NATIONAL OPEN UNIVERSITY OF NIGERIA

SCHOOL OF SCIENCE AND TECHNOLOGY

COURSE CODE: PHY 402

COURSE TITLE: NUCLEAR PHYSICS

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UNIT 1 NUCLEAR STRUCTURE

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1.0 INTRODUCTION

Matter was once considered to be made simply of atoms. It was soon discovered that atoms are made of elementary particles.

1.1 OBJECTIVES

After studying this unit, you will be able to:

- Highlight the constituents of an atom
- Explain nuclear size, nuclear masses and nuclear forces.

1.2 MAIN CONTENT

1.2.1 NUCLEAR STRUCTURE

Although there are numerous elementary particles, the only relevant particles in our earthly life and in nuclear reactors, which we are going to discuss, are photons and the particles that constitute material, that is, protons, neutrons, and electrons. Among these, the proton and neutron have approximately the same mass. However, the mass of the electron is only 0.05% that of these two particles. The proton has a positive charge and its absolute value is the same as the electric charge of one electron (the elementary electric charge). The proton and neutron are called nucleons and they constitute a nucleus. An atom is constituted of a nucleus and electrons that circle the nucleus due to Coulomb attraction.

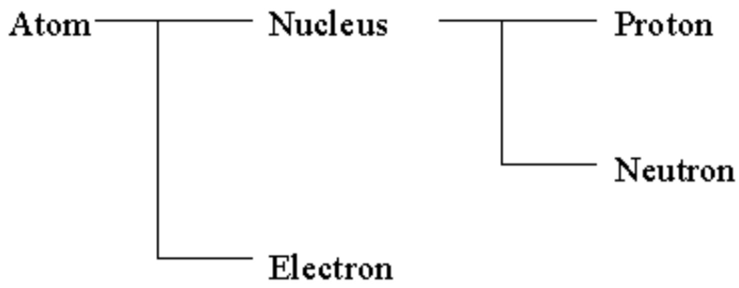


Fig.1 Constitution of an atom (No. 1)

Species of atoms and nuclei are called elements and nuclides, respectively. An element is determined by its proton number (the number of protons). The proton number is generally called the atomic number and is denoted by Z. A nuclide is determined by both the proton number and the neutron number (the number of neutrons denoted by N). The sum of the proton number and neutron number, namely, the nucleon number, is called the mass number and is denoted by A

(A=Z+N). Obviously, a nuclide can also be determined by the atomic number and mass number.

In order to identify a nuclide, A and Z are usually added on the left side of the atomic symbol as superscript and subscript, respectively. For example, there are two representative nuclides for uranium, described as ${}^{235}_{92}U$ and ${}^{238}_{92}U$. If the atomic symbol is given, the atomic number can be uniquely determined; thus Z is often omitted like ^{235}U and^{238}U. The chemical properties of an atom are determined by the atomic number, so even if the mass numbers of nuclei are different, if the atomic numbers are the same, their chemical properties are the same. These nuclides are called isotopic elements or isotopes. If the mass numbers are the same and the atomic numbers are different, they are called isobars. If the neutron numbers are the same, they are called isotones.

The above examples for uranium are isotopes. Summarizing these and rewriting the constitution of an atom, we obtain Figure 2.

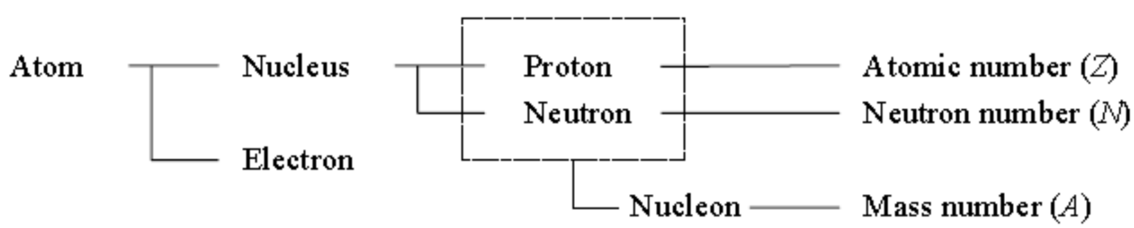


Fig.2 Constitution of an atom (No. 2)

A nuclide is any nuclear specie, with the combination of neutrons and protons i.e. ${}^{A}_{z}X$, A= atomic mass, Z = atomic number = number of proton = number of electrons while number of neutron, N = (A - Z).

- nuclides with the same Z = Isotopes
- nuclides with the same A= Isobars
- nuclides with the same N= Isotones
- nuclides with the same A and Z but different states of excitations = Isomers
- the charge distribution within the nucleus can be assumed uniform with charge density given by;

$P(r) = \frac{\rho_0}{1+e^{\frac{(r-R)}{b}}}$

Where ρ_0= density at the nuclear centre

R= radius at which ρ falls to $\rho_0/2$

b= measures how rapidly the density falls to zero at the nuclear surface.

R =$\rho_0 A^{\frac{1}{3}}$, r_o=1.23x10^{-15}m

R = radius of nucleus

This implies that

- the volume of nucleus is proportional to number of particle A
- charge density P(r) decreases slowly with increasing A.
- 1 a.m.u.= 1.6660053 x 10^{-27}kg.

  = 931MeV

To show that the electron is not a constituent of the nucleus.

Uncertainty principle is applied here.

- Typical nuclear are less than 10^{-14}m in radius
- Therefore for an electron to be confined within such nucleus, the uncertainty in its position may not exceed 10^{-14}m.
- The corresponding uncertainty in the electrons momentum is:

$$\Delta p \approx \frac{\hbar}{\Delta x}$$
$$\hbar = \frac{h}{2\pi} = \frac{6.625\times10^{-34}}{2\times3.14}$$
$$\cong 1.055\times10^{-34}\, JS^{-1}$$
$$and.\Delta x = 2r = 2\times10^{-14}\, m$$
$$\therefore\ \Delta p = \frac{\hbar}{\Delta x} = \frac{1.055\times10^{-34}}{2\times10^{-14}}$$
$$\approx 5.275\times10^{-21}\, kgms^{-1}$$

- If this is uncertainty in the momentum of the electron, the momentum itself must be at least comparable in magnitude.
- The K.E. of the electron of mass, m may be put as follows:

$$T = \frac{\rho^2}{2m}$$
$$where\ m = 9\times10^{-31}\, kg$$
$$= 9\times10^{-31}\, kg \times 1.6\times10^{-19}\, eV$$
$$= 1.44\times10^{-49}\, eV$$
$$\approx \frac{\left(5.275\times10^{-21}\right)^2}{2\times1.44\times10^{-49}}$$
$$\approx 9.7\times10^{7}\, eV$$
$$\approx 97 MeV$$

From the above, it follows that if the electrons are present inside the nucleus, their K.E must be of the order 97MeV. But experimental data reveal that no electron in the atom has energy greater than 4MeV. This clearly reveals that e-s do not exist in the nucleus.

Excess mass and Packing fraction

Excess mass is defined as the difference between the masses of the nucleons (M) and the atomic mass (A). This implies that excess mass = (M - A), which can either be either positive or negative.

Packing fraction (f) can be defined as the ratio of the excess mass to the atomic mass. This implies that packing fraction is expressed as (M – A)/A= f.

It is only Carbon -12 that has its M - A = 0.

1.2.2 THE NUCLEUS

The alpha (α) scattering experiment led to the discovery of a nucleus of an atom. The mass of the atom seems to be concentrated at the nucleus and it is surrounded by cloud of electrons which makes the entire atom electrically neutral.

One of the goals of Rutherford's α-scattering is the determination of the radius (R) of the nucleus that is as α-particle approaches the gold nucleus it slows down due to coulomb force but later speeds up on its way-out.

The coulomb repulsive force in the region close to the scattering gold nucleus is given by

$$F = \frac{2eZe}{4\pi\varepsilon_o b^2}$$

The time of operation of force

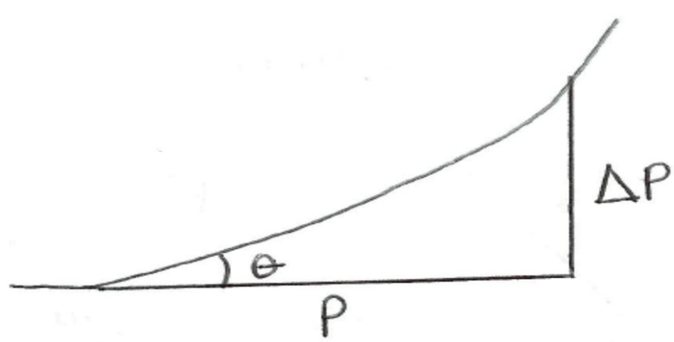
$$\Delta t = \frac{b}{v}$$

The force produces a momentum, Δp, which is perpendicular to the direction of α-particle.

$$F = \frac{\Delta p}{\Delta t}$$

$$\Delta p = f \cdot \Delta t$$

$$\Delta p = \frac{2eZe}{4\pi\varepsilon_0 b^2} \bullet \frac{b}{v}$$



$$\theta = \frac{\Delta p}{p} = \frac{2Ze^2}{4\pi\varepsilon_0 bv} \div mv$$

$$\theta = \frac{2Ze^2}{4\pi\varepsilon_0 bmv^2}$$

Making b the subject of formula

$$b = \frac{1}{4\pi\varepsilon_o} \bullet \frac{2Ze^2}{mv^2} \bullet \frac{1}{\theta}$$

Make b = R and putting $\theta = 1$

$$R = \frac{1}{4\pi\varepsilon_o} \bullet \frac{2Ze^2}{mv^2}$$

$$R \approx 10^{-14} m$$

Radius of nucleus, R is smaller than the radius of the atom.

Self Assessment Test 1

i. What do you understand by the term Nuclear force?

ii. Define the following terms:

(a) Excess mass (b) Packing fraction

iii. Show that the electron is not a constituent of the nucleus of an atom.

1.2.3 NUCLEAR BINDING ENERGY AND SEPARATION ENERGY

Binding energy is the energy that must be supplied to dissociate the nucleus into separate nucleus or the energy released when the separated nucleons were assembled into a nucleus.

B (A, Z) = [$ZM_H + NM_N - M(A,Z)$] 931MeV --------------------- (1)

Also, the difference between the actual nuclear mass and the mass of all the individual nucleus is called the mass defect (Md) which is equal to W – M.

Binding energy is a measure of cohesiveness of a nucleus that is between the proton and neutron. Also, a more useful measure of cohesiveness is the binding energy per nucleon

$$\frac{B(A,Z)}{A} = \left[ZM_H + NM_N - M(A,Z)931\right]\frac{MeV}{A} \quad \text{----------------- (2)}$$

From equation (1) the mass of a nucleon becomes.

$$\frac{B(A,Z)}{931} = \left[ZM_H + NM_N - M(A,Z)\right]MeV$$

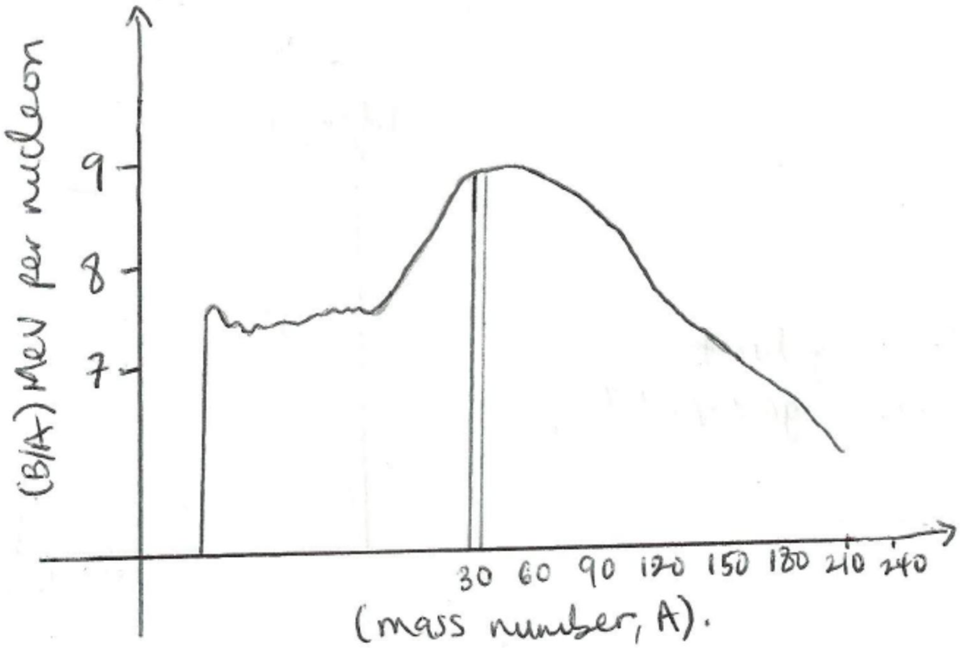
$$M(A,Z) = \left[ZM_H + NM_N - \frac{B(A,Z)}{931}\right]a.m.u$$

The binding energy can also be written in terms of the mass number or the atomic mass number

$$B = AM_N - Z(M_N - M_H) - M(A,Z)a.m.u$$

Dividing through by A

$$B = M_N - \frac{Z}{A}(M_N - M_H) - \frac{M}{A}$$



For A<28 there is a prominent cyclic re-occurrence because of strong binding energy and we give them A = 4n where n is integer.

