MAIN COURSE

MBA816
BASIC MATHEMATICS AND STATISTICS FOR MANAGERS

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UNIT 1: NUMBER SYSTEM

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1.0 INTRODUCTION

Number is one of the foundation concepts in mathematics and it is quite different in concept with numerals. Numerals are signs that serve as a means of representing numbers.

Number system generally is a technique of representing numbers by means of symbols. Modern number systems are value systems which an individual number value is determined in daily activities of life.

The history of number and numeration is as old as human history and civilization. At early civilization people used strokes, pebbles, or notches as a means of measuring the number of goods.

This is done by making strokes on walls, trees, stones or the notches made on a piece of wood to show the number. The process in which objects of one group are represented and compared with that of another group is called matching. The process of matching is also known as tallying. This tallying system of counting is still in use today. There are different ways that people in different communities use in counting, but what is common to every community is number. Number measures quantity and value and this remain the same all over the world.

2.0 OBJECTIVES

At the end of this unit, you should be able to

- explain the term number
- discuss natural numbers.
- Convert numbers to other bases

- Explain whole numbers
- Explain integers
- Discuss rational numbers
- Explain irrational numbers

3.0 MAIN CONTENT

3.1 NATURAL NUMBERS

All natural numbers are counting numbers that have definite beginning but no ending. The nature of natural numbers is said to be discrete. They are usually referred to as ordinal numbers. When they denote order, the order should be in magnitude, showing a unique pattern of increase or decrease in arrangement at any given time.

Anytime the natural numbers are used to show quantities such as 5 students, 4 cows, 17 cups, they are known as cardinal numbers. Natural numbers have some properties that make it unique. Some of the properties include

3.1.1 COUNTING NUMBERS.

They are used for counting in any community and any language. The process of counting is often done in various groups, example in group of 2_s, 5_s, 10_s 12_s or 20_s. Those number group form the number bases used for calculations. Some of the groups include, *even numbers*; they are natural numbers divisible by 2,4,6,8,10,12. *Odd numbers* are natural numbers that are not divisible by two, example 1,3,5,7,9,11,13 etc. *Prime Numbers* are natural numbers with no factor other than, unity or itself, example 3,5,7,11,13,15 *square numbers* are squares of natural numbers raised to the second power examples are 4,9,16,25. *Cubic numbers* are numbers that are third power of natural numbers, example 8,27,64.

3.1.2. CONVERSION OF NUMBERS TO OTHER BASES

Traditionally numbers can be converted from one base to another using different methods and techniques. The most common conversion is usually from base 10 to other bases through continuous division with the base in question and expressing the remainder as the digits of the required base in some definite order.

Example, change 86 to base 2.

2	86
2	43 R 0
2	21 R 1
2	10 R 1
2	5 R 0
2	2 R 1
2	1 R 0

$86_{10} = 10110_2$

The rule is to start expressing the digits from the remainders, beginning from the bottom to the first one i.e. 10110_2.

3.2 Whole Numbers

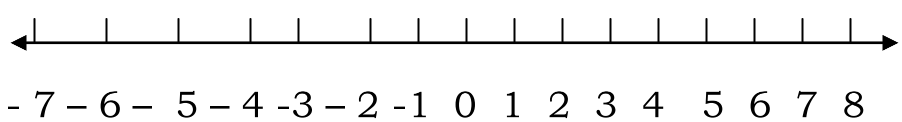
Given that natural number is a set of counting numbers beginning from one and the numbers continues without any limit. Originally zero was not a natural number so there were problems that became unresolved such as 2 minus 2, 3-3, 9-9. The discovery of zero in 600 AD help resolved the problem in numbers. This expanded the operation of number system. When zero is included to the set of natural numbers we have what is called whole numbers.

SELF ASSESSMENT EXERCISE

Explain the term whole number.

3.3 Integers

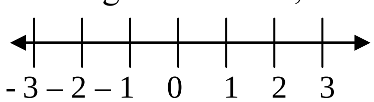
Integer is a whole numbers that do not have any form of fraction associated with it. An integer is a combination of positive, negative numbers together with zero. The positive numbers are usually called positive integers, the negative numbers are called negative integers, While the positive and negative numbers are called direct numbers. In mathematical analysis direct numbers can be represented on a number line.



written without attaching the positive sign before any of the numbers. However, negative integers are written with the negative sign attached before then or on top of each number distinguishing the negative integers from the positive integers. The only integer that is neither positive or negative is zero.

3.3.1 NUMBER LINE

A number line is a straight line that shows the ordering property and position of integers. The line is made up of arrows ending at one or both sides of the line indicating continuity in the numbers or integers. The number line is divided into equal parts to indicate the position of the integers. Usually only a small section of the integers can be represented on the line at a time. There are two strategies of drawing the number line. It can be drawn horizontally or vertically. The ordering property of integers is that numbers to the right of the line are always greater than those on the left, equally those to the left of zero are always less than those to the right of zero. The numbers are usually written in ascending order or descending. When they are in ascending order they increase from left to right. The signs used to show “greater than or less than” are “>” = greater than “<” = “less than” 5 is greater than 4, 6 < 7 “6 is less than 7”.



horizontal number line

3.3.2 ADDITION, SUBTRACTION AND MULTIPLICATION OF INTEGERS

In the addition of integers we count the positive numbers by moving to the right hand side or upwards, while the counting of negative numbers is by moving to the left of zero or downwards.

When subtracting numbers the following points should be noted. a) if we subtract or take away a positive integer from a smaller positive integer the answer is always a negative number. Example 1, 4 – 6 = - 2, 13 – 17 =- 4 etc. b) In order to subtract a negative integer from another integer, we add the absolute value of the negative integer to the other integer.

Example 2, 10 – (- 6) = 16 or – 10 - -15 = 5. Example 3, Evaluate the following (a) – 20 – (- 24) b) 12 – (- 12) c) – 13 – 16

SOLUTION

a) – 20 – (- 24) = - (20) + 24 = 4

b) 12 – (- 12) = 12 + 12 = 24

c) - 13 - 16 = - 29

In the division of integers, when integers are divided together two like sings give a positive result, while two unlike signs give a negative result.

Example 4.

(+ 15) ÷ (+ 3)	=	+5
(- 15) ÷ (- 3)	=	+5
(- 15) ÷ (+3)	=	-5
(+ 15) ÷ (-3)	=	-3

It should be noted that any number that is multiplied by zero equals zero, similarly a zero multiplied by any integer equals zero.

SELF ASSESSMENT EXERCISE

1. Evaluate the followings.

a) – 9 – (- 12) b) 28 - (- 28) (c) + 14 ÷ (-2) d) – 8 ÷ (-4).

3.4 RATIONAL NUMBERS

A rational number is an expression of a ratio of two whole numbers. It can take the form of $^{V}/_{Z}$ or V ÷ Z where V and Z are integers and Z is not equal to zero at any time. A set of rational numbers X include the set of integers as well as positive and negative fractions. Therefore, the set of integers is a proper subset of the rational numbers. Example 2/4, 1/5, 12/3, 7, 81/8, - 1/3 etc

The scope of rational numbers has no end in both positive and negative numbers and also within each numbers gap. Example between 0 and1, 1and 2, 2and 3, 0 and -1, -3 and -4

SELF ASSESSMENT EXERCISE

1. a) What is a rational numbers b) illustrate examples of rational numbers.

1. express the following rational numbers in

a) Ascending order – ½, - 3, 4, 2, - ¾

b) Descending order – 11, 9, - 4 – 17, 12, 3

3.5 IRRATIONAL NUMBERS

They are numbers that cannot be written as exact fractions nor expressed as terminating decimals. Irrational numbers usually do not have exact values, usually irrational numbers which are expressed in the form of roots are known as surds.

Example 5 $\sqrt{2}$, $\sqrt{3}$, $\sqrt{7}$

It should be noted that some numbers are expressed in form of roots and have exact terminating decimals are rational numbers and do not fall in the category of irrational numbers, example $\sqrt{4}$ $\sqrt{9}$ etc.

When two or more surds are to be multiplied together they should first be simplified. Whole numbers should be taken with whole number and surds with surds.

Example 6. simplify $\sqrt{27} \times \sqrt{50} =$

$$\sqrt{(9 \times 3)} \times \sqrt{(25 \times 2)}$$
$$= 3\sqrt{3} \times 5\sqrt{2}$$
$$= 15\sqrt{6}$$

Example 7. multiply the irrational numbers (surd)

$$\sqrt{12} \times 3\sqrt{60} \times \sqrt{45}$$
$$= \sqrt{(4 \times 3)} \times 3\sqrt{(4 \times 15)} \times \sqrt{(9 \times 5)}$$
$$= 2\sqrt{(3)} \times 3 \times 2\sqrt{15} \times 3\sqrt{5}$$
$$= 36 \sqrt{(3 \times 15 \times 5)}$$
$$= 36 \times 15$$
$$= 540.$$

Example 8. Simplify $3\sqrt{50} - 5\sqrt{32} + 4\sqrt{8}$

$$= 3\sqrt{(25 \times 2)} - 5\sqrt{(16 \times 2)} + 4\sqrt{(4 \times 2)}$$
$$= 3 \times 5\sqrt{2} - 5 \times 4\sqrt{2} + 4 \times 2\sqrt{2}$$
$$= 15\sqrt{2} - 20\sqrt{2} + 8\sqrt{2}$$
$$= 3\sqrt{2}$$

4.0 CONCLUSION

The above analysis shows that number system is the foundation of any mathematical analysis. It cut across all discipline, it is used daily by every individual in daily life be it in the home, office, or business. It is very essential to know the basics of numbers as a means of evaluation of any transaction; this is because numbers help us to measure quantity, price and other variables of life.

5.0 SUMMARY

The unit has thrown some light on the meaning and scope of numbers, even though the scope is wide and inexhaustive the basic foundational knowledge of numbers will help you cope with the challenges of other courses. The unit therefore examined the basic concepts of numbers as a means of launching you to study other units effectively.

6.0 TUTOR MARKED ASSIGNMENT

1. a)Simplify $\sqrt{45} \times \sqrt{27}$

   b) Evaluate (i) $-40 - (-28)$

   (ii) $48 - -11$

   (iii) $-10 - -18$

2)Write explanatory notes on the followings

a) Natural number
b) Whole number

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UNIT 2. SIMPLE FRACTIONS

CONTENTS

1.0 INTRODUCTION

A fraction is a part of a whole number. A whole number is called an integer, such as 1, 2, 3 - - - - 100. A fraction is a combination of pieces of a whole number. Example a thirty centimeter ruler can be cut into six equal parts, each part will be five centimeter long. Each of the pieces is a fraction of the whole ruler. The piece is called one – sixth and can be denoted $^1/_6$. Equally, each centimeter of the ruler is one – thirtieth($^1/_{30}$). In this fraction one (1) is called the numerator and thirty (30) is called the denominator.

2.0 OBJECTIVES

At the end of this unit, you should be able to

- define a fraction
- identify and discuss the concepts of fraction
- work some fractions involving addition, subtraction and multiplication
- work applications involving fractions.

3.0 MAIN CONTENT

3.1 TYPES OF FRACTIONS

There are three basic types of fraction in mathematical analysis. They are proper fraction, improper fraction and mixed numbers.

3.1.1 Proper Fraction

A fraction is classified as proper fraction when the numerator of a fraction is smaller than the denominator e.g. ½, $^{2}/_{3}$, Therefore anywhere you come across a fraction and the numerator is smaller than the denominator it is a proper fraction. In other words the denominator should be bigger than the numerator.

3.1.2 Improper Fraction

An improper fraction exist when the numerator of a fraction is greater than the denominator e.g. $^{3}/_{2}$, $^{5}/_{4}$, $^{8}/_{2}$

3.13 Mixed Number

If a number consists partly of an integer and partly of a fraction, this is called a mixed number e.g. $3 + {}^{1}/_{10}$ may be written as $3\,{}^{1}/_{10}$.

3.2 ADDITION AND SUBTRACTION OF FRACTIONS

The addition and subtraction of fractions comes in different ways addition and subtraction of fraction with the same denominator and fractions with different denomination.

3 .2.1 Addition and Subtraction of Fraction with the same denominators. Example addition of fraction

$$^{2}/_{7} + {}^{3}/_{7}$$

$$= \frac{2 + 3}{7}$$

$$= {}^{5}/_{7}$$

Example 2 subtraction of fraction with the same denominator

$$\frac{4y}{5} - \frac{2y}{5}$$

$$= \frac{4y - 2y}{5}$$

$$= \frac{2y}{5}$$

Example 3 $\frac{3x}{5} - \frac{8y}{5}$

$= \frac{3x - 8y}{5}$

Addition and Subtraction with different denominators.

3.2.2 Addition of fractions with different denominators. Once the fractions have different denominators, find a common factor as the lowest common multiple (LCM) as the common denominator. The lowest common multiple is the smallest number that can be divided without remainder by all the numbers of the given set of fractions.

Example 4. $^5/_6 + {}^3/_8$

Find the lowest common factor, which equals 24. It is the lowest number that constitute the exact multiple of 6 and 8.

$= \frac{20}{24} + \frac{9}{24} = \frac{20 + 9}{24} = \frac{29}{24} = 1^5/_{24}$

Example 5. Simplify the following fractions $^7/_{10} - {}^2/_{15}$

The lowest common factor of 10 and 15 is 30 Therefore we have

$\frac{21}{30} - \frac{4}{30} = \frac{21 - 4}{30} = \frac{17}{30}$

Example 6. Simplify the following

$\frac{5y}{6} - \frac{y}{3} = \frac{5y}{6} - \frac{2y}{6} = \frac{5y - 2y}{6} = \frac{3y}{6} = \frac{y}{2}$

SELF ASSESSMENT EXERCISE

Simplify the following fraction

a) $\frac{2x}{3} - \frac{3y}{5}$

b) $\frac{5m}{12} - \frac{3n}{8}$

c) $\frac{3y}{4} + \frac{y}{6}$

3.3 MULTIPLICATIONS OF FRACTIONS

In the multiplication of fractions the numerators are multiplied together and the denominators are also multiplied together to form a common whole fraction.

Example 7. Multiply the following fraction
$^{4}/_{6}$ x $^{8}/_{10}$ = $^{32}/_{60}$

If the fractions that would be multiplied have numerator and denominator that have common factors, it is more ideal to reduce them through division before the multiplication.

Example 8. Multiply the fraction below.

$^{4}/_{6}$ x $^{8}/_{10}$

The fractions can be reduced since they have a common factor of 2.

$^{4}/_{6}$ x $^{8}/_{10}$ = $^{2}/_{3}$ x $^{4}/_{5}$ = $^{8}/_{15}$

Example 9. Multiply the fraction below

$^{14}/_{10}$ x $^{6}/_{7}$ x $^{8}/_{9}$

= $^{2}/_{5}$ x $^{2}/_{1}$ x $^{4}/_{3}$ = $^{16}/_{15}$ = $1^{1}/_{15}$

When mixed numbers are given as part of a fraction, they should be all converted into improper fraction before multiplication is carried out.

Example 10. Solve the mixed numbers below 5½ x $2^{2}/_{7}$ x $^{5}/_{33}$.

The 5½ and $2^{2}/_{7}$ should be converted to improper fraction. They become $^{11}/_{2}$ and $^{16}/_{7}$

Collect the fraction together for multiplication

$\frac{11}{2}$ x $\frac{16}{7}$ x $\frac{5}{33}$

$^{11}/_{2}$ x $^{16}/_{7}$ x $^{5}/_{33}$ = $^{1}/_{1}$ x $^{8}/_{7}$ x $^{5}/_{3}$ = $^{40}/_{21}$ = $1^{19}/_{21}$

SELF ASSESSMENT EXERCISE

Solve the following fractions

1). $^{5}/_{6}$ + $^{2}/_{9}$, 2). $^{7}/_{12}$ – $^{3}/_{8}$, 3). 12 $^{1}/_{10}$ + 5 $^{4}/_{15}$ - $7^{3}/_{5}$, 4.) $^{4y}/_{9}$ x $^{3}/_{2}$, 5.) $^{5y}/_{6}$ x $^{9}/_{y}$

3.4 FRACTIONS INVOLVING BRACKETS

Fractions involving bracket is usually mixed equations. This is because it is made up of fractions and integers (whole numbers).

Example 11. Solve the following fractions with bracket

$^{3}/_{8}$ (y + 7) + $^{5}/_{6}$ (2y – 3)

= $^{3}/_{8}$ (y + 7) + $^{5}/_{6}$ (2y -3)

= $3\left(\frac{y-7}{8}\right) + 5\left(\frac{2y-3}{6}\right)$

Find a lower common multiple of 8 and 6 which is 24 the lower common multiple becomes a means of forming a common denominator as follows

$$\frac{9(y+7) \;+\; 20(2y-3)}{24}$$

$$= \quad \frac{9y + 63 \;+ 40y \;- 60}{24}$$

$$= \quad \frac{49y + 3}{24}$$

SELF ASSESSMENT EXERCISE.

Solve the following fractions

1. $^3/_8\,(4x - 5) - {}^5/_{12}\,(3x - 5) = {}^1/_6$
2. $^4/_5\,(2y + 5) = {}^2/_3\,(2y + 7) - {}^2/_{15}$

3.5 APPLICATION OF FRACTIONS

The application of fractions is an illustration of circumstances in real life that the knowledge of fractions can be used to solve daily problems.

Example 12. A cyclist made a journey of 152km in a total time of $3^1/_2$ hours. He went part of the way at an average speed of 40km/h and for the rest of the journey the cyclist average 48km/h. How many kilometers did the cyclist cover at 40km/h and 48km/h.

Solution

Assume the cyclist traveled y kilometers at 40km/h the time taken is $^y/_{40}$ hours ………… (1)

The remaining part of the journey was (152 - y) kilometers and he traveled this at 48km/hour.

The time taken for this journey = $^{152-y}/_{48}$ hours …..(2)

The total time for the cycling was 3½ hours.

Therefore: $\frac{y}{40} + \frac{152 - y}{48} = 3\frac{1}{2}$

$\therefore \frac{y}{40} \times 240 + \frac{152 - y}{48} \times 240 = 3\frac{1}{2} \times 240$

$6y + 5(152 - y) = {}^7/_2 \times 240$

$6y + 760 - 5y = 840$

$6y - 5y + 760 = 840$

$y = 840 - 760$

$y = 80$

The cyclist went 80km at 40km/hour the rest of the journey can also be determined

(152 – y)km

(152 – 80)km = 72km

The cyclist covered 72km at 48km/hour

4.0 CONCLUSION

The above analyses show that fractions are vital in business and daily life applications and should be encouraged. Therefore it is very essential for you to get involved in solving problems relating to fractions as it can be practically applied in your business transactions and daily living.

5.0 SUMMARY

In this unit we examined simple fractions, proper and improper fractions, addition, subtraction and multiplication of fractions. Fractions with bracket were also examined to give the you a broad knowledge of the topic. The application of fractions was also examined for the you to appreciate the fact that this arithmetic can be applied daily in life and business.

6.0 TUTOR MARKED ASSIGNMENT

1 a) (i). $y = 2^1/_2$, find the value of $2y^2 – 3y + 1$ (ii) if $y = 2^1/_4$, find the value of $^{2/}{}_3$ of y

2) In a 60km bicycle race a rider calculates that if he can increase his speed by 6km/h. he will cut his time for the distance by 20 minutes. What was his original speed?

3) A man bought a certain number of packets of matches at N1.26k. He kept 4 packets for his own use and sold the rest at 3k more per packet than he paid for them, making a total profit of 14k on the business. How many packets of matches did he buy.

7.0 REFERENCES/FURTHER READING

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UNIT 3. EXPONENTS AND ROOTS

CONTENT

1.0 INTRODUCTION

Exponential functions are commonly used in business and economics in growth theories. Exponential functions are applied to solve optimization equations and problems that use time as part of the choice variable. Therefore, they are used to express functions that grow overtime and the time is measurable through the application of the knowledge exponents and roots.

