Optimized Surface Acoustic Waves Devices With FreeFem++ Using an Original FEM/BEM Numerical Model

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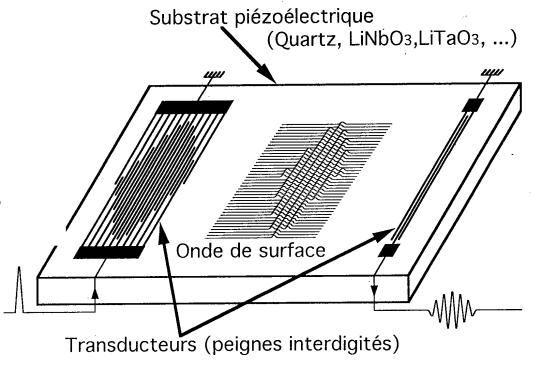
Outlines

- Introduction
- Physical model
- Electro Acoustical Computation : FreeFem++ Model
- **■** Include temperature variations
- Numerical and experimental results
- Conclusions
- Future works

SAW IDT components

How is built a SAW device

- Piezoelectric substrate
- SAW IDT transducer
- The Surface Acoustic Wave is
 - launched and detected
 - propagating



Geometry $\mathbf{\Omega}_d^{\infty}$ Dielectric medium Ω_d B_r^u electrode $B_r^{\stackrel{\mathsf{x}}{d}}$ B_l^d Piezoelectric medium down $\mathbf{\Omega}_{p}^{\infty}$

- Assumptions :
 - 2D analysis (very long electrode): plain strain approximation
 - p periodic along the x axis
 - harmonic electrical excitation of the electrodes: $V_n(\gamma) = V_0 e^{-j2\pi n\gamma}$
 - electrical assumption: no dielectric losses in the electrode
 - mechanical assumption : the metallic electrode are homogeneous isotropic, elastic materials

■ The Piezoelectric domain Ω_p and the Elastic domain Ω_E obeys Newton's second law:

$$\nabla \cdot \mathbf{T} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}$$

The Piezoelectric domain Ω_p , the Elastic domain Ω_B and the Dielectric domain Ω_D obeys the quasistatic Maxwell's equation:

$$\nabla \cdot \mathbf{D} = 0$$

■ The constitutives equations:

For
$$\Omega_P$$
For Ω_E

$$\begin{cases} \mathbf{T}_{ij} = \mathbf{C}_{ijkl}^{\mathbf{E}} \mathbf{S}_{kl} - \mathbf{e}_{ijk} \mathbf{E}_k \\ \mathbf{D}_i = \mathbf{e}_{kli} \mathbf{S}_{kl} + \varepsilon_{ik}^{\mathbf{S}} \mathbf{E}_k \end{cases}$$

$$\mathbf{T}_{ij} = (\lambda + \mu) \delta_{ij} \mathbf{S}_{kk} - 2\mu \mathbf{S}_{ij}$$
For Ω_D

$$\mathbf{D}_i = \varepsilon_{ik} \mathbf{E}_k$$

- \blacksquare γ periodic boundary conditions:
 - For the interfaces B_u^l and B_u^r : $\Phi(+p/2,y) = e^{-j2\pi y}\Phi(-p/2,y)$
 - For the interfaces B_d^l and B_d^r : $\begin{cases} \mathbf{u}(+p/2,y) = e^{-j2\pi\gamma}\mathbf{u}(-p/2,y) \\ \Phi(+p/2,y) = e^{-j2\pi\gamma}\Phi(-p/2,y) \end{cases}$