SEPARATION ENERGY

The work necessary to separate a proton, neutron, deuteron or α-particle from a nucleus is called Separation Energy.

$$Sn = M[A-1, Z] + M_N - M(A, Z)931MeV$$
$$Sp = M[A-1, Z-1] + M_H - M(A, Z)931MeV$$

Examples

$$For\ {}^{16}_{8}0$$
$$Sn = [M(A-1, Z) + M_N - M(A, Z)]931MeV$$
$$= (15.003070 + 1.008665 - 15.99491)931MeV$$
$$= 0.01682(931)MeV$$
$$= 15.65942MeV$$

$$Sp = [M(A-1, Z-1) + M_H - M(A, Z)]931MeV$$
$$= (15.000108 + 1.007825 - 15.99491)931MeV$$
$$= 12.18679MeV$$

$for\,{}_{8}^{17}0$

$$Sn = \left[M\left(A-1,Z\right)+M_{N}-M\left(A,Z\right)\right]931MeV$$
$$=\left(15.994915+1.008665-16.999133\right)931MeV$$
$$=4.140157MeV$$

$$Sp = \left[M\left(A-1,Z-1\right)+M_{P}-M\left(A,Z\right)\right]931MeV$$
$$=M\left(A-1,Z-1\right)+1.007825-16.999133)\,931MeV$$

Generally, Sp or Sn is large for nuclei with even N or even Z, then odd N or odd Z. If Sn or Sp is plotted against A, there are some areas of discontinuities at A=2, 8, 20, 50, 82, 126 (magic numbers). The energy required to remove magic numbers are higher than ordinary numbers.

Self Assessment Test II.

i. Calculate the binding energy and separation energy for the following atoms using the table in the appendix: (a) ${}_{8}^{16}O$ (b) ${}_{8}^{17}O$
ii. Define the following terms: (a) Nuclear binding energy (b) Separation energy.

1.3 CONCLUSION

In conclusion we have been able to examine the nuclear structure and its constituents. Also, we studied the nuclear size and the nuclear binding forces.

1.4 SUMMARY

In this unit, we have been able to understand that the nucleus of an atom consists of the proton and neutron which is surrounded by electrons. The nuclear binding force and separation energies were examined and calculated for some atoms.

1.5 TUTOR MARKED ASSIGNMENT

a) A nucleus with A = 235 splits into two nuclei whose mass numbers are in the ratio 2:1, find the radii of the new nuclei.

b) Calculate the binding energy and separation energy of protons and neutrons of the following atoms: (i) ${}_{12}^{24}Mg$ (ii) ${}_{13}^{27}Al$

c) Define the following terms: (i) isotopes (ii) isobars (iii) isotones.

1.6 REFERENCES/FURTHER READING

W. Greiner and J. A. Maruhn, Nuclear Models by Springer.

UNIT 2 NUCLEAR MODELS.

CONTENTS

2.0 INTRODUCTION

These models are proposed models that are used to explain the nuclear forces in the nucleus of an atom.

2.1 OBJECTIVES

After going through this unit, you will be able to;

- Highlight the properties of nuclear forces
- Understand all the proposed models used to describe the nuclei binding energy of forces.

2.2 MAIN CONTENT

2.2.1 NUCLEAR MODELS

These are just meant to explain the nuclear forces in the nucleus of an atom. All that is known about the nuclear force is that:

i. Short range of operation of the order of ≈10cm

ii. Independent of charge i.e. exists equally between proton and neutron.

iii. Strong force which can overcome the Coulomb force.

iv. Is a repulsive force to certain extent in order to prevent the collapse of the nucleus. This implies that the potential V is proportional to atomic mass A of the nucleus.

Different models were proposed to describe the nuclei binding energy of forces:

1. The uniform particle model.
2. The liquid drop model

3. The cluster α particle model
4. The shell model
5. The collective model
6. The optical model

2.2.2 LIQUID DROP MODEL

The property of a liquid having its energy binding molecules proportional to the mass has been likened to that of nuclei. Some other properties which liken a liquid drop to the nucleus are:

Liquid	**Nucleus**
Evaporation	Fission and radioactivity
Condensation	fusion
Constant density	Constant density

Using this likeness, an equation was developed for the binding energy of a nucleus.

1. Nuclear energy E_1:

$$E_1 \propto A$$
$$E_1 = a_1 A$$

a_1 is a constant

2. Surface energy (E_2)

$$E_2 \propto R^2$$
$$E_2 = -a_2 A^{2/3}$$

3. Coulomb energy, E_3

$$E_3 = -a_3 Z(Z-1)A^{-1/3}$$

4. Asymmetric energy E_4

$$E_4 = \frac{-a_4\left(Z - A/2\right)^2}{A}$$

5. Pairing energy E_5

This is a correction factor δ which accounts for the different stability characteristics observed in odd-even nucleon properties i.e.

$$E_5 = \delta a_5 A^{-3/4}$$
$$where\ \delta = 0\ for\ odd\ (N)\ and\ even\ (Z)\ or\ viceversa$$
$$\delta = +1\ for\ even, even$$
$$\delta = -1\ for\ odd, odd.$$

Therefore, $Binding\ energy = a_1 A - a_2 A^{2/3} - a_3 Z(Z-1)A^{-1/3}$

$$= -\frac{a_4 (Z - A/2)^2}{A} + \delta a_5 A^{-3/4}$$

This equation is known as Weizsacher's semi empirical formula or equation. The values of the constants in MeV are as follows:

$a_1 = 15.753$; $a_2 = 0.7102$; $a_3 = 17.80$; $a_4 = 94.77$; $a_5 = 33.60$.

Self Assessment Test I

i. List the properties of nuclear force.

ii. List the properties which liken a liquid drop to the nucleus of an atom.

2.3 CONCLUSION

In conclusion, we have been able to examine the different models proposed to describe the nucleus of an atom.

2.4 SUMMARY

In this unit, we have been able to understand that the nucleus of an atom can be likened to different forms of matter. This likeness led to the development of equations for the binding energies of a nucleus.

2.5 TUTOR MARKED ASSIGNMENT

a) What do you understand by the "Nuclear binding energy".

b) State the Weizsacher's semi empirical equation and explain each term of the equation.

2.6 REFERENCES/FURTHER READING

W. Greiner and J. A. Maruhn, Nuclear Models by Springer.

UNIT 3 RADIOACTIVITY

CONTENTS

3.0 INTRODUCTION

The decay of the nucleus of an atom can either be natural or artificial. This decay occurs in nuclide in order that they may attain stability. This disintegration or decay occurs with the emission of some particles or energy.

3.1 OBJECTIVES

After going through this unit, you will be able to:

- Explain radioactivity and its kinematics.
- Highlight and list the properties of particles released during radioactivity.
- Explain the radioactive series
- Explain age determination using radioisotopes.

3.2 MAIN CONTENT

3.2.1 STABILITY OF NUCLIDES

The stability of nuclides is mainly determined by the atomic mass (A) and the ($^N/_Z$) ratio. The condition for the stability of eight elements is $^N/_Z$=1, and for heavy elements, $^N/_Z \approx 1.5$ Nuclides that are not stable due to this ratio seek stability by undergoing inter-nuclear spontaneous transformation which shifts the $^{N}/_{Z}$ ratio to a more stable configuration. During this transformation, the following could occur:

1. ${}_{1}^{1}p \rightarrow {}_{0}^{1}n + {}_{1}^{0}\beta + V.$

2. ${}_{0}^{1}n \rightarrow {}_{1}^{1}p + {}_{1}^{0}\beta + V.$

3. α particles may be emitted (that is 2 protons and 2 neutrons)

4. Splitting of nucleus into two nearly equal fragments through nuclear fission.

Radioactivity can therefore be defined as the tendency of unstable nuclides, seeking to become stable through the emission of particles and energy.

The emitted particles during radioactivity are referred to as nuclear radiation.

Nuclear radiation can well be referred as ionizing radiation because they have sufficient energy to cause the production of ion pairs in any medium which they pass through. The most common particles usually emitted are $\beta^{+}, \beta^{-}, \alpha \; and \; \gamma$ particle.

$\boldsymbol{-\alpha - particles}$

The nuclear transformation equation for an α - particle is given by:

$${}_{Z}^{A}X \rightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}HE\left({}_{2}^{4}\alpha\right) + \gamma + Q$$
$${}_{92}^{238}U \rightarrow {}_{90}^{234}TH + {}_{2}^{4}HE\left({}_{2}^{4}\alpha\right) + \gamma + Q$$

Properties

1. α - particles can be stopped by a thin sheet of paper

2. They cause intense ionization in air

3. a group of α - particle emitted from the same type of nuclides usually have definite velocity and energy.

4. α - particles cover a definite distance in a given material practically without any loss of intensity and suddenly in a small distance are absorbed completely.

The definite distance traveled with a given material is called the RANGE

- β - Particles

The nuclear transformation equation for an β -particle is given by:

$${}_{Z}^{A}X \rightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}\beta^{-1} + \overline{V} + Q$$
$${}_{6}^{14}C \rightarrow {}_{7}^{14}N + {}_{-1}^{0}\beta^{-} + \overline{V} + Q$$

Properties

1. They cause less ionization in air
2. 100 times more penetrating than α particle and can penetrate a sheet of aluminum, a few millimeters thick.
3. A particular β - active element emits β - particles with energies varying between zero and certain maximum. This maximum energy is called the end point energy.

X-RAYS AND γ RAYS.

These are part of electromagnetic radiation.

γ RAYS.	x-RAYS
Short wavelength compared with X-ray	←
More energetic than x-rays and more penetrating than β RAYS. (≈ 100 times).	←
Owing to their large energies, they can dislodge electrons not only from outer orbits but from inner orbits.	←

The ability of the γ ray to be able to dislodge electrons from both the outer and inner orbit can be done in the following ways:

1. Photoelectric effect
2. Compton effect
3. Pair production

Self Assessment Test I

i. Define the term radioactivity
ii. What are the nuclear radiations?
iii. List and briefly explain the different forms of nuclear radiations.

3.2.2 KINEMATICS OF RADIOACTIVITY

When a nucleus disintegrates by emitting particle α, β, γ or capture an electron from an atomic shell (k-capture). This process is called *Radioactive decay*. All nuclear decay follows a single law called a *Decay law*.

The number of nuclei of a given radioactive sample disintegrating per second is called the *Activity* of the sample i.e. $A = \frac{dN}{dt}$.

The activity ($\frac{dN}{dt}$) at any instant of time is proportional to the number N of the parents type present at that time.

$$i.e. \frac{dN}{dt} \alpha\ N$$

$$A = \frac{dN}{dt} = -\lambda N \_\_\_\_\_\_(1)$$

Where λ is the decay constant or disintegration constant which only depends on the nature or characteristics of the radioactive sample and not on the amount of substance. Also λ - gives the probability of decay per unit interval of time.

From (1) above

$$\int_{No}^{N} \frac{dN}{N} = \int_{o}^{t} -\lambda dt$$

$$[\ln N]_{No}^{N} = -\lambda [t]_{o}^{t}$$

$$\ln N - \ln No = -\lambda t$$

$$\ln\left(\frac{N}{No}\right) = -\lambda t$$

$$\frac{N}{No} = e^{-\lambda t}$$

$$N = Noe^{-\lambda t} \_\_\_\_\_\_\_(2)$$

Multiply both sides of equation (2) by λ

$$\lambda N = \lambda Noe^{-\lambda t}$$

$$A = Aoe^{-\lambda t} \_\_\_\_\_\_\_\_(3)$$

From equation (1)

Where A stands for the activity. Activity is measured in Becquerel (1dps) and 1 curie (1ci) = 3.7×10^{10} Bq.

