Sixth Workshop on Generic Solvers for PDEs:

FreeFem++ and its Applications

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USING FREEFEM++ TO SOLVE AN INDUSTRIAL PROBLEM: HEAT TREATING OF A STEEL HELICAL GEAR

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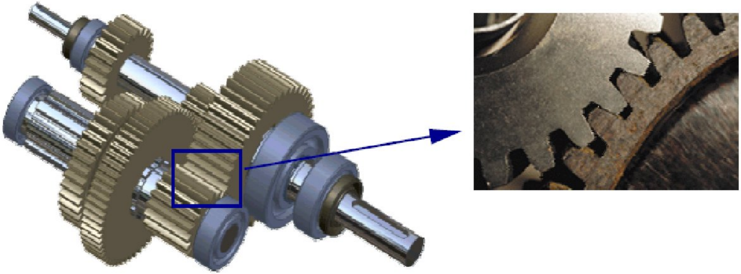
Steel hardening

In the automotive industry, many important moving (rotating/translating) pieces are in close contact in order to transmit the desired rotation/translation movement: gear wheels, toothed rings, bevel gears, rack and pinion, etc.



Steel hardening

These workpieces are made of **steel**.

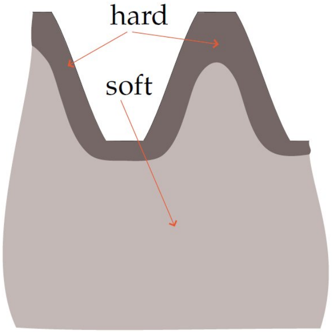
Prior to any hardening treatment, steel is a ductile material.

Rotating/translating workpieces in close contact are subject to stresses during its lifetime. **Hardening treatment** is necessary in order to avoid wear and abrasion.

Steel hardening

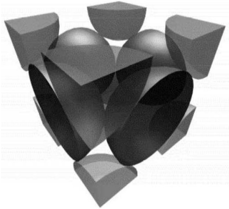
A convenient hardening treatment is then applied in order to produce:

- a hard boundary layer to hinder wear and abrasion, and
- a soft inner part to reduce fatigue.

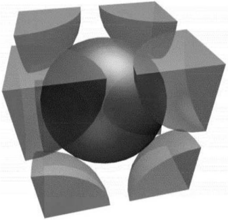


Some facts on steel

Steel is an iron based alloy.
Iron may appear in two type of crystal lattices:



face centered cubic (fcc)



body centered cubic (bcc)

Different solid phases in steel:

- **Austenite**: Solution of C in fcc iron. Only possible if concentration of C up to 2.11%; if so, only possible at a high temperature range.
- **Ferrite**: Nearly pure bcc iron.
- **Pearlite**: Lamellar structure of ferrite and cementite (Fe_3C).
- **Martensite**: Tetragonally bcc iron crystal distorted by C atoms. It can only stem from austenite.

Iron Carbide Phase Transitions

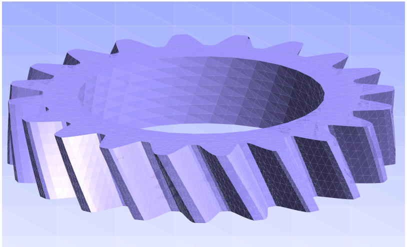
Phase transitions in hypo/hyper/eutectoid steel

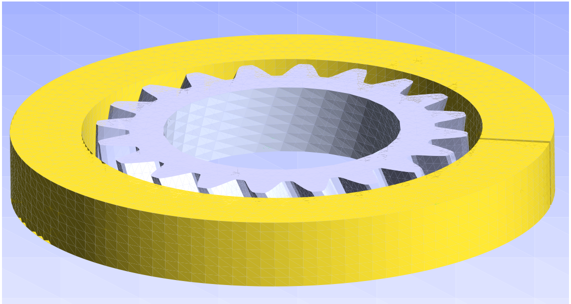
- Austenite $\rightarrow$ pearlite, bainite (slow cooling down temperature rate)
- Austenite $\rightarrow$ martensite (very rapid cooling down temperature rate)

