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Shape optimization for a heat exchanger in Navier-Stokes flow with dynamic pressure at the outlet

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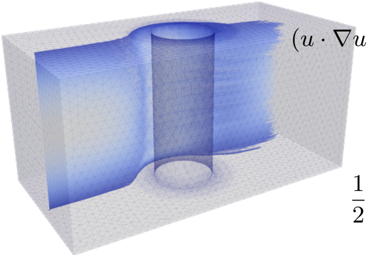
heat exchanger with cylinder shape in Navier-Stokes flow

boundary $\partial\Omega = \Gamma_{\text{in}} \cup \Gamma_{\text{out}} \cup \Gamma_{\text{side}} \cup \Gamma_{\text{t/b}} \cup \Gamma_{\text{cylinder}}$

stationary Navier-Stokes equations:



$$(u \cdot \nabla u) - \frac{1}{Re} \nabla \cdot \nabla u + \nabla p = 0 \text{ in } \Omega$$

$$\nabla \cdot u = 0 \text{ in } \Omega$$

$$u = u_0 \text{ on } \Gamma_0 := \Gamma_{\text{in}}$$

$$u = 0 \text{ on } \Gamma_1 := \Gamma_{\text{side}} \cup \Gamma_{\text{t/b}} \cup \Gamma_{\text{cylinder}}$$

$$\frac{1}{2} u \cdot u + p = p_0 \text{ on } \Gamma_2 := \Gamma_{\text{out}}$$

$$u \times n = 0$$

stationary heat convection equation without buoyancy term:

$$(u \cdot \nabla)\theta - \kappa \nabla \cdot \nabla \theta = 0 \text{ in } \Omega$$

$$\theta = 0 \text{ on } \widetilde{\Gamma}_0 := \Gamma_{\text{in}}$$

$$\theta = 1 \text{ on } \widetilde{\Gamma}_1 := \Gamma_{\text{t/b}} \cup \Gamma_{\text{cylinder}}$$

$$\nabla \theta \cdot n = 0 \text{ on } \widetilde{\Gamma}_2 := \Gamma_{\text{side}} \cup \Gamma_{\text{out}}$$

ϕ : shape deformation $\Omega(\phi) = \phi(\widehat{\Omega})$ from the reference domain $\widehat{\Omega}$

cost function:

$$J(\phi, \{u, p, \theta\}) = -\int_{\widetilde{\Gamma}_1(\phi)} \kappa \nabla \theta \cdot n \;\to \min$$

weak formulation for Navier-Stokes eqs. with dynamic pressure cnd.

$V = \{v \in H^1(\Omega)^3\,;\, v = 0 \text{ on } \Gamma_0 \cup \Gamma_1\,, v \times n = 0 \text{ on } \Gamma_2\}$, $Q = L^2(\Omega)$

$$\int_\Omega (\nabla \times u) \times u \cdot v - \frac{1}{2}\int_\Omega (u \cdot u)\nabla \cdot v + \frac{1}{Re}\int_\Omega \nabla \times u \cdot \nabla \times v$$
$$- \int_\Omega \nabla \cdot v p - \int_\Omega \nabla \cdot u q = -\int_{\Gamma_2} p_0\, v \cdot n \quad \forall (v, q) \in V \times Q$$

solution of the weak formulation $\Rightarrow$ strong form + boundary condition for velocity satisfying $\nabla \cdot u = 0$, applying integration by parts

$$\begin{aligned}
0 = &\int_\Omega (\nabla \times u) \times u \cdot v + \frac{1}{2}\int_\Omega \nabla(u \cdot u) \cdot v - \frac{1}{2}\int_{\partial\Omega} (u \cdot u) v \cdot n \\
&+ \frac{1}{Re}\int_\Omega \nabla \times (\nabla \times u) \cdot v + \frac{1}{Re}\int_{\partial\Omega} n \times (\nabla \times u) \cdot v \\
&+ \int_\Omega v \cdot \nabla p - \int_{\partial\Omega} v \cdot n p + \int_{\Gamma_2} p_0\, v \cdot n \\
= &\int_\Omega \left((\nabla \times u) \times u + \frac{1}{2}\nabla(u \cdot u) \right) \cdot v - \int_{\Gamma_0 \cup \Gamma_1 \cup \Gamma_2} \left(\frac{1}{2}(u \cdot u) + p - p_0 \right) (v \cdot n) \\
&+ \frac{1}{Re}\int_\Omega (\nabla(\nabla \cdot u) - \nabla \cdot \nabla u + \nabla p) \cdot v - \frac{1}{Re}\int_{\Gamma_0 \cup \Gamma_1 \cup \Gamma_2} (\nabla \times u) \cdot (v \times n)
\end{aligned}$$