Exponential functions can also be used to express and find solution to variables involving compound interest, annuities and sinking fund as it relate to business and economics.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- apply the laws of exponents
- explain the multiplication of exponents
- explain the division of exponent
- explain exponents involving roots.

3.0 MAIN CONTENT

3.1 THE LAWS OF EXPONENTS

The laws of the exponents can be expressed by the different use of exponents in different ways such as

1. When multiplication of two exponents are given, the exponents are added.

$$y^m\, y^n = y^{m+n}$$

Example 1. $5^2 \times 5^4 = 5^{2+4} = 5^6 = 15625$

2. When a number has an exponent and it is multiplied by another exponent then the product of it is multiplication of the two exponents.

$$(y^m)^n = y^{mn}$$

Example 2. $(3^2)^3 = 3^{2x3} = 3^6 = 729$

3. When an exponent is to be divided by another exponent the result is the subtraction of the exponential numerator from the denominator.

$$\frac{y^m}{y^n} = y^{m-n}$$

Example 3. $\frac{4^6}{4^2} = 4^{6-2} = 4^4 = 256$

4. Any variable that is raised to a zero exponent the product of it is one

$y^0 = 1$. Example 4. $88^0 = 1$

5. An exponent that is a product of two variables it is converted to the first variable multiplied by the second variable each raised to the same exponent $(xy)^n = x^n y^n$

Example 5. $(2x5)^2 = 2^2 \times 5^2 = 4 \times 25 = 100$.

6. When two variables that divide each other are raised to a common exponent it is converted to the two independent variable raised to the exponent $(x/y)^n = x^n/y^n$ where $y \neq 0$;

Example 6 solve the exponent $(2/5)^5 = 2^5/5^5 = {}^{32}/_{243}$

7 A negative exponent is the reciprocal of the number to be determined.

Example $y^{-n} = {}^1/_{y}{}^n$, $y \neq 0$ Example 7, solve $4^{-2} = {}^1/4^2 = {}^1/_{16}$

SELF ASSESSMENT EXERCISE

Discuss and illustrate the laws of exponents.

3.2 MULTIPLICATION OF EXPONENTS

When variables are raised to a given exponent that should be multiplied, the result is the sum of the given exponents.

Example 8. Solve the equation $x^4 \times x^7 = x^{4+7} = x^{11}$

Example 9. Solve the equation $4^3 \times 4^2 = 4^{3+2} = 4^5 = 1024$

Example 10. Multiply $3c^3e^2$ by $2c^2e^2 = 6c^{3+2}e^{2+2} = 6c^5e^4$

SELF ASSESSMENT EXERCISE

Given the following variables, find the solution with respect to the exponents.

a) $y^4 \text{ x } y^2$
b) $6^3 \text{ x } 6^2$

3.3 DIVISION OF EXPONENTS

Example 11. Simplify the following, divide $-12x^4y^3z^2$ by $-4x^3y$

$$= \frac{-12x^4y^3z^2}{-4x^3y}$$

$$= +\frac{12x\,x\,x\,x\,yyyzz}{4\,xxxy}$$

$$\mathbf{3xy^2z^2}$$

Example 12. Divide the following exponents y^5 by y^2

$$\frac{y^5}{y^2} = \frac{y\,y\,y\,y\,y}{y\quad y}$$

$$= y^3$$

Example 13. Divide the exponent 6^4 by 6^2

$$= \frac{6^4}{6^2} = 6^{4-2} = 6^2 = \mathbf{36}$$

3.4 EXPONENTS AND ROOTS

Sometimes exponents are expressed as roots or a product of some root. This can be solved using the same laws of exponents.

Example 14. Simplify $\sqrt{}y^{10}$

$= y^{10 \div 2}$

$= y^5$

Example 15. Simplify $3\sqrt{}y^{21}$

$= y^{21 \div 3}$

$= y^7$

Example 16. Solve $3\sqrt{}(-8y^{15}n^3)$

$= \quad -2y^{15 \div 3}\, n^{3 \div 3}$
$= \quad -2y^{5}\, n^{1}$
$= \quad -2y^{5}n$

Notice that $3\sqrt{8} = 2$ or $(-2)^3 = 8$

Example 17. Simplify $\dfrac{10x^2y + 6xy^2 - 8x^2y^2}{2xy}$

$$= \quad \frac{10x^2y}{2xy} \quad x \quad \frac{6xy^2}{2xy} \quad = \quad \frac{8x^2y^2}{2xy}$$

$$= \quad 5x + 3y - 4xy$$

SELF ASSESSMENT EXERCISE

Simplify a) $4\sqrt{(81x^8y^4)}$ c) $3\sqrt{(27y^{12})}$
b) $5\sqrt{(w^{15})}$ d) $3\sqrt{(125R^6)}$

3.5 FRACTIONAL EXPONENTS

There are circumstances in which the exponents can be in fractions. The solution follows the same rules of working exponents.

Example 18. $y^{3/4} = y^{1/4} \times y^{1/4} \times y^{1/4} = (y^{1/4})^3 = (4\sqrt{y})^3$

$16^{3/4} = \sqrt[4]{(16^3)} = \sqrt[4]{(2^4 \times 2^4 \times 2^4)} = \sqrt[4]{2^{12}} = 2^3$

Alternatively $16^{1/4} = 4\sqrt{16} = 2$
$16^{3/4} = 2^3$

Example 19. Simplify $10^{3/4}$
$= \quad 4\sqrt{10^3} = 4\sqrt{1000} = \sqrt{31.62} =$ **5.623**

SELF ASSESSMENT EXERCISE

Simplify a) $16^{1/2}$ b) $8^{2/3}$ c) $81^{3/4}$ d) $100^{2/3}$

4.0 CONCLUSION

Exponential functions are applied both in business arithmetic, economics and other social sciences. A good knowledge of exponents and roots can assist tremendously in enhancing your knowledge.

5.0 SUMMARY

The unit examined exponents and roots. We began by introducing the concept using simple symbols to assist the you and gradually we illustrated exponents using multiplication, addition and division. Ample examples were illustrated to drive home the context of the unit.

6.0 TUTOR MARKED ASSIGNMENT

1 Find x if $2^x \text{ x } 4^{2x+3} \text{ x } 8^{x-1} = 16$.
2 Find the value of x if $3^{2x+1} – 28 (3^x) + 9 = 0$

7.0 REFERENCES/FURTHER READING

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UNIT 4. RATIOS

CONTENTS

1.0 INTRODUCTION

A ratio shows the number of times one quantity or unit contains another. It is used to show the relationship between two amounts. Here the comparison is made in the form of a ratio that is the fraction which the first quantity is of the second. Suppose a company has 150 men and 200 women, then the number of men is ¾ of the number of women and we say that the ratio of the number of men to the number of women is 3 to 4, written 3:4 and this ratio can be represented by the fraction ¾.

2.0 OBJECTIVES

At the end of this unit, you should be able to

- Explain the term ratio
- Discuss the increase and decrease in ratio
- Explain and discuss comparison of ratio

3.0 MAIN CONTENT

3.1 Explanation of the term and concept of ratio. Ratio being a relationship between two amounts of quantity in which one relates to another. Ratios should be expressed as simply as possible, just as the fraction $^{8}/_{36}$ can be reduce to $^{2}/_{9}$, so the ratio 8:36 is equivalent to 2:9. Therefore a ratio is unaltered if the two numbers or quantities of the ratio are both multiplied, or both divided by the same number. Example, the ratio $^{5}/_{6}$: $^{3}/_{4}$ equals the ratio $^{5}/_{6}$ x 12 : ¾ x 12 that is 10:9

When we want to express the prices of two books x and y are N720, N960 respectively.

$$\frac{\text{Price of x}}{\text{Price of y}} = \frac{720}{960} = 4/3$$

Similarly

$$\frac{\text{Price of y}}{\text{Price of x}} = \frac{960}{720} = \frac{4}{3}$$

We write price of x : price of y as equal to 3:4 and the price of y : price of x as 4:3. Conversely, the statement that the ratio of the price of x to the price of y is 3:4 means that the price of x is ¾ of the price of y and that the price of y is $^4/_3$ of the price of x.

SELF ASSESSMENT EXERCISE

Explain the term ratio and illustrate it with examples.

3.2 INCREASE AND DECREASE IN RATIOS

Ratio can depict an increase and decrease in the occurrence of a given event, or numbers. If the daily price of a ticket is raised from N60 to N80, the ratio of the new price to the old price of ticket equals 80:60 = 4:3 we can say that the price of the ticket has been increase in the ratio 4:3. In other words, the new ticket price is $^4/_3$ times the old ticket price. If the daily price per ticket for entering a cinema is lowered fromN60 to N48, the ratio of the new ticket to the old ticket price would be 48:60 = 4:5, and we say that the ticket price has been reduced in the ratio of 4:5. In other words the new ticket price is $^4/_5$ times the old price.

The fraction $^4/_5$ by which the old ticket price N60 must be multiplied to give the new ticket price of N48 is called a multiplying factor.

$$\frac{\text{New quantity}}{\text{Old quantity}} = \text{Multiplying factor}$$

The multiplying factor is less than one if the new quantity is less than the old quantity, it is greater than one if the new quantity is greater than the old quantity.

Example 1: Umenemi Nig Ltd water wants to increase its water rate of 56k in the ratio 10:7. Determine the new water rate.

Increased value = 56k x $^{10}/_7$

$$= \frac{56 \text{ x } 10}{7}$$

= 80k

Example 2. Okewa bread wants to reduce the time taken for baking, of 2 hours in the ratio 5:6. What is the decreased time for baking?

Decreased time = 2hrs x $^{5}/_{6}$

$= \dfrac{2 \times 5}{6}$

= $^{5}/_{3}$ hours

= 1 hour 40 minutes

Example 3. In what ratio should N75 be increased to become N100?

The ratio $^{100}/_{75}$ = 100:75 = 4:3

3.3 COMPARISON OF RATIOS

We have stated that a ratio is a relationship, a ratio may be expressed in the form n:1, where n is a whole number, a fraction, or a decimal calculated to any required degree of accuracy. This is particularly important when comparing ratios.

Example 4. Express the ratio of 4.10:1.90 in the form n:1

$\dfrac{4.10}{1.90} = \dfrac{4.1}{1.9}$

$= \dfrac{41 \div 19}{1}$ dividing the numerator and denominator by 19

$= \dfrac{2.16}{1}$

:- The ratio is 2.16:1

Example 5: Find which ratio is greater 7:13 or 8:15

$^{7}/_{13}$ = 0.538 therefore 7:13 = 0.538:1

$^{8}/_{15}$ = 0.533 therefore 8:15 = 0.533:1

The first ratio is greater than the second. The first gives the value 0.538 while the second has the value 0.533.

Self assessment exercise

Find which ratio is greater from the following

9:16 or 7:1

3.4 APPLICATIONS OF RATIOS.

Ratio system is used by planners, geographers and geographical information system and other forms of surveys.

For map and plans the ratio is usually in the form 1:n. For example if the scale on a map is 5cm to the kilometer, 5cm on the map represents one kilometre on the ground based on survey specifications.

5cm:1km = 5cm:100000cm

$= 1:20000$
Therefore the scale of the map is 1:20000

The fraction $^1/_{20000}$ is called the representative fraction. Note that a scale of 1:16 is greater than a scale of 1:17 since $^1/_{16}$ is greater than $^1/_{17}$.
Example 6: Express the ratio 8:13 in the form 1:n
$^8/_{13} = {}^1/_{(13 \div 8)}$ dividing numerator and denominator y 8
$= {}^1/_{1.625}$
:- The ratio is 1:1.625.

Example 7. If 5 people dig the foundation of a house in 14 days, how long would 7 people take to dig the foundation.

Solution. Since the number of men have increased it will take them less days to dig the foundation. This can be expressed in a ratio
The number of men increased in ratio 7:5
Therefore the time taken is decreased in the ratio 5:7
What is to be calculated is the time
5 men = 14 days
7 men = 14 x $^5/_7$ days
= 10 days

Example 8. In a market 2¼kg of coffee cost N1.17 what quantity of coffee can be bought for N1.95 in the market?
Solution It is given that N1.17 is the cost of 2¼ kg. and N1.19 is the cost of 2¼ x $^{1.19}/_{1.17}$
9/4 x 195/117kg
3¾ kg

SELF ASSESSMENT EXERCISE

(1) If 10 people dig the foundation of a house in 28 days, how long would 16 people take to dig the foundation.

(2) In a market 4¼kg of coffee cost N118 what quantity of coffee can be bought for N295 in the market?

4.0 CONCLUSION

Analysis of ratio shows that it is important in all work of life where we need to make comparison of events. This range from daily comparison of sales, cost, work output to measurement of geographical area, and presentation of such data for human use. It is therefore important for you to learn about ratio practice the applications of ratios as well.

5.0 SUMMARY

The unit has shed some light on the meaning of ratio, comparison of ratios, increase and decrease of ratios, workings and application of ratios. The ratio system is important for every practicing manager.

6.0 TUTOR MARKED ASSIGNMENT

1. Find the ratio x:y if $6x^2 = 7xy + 20y^2$

2.(a) A man takes 18 minutes for a journey if he travels at 20km per hour. How long will he take if he travels at 24km per hour.

(b) A car takes 50 minutes for a journey if it runs at 72km/h at what rate must it run to do a journey of 40 minutes.

7.0 REFERENCES/FURTHER READING

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UNIT 5. ANALYSIS OF VARIATION

CONTENT

1.0 INTRODUCTION

Variation is a mathematical method of finding the rate of change in quantities, volumes, speed or any other event or group of events that depend on each other. Variation could be direct, inverse, joint or partial variation.

Variation as a unit embraces all aspects of daily life, as one activity depend on another. The ability to work depends on our health, energy and other utilities. The ability to drive depends on expertise, or experience, the type of vehicle and the nature of the road, the degree of concentration. This unit will attempt to examine some of these interrelationships and their applications.

2.0 OBJECTIVES

At the end of this unit, you should be able to

- Explain the term variation
- Discuss direct variation
- Discuss inverse variation
- Discuss joint variation

3.0 MAIN CONTENT

3.1 DIRECT VARIATION

If two variable quantities x and y are so related that the ratio of simultaneous values of x and y is constant than either quantity varies directly as the other quantity. Steps in solving variation problems.

a. Change the given statement into a mathematical expression involving α, where α is the proportionality symbols.
b. Replace α by k in the new mathematical expression, where k is a constant.
c. Find the value k using the initial values and substitute into the equation in step b
d. Solve the problem using equation in step b free of k.

Example 1
If y varies directly as the square root of x and y = 12 when x = 4. Find y when x = 9.
$Y \;\alpha\; \sqrt{X}$ …..(1)
$Y = k\sqrt{X}$ …..(2)
When y = 12, x = 4 substitute into equation (2)
$12 = K\sqrt{4}$
$12 = 2K$
Find K by divide through by 2
$^{12}/_{2} = {}^{2k}/_{2}$
$K = 6$
Substitute for k in equation 2 where x = 9
$Y = 6\sqrt{X}$
$Y = 6\sqrt{9}$
$= 6 \times 3$
$Y = 18.$

SELF ASSESSMENT EXERCISE

If W varies directly as the square root of V and w = 24 when v = 8 find w when v = 18.

3.2 INVERSE VARIATION

When two variables quantities x and y are related in such a way that the quotient obtained on dividing x by the corresponding value of $^{1}/_{y}$ is a constant, then x is said to vary inversely as y. Therefore if y varies inversely as x, y varies directly as $^{1}/_{x}$.

Example 2. The electrical resistance of a wire varies inversely as the square of its radius. Give that the resistance is 0.4 ohms when the radius is 0.3cm, find the resistance when radius is 0.45cm.
Let R be the resistance in ohms and r the radius in cm

Therefore $R \alpha {}^{1}/r^{2}$ ….. (1)

$R = {}^{K}/r^{2}$ …… (2) where r is a constant

When R = 0.4, r= 0.3 substitute into equation (2)

$0.4 = {}^{k}/_{(0.3)}{}^{2}$

$K = (0.4)(0.3)^{2} = 0.036$

$$R = \frac{0.036}{r^{2}}$$

When r = 0.45 substitute into equation (2)

$$R = \frac{0.036}{(0.45)^{2}}$$

$= 0.18$

Example 3. If y is inversely proportional to Z^2 and if y = 4 when Z = 3. (i) Find the value of y when Z = 4 and the positive value of Z in terms of y

$Y = {}^{K}/_{Z}{}^{2}$ when K is a constant

Since y = 4, when Z = 3.

$K = 4 \times 3^{2} = 36$

:- $y = {}^{36}/_{Z}{}^{2}$

When Z = 4, $y = {}^{26}/_{16} = 2\frac{1}{4}$

Since $Z^{2}\ y = {}^{36}/_{y}$

$Z^{2} = {}^{36}/_{y}$, :- $Z = {}^{6} / \sqrt{y}$

SELF ASSESSMENT EXERCISE

If y varies inversely as $\sqrt{x}$ and if y = 5 when x = 16, find y if x = 100 and find x if y = 60. Find also y in terms of x.

3.3 JOINT VARIATION

When one quantity varies as the product of two or more quantities, then it is called joint variation.

Example 4, if v values directly as the square of x and inversely as y and if v = 18 when x = 3 and y = 4. Find v when x = 5 and y = 2

$V \alpha x^{2} /y$……… (1)

:- $V = (Kx^{2})/y$ ……(2) where K is a constant

When v = 18, x = 3 and y = 4 then substitute into equation (2)

$$18 = \frac{K(3)^{2}}{4}$$

$$18 = \frac{9k}{4}$$

$$K = \frac{18 \times 4}{9} = 8$$

Therefore

$$V = \frac{8x^2}{Y}$$

When x = 5, y = 2

$$V = \frac{8 \times (5)^2}{2} = \frac{8 \times 25}{2} = 4 \times 25 = 100$$

V = 100

SELF ASSESSMENT EXERCISE

1. If y varies directly as the square of x and inversely as w and if y = 36 when x = 6 and y = 8 find y when x = 10 and y = 4

2. If w varies jointly as L and the square of r. find the percentage change in w if L increases by 20% and r increases by 50%. If w = 15 when h = 3 and r = 2½, find w when h=1 and r = 10; find also w terms of h, r.

3.4 PARTIAL VARIATION

This is a situation where a function varies partly as the sum or difference of two quantities. For partial variation there are at least two constants. These constants have to be found first before solving the question. For the computation of partial variation the procedures are slightly modified as follows:

a. Change the statement to a mathematical expression
b. The values given together with the mathematical expression formulate two equations, with two unknowns
c. Solve the two equations in step (b) simultaneously to obtain the values of the constants.
d. The problem can now be solved with the mathematical expression free of the constrains.

Example 5. Given that y is the sum of two quantities, one of which varies as x and the other of which varies inversely as x. If y = 20 when x = 1 and y = 12 when x = 3, find the values of y when x = 6.

Let y = a + b ………….. (1)

Then $a \alpha x$, a = cx where c is a constant

Also $b \ \alpha \ 1/c$

Then b = n/x, where n is a constant (n = c)

Substituting for a and b in equation (1)
$Y = cx + n/x$ ………. (2)
Substitute when $y = 20$, $x = 1$ into equation (2)
$20 = c + n$ ……………(3)
Substitute when $y = 12$, $x = 3$ into equation (2)
$12 = 3c + n/3$
$36 = 9c + n$……………… (4)
Now solve equation (3) and (4) simultaneously when $c = 2$ and $n = 18$ substitute into (2)
$Y = 2x + 18/x$ …………….(5)
Therefore when $x = 6$ substitute for x in equation (5)
$Y = 2(6) + 18/6$
$= 12 + 3$
$= 15$
When x 6, y = 15.