Phases have different physical properties

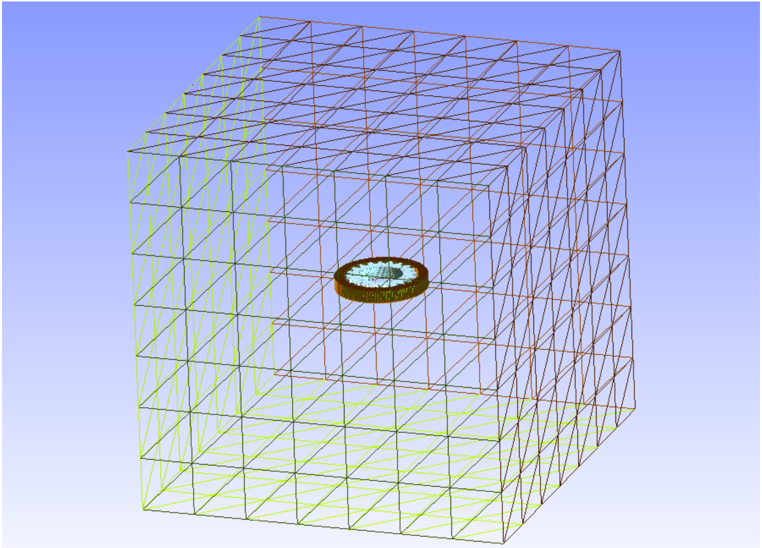
- Pearlite: soft and ductile.
- Martensite: hard and brittle.

