Resolution of PDE using domain decomposition methods on TeraFLOPic architectures

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Third workshop on `FreeFem++`

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Outline

Context

We want to solve large systems arising from the finite element method.

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What are the different alternatives ?

- parallel direct solvers (`MUMPS`, `SuperLU` ..),
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$\Longrightarrow$ high-performance algorithms on massively parallel distributed memory multiprocessor architectures.

Within FreeFem++

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C++ plus load function within FreeFem++:
$\Longrightarrow$ OpenMP (shared memory architectures), C for CUDA (GPGPU).

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`FreeFem++` is working on the following parallel architectures (amongst others):

	N° of cores	Memory	Peak performance	Compilers
hpc1@LJLL	64@2.00 Ghz	252 Go	$<$ 1 TFLOP/s	`Intel`
titane@CEA	12192*@2.93 Ghz	37 To	140 TFLOP/s	`Intel`
babel@IDRIS	40960@850 Mhz	20 To	139 TFLOP/s	`IBM+GNU`

* + 46080 CUDA cores

`http://www-ccrt.cea.fr`, Bruyères-le-Châtel, France.
`http://www.idris.fr`, Orsay, France.

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The original Schwarz method for a 2-way decomposition

Consider the following BVP in $\mathbb{R}^d$ ($d = 2, 3$):

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Then, solve in parallel:

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\qquad
\begin{aligned} \nabla \cdot \kappa \nabla u_2^{n+1} &= F(u_2^{n+1}) && \text{in } \Omega_2 \\ B(u_2^{n+1}) &= 0 && \text{on } \partial\Omega_2 \cap \partial\Omega \\ u_2^{n+1} &= u_1^n && \text{on } \partial\Omega_2 \cap \overline{\Omega_1} \end{aligned}$$

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   $\Longrightarrow$ the original Schwarz method is not a *scalable* algorithm.

2. Highly heterogeneous coefficient κ in the BVP,
   $\Longrightarrow$ long plateaux in the convergence of the algorithm.

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Construction of E

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2. use the low-frequency modes of the *Dirichlet-to-Neumann* operator of the BVP at the interface of each neighboring domains (Nataf *et al.* , 2011).

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Two-level methods

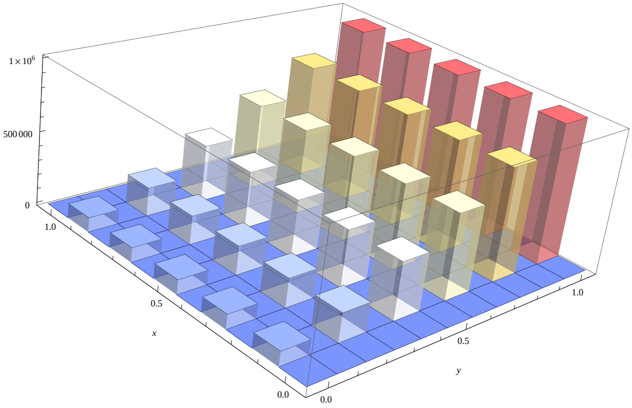
1. solve the global coarse problem $Ex = b$ on one node,
2. use the solution of the coarse problem on each local fine problem.

This is used to precondition a Krylov method (CG, GMRES) (Tang *et al.* , 2009).

Test case in $\mathbb{R}^2$

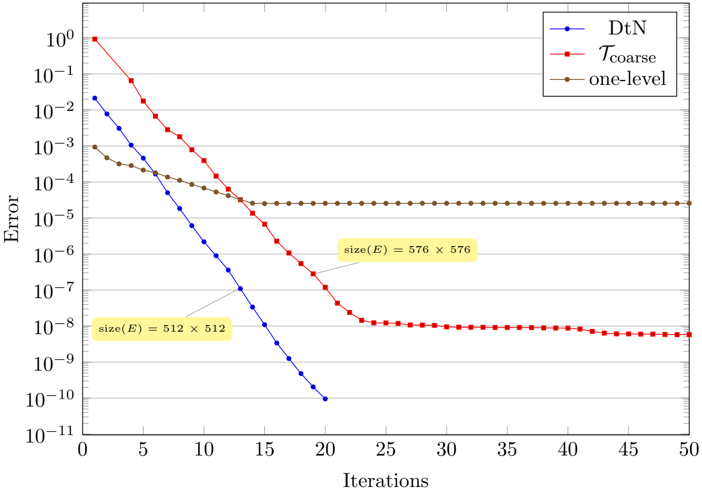
$$\begin{aligned} \nabla \cdot \kappa \nabla u &= 1 \quad \text{in } \Omega \\ u &= 0 \quad \text{on } \partial\Omega \end{aligned}$$

with a *skyscrapper* viscosity $\kappa(x, y)$:



