Mutual Admittances of an Infinite Periodic Surface Acoustic Waves Transducer using an Original Coupled FEM/BIE Numerical Model

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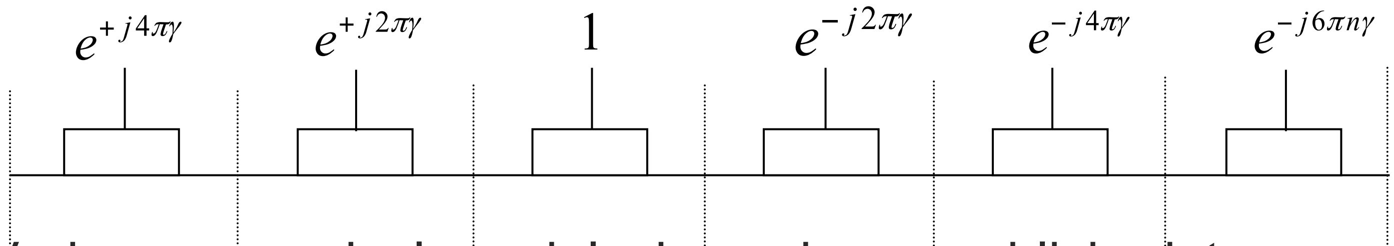
Introduction

- The mutual admittances of periodic SAW transducer can be easily computed from the harmonic admittance of the periodic array ?



$0 \leq \gamma \leq 1$

- Various numerical models have been published to compute the harmonic admittance of a periodic SAW transducer (Ballandras et alt. en 2002, Ventura et alt. en 2001, Hashimoto et alt. 2011).

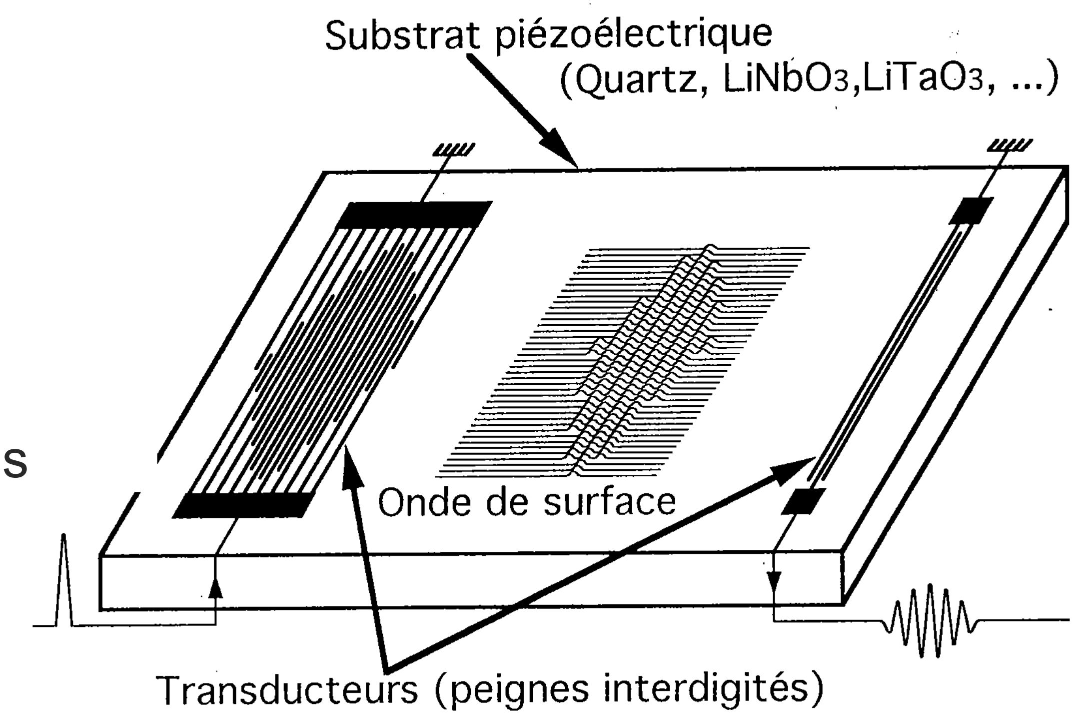
- In 2013, Hecht et alt. published an original numerical model that uses an efficient variational formulation to deal with the periodicity of the geometrical boundary conditions written using the powerful FreeFem++ environment.

