2D hierarchical dielectric analysis of finite electrode array using FreeFem++

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Outlines

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Introduction

- Various numerical models have been published to compute the harmonic admittance of a periodic SAW transducer (Ballandras et alt. en 2002, Ventura et alt. en 2001, Hashimoto et alt. 2011).

- In 2013, Hecht et alt. published an original numerical model that uses an efficient variational formulation to deal with the periodicity of the geometrical boundary conditions written using the powerful FreeFem++ environment.

- It has been able to derive the P-matrix parameters needed by SAW designers to analyze finite SAW transducers.

- Very good comparison between numerical simulations and experimental values for SAW resonators have been obtained.

Introduction

- Recently, in 2018, J. Koskela et alt. recently published a new technique which is very efficient to simulate finite SAW transducer based on a small set of elementary cells.

- This new technique consists in sub structuring each elementary cell, and using a tree concatenation to analyze the whole transducer. Perfectly Matched Layer technique is employed to account for semi-infinite piezoelectric and dielectric media.

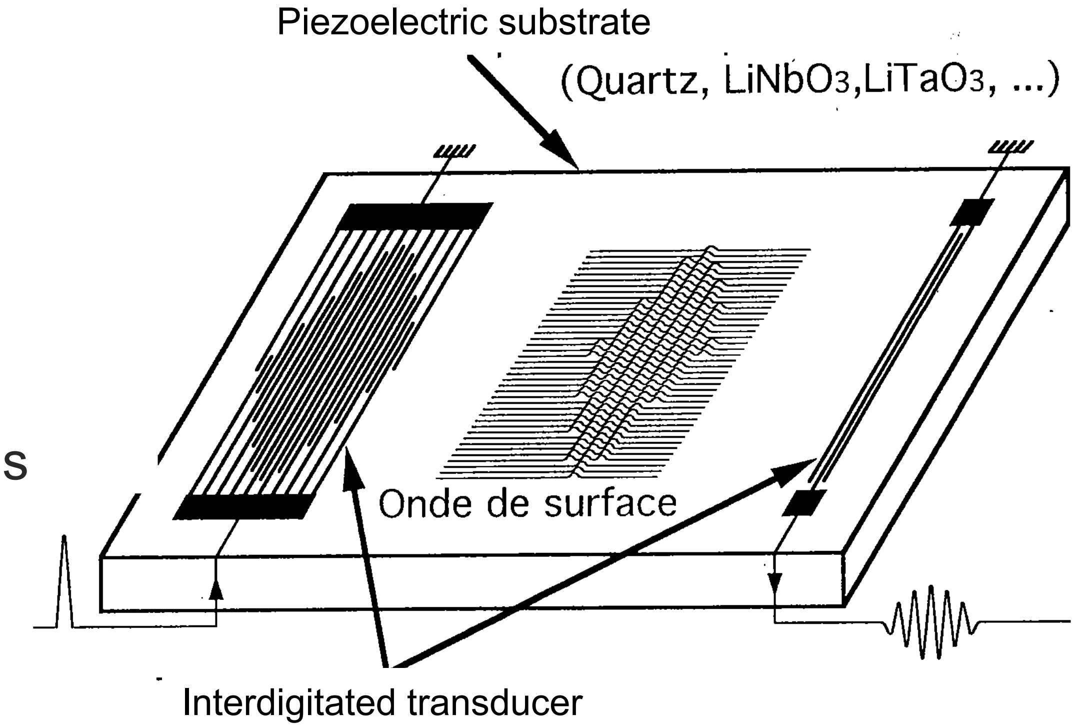
- As a first step, a 2D finite transducer FreeFem++ script has been developed that considers only dielectric effects. Static PML (1998, Bardi et alt.) has been used.