A time interval during which half of given sample of radioactive substance decays is referred to as the Half Life.

i.e.
$$A = \frac{Ao}{2} = Aoe^{-\lambda T_{1/2}}$$

$$1/2 = e^{-\lambda T_{1/2}}$$

Taking the log of both sides

$$-\ln 2 = -\lambda T_{1/2}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Since individual radioactive atoms may have life spans between o and ∞, we can then talk of average life or mean life.

$$T_{mean} = \frac{total\ life\ time\ of\ all\ nuclei\ in\ a\ given\ sample}{total\ number\ of\ nuclei\ in\ that\ sample}$$

$$= \frac{t_1 dN_1 + t_2 dN_2 ----+ t_N dN_N}{dN_1 + dN_2 +------ dN_N}]$$

$$T_{mean} = \frac{\int_{No}^{O} t dN}{\int_{No}^{O} dN} = \frac{-1}{No}\int_{No}^{O} t dN \_\_\_\_\_\_(1)$$

$$but\ N = Noe^{-\lambda t}$$

$$\lambda N = \lambda Noe^{-\lambda t}$$

Since $dN/dt = -\lambda N$

$$dN/dt = -\lambda Noe^{-\lambda t}$$

$$dN = -\lambda Noe^{-\lambda t} dt \_\_\_\_\_\_\_\_\_\_\_\_(2)$$

Substitute (2) into (1)

$$= -\frac{1}{No}\int_{No}^{o} t\,(+\lambda)\, Noe^{-\lambda t} dt$$

$$T_{mean} = \lambda \int_{o}^{\infty} te^{-\lambda t} dt \_\_\_\_\_\_\_\_\_(3)$$

$$it\ is\ o \to \infty\ because\ from$$

$$N = Noe^{-\lambda t}$$

$$for\ N = O = Noe^{-\lambda t}$$

$$O = e^{-\lambda t}$$

$$O = \frac{1}{e^{\lambda t}} \Rightarrow t \to \infty$$

$$and\ \ for\ N = N_o = N_o e^{-\lambda t}$$

$$1 = e^{-\lambda t}$$

$$1 = \frac{e}{e^{\lambda t}} = t \to 0$$

For the relationships to hold, then integrate equation (3) by parts.

From equation (3).

$$T_{mean} = \lambda \int_o^\infty t e^{-\lambda t} dt.$$

$$\Rightarrow \int u dv = uv - \int v du$$

$$where\ u = t\ and\ dv = e^{-\lambda t}$$

$$\Rightarrow du = 1dt\ and\ v = \int e^{-\lambda t} = -\frac{e^{-\lambda t}}{\lambda} + c$$

$$T_{mean} = \lambda \left[t\left(\frac{-e^{-\lambda t}}{\lambda}\right) - \int_o^\infty -\frac{e^{-\lambda t}}{\lambda} dt \right]$$

$$= \lambda \left[\frac{-te^{-\lambda t}}{\lambda} - \left(\frac{e^{-\lambda t}}{\lambda^2}\right) \right]_o^\infty$$

$$T_{mean} = \lambda \left[\frac{-te^{-\lambda t}(\lambda) - e^{-\lambda t}}{\lambda^2} \right]_o^\infty$$

$$= \frac{\lambda}{\lambda^2} \left[-te^{-\lambda t}(\lambda) - e^{-\lambda t} \right]_o^\infty + c$$

$$= \frac{1}{\lambda}[0+1] + c$$

$$= \frac{1}{\lambda}$$

Radioactive Equilibrium

Considering this decay process

$A \to B \to C$ **(**Stable**).** Since the number of nuclei entering B will be the decay of A.

$$\Rightarrow \frac{-dN_A}{dt} = \lambda_A N_A$$

The number of nuclei leaving B will be $\lambda_B N_B$

Therefore, the net charge in the number of nuclei per second of B is

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B \text{ \_\_\_\_\_\_\_\_(1)}$$

But this is a first order (linear d.e.) and $N_A = Noe^{-\lambda At}$

Rewriting equation (1)

$$\frac{dN_B}{dt} + \lambda_B N_B = \lambda_A N_A$$

$$\frac{dN_B}{dt} + \lambda_B N_B = \lambda_A N_o e^{-\lambda_A t} \text{ \_\_\_\_\_\_(2)}$$

Comparing this with $\frac{dy}{dx} + py = Q$

Where our integrating factor = $e^{\int pdx}$

Therefore, our integrating factor = $e^{\int \lambda_B dt = e^{\lambda_B t}}$

Multiply both sides of equation by integrating factor = $e^{\lambda_B t}$

$$e^{\lambda_B t}\frac{dN_B}{dt} + \lambda_B N_B (e^{\lambda_B t}) = \lambda_A N_O e^{-\lambda_A t}(e^{\lambda_B t}).$$

$$\frac{d}{dt}\left(e^{\lambda_B t} N_B\right) = \lambda_A N_o e^{-\lambda_A t}\left(e^{\lambda_B t}\right)$$

Now integrating both sides with t

$$e^{\lambda_B t} N_B = \int \lambda_A N_O e^{-\lambda_A t} e^{\lambda_B t} dt$$

$$= \lambda_A N_o \int e^{-\lambda_A t} . e^{\lambda_B t} dt$$

Using integration by parts

$$\int e^{-\lambda_A t} . e^{\lambda_B t} dt$$

$$u = e^{-\lambda_A t} \text{ and } dv = e^{\lambda_B t}$$

$$du = -\lambda_A e^{-\lambda_A t} \text{ and } v = \frac{e^{\lambda_B t}}{\lambda_B}$$

$$\int udv = uv - \int vdu$$

$$= e^{-\lambda_A t}\left(\frac{e^{\lambda_B t}}{\lambda_B}\right) - \int \frac{e^{\lambda_B t}}{\lambda_B} \cdot -\lambda_A e^{-\lambda_A t} dt$$

$$= \frac{e^{\lambda_A t} \cdot e^{\lambda_B t}}{\lambda_B} + \frac{\lambda_A}{\lambda_B}\int e^{\lambda_B t} \cdot e^{-\lambda_A t} dt - - - - - - - - - - - - (3)$$

Since we are back to the initial integral, put

$I = \int e^{\lambda_B t} \cdot e^{\lambda_A t}$

Then equation (3) is

$$I = \frac{e^{-\lambda_A t} \cdot e^{\lambda_B t}}{\lambda_B} + \frac{\lambda_A}{\lambda_B} I$$

$$I - \frac{\lambda_A}{\lambda_B}.I = \frac{e^{-\lambda_A t} \cdot e^{\lambda_B t}}{\lambda_B}$$

$$I\left(1 - \frac{\lambda_A}{\lambda_B}\right) = \frac{e^{-\lambda_B t} . e^{\lambda_B t}}{\lambda_B}$$

$$I = \frac{e^{-\lambda_A t} \cdot e^{\lambda_B t}}{\lambda_B} \div \left(1 - \frac{\lambda_A}{\lambda_B}\right) + C$$

$$= \frac{e^{-\lambda_A t} \cdot e^{\lambda_B t}}{\lambda_B} \div \left(\frac{\lambda_B - \lambda_A}{\lambda_B}\right) + C$$

$$= \frac{e^{-\lambda_A t} \cdot e^{\lambda_B t}}{\lambda_B} \times \frac{\lambda_B}{\lambda_B - \lambda_A} + C$$

$$= \frac{e^{-\lambda_A t} \cdot e^{\lambda_B t}}{\lambda_B - \lambda_A} + C$$

Then the equation will now be

$$e^{-\lambda_B t} N_B = \lambda_A N_O \left[\frac{e^{-\lambda_A t} \cdot e^{\lambda_B t}}{\lambda_B - \lambda_A}\right] + C$$

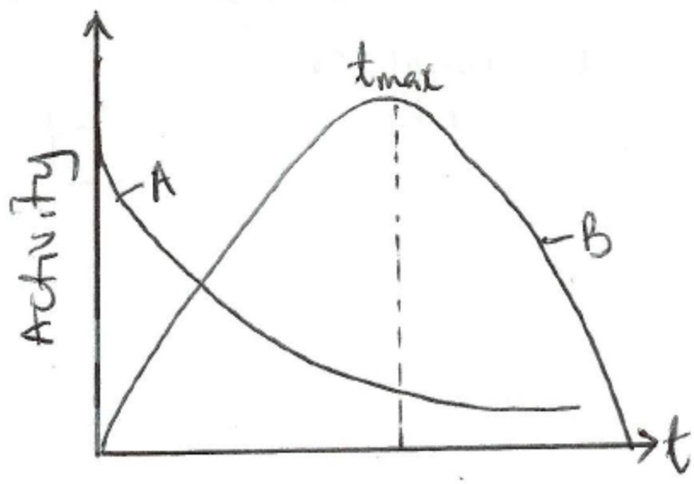
$$N_B = \frac{\lambda_A N_O}{e^{\lambda_B t}} \left[\frac{e^{-\lambda_A t} \cdot e^{\lambda_B t}}{\lambda_B - \lambda_A}\right] + C$$

$$N_B = \frac{\lambda_A N_O}{\lambda_B - \lambda_A} \left[e^{-\lambda_A t}\right] + C$$

$$N_B = \frac{N_O \lambda_A}{\lambda_B - \lambda_A} \left[e^{-\lambda_A t} - e^{-\lambda_B t}\right] ------------ (4)$$

Cases

1. At t maximum, $\frac{dN}{dt} = 0\, i.e\, \lambda_A N_A = \lambda_B N_B$ and the activity of the parent and daughter are said to be at equilibrium. This is called the *Ideal Equilibrium*



2. Considering a case whereby the daughter is short lived than the parent i.e. $T_A>T_B$ from the equation (1) above, the activity of B is

$$\lambda_B N_B = \frac{N_O \lambda_A}{\lambda_B - \lambda_A} \cdot \lambda_B \left[e^{-\lambda_A t} - e^{-\lambda_B t}\right] ---------(5)$$

$$but\, \lambda_A N_A = \lambda_A N_O e^{-\lambda_A t}$$

$$N_O \lambda_A = \frac{\lambda_A N_A}{e^{-\lambda_A t}} ---------(6)$$

Introducing (6) into (5)

$$\lambda_B N_B = \frac{\lambda_A N_A}{e^{-\lambda_A t}} \cdot \frac{\lambda_B}{\lambda_B - \lambda_A}\left[e^{-\lambda_A t} - e^{-\lambda_B t}\right]$$

$$= \frac{\lambda_A N_A \,.\boldsymbol{\lambda_B}}{\lambda_B - \lambda_A}. \left[1 - e^{-\lambda_B t} .e^{\lambda_A t}\right]$$

$$\lambda_A N_A \cdot \frac{\lambda_B}{\lambda_B - \lambda_A} = \left[1 - e^{-\lambda_B t + \lambda_A t}\right]$$

$$\lambda_B N_A = \lambda_A N_A \cdot \frac{\lambda_B}{\lambda_B - \lambda_A}\left[1 - e^{-(\lambda_B t - \lambda_A t)}\right]$$

$$\frac{\lambda_B N_B}{\lambda_A N_A} = \frac{\lambda_B}{\lambda_B - \lambda_A}\left[1 - e^{-(\lambda_B t - \lambda_A t)}\right]$$

Since $T = {}^{1}/_{\lambda}$ therefore $\lambda = {}^{1}/_{T}$

$$\Rightarrow \lambda_B = \frac{1}{T_B},\ \lambda_A = \frac{1}{T_A}$$

$$\frac{\lambda_B}{\lambda_B - \lambda_A} = \frac{\frac{1}{T_B}}{\frac{1}{T_B} - \frac{1}{T_A}} = \frac{\frac{1}{T_B}}{\frac{T_A - T_B}{T_B T_A}} = \frac{1(T_B T_A)}{T_B(T_A - T_B)} = \frac{T_A}{T_A - T_B}$$