- Theoretical basis of the coupled FEM/BIE model and derivation of mutual admittances will be given, numerical simulations are shown

Principles of SAW IDT device

How is built a SAW device

- Piezoelectric substrate
- SAW IDT transducer
- The Surface Acoustic Wave is
  - launched and detected
  - propagating



10 μm	2 μm	1 μm	0.5 μm	Typical Finger Width
100 MHz	500 MHz	1 GHz	2 GHz	Operating Frequency

Main dates of piezoelectricity and its applications

- 1880 : discovery of piezoelectricity(Pierre et Jacques Curie)
- 1885 : Rayleigh proved the existence of a surface acoustic wave propagating at a lower velocity than bulk acoustic waves
- 1915 : first use of piezoelectric effects in acoustic : SONAR
- 1920 : application to electronics : High quality coefficient Quartz resonators
- 1950 : fabrication of piezoelectric ceramics
- 1960 : piezoelectric thin film
- 1965 : first interdigitated SAW transducers (Hartmann patent)

Physical model

- The periodic array of electrodes is driven with an harmonic electrical potential $V_n(\gamma) = V_0 e^{-j2\pi n\gamma}$ $\gamma \in [0,1]$

  Electrode number

- Using Bloch-Floquet's theorem, it is possible to model only one period of the infinite periodic array of electrodes $\psi(x) = e^{-j2\pi\gamma x/p}\psi_p(x)$

- 2D analysis (very long electrode) : plain strain approximation

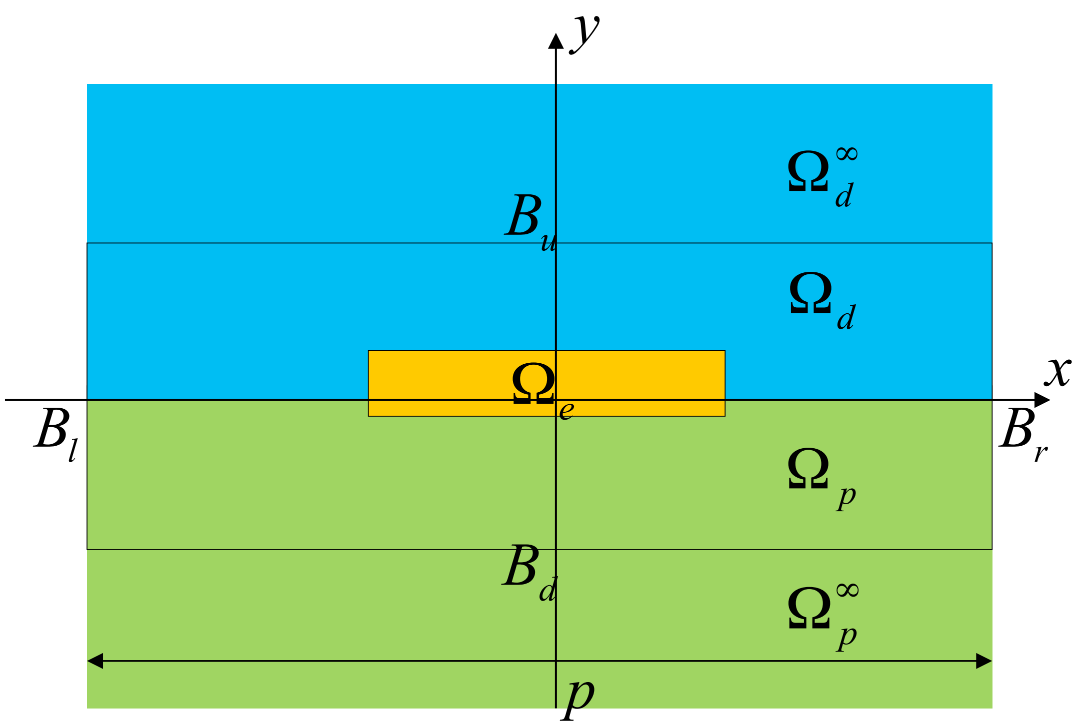
- electrical assumption: no dielectric losses in the electrode

- mechanical assumption : the metallic electrode are homogeneous isotropic, elastic materials

- Build a coupled FEM / BIE model which combines the Finite Element Model (finite geometries) and the Boundary Integral Equation (semi infinite domains)

Physical model

- The finite part of the model is bounded by the boundaries $\left(B_u, B_l, B_d, B_r\right)$
- The Green domain Ω_p is the piezoelectric substrate.
- The orange domain Ω_e is the partially buried metallic electrode.
- The blue domain Ω_d is the dielectric medium (air of vacuum) above the array.



Physical model

- Definitions:

  $\mathbf{u}$ mechanical displacement

  ρ mass density, ω pulsation

  ϕ electrical potential

  $\mathbf{E}$ electrical field, $\mathbf{D}$ electrical displacement field

  $\mathbf{T}$ stress tensor, $\mathbf{S}$ strain tensor

  $\mathbf{C}^{\mathbf{E}}$ elastic tensor for a constant electrical field

  $\mathbf{e}$ piezoelectric coupling tensor, $\boldsymbol{\varepsilon}^{\mathbf{S}}$ dielectric tensor for a constant strain

Physical model

- Differential equations:

  - The Piezoelectric domain Ω_p and the Elastic domain Ω_e obeys Newton's second law:

$$\nabla \cdot \mathbf{T} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}$$

  - The Piezoelectric domain Ω_p, the Elastic domain Ω_e, and the dielectric domain Ω_d obeys the quasistatic Maxwell's equation:

$$\nabla \cdot \mathbf{D} = 0$$

Physical model

- The constitutives equations:

In Ω_p (piezoelectric domain)

$$\begin{cases} \mathbf{T}_{ij} = \mathbf{C}^{\mathbf{E}}_{ijkl}\mathbf{S}_{kl} - \mathbf{e}_{ijk}\mathbf{E}_k \\ \mathbf{D}_i = \mathbf{e}_{kli}\mathbf{S}_{kl} + \varepsilon^{\mathbf{S}}_{ik}\mathbf{E}_k \end{cases}$$

In Ω_e (elastic domain)

$$\mathbf{T}_{ij} = (\lambda + \mu)\delta_{ij}\mathbf{S}_{kk} - 2\mu\mathbf{S}_{ij}$$

In Ω_d (dielectric domain)

$$\mathbf{D}_i = \varepsilon_{ik}\mathbf{E}_k$$

$$\mathbf{S}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad \mathbf{E}_i = -\frac{\partial \Phi}{\partial x_i}$$

Physical model

- The γ periodic boundary conditions:
  - For the interfaces B_l and $B_r\left(y>0\right)$: $\Phi\left(+p/2,y\right)=e^{-j2\pi\gamma}\Phi\left(-p/2,y\right)$
  - For the interfaces B_l and $B_r\left(y<0\right)$: $\begin{cases}\mathbf{u}\left(+p/2,y\right)=e^{-j2\pi\gamma}\mathbf{u}\left(-p/2,y\right)\\ \Phi\left(+p/2,y\right)=e^{-j2\pi\gamma}\Phi\left(-p/2,y\right)\end{cases}$

Variational Formulation

- Finds $(\mathbf{u},\phi)$ in $V_\gamma^3(\Omega_p \cup \Omega_e) \times V_\gamma^3(\Omega)$ (satisfying $\phi = 1$ in the electrode) such that for all $(\mathbf{v},\psi)$ in $V_\gamma^3(\Omega_p \cup \Omega_e) \times V_\gamma^3(\Omega)$ (satisfying $\phi = 0$ in the electrode)

$$\int_{\Omega_p \cup \Omega_E} \overline{\mathbf{S}}(\mathbf{v}) \cdot \mathbf{T}(\mathbf{u})\, d\Omega - \rho\omega^2 \int_{\Omega_p \cup \Omega_E} \overline{\mathbf{v}} \cdot \mathbf{u}\, d\Omega$$

$$-\int_{\Omega_p \cup \Omega_E \cup \Omega_D} \overline{\mathbf{E}}(\psi) \cdot \left(\mathbf{eS}(u) + \varepsilon \mathbf{E}(\phi)\right) d\Omega$$

$$= \int_{B_d} \overline{\mathbf{v}} \cdot (\mathbf{T} \cdot \mathbf{n})\, d\Gamma + \int_{B_u \cup B_d} \overline{\psi} \left(\mathbf{D}(\phi) \cdot \mathbf{n}\right) d\Gamma$$

  $V_\gamma(\Omega)$ is the mathematical space of $L^2(\Omega)$ satisfying γ-harmonic periodic boundary conditions

- It is possible to transform the weak formulation into a weak formulation with only periodic physical fields

Numerical implementation

- Numerical implementation in the powerfull FreeFem++ environment developped by F. Hecht, O. Pironneau, A. Le Hyaric, from Laboratoire Jacques Louis Lions, Université Pierre et Marie Curie in France, and O. Ohtsuka from Hiroshima university in Japan : http://www.freefem++.org/ff++/