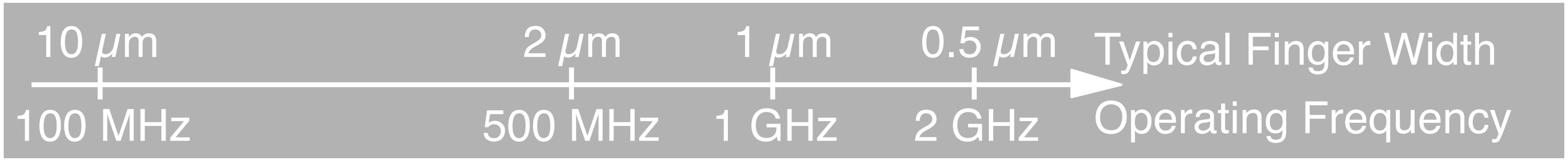
- Validations with a Green’s function software will be shown.

Principles of SAW IDT components

How is built a SAW device

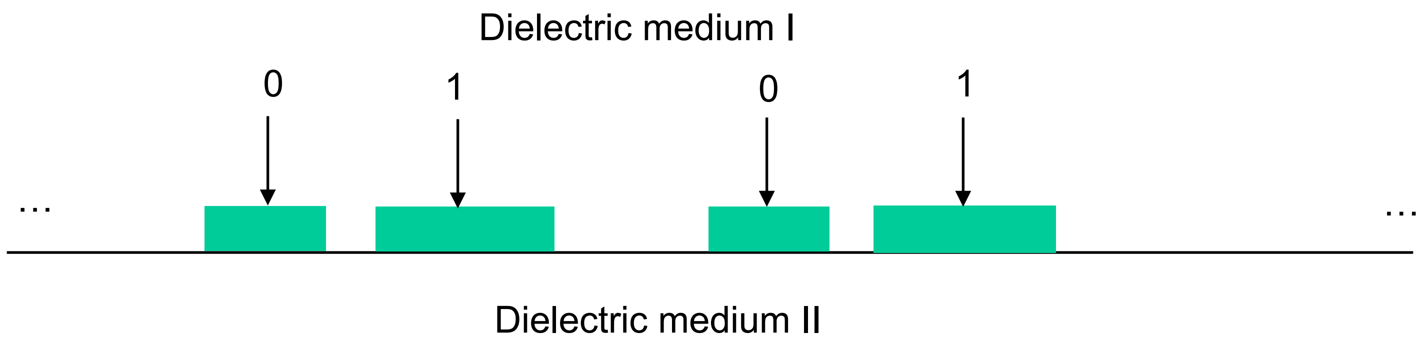
- Piezoelectric substrate
- SAW IDT transducer
- The Surface Acoustic Wave is
  - launched and detected
  - propagating





Physical model

- Finite electrode array based on a few basic cells which will be concatenaded



- Simplified assumptions:
  - Isotropic dielectric media are assumed above and below the electrodes.
  - 2D analysis is assumed (very long electrodes).
  - no dielectric losses in the electrode, and the thickness of the electrodes is neglected.

Physical model

- Notations:
  - ϕ electrical potential
  - $\mathbf{E}$ electrical field, $\mathbf{D}$ electrical displacement field
  - $\boldsymbol{\varepsilon}$ isotropic dielectric tensor

- Differential equations:
  - Both dielectric media obeys the Maxwell's quasistatic equation:

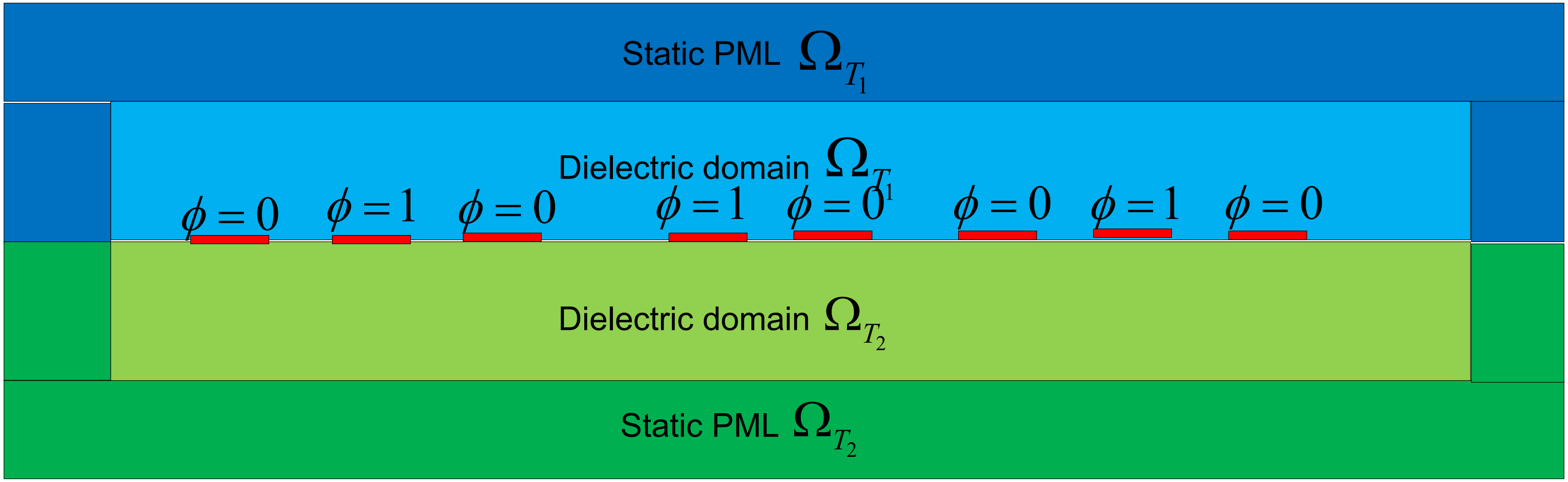
$$\nabla \cdot \mathbf{D} = 0$$

- Constitutive equations:

In the dielectric domains

$$\mathbf{D} = \boldsymbol{\varepsilon}\mathbf{E} \qquad\qquad \mathbf{E} = -\nabla\Phi$$

Variational formulation for the array of electrodes



The variational formulation for the whole array of electrodes $\Omega_T = \Omega_{T_1} \cup \Omega_{T_2}$

Finds ϕ in $V(\Omega_T)$), such that for all ψ in $V(\Omega_T)$

$$-\int_{\Omega_T} \mathbf{E}(\psi)\cdot\varepsilon\mathbf{E}(\phi)\,d\Omega = 0$$

