

NUMERICAL MODELING OF AIR FLOWS IN AN UNDERGROUND CAVITY CONNECTED TO THE SURFACE BY A SHAFT

Georges Edde

ENDSUM-Rouen team, CEREMA - Normandy
LMRS, University of Rouen Normandy

Supervised by :
Dr. Raphaël ANTOINE, Pr. Ionut DANAILA, Dr. Georges SADAKA

PLAN

GEOPHYSICAL INTRODUCTION: FRAMING OUR STUDY

MATHEMATICAL MODEL FOR NATURAL CONVECTION

APPLICATION TO THE BARCQ CAVITY

CONCLUSION AND PERSPECTIVES

PLAN

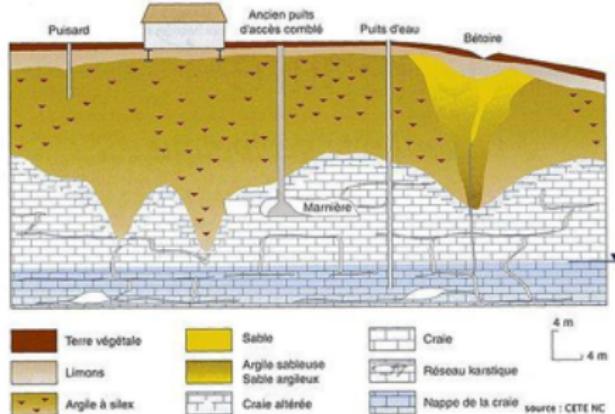
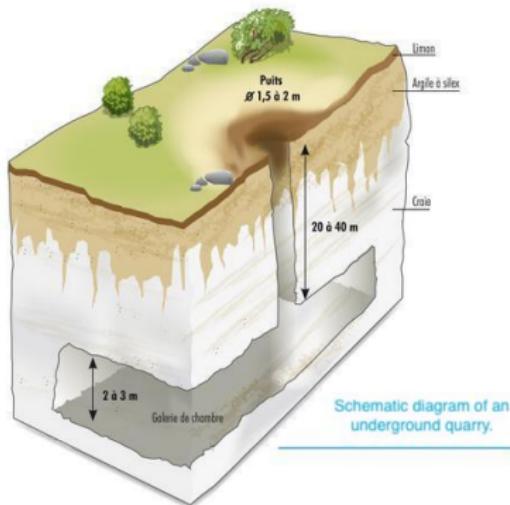
GEOPHYSICAL INTRODUCTION: FRAMING OUR STUDY

MATHEMATICAL MODEL FOR NATURAL CONVECTION

APPLICATION TO THE BARCQ CAVITY

CONCLUSION AND PERSPECTIVES

- Morphology of an underground cavity¹
 - Dug in the 19th century to extract chalk,
 - Approximately 120,000 in the Normandy region,



¹ C. Fauchard, P. Pothérat, P. Côte, and M. Mudet. Détection de cavités souterraines par méthodes géophysiques. Guide technique- Laboratoire central des ponts et chaussées, 2004.

- * Shaft filled with heterogeneous materials (planks, rocks, debris),
- * Climate change: acceleration of collapses.



well blocked by rocks (view from inside the underground cavity) - Collapse under a house.

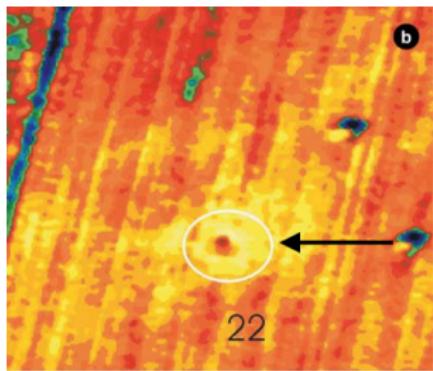
- Presence of multiple detection methods:
 - * Microgravimetry, geological radar, electrical methods, seismic methods,
 - * Each method has its own advantages and disadvantages.
- Search for a flexible method:
 - * Thermal camera carried by drone,
 - * A cost-effective and high-yield method.