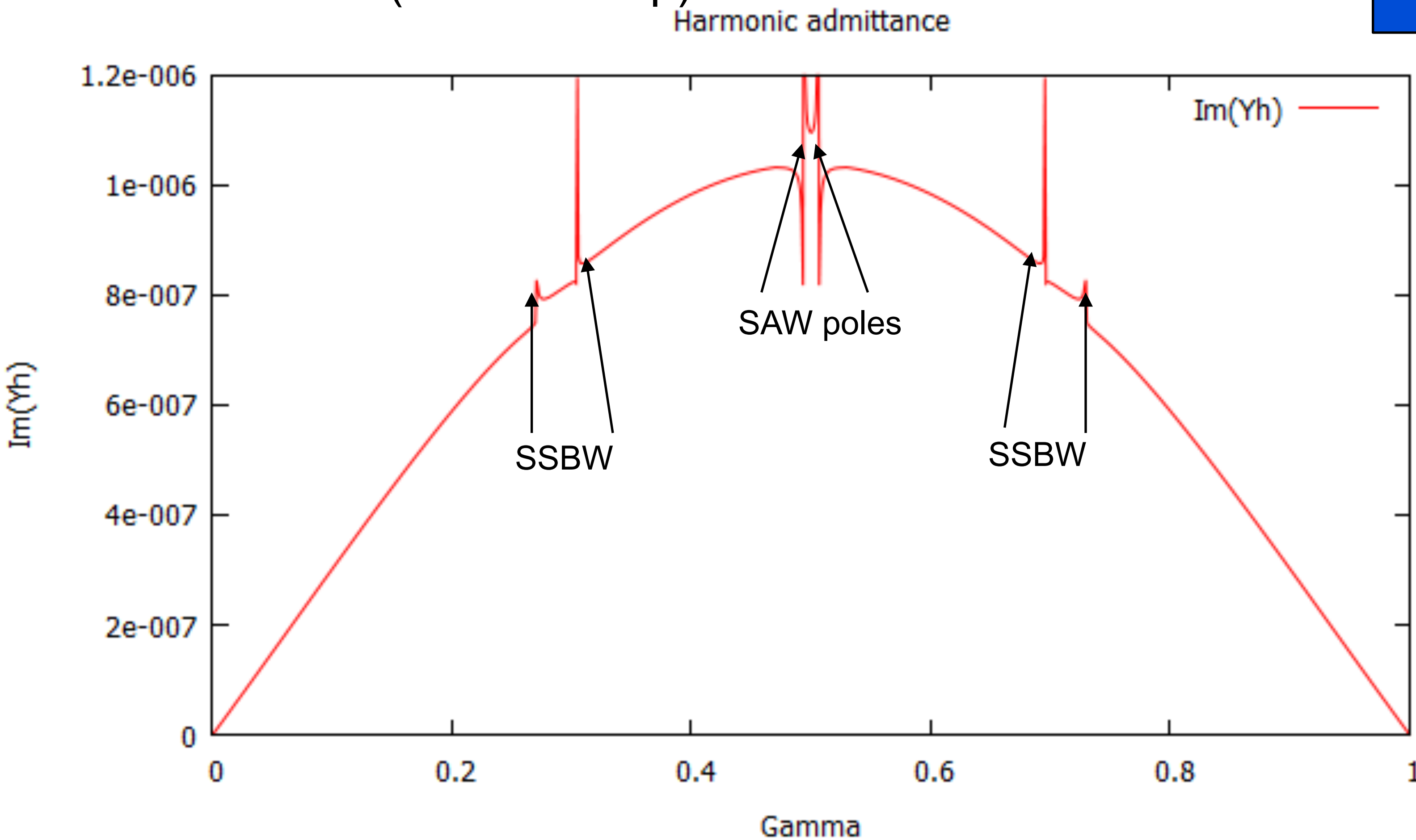
- In the variational formulation, the volume integrals are easily taken into account with FEM, special care is needed for the surface integrals (BIE part of the model) involving semi-infinite Green's function for the dielectric and the piezoelectric half spaces.

- All the mathematical details are given in:

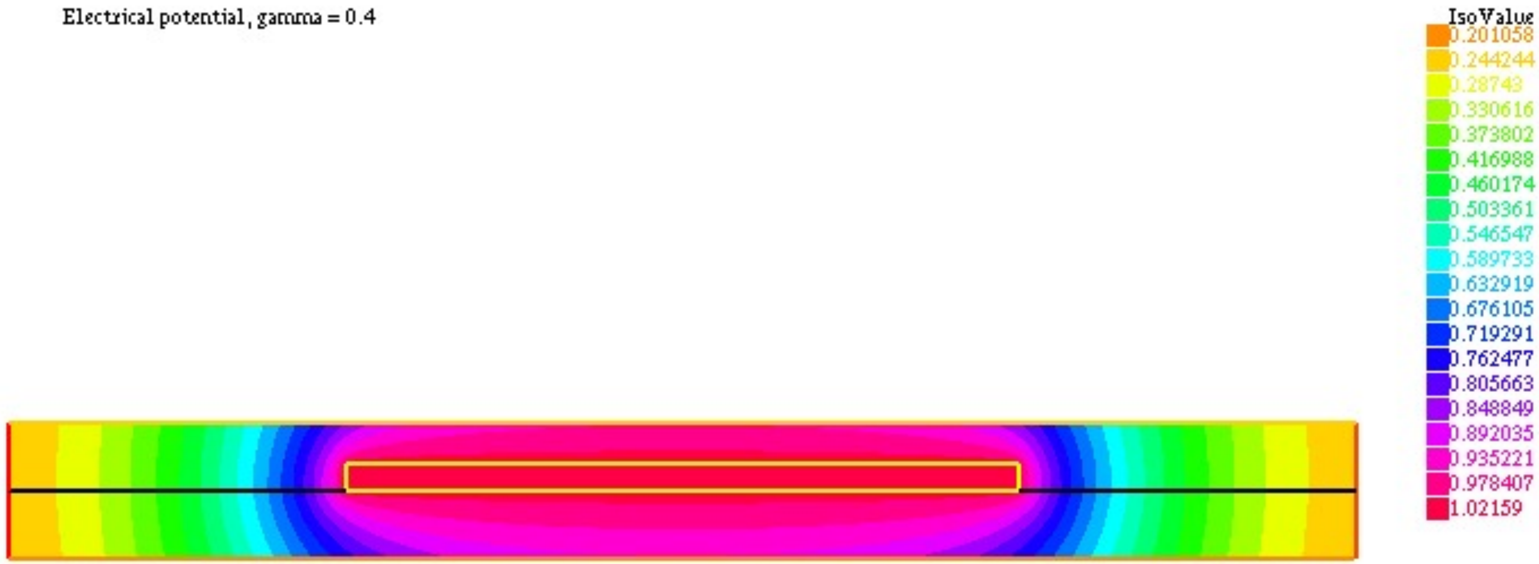
  F. Hecht, P. Ventura, and, P. Dufilié, “Original coupled FEM/BIE Numerical Model for Analyzing Infinite Periodic Surface Acoustic Wave Transducer”, Journal of Computational Physics, Vol. 246, August 2013, pp. 265-274.