NB: $(u \cdot \nabla u) u = (\nabla \times u) \times u + \dfrac{1}{2}\nabla(u \cdot u)$ and $(\nabla \times u) \times u = \nabla(\nabla \cdot u) - \nabla \cdot \nabla u$

finite element approximation to Navier-Stokes eqs. with curl-curl form

$V_h = \{v|_K \in P_1(K)^3\,;\, v = 0$ on $\Gamma_0 \cup \Gamma_1\,, v \times n = 0$ on $\Gamma_2\}$, $Q_h = \{q|_K \in P_1(K)\}$

two difficulties in P1/P1 approximation to curl-curl form

- $\int_\Omega \nabla\times u \cdot \nabla \times v$ is coercive on $\{u; \nabla\cdot u = 0\}$ $\Leftrightarrow (A_0 u, u) > 0\ \forall u \in \{u; B\,u = 0\}$
- P1/P1 approx. does not satisfy the inf-sup cond. $\Leftrightarrow \exists p \neq 0\ B^T p = 0$

$$\int_\Omega (\nabla\times u)\times u\cdot v - \frac{1}{2}\int_\Omega (u\cdot u)\nabla\cdot v + \frac{1}{Re}\int_\Omega \nabla\times u\cdot\nabla\times v + \int_\Omega \nabla\cdot u\nabla\cdot v$$
$$-\int_\Omega \nabla\cdot vp + \int_\Omega \nabla\cdot uq + \delta\sum_K h_K^2\int_k \nabla p\cdot\nabla q = -\int_{\Gamma_2} p_0\, v\cdot n \quad \forall(v,q)\in V_h\times Q_h$$

stiffness matrix for a linearlized equation

$$\begin{bmatrix} A_1 + A_0 + C & B^T \\ -B & \delta D \end{bmatrix} \quad \begin{cases} [A_0]_{i\,j} = \int_\Omega \nabla\times\varphi_j\cdot\nabla\times\varphi_i \\ C_{i\,j} \ = \int_\Omega \nabla\cdot\varphi_j\nabla\cdot\varphi_i \\ B_{i\,j} \ = -\int_\Omega \nabla\cdot\varphi_j\psi_i \\ D_{i\,j} \ = \sum_K h_K^2 \int_K \nabla\psi_j\cdot\nabla\psi_i,\ \delta > 0 \end{cases}$$

$A_0 = A_0^T$, $C = C^T$, Ker$C$ = Ker$B$, ImA_0 = Ker$B$

stiffness matrix of the Stokes eqs., without unsymmetric (nonlinear) term

$\begin{bmatrix} A_0 + C & B^T \\ -B & \delta D \end{bmatrix}$ is coercive $\Rightarrow$ additive Schawrz preconditioner in `hpddm`

- a proof of $(A_0 + C)u = 0 \Rightarrow u = 0$

  Ker$C$ = Ker$B \ni A_0 u = -C\,u \in$ ImC $\Rightarrow A_0 u = 0 \bigwedge u \in$ Ker$C$ = Ker$B$

  A_0 is coercive on Ker$B \Rightarrow u = 0$ $\quad C$ has the same effect as $B^T B$

weak formulation for thermal convection diffusion

$\Theta = \{\theta \in H^1(\Omega)\,;\, \theta = 0$ on $\widetilde{\Gamma}_0 \cup \widetilde{\Gamma}_1\}$

$$\int_\Omega (u\cdot\nabla)\theta\xi + \kappa\int_\Omega \nabla\theta\cdot\nabla\xi = 0 \quad \forall\theta\in\Theta$$

solution of the weak formulation $\Rightarrow$ strong form + boundary condition for given u

$$\int_\Omega \left((u\cdot\nabla)\theta - \kappa\nabla\cdot\nabla\theta\right)\xi + \kappa\int_{\widetilde{\Gamma}_0\cup\widetilde{\Gamma}_1\cup\widetilde{\Gamma}_2} \nabla\theta\cdot n\xi = 0 \quad \forall\xi\in\Theta$$

$$\Rightarrow \nabla\theta\cdot n = 0 \text{ on } \widetilde{\Gamma}_2$$

surface integration is converted to domain integration

η^* : harmonic extension of $\eta = 1$ on $\widetilde{\Gamma}_2$, $\eta = 0$ on $\widetilde{\Gamma}_0 \cup \widetilde{\Gamma}_1$

