Atsushi Suzuki
RIKEN Center for Computational Science/Cybermedia Center Osaka University Atsushi.Suzuki@cas.cmc.osaka-u.ac.jp

Fictitious domain method with implicit boundary definition by the level set method and a stabilization

For shape optimization problem, it is better to express the boundary of the target domain by the zero level set of a level set function, due to complicated geometry updating. In such case, Dirichlet boundary condition for the state and adjoint problems will be expressed in an implicit way. One idea is to generate a body fitted mesh to the boundary and to treat Dirichlet boundary conditions on the finite element nodes as usual. However we need to pay high cost to create such body fitted mesh for the zero level set obtained from finite element solution because of less regularity of the curve.
Another idea is to relax mesh distribution around the zero level set by acceptance non-aligned triangulation to the boundary. However we can expect much better alignment of the triangulation in the vicinity of the boundary than the uniform back ground mesh.
Fictitious domain method with a Lagrange multiplier is a strategy to impose Dirichlet boudnary conditions on the internal boundary. The original fictitious domain method consists of boundary integration for L2-inner product of the gap from given Dirichlet data and the Lagrange multiplier for their duality using precise geometrical information of the inner boundary. Here we can replace the boundary integration by element-wise one where the level set is crossing. By following a stabilization method by Burman-Hansho 2010, a stabilization term for the Lagrange multiplier allows to use more stable factorization of the indefinite system.