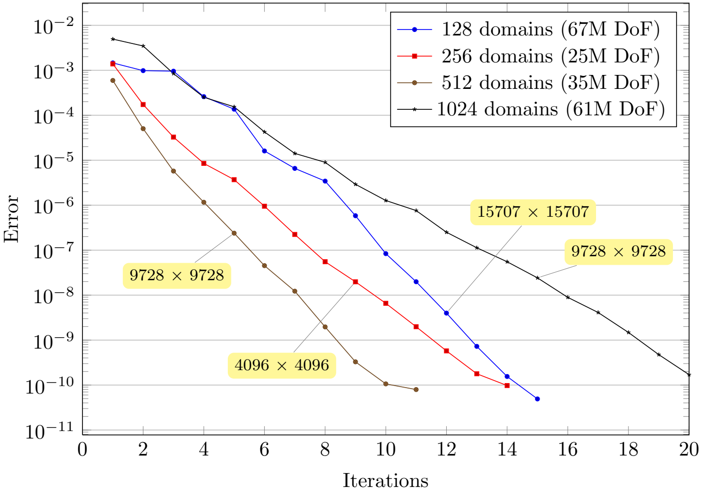
Convergence curves

2D test case running on 64 processors using $\mathbb{P}_2$ FE ($\varepsilon = 10^{-10}$, 450k DoF)



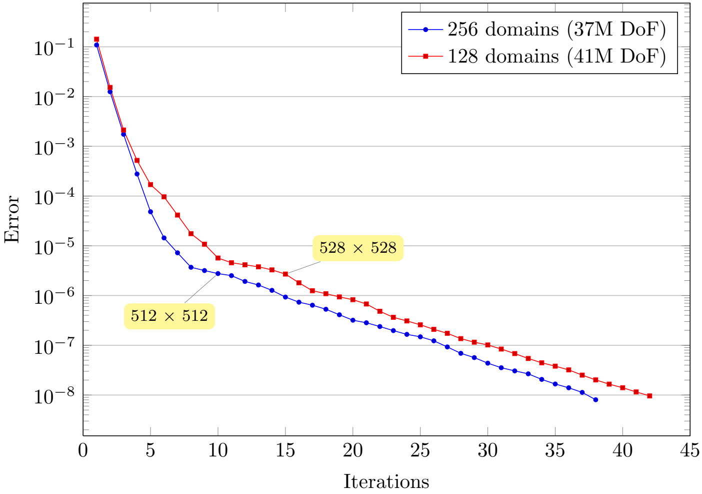
Some more 2D results

2D test case running on various number of processors using $\mathbb{P}_1$ FE ($\varepsilon = 10^{-10}$)



And some 3D results

3D test case running on various number of processors using $\mathbb{P}_2$ FE ($\varepsilon = 10^{-8}$)



Outline

Final words

Concerning `FreeFem++`:

1. efficient parallel algorithms (memory and CPU overhead with the `FreeFem++` - `MPI` bindings),
2. not very user-friendly interface at the moment (for DDM),
3. nice research tools (coarse operators, preconditionners ..).

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Concerning domain decomposition methods:

1. add another level,
2. do parallel computations on more complex systems (e.g. elasticity).

Thanks for your attention.

NATAF, F., XIANG, H., DOLEAN, V., & SPILLANE, N. 2011.
A coarse space construction based on local Dirichlet to Neumann maps, to appear.
SIAM Journal on Scientific Computing.

TANG, J.M., NABBEN, R., VUIK, C., & ERLANGGA, Y.A. 2009.
Comparison of two-level preconditioners derived from deflation, domain decomposition and multigrid methods.
Journal of Scientific Computing, **39**(3), 340–370.