Then the equation becomes

$$\frac{\lambda_B N_B}{\lambda_A N_A} = \frac{T_A}{T_A - T_B}\left[1 - e^{-(\lambda_B t - \lambda_B t)}\right]$$

Since $(\lambda_B - \lambda_A)_t = \left(\frac{T_A - T_B}{T_B T_A}\right) \cdot t$

$$= \left(\frac{T_A - T_B}{T_A}\right)\frac{1}{T_B} \cdot t$$

$$= \left(\frac{T_A - T_B}{T_A}\right)\lambda_B . t$$

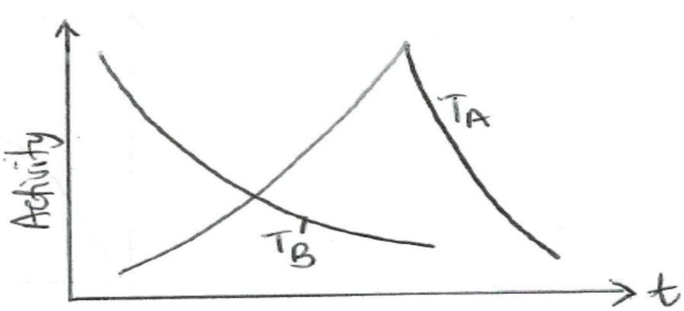
$$\frac{\lambda_B N_B}{\lambda_A N_A} = \frac{T_A}{T_A - T_B}\left[1 - e^{-[(T_A - T_B)\lambda_A]\lambda_B t}\right]$$

At large time, t

$$\frac{\lambda_B N_B}{\lambda_A N_A} = \frac{T_A}{T_A - T_B} ----------(7)$$

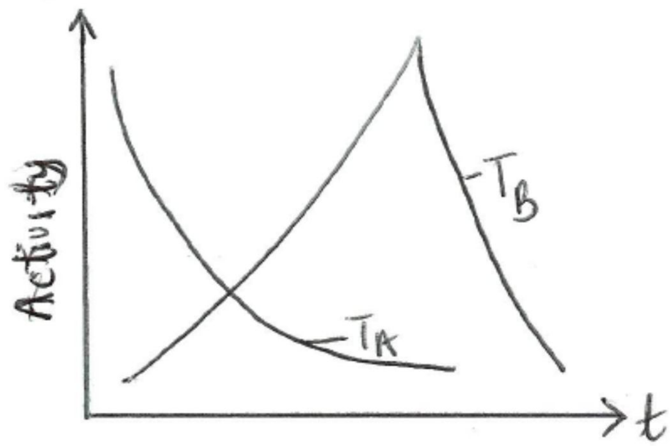
When this equation (7) holds, this implies that transient equilibrium exists between the parent and daughter, and its correlation is that

$\frac{T_A}{T_A - T_B} > 1$. For ($T_A > T_B$)



3. For the case in which the daughter is long-curved, then the parent i.e. ($T_A < T_B$). It follows from equation (7) that the ratio $\frac{\lambda_B N_B}{\lambda_A N_A}$ increases as t increases. Therefore after

sufficient time, the activity of the daughter becomes independent of that of the residual activity of the parents.



4. If $T_A \gg T_B$, $\lambda_A \ll \lambda_B$

$$then\ \lambda_B N_B = \lambda_A N_A \left[1 - e^{-\lambda_B t}\right]$$

$$for\, t = T_B$$

$$e^{-\lambda_B t} = 0$$

$$\lambda_B N_B = \lambda_A N_A \ (secular\ equilibrium)$$

$$i.e.\ \frac{\lambda_B}{\lambda_A} = \frac{T_A}{T_B} = \frac{N_A}{N_B}$$

All these can as well be called serial transformation.

Self Assessment Test II

i. Define the following terms:

(a) activity (b) half life (c) decay constant

ii. How many kinds of radioactivity equilibrium exist?

3.2.3 RADIOACTIVE SERIES

Radio nuclides that are related constitute a decay chain or series. The successive daughter products are formed through the emission of $\beta\ and\ \alpha$ particles leading to stable end-product. There are four known series in nature but that of neptinium is artificial.

They are

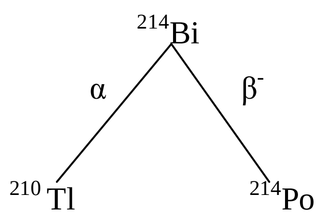
1. Thorium 4n

2. Neptinium 4n+1
3. Uranium 4n+2
4. Actinium 4n+3

There are also some, that are not of large atomic number e.g. ^{40}K and all these constitute a source of radioactivity in the earth crust.

Branching

Normally, a particular radio nuclide is supposed to decay through either $\alpha\, and\, \beta$ decay. In some cases some will decay through $\alpha\, and\, \beta$ or both. This phenomenon is known as branching. E.g.



Age Determination using Radioisotopes

Radioactivity is the best clock in determining or usually applied to estimate the absolute age of geological materials because it is totally not affected by environmental changes of natural processes like earthquakes, storms etc

Some of the radioisotopes that are useful for geological age dating include ^{87}Sr / ^{87}Rb, ^{14}C/ ^{12}C, Pb / U, Pb / Th etc.

The half life of these radioisotopes are usually use for determining the age ranges of interest.

The determination of geological ages is done very often by the *lead method* which involves

$$_{X}^{A}X \rightarrow _{Z-2}^{A-4}Y + _{z}^{4}\alpha + \gamma + Q$$
$$_{92}^{238}u \rightarrow emission\ of\ 8\left(_{z}^{4}x\right) \rightarrow {}_{82}^{206}pb$$
$$_{92}^{235}u \rightarrow emission\ of\ 7\left(_{z}^{4}x\right) \rightarrow {}_{82}^{206}pb$$
$$_{90}^{232}Th \rightarrow emission\, of\ 6\left(_{z}^{4}x\right) \rightarrow {}_{82}^{206}pb$$

These are natural decay series in which after sufficient time e.g. billion years only uranium and lead are the elements left in appreciable amounts. This is because all elements in the uranium series are in secular equilibrium with the parents except $_{82}^{206}Pb$ which is not in secular equilibrium.

From

$$N_B = \frac{N_O \lambda_A}{\lambda_B - \lambda_A}\left[e^{-\lambda_A t} - e^{-\lambda_B t}\right]$$

$$where\ N_B = N_{Pb}$$

$$\lambda_B = \lambda_{Pb} - 0(stable)$$

$$\lambda_A = \lambda_u$$

$$N_o = N_v$$

Then substitute

$$\Rightarrow N_{Pb} = \frac{N_V \lambda_A}{o - \lambda_A}\left[e^{-\lambda_u t} - e^{-0}\right]$$

$$= -N_V\left[e^{-\lambda_u t} - 1\right]$$

$$N_{Pb} = N_V\left[1 - e^{-\lambda_u t}\right] - - - - - - - - (1)$$

Therefore, number of uranium atoms originally present = present number of lead atoms + present number of uranium i.e.

$$N_V = N_{Pb} + N_u - - - - - - - - - (2)$$

Solving equation (1) and (2) simultaneously

$$N_{Pb} = N_V\left[1 - e^{-\lambda_u t}\right]$$

$$N_{Pb} = N_V - N_V e^{-\lambda_u t} - - - - - - - - (3)$$

$$N_{Pb} = N_V - N_u - - - - - - - - - - (4)$$

Then we have

$$0 = -N_V e^{-\lambda_u t} + N_u$$

$$N_V e^{-\lambda_u t} = N_u$$

$$e^{-\lambda_u t} = \frac{N_u}{N_v}$$

$$-\lambda_u t = \log\frac{N_u}{N_v}$$

$$t = -\frac{1}{\lambda_u}\log\frac{N_u}{N_v}$$

$$t = -\frac{1}{\lambda_u}\log\left[\frac{N_u}{N_v + N_{Pb}}\right]$$

$$t = -\frac{1}{\lambda_u}\left[\log N_u - \log\left(N_u + N_{Pb}\right)\right]$$

$$= \frac{1}{\lambda_u}\left[\log\left(N_u + N_{Pb}\right) - \log N_u\right]$$

$$t = \frac{1}{\lambda_u}\log\left[\frac{N_u + N_{Pb}}{N_u}\right]$$

Self Assessment Test III

i. What are radioactive series?

ii. List the four known radioactive series.

iii. Give the reasons for using radioactivity to determine the age of matter.

3.3 CONCLUSION

In conclusion, we have been able to examine radioactivity and its kinematics. Also, we examined the use of radioactivity such as in age determination.

3.4 SUMMARY

In this unit, we have been able to understand that nuclides decay to attain stability. Also, this decay is accompanished by the release of particles. This phenomenon can be used to determine the age of matter.

3.5 TUTOR MARKED ASSIGNMENT

i. Differentiate between the following radioactive particles (α, β and γ).

ii. Briefly explain the different kinds of radioactive equilibrium.

iii. Briefly explain Branching in radioactive decay.

iv. Explain age determination using radioisotopes.

3.6 REFERENCES / FURTHER READING

W. Greiner and J. A. Maruhn, Nuclear Models by Springer.

UNIT 4 THE ENERGETICS OF PARTICLE

CONTENTS

4.0 INTRODUCTION

Different particles are emitted during the disintegration of nuclides. These particles exhibit different characteristics when they are emitted. Therefore, it is important we study these particles in detail to enable us handle them properly when they are released.

4.1 OBJECTIVES

After through this unit, you will be able to:

- Explain the conditions necessary for α, β and γ decay to be possible.
- Understand the properties of these particles when they are released.

4.2 MAIN CONTENT

4.2.1 α-DECAY

α -particles are stable and exhibit a definite range when they transverse a medium. For an α decay to be possible, there is a minimum energy requirement.

$$X(A,Z) \rightarrow Y(A-Y, A-Z)$$

$$^{A}_{X}X \rightarrow ^{A-4}_{Z-2}Y + ^{4}_{z}\alpha + Q\alpha$$

Where $Q\alpha = \Delta mc^2$

Since is due to mass defect between m_i and m_f (initial and final masses).

$$m_i = [M\ (A,Z)] = ZM_P + NM_N - E_{Bi}$$
$$m_f = [M(A-4, Z-2)] = (Z-2)Mp + (N-2)M_N - E_{Bf}$$
$$M\alpha = [M(4,2)] = 2M_P + 2M_N - E_{B\alpha}$$

$$Q_\alpha = m_i - m_f - m_\alpha$$
$$= -E_{Bi} + E_{Bf} + E_{B\alpha}$$
$$= E_{B\alpha} + E_{Bf} - E_{Bi}$$

And $E_{B\alpha} = 28.3 MeV$

$$Q_\alpha = (28.3 + \Delta E_B)\, MeV$$

From semi-empirical formula

Binding energy $= E(Z, A)$

Therefore, the disintegration energy of nuclei Q_α or total energy released in α-decay is given as

$$Q_\alpha = 28.3 + \left(\frac{2E}{2A}\right)_2 \Delta A + \left(\frac{2E}{2Z}\right)_A \Delta Z$$

α emission is not possible if Q_α < O, that is Q_α must be > O.

From studies, it has been found that Q_α > O for nuclide for which Z > 82

Suppose, the mass of parents $=M_P$.