Example 6. The volume of a given mass of gas varies directly as the absolute temperature and inversely as the pressure. At absolute temperature of 360^0 and at pressure 736mm the volume is $450cm^3$; find a general formula and find the volume at absolute temperature 312^0 and pressure 960mm.

solution
If the volume is vcm^3 at absolute temperature T^0 and pressure Pmm
$V \alpha T/p$
$V = K \times T/p$
Where K is constant, when T = 360 and P = 736, V = 450
$450 = K \times 360/736$

$$:- K = \frac{450 \times 736}{360}$$

$$= \underline{920}$$

$$V = \frac{920T}{P}$$

When T = 312 and P = 960

$$V = \frac{920 \times 312}{960}$$

$$= \underline{299}.$$

The volume is $299cm^3$

SELF ASSESSMENT EXERCISE
If Z varies directly as the square of x and inversely as the square root of y, find the percentage change in Z if x increases by 20% and y decreases by

19%. If Z = 3 when x = 6 and y = 16, find Z when x = 12 and y = 25; find also Z in terms of x and y.

4.0 CONCLUSION

Variation has a wide range of usage and applications attempt has been made within the time limit and scope to present what can assist you in the analysis of mathematics in other levels of study.

5.0 SUMMARY

The unit examined variation combined theory of variation and practice or applications. The unit examined direct variation inverse, joint and partial variation to drive home the concept of variation. Examples used in the exercises were such that can assist you in independent studies.

6.0 TUTOR MARKED ASSIGNMENT.

1) If y varies directly as Z and y = 10 when Z = 6, find the value of Z when y = 12.5

2) (a) R α m and R = 6, when m = 16. Find the law connecting R and M. find R when m = 6¼ and m when R = 15.

Given that y varies directly as X^2. How is the value of y affected if the value of x decrease by 20%.

8.0 REFERENCES/FURTHER READING

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UNIT 6. LINEAR EQUATION

CONTENT

1.0 INTRODUCTION

A linear equation is a mathematical statement or an expression that has an unknown variable. The unknown variables are raised to the power of one. A linear equation usually may have a constant that connect the equation with unknown. The equation is usually connected by an equality (=) sign.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- Solve problems of linear equation involving addition and subtraction
- Solve problems of linear equation using multiplication method
- Solve linear equation using division
- Solve application problems involving linear equation.

3.0 MAIN CONTENT

3.1 ADDITION AND SUBTRACTION OF LINEAR EQUATION

We have earlier stated that any statement that two things are equal means it can be estimated quantitatively and the process of estimation is called equation.

Example 1. Find the value of the unknown variable

$14y = 28$

Find the value of y that is unknown

$14y = 28$

Divide both side by 14 and it becomes

$$\frac{14y}{14} = \frac{28}{14}$$

$Y = \underline{\underline{2}}$

Example 2. Find the value of x from the following

$3x + 2 = 2x + 10$

Collect like terms

$3x – 2x = 10 – 2$

$X = 8$

Example 3. Find the value of y form the following

$30x + 10 + 2x = 15x + x + 42$

Collect like terms

$30x + 2x – 15x – x = 42 – 10$

$32x – 16x = 32$

$16x = 32$

Divide through by 16 we have

$$\frac{16x}{16} = \frac{32}{16}$$

$X = \underline{\underline{2}}$

SELF ASSESSMENT EXERCISE

Find the value of the unknown in the following

a) $8 – 19 = 5 - 3y$ (b) $4 – 3x = -7x + 8$

c) $6x + 7 – 5x = 19 – 2x – 3$

3.2 MULTIPLICATION OF LINEAR EQUATION

In the multiplication of linear equation, the necessary expansion of the equation should first be carried out then the value of the unknown can be determined.

Example 4. Find the value of the unknown from the equation below.

$3(x + 3) = 2 (0.5x + 7)$

First clear the bracket through multiplication

$3(x + 3) = 2 (0.5x + 7)$

$3x + 9 = x + 14$

Collect like terms $3x – x = 14 – 9$

$2x = 5$

Divide through by 2 to find the value of x

$$\frac{2x}{2} = \frac{5}{2}$$

X = $\underline{\underline{2.5}}$

Example 5. Find the value of y in the following equation

y(10 – 2) = 80

:- 10y – 2y = 80

:- 8y = 80

:- $\frac{8y}{8} = \frac{80}{8}$

Y = 10.

Example 6. Solve the equation (5)y = 2y + 7

(5)y = 2y + 7

Subtract 2y form both side

5y – 2y = 2y + 7 – 2y

:- 3y = 7

Divide both sides by 3 to find the value of y

$\frac{3y}{3} = \frac{7}{3}$

Y = $2\,{}^{1}/_{3}$

Example 7. Solve the equation 22 = (7)y – 6

Add 6 to both sides of the equation

22 + 6 = 7y – 6 + 6

28 = 7y

divide both sides of the equation by 7 to find the value of y

28/7 = 7y/7, y = 4

SELF ASSESSMENT EXERCISE

Solve the following equations

(i) (3)x – 2 = 10 (ii) 10 (x – 2) = 2(x + 1)

(iii) x(15 + 4) = 5(x + 2)

3.3 DIVISION OF LINEAR EQUATION

In the division of linear equation an understanding of the process of multiplication linear equation is needed. |The understanding of the multiplication process help in simplifying the equation to determine the value of the unknown variable.

Example 8. Solve the equation $\frac{2y + 5}{y} = \frac{2y}{y-3}$

Cross multiply the equation to clear the division

$(2y + 5)(y - 3) = (2y)y$

Open the bracket and multiply out the variables

$2y^2 + 5y - 6y - 15 = 2y^2$

Collect the like terms

$2y^2 - 2y^2 + 5y - 6y = 15$

$5y - 6y = 15$

$-y = 15$

Example 9. Solve the equation

$\frac{10x + 4}{2} = \frac{2x}{4}$

Cross multiply

$\frac{10x + 4}{2} = \frac{2x}{4}$

Cross multiply

$\frac{10x + 4}{2} = \frac{2x}{4}$

$(10x + 4)4 = 2(2x)$

$40x + 16 = 4x$

Collect like terms $40x - 4 = -16$

$36x = -16$

$X = \frac{-16}{36}$

Example 10. Solve the equation $\frac{6x + 14}{x} = \frac{14x}{x-8}$

$\frac{6x + 14}{x} = \frac{4x}{x-8}$

Cross multiply the equation

$(6x + 14)(x - 8) = x(4x)$

$6x^2 - 14x - 48x - 112 = 4x^2$

Collect like terms

$6x^2 - 4x^2 + 14x - 48x - 112 = 0$

$2x^2 - 34x - 112 = 0$

Use the formula to solve the equation and find x

$\frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Where a=2, b=-34, c=-12

Substitute into the formula

$$\frac{-(-34) \pm \sqrt{\{(-34)^2 - 4 \times 2 \times -112\}}}{2 \times 2}$$

$$\frac{34 \pm \sqrt{\{1156 + 896\}}}{4}$$

$$\frac{34 \pm \sqrt{2052}}{4}$$

$$\frac{34 \pm 45.30}{4}$$

$$\frac{34 + 45.30}{4} \quad \text{or} \quad \frac{34 - 45.30}{4}$$

$$\frac{79.3}{4} \quad \text{or} \quad \frac{-11.3}{4}$$

19.83 or - 2.84

x = 19.83 or - 2.84

SELF ASSESSMENT EXERCISE

1. Solve equation

$$\frac{12y + 28}{y} = \frac{8}{16-y}$$

2. Solve the equation $\frac{4x + 5}{2} = \frac{4x}{8}$

3.4 APPLICATIONS OF LINEAR EQUATION

The equations that have been solved were necessary only to find the number represented by some letter. This section will show how practical problems that involve linear equation can be solved. In each case a letter is introduced to stand for the unknown variable to be calculated.

Example 11. Emma and Kehinde are to share N54 such that Kehinde has N8 less than Emma. Find the share of each person.
Lets denote Emma's share by x
Kehinde has N8 less than Emma = - 8
They share a total of N54
:- x + x - 8 = 54
Collect like terms x + x = 54 + 8
2x = 62
Divide through by 2 $\frac{2x}{2} = \frac{62}{2}$ x = 31.
Emma's share is N31
Kehinde's share = x – 8 31 – 8

$= 23.$

Kehinde has N23.

Example 12. Kufe drive for 3 hours at a certain speed and then double that speed for the next 2 hours if Kufe drove the car covering 63km altogether find the sped for the first 3 hours.

Let the speed that he started with x km/h

Then his speed later on was 2x km/h

Therefore in the first three hours he went 3x km

And in the next 2 hours he went 2 x 2x km = 4xkm

:- $3x + 4x = 63$

$7x = 63$

Divide through by 7 to find x $\frac{7x}{7} = \frac{63}{7}$ $x = 9$

He started at 9km/h

In 3 hours at 9km/h he went (9x3) = 27km

In 2 hour at 18km/h he went (2x18) = 36km

SELF ASSESSMENT EXERCISE

Emeka cycled for 6 hours at a certain seed and then double that speed for the next 2 hours. If the total distance covered was 126km altogether (1) Find the speed for the first three hour (2) find the distance covered for the period he doubled his speed.

4.0 CONCLUSION

The analysis in this unit demonstrate the fact that linear equation is important in business and managerial decisions. Linear equation can be used to solve problems relating to management practice in companies and even private business establishment. A knowledge of linear equation help to increase the practical application of quantitative reasoning in workplace.

5.0 SUMMARY

The unit has thrown light on the meaning and the application of linear equation. The unit examined the addition and subtraction of linear equation, multiplication and division of linear equation. This is to broaden the scope of your understanding of the unit. Applications were also treated so that you will not think linear equation is an abstract area of study in mathematics.

6.0 TUTOR MARKED ASSIGNMENT

1) Solve the equation (i) $4 - 3x = 17x + 8$

ii). $7 = 9 - 5y + 8$

iii). $\frac{4}{3} = \frac{x - 2}{(x + 4)}$

2.(a) Paul and peter received an award of N21,000 as a reward for their excellent performance with a condition that peter will receive N3000.00 more than Paul. Determine the amount Peter and Paul will receive.

(b). A certain number is multiplied by 8 and then 28 is added if the result is 100. Find the original number.

7.0 REFERENCES/FURTHER READING

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UNIT 7 SIMULTANEOUS LINEAR EQUATION

CONTENTS

1.0 INTRODUCTION

A simultaneous linear equation is a set of equations with more than one unknown variables, however the number of the unknown variables are usually as many as the set of equation.

2.0 OBJECTIVES

At the end of this unit, you should be able to

- Define a simultaneous equation
- Find the solution to any simultaneous equation by substitution
- Solve simultaneous equation by addition
- Solve simultaneous equation problem using elimination by subtraction
- Work practical problems involving simultaneous equation

3.0 MAIN CONTENT

3.1 SOLUTION BY SUBSTITUTION

This is a method of finding solution to simultaneous equation where one of the equation is rearranged such that one of the unknown is made the unit and becomes the subject of the equation where it is substituted into the remaining equation, this help provide a solution to one of the unknown.

Example 1. The demand for bread in market x and y is given as follows

$$3x - 4y = 19 \text{ ……..(1)}$$
$$x - 2y = 5 \text{ …….(2)}$$

Determine the value of x and y by substitution.

Make x the subject in equation (2)

$x = 5 + 2y$ …… (3)

Substitute the value of x into equation (1)

$3x – 4y = 19$

$3(5+2y) – 4y = 19$

$15 + 6y – 4y = 19$ … (4)

Rearrange equation (4) and collect like terms

$6y – 4y = 19 – 15$

$2y = 4$

$y = {}^{4}/_{2}.$

$= \underline{\underline{2}}$

Substitute the solution of y into equation (3) to determine the value of x.

$X = 5 + 2y$

$X = 5+2(2)$

$= 5 + 4$

$= 9$

Example 2. Solve the simultaneous equations

$3w + 2x = 21$ - - - (1)

$2w + 5x = 3$ - - - (2)

Using equation (1) solve for x

$2x = 21 – 3w$

$x = \frac{21 – 3w}{2}$ - - - (3)

Substitute equation (3) into equation (2) we have

$2w + 5\left(\frac{21 – 3w}{2}\right) = 3$ - - - (4)

:. $4w + 105 – 15w = 6$

$4w – 15w = 6 – 105$

$-11w = -99$

$W = \frac{99}{11}$

$= \underline{\underline{9}}$

Substitute for w into equation (3)

$$X = \frac{21 - 3w}{2}$$

$$= \frac{21 – 3(9)}{2}$$

$$= \frac{21 – 27}{2}$$

$$= -\underline{6}$$

$$2 \quad = \quad -3$$

SELF ASSESSMENT EXERCISE

1. Solve the following simultaneous equations

$$2x - 5y = -3$$
$$3x + 4y = 1$$

2. Solve the simultaneous equations

$$x - 2y = 27$$
$$7x + y = 9$$

3.2 SOLUTION BY ELIMINATION USING ADDITION

When the method by substitution involves awkward fractions, it is easier to use the method of elimination by addition or subtraction.

Example 3. Solve the simultaneous equations

$$3x - 2y = 11 \text{ - - - } (1)$$
$$5x + 2y = 29 \text{ - - - } (2)$$

Elimination by addition involves adding equation (1) and equation (2) together. When this is done the term y will disappear leaving only the x

$$3x - 2y = 11$$
$$\underline{5x + 2y = 29}$$
$$\underline{8x + 0 = 40} \quad - \text{ - - - } (3)$$

What is left from the equation after elimination by addition is $8x = 40$ the value of the unknown variable x can now be determined.

$$x = \frac{40}{8}$$
$$= \underline{\underline{5}}$$

Substitute $x = 5$ into equation (1) so that the y unknown can be calculated gives $3x - 2y = 11$

$$3(5) - 2y = 11$$
$$15 - 2y = 11$$
$$-2y = 11 - 15$$
$$-2y = -4$$
$$y = \frac{-4}{-2}$$
$$= \underline{\underline{2}}$$

SELF ASSESSMENT EXERCISE

Solve the following simultaneous equation by elimination using addition.

1. $x + y = 11$
   $x - y = 5$

2. Solve the simultaneous equation by addition
   $x - 4y = 2$
   $x + 4y = 28$

3.3 SOLUTION BY ELIMINATION USING SUBTRACTION

This method involves determining the value of the unknown in a simultaneous equation by subtracting one equation from the other, then determine the unknown variables.

Example 4. Solve the simultaneous equation by elimination using subtraction.

$2x + 5y = 28$ - - - (1)
$2x + 3y = 3$ - - - (2)

When equation (1) is subtracted from equation (2) the term x will become zero and therefore disappears from the equation system.

$2x + 5y = 28$ - - (1)
$-\ \underline{2x + 3y = 3}$ - - (2)
$0 \quad 2y = 26$
$2y = 26$
$y = \frac{26}{2}$
$= \underline{\underline{13}}$

Substitute $y = 13$ in equation (2) to determine the value of x, then we have

$2x + 3y = 3$
$2x + 3(13) = 3$
$2x + 39 = 3$

Collect like terms $2x = 3 - 39$
$2x = -36$
$X = -\frac{36}{2}$
$= \underline{\underline{-18}}$

The process of getting ride of one of the unknown variable is known as elimination. It does not matter which unknown is eliminated, the student should always start with the variable that is easy.

SELF ASSESSMENT EXERCISE

Solve the following simultaneous linear equation

a) $2x + 3b = 6$ b) $3a - b = 11$

$x + 2b = 6$ $2a - 3b = 5$

3.4 APPLICATION OF SIMULTANEOUS EQUATIONS

This involve solving problems that we commonly encounter in daily interaction, sometimes it may be in the business transactions and other activities.

Example 5.

In a market survey within Jos township, it was discovered that within Ahmadu Bello way, 6 exercise books and 12 biros cost N144. However at Rayfield 8 exercise books and 10 biros cost N132. Determine the price of a biro and an exercise book.

Solution: let exercise book be represented by x and biro by y we then have $6x + 12y = 144$ - - - (1)

$8x + 10y = 132$ - - - (2)

Determine the value of exercise book and a biro multiply equation (1) by 8 and equation (2) by 6 to bring x variable to the same unit

$6x + 12y = 144$ - - - (1) x 8

$8x + 10y = 132$ - - - (2) x 6

$48x + 96y = 1152$ - - (3)

$48x + 60y = 792$ - - (4)

Subtract equation (4) from equation (3)

$$\begin{array}{rrcr} & 48x + 96y & = & 1152 \\ - & \underline{48x + 60y} & \underline{=} & \underline{792} \\ & 0 \quad 36y & = & 360 \end{array}$$

$36y = 360$ - - - - (5)

$$y = \frac{360}{36}$$

$= 10$

Put the value of y into equation (1)

$6x + 12y = 144$

$6x + 12(10) = 144$

$6x + 120 = 144$

Collect like terms

$6x = 144 - 120$

$6x = 24$

$X = \frac{24}{6}$

$= \underline{\underline{4}}$

Example 6.

6years ago Edeh was 3 times as old as Ebo. Their combined age is 24. Determine the age of Edeh and Ebo

Solution $x + y = 24$ - - (1)

$x - 6 = 3(y-6)$

$x - 6 = 3y - 18$

$x - 3y = - 18 + 6$

$x - 3y = - 12$ - - - (2)

The simultaneous equations will be

$x + y = 24$ - - (1)

$x - 3y = - 12$ - - (2)

From equation (1) make x the subject

$x = 24 - y$ - - (3)

Substitute the value of x that is in equation (3) into equation (2)

$x - 3y = - 12$

$24 - y - 3y = - 12$

$24 + 12 = y + 3y$

$36 = 4y$

$Y = \frac{36}{4}$

$= \underline{\underline{9}}$

Substitute the value of y into equation (3)

$X = 24 - y$

$x = 24 - 9$

$x = 15$

Edeh is 15 years while Ebo was 9 years.

4.0 CONCLUSION

The above analysis shows that simultaneous linear equation is very vital in business practice and daily interactions. A good knowledge of the simultaneous equation will help us solve many common problems that we encounter.

5.0 SUMMARY

The unit has thrown some light to the meaning and scope of simultaneous linear equation. The method of finding solutions simultaneous equation examined are substitution, elimination by addition and subtraction. Attempt at working practical problems were also made for a better understanding of the unit.

6.0 TUTOR MARKED ASSIGNMENT

1. Solve the following simultaneous equations.

a) $6x - 5y = 27$ b) $3y + 2z = 12$

$3x + 4y = 16$ $5y - 3z = 1$

2. A certain number is formed of two digits, its value equals four times the sum of its digits. If 27 is added to it, the sum is the number obtained by interchanging the digits. What is the number?

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UNIT 8. QUADRATIC EQUATION

CONTENTS

1.0 INTRODUCTION

A quadratic equation is an equation of second degree, that is an equation in which 2 is the highest power of the letter or letter in the equation. There are different methods of determining the solution to quadratic equation. Some of the method includes factorization, completing the square, solution by formula and solution by graphical methods which is first worked in a table. The student is required to study each of the method so as to have adequate exposure in quantitative reasoning.

2.0 OBJECTIVE

At the end of this unit, you should be able to

† Solve the quadratic equation by factorization
† Solve quadratic equation by completing the square
† Solve quadratic equation by the formula
† Solve quadratic equation by graphical method

3.0 MAIN CONTENT

3.1 SOLUTION BY FACTORS

The method of finding solution to quadratic equation by factors requires looking for appropriate factors for the unknown and the integers within the equation.

Example 1. $x^2 – 25 = 0$
The factors of x^2 and 25 are x and 5 :- $x^2 – 25$;

$(x + 5)(x – 5) = 0$
:- either $x + 5 = 0$ or $x – 5 = 0$
:- $x = -5$ or 5
$x^2 = 25$
The second take the square root of each side; the square root of 25 which is either 5 or – 5 because $(+5)(+5) = 25$ and $(-5)(-5) = 25$ Therefore $x = 5$ or -5. the answer is usually written as $x = \pm 5$.

