Numerical simulations done with

FreeFem++

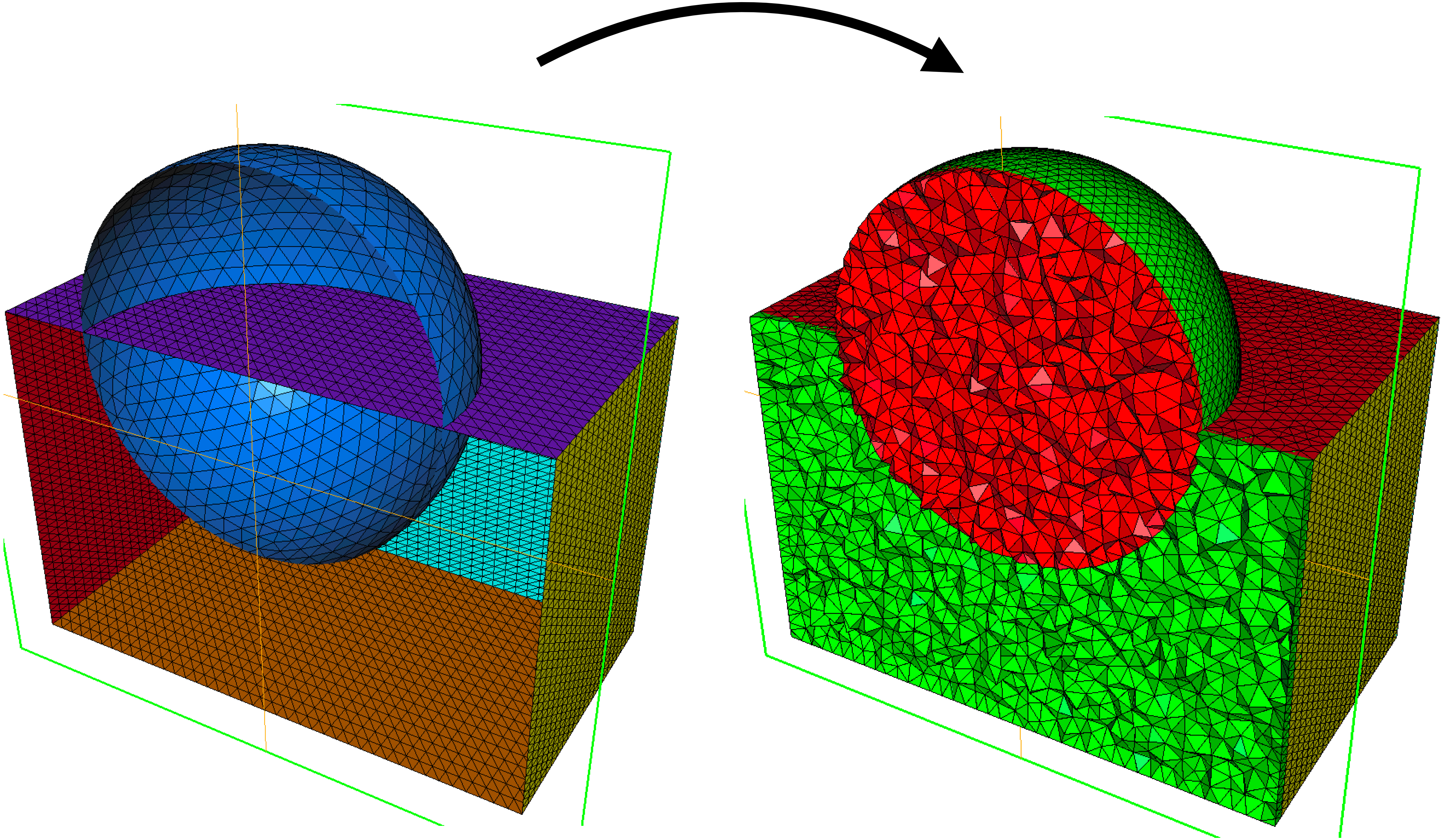
Georges SADAKA

IR - CNRS - LMRS - Rouen (Since 01-12-2023)

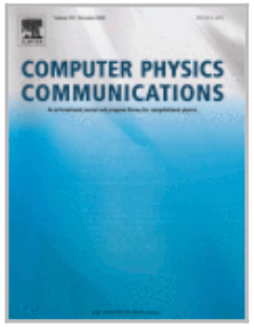
FreeFEM-DAYS 15th Edition, Paris : 07-12-2023

Build intesection of two surface

wait till the end



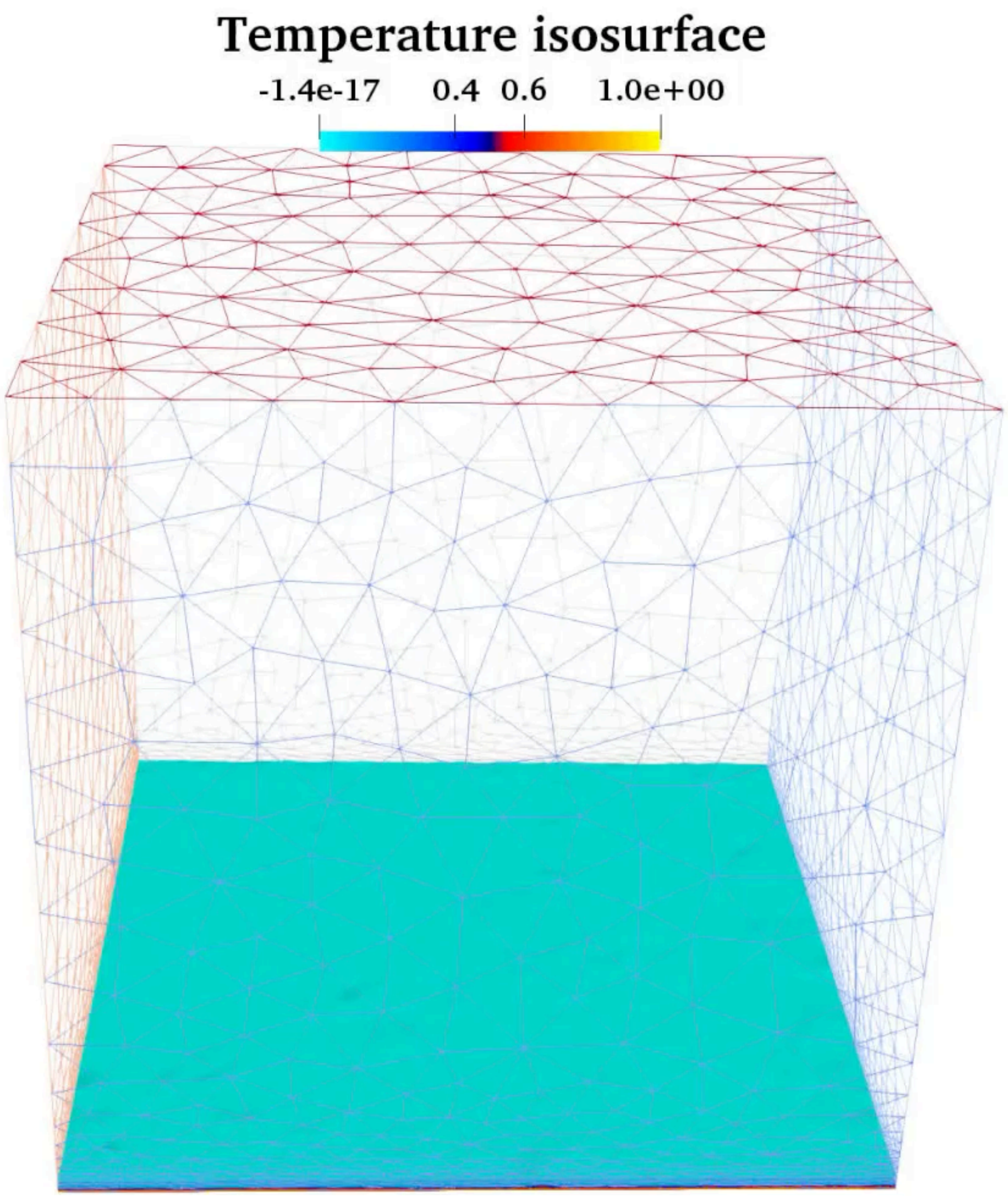
Parallel finite-element codes for the simulation of two-dimensional and three-dimensional solid–liquid phase-change systems with natural convection ☆, ☆☆



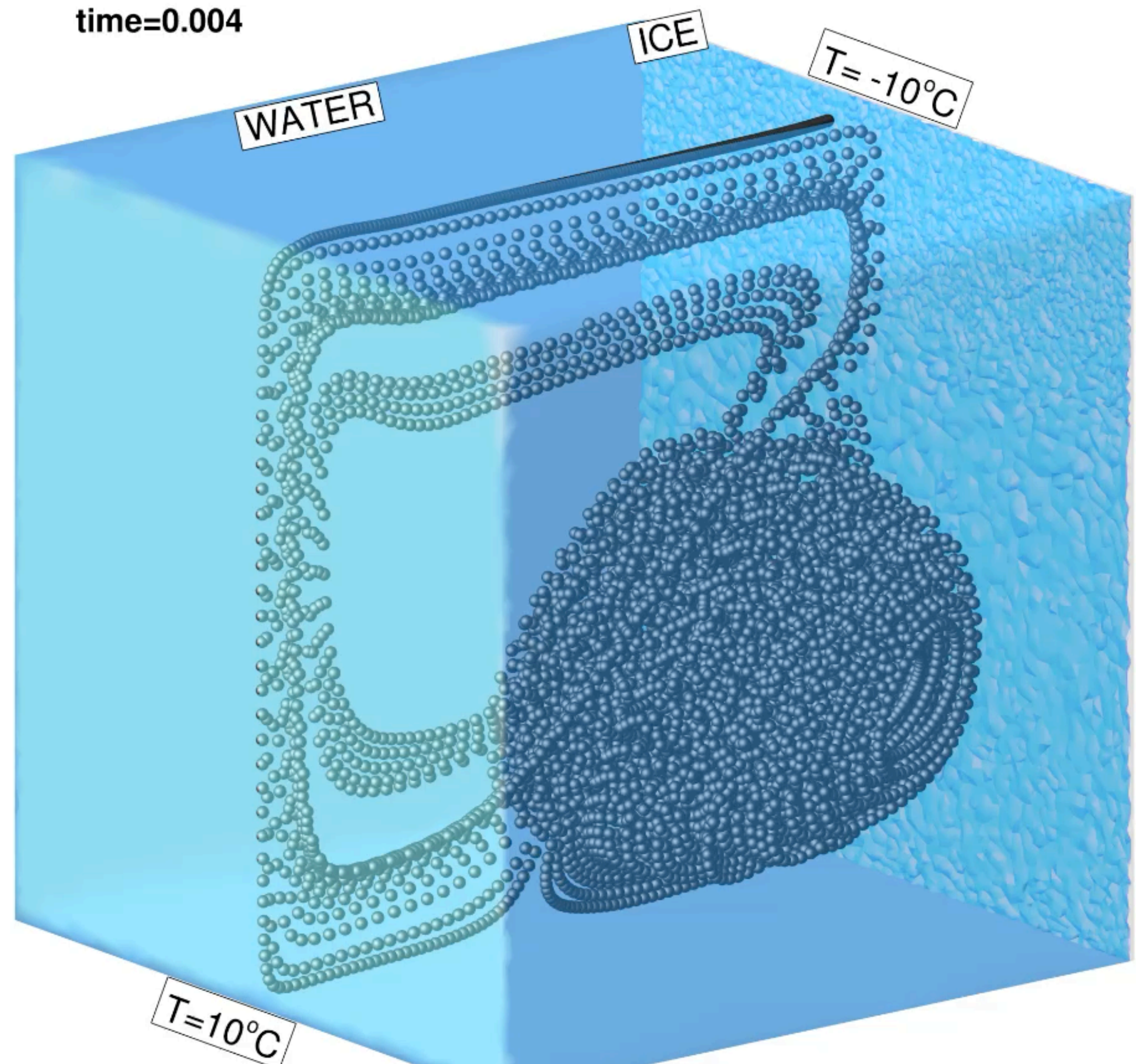
Georges Sadaka [a], Aina Rakotondrandisa [a], Pierre-Henri Tournier [b], Francky Luddens [a], Corentin Lothodé [a], Ionut Danaila [a]

Toolbox FREE

$$
\begin{aligned}
\nabla \cdot \mathbf{u} &= 0, \\
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla p - \frac{1}{Re}\nabla^2 \mathbf{u} - A_{mushy}(\theta)\mathbf{u} - f_B(\theta)\mathbf{e}_y &= 0, \\
\frac{\partial (C(\theta)\theta)}{\partial t} + \nabla \cdot (C(\theta)\theta \mathbf{u}) - \frac{K(\theta)}{Pr\ Re}\nabla^2 \theta + \frac{\partial (C(\theta)S(\theta))}{\partial t} &= 0.
\end{aligned}
$$



paraffine fusion



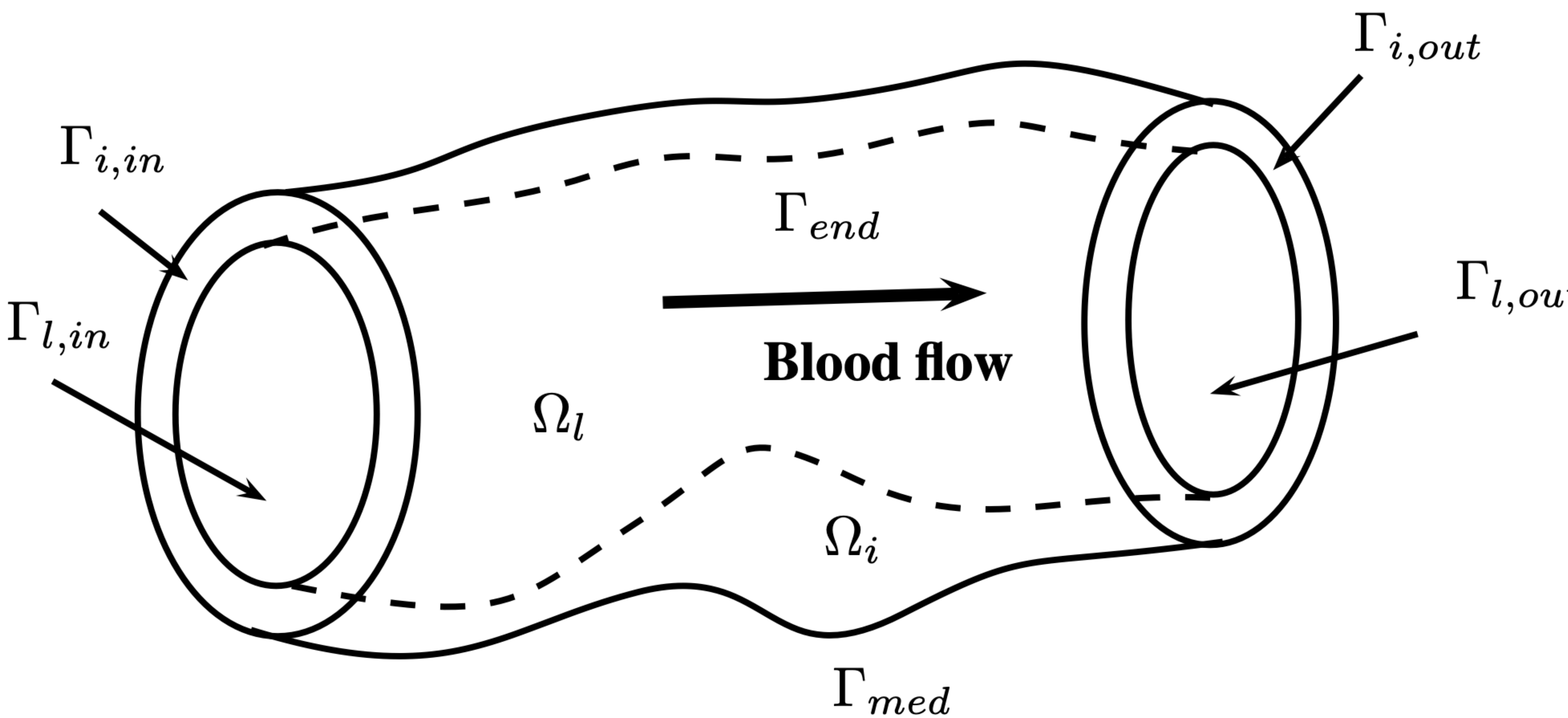
Water Solidification

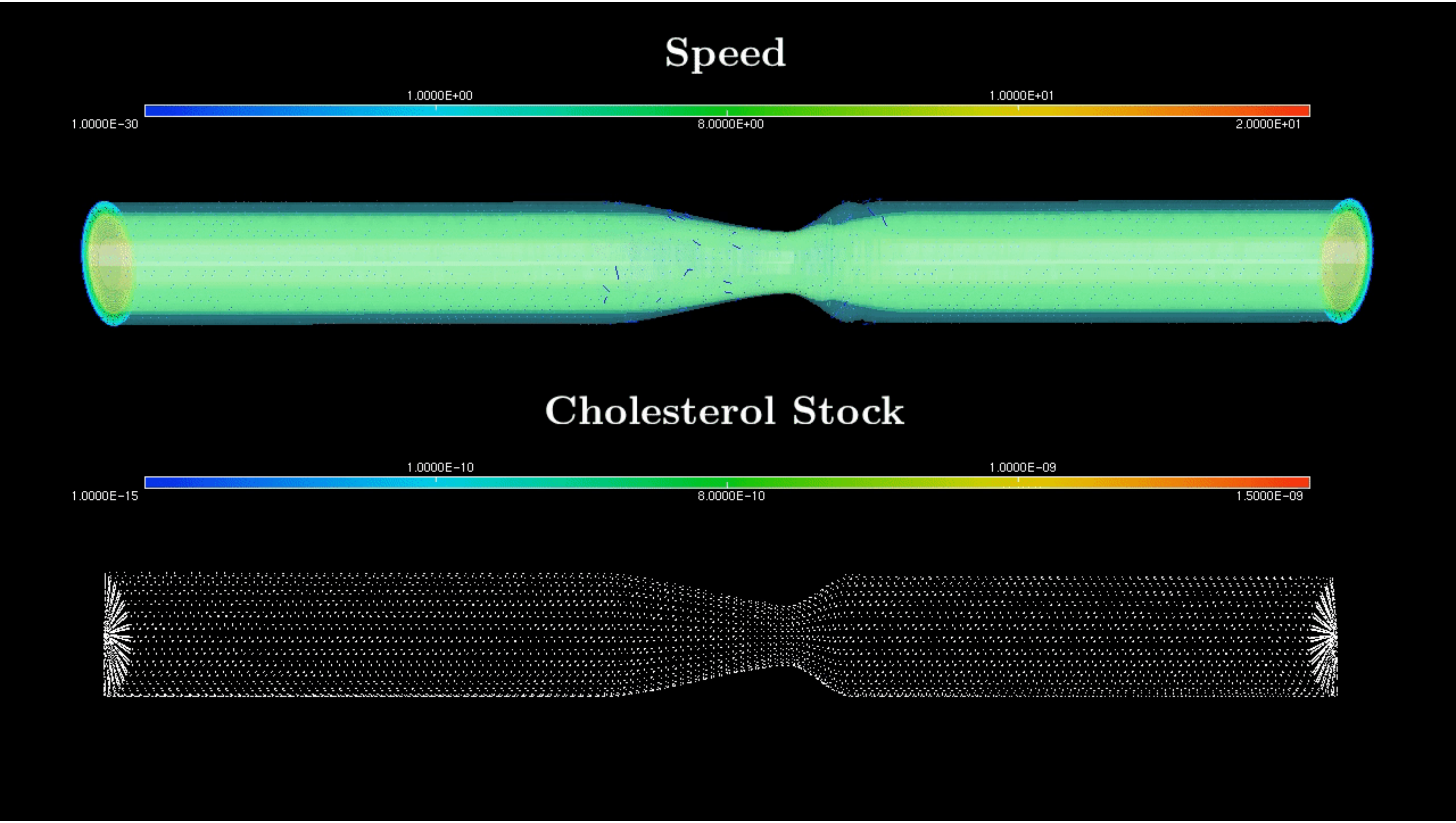
MATHEMATICAL AND NUMERICAL MODELING OF EARLY ATHEROSCLEROTIC LESIONS [*, **]