Mass of daughter= M_d

Mass of α particle= M_α

Velocity of α particle when emitted= V_α

Velocity of record of daughter = V_d

From the conservation of momentum

$M_\alpha V_\alpha = M_d V_d$ ----------- (1)

Total energy = Q_α =final kinetic energy – initial kinetic energy

$$Q_\alpha = \frac{1}{2} M_\alpha V_\alpha{}^2 + \frac{1}{2} M_d V_d{}^2 ------(2)$$

From (1)

$$V_d = \frac{M_\alpha V_\alpha}{M_d} --------(3)$$

Substitute (3) into (2)

$$Q_\alpha = \frac{1}{2} M_\alpha V_\alpha{}^2 + \frac{1}{2} M_d \left[\frac{M_\alpha V_\alpha}{M_d}\right]^2$$

$$Q_\alpha = \frac{1}{2} M_\alpha V_\alpha{}^2 + \frac{1}{2} M_\alpha{}^2 V_\alpha{}^2 / Md$$

$$= \frac{1}{2} M_\alpha V_\alpha{}^2 \left[1 + \frac{M_\alpha}{M_d} \right] ------(4)$$

For small approximation

$$\frac{M_\alpha}{M_d} \approx \frac{4}{A-4} ------(5)$$

Substitute (5) into (4)

$$Q_\alpha = \frac{1}{2} M_\alpha V_\alpha{}^2 \left[1 + \frac{4}{A-4} \right]$$

$$Q_\alpha = E_\alpha \left[\frac{4}{A-4} + 1 \right]$$

$$= E_\alpha \left[\frac{4 + A - 4}{A-4} \right]$$

$$Q_\alpha = E_\alpha \left[\frac{A}{A-4} \right]$$

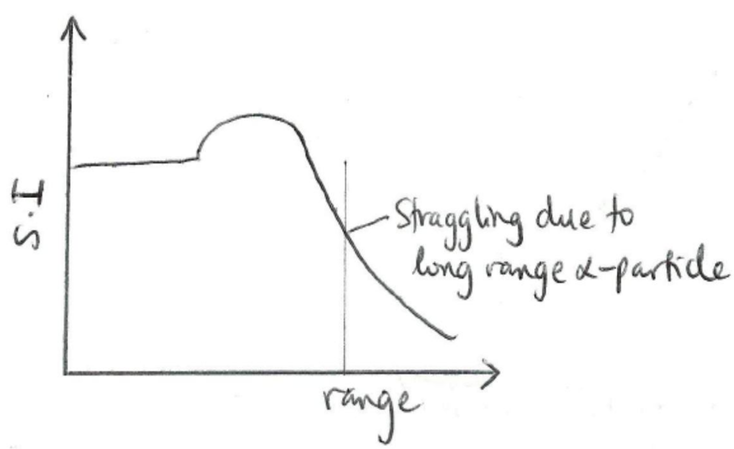
Since A is large

$$Q_\alpha \approx E_\alpha$$

This means most of the energy released is carried away by the α-particle.

RANGE OF α -PARTICLE

α - particles are densely ionizing and lose their energies in quick succession in air or any medium. The number of ions pairs produced per unit length is called *specific ionization* (S.I)



The mean distance travelled by α -particle before absorption is called *range*.

The intersection of α -particle with atoms or molecules of a medium are purely statistical and as a result they do not have the same range as in air.

Range, R = $318E^{3/2}$

An empirical relation between the range of α -particle and disintegration constant is given by Geiger Nuttal Law

$log\lambda = AlogR + B$

α -decay paradox

Because α -particle is a tightly bound entity we can assure it pre-exists in the nucleus before its emission.

For a α -particle to come out or go into the nucleus it implies it must have an energy in the neighborhood of the potential well of the nucleus.

The energy of α -particle usually ranges between 4-8Mev which is far less than what is required to surmount the potential barrier. Classically it is impossible to understand this because it has no chance of leaving the nucleus.

In 1928, George Gamow and independently with others applied wave mechanics to the problem of α -decay paradox and they were able to resolve it. They considered an α -particles as a matter wave. This implies that α -particle as a finite probability of penetrating the wall of thickness where it undergoes series of collisions per second.

It was also discovered that the probability of finding the α -particle outside the nucleus is small but not zero.

Self Assessment Test I

i. Show that ${}^{236}_{94}Pu$ is unstable against α-decay.
ii. List the conditions necessary for an α-decay to occur.
iii. What is meant by the range of an α-particle?

4.2.2 β DECAY

A decay process in which the charge of the nucleus changes without a change in the number of nucleons

There are three types of β decay

i. β^- decay e.g.

$$ {}_{Z}^{A}X \rightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}\beta + \bar{v} $$
$$ {}_{5}^{12}B \rightarrow {}_{6}^{12}C + \beta + \bar{v} $$

ii. β^+ decay e.g.

$$ {}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + {}_{-1}^{0}\beta + \bar{v} $$
$$ {}_{7}^{12}N \rightarrow {}_{6}^{12}C + \beta + \bar{v} $$

iii. Electron capture or k-capture

A process through which the nucleus captures an orbital electron, most often from the closest shell to convert a proton to neutron

$$ {}_{Z}^{A}X + {}_{-1}^{0}e \rightarrow {}_{Z-1}^{A}Y + \bar{v} $$
$$ {}_{4}^{7}Be + {}_{-1}^{0}e \rightarrow {}_{3}^{7}li + \bar{v} $$

Energetic of β^- decay

$$ {}_{Z}^{A}X \rightarrow {}_{z+1}^{A}Y + \beta^- + \bar{v} $$

In terms of nuclear masses

$$ Q/C^2 = Mn\left({}_{Z}^{A}X\right) - Mn\left({}_{z+1}^{A}Y\right) - Me $$

And in terms of atomic masses

$$ Q/C^2 = Ma\left({}_{Z}^{A}X\right) - Ma\left({}_{z+1}^{A}Y\right) $$

For β^- to be possible, $Q > 0$

β^+ Decay

$$ {}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + \beta^{-1} + \bar{v} $$

Nuclear masses

$$ Q/C^2 = Mn\left({}_{Z}^{A}X\right) - Mn\left({}_{z-1}^{A}Y\right) - Me $$

Atomic masses

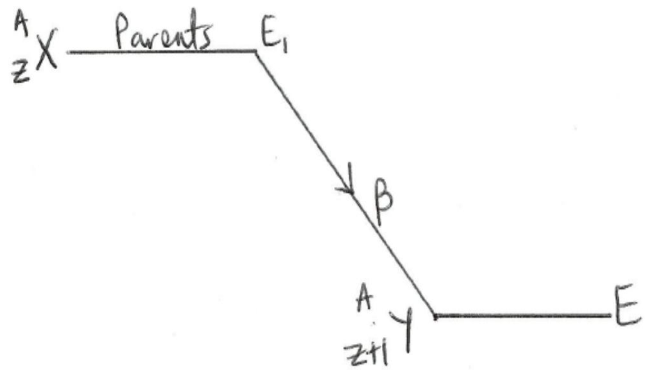
$$ Q/C^2 = Ma\left({}_{Z}^{A}X\right) - Ma\left({}_{z-1}^{A}Y\right) - 2Me $$

Electron capture

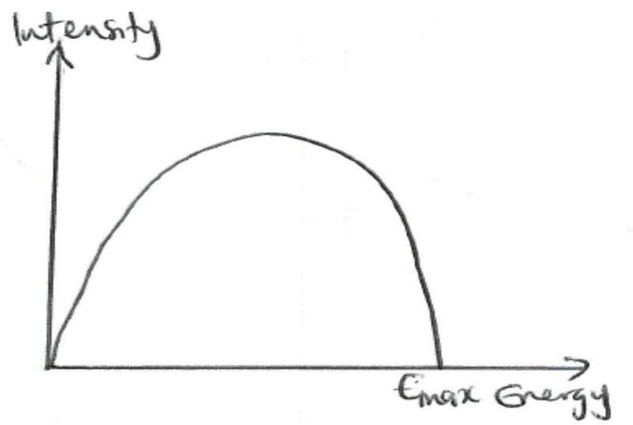
$$Q/C^2 = Ma\left({}^{A}_{Z}X\right) - Ma\left({}^{A}_{z-1}Y\right)$$

$\beta - Spectrum$

1) Unlike α-rays, the spectrum of β-rays happens to be continuous that is the electrons emitted have different kinetic energies.
2) It is also an energy transition between two definite energy states.



3) Mono-energetic β-rays forming a line spectrum are expected.



From the figure above, most of the electrons are emitted with only $1/3$ of the energy.

Therefore, this makes one to imagine where the remaining of the $2/3$ of the maximum energy would have gone to.

Since measurements like momentum and angular momentum are not conserved. These suggest that a third particle must exist that always accompany the β-decay. It was detected to be neutrino(μ).

Neutrino $(\boldsymbol{\mu})$

1. Carries away energy equal to the energy different between the observed energy for a $\beta -$ decay and the maximum energy of the continuous spectrum.

2 .To maintain the principle of conservation of energy, neutrino was given the following properties

a). 0 charge b).0 mass c). Moves with speed of light d). Spin of ½$\hbar$.

3. The antiparticle of neutrino, (antineutrino) has the following properties

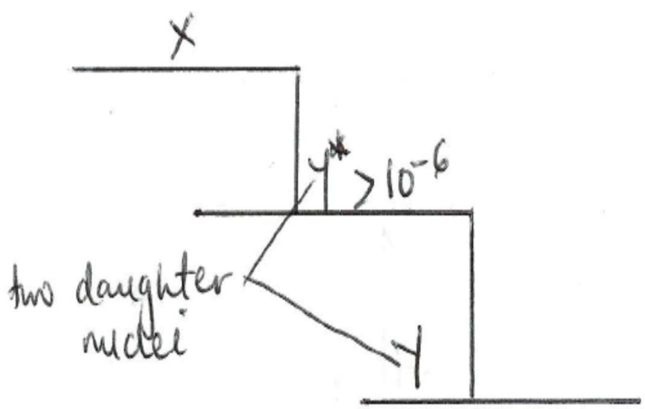
a). 0 charge b). 0 mass c). spin of ½$\hbar$

4.2.3 γ - DECAY

When a nucleus is in an excited gamma rays are emitted and it is brought to the ground state.

A nucleus is usually left in an excited state after emitting either $\alpha\, or\, \beta$ rays then it is de-excitated by emitting gamma rays. Gamma rays are emitted with discrete and definite energies which is an indication of the nuclear structure. The energy carried away is $\Delta E = hf$.

When the mean life time of the excited nucleus is >10^{-65}, the daughter nucleus is said to exhibit nuclear isomerism.



Y^k and Y are nuclear isomers and are chemically and physically the same. The difference is that Y^k is more energetic than Y and it eventually emits the energy as γ ray and returns to ground state.

Sometimes, instead of γ ray being emitted, this excess energy of the excited nucleus may be transferred to an extra nuclear electron to get it from its shell (usually K or L shell). This process is called *internal conversion*.

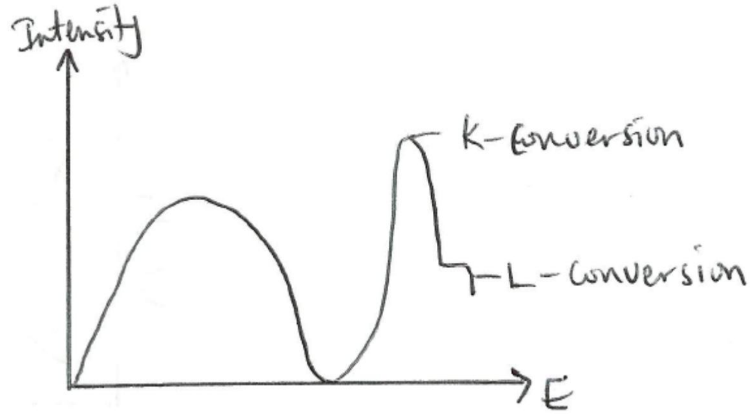
Kinetic energy of converted electron is

$Ke = \Delta E - Be$

$Be - binding\ energy\ of\ electron$

$\Delta E = E_i - E_f$

Generally, because of internal conversion, the yield of γ rays in a particular decay <100%. Some of the spikes observed in continuous β – spectrum is usually due to internal conversion process.



Self Assessment Test II

i. Explain the processes involved in a γ decay scheme of a nuclide.
ii. What is nuclear isomerism?

4.3 CONCLUSION

In conclusion, we have been able to examine the particles emitted during the disintegration of nuclides as well as their energetic.

4.4 SUMMARY

In this unit, we have been able to understand that different particles and antiparticles are emitted during disintegrations. Also, we have understood that conditions surrounding disintegration determines which of the particles are released during any disintegration.

4.5 TUTOR MARKED ASSIGNMENT

i. What is meant by the term specific ionization?
ii. Write a formula relating the range of α particle and the disintegration constant.
iii. Briefly explain the α decay paradox.

iv. Show that $^{12}_{7}N \rightarrow \; ^{12}_{6}C + \beta^{+} + Q$ is energetically possible for β^{+} decay.

v. What happens to the remaining energy in β decay after the emission of a β particle?

vi. What is internal conversion?

4.6 REFERENCES / FURTHER READING

R. Gautreau and W. Savin, Schaum's outline of theory and problems of modern physics, 1999 edition.

UNIT 5 NUCLEAR REACTIONS

CONTENTS

5.0 INTRODUCTION

In this unit, nuclear reactions are briefly explained. Here nuclei are bombarded with known projectiles and the final products are observed.