θ is the solution of the strong form for given u

$$\begin{aligned}
&-\kappa\int_{\widetilde{\Gamma}_2} \nabla\theta\cdot n\eta \\
&= -\kappa\int_{\widetilde{\Gamma}_2} \nabla\theta\cdot n\eta - \int_\Omega \left((u\cdot\nabla)\theta - \nabla\cdot\nabla\theta\right)\eta^* \\
&= -\kappa\int_{\widetilde{\Gamma}_2} \nabla\theta\cdot n\eta - \int_\Omega (u\cdot\nabla)\theta\eta^* - \kappa\int_\Omega \nabla\theta\cdot\nabla\eta^* + \int_{\widetilde{\Gamma}_0\cup\widetilde{\Gamma}_1\cup\widetilde{\Gamma}_2} \kappa\nabla\theta\cdot n\eta \\
&= -\int_\Omega (u\cdot\nabla)\theta\eta^* - \kappa\int_\Omega \nabla\theta\cdot\nabla\eta^* = -\int_\Omega \nabla\theta\cdot u\eta^* - \kappa\int_\Omega \nabla\theta\cdot\nabla\eta^*
\end{aligned}$$

cost function and Lagrangean

$$
\begin{aligned}
J(\phi, \{u, p, \theta\}) &= -\int_{\Gamma_2(\phi)} \kappa \nabla\theta \cdot n \\
&= -\int_{\Omega(\phi)} \nabla\theta \cdot u\eta^* - \kappa \int_{\Omega(\phi)} \nabla\theta \cdot \nabla\eta^*
\end{aligned}
$$

constraint for the thermal fluid

$$
\begin{aligned}
&A(\phi, \{u, p, \theta\}, \{v, q, \xi\}) \\
&= \int_\Omega (\nabla \times u) \times u \cdot v - \frac{1}{2}\int_\Omega (u \cdot u)\nabla \cdot v + \frac{1}{Re}\int_\Omega \nabla \times u \cdot \nabla \times v + \int_\Omega \nabla \cdot u \nabla \cdot v \\
&- \int_\Omega \nabla \cdot vp + \int_\Omega \nabla \cdot uq + \delta \sum_K h_K^2 \int_k \nabla p \cdot \nabla q + \int_{\Gamma_2} p_0\, v \cdot n \\
&+ \int_{\Omega(\phi)} (\nabla\theta \cdot u)\xi + \kappa \int_{\Omega(\phi)} \nabla\theta \cdot \nabla\xi
\end{aligned}
$$

Lagrangean is defined as

$$
\mathcal{L}(\phi, \{u, p, \theta\}, \{v, q, \xi)\} = J(\phi, \{u, p, \theta\}) + A(\phi, \{u, p, \theta\}, \{v, q, \xi\})
$$

state problem

$$
d\mathcal{L}_{\{v,q,\xi\}}(\phi, \{u, p, \theta\}, \{v, q, \xi\})[\{\delta v, \delta q, \delta\xi\}] = 0
$$

adjoint problem using solution $\{u, p, \theta\}$ of the state problem

$$
d\mathcal{L}_{\{u,p,\theta\}}(\phi, \{u, p, \theta\}, \{v, q, \xi\})[\{\delta u, \delta p, \delta\theta\}] = 0
$$