Physical model – Weak formulation

Finds (\mathbf{u},ϕ) in $V_{\gamma}^{3}(\Omega_{p}\cup\Omega_{e})\times V_{\gamma}^{3}(\Omega)$ (satisfying $\phi=1$ in the electrode) such that for all (\mathbf{v},ψ) in $V_{\gamma}^{3}(\Omega_{p}\cup\Omega_{e})\times V_{\gamma}^{3}(\Omega)$ (satisfying $\phi=0$ in the electrode)

$$\int_{\Omega_{p} \cup \Omega_{E}} \overline{\mathbf{S}}(\mathbf{v}) \cdot \mathbf{T}(\mathbf{u}) d\Omega - \rho \omega^{2} \int_{\Omega_{p} \cup \Omega_{E}} \overline{\mathbf{v}} \cdot \mathbf{u} d\Omega
- \int_{\Omega_{p} \cup \Omega_{E} \cup \Omega_{D}} \overline{\mathbf{E}}(\psi) \cdot (\mathbf{eS}(u) + \varepsilon \mathbf{E}(\phi)) d\Omega
= \int_{B_{d}} \overline{\mathbf{v}} \cdot (\mathbf{T} \cdot \mathbf{n}) d\Gamma + \int_{B_{u} \cup B_{d}} \overline{\psi}(\mathbf{D}(\phi) \cdot \mathbf{n}) d\Gamma$$

 $V_{\scriptscriptstyle\gamma}(\Omega)$ is the mathematical space of $L^2\left(\Omega\right)$ satisfying γ -harmonic periodic boundary conditions

Numerical model

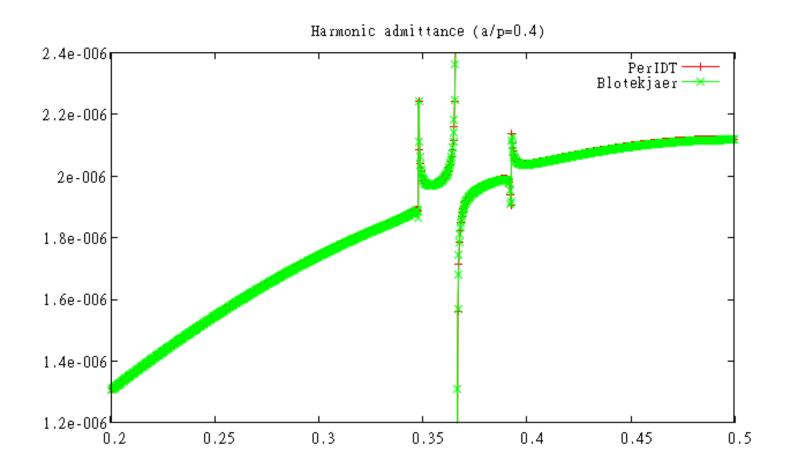
- Incorporates γ periodic boundary conditions in the variational formulation:
- The idea is to transform a γ periodic problem into a periodic problem using the relationship:

$$u(x,y) = \varphi_{\gamma}(x) \varphi(x,y) \qquad \varphi_{\gamma}(x) = e^{-j2\pi\gamma \frac{x}{p}}$$
Periodic function

Modify the variational formulation which allows to use only the periodic option in FreeFem++

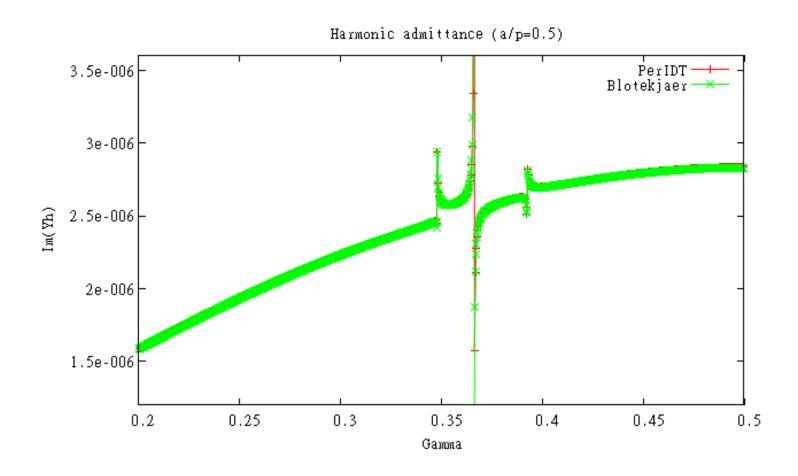
First Results

 Harmonic admittance computation of buried IDT (comparison with published analytical models)



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 Harmonic admittance computation of buried IDT (comparison with published analytical models)