Plot of the harmonic admittance

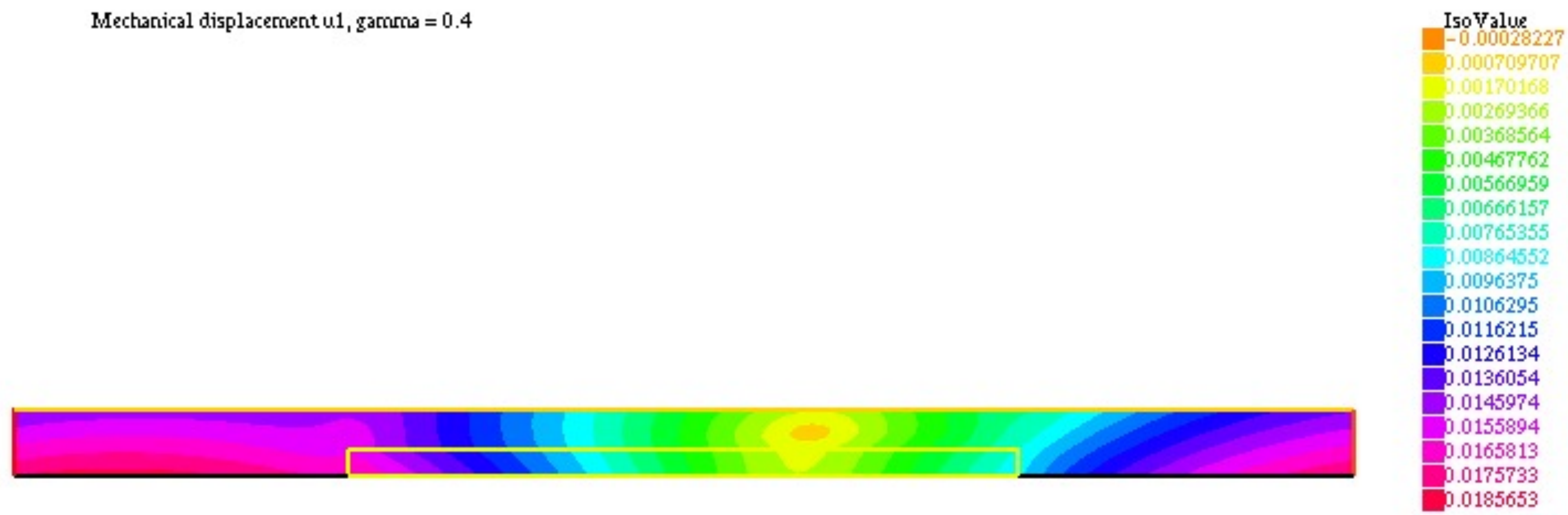
- Fixed frequency (0.499 MHz). The piezoelectric substrate is Y+39° X propagation Quartz, the period of the transducer is 3.1145 mm, the electrode are assumed rectangular with 2% buried depth and 2% metal thickness (relative to 2p).