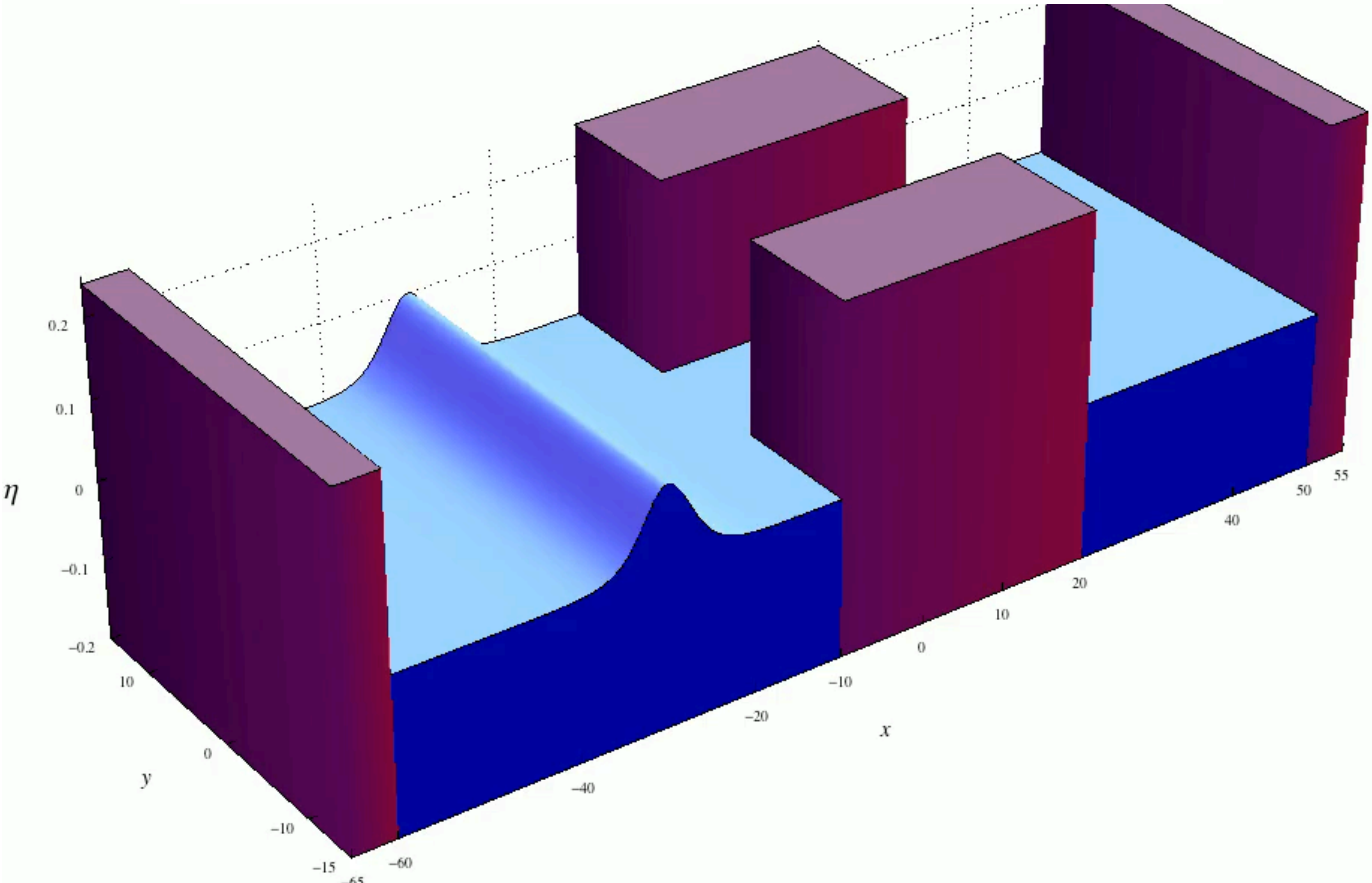
Vincent Calvez[1], Jean Gabriel Houot[2], Nicolas Meunier[2], Annie Raoult[2] and Gabriela Rusnakova[3]

$$\rho\,[\partial_t \mathbf{u}_l + (\mathbf{u}_l\cdot\nabla)\mathbf{u}_l] - \nu\Delta\mathbf{u}_l + \nabla p_l = 0,$$
$$\nabla\cdot\mathbf{u}_l = 0,$$
$$\partial_t c_l + \nabla\cdot(-D_l\nabla c_l + \mathbf{u}_l c_l) = 0,$$

$$\mathbf{u}_i = -\frac{K}{\mu}\nabla p_i,$$
$$\nabla\cdot\mathbf{u}_i = 0,$$
$$\partial_t c_i + \nabla\cdot(-D_i\nabla c_i + \mathbf{u}_i c_i) = -r_{ox} c_i,$$



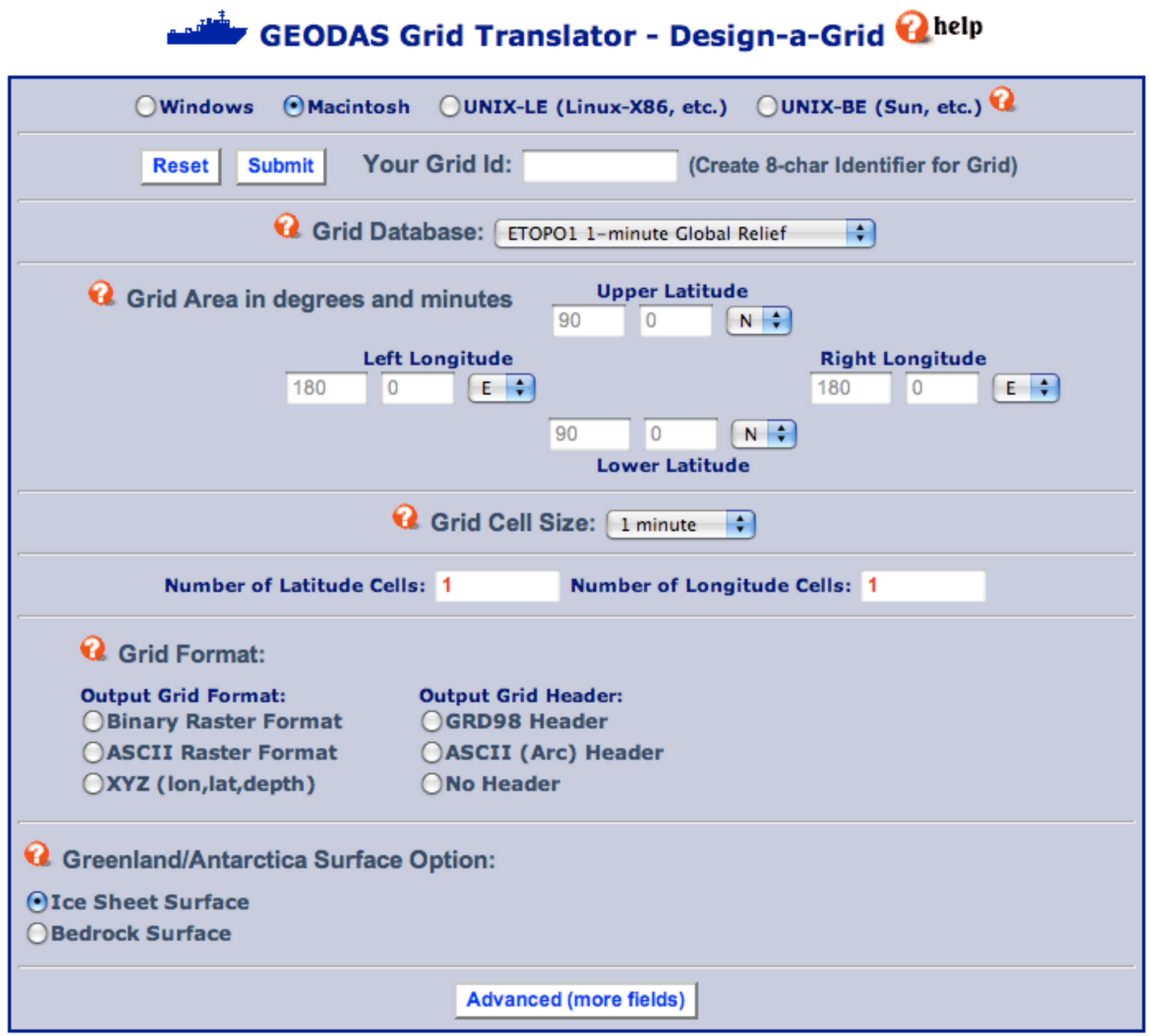

Speed
1.0000E+00
1.0000E+01
1.0000E-30
8.0000E+00
2.0000E+01
Cholesterol Stock
1.0000E-10
1.0000E-09
1.0000E-15
8.0000E-10
1.5000E-09



$$\eta_t + \nabla \cdot ((D+\eta)V) - b\nabla \cdot \left(D^2 \nabla \eta_t\right) = 0,$$

$$\mathbf{V}_t + \nabla\eta + \tfrac{1}{2}\nabla|\mathbf{V}|^2 - dD^2\Delta\mathbf{V}_t = 0,$$

Real Topography Data



$$\eta_t + \nabla \cdot ((D+\eta)V) - b\nabla \cdot \left(D^2 \nabla \eta_t\right) \;=\; 0,$$

$$\mathbf{V}_t + \nabla \eta + \tfrac{1}{2}\nabla |\mathbf{V}|^2 - dD^2 \Delta \mathbf{V}_t \;=\; 0,$$

$$f(\xi,\eta)\,||= f(\xi,p) - f(\xi,p-W) - f(\xi-L,p) + f(\xi-L,p-W)\ ,$$

$$\mathcal{O}(x,y) = -\frac{U}{2\pi}\left(\frac{\tilde{d}q}{R(R+\xi)} + \sin\delta \arctan\frac{\xi\eta}{qR} - I\sin\delta\cos\delta\right)\Bigg\|,$$

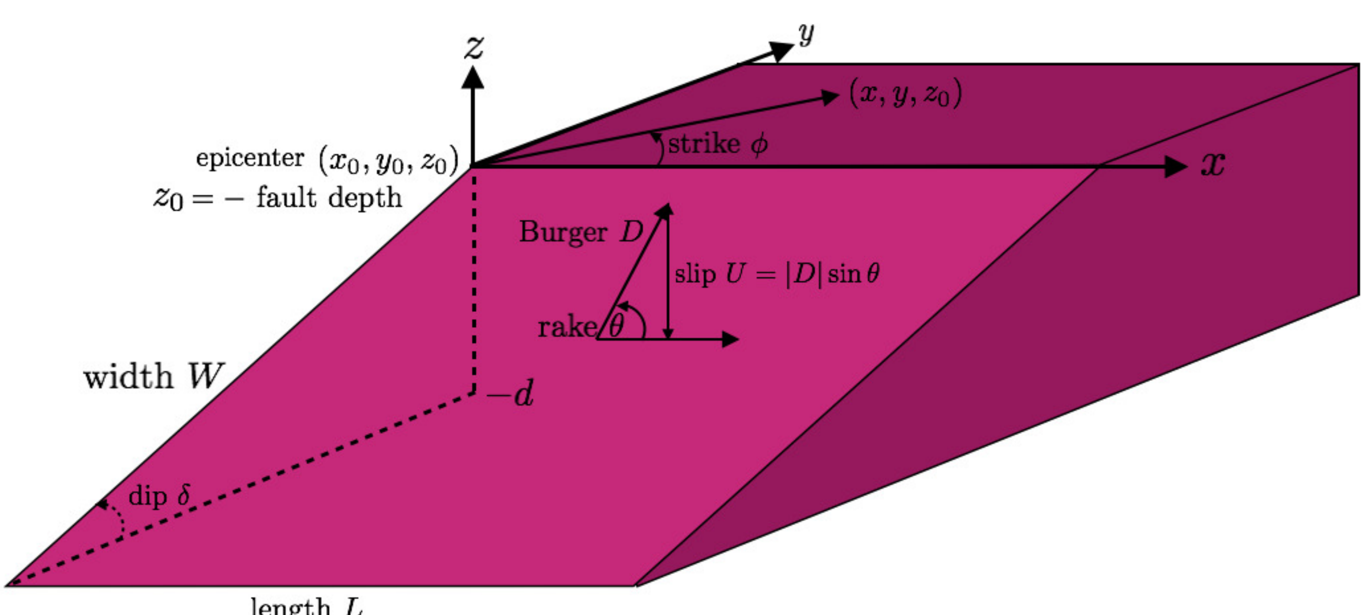
$$\xi = (x-x_0)\cos\phi + (y-y_0)\sin\phi,\ Y = -(x-x_0)\sin\phi + (y-y_0)\cos\phi,$$

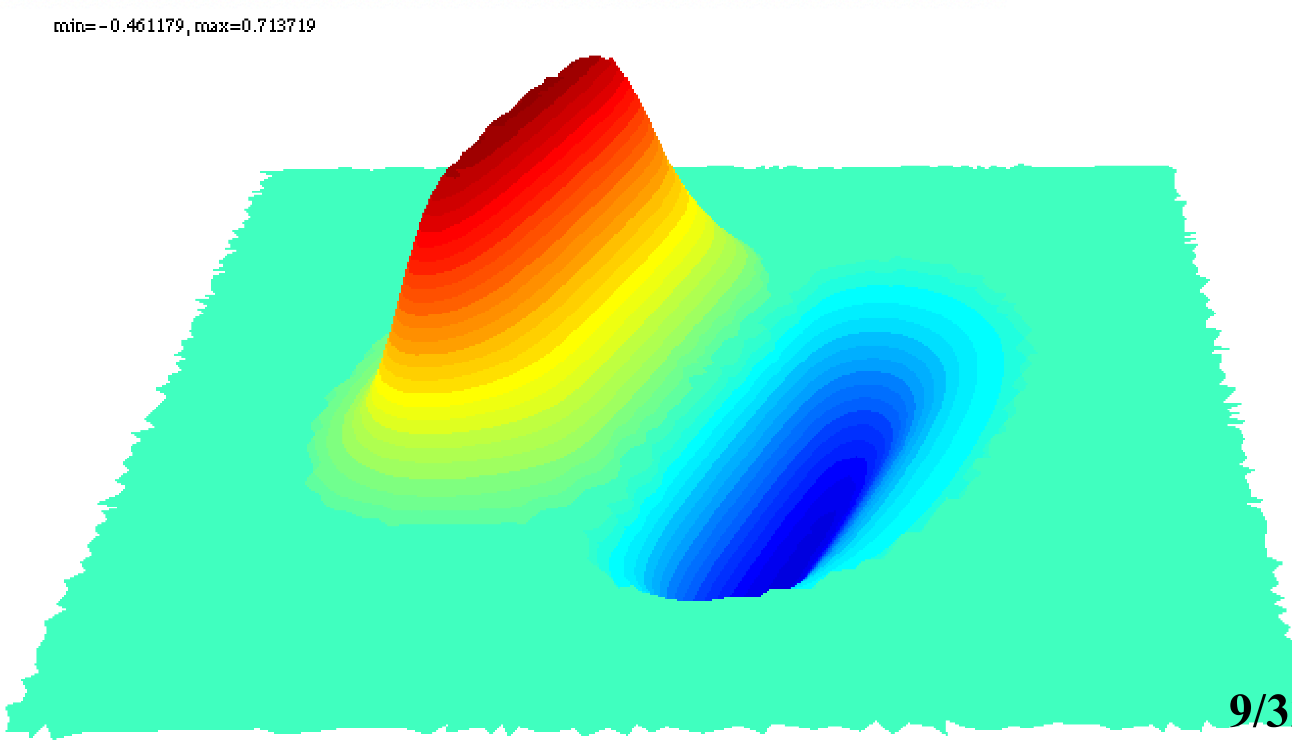
$$p = Y\cos\delta + d\sin\delta,\ q = Y\sin\delta - d\cos\delta,$$

$$\tilde{y} = \eta\cos\delta + q\sin\delta,\ \tilde{d} = \eta\sin\delta - q\cos\delta,$$

$$R^2 = \xi^2 + \eta^2 + q^2 = \xi^2 + \tilde{y}^2 + \tilde{d}^2,\ X^2 = \xi^2 + q^2$$