$V(\Omega_T)$ is the mathematical space $L^2(\Omega_T)$

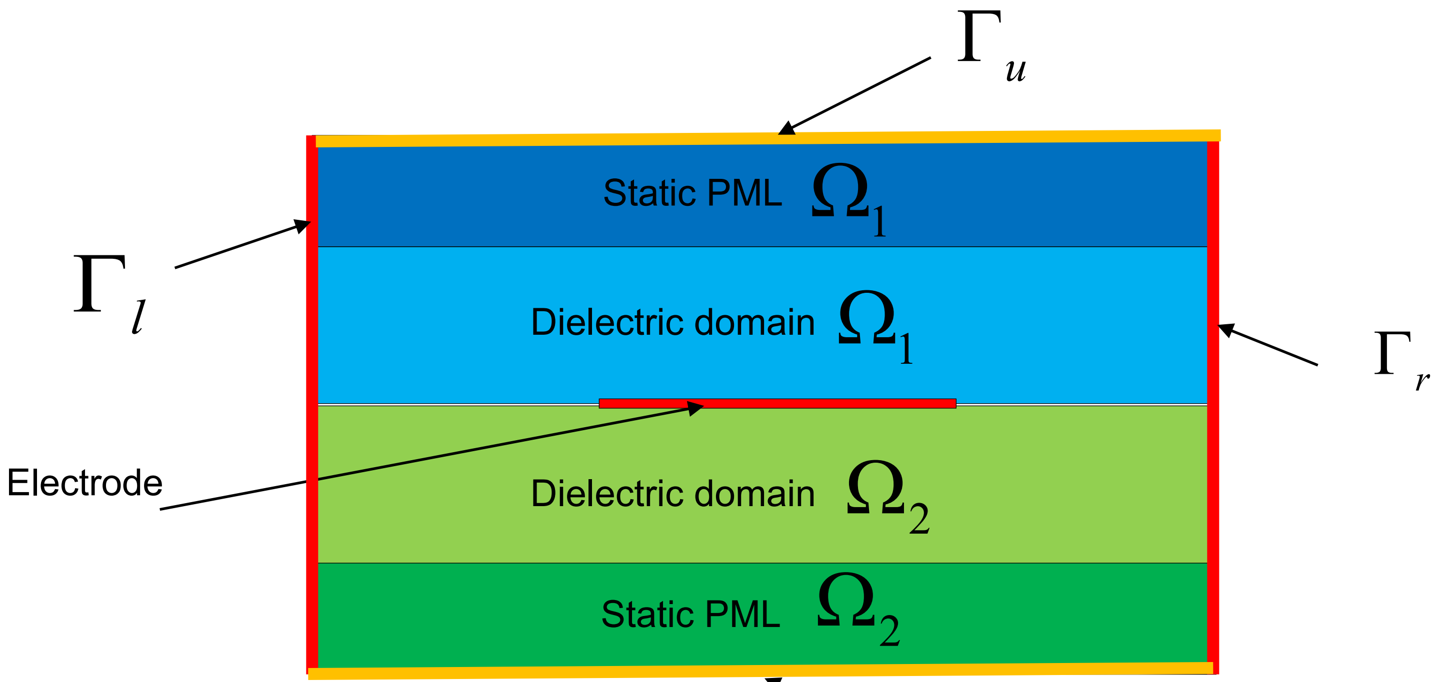
add Dirichlet boundary conditions at the boundary of each electrode (applied voltage)

Hierarchical numerical model

- The basic steps:
  - Step 1: write the variational formulation for each elementary cell (contains only one electrode) and compute the FEM stiffness matrix.
  - Step 2: remove internal degrees of freedom, keeping the electrical potential degrees of freedom of the lateral boundaries and of the electrode (assumed equipotential).
  - Step 3: apply the needed concatenations required to simulate the whole transducer.
  - Step 4: apply Dirichlet boundary conditions given by the applied voltage to the electrical port.
  - Step 5: compute the charge distribution at the boundaries of the electrodes.

Hierarchical numerical model

- Step 1: define an elementary cell



$$\Omega = \Omega_1 \cup \Omega_2$$

Hierarchical numerical model

- Step 1: write the variational formulation of the elementary cell Ω

Finds ϕ in $V(\Omega)$), such that for all ψ in $V(\Omega)$

$$-\int_{\Omega} \mathbf{E}(\psi)\cdot\varepsilon\mathbf{E}(\phi)\,d\Omega = \int_{\Gamma_l\cup\Gamma_r} \psi\left(\mathbf{D}(\phi)\cdot\mathbf{n}\right)d\Gamma$$

$V(\Omega)$ is the mathematical space $L^2(\Omega)$

add Dirichlet boundary conditions at the boundary of the electrode

- Compute the FEM matrix: $[\mathbf{K}]$

No need to consider the second member (removed in the concatenations processes)

Hierarchical numerical model

- Step 2: sort on one part the internal potential degrees of freedom (index i) and on the other part left and right boundaries and the electrode potential degree of freedom (index e). Of course, only one degree of freedom for the electrode potential is kept.

$$\left[\tilde{\mathbf{K}}\right] = \begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{ie} \\ \mathbf{K}_{ei} & \mathbf{K}_{ee} \end{bmatrix}$$

In order to remove internal degrees of freedom, compute the Schur Complement :

$$[\mathbf{B}] = -[\mathbf{K}_{ii}]^{-1}[\mathbf{K}_{ie}] \qquad [\mathbf{S}] = [\mathbf{K}_{ee}] - [\mathbf{K}_{ei}][\mathbf{K}_{ii}]^{-1}[\mathbf{K}_{ie}]$$

$$\{\mathbf{u}_i\} = [\mathbf{B}]\{\mathbf{u}_e\}$$

Hierarchical numerical model

- Step 3: concatenate a left and a right cell
  the boundary belonging to both cells is called middle boundary

$\left[\mathbf{S}_l\right]$ and $\left[\mathbf{S}_r\right]$ are the $\left[\mathbf{S}\right]$ matrix of left and right cells.

derive $\left[\mathbf{S}_{l\cup r}\right]$ matrix with all degrees of freedom of the whole cell, considering the common degrees of the freedom of the common middle boundary and the connected electrical ports.