Example 2. Solve the following quadratic equation
$(x + 3)(x – 5) = 20$
Multiply out the equation to form the quadratic equation as follow
$x^2 – 2x – 15 = 20$
:- $2^2 – 2x – 35 = 0$
$(x – 7 (x + 5) = 0$
either $x – 7 = 0$ or $x + 5 = 0$
$x = 7$ or $x = -5$

Example 3. Solve the following quadratic equation $8x^2 + 6x = 9$
$8x^2 + 6x – 9 = 0$
Find the factors $(2x + 3)(4x – 3) = 0$
:- either $2x + 3 = 0$ or $4x – 3 = 0$
:- $2x = -3$ or $4x = 3$
:- $x = {}^3/_2$ or ¾

SELF ASSESSMENT EXERCISE

Solve the following equations
a) $x^2 – 6x + 9 = 0$, (b) $x^2 – 5x – 6 = 0$, (c) $x^2 + 9x + 14 = 0$

3.2 SOLUTION BY COMPLETING THE SQUARE

This involves a process of converting the equation into perfect square and taking the root of each side. Example to convert $x^2 + 6x$ into a perfect square we add to it $(½ \text{ of } 6)^2 = 3^2$ because $x^2 + 6x + 3^2 = (x + 3)^2$ similarly to convert $y^2 – 7y$ into a perfect square, we add to it $(½ \text{ of } 7)^2 = ({}^7/_2)^2$, because $y^2 – 7y + ({}^7/_2)^2 = (y – {}^7/_2)^2$

Generally $y^2 + bx$ becomes a perfect square if we add $(½b)^2$ to the equation
$y^2 + by + (½b)^2 = (y + ½)^2$

Example 4. Solve the following equation by completing the square $y^2 - 6y = 27$.

Add 3^2 to each side of the equation $y^2 + 6y + 3^2 = 27 + 9$

$:- (y + 3)^2 = 36.$

Take the square root of each side: the square root of 36 is either + 6 or -6.

$:- y + 3 = +6$ or $y + 3 = -6$

$:- y = 3$ or -9.

Example 5. What should be added to $y^2 + 6y$ to make the expression a perfect square?

Suppose $y^2 + 6y + k$ is a perfect, and that it is equal to $(y + a)^2$. It is known by expansion that $(y + a)^2 = y^2 + 2ay + a^2$ therefore $y^2 + 2ay + a^2$ and $y^2 + 6y + k$ are identically equal. If we compare the coefficient of y.

$2a = 6$

$:- a = 3.$

Therefore $y^2 + 6y + k = (y + 3)^2$

$= y^2 + 6y + 9.$

This shows that 9 should be added and k equals 9. Then the equation is $y^2 + 6y + 9 (y + 3)^2$. In practice the quantity to be added is the square of half of the coefficient of y (or any other letter that may be involved in example 5 above the coefficient of y is 6, half of 6 is 3, and the square of 3 is 9 that is why 9 should be used to make it a perfect square.

Example 6. Solve the equation by completing the square $y^2 – 8y + 3 = 0$.

The left hand side of the equation does not factorise, therefore the equation is first rearranged to make the left hand side a perfect square.

$y^2 – 8y + 3 = 0$

Subtract 3 form both sides

$y^2 – 8y = -3$

Add 16 to both sides of the equation

$y^2 – 8y + 16 = -3 + 16$

$y^2 -8y + 6 = 13$

$(y – 4)^2 = 13$

$:- y – 4 = \pm \sqrt{13}$

$:- y = 4 \pm \sqrt{13}$

SELF ASSESSMENT EXERCISE

1. From the following add the term that will make each expression a perfect square.
   (a) $w^2 – 4w$ (b) $y^2 – 7y$ (c) $x^2 + 5x$
2. Solve the equation below
   (a) $x^2 + 18 = 9x$ (b) $x^2 + 10x + 21 = 0$ (c) $9y^2 + 6y + 1 = 0$

3.3 SOLUTION TO QUADRATIC EQUATION BY FORMULA

Mathematically, any quadratic equation can be reduced to the form of expression as $ax^2 + bx + c = 0$. The formula for the values of x is often called almighty formula or the formula. It can be expressed as follows

$$\frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Example 7. Solve the quadratic equation $5x^2 = 9x \_ 6$. the equation $ax^2 + bx + c = 0$ is equivalent to $5x^2 - 9x - 6 = 0$ through rearrangement. Therefore a=5, b=-9, c=-6. It can now be substituted into the formula as follows:

$$x = \frac{-(-9) \pm \sqrt{\{(-9)^2 - 4\,(5)\,(-6)\}}}{2\,(5)}$$

$$= \frac{9 \pm \sqrt{(81 + 120)}}{10}$$

$$\frac{9 \pm \sqrt{201}}{10}$$

$$x = \frac{9 \pm 14.8}{10}$$

$$x = -\frac{23.18}{10} \quad \text{or} \quad -\frac{5.18}{10}$$

$$= 2.318 \quad \text{or} \; -0.518$$

SELF ASSESSMENT EXERCISE

Solve the following equation
(a) $x^2 + 7x = 5$ (b) $5x^2 - 7x - 4 = 0$ (c) $2x^2 - 5x = 4$

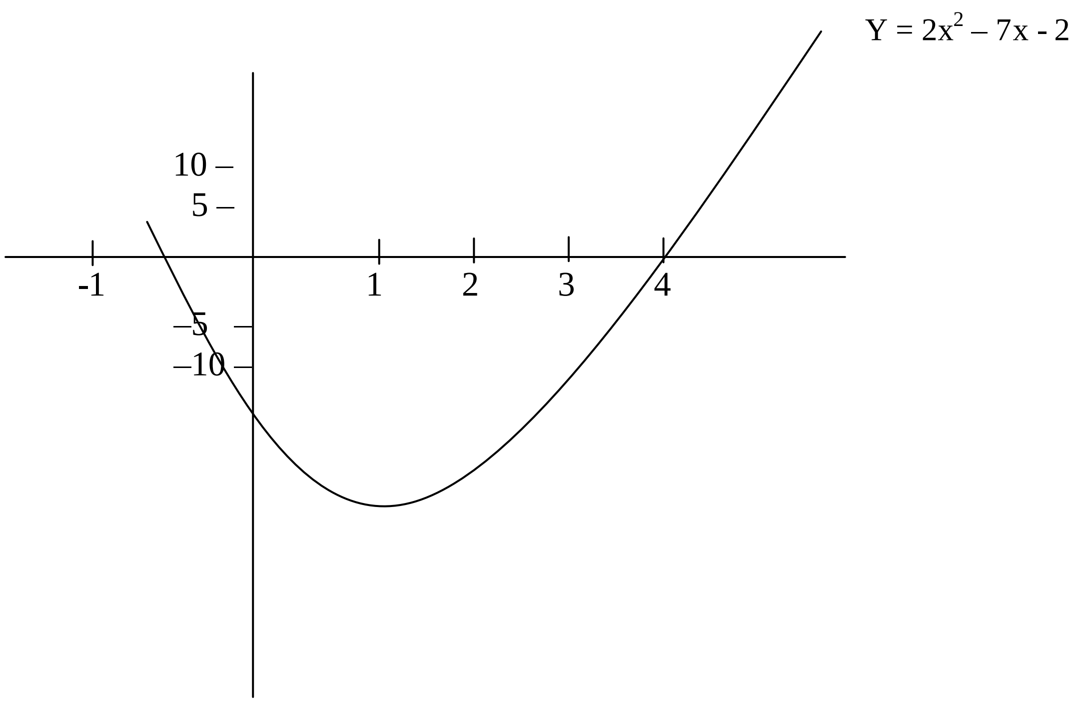
3.4 SOLUTION BY GRAPH

Quadratic equation can be solved using graphical method. In this method the equation and the range for the graph would be given.

Example 8. Given the following quadratic equation $y = 2x^2 - 7x - 2$, draw a graph for values of x range from -1 to +4.

The first step is to make a table, work by rows $y = 2x^2 – 7x – 2$

X	-1	0	1	2	3	4
-2	-2	-2	-2	-2	-2	-2
$2x^2$	2	0	2	8	18	32
-7x	7	0	-7	-14	-21	-28
$Y=2x^2-7x-2$	7	-2	-7	-8	-5	2



Scale 2cm on x-axis represents 1 unit.
5cm on y a-axis represent 1 unit

The solution are at point 'A' and 'B' It can be read to determine the actual points that are optimal.

SELF ASSESSMENT EXERCISE

Draw the graph of y where $y = 4x^2 + 6x – 7$ for values of x range from – 3 to + 2

4.0 CONCLUSION

Quadratic equation from the above analysis can be solved using different methods so as to enrich our knowledge of algebra in business and planning. It is important to use different options in solving the same problem. It could be in your business or daily transactions.

5.0 SUMMARY

The unit has thrown some light on the operations of quadratic equation using completing the square method, factorization. Where this seem to be difficulties the formula or the graphical method can be used. Any of the method will give the same solution. However the choice is for the student to determine the approach that he understands best.

6.0 TUTOR MARKED ASSIGNMENT

1. Given the following quadratic equation $y = 2x^2 - 3x - 7 = 0$ using the range of x = -1 to +4. Plot the graph and read the roots.
2. (a) Solve the quadratic equation $3y^2 - 4y + 5 = 0$
   (b) Solve the equation $y^2 - 4y + 13 = 0$
   (c) Solve the equation $x^2 - 7x + 10 = 0$

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UNIT 9. PROGRESSIONS

CONTENT

1.0 INTRODUCTION

A progression is a set of numbers in some definite order in successive terms or numbers of a sequence formed according to a given number of rules or conditions. The progression at any given time is an integer, a real number. The number could be positive or negative depending on the circumstance and the question that would be solved.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- Explain arithmetic progression
- Discuss geometric progression
- Discuss the application of progressions

3.0 Main Content

3.1 ARITHMETIC PROGRESSION CONCEPTS

An arithmetic progression is a sequence in which quantities increase or decrease by a common difference. The sequence 5, 7, 9, 11 … n is an arithmetic progression since the difference between any two consecutive terms is 2. The sequence 3, 7, 11, 15, 19, 23 … n is an arithmetic progression where the difference between any two consecutive terms is 4. Arithmetic progression can occur in the form of negative integers such as -2, -5, -8, -11 … n, as the difference between any two consecutive term is -

3. An arithmetic progression can also have a combination of positive and negative integers such as 14, 8, 2, -4, -10, -16.
Given the sequence 1, 3, 5, 7, …n the you would observe that there is a rule governing the sequence as each number other than the first can be obtained from the preceding one by adding a fixed number 2. Each number or quantity in a progression is called a term, the difference between one term and the preceding one is called common difference denoted ‘d’. The first term in a progression is conventionally denoted by ‘a’.

The terms generally of an arithmetic sequence can be written as, a, a + b, a + 2d, a + 3d … a + (n–1)d. Therefore the nth term of an arithmetic progression is given by t = a + (n–1)d

The sum of an arithmetic progression (ap) is given by

$Sn = {}^{n}/_{2}\{2a + (n-1)d\}$

Or

$Sn = {}^{n}/_{2}\,(a + L)$

Here L is the last term in the arithmetic progression.

3.2 FINDING THE NTH TERM OF ARITHMETIC PROGRESSION

Based on the introduction we stated that the nth term of an arithmetic progression can be computed $S_n = a + (n - 1)d$. where ‘a’ is the first term, ‘d’ = common difference.

Example 1. Find which term is 383 from the following series, 5 + 8+ 11 + … n.

Solution

Based on the series the first term ‘a’ = 5, the common difference ‘d’ = 3, the nth term = 383

t = a + (n – 1)d

Substitute the variables

t = 5 + (n – 1)3 = 383

5 + 3n – 3 = 383

5 – 3 + 3n = 383

2 + 3n = 383

3n = 383 – 2

3n – 381

Divide through by 3 $\frac{3n}{3} = \frac{381}{3}$

n = 127.

Example 2. In an arithmetic progression the third term is 10 the 7^{th} term of this progression is 34. Find the first term and the common difference.

Solution The first term = a, the common difference = d

Therefore the 3^{rd} term equation is $= a + 2d = 10$ …. (1)

The 7^{th} term equation is $= a + 6d = 34$ …….. (2)

Solve the equations simultaneously

$a + 2d = 10$

$\underline{a + 6d = 34}$

$\underline{\quad 4d = 24}$

$4d = 24$

$$\frac{4d}{4} = \frac{24}{4}$$

$d = 6$

Substitute $d = 6$ in equation (1) we have

$a + 2(6) = 10$

$a + 12 = 10$

$a = 10 - 12$

$a = -2.$

SELF ASSESSMENT EXERCISE

How many terms of the series 24, 20, 16 should be so that the sum may be 72?

3.3 COMPUTATION OF THE SUM OF ARITHMETIC PROGRESSIONS.

Example 3. Find the sum of the first 28 terms of an arithmetic progression whose series is give as $3 + 10 + 17 + \ldots n$

$Sn = {}^{n}/_{2}\{2a + (n - 1)d\}$

$= 14\{6 + (27)7\}$

$= 14\{6 + 189\}$

$= 14(195)$

$= 2730.$

SELF ASSESSMENT EXERCISE

a. Find the sum of the first 42 terms of an arithmetic progression whose first term is 3, and the common difference is 7.

3.4 GEOMETRIC PROGRESSION

If the consecutive terms of a sequence differs by a common ratio, the term are said to form a geometric progression. In other words, this is a type of progression in which one term other than the first can be obtained from the preceding on by multiplying or dividing by a constant quantity known as the common ratio denoted by 'r' The first term of a geometric progression is conventionally denoted by 'a'.

The general form of geometric progression is given by as, a, ar, ar^2, ar^3 ….ar^{n-1}

The nth term of a geometric progression is given by the formula

$GP_n = ar^{n-1}$

Example 4. The third term of a geometric progression is 20 and the seventh term is 320. What is the first term and the common ratio.

The Third Term = Gp_3 is $ar^{3-1} = 20$

$= ar^2 = 20$ …… (1)

The Seventh Term = is $ar^{7-1} = 320$

$Ar^6 = 320$…… (2)

Divide equation (2) by equation (1)

$$\frac{ar^6}{ar^2} = \frac{320}{20}$$

$r^4 = 16$

$r = {}^4/_{16}$

$r = 2$

Substitute for r in equation (1)

$ar^2 = 20$

$a2^2 = 20$

$a4 = 20$

$a = 20/4 = \underline{\underline{5}}$

The sum of a geometric progression give that the geometric progression series

$Sn = a + ar + ar^2 + ar^3 + \dots ar^{n-1}$ …. (1)

Multiply through by r the common ratio

$rsr = ar + ar^2 + ar^3 + ar^4 + \dots + ar^n$ …. (2)

Subtract (1) from (2)

$Sn = rsn = a - ar^n$

$S (1 - r) = a (a - r^n)$

$Sn = \frac{a(1 - r^n)}{1 - r}$ Used when r < 1

$Sn \frac{a(r^n - 1)}{r - 1}$ Used when r > 1

Example 5. The second and third term of a geometric progression are 16 and 64 respectively. Find the first term and the common ratio of the progression.

The second term ar = 16

The third term $ar^2 = 64$

$$\frac{ar}{16} = \frac{ar+2}{64}$$

$$\frac{64}{16} = \frac{ar^2}{ar}$$

$4 = r$

$r = 4$

But the second term $ar = 16$

$a(4) = 16$

$4a = 16$

$a = 16/4$

$a = 4$

Example 6. The third term of a geometric progression is 20 and the seventh term is 320, what is the sum of it first nine terms?

Gp_3 :- $ar^2 = 20$

Gp_4 :- $ar^6 = 320$

$$\frac{ar^6}{ar^2} = \frac{320}{20}$$

$r^4 = 16$

$r = 2$

Substitute and find common ratio

$ar^2 = 20$

$a2^2 = 20$

$4a = 20$

$A = 20/4 = 5$

$$Sn = \frac{a(r^n - 1)}{r - 1} = \frac{5(2^9 - 1)}{2 - 1} = \frac{5(512 - 1)}{1} = \underline{\underline{2555}}$$

3.5 APPLICATIONS OF PROGRESSIONS

A man starts work with a salary of N14,000 a year and receives annual increase of N480 a year (a) How much does he receive for the first four years. (b) How much will he receive in the tenth year of employment.

$Ap_n = a + (n - 1)d$

$Ap_1 = 1400 + (1 - 1)\ 480$

$1400 + (0)\ 480$

$= 14,000.$

$Ap_2 = 14,000 + (2 - 1)\ 480$

$= 1400 + (1)\ 480$

$= 1400 + (1)\ 480$

$1400 + 480$

14480.
$Ap_3 = 1400 + (3 – 1)\ 480$
$= 1400 + (2)\ 480$
$1400 + 960$
14960.
$Ap_4 = 1400 + (4 – 1)\ 480$
$1400 + (3)\ 480\ 1400 + 1440$
$= 15440$
The total amount for the first four years will be 1400 + 14480 + 14960 + 15440 = $\underline{\underline{58880}}$

b. In the tenth year n = 10
$Ap_{10} = 1400 + (10 – 1)\ 480$
$1400 + 9\ (480)$
$1400 + 4320$
18320

SELF ASSESSMENT EXERCISE

Umenemi was employed earning N12000 annually. He is offered a choice between a yearly increment of N150 and an increment of N420 every two years. Calculate the total sum he will earn in the course of 20 years under each option offered to him.

4.0 CONCLUSION

The above analyses show that arithmetic and geometric progression form an integral part of business mathematics. It equally has wide applications in business and economics. Therefore it is very essential for the you to get involved in the learning of progressions as a means of enhancing quantitative reasoning.

5.0 SUMMARY

The unit examined the meaning and scope of progressions. The concept of arithmetic progression, geometric progressions and the process of deriving the formula and application of the equations used in progressions. The application of progression in business life was also highlighted and illustrated to give a balanced knowledge of the unit.

6.0 TUTOR MARKED ASSIGNMENT

1. Find the values of x, y, z if 12, x, y, z, -4 form an arithmetic progression.

2. (a) The third term of an arithmetic progression 42 and the 13^{th} term is 182 find the first term and the common difference.

(b) A man was employed with an annual salary of N280,000 and receives an annual increment of N820 (i) How much does he receives for the first three years.

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UNIT 10. STATISTICAL INVESTIGATION AND DATA COLLECTION

CONTENT

1.0 INTRODUCTION

Statistics is a process of factual data collection and analysis of the data. It involves collection of numerical facts in a systematic way. Statistics also involves the careful analysis of the data collected in form of tables and the interpretation of such data. It also involves the use of scientific method of collecting, organizing, summarizing, presenting, analyzing data as well as drawing conclusions so as to take reasonable decision concerning a given phenomenon.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- Explain the basic concept of descriptive and inferential statistics.
- Discuss strategies of statistical enquiries.
- Discuss and apply the data collection method in statistics.

3.0 MAIN CONTENT

3.1 DESCRIPTIVE AND INFERENTIAL STATISTICS.

(1) **Descriptive Statistics:**

In descriptive statistics, the data collected describes the situation that existed at the point in time when the census was taken. It provides a step by step detail of data available and collected at any given period. The important characteristics of descriptive statistics is that population to be studied are included. If the University of Jos for example is taken as a point of discussion, the Vice Chancellor should know the number of Deans of faculties, the Heads of departments, the heads of units which are regarded as administrative instruments in the institution. The Deans should know the number of Heads of departments and other staff. The knowledge of the categories of staff is also important. So also does it apply to the Head of department and lecturers who need to know the number of students and the score of each student in each course. Events are described as they happen and these could be presented in;

i. Bar charts
ii. Pie charts or in pictorial form.