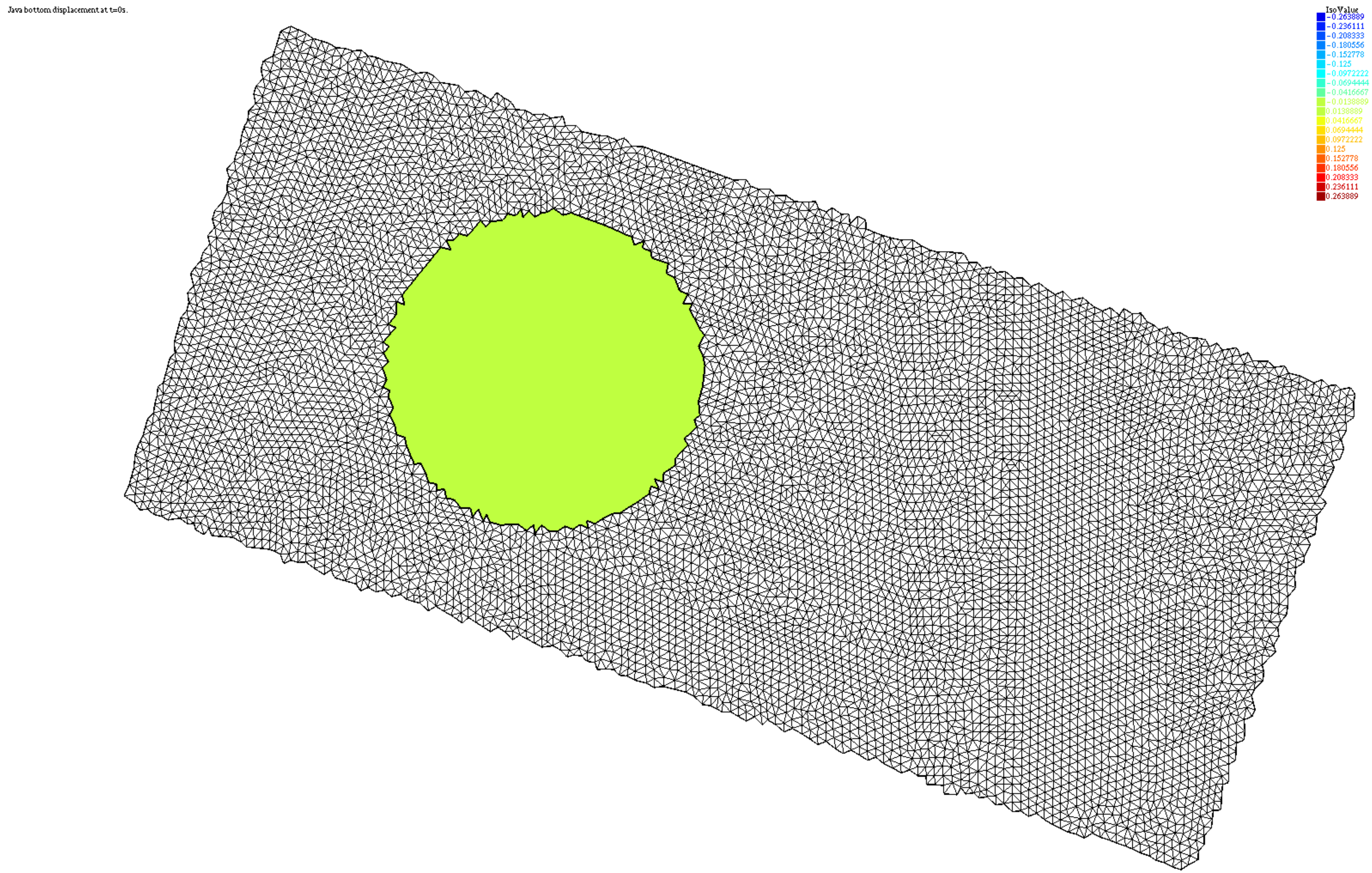
$$I = \begin{cases} \dfrac{\mu}{\lambda+\mu}\dfrac{2}{\cos\delta}\arctan\dfrac{\eta(X+q\cos\delta)+X(R+X)\sin\delta}{\xi(R+X)\cos\delta} & \text{if } \cos\delta\neq 0,\\ \dfrac{\mu}{\lambda+\mu}\dfrac{\xi\sin\delta}{R+\tilde{d}} & \text{if } \cos\delta = 0. \end{cases}$$





finite fault bottom displacement near Java Island in 2006

$$\zeta(x,y,t) = \sum_{i=1}^{N_x \cdot N_y} \mathcal{H}(t - t_i) \cdot \left(1 - e^{-\alpha(t-t_i)}\right) \cdot \mathcal{O}_i(x,y)$$



https://Earthquake.usgs.gov/archive/product/finite-fault/usp000ensm/us/1486510367579/web/p000ensm.param

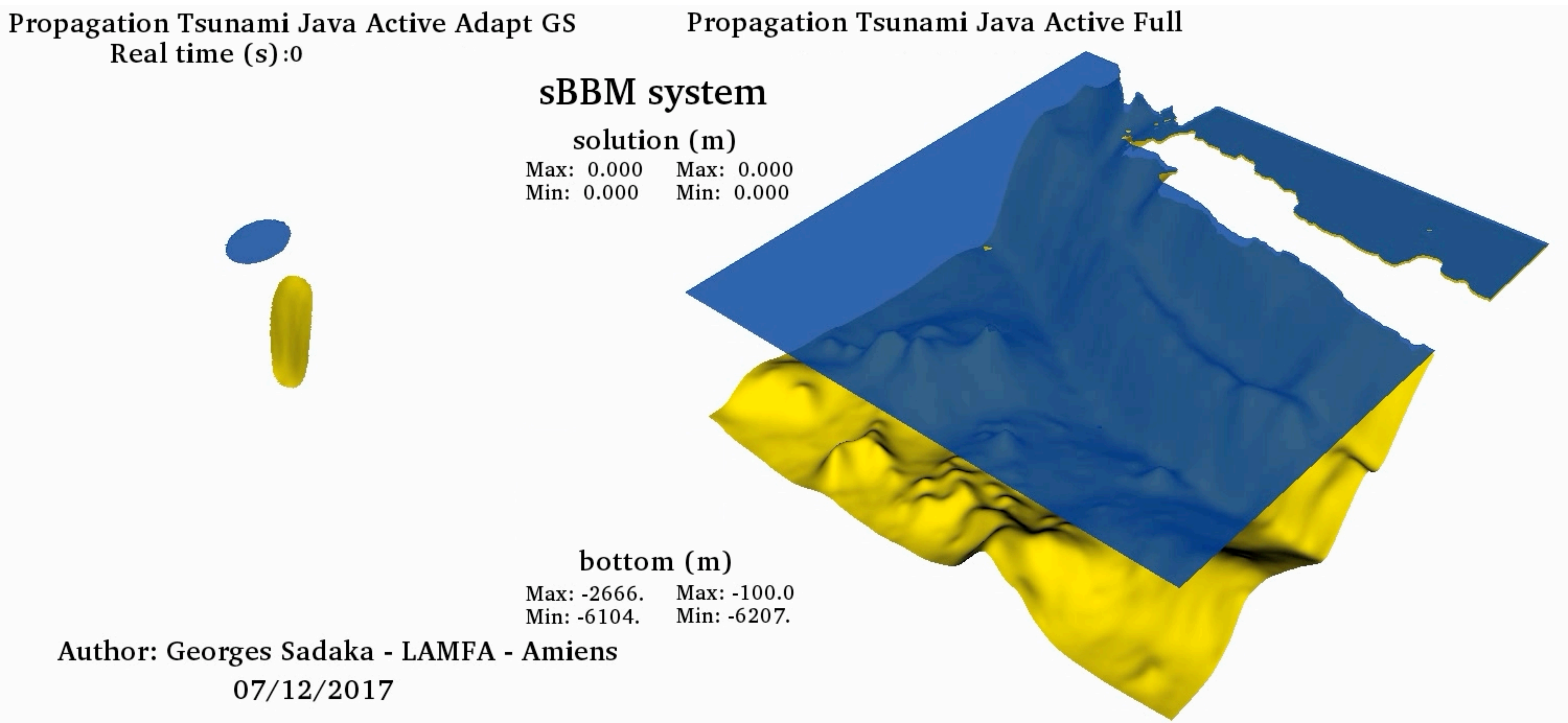
Adaptive numerical modelling of tsunami wave generation and propagation with FreeFem++

Toolbox FREE

Georges SADAKA [1,†,*], Denys DUTYKH [2,†]

$$\eta_t + \nabla \cdot ((h+\eta)V) + \zeta_t + \tilde{A}\nabla \cdot \left(h^2 \nabla \zeta_t\right) + \nabla \cdot \left\{ Ah^2 \left[\nabla\left(\nabla h \cdot V\right) + \nabla h \nabla \cdot V\right] - bh^2 \nabla \eta_t \right\} = 0$$

$$V_t + g\nabla\eta + \frac{1}{2}\nabla|V|^2 + Bgh\left[\nabla\left(\nabla h \cdot \nabla \eta\right) + \nabla h \Delta \eta\right] - dh^2 \Delta V_t - Bh\nabla\zeta_{tt} = 0$$

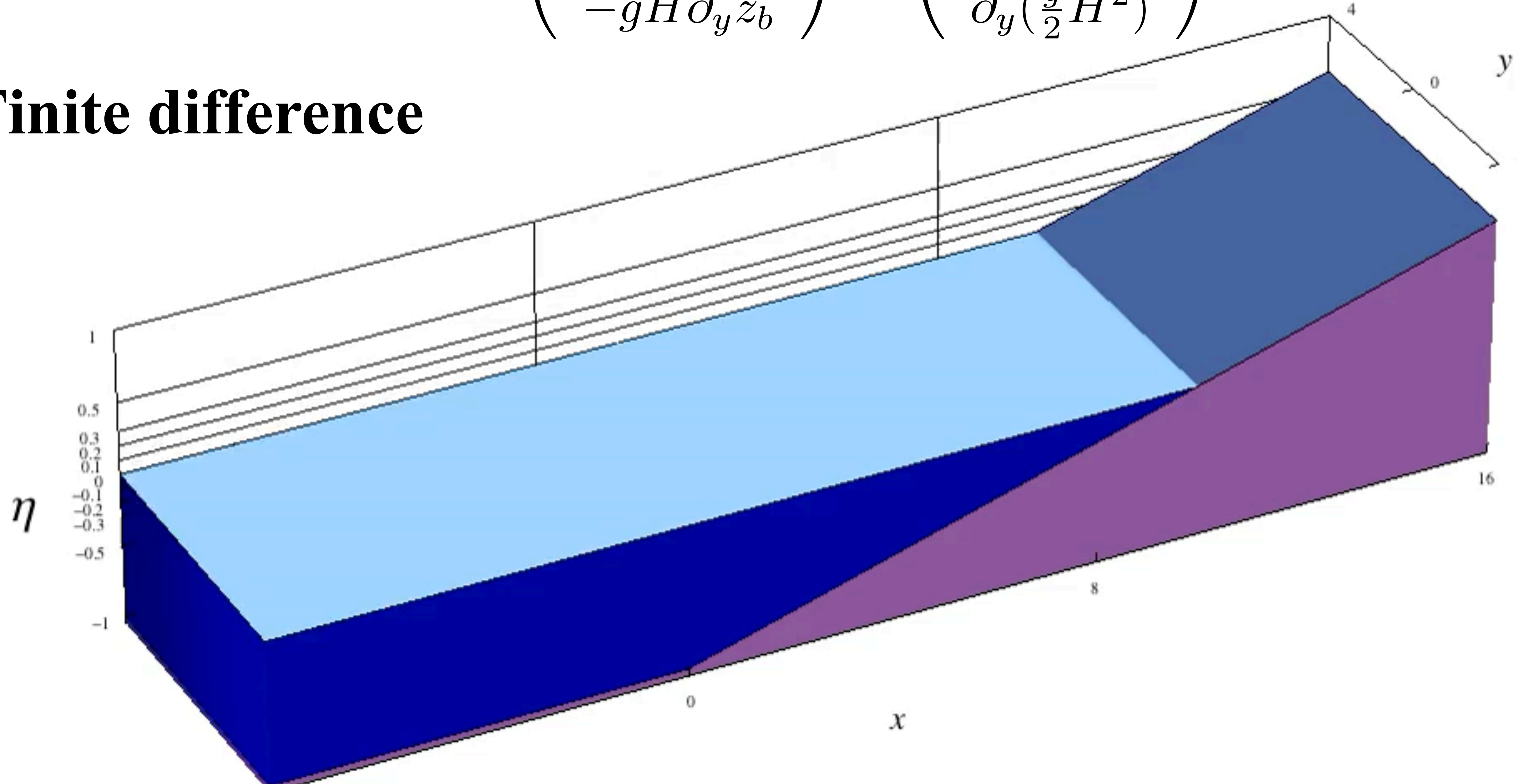


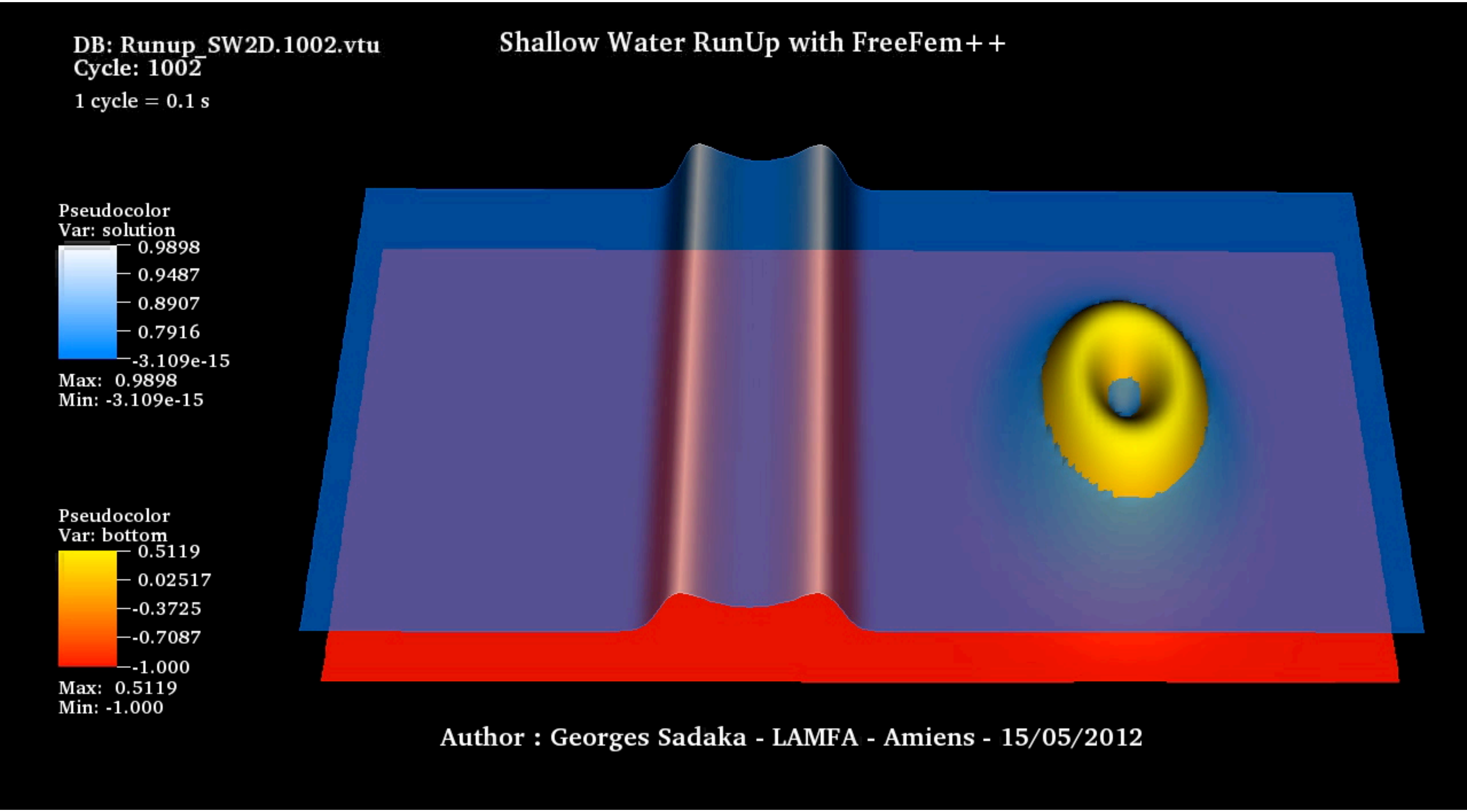
$$\partial_t \mathbf{U} + \mathbf{div}([F(\mathbf{U}), G(\mathbf{U})]) = S(\mathbf{U}),$$