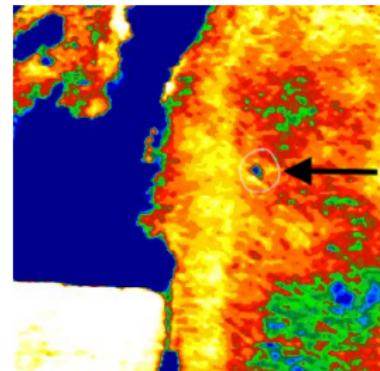
Drone DJI M600 Pro + Thermal camera²

²ENDSUM-Rouen Team, Cerema

Objective: Characterize heat transfer between the cavities and the atmosphere; evaluate the potential of thermal infrared observation.



winter



summer

Surface temperatures observed by airplane using a thermal camera showing the thermal signature of two invisible wells, with a contrast between 2 and 4 degrees³

³ Fauchard, 2004, Pothérat, 2000

PLAN

GEOPHYSICAL INTRODUCTION: FRAMING OUR STUDY

MATHEMATICAL MODEL FOR NATURAL CONVECTION

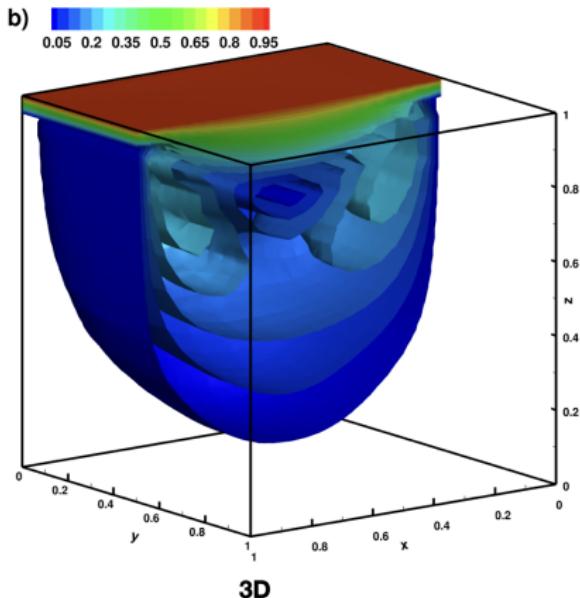
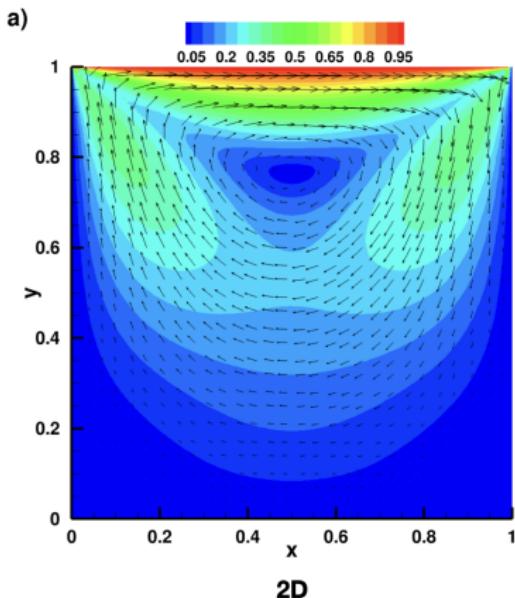
APPLICATION TO THE BARCQ CAVITY

CONCLUSION AND PERSPECTIVES

Concepts of natural convection

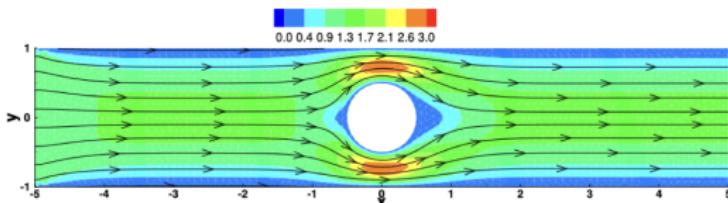
Forging the numerical solutions of progressively complex PDEs using FreeFem++⁴ to craft a model of an underground cavity.

