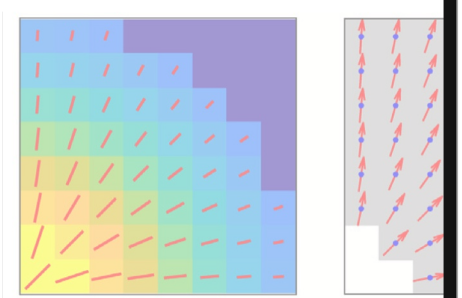
[Back]

Appendix E: Phasor-based De-homogenisation

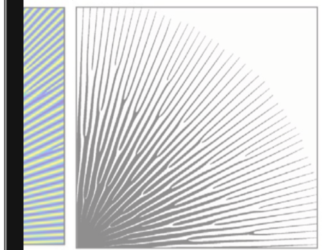
- Phasor noise
  - Smoothly varying loc
    orientation and peric



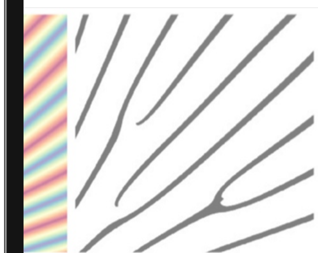
- Layer-wise translation fr
  set of phasor kernels



ons and thickness



ons and phasor



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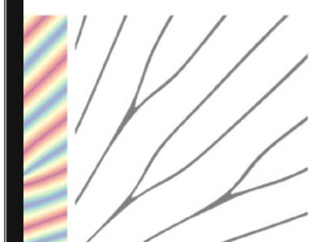