5.1 OBJECTIVES

After going through this unit, you will be able to:

- Explain process involved in nuclear reactions.
- Explain nuclear fission and fusion reactions.
- Explain the types of nuclear reactions which exist.

5.2 MAIN CONTENT

5.2.1 NUCLEAR REACTION

A nuclear reaction is a process whereby the mass number or the atomic number of target nuclei changes as a result of bombardment with projectile particles resulting in the release of energy.

The necessary things for a nuclear reaction are:

1. A target nucleus
2. A projectile example ${}_0^1n, {}_1^1H, {}_2^4He, {}_1^2H$.

$X + x \rightarrow Y + y + Q$ or $X\,(x, y)\,Y$

e.g. ${}^{14}_{7}N\ (\alpha, p)\ {}^{17}_{7}O$

$\Longrightarrow\ {}^{14}_{7}N + {}^{4}_{2}He \longrightarrow {}^{17}_{8}O + {}^{1}_{1}H + Q.$

Types of Nuclear Reaction

1) F - Fission - $X + x \to y_1 + y_2$
2) I - Inelastic nuclear reaction - X + x →X^k +x
3) T - Transmutation – X + x → Y + y
4) E - Elastic nuclear reaction – X + x → X + x
5) C - Capture – X + x → Y^k

The physical qualities which are conserved in any nuclear reaction include:

1) Total electric change EZ = K
2) Total number of nuclei EA = K
3) Linear momentum EP = K
4) Sum of mass and energy E(mass + Ke)
5) Parity K

5.2.2 Q - VALUE EQUATION

This is the nuclear change or the amount of energy released in a nuclear reaction. For a nuclear reaction, the total rest mass and kinetic energy are conserved

Example $X + x \to Y + y + Q$

$i.e\ \left[E_X + m_X C^2\right] + \left[E_x + m_x C^2\right]$

$= [E_Y + m_Y C^2] + \left[E_y + m_y C^2\right] + Q$

Since the target nucleus X is at rest, then the equation turns to

$$\left[M_X C^2\right] + \left[E_x + m_x C^2\right] = \left[E_Y + M_Y C^2\right] + \left[E_y + M_y C^2\right] + Q$$

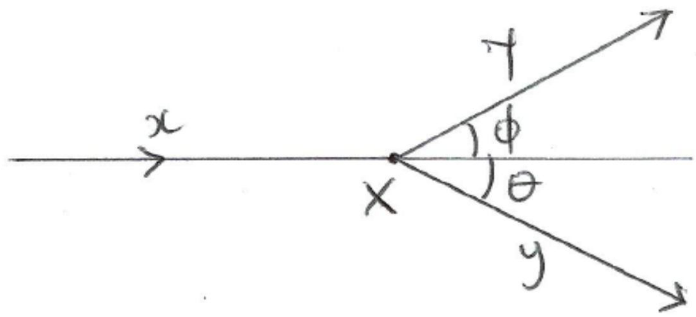
But Q = change in energy

Therefore Q = E_x - (E_Y + E_y)

$$Q = \left[(M_X + m_x) - (M_Y + M_y)\right]C^2$$

$$Q = \Delta mc^2$$

Conventional Q-value Equation



Applying the principle of the conservation of momentum

$$M_xV_x = M_YV_Y\cos\phi + M_yV_y\cos\theta\,(x - directon) - - - - (1)$$

$$O = M_YV_Y\sin\phi + M_yV_y\sin\theta\,(y - direction) - - - - - - - - - - (2)$$

$$from\,E = \frac{p^2}{2m}, p^2 = 2mE$$

Therefore, momentum mv = $\sqrt{2ME}$

Equation (1) and (2) turns into

$$(M_xE_x)^{1/2} = (M_YE_Y)^{1/2}\cos\phi + (M_yE_y)^{1/2}\cos\theta - - - - - - - - (3)$$

$$O = (M_YE_Y)^{1/2}\sin\phi + (M_yE_y)^{1/2}\sin\theta - - - - - - - - - - - (4)$$

Square equation (3) and (4) and add

$$M_xE_x = M_YE_Y\cos^2\phi + M_yE_y\cos^2\theta$$

$$O = M_YE_Y\sin^2\phi + M_yE_y\sin^2\theta$$

$$M_xE_x = M_YE_Y(\cos^2\phi + \sin^2\phi) + M_yE_y(\cos^2\theta + \sin^2\theta)$$

$$M_xE_x = M_YE_Y + M_yE_y\,.$$

The minimum energy a projectile must have before it can induce a nuclear reaction is called the *threshold energy*

Conservation of linear momentum

$$M_xV_x = M_cV_c$$

$$V_c = \frac{M_xV_x}{M_c}$$

But $-Q = {}^1/_2\, M_xV_x{}^2 - {}^1/_2\, M_cV_c{}^2$

$$-Q = {}^1/_2\, M_xV_x{}^2 - {}^1/_2\, M_c\frac{M_x{}^2V_x{}^2}{M_c{}^2}$$

$$-Q = {}^1/_2 M_x V_x^{\ 2} \left[1 - \frac{M_x}{M_c}\right]$$

But $M_c = M_X + M_x$

$$-Q = {}^1/_2 M_x V_x^{\ 2} \left[1 - \frac{M_x}{M_X + M_x}\right]$$

$$-Q = {}^1/_2 M_x V_x^{\ 2} \left[\frac{M_X + M_x - M_x}{M_X + M_x}\right]$$

$$-Q = {}^1/_2 M_x V_x^{\ 2} \left[\frac{M_X}{M_X + M_x}\right]$$

Take ${}^1/_2 M_x V_x^{\ 2} = E_{thr}$

$$-Q = E_{thr} \left[\frac{M_X}{M_X + M_x}\right]$$

$$E_{thr} = -Q \times \left[\frac{M_X + M_x}{M_X}\right]$$

$$E_{thr} = -Q \times \left[1 + \frac{M_x}{M_X}\right]$$

Self Assessment Test I

i. Define what is meant by a nuclear reaction and Q – value energy.

ii. What is meant by threshold energy?

iii. When ${}^6_3Li$ is bombarded with 4MeV deuterons, one reaction that is observed is the formation of two α-particles, each with 13.2MeV of energy. Find the Q-value for this reaction.

iv. Determine the unknown particle in the following nuclear reactions: (a) ${}^{18}_{8}O$(d,p)X, (b) X(p,α)${}^{87}_{39}Y$, (c) ${}^{122}_{52}Te$(X,d)${}^{124}_{53}I$.

v. Calculate the Q – values for the reactions (a) ${}^{16}_{8}O$(γ,p) ${}^{15}_{7}N$ (b) ${}^{150}_{62}Sm$(d,p) ${}^{147}_{61}Pm$.

5.2.3 NUCLEAR FISSION REACTION

This is a reaction which involves the splitting of heavy nuclei into two or more lighter nuclei by bombarding the heavy nuclei with thermal neutrons and it is usually accompanished with a high energy released. Nuclides that can be fissioned by thermal neutrons as follows ^{235}U, ^{237}U and ^{239}Pu.

Only ^{235}U occurs naturally while others are gotten from fertile materials. The process of conversion of fertile materials to fissionable material is referred to as *breeder reaction.*

Energy released in a fission reaction is $Q = \Delta mc^2 = \left(\Sigma m_i - \Sigma m_f\right)c^2$

5.2.4 NUCLEAR FUSION REACTION

This is the combination of two or more light nuclei to form a heavier one and this involves the supply of high energy. In which this energy will be able to overcome the coulombs force between them,

$${}_1^1H + {}_1^2H \rightarrow {}_1^3H + \gamma + Q.$$

And in doing so they must overcome the potential barrier which is equal to

$$V = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_o r}$$

$$V = 0.15 Z_1 Z_2 MeV,\ for\, r = 10^{-14}\, m$$

This sort of energy (10^9k) can only be acquired during nuclear explosions which is not practicable. Usually at this temperature, atoms exist as ions and are collected as plasma. Because of the high temperature, it is also referred to as thermonuclear reaction.

Self Assessment Test II

i. Distinguish between nuclear fusion and nuclear fission.

ii. What is the kinetic energy of a 300K thermal neutron?

iii. On the average, neutrons lose half their energy per collision with quasi-free protons. How many collisions, on the average, are required to reduce a 2MeV neutron to a thermal energy of 0.04eV?

iv. About 185MeV of usable energy is released in the neutron-induced fissioning of a ${}_{92}^{235}U$ nucleus. If ${}_{92}^{235}U$ in a reactor is continuously generating 100MW of power, how long will it take for 1kg of the uranium to be used up?

v. Estimate the temperature required to produce fusion in deuterium plasma (a neutral mixture of negatively charged electrons and positively charged deuterium nuclei).

vi. What will be the energy released if two deuterium nuclei fuse into an α-particle?

5.3 CONCLUSION

In conclusion, we have been able to examine nuclear reactions in its different forms. Also nuclear fusion and nuclear fission were examined.

5.4 SUMMARY

In this unit, we have been able to understand the conditions necessary for nuclear reactions to take place. Also, the importance of the Q-value was examined. Also nuclear fusion and nuclear fission were examined with their kinematics.

5.5 TUTOR MARKED ASSIGNMENT

i. Determine the unknown particle in the nuclear reactions: (a) ${}^{182}_{74}W({}^{3}_{2}He, n)X$, $(b)\ {}^{42}_{20}Ca({}^{6}_{3}Li, X){}^{45}_{21}Sc$.

ii. Calculate the Q-value for the reaction ${}^{42}_{20}Ca(p, d){}^{41}_{20}Ca$.

iii. Calculate the Q-value for the D-T fusion reaction ${}^{3}_{1}H(d, n){}^{4}_{2}He$.

iv. Find the Q-value for the D-D reactions (a) ${}^{2}_{1}H(d, n){}^{3}_{2}He$, (b) ${}^{2}_{1}H(d, p){}^{3}_{1}H$.

v. Calculate the energy released in the fusion process ${}^{4}_{2}He + {}^{4}_{2}He + {}^{4}_{2}He \rightarrow {}^{12}C$.

5.6 REFERENCES / FURTHER READING

Schaum's outline of theory and problems of modern physics. (2nd edition) by R. Gautrean and W. Savin.

UNIT 6 INTERACTION OF RADIATION WITH MATTER

CONTENTS

6.0 INTRODUCTION

Radiation measurements are possible through interaction with matter either living or dead. This interaction has made of possible to be used in diagnosis, researchers industries X-ray and radiotherapy. Because of the fundamental difference in energy transfer. Radiation can be categorize into four

1. Heavy charged particles
2. Fast electrons particles
3. Neutrons electrons particle
4. Protons electrons particles

Cross section and interaction co-efficient

The probability that an interaction will take place is expressed in cross sections. Cross section actually describes the effective area which the interaction center or entities presents to the radiation which if traversed by the radiation, it ensures that an interaction occur.

$$\delta = \frac{prob\, of\ \text{int}\, eraction\ p}{no.of\ conc.center\ area}$$

$$\delta = \frac{prob\, of\ \text{int}\, eraction\ p}{particle\ fluence, Q}$$

δ is expressed in m^2 or in barns

$1 barns = 10^{-28} m^2$. Linear attenuation co-efficient is given as

$$\frac{d\emptyset}{\emptyset} = -\mu dl$$

$$\emptyset = \emptyset_o e^{-\mu dl}$$

Where $\mu = p\delta$ that is probability of interaction per unit length.

Also number of atoms per unit volume of substances

$$n = \frac{N_A \rho}{M} = \frac{N}{V}$$

6.1 OBJECTIVES

After going through this unit, you will be able to:

- Explain parameters used in quantifying the amount of radiation interacting with matter.
- Explain how the different categories of radiation interact with matter.

6.2 MAIN CONTENT

6.2.1 HEAVY CHARGED PARTICLE INTERACTION

This interaction can be divided into 3 broad groups which are:

a. interaction with individual electron of the atoms which leads to excitation or ionization of atoms and this collision can be;

i. *Inelastic* - sufficient energy received for excitation.

ii. *Elastic* - energy received is less than the smallest energy difference of the atomic level. It may be hard or soft collision. It may also be fast or slow depending on projectile velocity and orbital velocity

b. Interaction with nuclei if incoming particle is heavy compared with electrons.

c. Interaction with the whole coulomb field surrounding an atom. This occurs when the incident particle is of low energy or heavy particle with low velocities.