- Stokes equation: $\begin{cases} -\nu \Delta \mathbf{v} + \nabla p = 0 \text{ in } \Omega, \\ \nabla \cdot \mathbf{v} = 0 \text{ in } \Omega. \end{cases}$

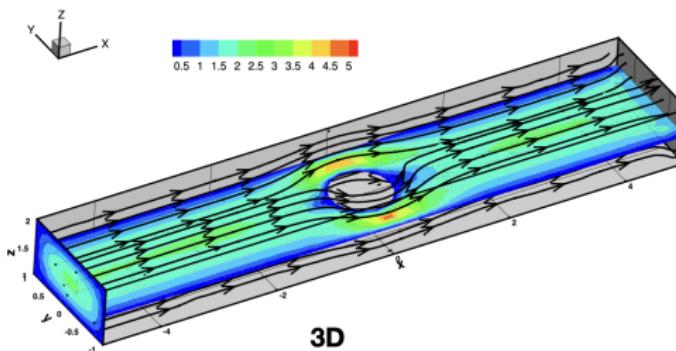


⁴ <https://freefem.org>

- Navier-Stokes:
$$\begin{cases} \frac{\partial \mathbf{v}}{\partial t} - \nu \Delta \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \nabla p = 0 \text{ in } \Omega, \\ \nabla \cdot \mathbf{v} = 0 \text{ in } \Omega. \end{cases}$$

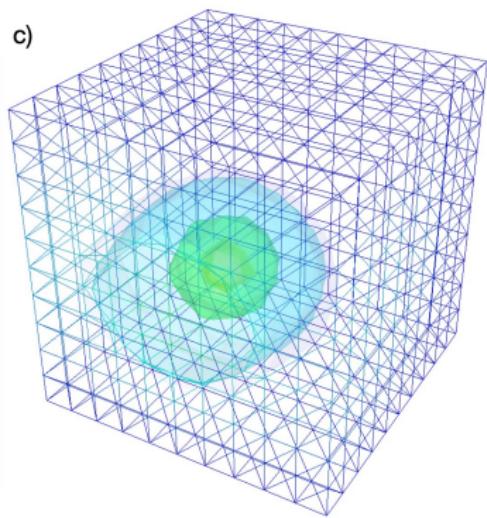
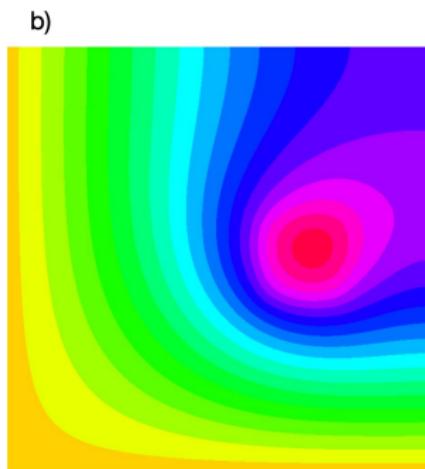
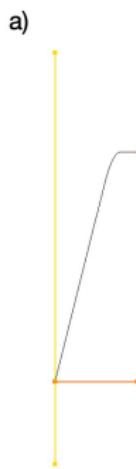


2D



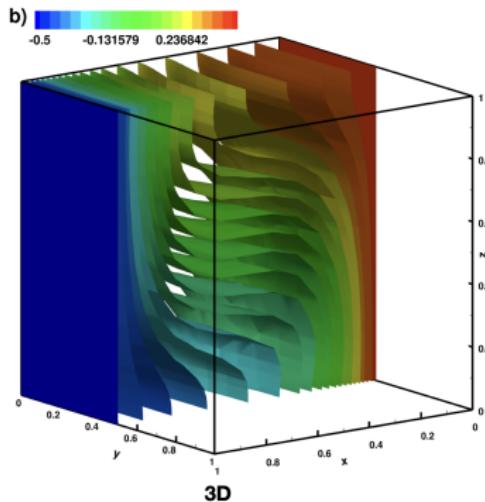
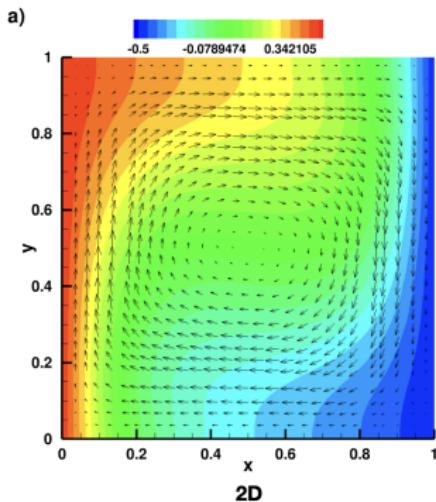
3D