Remove the degrees of freedom of the middle boundary with a Schur-Complement as previously:

New $\left[\mathbf{S}_c\right]$ and $\left[\mathbf{B}_c\right]$ matrices are obtained for the concatenated cell

A $\left[\mathbf{M}\right]$ matrix is also derived to be able to compute internal degrees of freedom of left and right cell knowing the remaining degrees of the freedom of the concatenated cell.

Hierarchical numerical model

- Step 4: at the end of the concatenation processes only the electrical port degrees of freedom are remaining.

  Apply Dirichlet boundary conditions on the electrical ports 1, …, N

$$\{\mathbf{\Phi}\} = [\mathbf{M}]\begin{Bmatrix} V_1 \\ \vdots \\ V_N \end{Bmatrix}$$

Potential degrees of freedom on the whole electrode array

Applied electrical potential

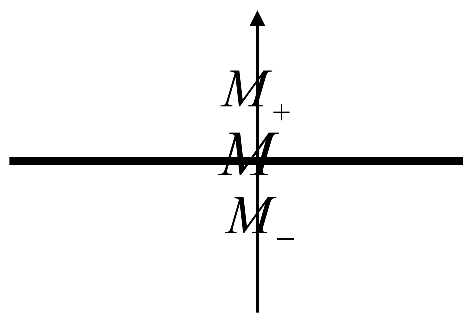
Hierarchical numerical model

- Step 5: the charge distribution at the boundaries of the electrodes is the jump $[\mathbf{D}\cdot\mathbf{n}]$ when crossing the boundary.

once the electrical potential is known, it is possible to compute the jump at a given point M of the electrode boundary :