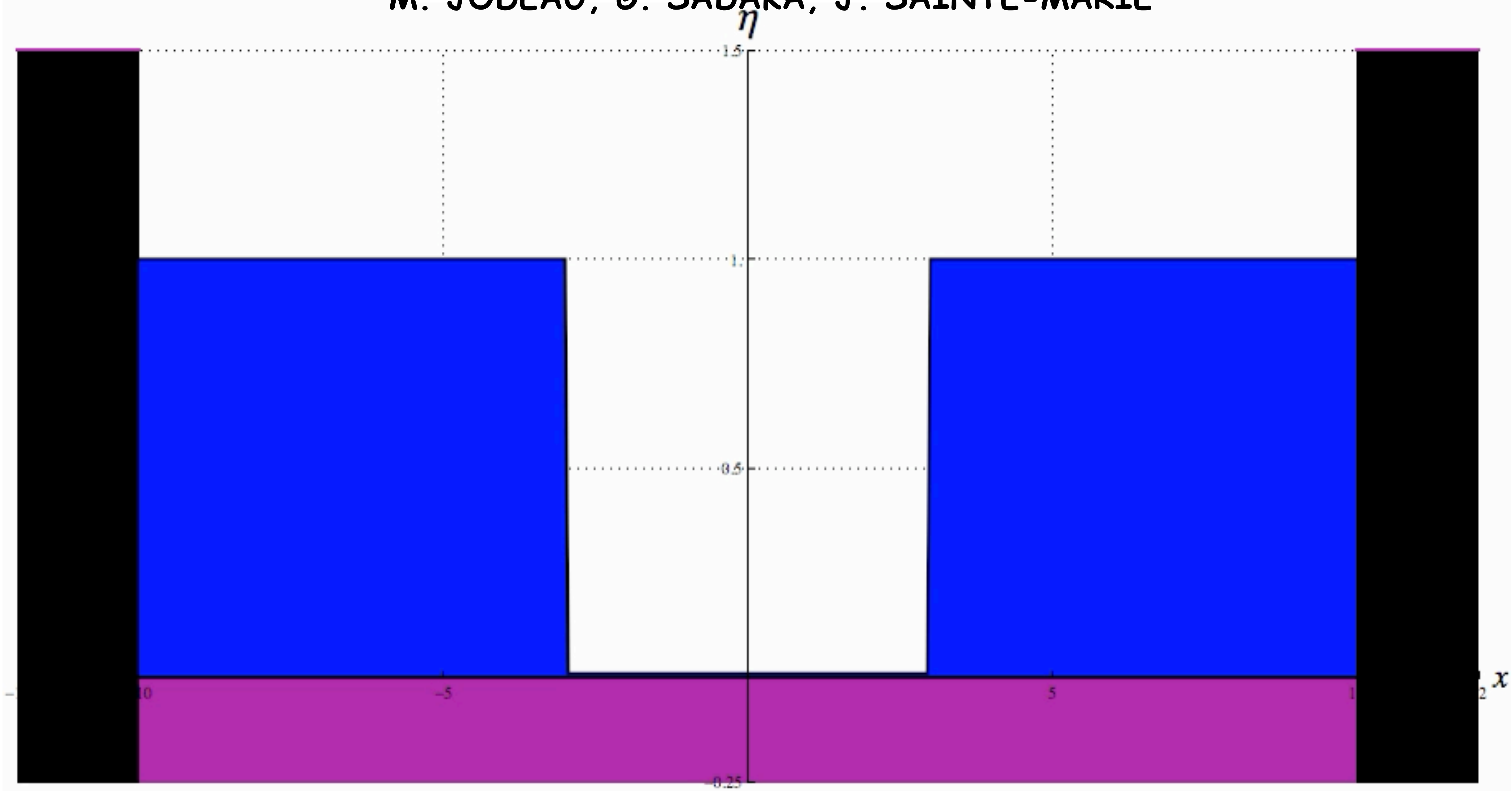
$$\mathbf{U} = \begin{pmatrix} H \\ Hu \\ Hv \end{pmatrix}, F(\mathbf{U}) = \begin{pmatrix} Hu \\ Hu^2 + \frac{g}{2}H^2 \\ Huv \end{pmatrix}, G(\mathbf{U}) = \begin{pmatrix} Hv \\ Huv \\ Hv^2 + \frac{g}{2}H^2 \end{pmatrix},$$

$$S(\mathbf{U}) = \begin{pmatrix} 0 \\ -gH\partial_x z_b \\ -gH\partial_y z_b \end{pmatrix} = \begin{pmatrix} 0 \\ \partial_x(\frac{g}{2}H^2) \\ \partial_y(\frac{g}{2}H^2) \end{pmatrix}$$

Finite difference



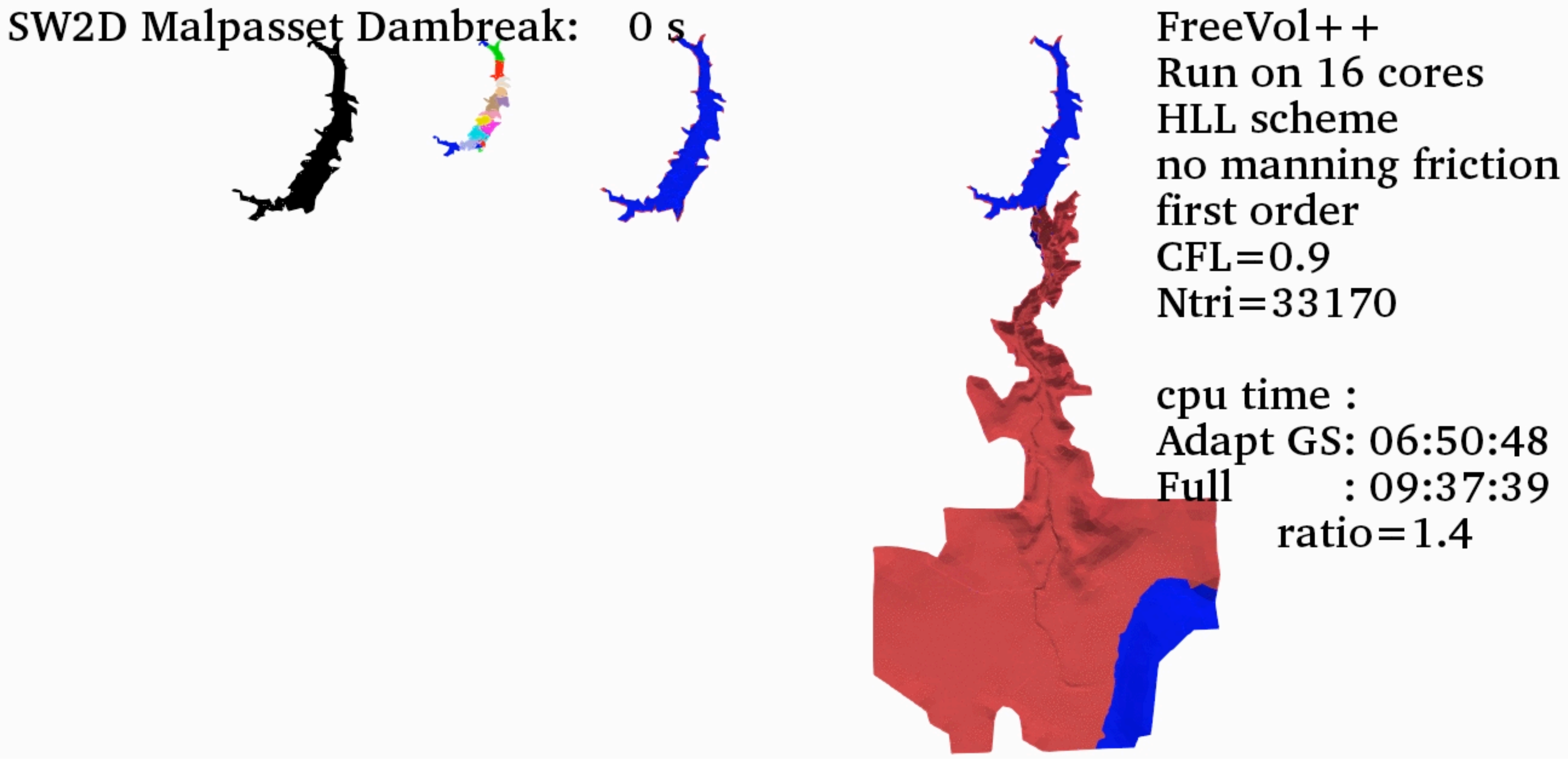

DB: Runup_SW2D.1002.vtu
Cycle: 1002
1 cycle = 0.1 s
Shallow Water RunUp with FreeFem++
Pseudocolor
Var: solution
0.9898
0.9487
0.8907
0.7916
-3.109e-15
Max: 0.9898
Min: -3.109e-15
Pseudocolor
Var: bottom
0.5119
0.02517
-0.3725
-0.7087
-1.000
Max: 0.5119
Min: -1.000
Author : Georges Sadaka - LAMFA - Amiens - 15/05/2012



ρ_s : density
p : porosity
Q_s : the bed load
$Q = Hu$: the water discharge

$$\begin{aligned}
\frac{\partial H}{\partial t} + \frac{\partial Q}{\partial x} &= 0, \\
\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{H} + \frac{g}{2}H^2\right) &= -gH\frac{\partial z_b}{\partial x}, \\
\rho_s(1-p)\frac{\partial z_b}{\partial t} + \frac{\partial Q_s}{\partial x} &= 0,
\end{aligned}$$

Finite Volume



Georges SADAKA, Emmanuel AUDUSSE, Nicolas SEGUIN,
Denys DUTYKH, Pierre JOLIVET, Frédéric HECHT

Mixing Finite Element and Finite Volume

$$\eta_t + \nabla \cdot \mathcal{V} + \nabla \cdot (\eta \mathcal{V}) - b \Delta \eta_t = 0;$$

$$\mathcal{V}_t + \nabla \eta + \tfrac{1}{2} \nabla |\mathcal{V}|^2 - d \Delta \mathcal{V}_t = 0,$$

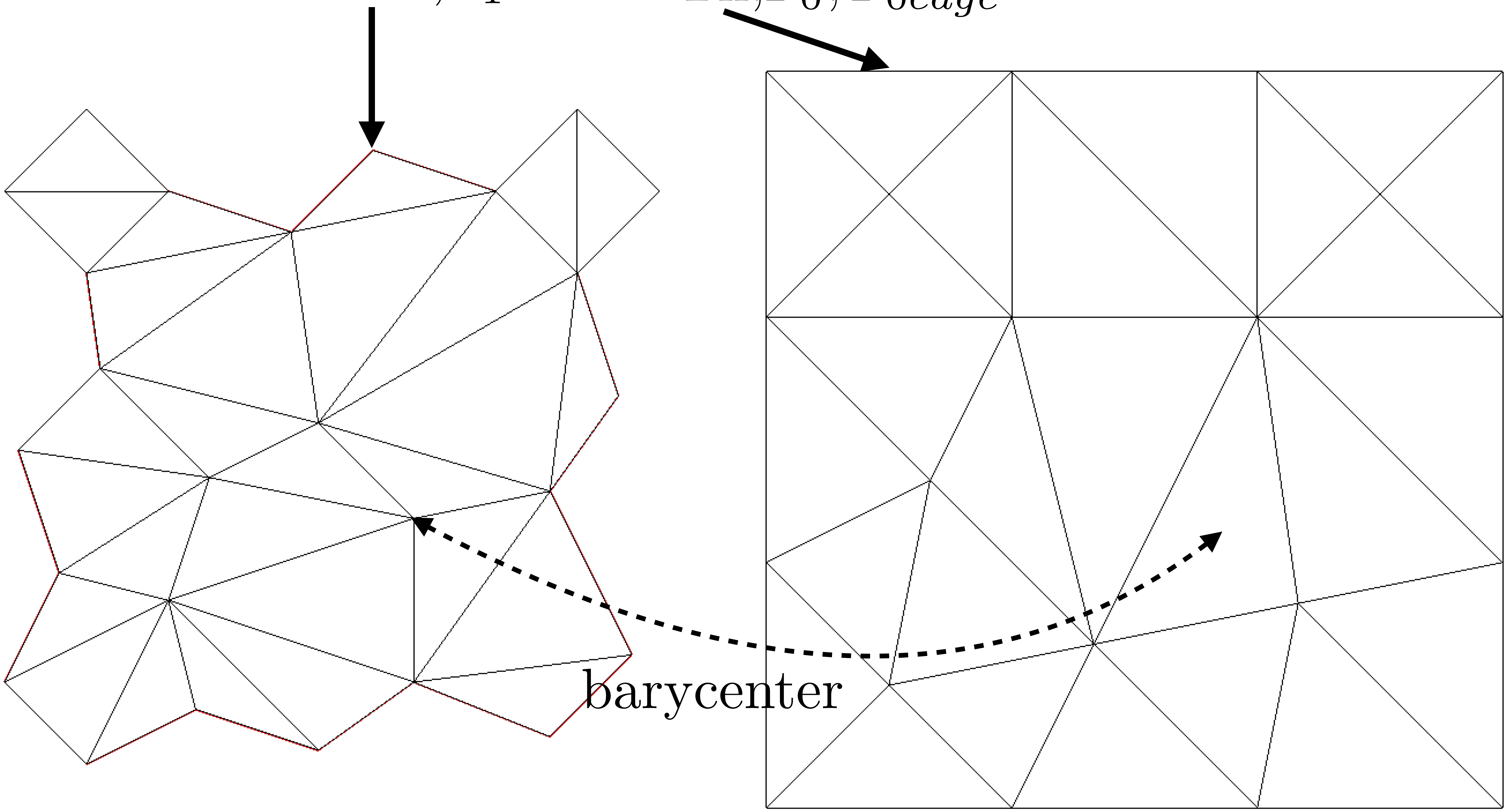
where b and d are positive parameters such that $b + d = 1/3$.

$$\mathcal{A}_{BBM} \partial_t \mathbf{E} + \mathbf{div}([F(\mathbf{E}), G(\mathbf{E})]) = 0,$$

$$\mathcal{A}_{BBM} = \begin{pmatrix} \bullet - b\Delta \bullet & 0 & 0 \\ 0 & \bullet - d\Delta \bullet & 0 \\ 0 & 0 & \bullet - d\Delta \bullet \end{pmatrix},$$

$$\mathbf{E} = \begin{pmatrix} \eta \\ u \\ v \end{pmatrix}, F(\mathbf{E}) = \begin{pmatrix} (1+\eta)u \\ \eta + \dfrac{1}{2}(u^2 + v^2) \\ 0 \end{pmatrix}, G(\mathbf{E}) = \begin{pmatrix} (1+\eta)v \\ 0 \\ \eta + \dfrac{1}{2}(u^2 + v^2) \end{pmatrix}.$$

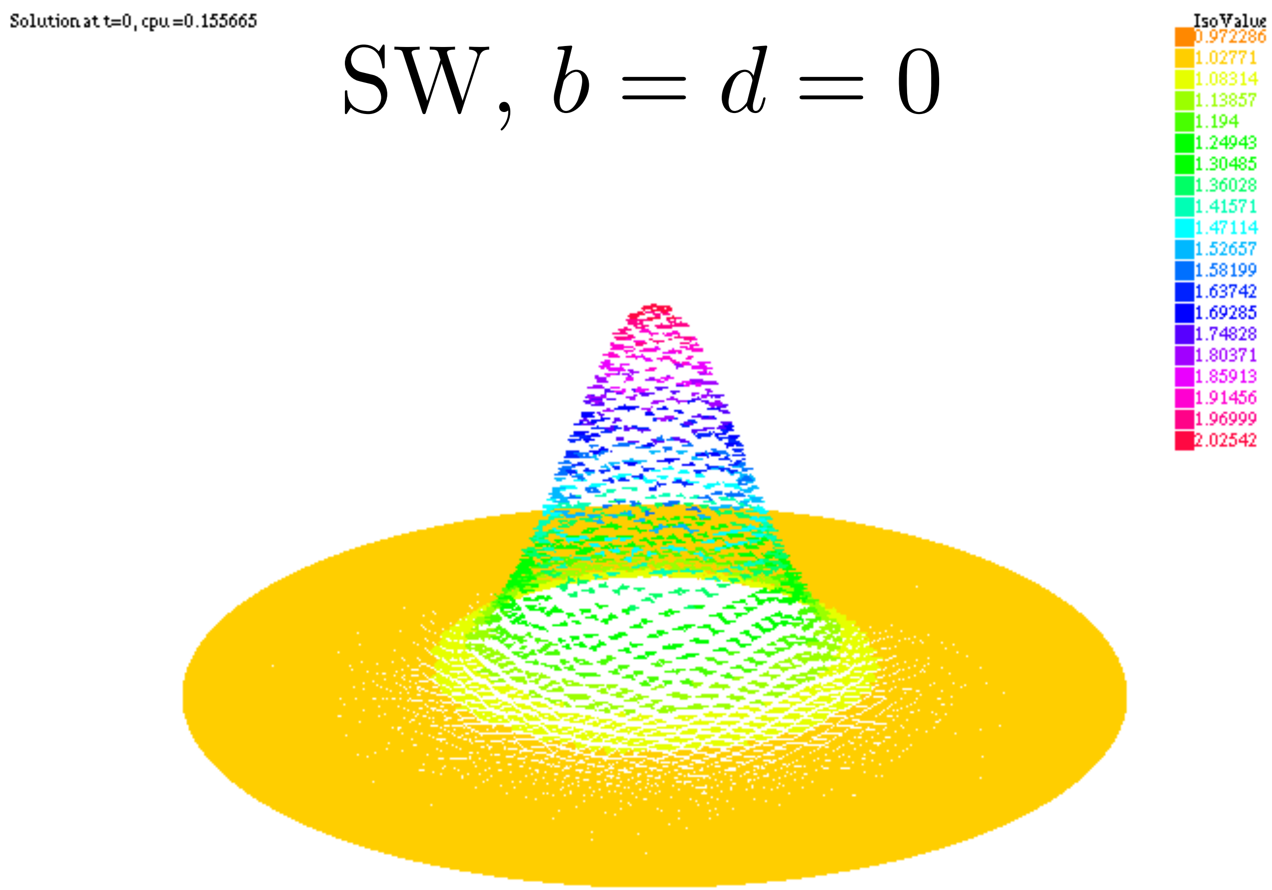
$$\underbrace{\mathcal{A}_{BBM}\partial_t \mathbf{E}}_{\underbrace{\mathbf{FE}}_{Th1,\mathbb{P}_1}} + \underbrace{\mathbf{div}([F(\mathbf{E}), G(\mathbf{E})])}_{\underbrace{\mathbf{FV}}_{Th,\mathbb{P}_0,\,\mathbb{P}_{0edge}}} = 0,$$