- Heat equation:
$$\begin{cases} \frac{\partial T}{\partial t} - K\Delta T = f & \text{in } \Omega, \\ T(t=0) = T_0, \\ K \cdot \frac{\partial T}{\partial n} + \alpha \cdot (T - T_e) = 0 & \text{on } \Gamma_2 \cup \Gamma_3, \\ T = T_e & \text{sur } \Gamma_1 \cup \Gamma_4. \end{cases}$$



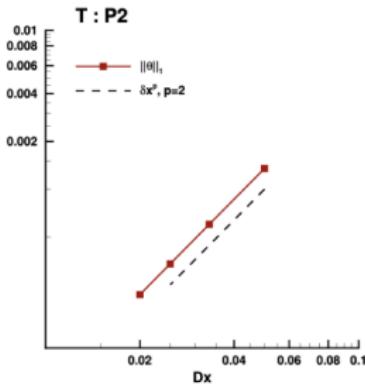
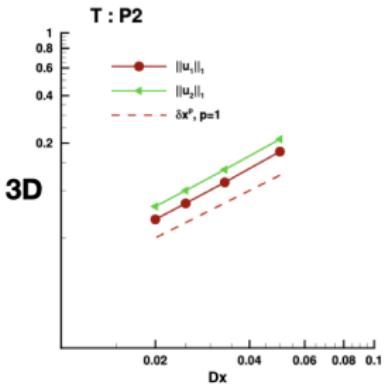
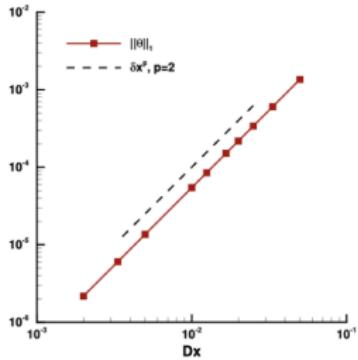
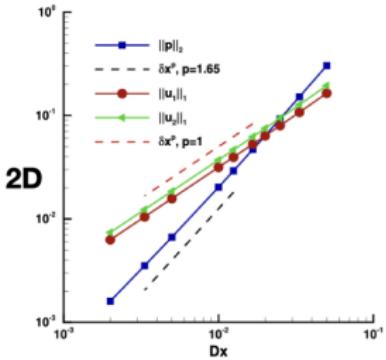
- System of equations for natural convection⁵:

$$\left\{ \begin{array}{l} \nabla \cdot \mathbf{v} = 0, \\ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} + \nabla p - \frac{1}{Re} \nabla^2 \mathbf{v} - f_B(T) \mathbf{e}_y = 0, \\ \frac{\partial T}{\partial t} + \nabla \cdot (T \mathbf{v}) - \frac{1}{Re \cdot Pr} K \Delta T = 0. \end{array} \right.$$

⁵

Georges Sadaka, Aina Rakotondrandisa, Pierre-Henri Tournier, Francky Luddens, Corentin Lothode, and Ionut Danaila. Parallel finite-element codes for the simulation of two-dimensional and three-dimensional solid-liquid phase-change systems with natural convection. Computer Physics Communications, 257:107492, 2020.

- Mathematical validation of methods, spatial convergence using the Burggraf solution (Velocity: P1b, Pressure: P1, Temperature: P2).



