Vortices in Bose-Einstein condensates: simulations and identification of vortex lines

Victor Kalt[1]

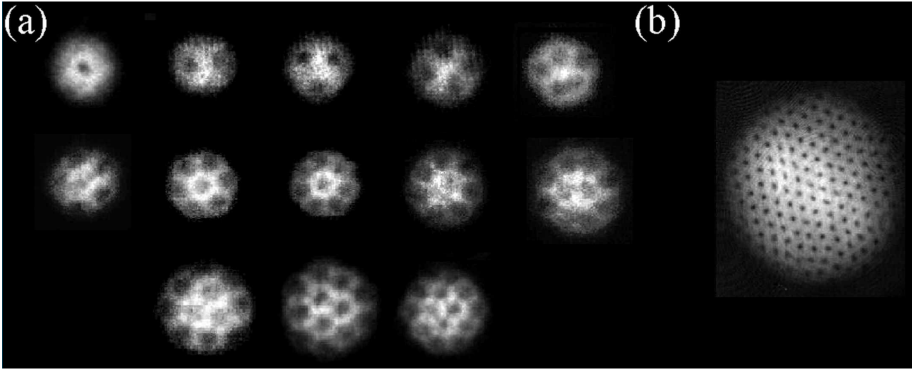
Thesis advisors: Ionut Danaila[1], Marc Brachet[2]

[1]Laboratoire de Mathématiques Raphaël Salem
Université de Rouen Normandie

[2]Laboratoire de Physique de l'École Normale Supérieure
École Normale Supérieure

Context

- Studying the behaviour of vortices is key to understand phenomena observed in quantum fluids.
- Work in collaboration with G. Sadaka on a basis of a previous study by P.E. Emeriau, F. Hecht and I. Danaila.



a) Abrikosov lattice in a fast rotating Bose-Einstein condensate (ENS).
b) Vortex lattice (MIT).

Bose-Einstein Condensates and superfluid helium

Bose-Einstein Condensates (BECs)

A new state of matter obtained from a dilute gas of bosons cooled to temperatures near 0 K. All particles are in the same (lowest energy) state.

Superfluid helium

Superfluids are fluids with vanishing viscosity when cooled at low temperature ($\sim 1K$). They display similar phenomena as BECs, although the physics is different.

The Gross-Pitaevskii equation (GPE)

- BECs are described by a complex wavefunction $\psi = \sqrt{\rho}e^{iS}$
  - $\rho = |\psi|^2$ corresponds to the atomic density,
  - S is the phase,
  - $\mathbf{v} = \nabla S$ is the fluid velocity.
- The Gross-Pitaevskii equation, in dimensionless form:

$$i\partial_t \psi = -\frac{1}{2}\nabla^2\psi + V\psi + g|\psi|^2\psi - \Omega\ell_z\psi \tag{1}$$

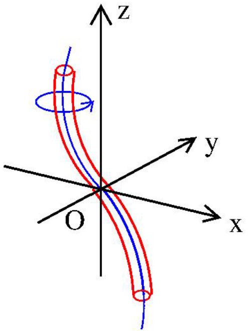
- V: trapping potential
  - harmonic potential for BECs: $V = a_x x^2 + a_y y^2 + a_z z^2$
  - no potential for superfluid helium: $V = 0$
- g: interaction between atoms
- $\ell_z = iA^T\nabla,\ A^T = (y, -x, 0)$: angular momentum operator
- Ω: angular velocity

Vortices

- On a closed path C around the vortex, the circulation is:

$$\Gamma = \oint_C \mathbf{v} \cdot \mathbf{dl} = 2\pi q, \;\; q \in \mathbb{Z} \text{ (winding number)} \tag{2}$$

- $\psi = 0$ along the vortex line



Circulation around a vortex line

Vortices

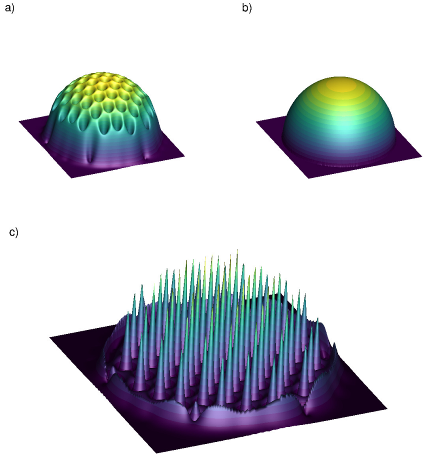
Vortices are points (in 2D) or lines (in 3D) where the density is $\rho = 0$. The circulation on a closed loop around the vortex is $\Gamma = \pm 2\pi$.

Simulations of BECs

- Simulations are performed with the Fortran code GPS (Gross-Pitaevskii simulator) using a backward Euler method for the stationnary GP equation and a time-splitting method for the time-dependant GP equation.
- The GPE can also be solved with the FreeFem toolbox GPFEM using a Sobolev gradient descent or the minimization of the energy with Ipopt.
- Vortex line identification is necessary to obtain information about vortices from simulation data. We use a finite element approach using FreeFem.