The industrial procedure: Induction heating



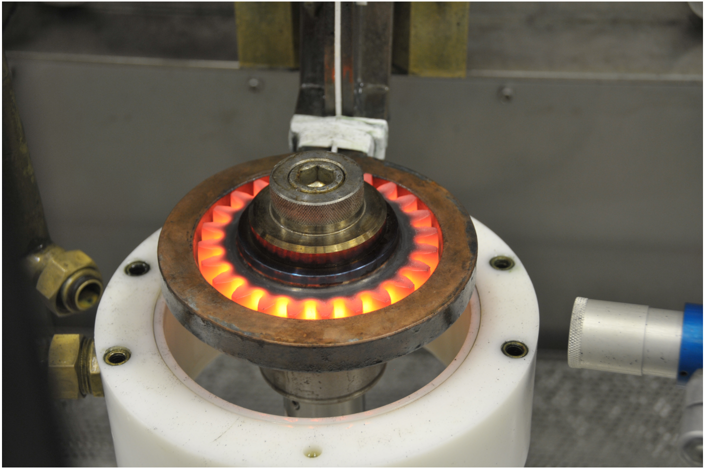


The industrial procedure: Induction heating

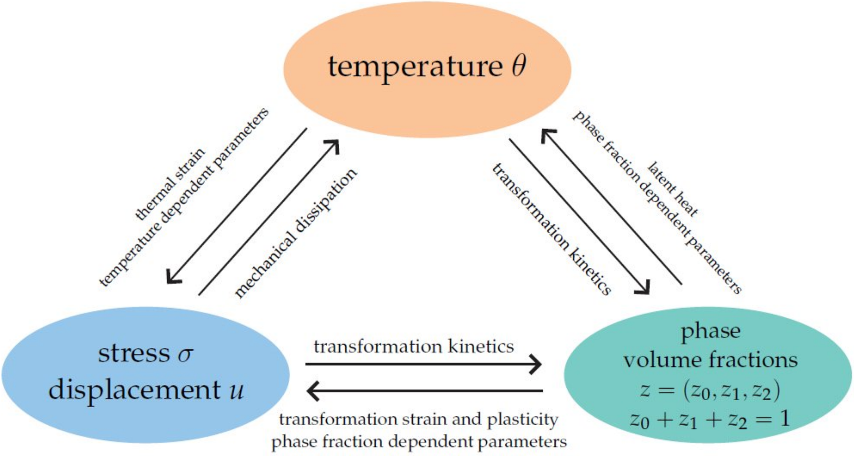


The industrial procedure: Induction heating

Real machine at work



Thermomechanical phenomena



EM + thermal + phase fractions modeling

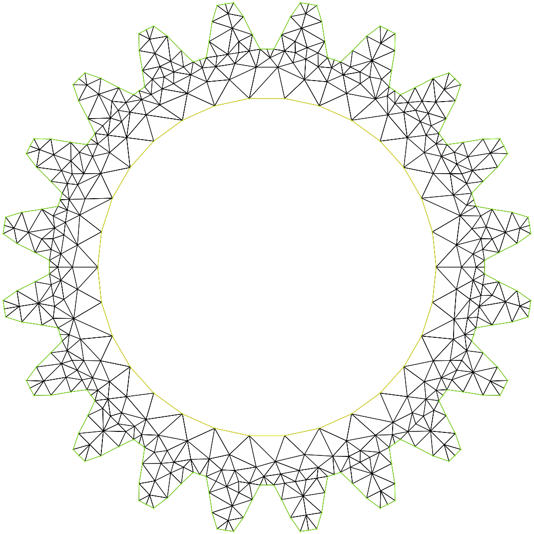
Heating stage in the harmonic regime

$$
\begin{aligned}
\nabla \cdot (b(\theta)\nabla\varphi) &= 0 \text{ in } \Omega \times (0, T_{\mathrm{h}}),\\
\frac{\partial\varphi}{\partial n} &= 0 \text{ on } \partial\Omega \times (0, T_{\mathrm{h}}),\\
\left[b(\theta)\frac{\partial\varphi}{\partial n} \right]_{\Gamma} &= \boldsymbol{j} \text{ on } \Gamma \times (0, T),\\
i\omega b_0(\theta)\boldsymbol{A} + \nabla \times \left(\frac{1}{\mu}\nabla \times \boldsymbol{A} \right) - \delta\nabla(\nabla \cdot \boldsymbol{A}) &= -b_0(\theta)\nabla\varphi \text{ in } D \times (0, T_{\mathrm{h}}),\\
\boldsymbol{A} &= 0 \text{ on } \partial D \times (0, T_{\mathrm{h}}),\\
z_t &= F(\theta, z) \text{ in } \Omega^{\mathrm{s}} \times (0, T_{\mathrm{h}}),\\
z(0) &= z_0 \text{ in } \Omega^{\mathrm{s}},
\end{aligned}
$$

$$
\begin{aligned}
\rho c_\epsilon \theta_t - \nabla \cdot (k(\theta)\nabla\theta) &= \frac{1}{2} b(\theta)|i\omega\boldsymbol{A} + \nabla\varphi|^2 + \rho L z_t + G \text{ in } \Omega \times (0, T_{\mathrm{h}}),\\
\frac{\partial\theta}{\partial n} &= 0 \text{ on } \partial\Omega \times (0, T_{\mathrm{h}}),\\
\theta(\cdot, 0) &= \theta_0 \text{ in } \Omega,
\end{aligned}
$$

An existence result by M. T. González Montesinos and FOG.

Geometry: building a mesh to the helical gear



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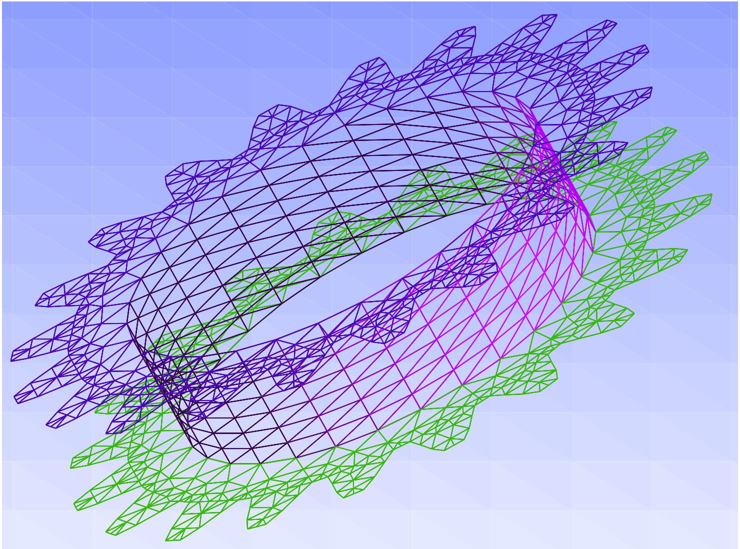

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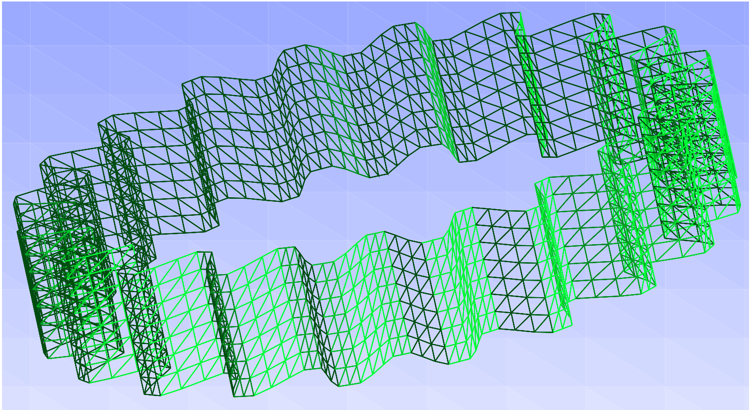
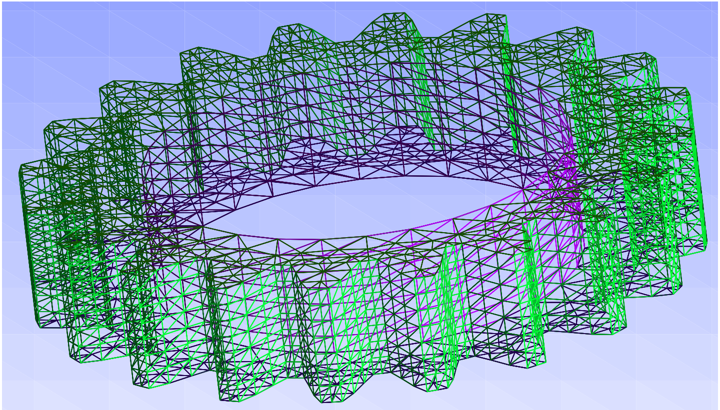