PLAN

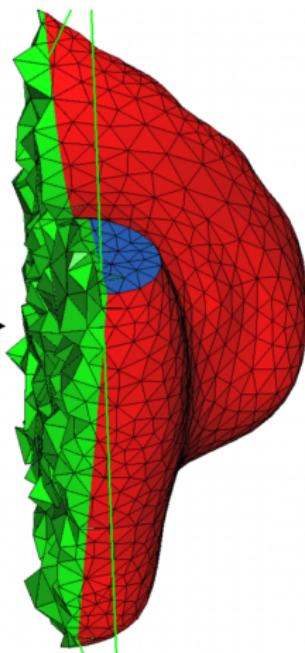
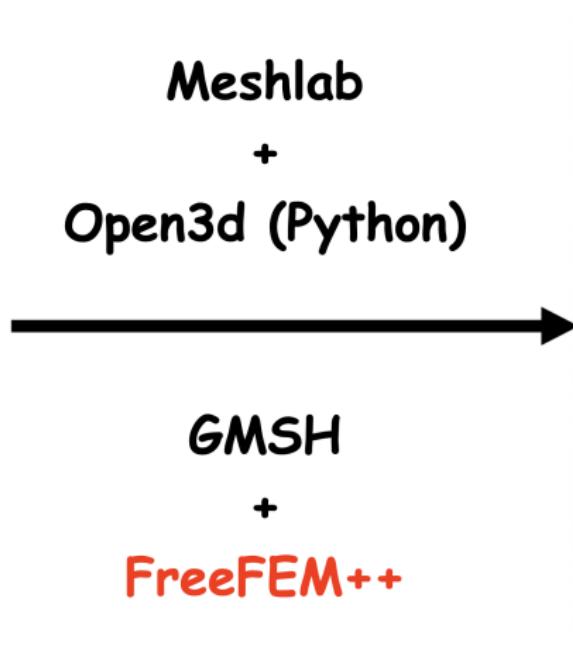
GEOPHYSICAL INTRODUCTION: FRAMING OUR STUDY

MATHEMATICAL MODEL FOR NATURAL CONVECTION

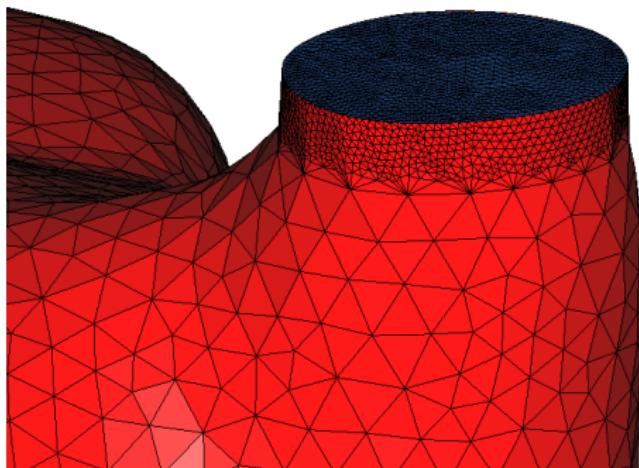
APPLICATION TO THE BARCQ CAVITY

CONCLUSION AND PERSPECTIVES

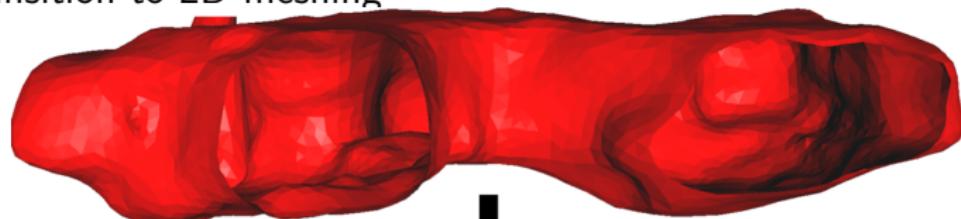
- Generation of the 3D mesh of the Barcq⁶ underground cavity:



```
int[int] labs = [0];
meshL ThL = extract(ThSBarc, label=labs);
// compute the barycenter of the meshL ThL
real bx=int1d(ThL)(x)/ThL.measure;
real by=int1d(ThL)(y)/ThL.measure;
real bz=int1d(ThL)(z)/ThL.measure;
meshL ThLup = movemeshL(ThL,transfo=[(x-bx)*.97+bx,(y-by)*.97+by,bz+2.],region=labS);
meshL ThLmiddle = ThL + ThLup;
include "buildmeshS.idp"
meshS Thcylmiddleup = buildmeshSLap(ThLmiddle,1);
meshS Thcyldown = buildmeshSLap(ThL,1);
meshS ThSfinal = Thcyldown + Thcylmiddleup + ThSBarc;
```