Vortex line identification in 2D

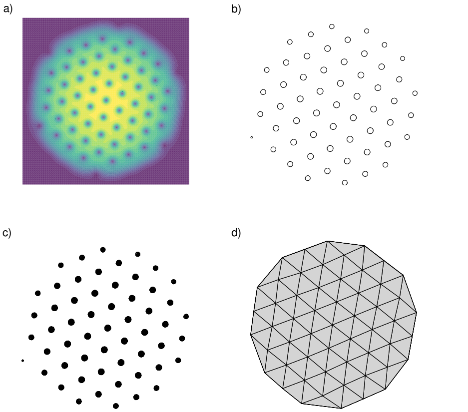
- Remove the Thomas-Fermi density.
- Compute isolines.
- Extract the enclosed mesh.
- Compute the circulation on the isoline.
- Fit a Gaussian ansatz with Ipopt to compute the vortex radius and center.



a) Simulated atomic density,
b) Thomas-Fermi density,
c) Vortex spikes.

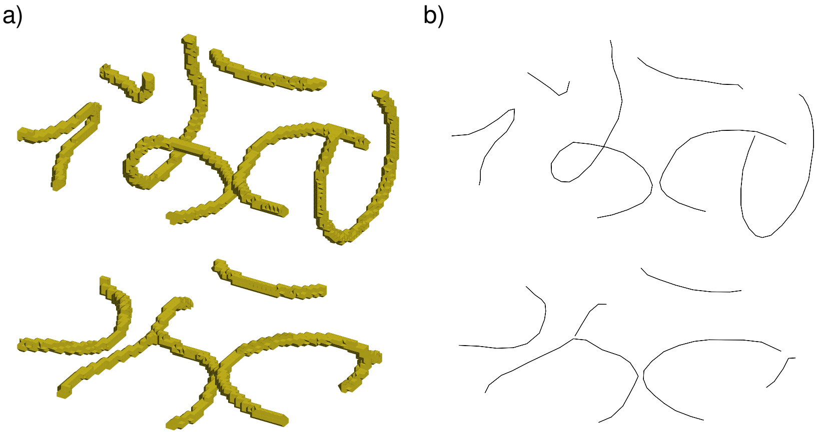
Vortex line identification in 2D

- Remove the Thomas-Fermi density.
- Compute isolines.
- Build the enclosed meshes.
- Compute the circulation on the isoline.
- Fit a Gaussian ansatz with Ipopt to compute the vortex radius and center.



a) Plot of the density, b) Isolines c) Extracted meshes d) Vortex lattice.

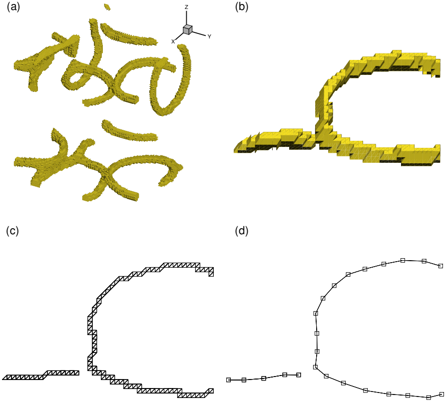
Vortex line identification in 3D



Example of the identification process. a) isosurface of density, b) extracted vortex lines.

Identification process

- Adaptation of a method presented by Liu et al, 2019
- Step 1: reduce the mesh size:
  - Remove zones of high density $\rho > \rho_{threshold}$.
  - Remove zones outside the Thomas-Fermi radius.
  - Approximate the circulation on the triangles of the mesh.
  - Remove the zones of circulation 0.
  - Separate the regions.



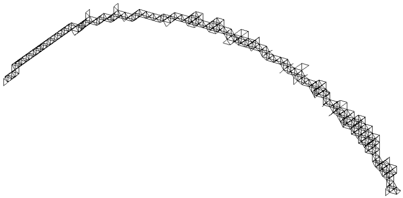
Steps of the identification process

Approximating the circulation

- The phase can be written as $S = \Im(\log(\psi))$.
- Approximation on the mesh triangles:

$$\Gamma = \Im\left(\log\frac{u_1}{u_0} + \log\frac{u_2}{u_1} + \log\frac{u_0}{u_2}\right)$$

- Obtain a FE function representing the circulation
- Approximation artefacts must be removed separately