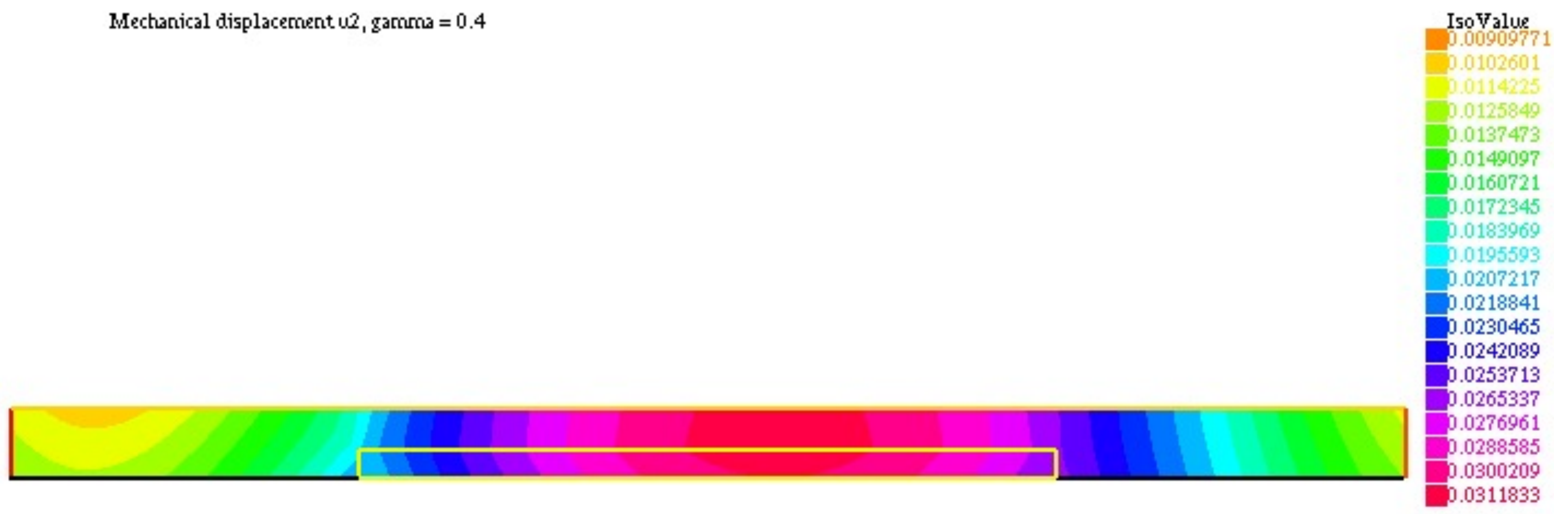
Plot of the electrical field



Plot of u1


Mechanical displacement u1, gamma = 0.4
IsoValue
-0.00028227
0.000709707
0.00170168
0.00269366
0.00368564
0.00467762
0.00566959
0.00666157
0.00765355
0.00864552
0.0096375
0.0106295
0.0116215
0.0126134
0.0136054
0.0145974
0.0155894
0.0165813
0.0175733
0.0185653

Plot of u2


Mechanical displacement u2, gamma = 0.4
IsoValue
0.00909771
0.0102601
0.0114225
0.0125849
0.0137473
0.0149097
0.0160721
0.0172345
0.0183969
0.0195593
0.0207217
0.0218841
0.0230465
0.0242089
0.0253713
0.0265337
0.0276961
0.0288585
0.0300209
0.0311833

Mutual Admittance

- The mutual admittance Y_m is related to the harmonic admittance $Y(\gamma)$

$$Y_m = \int_0^1 Y(\gamma) e^{j2\pi m\gamma} d\gamma$$

- because of very fast variation of $Y(\gamma)$ close to the SAW pole, a special mathematical treatment is needed.

- SAW contribution to the mutual admittance:

$$\begin{cases} Y_{SC} \exp\left(-j|m|\phi_{SC}\right) \text{ for } m \neq 0 \\ Y_{SC}\left(1 + j/\tan\left(\phi_{SC}/2\right)\right) \text{ for } m = 0 \quad \text{(to satisfy charge conservation)} \end{cases}$$

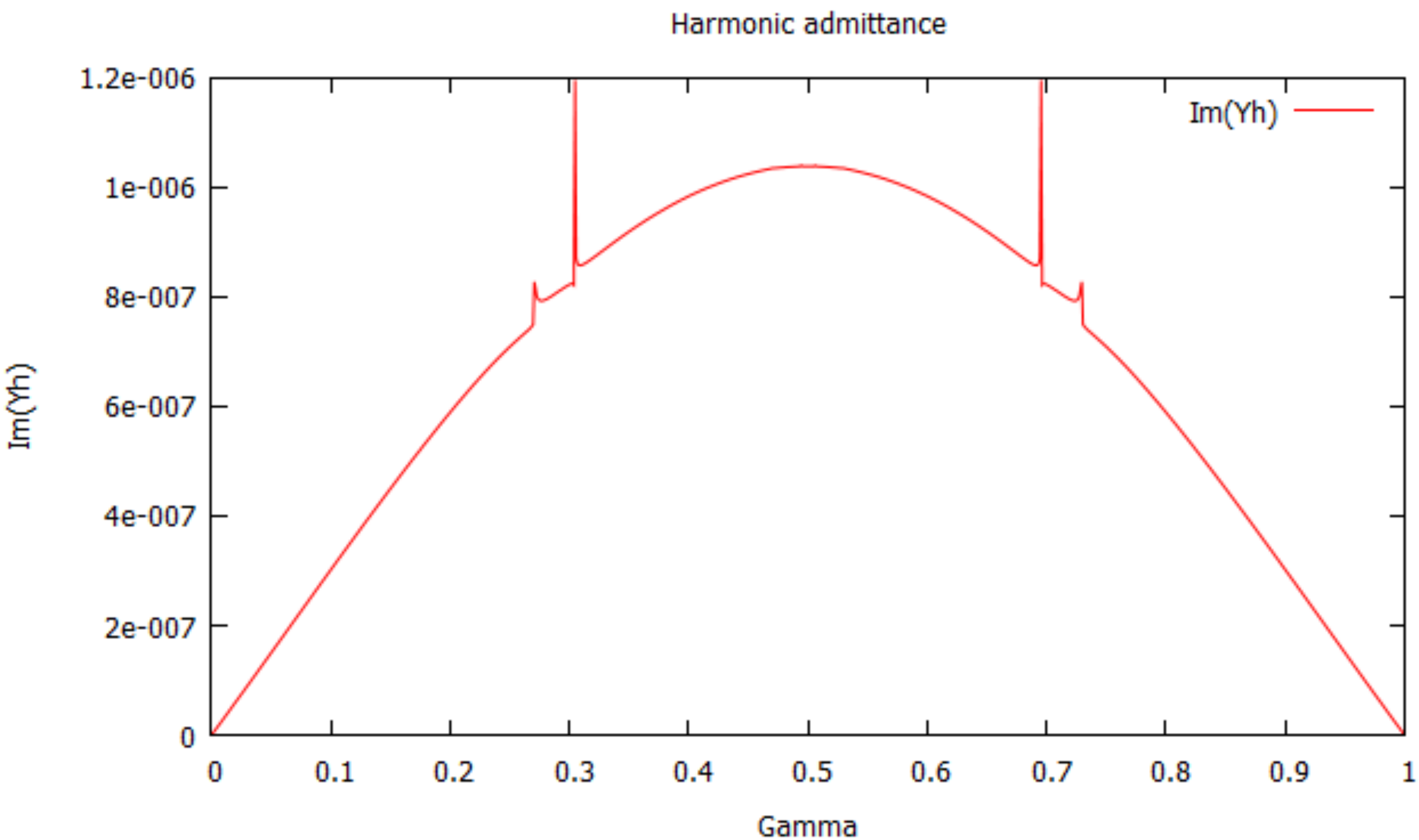
- SAW contribution to the harmonic admittance