Supposing in the faculty of social sciences, each student is allocated a file and all information required of them is included in these files, a lot of things can be done with the information; If we want to consider the scores of female and male students on statistics, we can draw graphs to represent the information. If we are concerned with the relationship of individual scores to the averages that have been computed we can change the raw scores to standard score, all of the foregoing operations are included in what is referred to as descriptive statistics.

Descriptive statistics presents information in a convenient format usable and understandable form in words with little numeric included in the description of the data.

(2) **Inferential Statistics**

This is a data collected and used to make inference as related to the occurrence of events. Inferential statistics which is mostly linked with probability theory, it involves estimate outcomes of events. According to "Philips (1980) "it is that measure you have gotten that could have occurred by chance". Therefore statistics of inference essentially has to do with the measurement of chance.

We usually start with setting up a hypothesis or a number of hypotheses specifying our assumption(s) at the beginning of a study. For e.g. we can say that most members of the PDP are conservative in respect of economic policies while ANPP members take a liberal approach to economic policies. We are able to make these assumptions from selected samples of members of the two parties. Sometimes this hypothesis may turn out to be true but it may also be false when this happens, it is not possible to generalise because we may not be correct and a problem might arise.

This problem has been acknowledged in statistics long time ago and what to do is to make sure we use the most appropriate method and measurement/analytical procedure so that the result we get will approximate the real population characteristic. Making inferences is a question of chance, however there are methods available for us to determine whether the results we obtain from a statistical investigation could be attributed to a chance occurrence even if its opposite were generally true. On the other hand we could also measure the odds that the result of our investigation is false. This will place us in a position to make right conclusion on a particular social or political phenomenon when we do this there is to a certain measure the possibility of the truth or the possibility of falsehood of our result ; we accept to measure or test, the statistical significance of our result.

Inferential statistics can be divided into two namely:

- Deductive statistics
- Inductive statistics

(A) Deductive Statistics: Is the act of drawing inference about a sample using our knowledge of the population. The process involves arguing from the general (the population) to specific (the sample). It is deductive inference when probability of an event within a population context is obtained from a prior knowledge of the parameters of the distribution.

(B) Inductive Statistics: Is the process of drawing inference about the population from the sample. It is arguing from the specific to the general. Reason of cost, time factor, accuracy and other constraints may make a complete enumeration (census) of the population impossible. The alternative is the use of concepts in probability to draw a sample from the population obtain estimate of the population parameters and test statements (hypothesis) about the parameters.

(C) **Correlation Statistics** This is a statistical method that involves a comparison between two events. Example the first Semester test scores in statistics can be compared with the second semester statistics scores in year two. In statistics, this study of prediction is referred to as “Regression Analysis” the results of correlation analysis are used to study the reliability and validity of the test. Correlation analysis is a major part of statistical methodology.

SELF ASSESSMENT EXERCISE

Carefully expain the inductive and deductive statistics.

3.2 STATISTICAL INQUIRIES

The businessman and indeed anyone who has to administer any organization is concerned with inquiries of many kinds. Some of these are capable of being treated and tested statistically and statistical evidence can be provided in respect of the information desired in any given situation.

The steps in a statistical inquiry are as follows:

1. **The Problem should be clearly stated**

It is necessary to know the purpose of the investigation as this will influence the type of information to be obtained. Suppose the problem concerns wages in a factory, is it about wages earned or wage grade?, does the statistics concern all employees or only (women or men)?, Should lost time, overtime, piece work and bonus payments be included or allowed for?, Should receipts in kind be included in the wage? The purpose of the investigation will provide guidance as to the exact information that ought to be obtained.

2. **Selection of the Sample**

In most problems concerning the administration of business, governmental or personal affairs or in making scientific generalization complete information cannot be obtained hence incomplete information must be used and these means taking the sample. The size of the sample and the sampling method will have to be determined. The best example of a sample inquiry in business is market research.

3. **Drafting the Questionnaire**

This is quite a difficult job if answers are to be of value. Usually a number of questions have to be drafted to get the exact information needed for a given time. A pilot survey is useful to enable a satisfactory questionnaire to be obtained. A great deal of information in business, however is already available in form of accounting records, costing and administrative information about personnel, questionnaires apart from market research are therefore useful only for special inquiries.

4. **Data Collection**

Where not available as administrative published records, the most satisfactory way to obtain information is by means of enumerators or postal questionnaires. The types of questions to be asked depends on whether postal questionnaires are going to be used or enumerators are to be sent out. Questions for postal questionnaires are usually simple and easy to understand but those question given to enumerators may be complex:

Because enumerators would be there themselves and would be able to explain it clearly. Enumerators are usually trained on how to ask the questions before they are sent to the field. They also understand the objective of the research so that they work with a clear vision and focus.

5. **Editing the Schedule**

Questionnaires require to be checked sometimes coded and calculations made before conclusion can be drawn to picking relevant information from the study.

6. **Organization of Data**

The items required to be counted or the value summed either in quota or in various categories before they can be calculated.

7. **Analysis and Interpretation**

Before the information acquired can be used, it is analysed and then interpreted, this requires a sound knowledge of statistical methods and also a sound knowledge of the subject for which statistical evidence has been obtained.

8. **Presentation**

This might take the form of tables, charts and graphs that will give a picture of the data under study. The presentation of statistical data helps to give an idea of the outcome of the study.

9. **Writing of the Report**

This gives the result of the investigation and where required or necessary we make recommendations. Tables and charts usually play an important part in biz reports such reports are sent to government department, business (biz) organization and private bodies to be used for planning.

3.2.1 USES OF STATISTICS AND STATISTICAL INFORMATION

Although statistics is a powerful tool for analyzing numerical data, its application is widely seen in all fields of human Endeavour. For instance we apply statistics to;

A. Physical Science: It can be used to determine whether or not experimental results should be incorporated into the general body of knowledge.
B. Biological and medical science: Statistics guides the researchers in determining which experimental findings are significant enough to demand further study, or to be tested more to meet human needs.

Thus the physician uses statistics to access the effectiveness of a particular treatment and statistics also helps the pharmacologists to evaluate a proposed drug. In some fields such as genetics statistics is thoroughly integrated into the field to study the multiplication of cells and other variables.

C. Social Sciences: The role of statistics in the social sciences cannot be ignored especially in business administration. Accounting, political science, psychology, sociology and economics. The behavior of individuals and organizations can be monitored through "numerical data" to lend credence to models and theories that are applicable to man.

D. Engineering, Education and Business: The professional fields of engineering education and business all employ statistics in planning establishing policies and setting standards. The headmasters or principal of a school may use statistics to write the curriculum, the school enrolment, teachers required, the civil engineer can use statistics to determine the properties of various materials and perform some durability test. The company manager may employ statistics to forecast sales, design products and produce goods more efficiently.

E. Meteorology: Statistical information is also used in meteorology i.e. the science of weather prediction. In fact the application of statistical techniques is so wide spread and the influence of statistics in our lives and habits is so great that the importance of statistics can hardly be over emphasized. There can be a little doubt then on the effect of statistics and statistical techniques on each of us. The result of statistical studies are seen but perhaps not realized.

In scientific and behavioral research, statistical tools enable success of research results. In business and economic situations, it's use is highly appreciated. Below is the summary of some of the uses of statistics in every day life.

1. For summarizing large mass of data into concise and meaningful form leading to a better understanding of condensed data.
2. Giving visual impact on data especially when presented in diagrams and charts.
3. Enables comparison to be made among various types of data
4. Making conclusions from data generated in pure experimental, social and behavioural research.
5. Enabling a business establishment to make accurate, reasonable and reliable policies based on statistical data.
6. Predicting future events in daily life and business.
7. For the formation as well as testing of hypothesis.

8. For prediction of Gross National Product (comparing it with that of other countries) input-output analyses, public finance and consumer finance.
9. For budgeting and planning
10. Widely used in industrial and commercial dealings as well as government establishment.
11. It's knowledge enables one to understand relevant articles in scientific journals and books.

3.2.2 Problems of Data Collection

Data collection can be difficult or inaccurate sometimes. The absence and unavailability of accurate statistical data may be due to all or some of the following reasons;

1. Lack of proper communication between users and producers of statistical data.
2. Difficulty in estimating variables which are of interest to planners.
3. Ignorance and illiteracy of the respondent.
4. High proportion of non response due to suspicion on the part of respondent.
5. Lack of proper framework from which samples can be selected.
6. The wrong ordering of priorities including misdirection of emphasis and bad utilization of human and material resources.

3.2.3 Limitations of statistics

1. Statistical data or result is only an approximation of the total and therefore not entirely accurate in some cases. This is because not all the population will be covered for any sample study.
2. Statistics if not carefully used can establish wrong conclusion and therefore it should only be handled by expert. Where experts are not available some form of training should be conducted for those that may be required to carry out any statistical research.
3. Statistics deals only with aggregate of fact as no importance is attached to individual items.

SELF ASSESSMENT EXERCISE

(1) List and explain the steps of statistical enquiry.
(2) List the uses of statistics and statistical information

3. 3 METHODS OF DATA COLLECTION

Business data are collected in the normal cause of administration and not specifically for statistical purpose, however, there is no reason why records should not serve the two purposes and in such cases, care should be taken to ensure that the record is accurate statistically as well as administratively.

The following list covers some of the important methods of collecting data;

1. **Postal Questionnaire**

This takes the list of questions sent by post, unless of course the respondent has an interest in answering it or is under legal compulsion. The postal questionnaire is generally unsatisfactory producing few replies and those of a bias nature, the postal questionnaire is also satisfactory when sent by trade associations to the members, since the members have interest in answering it. Some firms have tired to get answers by offering small gift, this is not a very good idea, since it will produce bias answers that is the respondent tires to please the donor.

2. **Questionnaire to be filled by the enumerator**

This is the most satisfactory method. The enumerator or field workers can be briefed so that they understand exactly what the questions mean. They get the right "answers" and they fill in the questionnaire more accurately than would be the respondent themselves.

3. **Telephone**

This involve asking question by phones call to the respondent. Asking questions by telephone is not usually a very good method. People who posses telephones, form a biased sample. Telephone interviews are useful for certain kinds of radio research.

4. **Direct Observation**

This method entails sending observers to record what actually happens while it is happening at the current period, an example of these method being suitable is in the case of traffic census. Actual measurement or counting also comes under the heading of direct observation, examples occur in statistical quality control. It can either be participatory or non participatory.

5. **Reports**

This is method that may be based on observations or informal conversations. These are usually incomplete and biased but in certain cases it may be useful.

6 **Results of Experiments**

This method is of interest to the production manger, engineer, the agronomist and other applied scientist. It require carrying out experiments, sample test, laboratory examination to determine the behaviour of certain events.

7. **Interview**

The researcher asks the respondent the questions listed. The listed questions provide a guide to what is being investigated. He or she also fills in the answers. The researcher now has the duty of obtaining accurate information form the respondent. The interviewer must barely records facts accurately, all the answers as given. The interviewer must be sure of interviewing the right person or sample.

8. **Personal Investigation**

The personal investigation involves the researcher using direct contact with the respondent. These tends to be time consuming, expensive and limited in size of field, but the data collected will be complete and reliable. It is useful for pilot survey.

9. **Team of Investigation**

It is the same as the personal investigation method. The only difference is that the investigators go to the respondents in a group. The group can cover a large field than the personal investigation, but it will be more expensive. The members of the team should obviously be carefully briefed to ensure that the data they collect is satisfactory. This method is sometimes called "delegated personal investigation".

10. **Registration Method**

This method involves the recording of vital events as they are taking place within a given time, vital events include statistics on births, deaths, migration immigration, separation and adoption.

11. **Panel Method**

This method is commonly used in interviewing job seekers in Nigeria. Under this method, certain groups of people that are specifically trained interview certain people. This is to determine the true position of events under study.

Self assessment Exercise

List and explain the various methods of statistical data collection.

3.4 SAMPLING PROCESS IN DATA COLLECTION

Instead of obtaining data from the whole of the material being investigated, sampling methods are often used, in which only the sample selected from the whole is dealt with and from these samples conclusions are drawn relating to the whole population. If conclusions are to be valid, the sample should be the representative of the whole, the selection of these sample should therefore be made with great care.

3.4.1 Reasons for Using Samples

1. It is used in collecting data based on certain characteristics of a group of individuals or objects it is often impossible or impracticable to observe the entire group, especially if it is large. Instead of examining the entire group called the population or universe, one examines a small part of the group called "a sample"
2. Even where complete inspection is possible, sampling may have economic advantages. Resources such as materials, time, personnel and equipments, constitute a limitation in any investigation. It is then necessary to use the available resources to get necessary information by selecting a sample instead of the entire population.
3. Another reason for using sample is that for making data, the population is inaccessible.

In any case, the sample chosen must be representative of the whole population as the sample would provide information about the population characteristics, which are being examined. The population refers to the whole of the material from which the sample is taken. The frame will consist of a list of all the items in the population or some means of identifying any particular item in the population. This frame is necessary so that any item in the population can be part of the sample. The frame must be complete, i.e. no item of the population should be left out and it should not be defective, because of being out of date or contain inaccurate, duplicate items or inadequate because it does not cover all the categories required to be included in the investigation.

3.4.2 Sampling Techniques.

Some of the basic techniques used in statistical sampling include the following,

1.Random Sampling The word "random" does not mean haphazard it refers to a definite method of selection. A random sample therefore is one in which every member of the population has an equal chance of being selected in the sample.

A technique for obtaining a random sample is to assign numbers or names to each member of the population. Write these numbers on small pieces of paper, place them in a box, and after mixing thoroughly, draw from the box in lottery fashion. Another method is to use a table of random sampling numbers when the random sample of names have been drawn, interviews or enumerators would be sent to the people to collect all the necessary information. Although random sampling is a long, expensive operation, it does give a reliable, unbiased picture of the whole population. This method is suitable where the population is relatively small and where the sampling frame is complete.

2. **Systematic Sampling.** For practical work, it is easier to select every earned item in a list of the population. This method is termed "systematic sampling". The 1^{st} of the sample unit being selected by some random process for instance if the list comprises a population of say 25,000 and the sample required is 500, then the selection of every 50^{th} item will yield the required sample.

Systematic sampling is not random because once the initial starting point has been determined, it follows that the remainder of the item selected for the sample are predetermined by the constant interval (i.e. 50). In Random/systematic, the samples are believed to be homogenous

3. **Stratified Sampling.** So far we assume that the population to be sampled consist of a single homogenous groups, i.e. people with the same characteristics where the population is heterogeneous i.e. comprises men and women in different age groups, in different social circumstances or of different backgrounds, a stratified sample is taken, this is because people in different social groups will think differently from other groups. In stratified sampling the population is divided into strata, groups or blocks of units in such a way that each group is as homogenous as possible (hence, same characteristics). Each group, block or stratum is then sampled at random. The stratified sample would be representative of the whole population.

4. **Multi-stage Sampling.** This is where a series of samples are taken at successive stages. For instance in the case of a national sample, the 1^{st} stage will be to breakdown the sample into the main geographical areas. In the 2^{nd} stage a limited number of towns and rural districts in each of the states will be selected. In the 3^{rd} stage within the selected towns and rural districts, a sample of respondents allocated to each state is drawn. This may also involve the list in which certain households are selected and many more stages may be added.

5. **Quota Sampling.** To economist and business managers, time and cost is taken into consideration in sample data, for this reason a method of sampling known as quota sampling is extensively employed by many organizations. The essence of quota sampling is that the final choice of the

respondents to be interviewed lies with the interviewer, this of course introduces bias. The quotas are chosen so that the sample is representative of the population in a number of respects according to the controls chosen. The interviewer is instructed to carry out a number of interviews with individuals who conforms to certain requirements. Some of the requirements often used are age, sex and social class.

6. **Cluster Sampling.** In this technique, the country is divided into small areas almost similar like multi-level sampling method. The interviewers are sent to the areas with instruction to interview every person they can find those who fits the definition given. Generally, cluster sampling is used when it is the only way a sample can be found.

3.4.2 Errors in sample Data Collection

It is believed that the larger the sample the smaller the error. There could be sampling bias because the sample is too small. In order to reduce bias, it is being approved that the sample should be large enough to provide a clear information on the topic to be studied.

1. Errors due to bias Deliberate selection can introduce bias in a sample, Substitution also introduces bias, Failure to cover the sample (not being consistent) introduce errors, Haphazard selection is also prone to errors.

2. The Questionnaires: In drafting questionnaires, direct answers like "yes" or "No" should be minimal and the respondent given an opportunity to view out his or her understanding.

3. Memory Error: The respondent may give a wrong information when the event being investigated has taken a long time. To minimize such errors, interviews should prevent asking questions on events that happened long ago, bringing events that will help the respondent recall would be of help in providing accurate information.

4. Coding Error: There are circumstances where we may use a wrong code in the process of carrying out a statistical survey. This give rise to errors in statistical data collection.

5. Editing error: Some times errors emerge during coding, it entails writing the wrong thing when compiling the result e.g. writing "1997" instead of "1977"

6. Error due to tabulation. Sometimes errors emerge as a result ofwrong tabulation of statistical information.

7. Respondents error: Due to poor education background and illiteracy, the respondent may give a wrong information. Also because of ignorance or lack of understanding of the context of the questionnaire and sample questions the respondent may give a wrong information.

Error in the sharing of questionnaires. There are situations where the individual carrying out the statistical survey may administer the questionnaire wrongly. Example a questionnaire meant for the working class and it is given to the student. The student may not provide the work experience aspect to the questionnaire.

SELF ASSESSMENT EXERCISE

List and explain the various methods of sampling in data collection.

4.0 CONCLUSION

The analysis shows that statistics is a vital course that we should take seriously due to the wide application of the subject in all areas of life. It is therefore important for every body to get involved in learning the subject. The reason being that it will help us in the studies of other course that are quantitative.

5.0 SUMMARY

The unit has thrown more light on the meaning and scope of statistics. The concept of gathering statistical information, the merit of such a process was also analysed. Sampling as a major strategy for data collection and the likely errors associated with it were highlight and discussed. In the next study unit, you will be taken through the discussion on statistical data presentation.

6.0 **TUTOR MARKED ASSIGNMENT**

1. List and discuss the various techniques of statistical enquiries.

2. Mention and explain types of techniques used in data collection.

7.0 **REFERNCES/ FURTHER READING**

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UNIT 11. DATA PRESENTATION IN STATISTICS.

Content

1.0 INTRODUCTION

In statistical analysis information gathered can be presented in the form of tables, pie chart, bar charts and frequency distribution. The presentation in the tabular form gives an idea of the distribution of the information gathered for further evaluation. The pie chart and the bar chart are pictograms that give a quick understanding of the statistical information obtained.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- Explain the basic statistical concepts with table
- Discuss and present data in pie charts
- Discuss present data in bar charts
- Discuss and present data in frequency table and graphs

3.0 Main Content

3.1 THE PRESENTATION OF STATISTICAL DATA USING TABLES.

In statistical analysis, it is often simpler and quicker to illustrate ideas with tables, charts and graph than with endless written pages. Statistical tables, graphs and diagrams are visual aids to a quick understanding of information. Such visuals aid are condensed ideas are more meaningful and comprehensive to readers who have difficulty in interpreting statistics from printed words or who have less time to read volumes. Visual comparison become important in economic and business analysis, it is also an acceptable norm in modern writing.

It is an orderly arrangement of information showing the relationship between variables. Consider the following marks obtained by students in an entrance examination. Example1.