state and adjoint problems

state problem to find $\{u, p, \theta\}$

$$
\begin{aligned}
0 =& d\mathcal{L}_{\{v,q,\xi\}}(\phi, \{u, p, \theta\}, \{v, q, \xi\})[\{\delta v, \delta q, \delta \xi\}] = A(\varphi, \{u, p, \theta\}, \{\delta v, \delta p, \delta \xi\}) \\
=& \int_\Omega (\nabla \times u) \times u \cdot \delta v - \frac{1}{2}\int_\Omega (u \cdot u)\nabla \cdot \delta v + \frac{1}{Re}\int_\Omega \nabla \times u \cdot \nabla \times \delta v + \int_\Omega \nabla \cdot u \nabla \cdot \delta v \\
& - \int_\Omega \nabla \cdot \delta v p + \int_\Omega \nabla \cdot u \delta q + \delta \sum_K h_K^2 \int_k \nabla p \cdot \nabla \delta q + \int_{\Gamma_2} p_0 \, \delta v \cdot n \\
& + \int_{\Omega(\phi)} (\nabla \theta \cdot u)\delta \xi + \kappa \int_{\Omega(\phi)} \nabla \theta \cdot \nabla \delta \xi
\end{aligned}
$$

adjoint problem to find $\{v, q, \xi\}$ using solution $\{u, p, \theta\}$ of the state problem

$$
\begin{aligned}
0 =& d\mathcal{L}_{\{u,p,\theta\}}(\varphi, \{u, p, \theta\}, \{v, q, \xi\})[\{\delta u, \delta p, \delta \theta\}] \\
=& \int_\Omega (\nabla \times \delta u) \times u \cdot v + \int_\Omega (\nabla \times u) \times \delta u \cdot v - \int_\Omega (\delta u \cdot u)\nabla \cdot v \\
& + \frac{1}{Re}\int_\Omega \nabla \times \delta u \cdot \nabla \times v + \int_\Omega \nabla \cdot \delta u \nabla \cdot v \\
& - \int_\Omega \nabla \cdot v \delta p + \int_\Omega \nabla \cdot \delta u q + \delta \sum_K h_K^2 \int_k \nabla \delta p \cdot \nabla q \\
& + \int_{\Omega(\phi)} (\nabla \delta\theta \cdot u)\xi + \int_{\Omega(\phi)} (\nabla \theta \cdot \delta u)\xi + \kappa \int_{\Omega(\phi)} \nabla \delta \theta \cdot \nabla \xi \\
& - \int_{\Omega(\phi)} \nabla \theta \cdot \delta u \eta^* - \int_{\Omega(\phi)} \nabla \delta\theta \cdot u \eta^* - \kappa \int_{\Omega(\phi)} \nabla \delta \theta \cdot \nabla \eta^*
\end{aligned}
$$

shape derivative : assuming material derivative vanished

shape derivative (material derivative)

$$u(\phi+\varphi)(x+\varphi(x)) - u(\phi)(x) \simeq u'(\phi)[\varphi](x)$$

shape derivative of gradient with $F(\varphi) = I + \nabla\varphi$

$$\nabla_z u(\phi+\varphi)(x+\varphi(x)) \simeq F^{-T}(\varphi)\nabla_x u(\phi)(x) + (\nabla_x u)'(\phi)[\varphi](x)$$

derivative of domain integral of scalar function u

$$f(\phi, u(\phi)) = \int_{\Omega(\phi)} u(\phi)(x)dx \ \Rightarrow\ d\,f_\phi(\phi,u)[\varphi] = \int_{\Omega(\phi)} (u' + u(\nabla\cdot\varphi))dx$$

derivative of domain integral of ∇u

$$f(\phi, \nabla u(\phi)) = \int_{\Omega(\phi)} \nabla u(\phi)(x)dx$$

$$\Rightarrow \quad d\,f_\phi(\phi,\nabla u)[\varphi] = \int_{\Omega(\phi)} \left\{(\nabla u)'[\varphi] - (\nabla\varphi)^T\nabla u + (\nabla\cdot\varphi)\nabla u\right\} dx$$

- differential operators of 1st order, $\nabla\cdot$, $\nabla\times$ to vector valued u are written with tensor of $(\nabla u)^T$ like $\nabla\cdot u = (\nabla u) : I = (\nabla u)^T : I$
- $\nabla \to -(\nabla\delta\varphi)^T\nabla$ and multiplying $\nabla\cdot\delta\varphi$ to the integrand

shape derivative of integration of curl-curl form

$$\int_{\Omega(\phi+\varphi)} (\nabla_z \times u(\phi+\varphi)) \cdot (\nabla_z \times v(\phi+\varphi)) - \int_{\Omega(\phi)} (\nabla \times u(\phi)) \cdot (\nabla \times v(\phi))$$