Include temperature dependence

■ Taylor expansion of the piezoelectric elastic tensor and mass density around nominal temperature:

$$\mathbf{C}_{p}^{\mathbf{E}} = \mathbf{C}_{0} \left(1 + \alpha_{1}^{\mathbf{C}} \left(T - T_{0} \right) + \alpha_{2}^{\mathbf{C}} \left(T - T_{0} \right)^{2} + \alpha_{3}^{\mathbf{C}} \left(T - T_{0} \right)^{3} \right)$$

$$\rho = \rho_{0} \left(1 + \alpha_{1}^{\rho} \left(T - T_{0} \right) + \alpha_{2}^{\rho} \left(T - T_{0} \right)^{2} + \alpha_{3}^{\rho} \left(T - T_{0} \right)^{3} \right)$$

Piezoelectric substrate expansion:

$$L_{S} = 1 + \alpha_{1}^{L_{S}} (T - T_{0}) + \alpha_{2}^{L_{S}} (T - T_{0})^{2} + \alpha_{3}^{L_{S}} (T - T_{0})^{3}$$

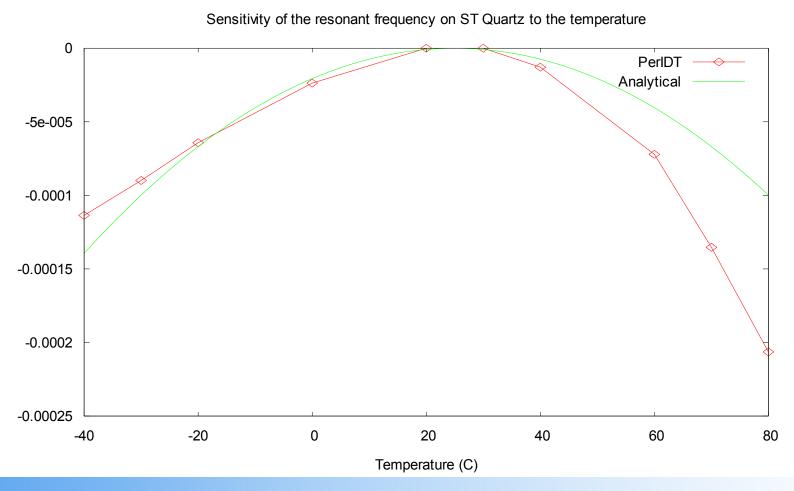
Temperature effect on the Young's modulus and Poisson's ratio $\begin{cases} \frac{dE}{dT} = \alpha_E = -0.0375 \cdot 10^9 \,\text{Pa/}^{\circ}\text{C} \\ \frac{d\mu}{dT} = \alpha_{\mu} = -0.0149 \cdot 10^9 \,\text{Pa/}^{\circ}\text{C} \end{cases}$ (first order

$$\frac{d\mu}{dT} = \alpha_{\mu} = -0.0149 \cdot 10^9 \,\text{Pa/°C}$$

- Electrode width follows the substrate expansion
- Electrode thickness follows the metal expansion $L_m = 1 + \alpha_1^{L_m} (T T_0)$