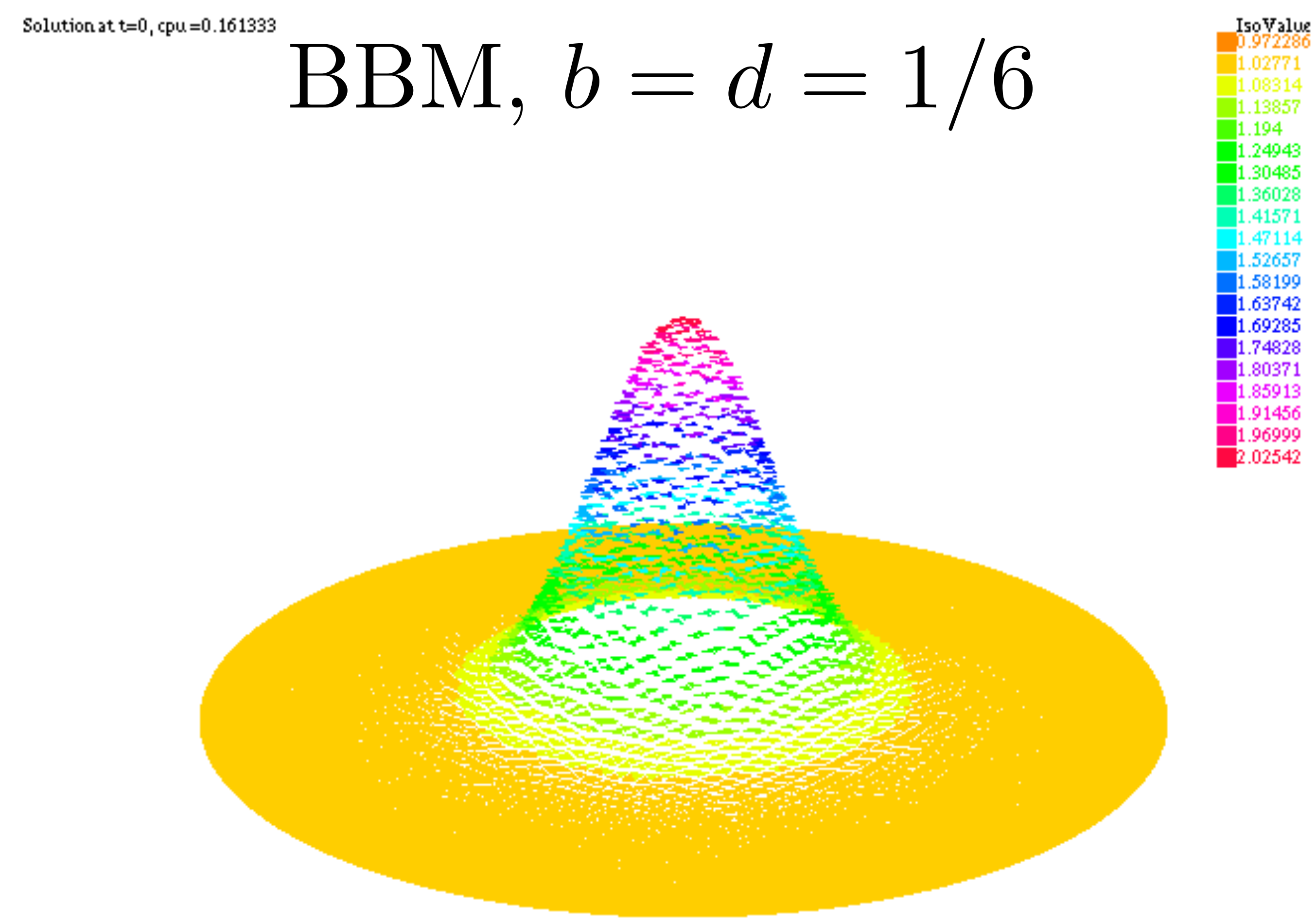
$$\eta_t + \nabla\cdot\mathcal{V} + \nabla\cdot(\eta\mathcal{V}) - b\Delta\eta_t = 0;$$

$$\mathcal{V}_t + \nabla\eta + \frac{1}{2}\nabla|\mathcal{V}|^2 - d\Delta\mathcal{V}_t = 0,$$

SW, $b = d = 0$

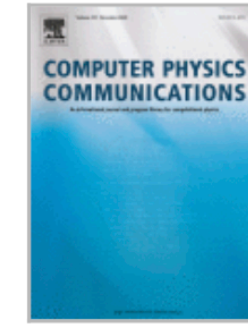


BBM, $b = d = 1/6$



A finite element toolbox for the Bogoliubov-de Gennes stability analysis of Bose-Einstein condensates ☆,☆☆

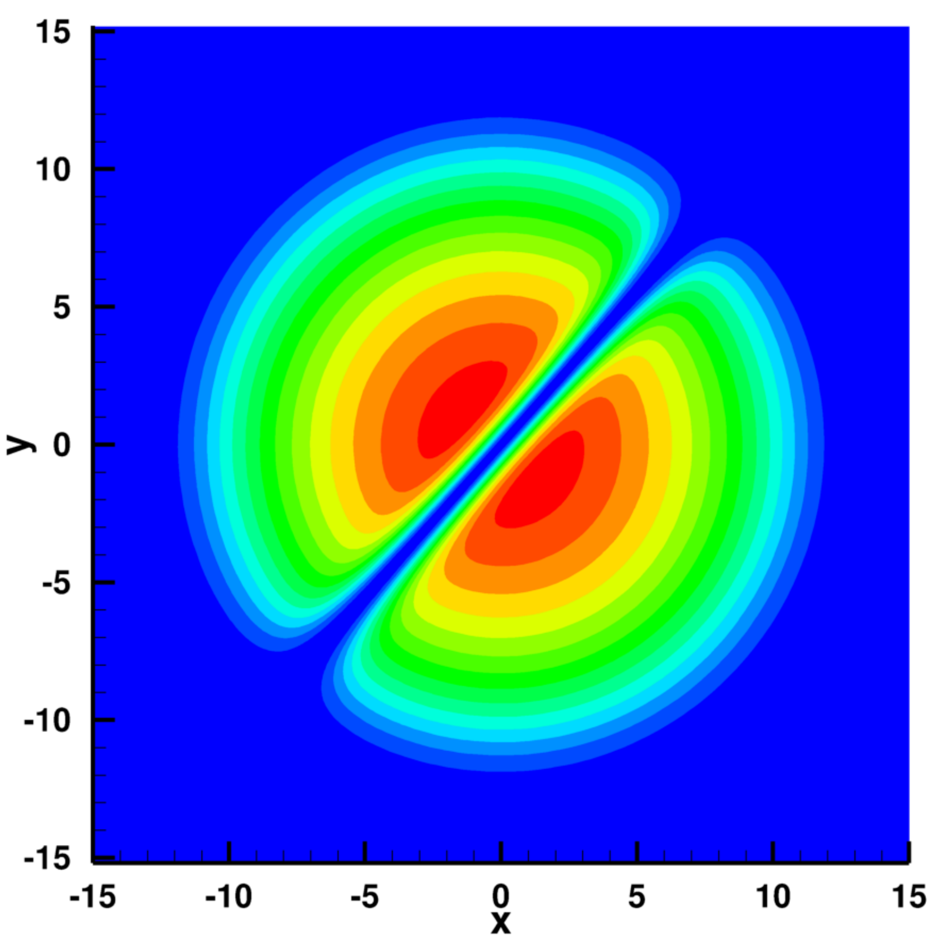
Toolbox FREE

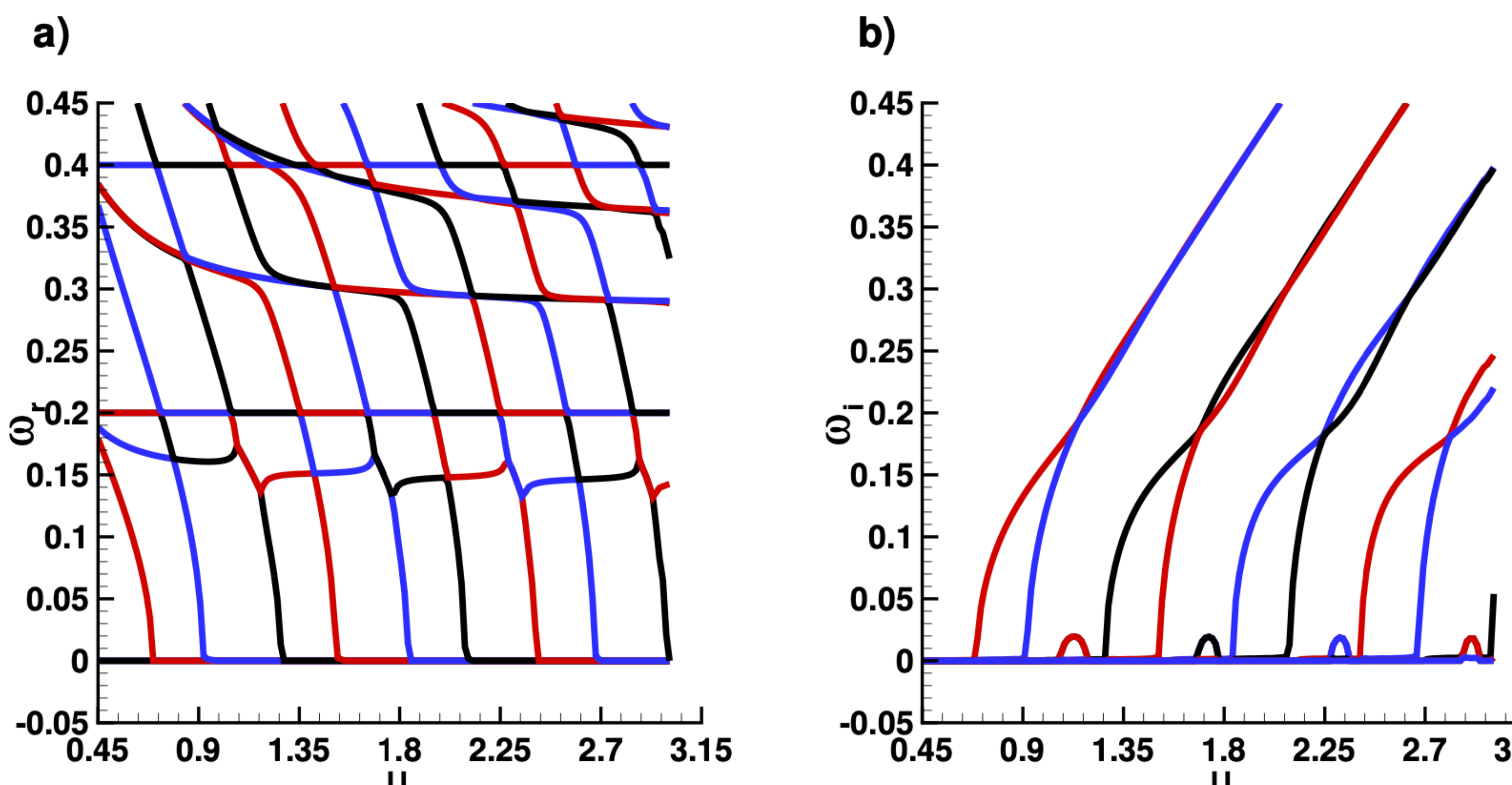


Georges Sadaka [a], Victor Kalt [a], Ionut Danaila [a,*], Frédéric Hecht [b]

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V_{\text{trap}}\psi + g|\psi|^2\psi,$$

$$\begin{pmatrix} \mathcal{H} - \mu + 2g|\phi|^2 & g\phi^2 \\ -g\overline{\phi}^2 & -(\mathcal{H} - \mu + 2g|\phi|^2) \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \hbar\omega \begin{pmatrix} A \\ B \end{pmatrix}.$$

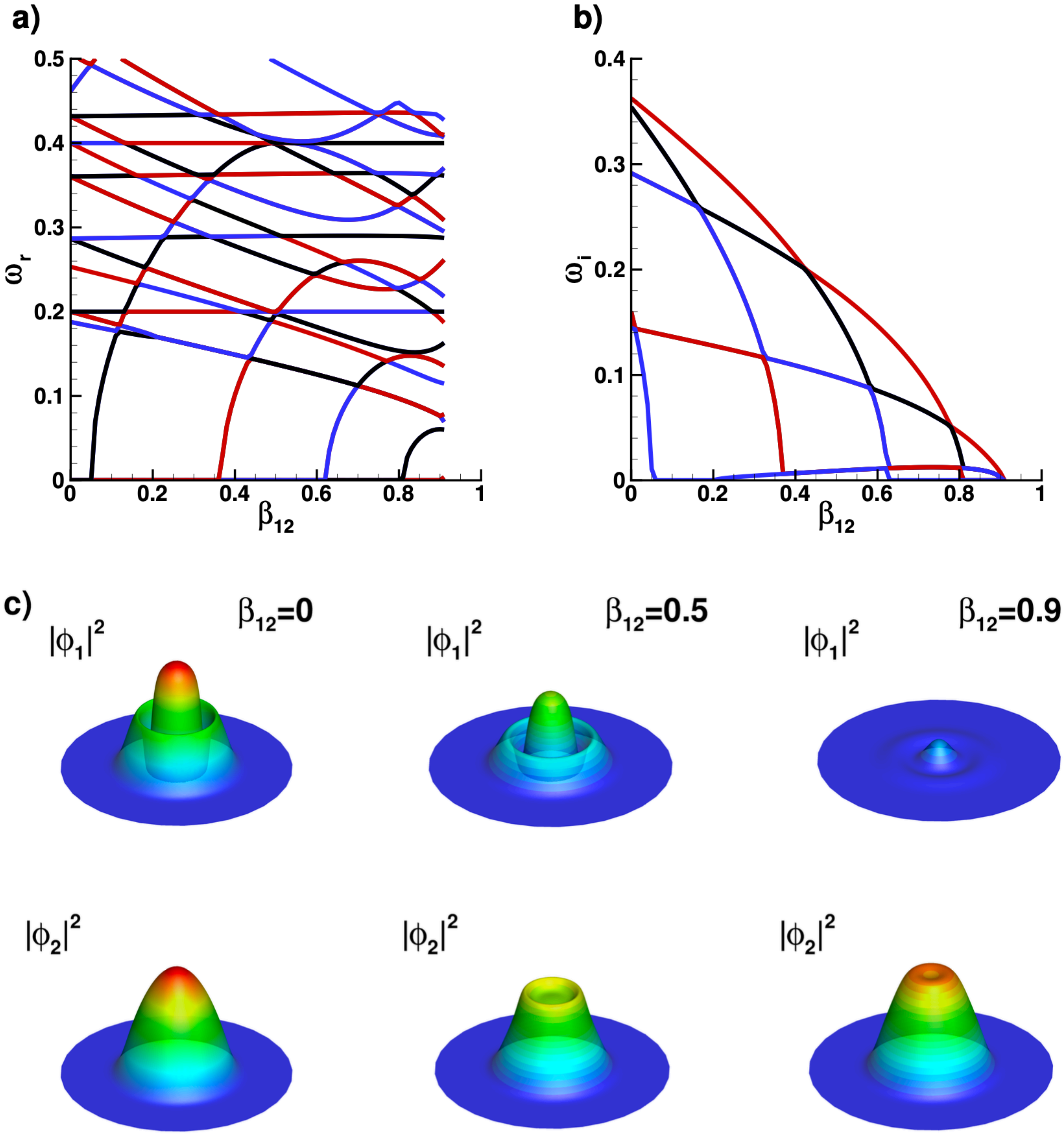




$$
\begin{cases}
i\hbar\dfrac{\partial\psi_1}{\partial t} = \left(-\dfrac{\hbar^2}{2m}\nabla^2 + V_{\text{trap}} + g_{11}|\psi_1|^2 + g_{12}|\psi_2|^2\right)\psi_1, \\
i\hbar\dfrac{\partial\psi_2}{\partial t} = \left(-\dfrac{\hbar^2}{2m}\nabla^2 + V_{\text{trap}} + g_{21}|\psi_1|^2 + g_{22}|\psi_2|^2\right)\psi_2.
\end{cases}
$$

$$
M = \begin{pmatrix}
M_{11} & g_{11}\phi_1^2 & g_{12}\phi_1\overline{\phi_2} & g_{12}\phi_1\phi_2 \\
-g_{11}\overline{\phi_1}^2 & M_{22} & -g_{12}\overline{\phi_1\phi_2} & -g_{12}\overline{\phi_1}\phi_2 \\
g_{21}\overline{\phi_1}\phi_2 & g_{21}\phi_1\phi_2 & M_{33} & g_{22}\phi_2^2 \\
-g_{21}\overline{\phi_1\phi_2} & -g_{21}\phi_1\overline{\phi_2} & -g_{22}\overline{\phi_2}^2 & M_{44}
\end{pmatrix},
$$

$$
\begin{cases}
M_{11} &= \mathcal{H} - \mu_1 + 2g_{11}|\phi_1|^2 + g_{12}|\phi_2|^2, \\
M_{22} &= -M_{11}, \\
M_{33} &= \mathcal{H} - \mu_2 + g_{21}|\phi_1|^2 + 2g_{22}|\phi_2|^2, \\
M_{44} &= -M_{33}.
\end{cases}
\qquad
M\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \hbar\omega\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix},
$$



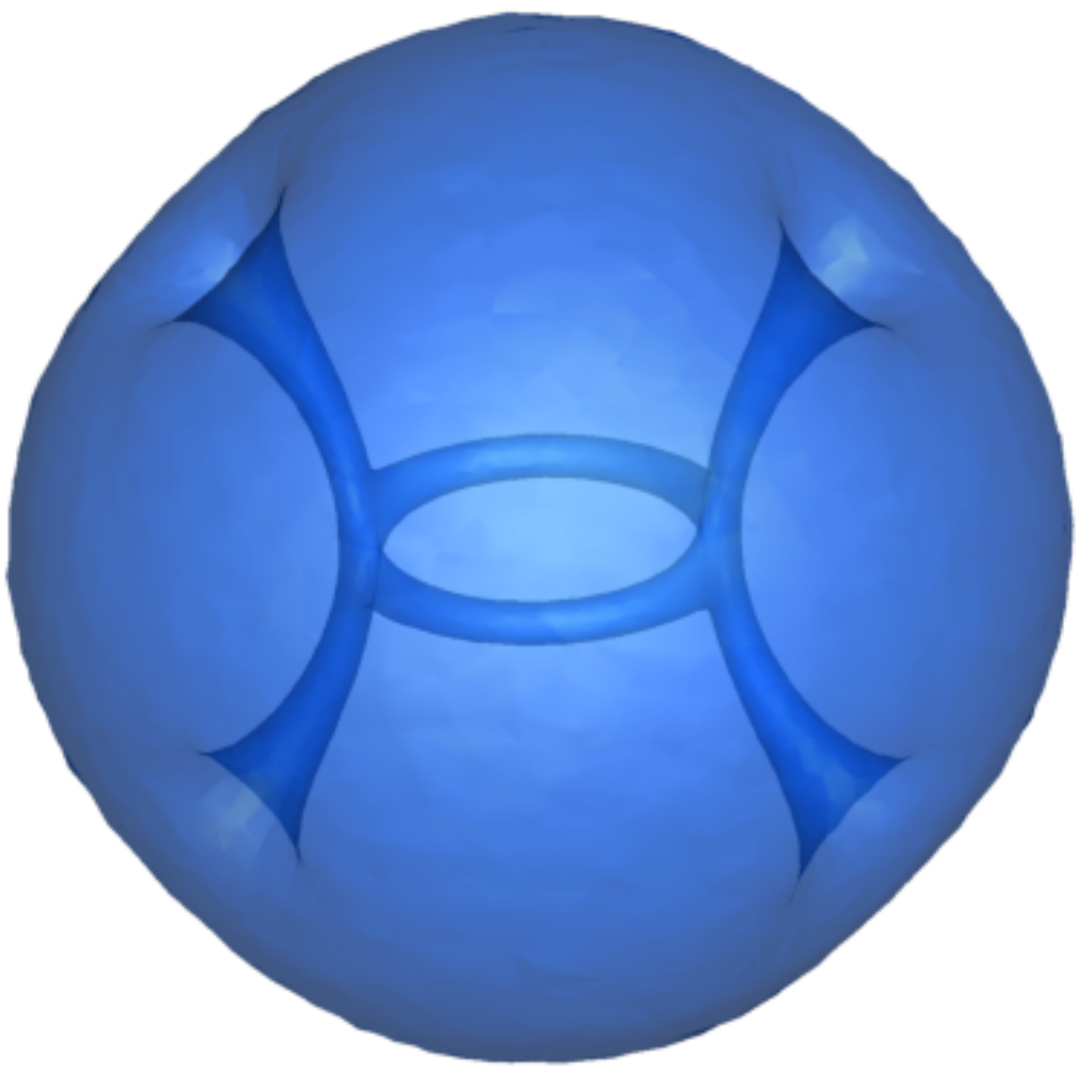
a)
b)
0.5
0.4
0.3
0.2
0.1
0
ω_r
0
0.2
0.4
0.6
0.8
1
β_{12}
0.4
0.3
0.2
0.1
0
ω_i
0
0.2
0.4
0.6
0.8
1
β_{12}
c)
β_{12}=0
β_{12}=0.5
β_{12}=0.9
$|\phi_1|^2$
$|\phi_1|^2$
$|\phi_1|^2$
$|\phi_2|^2$
$|\phi_2|^2$
$|\phi_2|^2$