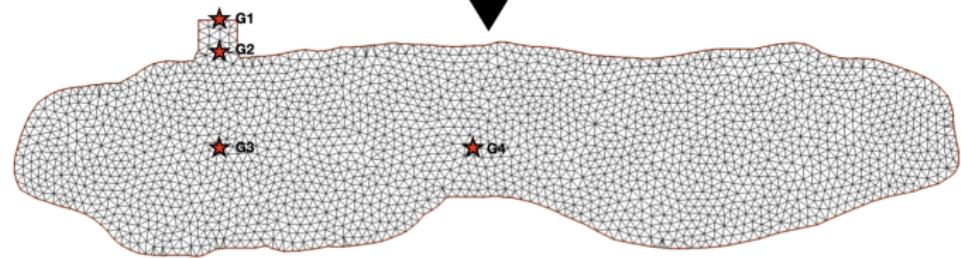
- Transition to 2D meshing



Gimp



FreeFem++



- Concepts about the Rayleigh number: $\mathcal{R}a = \frac{g\beta H^3 \delta T}{\nu\alpha}$
 - * $g, \beta, \nu, \alpha \rightarrow$ Constants (gravitational acceleration, thermal expansion coefficient, viscosity, thermal diffusivity),
 - * $\delta T \rightarrow$ Low temperature difference,
 - * $H^3 \rightarrow$ volume of the cavity,
- Example of Rayleigh:
 - * $\mathcal{R}a = 10^9 \iff \delta T = 1^\circ$ et $H^3 = 10m^3$
- Following simulations carried out in non-dimensional case.

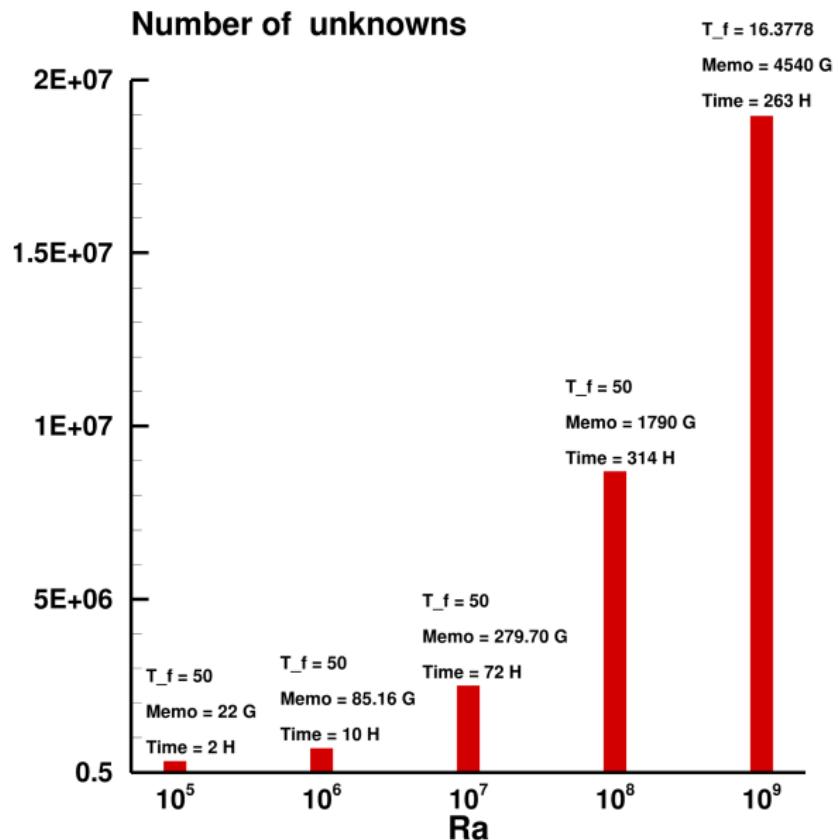
Solvers used from the PETSc⁷ library ((Portable, Extensible Toolkit for Scientific Computation)