$\nabla \times u$ is computed by component-wise as

$$\nabla \times u = \left[(\nabla u)^T : \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix},\ (\nabla u)^T : \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix},\ (\nabla u)^T : \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right]^T$$

$$\int_{\Omega(\phi+\varphi)} (\nabla_z u(\phi+\varphi))^T : B_1 - \int_{\Omega(\phi)} (\nabla u(\phi))^T : B_1$$

$$\simeq \int_{\Omega(\phi)} \big((F^{-T}(\phi)+F^{-T\prime}(\phi)[\varphi])\,(\nabla u(\phi))^T + (\nabla u)^{T\prime}(\phi)[\varphi]\big) : B_1 \big(\omega(\phi)+\omega'(\varphi)\big) - (\nabla u(\phi))^T : B_1$$

$$\simeq \int_{\Omega(\phi)} (\nabla u(\varphi))^T : B_1 (\nabla \cdot \varphi) - (\nabla \varphi)^T\,(\nabla u(\phi))^T : B_1 + (\nabla u)^{T\prime}(\phi)[\varphi] : B_1$$

supposing $(\nabla u)^{T\prime}(\phi) = 0$, and using

$$[(\nabla \varphi)^T (\nabla u)^T : B_1,\ (\nabla \varphi)^T (\nabla u)^T : B_2,\ (\nabla \varphi)^T (\nabla u)^T : B_3]^T = \big((\nabla \varphi)^T \nabla\big) \times u$$

shape derivative is calculated as

$$\int_{\Omega(\phi)} (\nabla \times u) \cdot (\nabla \times v)\,(\nabla \cdot \varphi) - \Big(\big(\big((\nabla \varphi)^T \nabla\big) \times u\big) \cdot (\nabla \times v) - (\nabla \times u) \cdot \big(\big((\nabla \varphi)^T \nabla\big) \times v\big)$$

shape derivative of the Lagrangean of the NS eqs.

$$
\begin{aligned}
& d\mathcal{L}_\phi(\phi, \{u, p, \theta\}, \{v, q, \xi\})[\varphi] \\
& = -\int_{\Omega(\phi)} \nabla\theta \cdot u\eta^* (\nabla \cdot \varphi) - (\nabla\varphi)^T \nabla\theta \cdot u\eta^* \\
& - \kappa \int_{\Omega(\phi)} \nabla\theta \cdot \nabla\eta^* (\nabla \cdot \varphi) - (\nabla\varphi)^T \nabla\theta \cdot \nabla\eta^* - \nabla\theta \cdot (\nabla\varphi)^T \nabla\eta^* \\
& + \int_{\Omega(\phi)} ((\nabla \times u) \times u) \cdot v (\nabla \cdot \varphi) + \Big(\Big(((\nabla\varphi)^T \nabla) \times u\Big) \times u\Big) \cdot v \\
& - \frac{1}{2} \int_{\Omega(\phi)} (u \cdot u)(\nabla \cdot v)(\nabla \cdot \varphi) + (u \cdot u)(((\nabla\varphi)^T \nabla) \cdot v) \\
& + \frac{1}{Re} \int_{\Omega(\phi)} (\nabla \times u \cdot \nabla \times v)\, (\nabla \cdot \varphi) - \Big((\nabla\varphi)^T \nabla\Big) \times u \cdot \nabla \times v - \nabla \times u \cdot \Big((\nabla\varphi)^T \nabla\Big) \times v \\
& + \int_{\Omega(\phi)} \nabla \cdot u \nabla \cdot v (\nabla \cdot \varphi) - (\nabla\varphi)^T \nabla \cdot u \nabla \cdot v - \nabla \cdot u (\nabla\varphi)^T \nabla \cdot v \\
& - \int_{\Omega(\phi)} \nabla \cdot v\, p (\nabla \cdot \varphi) - \Big((\nabla\varphi)^T \nabla\Big) \cdot v\, p + \int_{\Omega(\phi)} \nabla \cdot u\, q (\nabla \cdot \varphi) - \Big((\nabla\varphi)^T \nabla\Big) \cdot u\, q \\
& + \delta \sum_K h_K^2 \int_{K(\phi)} \Big\{ \nabla p \cdot \nabla q (\nabla \cdot \varphi) - (\nabla\varphi)^T \nabla p \cdot q - \nabla p \cdot (\nabla\varphi)^T \nabla q \Big\} \\
& + \kappa \int_{\Omega(\phi)} \nabla\theta \cdot \nabla\xi (\nabla \cdot \varphi) - (\nabla\varphi)^T \nabla\theta \cdot \nabla\xi - \nabla\theta \cdot (\nabla\varphi)^T \nabla\xi \\
& + \int_{\Omega(\phi)} \nabla\theta \cdot u\xi (\nabla \cdot \varphi) - (\nabla\varphi)^T \nabla\theta \cdot u\xi
\end{aligned}
$$