Graph of points with non-zero circulation

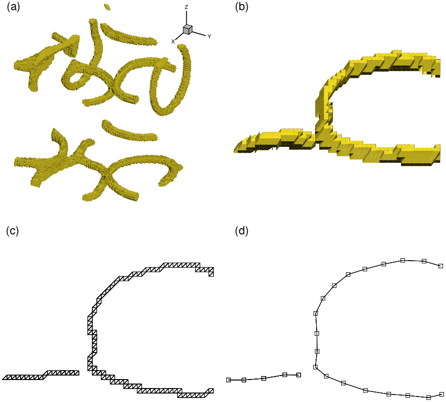
Separating the regions

- Solve $-\Delta u = u_0$ with $u_0 = \begin{cases} 1 & \text{on 1 vertex} \\ 0 & \text{elsewhere} \end{cases}$
- Obtain a non-zero value in the region
- Truncate the region from the mesh

```
while(Th.nt != 0){
    Vh u0rn,urn,vrn;
    u0rn[](0)=1;
    solve lap(urn,vrn) = int3d(Th)(grad(urn)'*grad(vrn)) - int3d(Th)(u0rn*vrn);
    urnmin = urn[].min; urnmax = urn[].max;
    REG++;
    Threg.resize(REG);
    Threg[REG-1] = trunc(Th,((urnmin<0.) ? urn<urnmax-1. : urn>urnmax-1.),fregion=REG);
    Th = trunc(Th,((urnmin<0.) ? urn>urnmax-1. : urn<urnmax-1.),fregion=REG+1);
}
```

Identification process

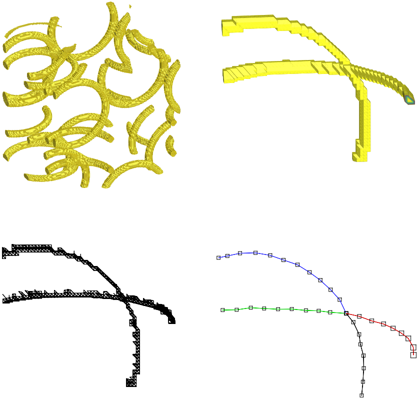
- Step 2: isolate the lines:
    - Extract the connected points of circulation 2π as a graph.
    - Compute mid-points.



Steps of the identification process

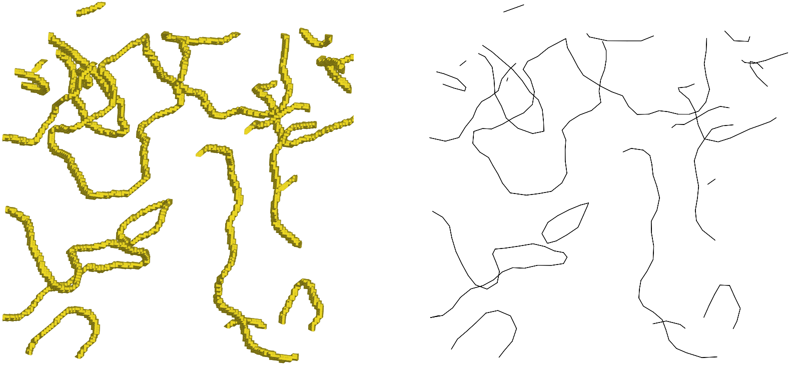
Identification process

- Step 3: construct the lines:
  - Multiple non-connected lines in a single region.
  - Vortex rings.
  - Vortex reconnection.
- Vortex reconnection:
  - Intersection of two vortex lines.
  - Separate the points based on connectivity.
- Save the line as a meshL



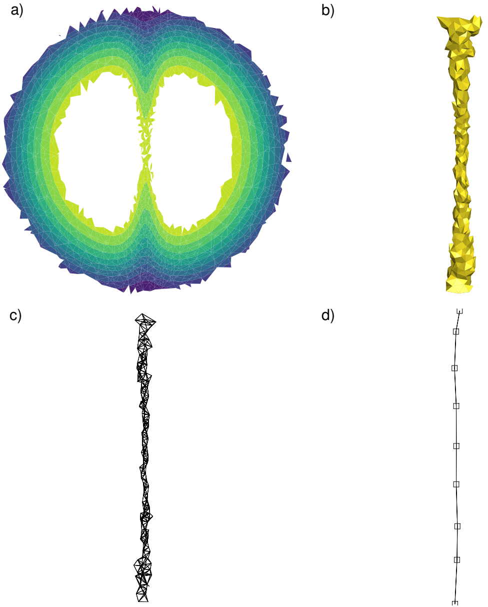
Steps of the identification process

Results in superfluid helium



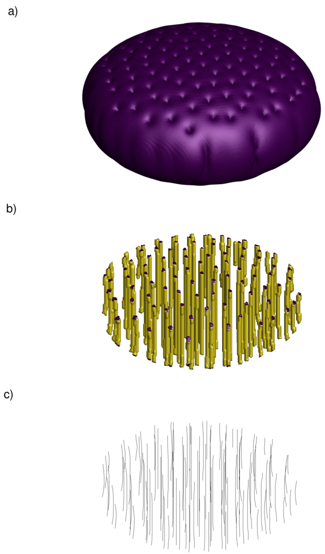
Extracted lines in a simulation of superfluid helium

Results in BEC



Vortex line in a BEC simulation

Results in BEC



Vortex lattice in a BEC simulation

a) Isosurface of atomic density
b) Separated regions
c) Identified vortex lines

Future work

- Parallelize the initial data loading
- Simulate condensates with many vortices (quantum turbulence).
- Study vortex line oscillations (Kelvin waves) and vortex lattice oscillations (Tkachenko waves).