The outcome of a collision or interaction is determined by:

i. velocity of collision, V

ii. Distance of closest approach of the participants.

iii. The range of the potential which governs the interaction between the incident particle and the target.

For heavy particles like α particles interacts with the coulomb forces between positive and negative charges in matter. It then dissipates its energy in succession to the electron through inelastic collision which then results in either excitation or ionization. The number of ion produced per unit distance is called *specific ionization*.

The linear rate of energy loss or linear stopping power S for heavy charged particle in a given absorber is

$$S = -\frac{dE}{dX}$$

S increase as the particle velocity decreases that is $S \propto \frac{1}{V}$

For a non-relativistic particle,

$$S = \frac{4\pi e^4 Z^2 NB(Z_1V)}{M_o V^2} \left(Beth's\ formula\right)$$

Where M_0 - rest mass of electron

V-velocity of heavy particle

e - Electronic charge

Z - Atomic number of the absorber atom

$B(Z_1V)$ - Beth's formula

$$B(Z_1V) = Z\left[\frac{\ln 2M_o V^2}{I} - \ln\left(1 - \frac{V^2}{C^2} - \frac{V_o^{\ 2}}{C^2}\right)\right]$$

I = average ionization potential of absorption

While for a non-relativistic charged particle

$$B(Z_1V) = Z\left(\frac{\ln MoV^2}{I}\right), S \propto \frac{1}{V^2} \propto \frac{1}{E}$$

6.2.2 BETA RAYS (FAST ELECTRONS)

The energy lost by fast electrons is due to excitation and ionization as well. Majorly, energy are usually lost due to;

a). Scattering of the fast electrons because they are colliding with another electron in target. The energy loss per unit length D

$$-\frac{dE}{dx}\Big/C = collision$$

b). Through radiation process - $-\frac{dE}{dx}\Big/ r = radiation$

Which take place in the form of Brennstraching (e-m radiation). (Usually for electrons with energy greater than rest mass energy).

Therefore, the total linear specific energy loss is;

$$\frac{dE}{dX} = \frac{dE}{dX}\Big/ c + \frac{dE}{dX}\Big/ r$$

$$and\ \frac{\frac{dE}{dX}\Big/ c}{\frac{dE}{dX}\Big/ r} = \frac{E8}{7w}$$

c). Cerenkov radiation which is negligible at lower energy.

Self Assessment Test I

i. What do you understand by the term "specific ionization"?
ii. Write the equation relating the specific ionization and the velocity of heavy particles.
iii. Describe one of the ways by which energy is lost when an electron interact with matter.

6.2.3 PHOTONS

Interaction of E-M radiation (i.e. X and γ rays photons)

Under an energy region of 0.01 - 10MeV, most interactions γ and X-rays can be explained under three different modes

a. Photoelectric effect

b. Compton effect

c. Pair production

a). Photoelectric Effect

This is a kind of interaction whereby an incident photon transfers all its energy to the electron in a target and therefore these electrons are emitted as photoelectrons with a kinetic energy given as below;

$E_e = E_\gamma - E_B$

The incident photon must have an energy greater than or equal to E_B of the electron to the nucleus of the target.

Therefore, vacancies are created at the K-shells and filled by electrons from higher shells which results into X-rays. The cross section of the photoelectric effect is given as;

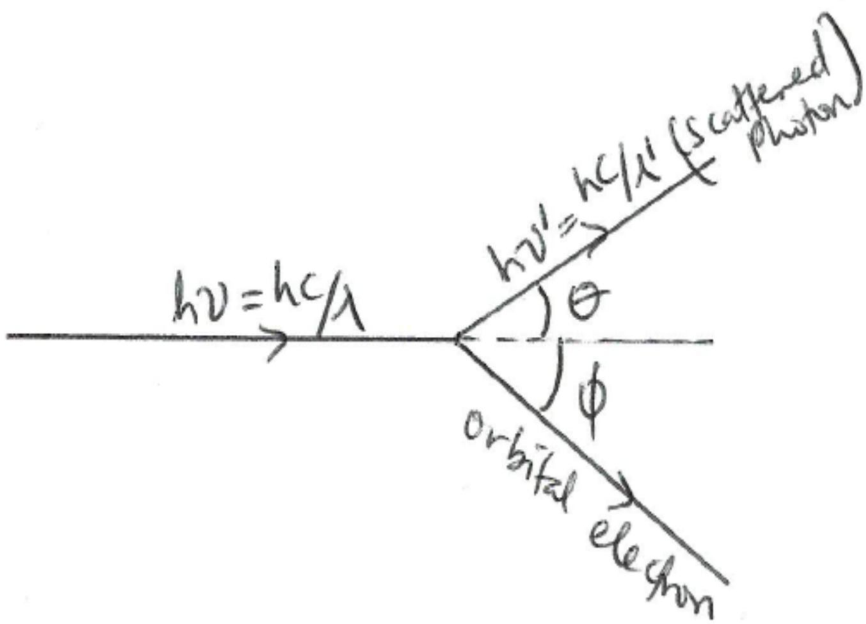
$\sigma_{P.E} = \delta E_{\gamma}^{-7/2} \rho Z^5$

ρ *and* Z^5 are density and atomic number of the absorbing material or target.

δ is constant

b). Compton Effect

This is a situation whereby the incident photon collides with the orbital electron of the target and the incident photon and orbital electrons are scattered at different angles.



Observations

i. Reduction in photon energy from hv → hv^1.

ii. Frequency is changed from v → v^1 (reduced).

iii. The wavelength of photon increases from $\lambda \rightarrow \lambda^1$.

iv. Energy of scattered electron is ($hv - hv^1$).

v. The increase in wavelength is given as

$$\Delta\lambda = \lambda^1 - \lambda = \frac{h}{M_o C}[1 - \cos\theta]$$

Where M_o is the rest mass of the atom of which the electron is used.

vi. The energy of the scattered photon is;

$$E_{\gamma^1} = \frac{E_\gamma}{1+\left[\frac{E_\gamma}{M_o C^2}\right][1-\cos\theta]}$$

vii. The kinetic energy of the photo electron ($E_{K.E.}$) or ejected electron is

$$E_{K.E.} = \frac{\left[\frac{E_\gamma}{M_o C^2}\right][1-\cos\theta]}{1+\left[\frac{E_\gamma}{M_o C^2}\right][1-\cos\theta]}$$

Where M_oC^2 is the rest energy of the electron

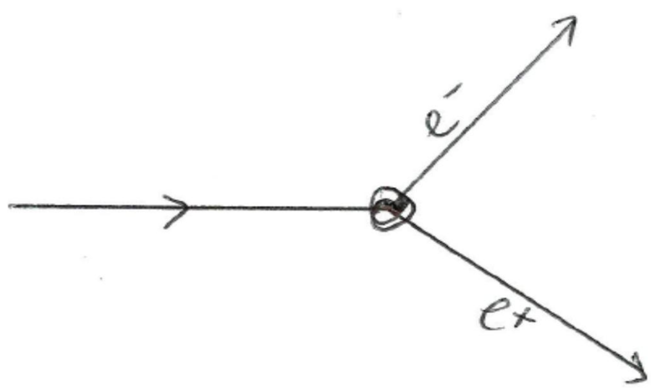
$E_{K.E.}$ is at minimum when θ=0 and $E_{K.E.}$ is maximum when θ=180^{o}and this is called the *Compton edge energy.*

$$E_c = E_\gamma\left[\frac{2E_\gamma}{-M_o C^2 + 2E_\gamma}\right]$$

That is when photon is scattered backwards at θ=180^{0}which explain Compton plateau in γ spectroscopy.

c). Pair Production.

This occurs when γ rays with sufficient energy interacts with the atom of the target in the coulomb field of the nucleus and disappears with electron position pair in place of it.



The energy equation of the process is given as hv = E_{e^-} +E_{e^+} + $2M_oC^2$

Pair production can only take place if greater than or equal to $2M_oC^2$ = 1.02MeV.

The position is an unstable particle once its kinetic energy = 0, it annihilates with the electron to form a photon which either escapes from the medium or interacts with the medium either photoelectrically or photocompton.

$\sigma_{p.p} = CZ^2 \delta ln E_\gamma$

The net effect of the three in γ-ray passing through an absorbing material or the linear cross-section is an exponential attenuation given by

$I = I_O e^{-\sigma x}$

6.2.4 NEUTRONS

The kind of reaction a neutron undergoes depends on its energy. Neutrons are classified according to their energy;

1. High energy neutron greater than 10MeV
2. Thermal neutrons is the same as average kinetic energy of gas molecules = 0.025eV.

All neutrons at the time of their birth are fast but are slowed down (thermalizing) by colliding them elastically with atoms in their environment. After the slowing down, they are now been absorbed by nuclei of the absorbing material and interaction takes place. The interaction of neutron is different from past discussion because, it does not show a variation between the atomic mass and energy. The interactions instead produce;

1. Recurring nuclei
2. Subatomic particles
3. Photons which undergo previous processes.

Generally neutrons may collide with nuclei and undergo;

1. Elastic collisions

Fast neutrons react with low atomic number absorbers. The neutron is scattered with a reduced energy because part of the energy have been transferred to the recurring nuclei

e.g. ^{1}H (n, n) ^{1}H. (Here the nucleus moves).

2. Inelastic collisions

If the neutron is of low energy, the neutron may be momentarily captured by the nucleus and then emitted with diminished energy leaving the nucleus in an excited state and may return to its ground state with the emission of photon e.g. ^{16}O(n, n)^{16}O. (Here the nucleus does not moved but brushed). If another particle is produced after interaction, it is called non-elastic collision e.g. ^{16}O (n,α) ^{13}C. Thermal neutrons are captured by nuclei with reaction cross section as

$$\sigma \propto \frac{1}{v} \propto \frac{1}{\sqrt{E}} \Rightarrow \frac{\sigma}{\sigma_o} = \sqrt{\frac{E_o}{E}}$$

Passage of neutrons through a moderating material

The process of slowing down fast neutrons is known as moderation or thermalization and this is done by making use of moderators (element with low atomic mass) e.g. graphite and heavy water, in such a way that no reaction is lost by absorption but merely have their kinetic energy being reduced by elastic collision with nuclei of the moderator.

Velocity V_1 of a neutron after collision is

$$V_1^2 = V^2\left(1 + A^2 + 2A\cos\phi\right) - - - - - - - (1)$$

and V is the velocity of neutron before collision also

$$V_0 = V(1 + A) - - - - - - (2)$$

And V_0 is the velocity of neutron in real frame.

$E_o = {}^1\!/_2\, mv_o{}^2$ = incident energy of neutron before collision

$E_1 = {}^1\!/_2\, mv_1{}^2$ = energy of neutron after collision.

Therefore the fractional energy e_o is

$$\frac{E_1}{E_o} = \frac{{}^1\!/_2\, mv_1^2}{{}^1\!/_2\, mv_0^2} = \frac{v^2\left(1 + A^2 + 2A\cos\phi\right)}{v^2(1 + A)^2}$$

$$\frac{E_1}{E_0} = \frac{1 + A^2 + 2A\cos\phi}{(1 + A)^2} - - - - - - - (3)$$

Cases

1. Glacing angle i.e. where $\phi = 0$

From $\dfrac{E_1}{E_0} = \dfrac{1 + A^2 + 2A\cos\phi}{(1 + A)^2}$

$$\frac{E_1}{E_o} = 1 - - - - - - - - (4)$$

2. Head-on-collision where $\phi = \pi = 180^0$

$$\frac{E_1}{E_o} = \frac{(A-1)^2}{(A+1)^2} - - - - - - (5)$$ (Neutron energy loss here is maximum)

$$let\,\alpha = \left(\frac{A-1}{A+1}\right)^2$$

$$\frac{E_1}{E_o} = \alpha$$

From $\Delta E_{\max} = (E_0 - E_1)\max$

$$= E_0\left(1 - \frac{E_1}{E_0}\right)\max$$

$$\Delta E_{\max} = E_0(1-\alpha)\max$$

The maximum fractional energy loss can be deformed as

$$\frac{\Delta E_{\max}}{E_0} = 1-\alpha = 1 - \frac{(A-1)^2}{(A+1)^2} - - - - - - - - (6)$$

For a good moderator, $\Delta E_{\max}$ must be large and therefore A must be small. From past discussion we have been dealing with flux of neutron, but now we want to discuss about one which will now be a statistical problem.