- A's scores: English 40%, Maths 60%, General Knowledge 80%
- B's scores: English 80%, Maths 60%, Gen. Knowledge 60%
- C's scores: English 80%, Maths 40% Gen. Knowledge 60%
- D's scores: English 60%, Maths 50% General Knowledge 80%

The piece of information does not make for easy comprehension,

however, we can prepare a table to show each students marks under each subject as shown in the table below;

Performances of four students in an entrance examination

Students	Subjects			
Students	Maths %	English %	Gen. Know %	Total marks
A	60	40	80	180
B	60	80	60	200
C	40	80	60	180
D	50	60	80	190
Total Score	210	260	280	750

From the table, we can see at a glance the relative performance of the four students, we can also interpret the relative performance in the 3 subjects. Based on the table we can draw some inferences. The aggregate marks in the last column shows that "student B" has the highest total marks (200%) while those for the subject shows that the General knowledge has the highest total score (280 marks). We can state that the students performed better in the general studies among the three courses, and the students' performance is lowest in maths. We could not have seen this easily without the table.

IMPORTANCE OF TABLES

- It is used to interpret data as shown in the table.
- Data in the table can be used for comparative analysis.
- Quick decisions can be taken based on information derived from the table.
- Information from tables occupies less space.
- It reveals to us at a glance, the information conveyed on the data.
- The data given can be used in forecasting the future performance of events.

SELF ASSESSMENT EXERCISE

Consider the following information on the performance of some students in the post ume examination. Matta, English 57%, maths 50%, Current Affairs 81%. Idoko, English 87%, Maths 79%, Current Affairs 67%. Dawang, English 69%, Maths 62%, Current Affairs 61%. Present the performance of the students in a table. Interpret your findings.

3.2 PIE CHARTS

It is a circular representation of data and it is based on the fact that the sum of angles about a point is 360^0. That means a pie chart is a pictograph drawn in a circle to represent relative performance of a given variable in relation to the total value. We know a circle has a total angle of 360^0, the pie chart is constructed by dividing 360^0 proportionately. The information collected for analysis is converted in the 360^0 proportionally.

Example 2.

1. A pure water company awarded contracts to various contractors for constructing a factory house as follows:

Carpenter	25,000
Bricklayer	20,000
Painter	5,000
Decorator	10,000
Total	60,000

Represent the above information on a pie chart

Solution The first step is to find the total spent on the contract award, the divide each by the total and multiplied by 360^0 as follows.

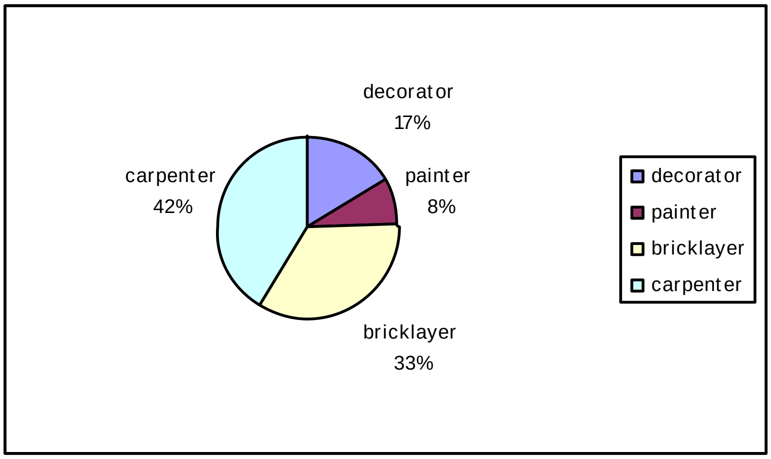
Carpenter $= \dfrac{25000}{60000} \times 360 = 150^0$

Bricklayer $= \dfrac{20000}{60000} \times 360 = 120^0$

Painter $= \dfrac{5000}{60000} \times 360 = 30^0$

Decorator $= \dfrac{10,000}{60,000} \times 360 = 60^0$

pie chart showing the proportion of contract awarded to the various contractors



The pie chart shows that the carpenter received the highest allocation from the contract, this is followed by the bricklayer, the decorator and the painter had the lowest allocation.

Example 3. The following information shows the contribution of a family in the upkeep of their school child in the first and second term at school.

2. Contributions to a students pocket money

Contributors	1st term(₦)	2nd term(₦)
Father	15	25
Mother	20	20
Friends	10	5
Total	45	50

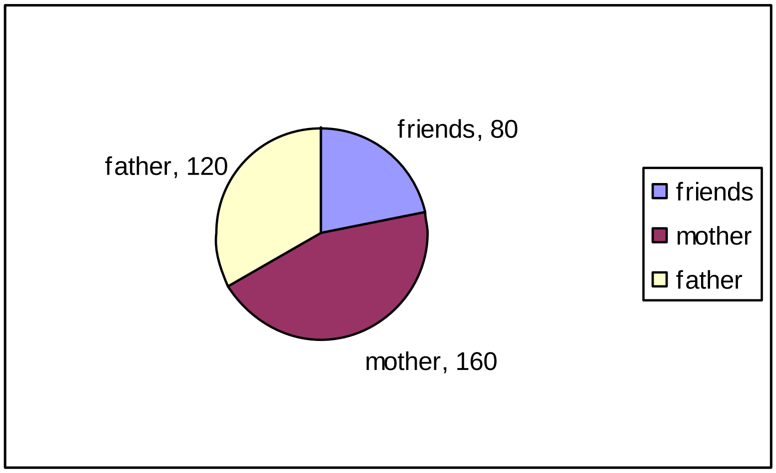
Present the information on a pie chart.

1st Term

Father $= \frac{15}{45} \times 360 = 120^0$

Mother $= \frac{20}{45} \times 360 = 160^0$

Friends $= \frac{10}{45} \times 360 = 80^0$



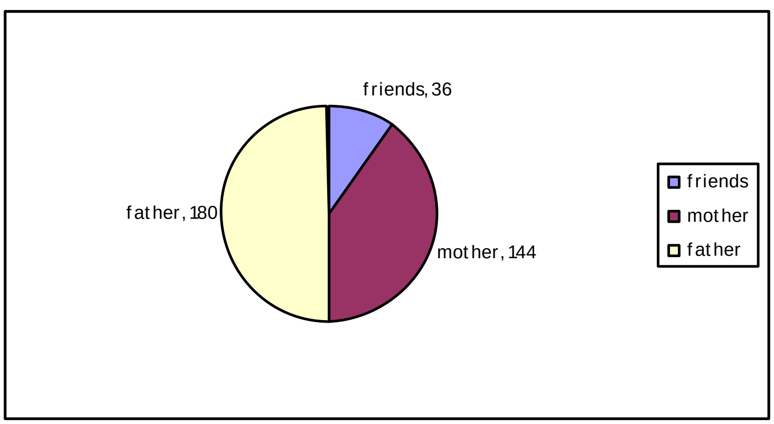
The pie chart shows graphically the contribution by the three individuals in the upkeep of the student. We can see from the diagram that the mother contributes more toward the upkeep than the father while the friends made the least contribution in the first semester.

2^{nd} term

Father = $\frac{25}{50} \times 360 = 180^0$

Mother = $\frac{20}{50} \times 360 = 144^0$

Friends = $\frac{5}{50} \times 360 = 36^0$



Using the pie chart, in the second semester the father contributed more toward the upkeep of the student than the mother and friends.

SELF ASSESSMENT EXERCISE

Given the following information on the performance of a student in five courses registered in the university in the first year. Business admin 86, Economics 72, Political science 52, General studies 49, Accounting 57. Use a pie chart show the performance of the student in the first year.

3.3 THE BAR CHART

It is a chart in which data is presented in the form of a bar and it is used to show magnitude, usually there are three types of bar charts namely:

1. Simple Bar Chart
2. Component, Bar Chart
3. Compound or multiple Barr Chart

A bar chart can also be defined as a series of rectangles whose heights are plotted proportionally to the values that are being represented or assessed. The height of the bars should be plotted to scale to show relative measurement. The width of the rectangle could be of any size but all the bars must have the same width.

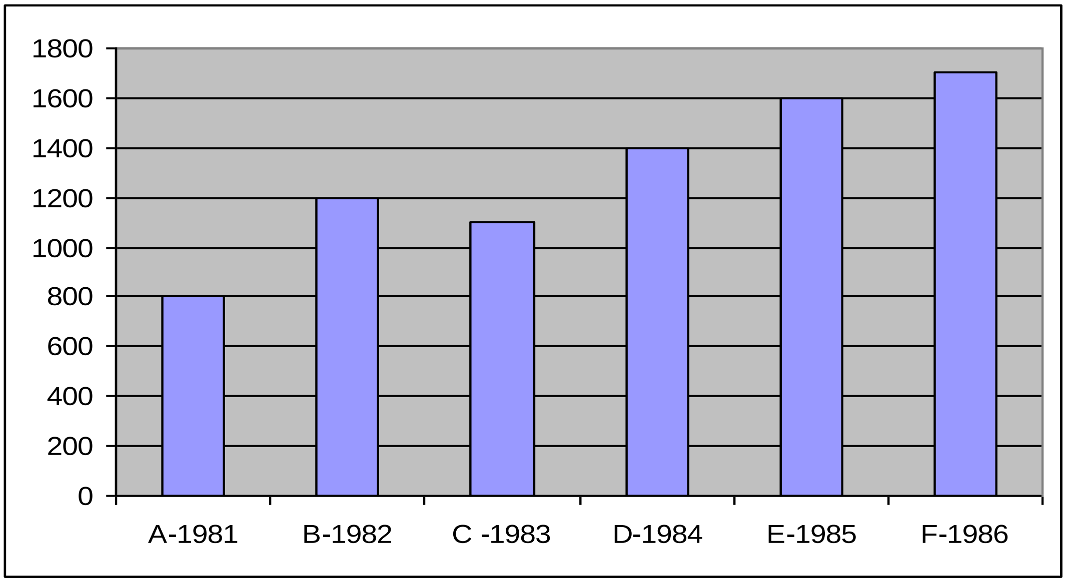
1. **Simple Bar Chart**

The simple bar chart is the chart of one or more bars in which the length of the bars indicate the magnitude of the data. Each shows the magnitude of the occurrence of the situation under study.

Example 4: Peace House have the following 6 years projection for those that will attend its annual teachers conference. Present the data in a bar chart.

Year	Attendance	Year	Attendance
1981	800	1984	1400
1982	1200	1985	1600
1983	1100	1986	1700

Representation on the bar chart.



Using the bar chart it can easily be inferred that the highest attendance for the conference is 1986 and the lowest is 1981.

2. **Component Bar Charts**

A component bar chart shows the breakdown of the total values for a given information into their component parts. There are three types of component bar charts.

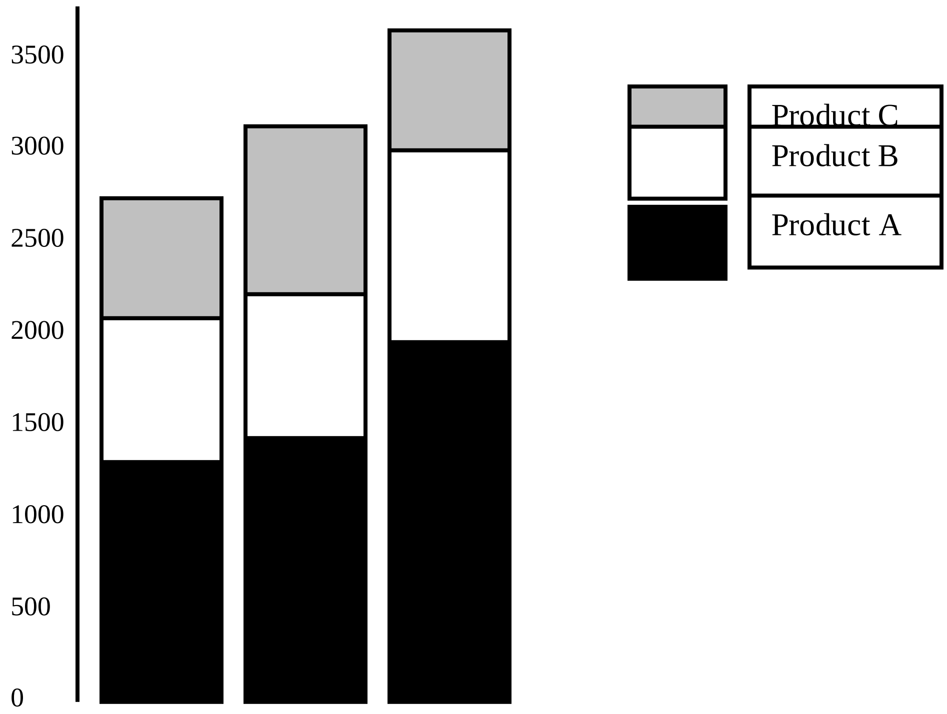
a. Multiple bar charts
b. Bar charts studying relative value
c. Percentage component bar chart

Example 5

Sam and Sam Ltd have the following sales of 3 products in the market. Present the information on a component bar chart

	1997 (sales)	1998 (sales)	1999 (sales)
Product A	₦1000	₦1200	₦1700
Product B	₦900	₦1000	₦1000
Product C	₦500	₦600	₦700
Total	₦2400	₦2800	₦3400

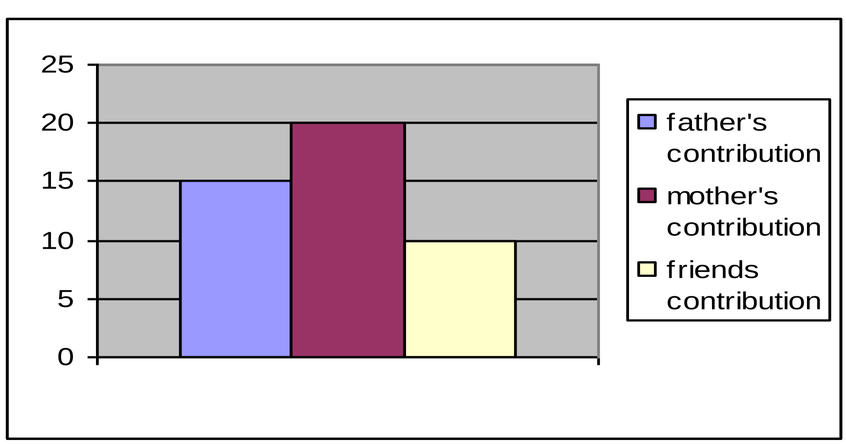
Sam and Sam Ltd company component bar charts for products A,B, and C



3. MULTIPLE BAR CHART

They are charts in which the component part are represented separately to show the total values. Example 6, suppose your father, mother and friends gave you N15, N20 and N10 respectively for your pocket money for the first term in school. The multiple bar chart of this sum is shown below.

Contributions to 1st Term Pocket Money



Here, the total sum contributed by the father, mother and friends is the sum of the height of the rectangle, each portion is represented separately.

4 Percentage multiple Bar Chart

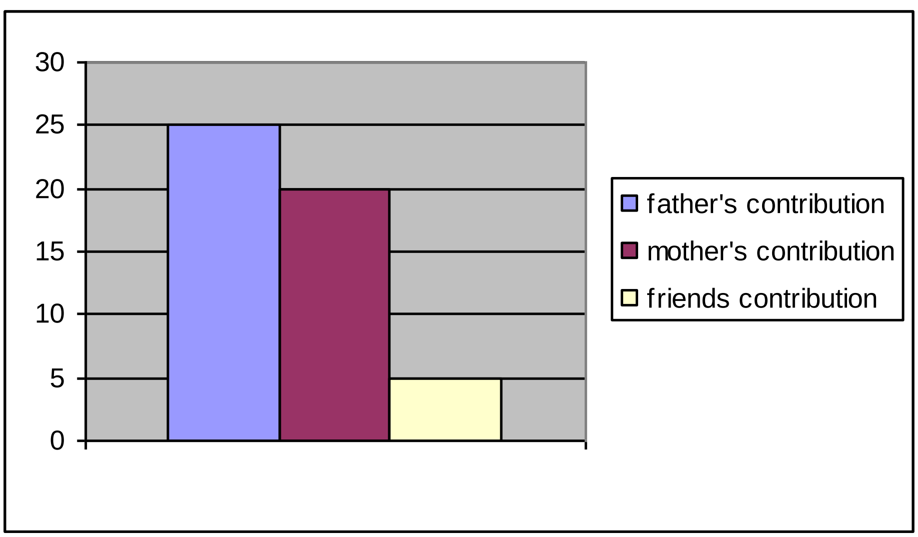
This mainly shows relatively the values that are expressed as percentages of the totals. Although the total contribution for the two terms are different, to construct a percentage component bar chart the information would be represented as 100% their component therefore will add up to 100%.

Example 7. Given the following information on the contribution for a student upkeep, Father = N25, Mother = N20, Friends = N5. Present the data on percentage multiple bar charts. The first step is to add the total contribution, then divide each contribution by the total and multiply by 100% as follows.

$$\frac{25}{50} \times 100 = 50\%$$

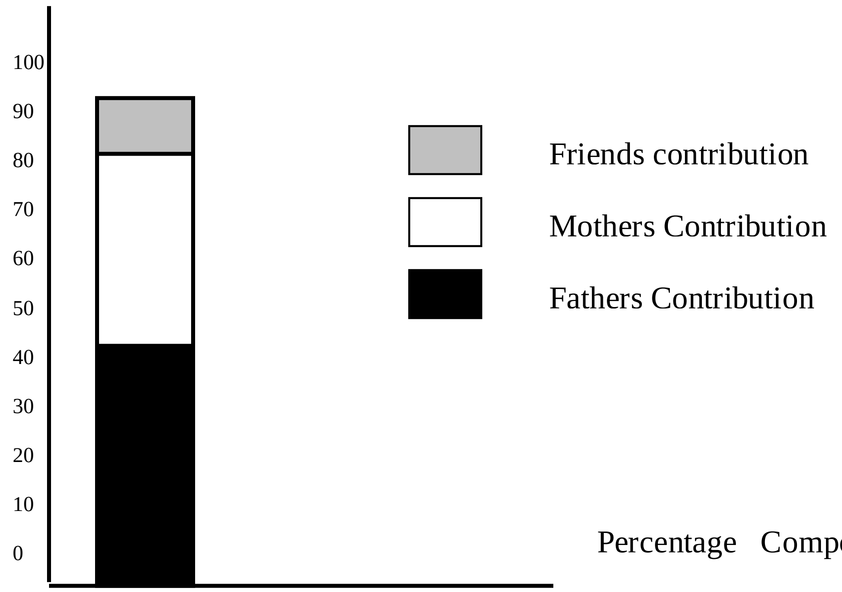
$$\frac{20}{50} \times 100 = 40\%$$

$$\frac{5}{50} \times 100 = 10\%$$



5 Percentage Component Bar Chart

This mainly shows the values expressed as percentages of the totals. In attempt to construct a percentage component bar chart on the information would be presented as 100% their component therefore will add up to 100%.



Percentage Component bar chart

SELF ASSESSMENT EXERCISE

The following data shows the performance of a student in four courses in first semester. Economics 75, sociology 63, maths 48, Accounting 61. (1) Present the information in a bar chart (2)Present the data in multiple bar chart (3) present the information in a percentage component bar chart.

3.4 FREQUENCY DISTRIBUTION.

A frequency distribution is an array of numbers. The unorganized data collected during investigation is known as "raw data" You can also arrange the data in ascending or descending order. The data that is arranged in such order is called an array of data.
Example 7. Given the following data on the number of vehicles that are parked daily in a parking lot. 92, 78, 68, 58, 45, 89, 75, 68, 58,45, 43, 57, 67, 75, 87, 42, 57, 65, 74, 87, 85, 73, 65, 56, 41, 36, 56, 61, 69,84,32, 55, 61, 69, 84, 81, 69, 60, 52, 32, 25, 49, 58, 69, 79, 15, 49, 58, 69, 79. Arrange the data in an ascending order

Solution is as follows.