Geometry: building a mesh to the helical gear



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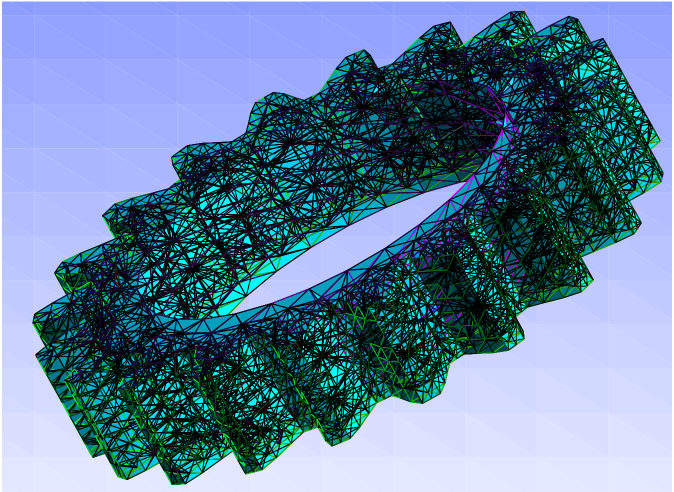

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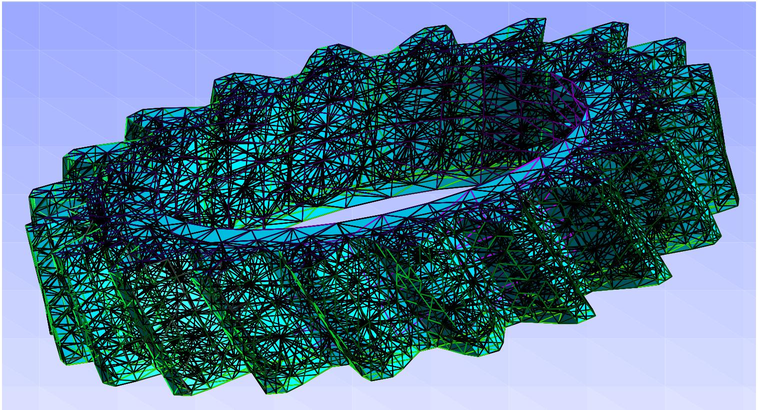
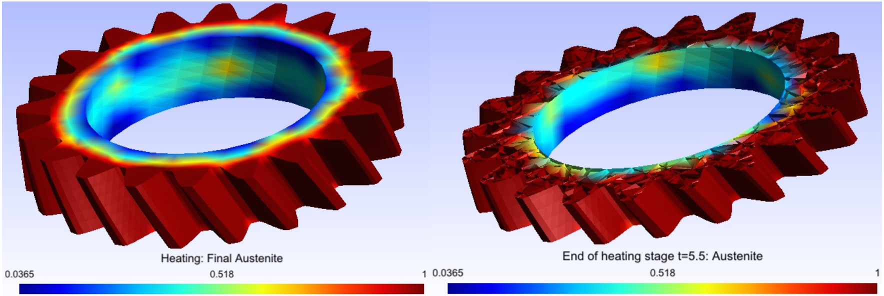
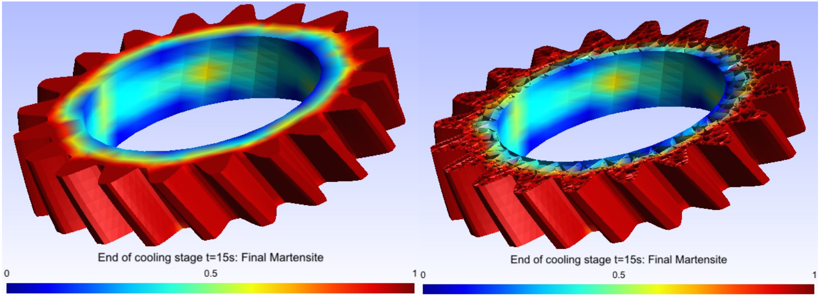


Numerical simulation



Left: austenite at the end of the heating stage $t = 5.5$ s.
Right: austenite at the end of the heating stage $t = 5.5$ s.
Clipping image.

Numerical simulation



Left: martensite at the end of the cooling stage $T = 15$ s.
Right: martensite at the end of the cooling stage $T = 15$ s. Clipping image.

Some references

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