gradient flow solver with inner product by elasticity equations

gradient flow solver with H^1-norm

$$((\frac{\varphi^{k+1}-\varphi^k}{\Delta\tau},\varphi)) = -d\mathcal{L}_\phi\left(\varphi^k,\{u^k,p^k,\theta^k\},\{v^k,q^k,\xi^k\}\right)[\varphi]\ \ \forall\varphi$$

$$((\psi,\varphi)) = d\mathcal{L}_\phi\left(\varphi^k,\{u^k,p^k,\theta^k\},\{v^k,q^k,\xi^k\}\right)[\varphi]\ \ \forall\varphi$$

$$\varphi^{k+1} = \varphi^k - \Delta\tau\psi$$

$\{u^k,p^k,\theta^k\}$: solution of the state problem at k-th pseudo time step
$\{v^k,q^k,\xi^k\}$: solution of the adjoint problem at k-th pseudo time step
elasticity equations to update domain with Lamé constants $\lambda=\mu=1$

$$2\mu\int_\Omega D(\psi):D(\varphi)+\lambda\int_\Omega\nabla\cdot\psi\nabla\cdot\varphi = d\mathcal{L}_\phi\left(\varphi^k,\{u^k,p^k,\theta^k\},\{v^k,q^k,\xi^k\}\right)[\varphi]$$

boundary conditions for moving surface of the cylinder whose edges are fixed on top/bottom surfaces within a box

$$\psi = 0 \text{ on } \Gamma_{\text{in}}\cup\Gamma_{\text{out}}\cup\Gamma_{\text{side}}$$

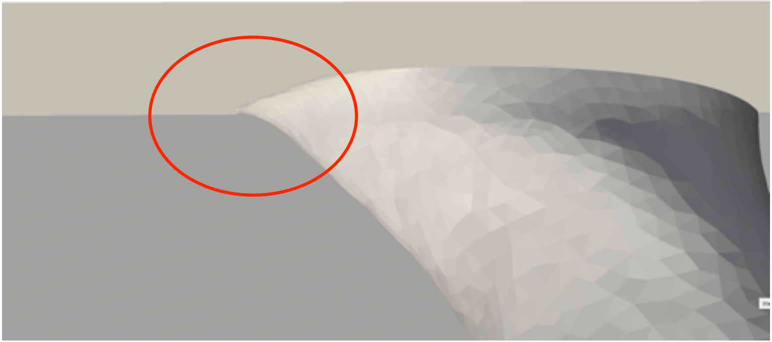
$$\psi\cdot n = 0 \text{ on } \Gamma_{\text{t/b}}$$

$$D(\psi)n = 0 \text{ on } \Gamma_{\text{cylinder}}$$

- inequality condition to keep cylinder surface within upper/bottom boundary

inequality conditions to keep the upper/bottom surfaces

surface Γ_{cylinder} is updated by $\vec{x} - \Delta\tau\psi(\vec{x})$



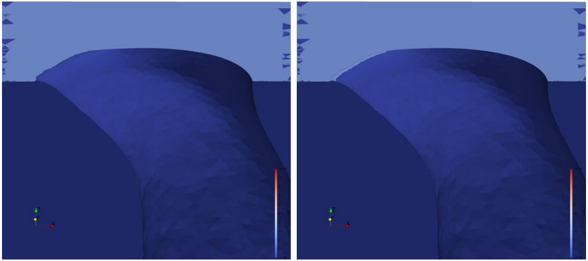
inequality condition to keep cylinder surface within upper/bottom boundary

$$-1 \leq (\vec{x} - \Delta\tau\psi(\vec{x}))|_z \leq 1 \iff \frac{z-1}{\Delta\tau} \leq \psi(\vec{x})|_z \leq \frac{z+1}{\Delta\tau} \text{ on } \Gamma_{\text{cylinder}}$$