$$Y_S(\gamma) = \frac{jY_{SC}}{\tan\left(\phi_{SC}/2\right)} \frac{1 - \cos\left(2\pi\gamma\right)}{\cos\left(\phi_{SC}\right) - \cos\left(2\pi\gamma\right)}$$

- Y_{SC} and ϕ_{SC} are computed using an iterative algorithm

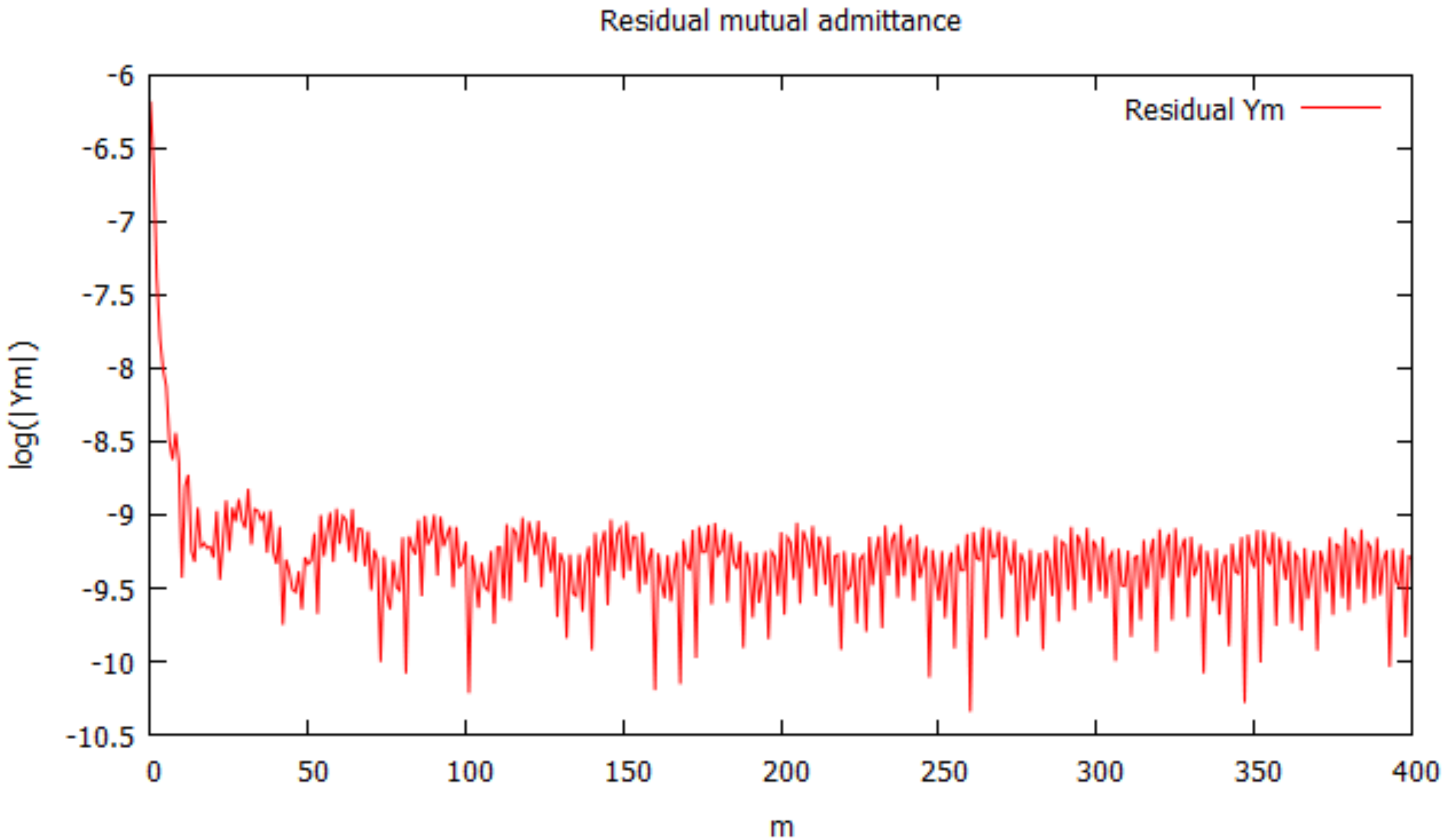
Mutual Admittance

- residual harmonic admittance



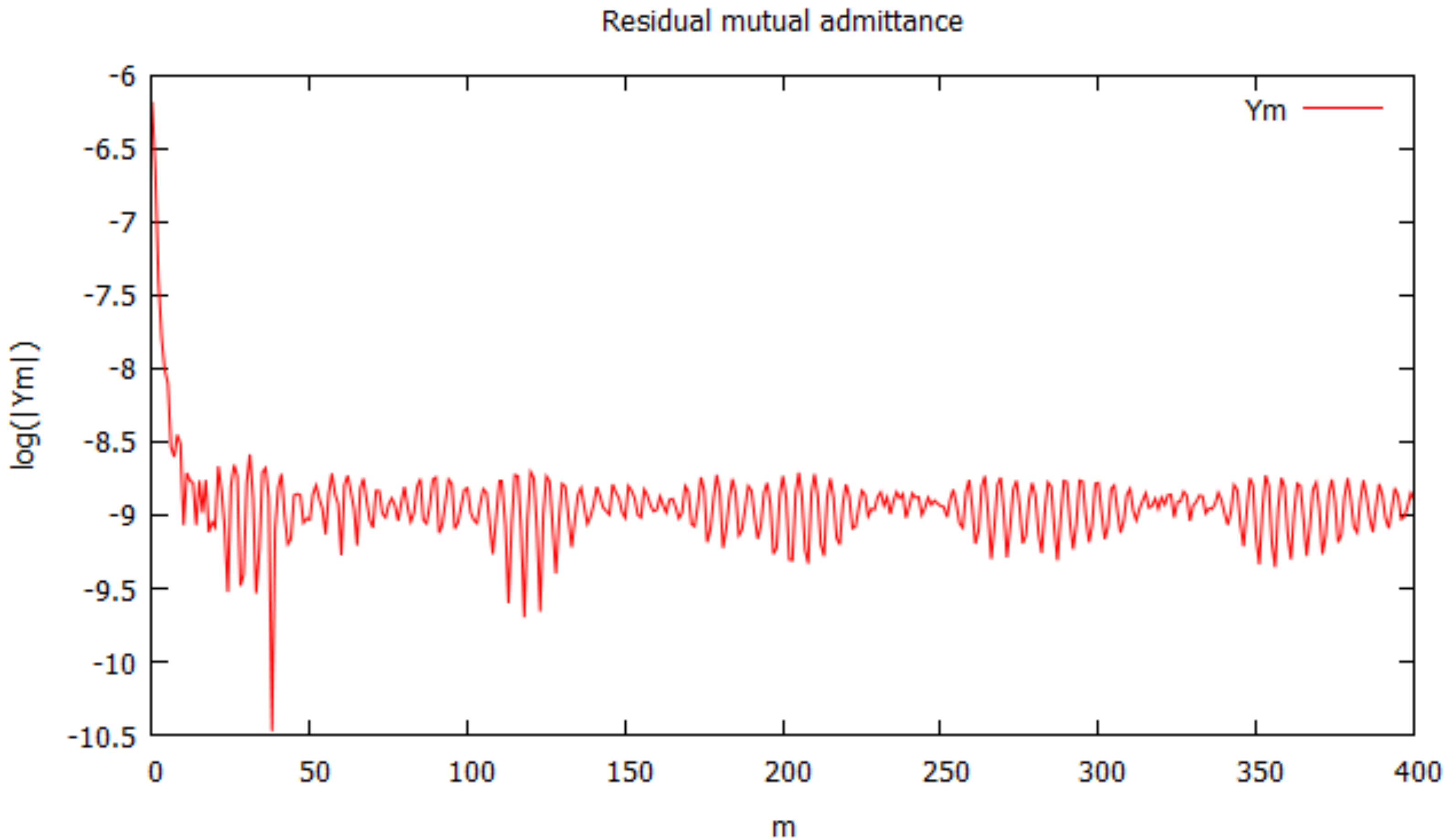
Mutual Admittance

- residual mutual admittance computed by FFT



Mutual Admittance

- mutual admittance after adding the analytical expression of the SAW contribution



Conclusions

- A new coupled FEM/BIE numerical model that computes the harmonic admittance of an infinite periodic SAW transducer has been developped in the powerful FreeFem++ environment.

- Using an efficient numerical technique to extract the SAW contribution, we have been able to compute the mutual admittance of the SAW transducer with a very good accuracy.

- This numerical technique can be generalized for various periodic physical problems (acoustics, electromagnetics, …)