15	25	32	32	36	41	42	43	45	45
49	49	52	55	56	56	57	57	58	58
58	58	60	61	61	65	65	67	68	68
69	69	69	69	69	73	74	75	75	78
79	79	81	84	84	85	87	87	89	92

From the array, we can easily identify the highest number as 92 and the lowest as 15. The difference between these numbers is known as the range i.e 92 – 15 = 77 (The range is the highest value minus the lowest value). We can still organize the data further because it is not in categories or groups or classes, normally we expect that the classes should be between 5 & 20 or 11 & 20. In this case we are using 11 – 20, 21 – 30, 31 – 40 etc.

We use tallies to form the frequency table. Tallies are strokes used for counting and value for 5 tallies is denoted by four vertical strokes and one diagonal stroke (||||) this is to facilitate the counting. So for the strokes score, we obtain the following frequency table.

<table>
<tr><th>Classes</th><th>Tally</th><th>Frequency</th><th>Cum. F</th></tr>
<tr><td>11 – 20</td><td>|</td><td>1</td><td>1</td></tr>
<tr><td>21 – 30</td><td>|</td><td>1</td><td>2</td></tr>
<tr><td>31 – 40</td><td>|||</td><td>3</td><td>5</td></tr>
<tr><td>41 – 50</td><td>||||| ||</td><td>7</td><td>12</td></tr>
<tr><td>51 – 60</td><td>||||| ||||| |</td><td>11</td><td>23</td></tr>
<tr><td>61 – 70</td><td>||||| ||||| |</td><td>11</td><td>34</td></tr>
<tr><td>71 – 80</td><td>|||| |||</td><td>8</td><td>42</td></tr>
<tr><td>81 – 90</td><td>||||| ||</td><td>7</td><td>49</td></tr>
<tr><td>91 – 100</td><td>|</td><td>1</td><td>50</td></tr>
<tr><td></td><td></td><td>50</td><td></td></tr>
</table>

Relative freq.	Cumulative relative freq.	Class Boundary	Class Mark
1/50 = 0.02	1/50 = 0.02	10.5-20.5	15.5
1/50 = 0.02	2/50 = 0.04	20.5-30.5	25.5
3/50 = 0.06	5/50= 0.1	30.5-40.5	35.5
7/50 = 0.14	12/50 = 0.24	40.5-50.5	45.5
11/50 = 0.22	23/50 = 0.46	50.5-60.5	55.5
11/50 = 0.22	34/50 = 0.68	60.5-70.5	65.5
8/50 = 0.16	42/50 = 0.84	70.5-80.5	75.5
7/50 = 0.14	49/50 = 0.98	80.5-90.5	85.5
1/50 = 0.02	50/50 = 1	90.5-100.5	95.5

Note: Class boundaries are used for histogram

- Class midpoint is obtained by $\frac{11+20}{2}$ or using class boundaries

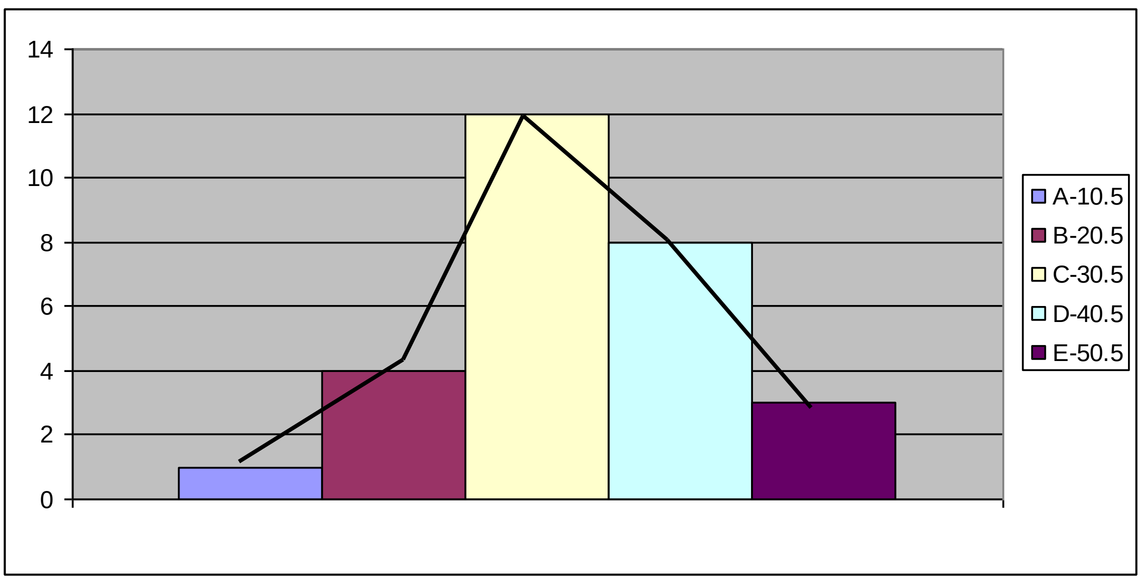
$$\frac{10.5 + 20.5}{2} = \frac{31}{2} = 15.5$$

Draw also the frequency polygon on the histogram

Group	1-10	11-20	21-30	31-40	41-50
Frequency	1	4	12	8	3
Class boundaries	0.5-10.5	10.5-20.5	20.5-30.5	30.5-40.5	40.5-50.5

Solution

- We first calculate the class boundaries and obtain result as shown above:



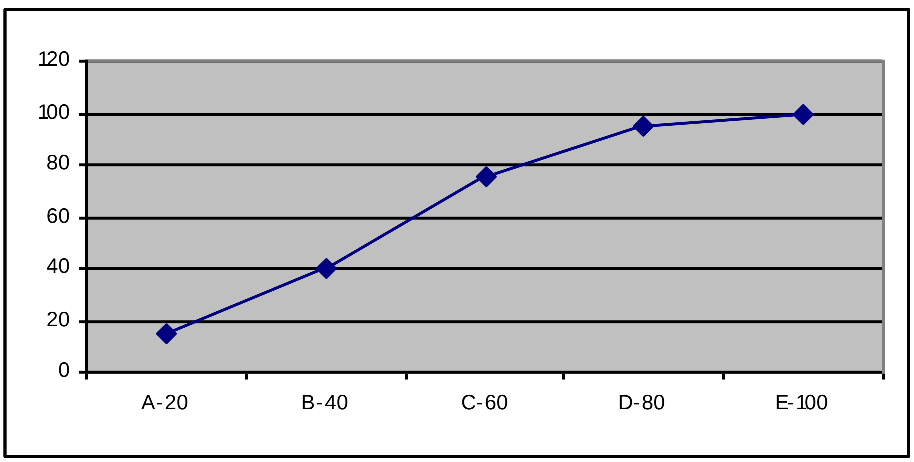
The rectangles can be used to form a line graph. The graph that results as shown above is called a frequency polygon.

A frequency polygon is a line graph of class frequencies obtained by connecting the midpoints of the tops of the rectangles forming the histogram (the graph of the cumulative frequency distribution is called an "Ogive".

Example 9.
Frequency distribution of marks of a class test

Marks	F	Cum. F
1-20	15	15
21-40	25	40
41-60	35	75
61-80	20	95
81-100	5	100
	100	

Use upper class marks for x axis and cumulative frequency for y axis.



Self Assessment exercise

Given the following data on the number of vehicles that are cross a traffic point daily. 92, 78, 68, 58, 45, 89, 75, 68, 58,45, 43, 57, 67, 75, 87, 42, 57, 65, 74, 87, 85, 73, 65, 56, 41, 36, 56, 61, 69,84,32, 55, 61, 69, 84, 81, 69, 60. Present the information in a frequency table.

4.0 CONCLUSION

The above analyses show that pie chart, bar charts form an important part of statistical analysis. It equally has wide applications in business and economics. Therefore it is very essential for you to get involved in learning data presentation in tables, pie and bar charts as a means of enhancing quantitative reasoning.

5.0 SUMMARY

The unit examined the meaning and scope of tables, pie and bar charts. There were also provided illustrations on how to compute tables, pie chart, bar chart, frequency polygon. The application of each in business life was also highlighted and illustrated to give an adequate knowledge of the unit.

6.0 TUTOR MARKED ASSESSMENT

1. The following shows the number of stores that purchase swan water in Jos, Abuja, Akure, and Akwa Ibom respectively:

Jos	25,000	Abuja	20,000
Akure	10,000	Akwa Ibom	5,000

Present the above purchases of Swan water information on a pie chart

2. Agada foods is projecting the demand for its product as follows. Present the data in a bar chart.

Year	Demand	Year	Demand
1981	800	1984	1400
1982	1200	1985	1600
1983	1100	1986	1700

7.0 **REFERNCES/ FURTHER READING**

Frank, O and Jones, R. (1993), Statistics. Pitman Publishing, London.

Levin, R.I. (1988), Statistics for managers. Eastern Economy Edition. Prentice- Hall of India Private Limited.

Nwabuokei, P.O. (1986), Fundamental of Statistics. Koruna Books, Enugu, Nigeria.

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UNIT 12. MEASURES OF CENTRAL TENDENCY

CONTENT

1.0 INTRODUCTION

The mean, median and mode are measures of central tendency, showing the average, the central number and the most frequent occurrence variable. Therefore the relationship between these three statistical variables is very important in the understanding of the unit. The descriptive statistics is aimed at describing data through summarizing the values in the data set. One method of doing this is by finding a single value that will describe the general notation of the data. This single value which is the central point of the distribution is known as a measure of central tendency or location. Measures of central tendency are typical and representative of a data set. Every value in the distribution clusters around the measures of location. The population average which in statistics is called the arithmetic mean is of such measure. Others are the median and the mode.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- Explain the basic statistical concept of the mean and its illustrations
- Discuss the median
- Discuss the mode

3.0 MAIN CONTENT

3.1 THE MEAN

The measure of central tendency most widely used is the ‘Arithmetic mean’ usually shortened to the mean. For raw data, i.e. ungrouped data, the mean is the sum of all the values divided by the total number of values. To find the mean for example, we use the following formula;

Sample mean ($\overline{X}$) = $\dfrac{\text{sum of all values in the sample}}{\text{No. of values in the sample}}$

Symbolically, it is $\overline{X} = {}^{\Sigma x}/_{n}$

Where $\overline{X}$ = sample mean read as 'x bar'

$\overline{X}$ = a particular value

Σ = sigma indicating addition

Σx's = the sum of all the x's

N = total number of values in the sample

The mean of a sample or any other measure based on sample data is called 'A statistic i.e. A measurable characteristics of a sample.

Example 1

1. The net weight of the contents of 5 coke bottles selected 85.4,84.9,85.3,85.4,85.0. from the production. What is the arithmetic mean weight of the sample observation?

solution

$\overline{X} = {}^{\Sigma X}/_{n}$

$\overline{X} = \dfrac{85.4+84.9+85.3+85.4+85.0}{5}$

$\overline{X}$ = 426/5 = 85.2

$\overline{X}$ = 85.2kg

The mean weight is 85.2kg

Many studies involve all the population values. The mean of the population in terms of symbols is $\mu = {}^{\Sigma x}/_{n}$ where

$\mu = {}^{\Sigma x}/_{n}$ = population mean

N = total number of observations in the population

As noted earlier, a measurable characteristic of a sample is called the statistic. Any measurable characteristics of a population. Such as the mean is called a parameter. A sample statistics is used to estimate a population parameter.

PROPERTIES OF THE ARITHMETIC MEAN

As noted earlier, the arithmetic mean is a widely used measure of central tendency. It has several properties which includes

1. Every set of interval level and ratio level data has a mean.
2. All the values are included in computing the mean.
3. A set of data has only one mean: It is unique

4. The mean is a very useful measure for comparing 2 or more populations.
5. They arithmetic mean is the only measure of central tendency where the sums of the deviation of each value from the mean will always be zero (0). Expressed, symbolically

   $\Sigma (x - \bar{X}) = 0$

Example 2 Find the Mean of 3, 8, 4

$$\bar{X} = \frac{3 + 8 + 4}{3} = \frac{15}{3} = 5$$

Deviation = (3 – 5) + 8 – 5) + (4 – 5)
= -2 + 3 + (-1) = 0
= -2 + 3 – 1 = 0

WEIGHTED MEAN

Masco company pays its sales people either N6.50k, N7.50K or N8.50k an hour it might be concluded that the arithmetic mean hourly wage is N7.50k, found by the sum of $\bar{X}$ give us the following.

$$\frac{N6.50k + N7.50k + N8.50k}{3} = 7.50k$$

However, this is true only if there are the same number of sales people, earning N6.50k, N7.50k and N8.50k an hour. However, suppose 24 sales persons earn N6.50k an hour, 10 are paid N7.50k and 2 get N8.50k. To find the mean, N6.50k is weighted or multiplied by 14 (6.50 x 15); N7.50 is weighted by 10(7.50x10); N8.50 is weighted by 2 (8.50 x 2). The resulting average is called the weighted mean.

In general, the weighted mean of a set of numbers designated x_1, x_2, x_3, …… x_n with corresponding weighted, w_1, w_2, w_3,….. w_n is computed by

$$\bar{X}_w = \frac{w_1x_1 + w_2x_2 + w_3x_3\ldots.. + w_nx_n}{W_1 + w_2 + w_3 \ldots.. + w_n}$$

This may be shortened to

$$\bar{X}w = \frac{\Sigma(w.x)}{\sum w}$$

Example 3 Masco company pays its sales people either N6.50k, N7.50K or N8.50k. The corresponding weight is 14,10 and 2 respectively. Determine the weighted mean?.

$$\bar{X}w = \frac{14 \times N6.50k + 10 \times N7.50k + 2 \times N8.50k}{14 + 10 + 2}$$

$$= \underline{41 + 75 + 17} = \underline{183}$$

$$\frac{\quad}{26} \qquad \frac{\quad}{26}$$

$\overline{X}$ w = N7.038

= N7.04K

SELF ASSESSMENT EXERCISE

Bata nig Ltd pay the following to its workers 2000, 3789, 4302, 2118, 5002. Determine the mean wage for the workers.

3.2 MEDIAN

When the data contains one or two very large or very small values, the arithmetic mean may not be representative. The centre point for such problems can be better described using a measure of central tendency called "A median". For ungrouped data, the median is the midpoint of the values after they have been ordered from the smallest to the largest. There are as many values above the median as there are below it in the data array.

Example 4

Given the following set of numbers 60, 65, 70, 80, 275. The median is 70. In this example, there is an odd number observation "5". For an even number of observations, the observations are ordered. Then the usual practice is to find the arithmetic mean of the two middle observations. Note that for an even number of observations, the median may not be one of the given values.

Example 5 A sample of a paramedical fees charged by U.J's clinic revealed these amounts; N35, N29, N30, N25, N32, N35. What is the mean charge.

Solution Arrange the paramedical fees from lowest to highest charge.
25, 29, 30, 32, 35, 35

The median fee is N31, found by determining by the arithmetic mean of the a centre observation i.e.

$$\frac{30 + 32}{2} = \frac{62}{2} = 31$$

An easy way to locate the position of the middle items for raw ungrouped data is by the formula;

Location of the median value = $^{n+1}/_{2}$

Where n = total no. of items

For example above, there are 6 items, so $^{n+1}/_2 = {}^{6+1}/_2 = {}^7/_2 = 3.5$. after arranging the data from the lowest – highest, we locate the middle item by counting to the 3.5th item and then determine its values.

PROPERTIES OF THE MEDIAN

1. It is unique i.e. like the mean, there is only one median for set of data.
2. To determine the median, arrange the data from lowest to highest and find the value of the middle observation.
3. The median is not affected by extremely large or small values, therefore a valuable measure of central tendency.
4. It can be computed for ratio, interval level and ordinal level data.

SELF ASSESSMENT EXERCISE

A sample of fees paid by commuters from Ikeja to Ogba are as follows. N150, N120, N130, N120, N100. Find the median values for the fees.

3.3 MODE

It is another measure of central tendency. It is the value of the observation that appears most frequency.

Example 5 Given the following set of data;

35, 49, 50, 50, 40, 58, 50, 60, 50, 65, 50, 71, 50, 55, what is the mode?

Solution

A look at the values reveals that 50 appears more than any other. The mode is therefore 50.

We can determine the mode for all levels of data whether they are nominal, ordinal, interval or ratio. 1).The mode also has the advantage of not being affected by extremely high or low values like the median. 2). It can be used as a measure of central tendency for open-ended distributions. The mode has a number of disadvantages however, that causes it to be used less frequently than the mean or median. For many sets of data, there is no mode if no value appears more than once.

SELF ASSESSMENT EXERCISE

The following information represent the distribution of pencils in a market 40, 51, 39, 53, 51, 85, 75, 53, 44, 53, 90, 53. Find the mode of pencil distribution.

3.4 THE MEAN, MEDIAN AND MODE OF GROUPED DATA

In a grouped data the information is presented in the form of a frequency distribution. It is usually impossible to secure the mean at face value using the original raw data. Thus if we are interested in estimating the mean value to represent the data, we must estimate it based on a frequency distribution.

1. THE ARITHMETIC MEAN OF GROUPED DATA

To approximate the arithmetic mean of data organized into a frequency distribution, the observations in each class are represented by the midpoints of the class. The mean of a sample of data organized in a frequency distribution is computed by: $\overline{X} = {}^{\Sigma fx}/_{n}$ where $\overline{X}$ = mean

X = mid value or midpoint of each class, F = frequency in each class

Fx = frequency in each class midpoints of class. Σfx = sum of these products

n = number of frequencies

Example 6

Given the data group into a frequency distribution below

Monthly Rentals of Halls	No. of units
600-799	3
800-999	7
1000-1199	11
1200-1399	22
1400-1599	49
1600-1799	24
1800-1999	9
2000-2199	4
	120

What is the mean monthly rental for the group data?

Solution

Monthly rentals of halls	No. of units F	X (midpoints)	Fx
600-799	3	699.5	2098.5
800-999	7	899.5	6296.5
1000-1199	11	1099.5	12094.5
1200-1399	22	1299.5	28589
1400-1599	40	1499.5	59980
1600-1799	24	1699.5	40788
1800-1999	9	1899.5	17095.5
200-2199	4	2099.5	8398
	120		175340

$$\overline{X} = {}^{\Sigma fx}/_{n}$$

$$= \frac{175340}{120} = 1461.16667$$

2. THE MEDIAN OF GROUPED DATA

Recall that the median is defined as the value below which half of the value lie and above which the other half of the values lie. Since the raw data has been organized into a frequency distribution, some of the information is not identifiable. As a result, we cannot determine an exact median it can be estimated however by

i. Locating the class in which the median lies
ii. Interpolating within that class to arrive at the median

The rationale for this approach is that the members of the median class are assumed to be evenly spaced throughout the class. The formula is

$$\text{Median} = \frac{L + {}^{n}/_{2} - cfi}{f}$$

where, L = lower true limit of the class containing the median N = total number of frequencies, F = frequency of the median class Cf = cumulative no of frequencies in all classes immediately, proceeding the class containing the median, I = width of the class in which the median lies

Example 7 Using the frequency in question 6 determine the median

Solution

Monthly rentals	F	Cumulative frequency
600-799	3	3
800-999	7	10
1000-1199	11	21
1200-1399	22	43
1400-1599	40	83
1600-1799	24	107
1800-1999	9	116
2000-2199	4	120

The monthly rentals have already been arranged in ascending order from 600-2199. It is common practice to locate the middle observation by dividing the total number of observations by 2. In this case, $^n/_2 = {}^{120}/_2 = 60$. The class containing the 60th unit is located by referring to the cumulative frequency column in the table above. The 60^{th} rental is in the 1400-1599 class. Recall that the lower unit of the class is really 1399.50 and the upper limit is 1599.50.

To interpolate in the 1399.50 to 1599.50 class, recall that the monthly rentals are assumed to be evenly distribution between the lower and upper true limits, there are 17 rentals between the 43^{rd} and 60^{th} units. And there are median is therefore $^{17}/_{40}$. The distance between 1399.50 and 1599.50, that distance is 200, thus $^{17}/_{40}$ of 200 or 85 is added to the lower true limit 1399.50 to give '1484.50'. The estimated median rental.