- Direct solver used for the 2D case:
"-pc_type lu -ksp_type preonly",
- For the 3D problem, we used the following set of parameters:
"-ksp_converged_reason -pc_type asm -pc_asm_overlap 1
-ksp_pc_side left -ksp_type gmres -ksp_gmres_restart 50
-ksp_max_it 100 -ksp_atol 1e-5 -sub_pc_type lu
-sub_pc_factor_mat_solver_type mumps -ksp_rtol 1e-6".

⁷<https://www.mcs.anl.gov/petsc/>

2D Natural convection with $Ra = 10^9$.

Convection naturelle en 3D avec $Ra = 10^8$.



PLAN

GEOPHYSICAL INTRODUCTION: FRAMING OUR STUDY

MATHEMATICAL MODEL FOR NATURAL CONVECTION

APPLICATION TO THE BARCQ CAVITY

CONCLUSION AND PERSPECTIVES

- **Conclusion :**

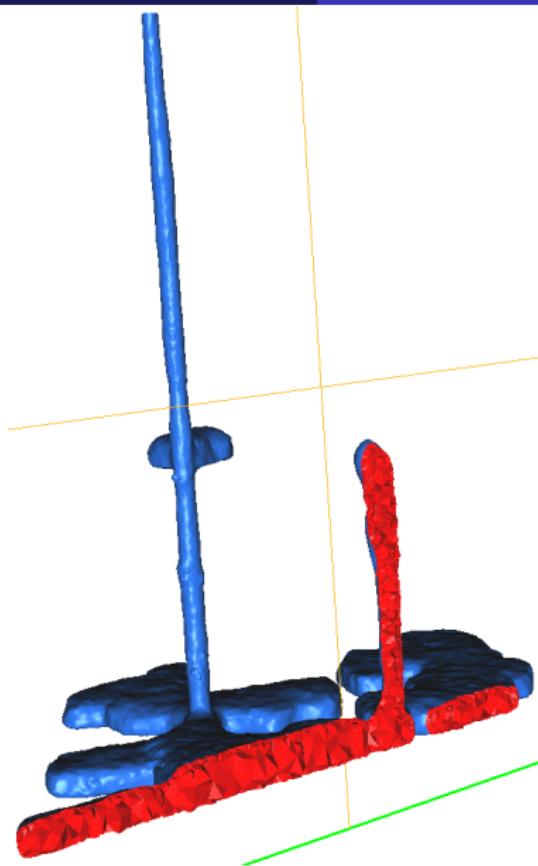
- * The geological context in the Normandy region obstructs the discovery of cavities. The thermal infrared method shows great potential for overcoming these challenges.
- * Simulating natural convection within the cavity using FreeFem++ .
- * Observing the initiation of natural convection, as well as changes in heat flux and the quantity of convection cells, based on the Rayleigh number.

- **Conclusion :**

- * The geological context in the Normandy region obstructs the discovery of cavities. The thermal infrared method shows great potential for overcoming these challenges.
- * Simulating natural convection within the cavity using FreeFem++ .
- * Observing the initiation of natural convection, as well as changes in heat flux and the quantity of convection cells, based on the Rayleigh number.

- **Perspectives :**

- * The 3D case requires improvement (PETSc - P. Jolivet we need help).
- * Integration of geophysical parameters into the simulation for enhanced realism.
- * Incorporating data from thermocouples and radiation sensors to validate and compare our simulation results with actual parameters.



Cavity of Goderville (Normandy).

Thank you for your attention.