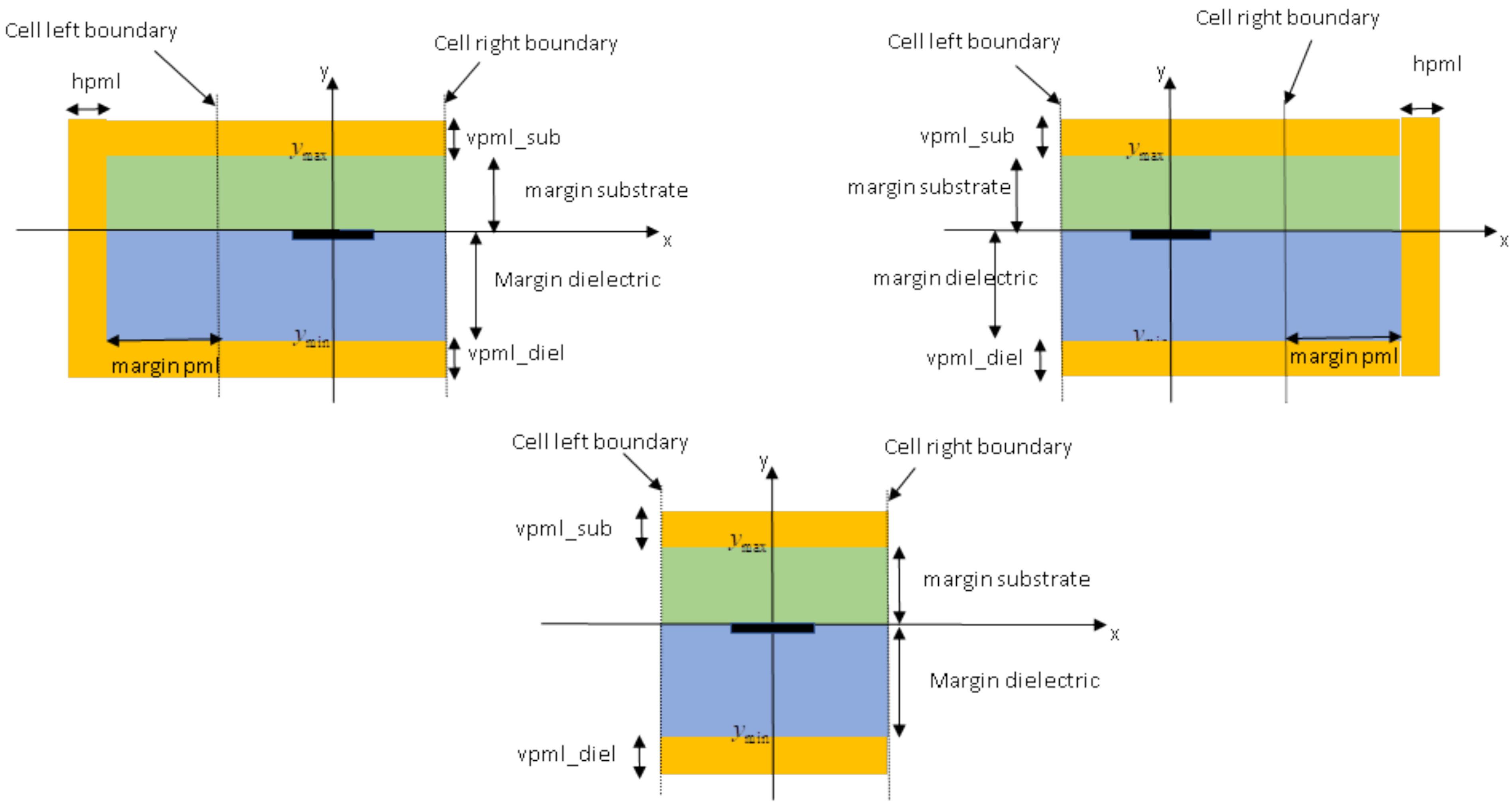
$$[\mathbf{D}\cdot\mathbf{n}](M) = -\boldsymbol{\varepsilon}\nabla\phi(M_+) + \boldsymbol{\varepsilon}\nabla\phi(M_-)$$

(M_+ and M_- are opposite points on the normal direction, very close to M)



FreeFem++ code (input file)

- Three kind of elementary cell: left boundary (type 0), intermediate (type 1) and right boundary (type 2).



FreeFem++ code (input file)

- Example of transducer [1 1 0 1 1]: the input file

  - elementary cell geometry and margin are given (first part of the input file):

```
3 1 0.4 1.0 1.0 0.6 0.000 0.6      # Number of elementary cells, and margins
50.1 50.1                          # epsrx epsry  (substrate)
0 0 0.229 0.5 0.50 -0.0004 0.      # [cell_no] [left boundary cell] p x/p a/p hei hb
1 2 0.229 0.5 0.50 -0.0004 0.      # [cell_no] [right boundary cell] p x/p a/p hei hb
2 1 0.229 0.5 0.50 -0.0004 0.      # [cell_no] [intermediate cell] p x/p a/p hei hb
```