To summarize

$$\text{Median} = 1399.50 + \frac{60 - 43}{40}(200)$$

$$= 1399.50 + \frac{17}{40}(200)$$

$$= 1399.50 + 85$$

$$= 1484.50$$

Using the median formula, we have

$$\text{Median} = \frac{L1 + {}^n/_2 - CF\ i}{F}$$

$$= \frac{1399.50 + {}^{120}/_2 - 43\ (200)}{40}$$

$$= \frac{1399.50 + {}^{17}/_{40}\ (200)}{40}$$

$$= 1399.50 + 85$$
$$= 1484.50$$

3. THE MODE OF GROUPED DATA

Recall that the mode is defined as the value that occurs most frequently. For data grouped into a frequency distribution, the mode can be approximated by the midpoint of the class containing the largest number of frequencies. Using the table in example 5 the class with the highest number of frequency has the mode. Therefore the mode is 40 in the class 1400 – 1599.

4.0 CONCLUSION

The above analysis shows that the mean, median and the mode are crucial in statistical analysis. It cut across all discipline, it is used daily by every individual in daily life be it in the home, office, or business. It is very essential to know the basics of the measures of central tendency as a source of evaluation of many activities in business and other variables of life.

5.0 SUMMARY

The unit has thrown some light on the meaning and scope of the mean, median, and mode, even though the scope is wide and in exhaustive the basic foundational knowledge of the measures of central tendency will help you cope with the challenges of other courses.

6.0 Tutor Marked Assignment

1) The following data represent the distribution of pure water in Jos Nigeria. 44,56,42,56,76,34,80,37,56,45,46,92,56,40,49,44,68. Find the mean, the mode and the median.

2. Find the mean and mode from the following data. 1000, 2000, 4000,3000,6000,4000,6000,3000,9000.list the properties of the median.

7.0 REFERNCES/ FURTHER READING

Frank, O and Jones, R. (1993), Statistics. Pitman Publishing, London.

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UNIT 13 MEASURES OF DISPERSION

CONTENT

1.0 INTRODUCTION

In this section, we will examine several measure that describe the dispersion, variability or spread of the data. The measures include; the range, average, deviation, variance, standard deviation among others.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- Explain the range
- Explain variance
- Discuss the standard deviation

3.0 MAIN CONTENT

3.1 RANGE: This is the simplest of the measures of dispersion, it is the different between the highest and the lowest values in a set of data, in the form of equation.

Range = highest values – lowest value

Example 1

The capacities of metal drum container are 38, 20, 37, 64 and 27 litres respectively. What is the range?

Solution. Find the difference between the highest and the lowest numbers in the data. Range = 64 – 20 = 44

Mid range**:** It is the middle of the range.

[Highest value – lowest value ÷ 2] = 44/2 = 22

SELF ASSESSMENT EXERCISE

Find the range and the mid range from the following data, 2004, 1988, 2030, 2011,1981.

3.2 **AVERAGE DEVIATION:**

A serious defect of the range is that it is based only on two values; the highest and the lowest. It does not take into consideration all of the values. The average deviation does often referred to as the mean deviation, it measures the average amount by which the values in the population or sample vary from their mean, in terms of definition, average deviation(A. D) is the arithmetic mean of the absolute values of the deviation from the arithmetic mean.

In terms of the formula, the A.D is computed for a sample by

$$A.D = \frac{\Sigma|X - \overline{X}|}{n}$$

Where; X = value of each observation, $\overline{X}$ = the arithmetic mean of the values

n = number of observation in the sample, || = the absolute value i.e. the signs of the deviations from the mean are disregarded.

Because, we use absolute deviation, the mean deviation is often called the Mean Absolute deviation (M.A.D).

Example 2. The height of sample of carton of sweets is given as follows. 103, 97, 101, 106, 103. What is the mean deviation. How it is interpreted?

Solution The arithmetic mean weight is 102, found by the sum of all observations divided by 5. To find the A.D, take the following steps

i. The mean is subtracted from each value
ii. The absolute deviation are summed
iii. The sum of A.D is divided by the number of values

Weights	X- $\overline{X}$	A.D $\lvert X-\overline{X}\rvert$
103	1	1
97	-5	5
101	-1	1
106	4	4
103	1	1
		$\Sigma\lvert x-x\rvert = 12$

$$A.D = \frac{\Sigma|x-\bar{x}|}{n}$$

$$= {}^{12}/_{5}$$

$$= 2.4$$

The average deviation of the sample is 2.4, the interpretation is that the height of the carton deviate on the average 2.4 from the mean weight of 102.

The A.D does have 2 advantages

i. It uses the value of every item in a set of data, in its compilation.

ii. It is easy to understand

However, absolute values are difficult to work with, so the average deviation is not frequently used.

3.3 THE VARIANCE

The variance is the arithmetic mean of the squared deviation from the mean.

1. Population variance

The formula for the population variance and a sample variance are slightly different. The population variance is found by;

$$F^2 = \frac{\Sigma(x-U)^2}{N} \quad \text{or} \quad \frac{\Sigma x^2}{N} - \frac{(\Sigma x)^2}{N}$$

Where; F^2 = population variance, X = the value of the observation in the population, U = the mean of the population, N = total number of observations in the population.

Example 3. The ages of all patients in ward A in Abuja clinic are 38, 26, 13, 41, 22 years. What is the population variance?

Solution

Ages x	**x-µ**	**$(x-\mu)^2$**	**X^2**
38	10	100	1444
26	-2	4	676
13	-15	225	169
41	13	169	1681
22	-6	36	484
$\Sigma x = 140$		$\Sigma(x-\mu)^2 = 534$	$\Sigma x^2 = 4454$

$$\mu = {}^{140}/_{5} = 28$$

$$\text{Variance} = \frac{\Sigma(x-\mu)^2}{N} \quad \text{or} \quad \frac{\Sigma x^2}{N} - \frac{(\Sigma x)^2}{N}$$

$$= {}^{534}/_{5} = {}^{4454}/_{5} - ({}^{140}/_{5})^2$$

$$= 106.8 = 890.8 - (28)^2$$

$$= 890.8 - 748$$
$$= 106.8$$

Like the range and A.D, the variance can be used to compare the dispersion in 2 or more sets of observation.

3.4 SAMPLE STANDARD DEVIATION

A sample S.|D is used as an estimator of the population standard deviation. The sample deviation is the square root of the sample variance. It is found by the formula;

$$S = \sqrt{\frac{\Sigma(x-x)^2}{n-1}}$$

or using the more direct formula the standard deviation can be given as

$$S = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{N}}{n-1}}$$

Using the previous example 5 the standard deviation can be determined as the square root of the variance as follows.

$S = \sqrt{10} = 3.16$

4.0 CONCLUSION

The above analysis shows that the measures of dispersion determine the rate at which the data spread. It is of importance to know the rate of variability of any given data, this help us to ensure that there is even distribution in data collection as it is used daily by every individual in life be it in the home, office, or business. Example it is very essential to know the rate of spread of customers, consumer for a business man.

5.0 SUMMARY

The unit has thrown some light on the meaning and application of range, deviation, standard deviation. The unit therefore examined the basic concepts of measures of dispersion as a means of launching you to study other units effectively.

6.0 TUTOR MARKED ASSIGNMENT

Question 1. The following represent the number of students that were expelled in a given session 38, 26, 13, 41, 22 years. What is the population variance?.

2)The following data represent the performance of some students in some courses Mr Abo.56,67,46,80,52,48,68,74. Mrs Walter 72,43,59,71,50,58,90,44. Determine the range and the midrange in the performance of the two students

7.0 **REFERNCES/ FURTHER READING**

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UNIT 14 ANALYSIS OF CORRELATION

CONTENT

1.0 INTRODUCTION

Correlation is a statistical method used to define or establish a relation that exist between two or more variables. This relationship could be positive or negative or zero. The method of establishing this relationship is also known as scatter diagram. In this unit you will learn how to determine correlation using statistical equations.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- Explain the basic concept of correlation
- Discuss rank correlation
- Explain correlation coefficient

3.0 MAIN CONTENT

3.1 Correlation is a statistical method used to define or establish a relation that exists between 2 or more variables. This relationship could be positive or negative or zero. The method of establishing this relationship is also known as scatter diagram. A scatter diagram measures the closeness of variable among themselves and vice-versa.

The formula for correlation coefficient is as follows.

$$r = \frac{\sum xy}{\sum x^2 \ \sum y^2}$$

Y	X	Y	$x = X - \overline{X}$	$Y = Y - \overline{Y}$	XY	X^2	Y^2
1	6	30	-6	11	-66	36	21
2	9	15	-3	-4	12	9	16
3	15	16	3	-3	-9	9	9
4	18	15	6	-4	-24	36	16

Calculate the correlation co-efficient for the above data and interpret your result.

$\overline{X} = 48 = 12, \quad \overline{Y} = 19$

$$r = \frac{-84}{\sqrt{90 \times 162}} = \qquad r = \frac{-84}{120.74} \qquad = -0.70$$

The above shows that there is negative correlation.

$r > 0.5$ = Strong correlation

$r < 0.5$ = Weak correlation

$r = 0$ – No correlation

If $r = 1$ there is Perfect correlation between the variables

r = negative = There is inverse relationship

SELF ASSESSMENT EXERCISE

The following data represent the performance of some students in some courses

Mr Abo.56,67,46,80,52,48,68,74. Mrs Walter 72,43,59,71,50,58,90,44. Determine the correlation between the performance of the two students.

3.2 RANK CORRELATION

This method involves ranking variables according to the magnitude of their occurrence without altering the format with which the observations occur in a given data. The ranks that are allocated to each of the observation is used to measure the level of correlation between the variables that occur in that data. The formula for rank correlation is as follows.

$$1 - \frac{6\sum d^2}{n(n^2 - 1)}$$

Where d =deviation n = No. of observation

Seven students have the following as their scores in GST and statistics

X	GST	Statistics	Rank (x)	Rank Y	d = x-y	d^2
X_1	70	86	4	1	3	9
X_2	92	71	1	3	-2	4
X_3	89	80	2	2	0	0
X_4	50	63	5	4	1	1
X_5	41	50	6	5	1	1
X_6	82	34	3	6	-3	9
X_7	40	31	7	7	0	0
						24

Compute the Rank correlation of student performance in each subject and explain whether there is any relationship between the two course.

$$1 - \frac{6(24)}{7(49-1)} = 1 - \frac{144}{336}$$

$$1 - 0.43$$

$$= 0.57$$

This shows that the correlation is strong since the value of the correlation is greater than 0.5

Ties in rank for correlation, the method of solving the problem remain the same.

	A	B	C	D	E	F	G	H	I	J
Rank by X	4	2	8	4	7	6	10	1	3	9
Rank by Y	3	1	5	8	5	9	5	2	4	10
Rx	4.5	2	8	4.5	7	6	10	1	3	9
Ry	3	1	6	8	6	9	6	2	4	10
D	1.5	1	2	-3.5	1	-3	4	-1	-1	-1
D^2	2.25	1	4	12.25	1	9	16	1	1	1

$\sum d^2 = 48.50$

$$R = 1 - \frac{6\sum d^2}{10(100-1)}$$

$$= 1 - \frac{6 \times 48.5}{99}$$

$$= 1 - \frac{9.70}{33}$$

$$1 - 0.29$$

$$= 0.71$$

SELF ASSESSMENT EXERCISE

Given the following performance of some contractors of some projects completion.
Contractor X's performance, 82%, 57%, 74%, 40%, 52%, 51%. Contractor Y, 71%, 34%, 50%, 81%, 62%, 54%. Calculate the ranked correlation.

4.0 CONCLUSION

The above analysis shows that correlation is a statistical method that is used daily by people in transaction and business, as long as it involves comparison. . It cut across all discipline, it is used daily by every individual in daily life be it in the home, office, or business. It is very essential to know the basics of correlation as a means of evaluation of any transaction that involves comparative analysis..

5.0 SUMMARY

The unit has thrown some light on the meaning and scope of correlation, even though the scope is wide, the basic foundational knowledge of correlation will help you cope with the challenges in business as managers. The unit therefore examined the basic concepts of correlation, correlation coefficient, rank correlation.

TUTOR MARKED ASSIGNMENT

1) The following data shows the attendance of lectures and the rate of passing the examination in a given course.
Attendance 20,19,34,25,18,22. Pass rate 170,170,230,200,180,190. calculate the correlation between attendance and the pass rate.

2) Seven dealer have the following as their scores in the distribution of Gt and St

W	Gt	St
w_1	70	86
w_2	92	71
w_3	89	80
W_4	50	63
W_5	41	50
W_6	82	34
W_7	40	31

a) Compute the rank correlation of the distributors in product Gt and St and explain whether there is any relationship between the two products.

7.0 **REFERNCES/ FURTHER READING**

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UNIT 15 ANALYSIS OF REGRESSION
CONTENT

1.0 INTRODUCTION

Regression is a statistical method of determining the best fit. It is used to measure the rate of dispersion along any given curve.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- Explain the concept of regression
- Estimate a regression equation
- Forecast using regression equation.

3.0 MAIN CONTENT

3.1 Analysis of Regression

Regression is a quantitative ways of arriving at the best fit. It is also referred to as the line of average in any given data.

Example 1. The following data represent sales from a particular company as a result of advertising his product.

Sales(Y)	Advert(X)	$x=X - \bar{X}$	$Y = Y - \bar{Y}$	xy	X^2
82	20	0	2	0	0
70	46	-4	-10	40	16
90	24	4	10	40	16
85	22	2	5	10	4
73	18	-2	-7	14	4
= 400	= 100			104	40

$\bar{Y} = \frac{400}{5}$ $\bar{X} = \frac{100}{5}$

$\bar{Y} = 80$ $\bar{X} = 200$

Attempt a regression of Y on X

$Y = a + bx$

$$b = \frac{\sum[(x - \bar{X})(y - \bar{X})]}{\sum (x - \bar{X})^2} = \frac{\sum xy}{\sum x^2}$$

$b = \frac{104}{40} = 2.6$

$a = \bar{Y} - b\bar{X}$

$a = 80 - (26)(20)$

$a = 80 - 52$

$a = 28$

$\bar{Y} = 28 + 2.6\hat{x}$

There is a positive relationship between x and y, that is as x increases y also increases. It means advert leads to increase in sales.

3.2 Estimate the regression equation.

E.xample 2 An employer of labour wants to find the relationship between the labour input employed and the total output using the following hypothetical data.

Labour input	Output
0.8	28
1.1	31
1.6	29
2.3	20
2.2	37
3.1	35
3.0	40
4.6	56

(a) Establish the least square regression for the above data (b) using the average in the data. Sketch the regression line (c) Assuring the labourer decide to employ 80 workers estimate the output for his firm. Y on X.

Output(Y)	Input(X)	XY	X^2
28	0.8	22.4	0.64
31	1.1	34.1	1.21
29	1.6	46.4	2.56
20	2.3	46.0	5.29
37	2.2	81.4	4.84
35	3.1	108.5	9.61
40	3.0	120.0	9.00
56	4.6	257.6	21.16
= 276	= 18.7	= 716.4	= 54.31

Y = mx + c

Y = a,x + ao

Y = bx + a

$$Y = \frac{\sum Y}{N} = \frac{276}{8}$$

$$= 34.5$$

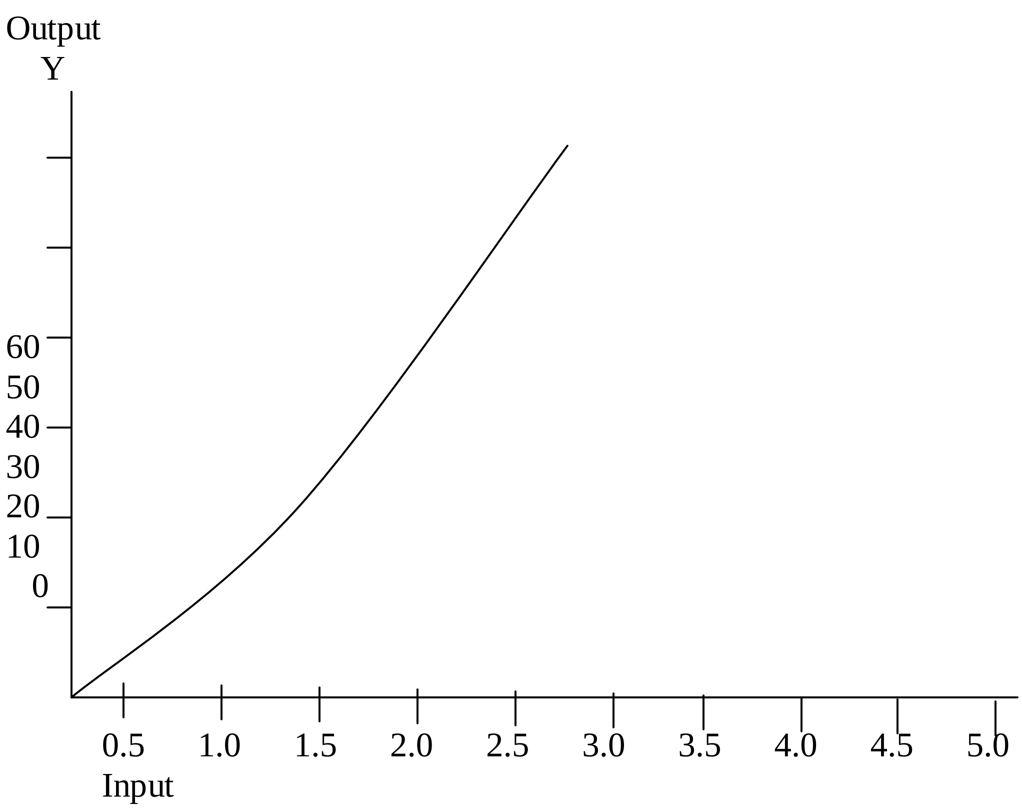
$$X = \frac{\sum X}{N} = \frac{18.7}{8}$$

$$= 2.34$$

Last square equation

$nc + m\sum x = \sum y$
$c\sum x + m\sum x^2 = \sum xy$

where
$18.7\ (8c) + 18.7m = 276.0$ ……………….. (I)
$8\ (18.7c) + 54.31m = 716.4$ ……………….. (II)
$149.6c + 349.69m = 5161.2$ ……………….. (III)
$149.6c + 434.48m = 5731.2$ ………………… (IV)
$84.79m = 570$
$m = 6.72$
substitute 6.72 for m in equation (1)
$8C + 18.7\ (6.72) = 276.0$
$8C + 125.66 = 276$
$8C = 276 – 125.66$
$8C = 150.34$
$C = 18.78$
$Y = 6.72X + 18.79$



3.3 FORECASTING USING REGRESSION

Using the question in 3.2 above we can forecast based on the estimated regression equation.

Assuming the labour decide the employ 80 workers when the input =

$Y = 6.72(80) + 18.79$

$Y = 537.6 + 18.79$

$Y = 556.39$

4.0 CONCLUSION

The above analysis shows that regression is one the fundamental method of estimating any statistical equation. It cut across all discipline, it is used also in forecasting in business. Therefore a good knowledge of regression can help a manager to forecast sales, market performance etc.

5.0 SUMMARY

The unit has thrown some light on the meaning and scope of regression, even though the scope and application is wide the basic foundational knowledge of the unit will help you cope with the challenges of other in business management and forecasting. The unit therefore examined the basic concepts of best fit as a means of graphical exposition.

6.0 TUTOR MARKED ASSIGNMENT

(1)The following data shows the death and the birth rate in a given city.
Death rate 20,19,34,25,18,22. Birth rate 170,170,230,200,180,190.
Estimate the regression equation for death and the birth rate of the city.

(2) Forecast the birth when the death rate is 58, 66 and 80.

7.0 REFERNCES/ FURTHER READING

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