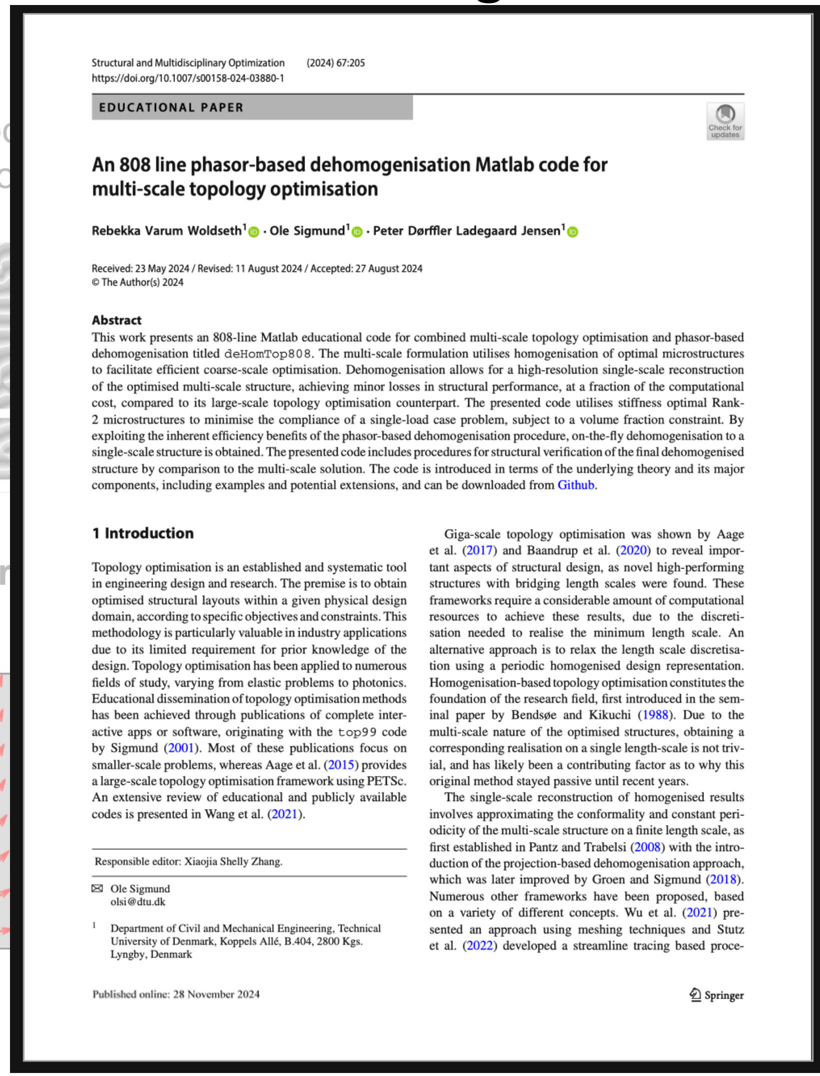

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EDUCATIONAL PAPER

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An 808 line phasor-based dehomogenisation Matlab code for multi-scale topology optimisation

Rebekka Varum Woldseth[1] · Ole Sigmund[1] · Peter Dørffler Ladegaard Jensen[1]

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Abstract
This work presents an 808-line Matlab educational code for combined multi-scale topology optimisation and phasor-based dehomogenisation titled `deHomTop808`. The multi-scale formulation utilises homogenisation of optimal microstructures to facilitate efficient coarse-scale optimisation. Dehomogenisation allows for a high-resolution single-scale reconstruction of the optimised multi-scale structure, achieving minor losses in structural performance, at a fraction of the computational cost, compared to its large-scale topology optimisation counterpart. The presented code utilises stiffness optimal Rank-2 microstructures to minimise the compliance of a single-load case problem, subject to a volume fraction constraint. By exploiting the inherent efficiency benefits of the phasor-based dehomogenisation procedure, on-the-fly dehomogenisation to a single-scale structure is obtained. The presented code includes procedures for structural verification of the final dehomogenised structure by comparison to the multi-scale solution. The code is introduced in terms of the underlying theory and its major components, including examples and potential extensions, and can be downloaded from Github.

1 Introduction

Topology optimisation is an established and systematic tool in engineering design and research. The premise is to obtain optimised structural layouts within a given physical design domain, according to specific objectives and constraints. This methodology is particularly valuable in industry applications due to its limited requirement for prior knowledge of the design. Topology optimisation has been applied to numerous fields of study, varying from elastic problems to photonics. Educational dissemination of topology optimisation methods has been achieved through publications of complete interactive apps or software, originating with the `top99` code by Sigmund (2001). Most of these publications focus on smaller-scale problems, whereas Aage et al. (2015) provides a large-scale topology optimisation framework using PETSc. An extensive review of educational and publicly available codes is presented in Wang et al. (2021).

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✉ Ole Sigmund
olsi@dtu.dk

1 Department of Civil and Mechanical Engineering, Technical University of Denmark, Koppels Allé, B.404, 2800 Kgs. Lyngby, Denmark

Giga-scale topology optimisation was shown by Aage et al. (2017) and Baandrup et al. (2020) to reveal important aspects of structural design, as novel high-performing structures with bridging length scales were found. These frameworks require a considerable amount of computational resources to achieve these results, due to the discretisation needed to realise the minimum length scale. An alternative approach is to relax the length scale discretisation using a periodic homogenised design representation. Homogenisation-based topology optimisation constitutes the foundation of the research field, first introduced in the seminal paper by Bendsøe and Kikuchi (1988). Due to the multi-scale nature of the optimised structures, obtaining a corresponding realisation on a single length-scale is not trivial, and has likely been a contributing factor as to why this original method stayed passive until recent years.

The single-scale reconstruction of homogenised results involves approximating the conformality and constant periodicity of the multi-scale structure on a finite length scale, as first established in Pantz and Trabelsi (2008) with the introduction of the projection-based dehomogenisation approach, which was later improved by Groen and Sigmund (2018). Numerous other frameworks have been proposed, based on a variety of different concepts. Wu et al. (2021) presented an approach using meshing techniques and Stutz et al. (2022) developed a streamline tracing based proce-

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