Parallel finite-element codes for the Bogoliubov-de Gennes stability analysis of Bose-Einstein condensates

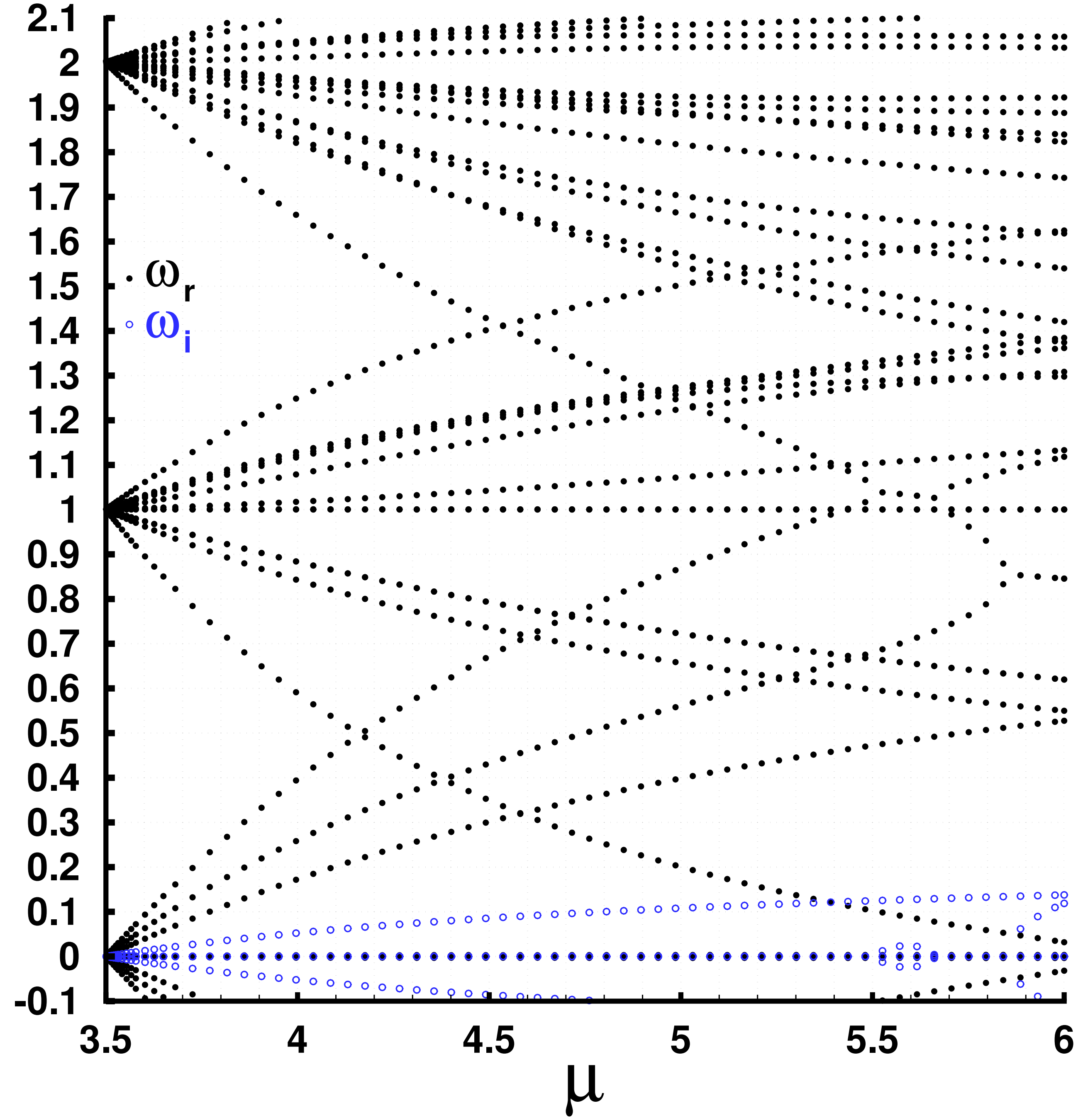
To be published

Toolbox FREE

Georges Sadaka[a], Pierre Jolivet[b], Efstathios G. Charalampidis[a,c], Ionut Danaila[a,*]



3E+05 unknown

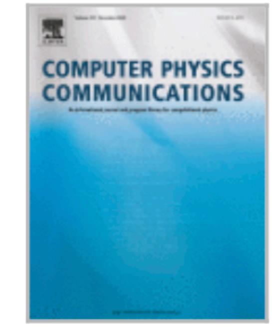


50E+06 non zero element

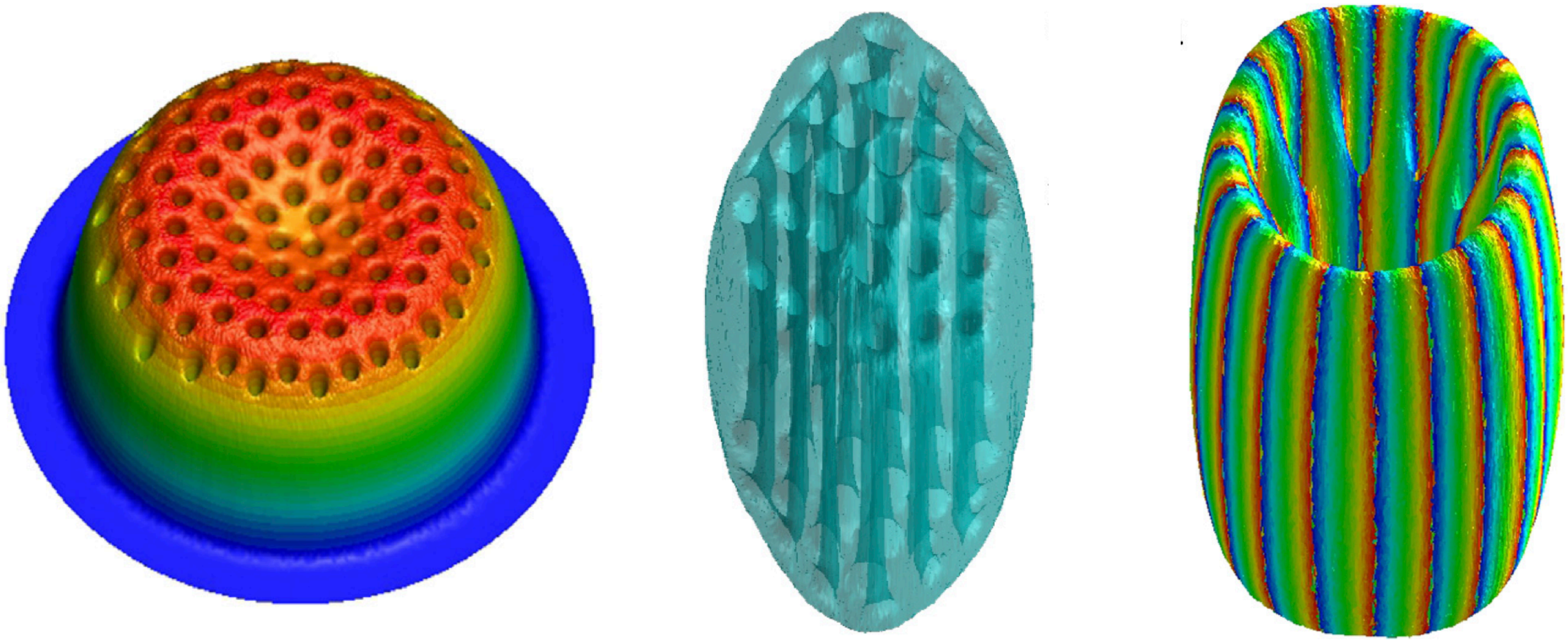
A finite-element toolbox for the stationary Gross–Pitaevskii equation with rotation☆

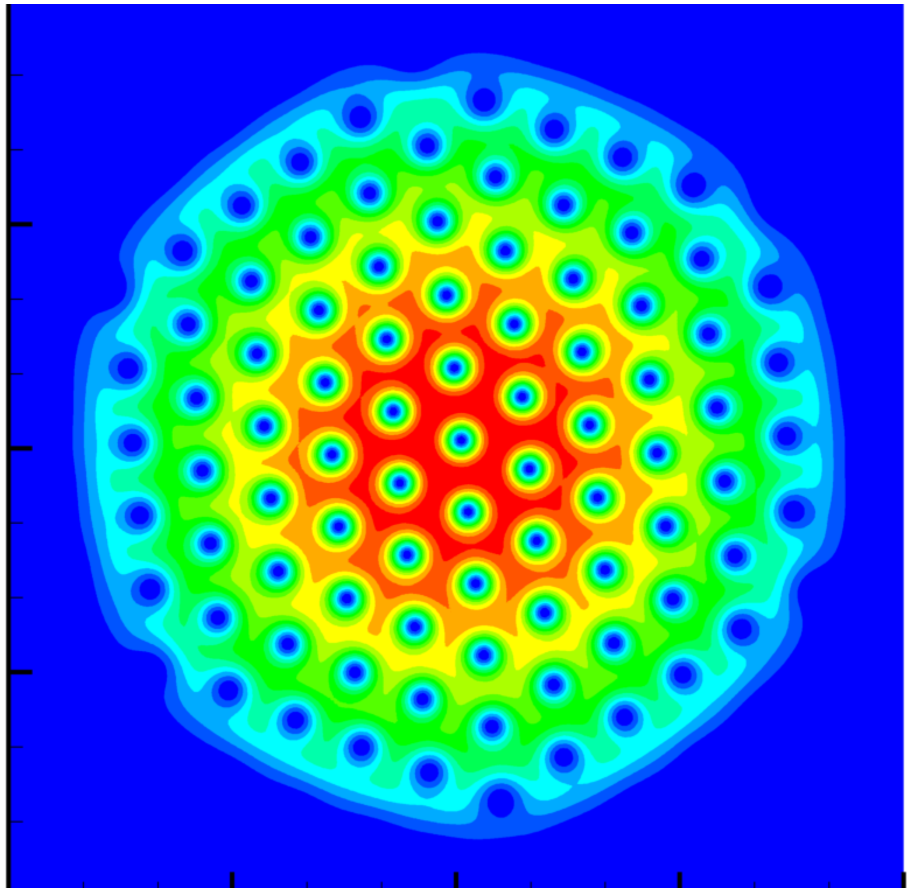
Guillaume Vergez[a,b], Ionut Danaila[a,*], Sylvain Auliac[b], Frédéric Hecht[b]

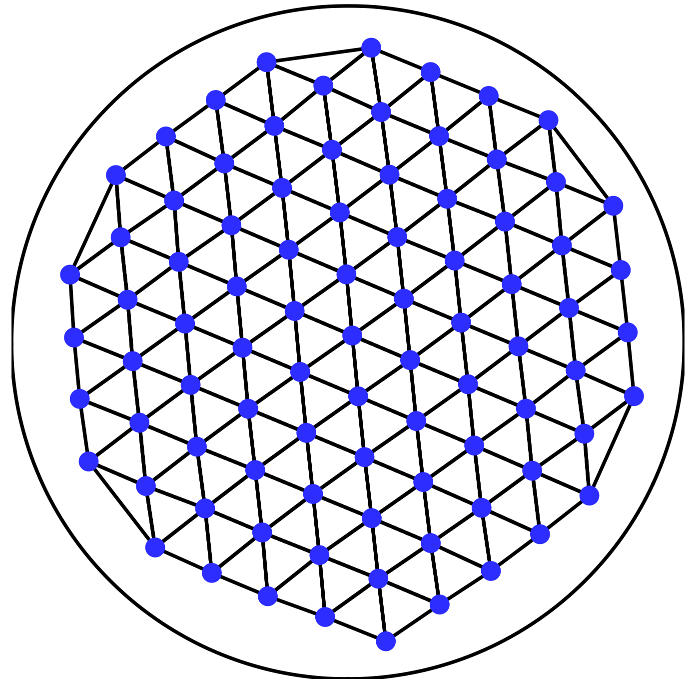
Toolbox FREE

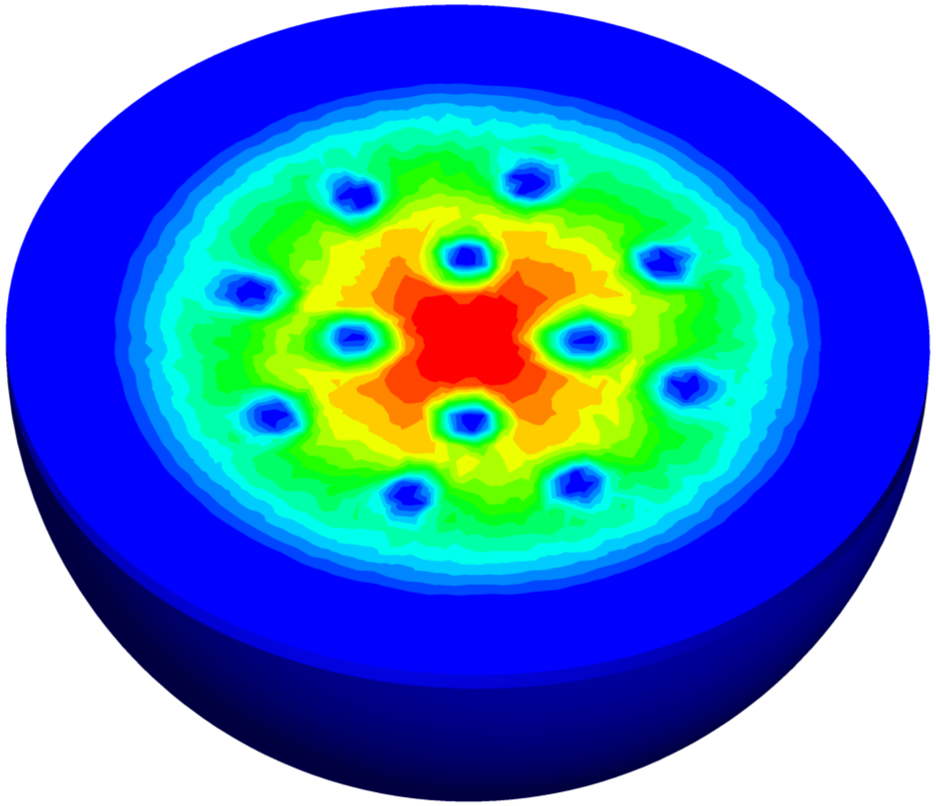


$$-\frac{\hbar^2}{2m}\nabla^2\psi + V_{\text{trap}}\psi + g|\psi|^2\psi - i\hbar\Omega A^t\nabla\psi = \mu\psi$$



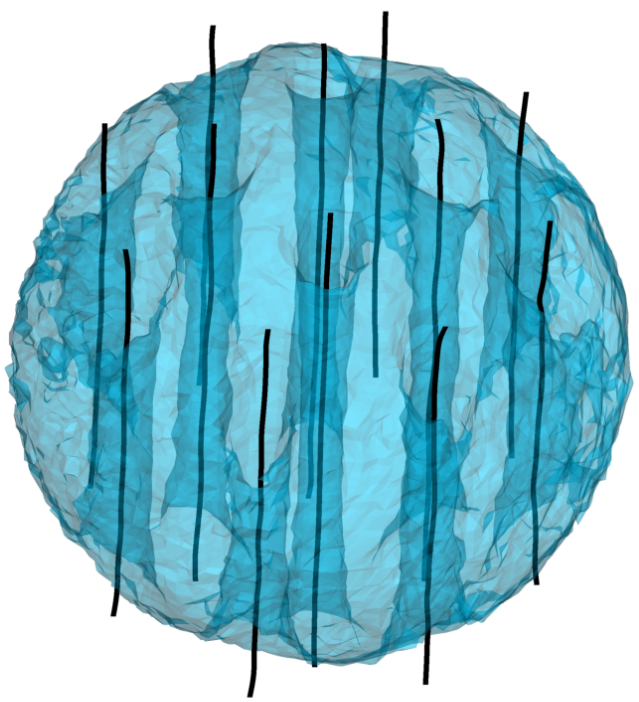






Identification of vortices in quantum fluids: Finite element algorithms and programs ☆, ☆☆

Victor Kalt [a], Georges Sadaka [a], Ionut Danaila [a], Frédéric Hecht [b]