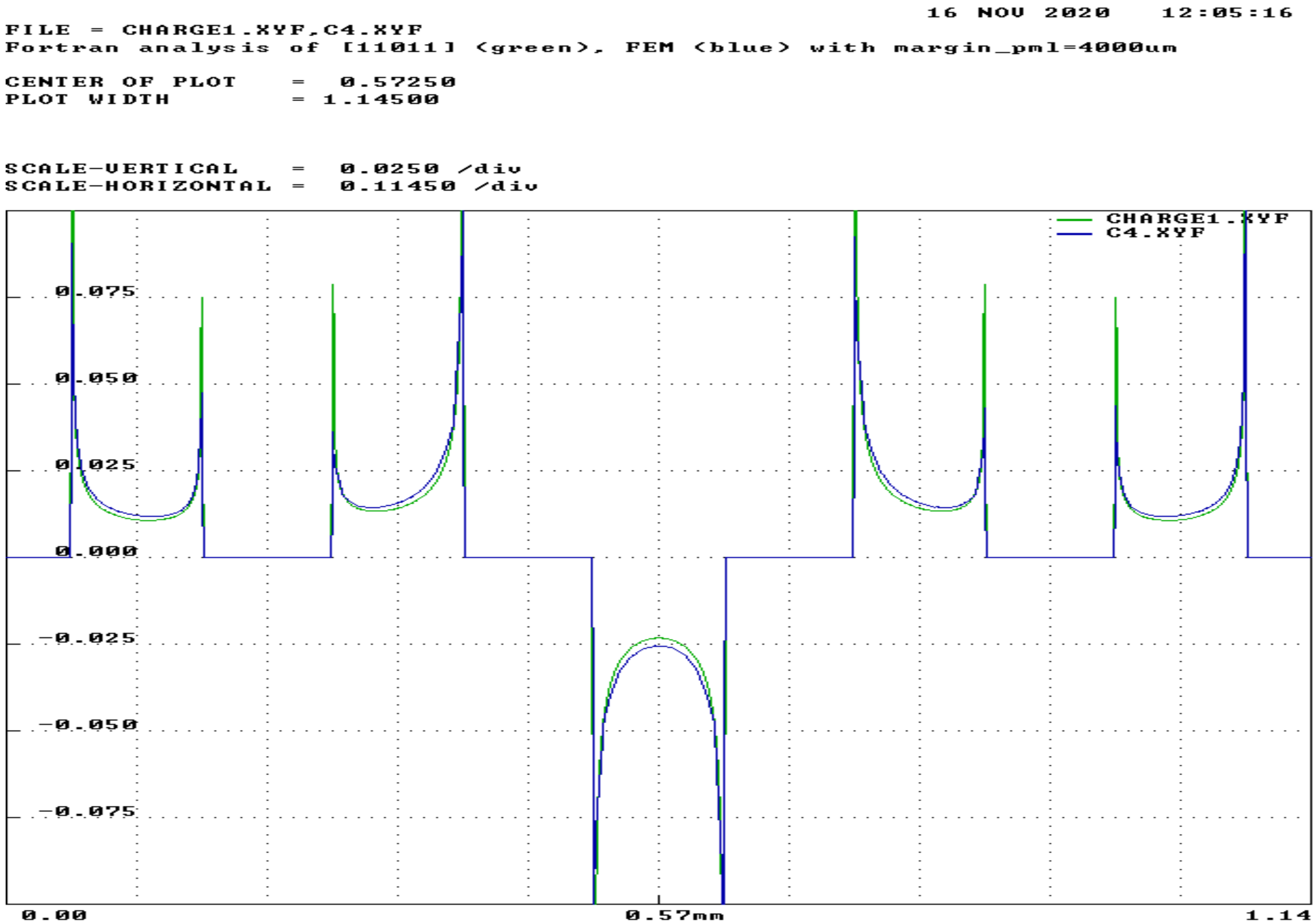
  - description of each concatenation (second part of the input file):

An intermediate cell is concatenated to the right boundary cell, the electrical port 0 of both cells are connected to the electrical port 0 of the concatenated cell.

```
2 1 3   #  3 concat(2,1)
1       # number of ports of the concatenated cells
0  0    #

…
```

Validations using a Green's function code



Charge distribution for a 5 strip transducer with ++-++ polarity sequence and margin_pml = 4.00mm . FEM analysis is the blue curve

Conclusions

- A new hierarchical numerical model has been developed successfully in the FreeFem++ environment to simulate 2D finite array of electrodes within dielectric media.

- The electrical charge distribution has been compared with a Green's function computation and give excellent fits.

- For the future : generalize the hierarchical numerical code to deal first 2D Surface Acoustic Waves transducers, followed with a 3D simulation.