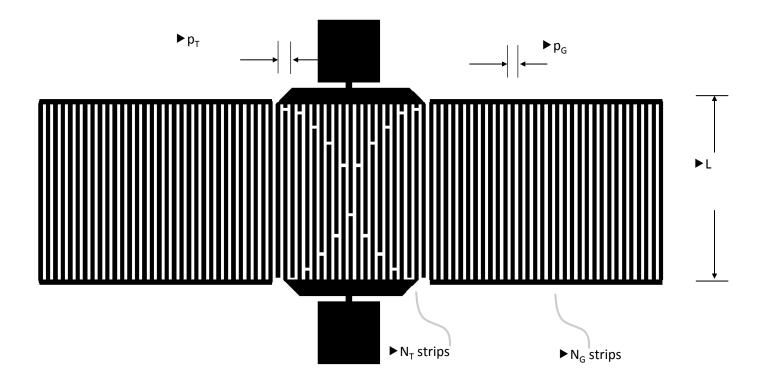
Numerical results

Quartz cut turnover temperature computations



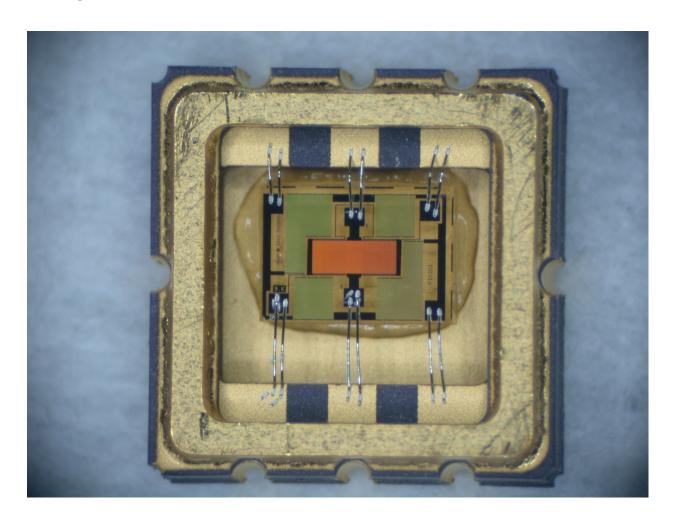
Experimental results

■ Schematic of a one port STW Single port resonator



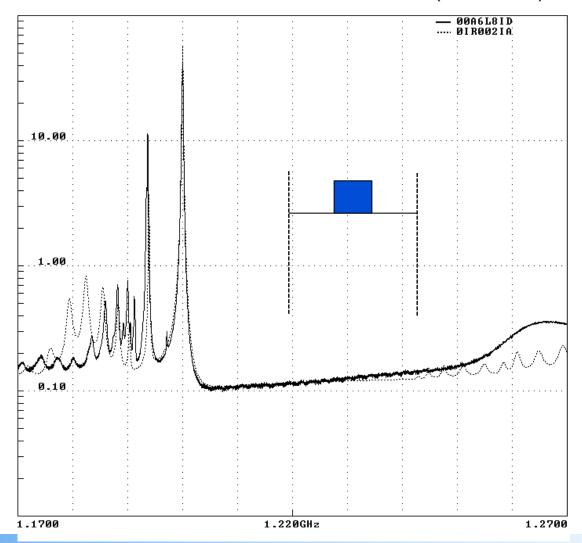
Experimental results

■ Actual packaged STW resonator



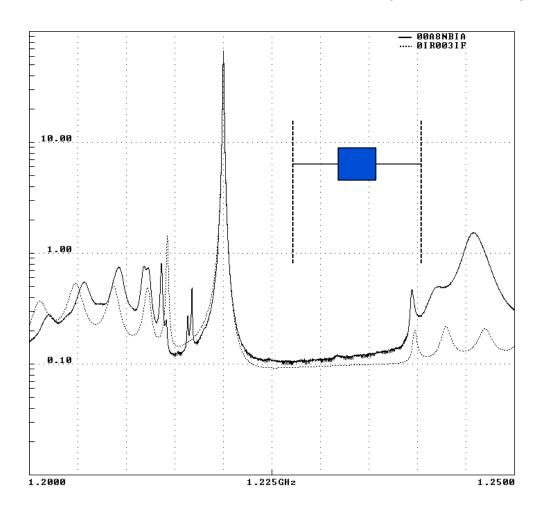
Numerical results

■ Conductance of a non buried STW resonator (Q=6300)



Numerical results

■ Conductance of a buried STW resonator (Q = 10700)



Conclusions

- A periodic Analysis of Surface Acoustic Waves Transducer has been developed using FreeFem++
- The variational formulation incorporates:
 - The Green's function for the Piezoelectric semi-space
 - \circ The γ periodic boundary conditions
- The temperature dependence has been included using simplified assumptions
- Partially buried electrode STW resonators with improved Q have been developed with the aid of the mixed FEM/BEM numerical model PerIDT

Future works

- Develop a 3D periodic model using FreeFem++ 3D to take into account transverse wave guiding effects
- code parallelization