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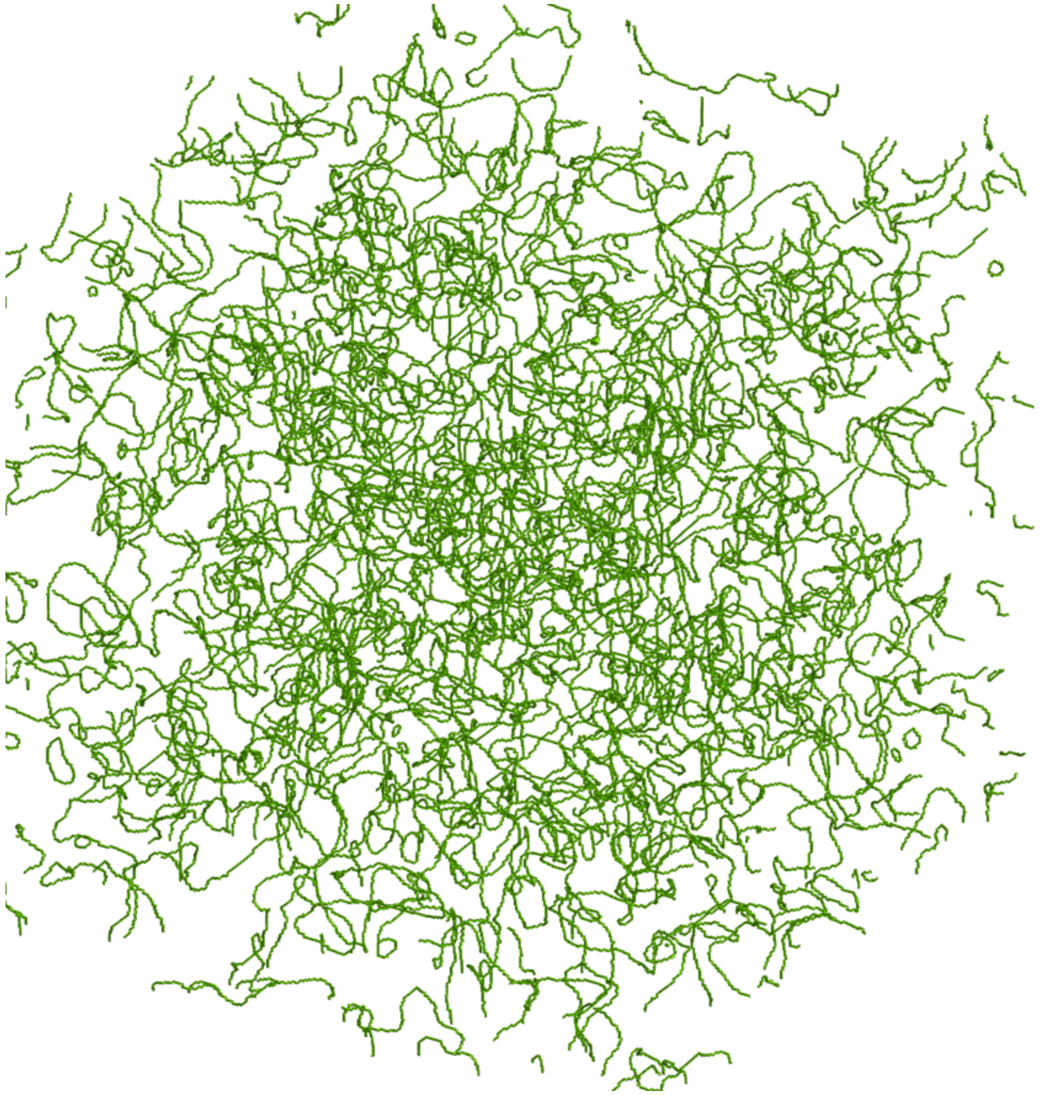
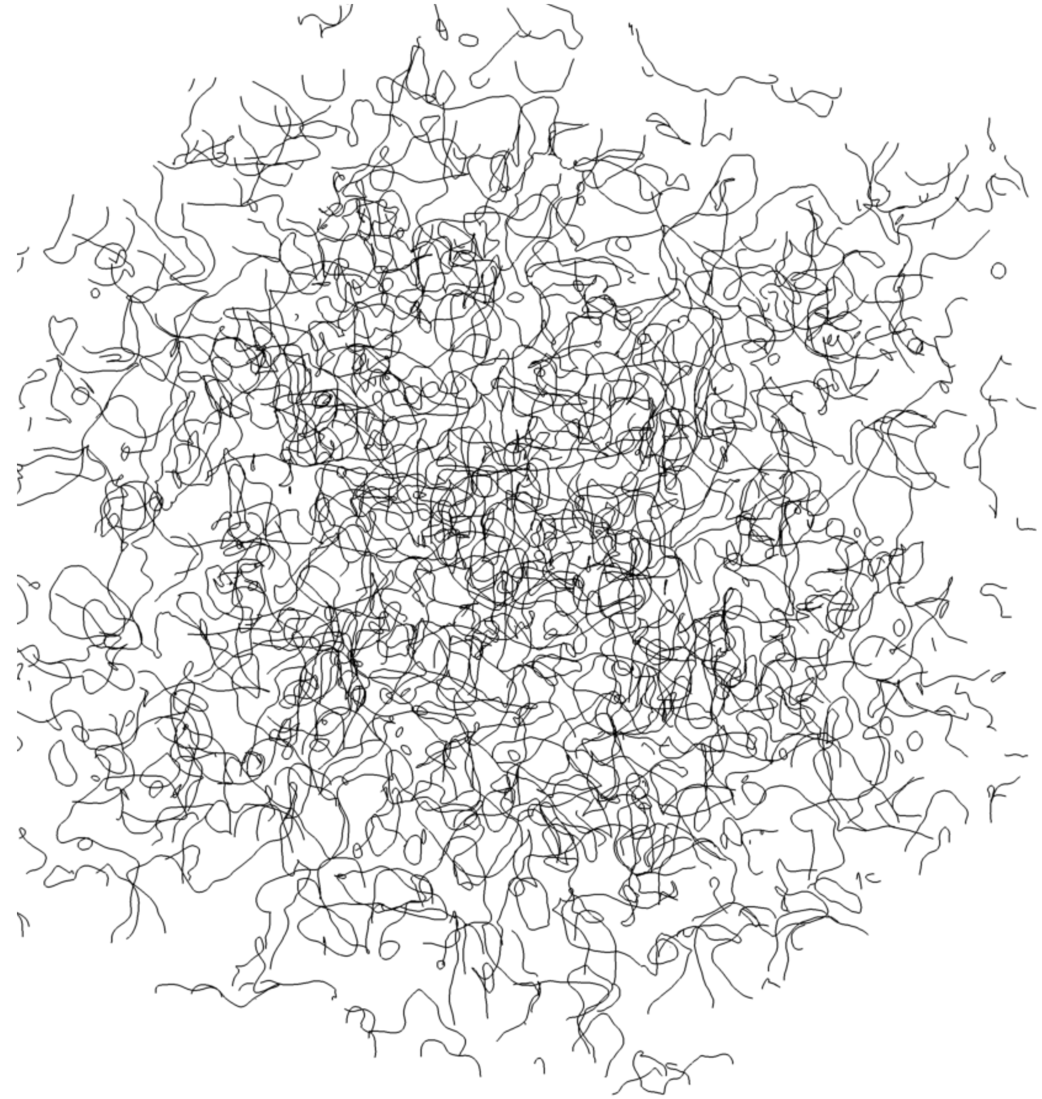
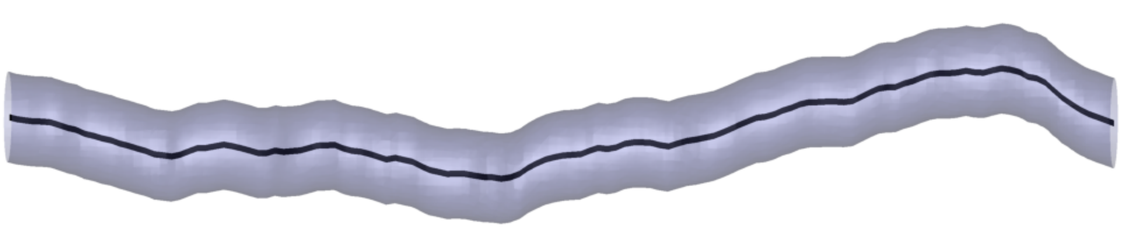
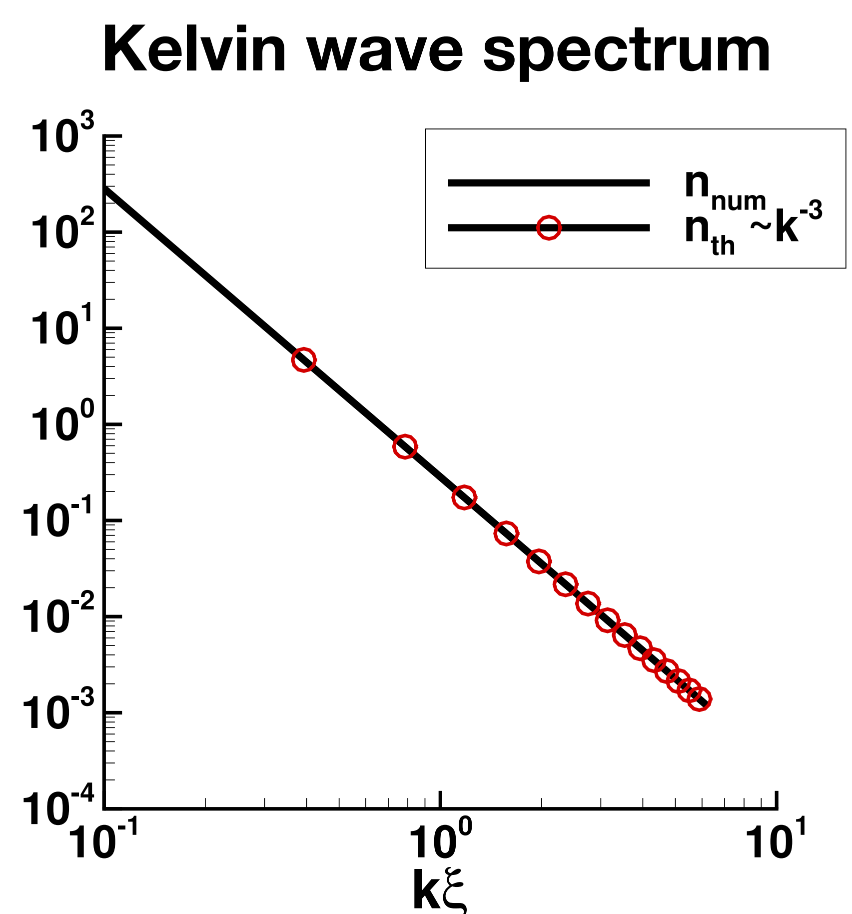


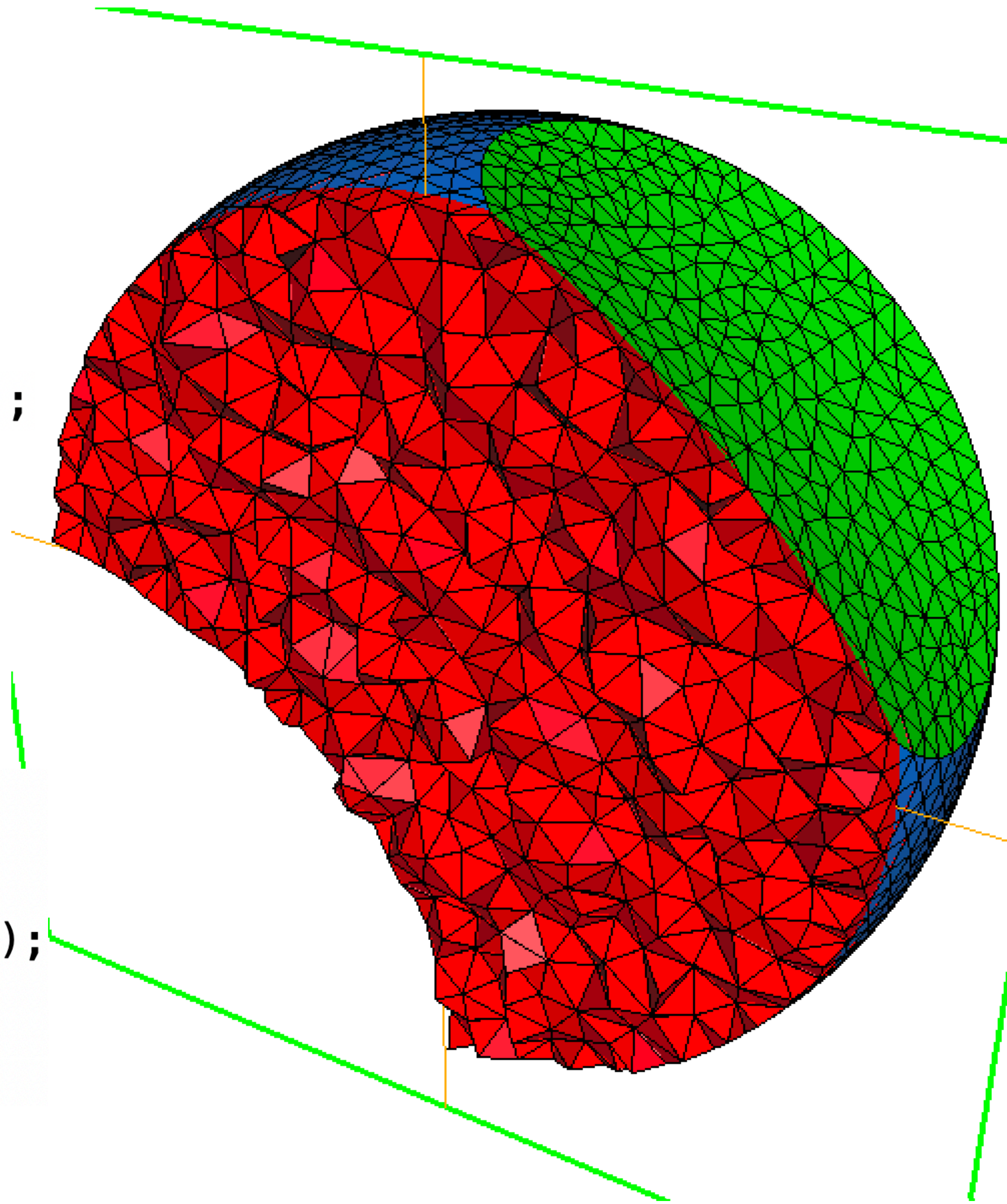





Kelvin wave spectrum
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-1}
10^{0}
10^{1}
$k\xi$
n_{num}
$n_{th} \sim k^{-3}$

Some tricks with FreeFEM

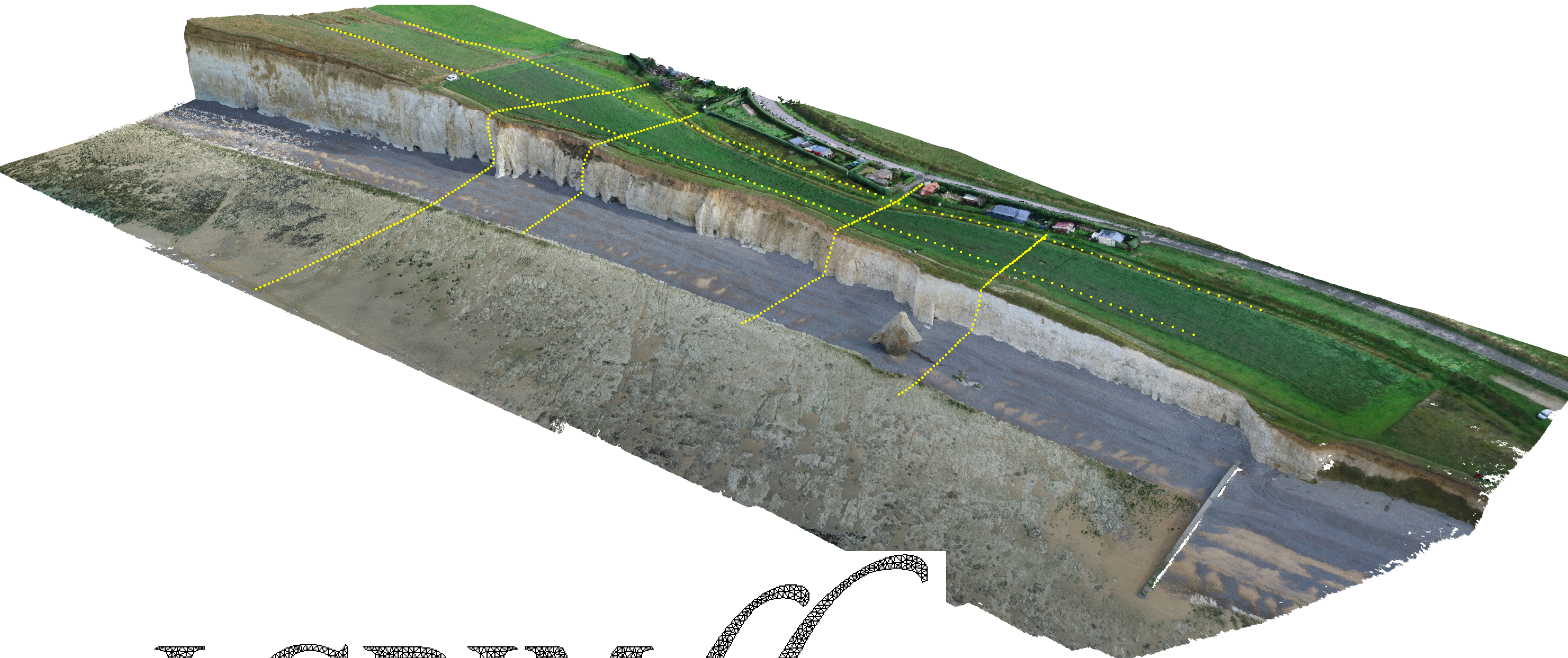
```
include "MeshSurface.idp"
meshS ThS=Sphere20(1.,17,1,0);
ThS=change(ThS,rmInternalEdges=1,fregion=0);

fespace Vh(ThS,P1);
Vh u=abs(-x^2+y+z);

ThS = mmgs(ThS,ls=1.,iso=1,hmin=.1,
      hmax=.1,hausd=1.e+0,metric=u[]);

ThS = trunc(ThS,region==3,fregion=0);

meshL ThL = extract(ThS,label=labels(ThS));

load "meshtools"
fespace Ph(ThL,P0);
Ph cste;
int nbc = ConnectedComponents(ThL,
                    cste[],closure=1);

include "buildmeshS.idp"
for(int i=0;i<nbc;i++){
   meshL ThLtemp = trunc(ThL,cste==i);
   meshS ThStemp = buildmeshSLap(ThLtemp,1);
   ThStemp = change(ThStemp,fregion=i+1);
   ThS = ThS + ThStemp;
}

load "tetgen"
real[int] domain = [0.,0.,0.,1,.1^3/6];
mesh3 Th3=tetg(ThS,switch="paAAQYY",
      nbofregions=1,regionlist=domain);
```