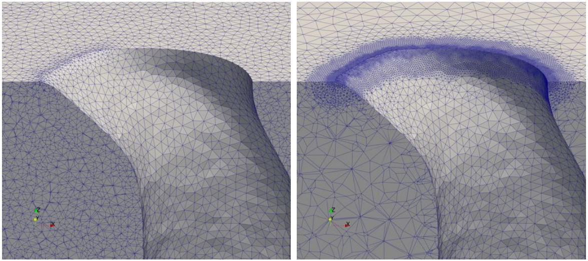
variational inequality can be solved by

- IPOPT : easy to implement but in sequential not in distributed way
- primal-dual active set method : need to be written by FreeFEM script but can be parallelized

re-meshing procedure with mmg3d and Gmsh



- FEM node Γ_{cylinder} is moved to Γ_{top} with $z = 1$



- mmg3d to coarsen the surface triangulation
- Gmsh to generate refined mesh for deformed surface with updated edge

FreeFEM script to change surface label

thanks to Dr. P. Jolivet

```
mesh3 Th;
int[int] ifaces(ThG.nt * 4);
// retrieving the original label of the boundary surface
for (int k = 0; k < Th.nbe; k++) {
  int kk = Th.be(k).Element;
  int ll = Th.be(k).whoinElement;
  ifaces[kk * 4 + ll] = Th.be(k).label;
}
// set new label
{
  int k = targetsurfacelement;
  int kk = Th.be(k).Element;
  int ll = Th.be(k).whoinElement;
  int newlabel;
  ifaces[kk * 4 + ll] = newlabel;
}
Th = change(Th, flabel = ifaces[nuTet * 4 + nuFace]);
```

- `whoinElement` returns vertex id, which is the opposite side of the boundary surface and inside of the domain, and also indicates element surface id

technique to write shape gradient operator in FreeFEM

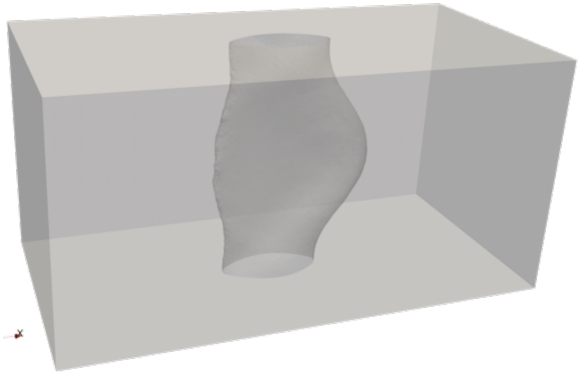
for shape derivative of curl-curl form

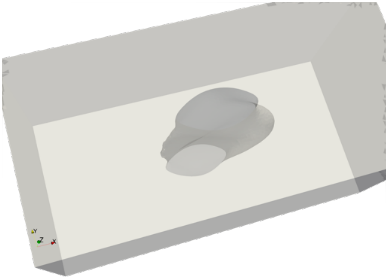
$$\int_{\Omega} \left((\nabla\varphi)^T \nabla \right) \times u \cdot \nabla \times v$$

