Topology optimization in plasticity and quasi-brittle fracture using a level-set method on FreeFEM++

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In structural design involving steel, the plasticity model is quite often used. Using this model, one can determine the plastic strain or the permanent deformation, which occurs in the solid when it undergoes a stress that exceeds a value known as the yield strength. The plasticity model written using a variational inequation can be resolved numerically, using either the radial-return algorithm or the incremental approach [1]. We show that both of these approaches can be implemented on `FreeFEM++` in a robust manner.

Fracture mechanics plays a crucial role in the design of concrete structures. The laws governing brittle fracture, proposed by A.A. Griffith are reformulated as minimization of the total mechanical energy [2]. A damage gradient model governs the progressive degradation in solids. Such a model can be effectively used to approximate the total mechanical energy. The minimization of the total energy written in its variational form, is an inequation that can be solved using the penalization approach. We show effectiveness of the `macros` of `FreeFEM++` in the numerical implementation of the penalized inequation.

Shape or topology optimization of structures using level-sets is a well-known technique where a scalar function describing a shape implicitly is advected along a shape derivative [3]. The shape derivative can be easily computed using the Céa's technique [4].

We apply Céa's technique to the plasticity and the damage model, approximated by penalization. The resulting shape derivative demands the resolution of an adjoint problem that is shown to be well posed. The shape derivative paves the way for an iterative algorithm devoted to minimize the total compliance or the fracture energy. In our numerical implementation, the shape evolution is driven by a body-fitted discretization strategy [5]. Numerical examples in $2D$ and $3D$ will be proposed to show that the algorithm results in a shapes with minimal plastic deformation and no crack shown in the figure below.

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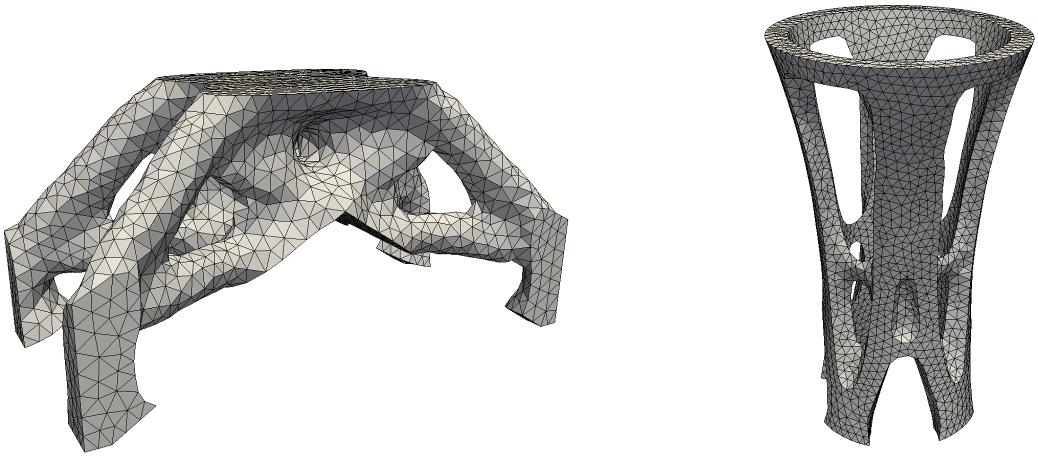

(a) Wedge optimized for plasticity (b) Column optimized for fracture energy

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