Sainte-Marguerite Sur-Mer cliff - Normandie

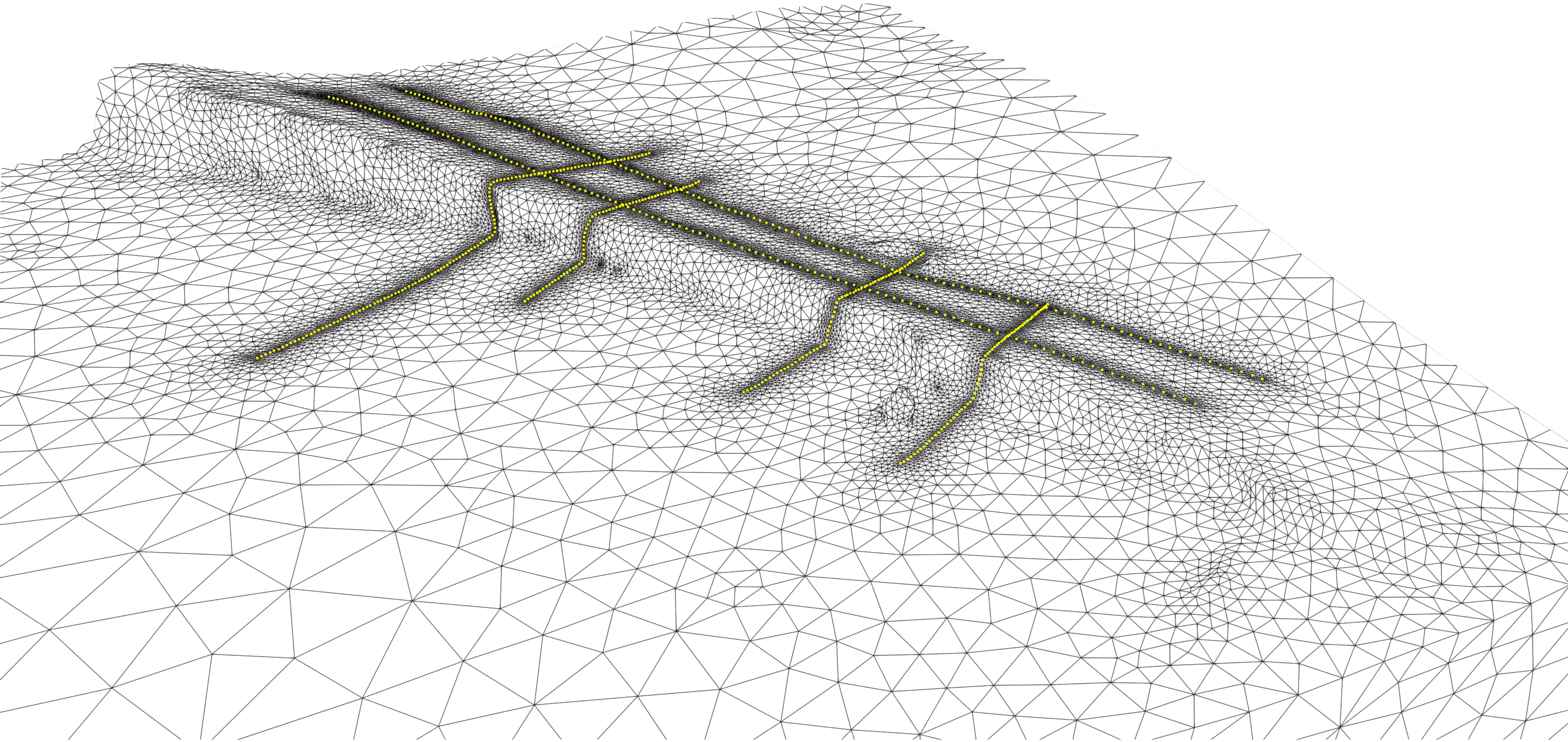
Cerema ENDSUM

Georges SADAKA, Cyrille FAUCHARD, Antoine TONNOIR, Raphaël ANTOINE,
Vincent GUILBERT, Bruno BEAUCAMP, Theau COUSIN, Victor KALT, Cyril LEDUN



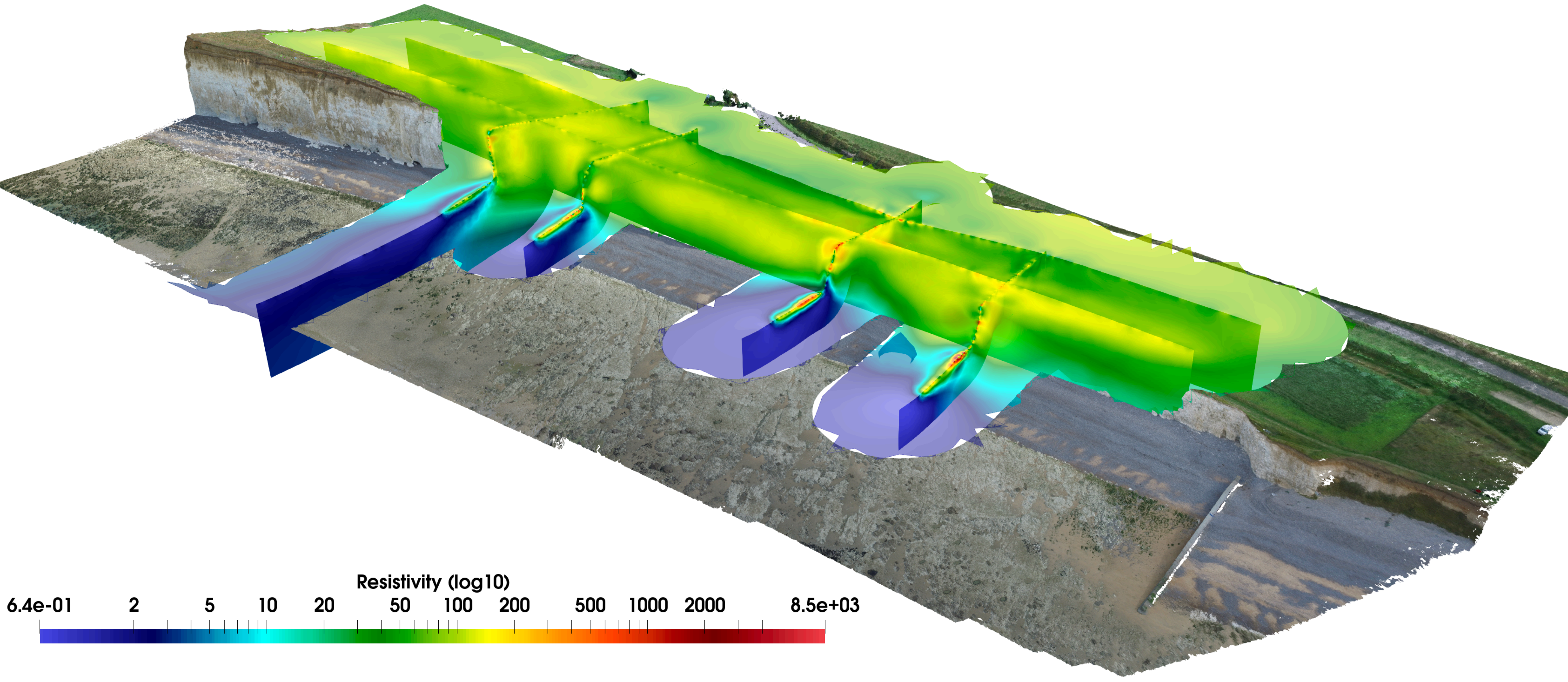
pyLGRIMff

Building Mesh with open3D, meshlab and FreeFem



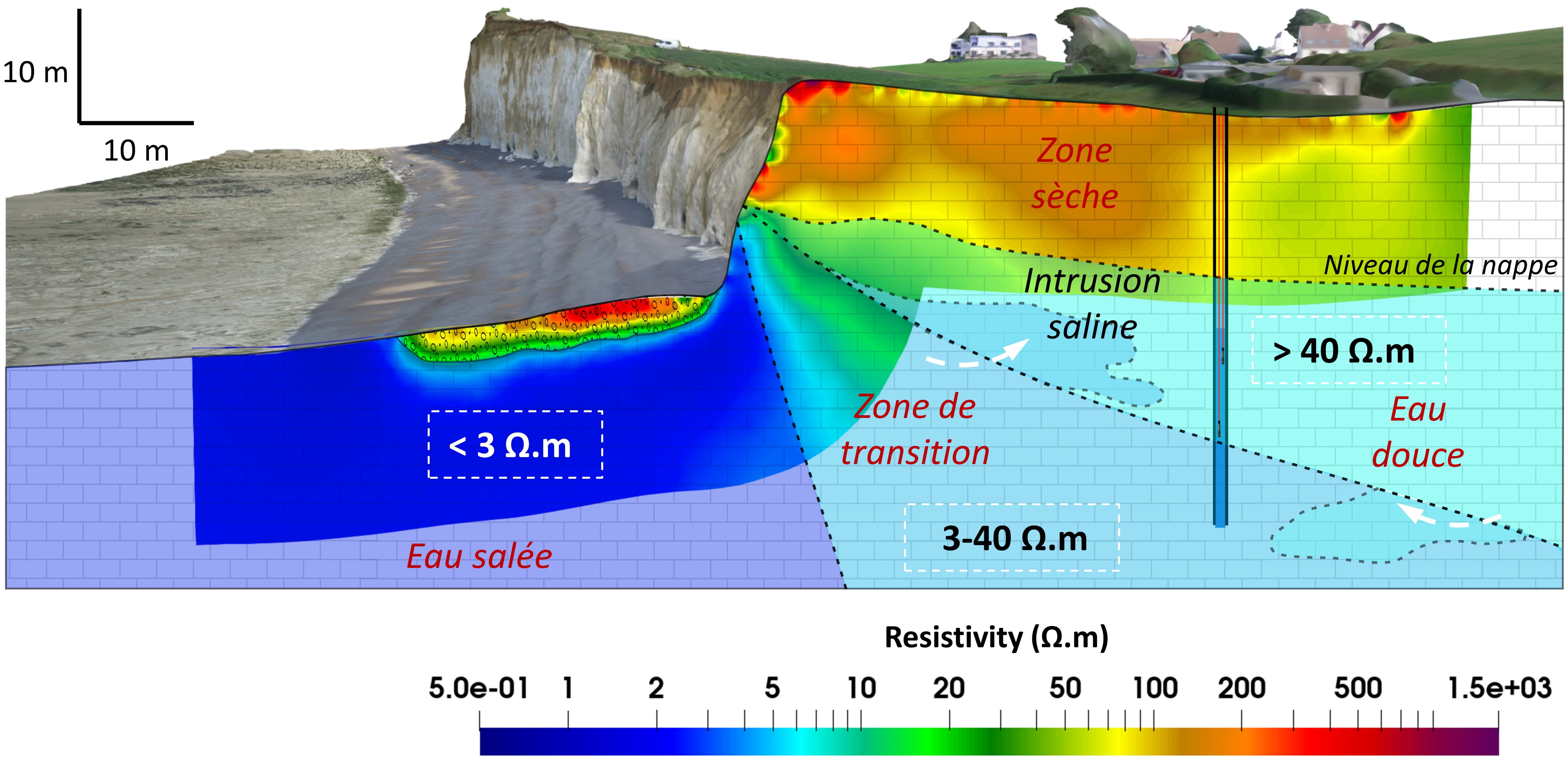
Electrical Resistivity Imaging of SMSM 2022

Cerema
ENDSUM



Electrical Resistivity Imaging on one profile

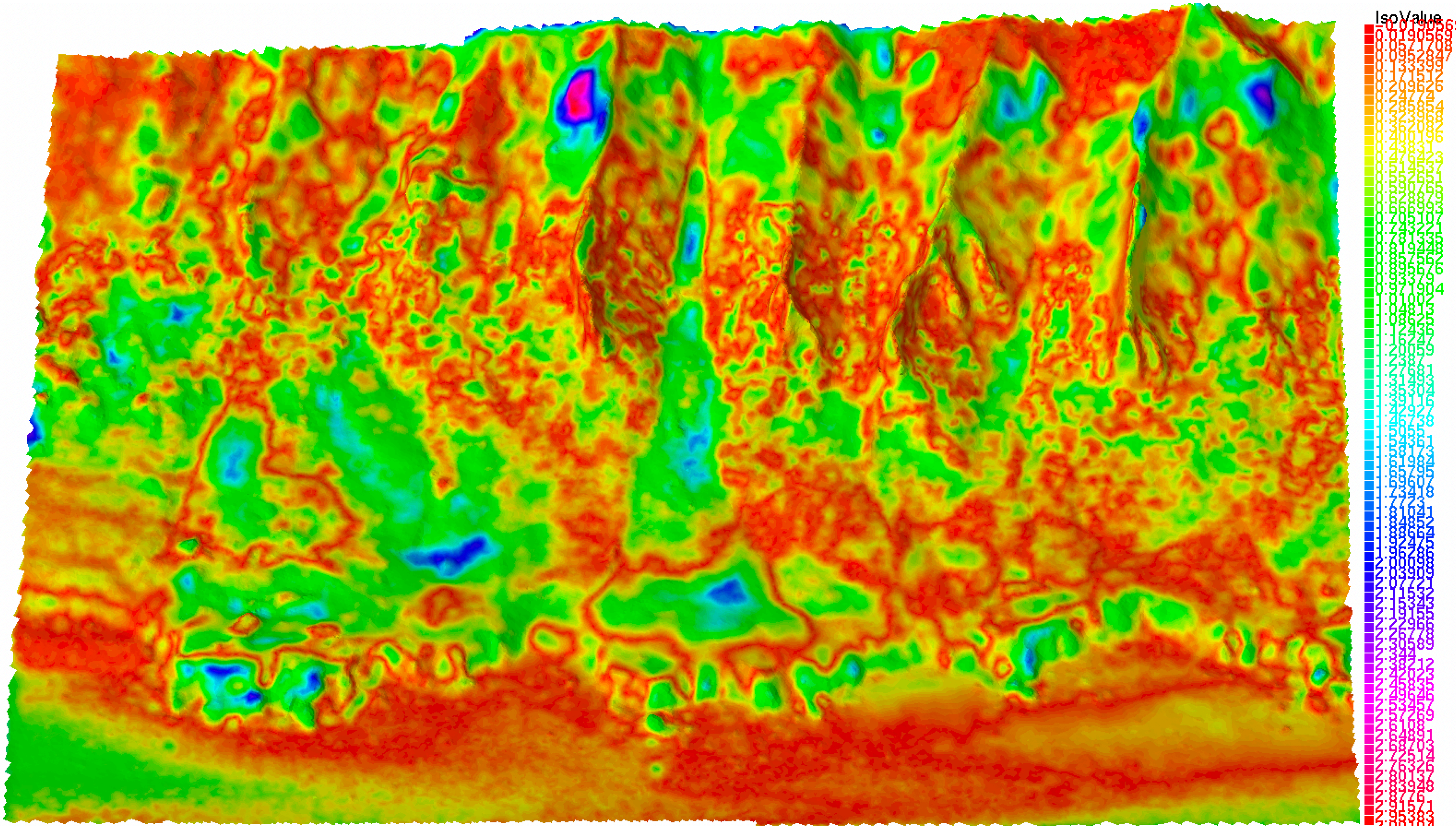
Cerema
ENDSUM



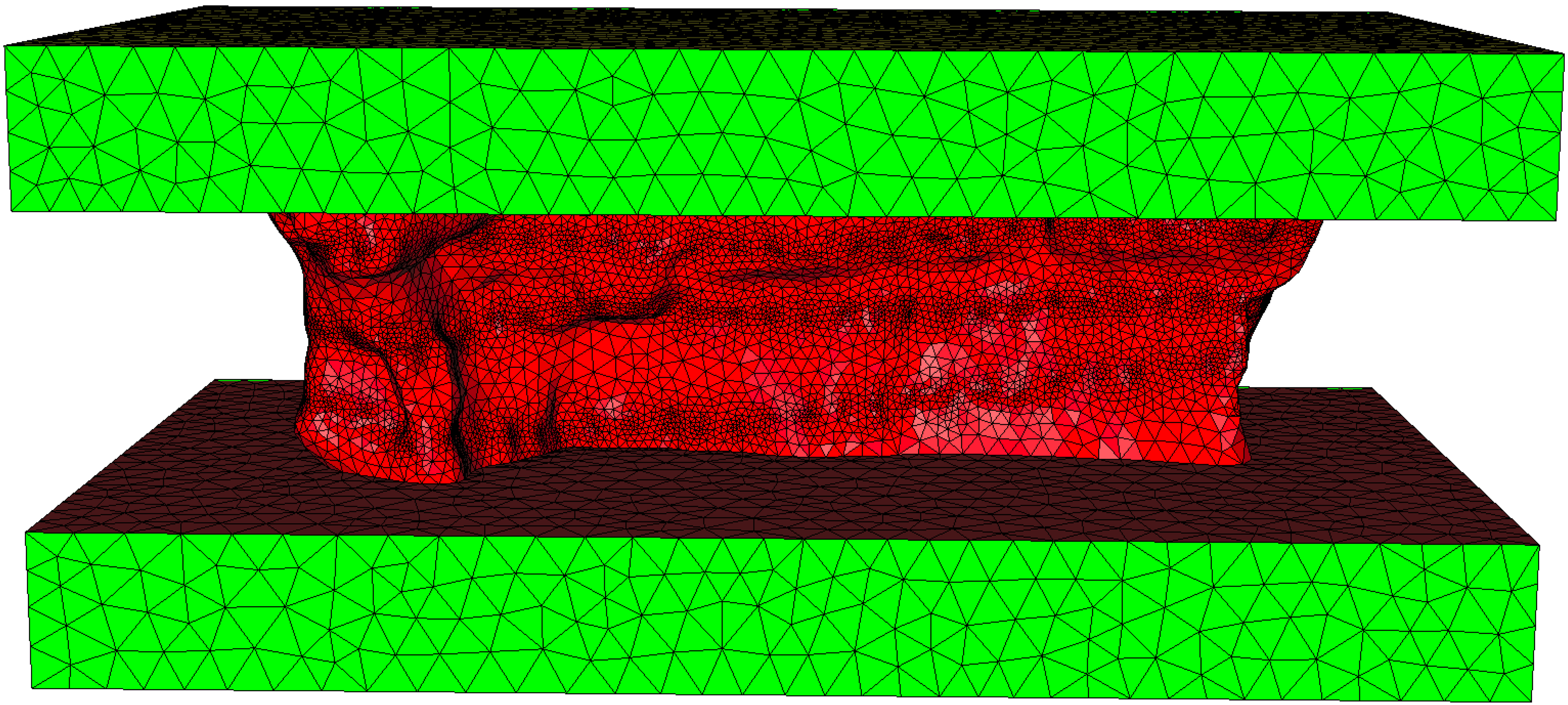
Les Vaches Noirs comparison 2016 - 2023

Cerema ENDSUM

```
meshS ThS2016, ThS2023;
fespace Vh(ThS2016,P1);
Vh u=dist(ThS2023);
```



Pilier Saint-Martin Le-Noeud Beauvais



Thanks for your attention!

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