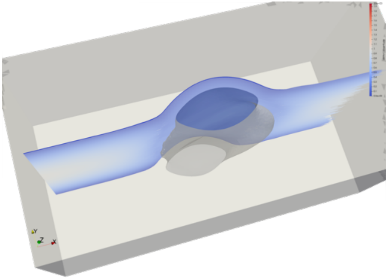
```
macro DOT(uu, vv) ((uu)' * (vv)) //'
macro Rot(uu) [(dy(uu[2]) - dz(uu[1])),
               (dz(uu[0]) - dx(uu[2])),
               (dx(uu[1]) - dy(uu[0]))] //
macro ShapeTensor(ww) [[dx(ww[0]), dx(ww[1]), dx(ww[2])],
                       [dy(ww[0]), dy(ww[1]), dy(ww[2])],
                       [dz(ww[0]), dz(ww[1]), dz(ww[2])]] //
macro ShapeRot(aa, uu)
[((aa[1][0])*dx(uu[2])+(aa[1][1])*dy(uu[2])+(aa[1][2])*dz(uu[2])
 -(aa[2][0])*dx(uu[1])+(aa[2][1])*dy(uu[1])+(aa[2][2])*dz(uu[1])),
 ((aa[2][0])*dx(uu[0])+(aa[2][1])*dy(uu[0])+(aa[2][2])*dz(uu[0])
 -(aa[0][0])*dx(uu[2])+(aa[0][1])*dy(uu[2])+(aa[0][2])*dz(uu[2])),
 ((aa[0][0])*dx(uu[1])+(aa[0][1])*dy(uu[1])+(aa[0][2])*dz(uu[1])
 -(aa[1][0])*dx(uu[0])+(aa[1][1])*dy(uu[0])+(aa[1][2])*dz(uu[0]))] //
func Pk4 = [P1, P1, P1, P1];
fespace Wh(Th, Pk4);
Wh [uu, uu, uu, pp], [vv, vv, vv, qq]; // state/adjoint sols.

varf SensitivityRHS([w1, w2, w3], [phi1, phi2, phi3]) //
= int3d(Th,qfV=qfV1)( // lowest order
   DOT(ShapeRot(ShapeTensor([phi1, phi2, phi3]), [uu1, uu2, uu3]),
       Rot([vv1, vv2, vv3]))
```

- `Rot` macro receives and returns 3-component vector
- possible application of successive multiplication of `ShapeTensor`
- lowest order numerical quadrature for constant value of integrand in an element

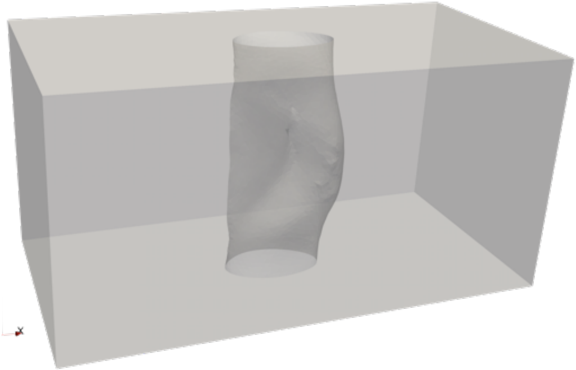
preliminary results: $Re = 1$

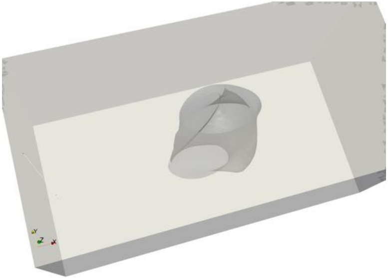


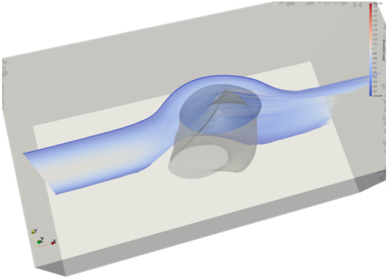




preliminary results: $Re = 50$







summary

- weak formulation with curl-curl is used for the Navier-Stokes equations with dynamic pressure condition to perform engineering setting.
- P1/P1 approximation with adding div-div form for the velocity and element-wise grad-grad form for the pressure leads to coercive stiffness matrix for low Reynolds number, which is easily solved by additive Schwarz preconditioner in `hpddm`.
- shape derivative for domain variations is calculated by using the solutions of state/adjoint problems.
- gradient solver is used to find extremal of the Lagrangean using shape derivative.
- inequality conditions is supposed to keep the deformed domain within a certain box, which is solved by IPOPT.
- `mmg3d` is used to coarsen the mesh of the cylinder surface and `Gmsh` is used to regenerate the whole mesh with mesh refinement in the vicinity of the cylinder edge with updating of surface label.

on-going

- 2nd order shape derivative, which is useful for backward Euler method for the gradient flow solver and for Newton iteration
- primal-dual active set method, which can be run on distributed memory environment, to solve variational inequality

references

- The Stokes and Navier-Stakes equations with boundary conditions involving the pressure, C. Conca, F. Murat and O. Pironneau, *Japan J. Math.*, Vol 20, 1994, pp.279-318
- *Shape Optimization Problems*, H. Azegami, Springer, 2020