Let's assume the neutron is scattered between angle $\phi\, and\, \phi + d\phi$ and the energy (that is E and E + dE) between E and E+dE. It will be observed that the entire range of energy through which the neutron can be scattered is between 1. $E_1 = E_0$(from glancing) and (2) $E_1 = \alpha E_0$ (from head on collision). This implies that $E_o - \alpha E_o = E_o(1-\alpha)$

The probability P(E)dE that a neutron will have an energy E between E_0 and αE_o is 1.

The probability that it will lie between E and E+dE

= P(E) = $\frac{1}{E_o(1-\alpha)}$

$$\int_{\alpha E_o}^{E_o} P(E)dE = \int_{\alpha E_o}^{E_o} \frac{dE}{E_o(1-\alpha)} = 1$$

Therefore, the average energy (E) of a neutron after a series of scattering or the probability that a single collision will make a neutron have energy E is

$$(E) = \frac{\int_{\alpha E_o}^{E} EP(E)dE}{\int_{\alpha E_o}^{E} P(E)dE} = \int_{\alpha E_o}^{E} \frac{EdE}{E_0(1-\alpha)}$$

$$= \frac{1}{E_0(1-\alpha)}\int_{\alpha E_o}^{E_o} EdE$$

$$= \frac{1}{E_0(1-\alpha)}\left[\frac{E^2}{2}\right]_{\alpha E_0}^{E_0}$$

$$= \frac{1}{2E_0(1-\alpha)}\left[E_0^{\,2} - \alpha^2 E_0^{\,2}\right]$$

$$=\frac{1}{2E_0(1-\alpha)}E_0{}^2\left[1-\alpha^2\right]$$

$$=\frac{1}{2(1-\alpha)}E_0\left[(1-\alpha)(1+\alpha)\right]$$

$$= {}^1\!/_2\, E_0(1+\alpha)$$

Average log energy decrement

This is used to obtain the average number of collisions which a fast neutron, will make before its energy E_0 is reduced to thermal energy E_t.

Take $E_t = E$

Log energy decrement is $\log_e E_o - \log_E E = \log_e (E_o/E)$

Therefore average log = $[\log_e (E_o/E)]$

$$\xi = \int_{\alpha E_o}^{E_o} \log\left({}^{E_0}\!/_{E}\right) P(E)dE$$

$$= \int_{\alpha E_0}^{E_0} log\left({}^{E_0}\!/_{E}\right)\frac{dE}{E_0(1-\alpha)} - - - - - - - - (1)$$

Since $\int_{\alpha E_o}^{E_o} \frac{dE}{E_o(1-\alpha)} = 1$

Then put x = ${}^{E}\!/_{E_0}$

For limits to change

For E = E_o; x = 1

And E = αE_o = x = α

From x = ${}^{E}\!/_{E_0}$, $dE = E_0 dx$

And log ${}^{E_0}\!/_{E}$ = -log x.

Then integral (1) turns into

$$=\int_{\alpha}^{1}(-\log x)\frac{E_0 dx}{E_0(1-\alpha)}$$

$$=-\frac{1}{1-\alpha}\int \log xdx$$

$$=1+\frac{\alpha}{1-\alpha}\log\alpha$$

Then substituting for $\alpha = \left(\frac{A-1}{A+1}\right)^2$

$$\xi = 1 + \frac{\frac{(A-1)^2}{(A+1)^2}\log\frac{(A-1)^2}{(A+1)^2}}{1-\frac{(A-1)^2}{(A+1)^2}}$$

$$\xi = 1 - \frac{(A-1)^2}{2A}\log\frac{A-1}{A+1}$$

For A > 1

$$\xi = \frac{2}{A+{}^{2}\!/_{3}}$$

Generally the number of collision required to reduce $E_o + E_t$ is given by

$$n = \frac{1}{\xi} log\, {}^{E_0}/_{E_t}$$

The distance travelled by a fast neutron between its introduction into a slowing down medium and its thermalization is called *fast diffusion length* or *slowing down length* and square of the fast diffusion length is the *Fermi-age.*

Also, the distance travelled by the thermalized diffusion length and it is defined as the thickness of the slowing down medium.

$$n = n_0 e^{-t/_l}$$

n and n_0 are number of neutrons before and after collision and L is thermal diffusion length. But for large absorption cross section

$$I = I_0 e^{-\sigma N t}.$$

Self Assessment Test II

i. Name the electromagnetic radiations (photons) which can interact with matter.
ii. What is photoelectric effect?
iii. List the resulting effect of neutron interacting with matter.

6.3 CONCLUSION

In conclusion, we have been able to examine how the different radiations interact with matter as well as their resulting effect.

6.4 SUMMARY

In this unit, we have been able to understand that radiations which interact with matter can be categorized into four. The effect of each of these radiations were examined and quantified.

6.5 TUTOR MARKED ASSIGNMENT

i. Mention five important applications of the interaction of radiation with matter.
ii. Define the following terms:
   - Cross section
   - Cerenkov radiations
   - Moderation
   - Bremstrauhlung
iii. Briefly explain Compton effect.
iv. Distinguish between Compton effect and pair production.

6.6 REFERENCES

W. Greiner and J. A. Maruhn, Nuclear Models by Springer.

Solutions and Answers

Unit 1

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Unit 4

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Unit 6

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APPENDIX

APPENDIX III

MASSES OF NEUTRAL ATOMS

In the fifth column of the table an asterisk on the mass number indicates a radioactive isotope, the half-life of which is given in the seventh column.

Z	Element	Symbol	Chemical Atomic Weight	A	Mass (u)	$T_{1/2}$
0	(Neutron)	n		1*	1.008665	12 min
1	Hydrogen	H	1.0079	1	1.007825	
	Deuterium	D		2	2.014102	
	Tritium	T		3*	3.016030	12.26 y
2	Helium	He	4.0026	3	3.016030	
				4	4.002603	
				6*	6.018892	0.802 s
3	Lithium	Li	6.939	6	6.015125	
				7	7.016004	
4	Beryllium	Be	9.0122	7*	7.016929	53.4 d
				9	9.012186	
				10*	10.013534	2.7×10^6 y
5	Boron	B	10.811	10	10.012939	
				11	11.009305	
6	Carbon	C	12.01115	12	12.000000	
				13	13.003354	
				14*	14.003242	5730 y
7	Nitrogen	N	14.0067	14	14.003074	
				15	15.000108	
8	Oxygen	O	15.9994	15*	15.003070	122 s
				16	15.994915	
				17	16.999133	
				18	17.999160	
9	Fluorine	F	18.9984	19	18.998405	
10	Neon	Ne	20.183	20	19.992440	
				21	20.993849	
				22	21.991385	
11	Sodium	Na	22.9898	22*	21.994437	2.60 y
				23	22.989771	
12	Magnesium	Mg	24.312	23*	22.994125	12 s
				24	23.985042	
				25	24.986809	
				26	25.982593	
13	Aluminum	Al	26.9815	26*	25.986892	7.4×10^5 y
				27	26.981539	
14	Silicon	Si	28.086	28	27.976929	
				29	28.976496	
				30	29.973763	
				32*	31.974020	~ 700 y
15	Phosphorus	P	30.9738	31	30.973765	
16	Sulfur	S	32.064	32	31.972074	
				33	32.971462	
				34	33.967865	
				36	35.967089	

Z	Element	Symbol	Chemical Atomic Weight	A	Mass (u)	$T_{1/2}$
17	Chlorine	Cl	35.453	35	34.968851	
				36*	35.968309	3×10^5 y
				37	36.965898	
18	Argon	A	39.948	36	35.967544	
				38	37.962728	
				39*	38.964317	270 y
				40	39.962384	
				42*	41.963048	33 y
19	Potassium	K	39.102	39	38.963710	
				40*	39.964000	1.3×10^9 y
				41	40.961832	
[illegible]	Calcium	Ca	40.08	39*	38.970691	0.877 s
				40	39.962589	
				41*	40.962275	7.7×10^4 y
				42	41.958625	
				43	42.958780	
				44	43.955492	
				46	45.953689	
				48	47.952531	
21	Scandium	Sc	44.956	45	44.955920	
				50*	49.951730	1.73 min
22	Titanium	Ti	47.90	44*	43.959572	47 y
				46	45.952632	
				47	46.951768	
				48	47.947950	
				49	48.947870	
				50	49.944786	
23	Vanadium	V	50.942	50*	49.947164	$\sim6\times10^{15}$ y
				51	50.943961	
24	Chromium	Cr	51.996	50	49.946055	
				52	51.940513	
				53	52.940653	
				54	53.938882	
25	Manganese	Mn	54.9380	50*	49.954215	0.29 s
				55	54.938050	
26	Iron	Fe	55.847	54	53.939616	
				55*	54.938299	2.4 y
				56	55.939395	
				57	56.935398	
				58	57.933282	
				[illegible]	59.933964	$\sim10^5$ y
27	Cobalt	Co	58.9332	59	58.933189	
				60*	59.933813	5.24 y
28	Nickel	Ni	58.71	58	57.935342	
				59*	58.934342	8×10^4 y
				60	59.930787	
				61	60.931056	
				62	61.928342	
				63*	62.929664	92 y
				64	61.927958	
29	Copper	Cu	63.54	63	62.929592	
				65	64.927786	
30	Zinc	Zn	65.37	64	63.929145	
				66	65.926052	
				67	66.927145	
				68	67.924857	
				70	69.925334	
31	Gallium	Ga	69.72	69	68.925574	

APPENDIX III

MASSES OF NEUTRAL ATOMS

In the fifth column of the table an asterisk on the mass number indicates a radioactive isotope, the half-life of which is given in the seventh column.

Z	Element	Symbol	Chemical Atomic Weight	A	Mass (u)	$T_{1/2}$
0	(Neutron)	n		1*	1.008665	12 min
1	Hydrogen	H	1.0079	1	1.007825	
	Deuterium	D		2	2.014102	
	Tritium	T		3*	3.016030	12.26 y
2	Helium	He	4.0026	3	3.016030	
				4	4.002603	
				6*	6.018892	0.802 s
3	Lithium	Li	6.939	6	6.015125	
				7	7.016004	
4	Beryllium	Be	9.0122	7*	7.016929	53.4 d
				9	9.012186	
				10*	10.013534	2.7×10^6 y
5	Boron	B	10.811	10	10.012939	
				11	11.009305	
6	Carbon	C	12.01115	12	12.000000	
				13	13.003354	
				14*	14.003242	5730 y
7	Nitrogen	N	14.0067	14	14.003074	
				15	15.000108	
8	Oxygen	O	15.9994	15*	15.003070	122 s
				16	15.994915	
				17	16.999133	
				18	17.999160	
9	Fluorine	F	18.9984	19	18.998405	
10	Neon	Ne	20.183	20	19.992440	
				21	20.993849	
				22	21.991385	
11	Sodium	Na	22.9898	22*	21.994437	2.60 y
				23	22.989771	
12	Magnesium	Mg	24.312	23*	22.99[illegible]125	12 s
				24	23.985042	
				25	24.986809	
				26	25.982593	
13	Aluminum	Al	26.9815	26*	25.986892	7.4×10^5 y
				27	26.981539	
14	Silicon	Si	28.086	28	27.976929	
				29	28.976496	
				30	29.973763	
				32*	31.974020	~ 700 y
15	Phosphorus	P	30.9738	31	30.973765	
16	Sulfur	S	32.064	32	31.972[illegible]74	
				33	32.971462	
				34	33.967865	
				36	35.967089	

Z	Element	Symbol	Chemical Atomic Weight	A	Mass (u)	$T_{1/2}$
17	Chlorine	Cl	35.453	35	34.968851	
				36*	35.968309	3×10^5 y
				37	36.965898	
18	Argon	A	39.948	36	35.967544	
				38	37.962728	
				39*	38.964317	270 y
				40	39.962384	
				42*	41.963048	33 y
19	Potassium	K	39.102	39	38.963710	
				40*	39.964000	1.3×10^9 y
				41	40.961832	
[illegible]	Calcium	Ca	40.08	39*	38.970691	0.877 s
				40	39.962589	
				41*	40.962275	7.7×10^4 y
				42	41.958625	
				43	42.958780	
				44	43.955492	
				46	45.953689	
				48	47.952531	
21	Scandium	Sc	44.956	45	44.955920	
				50*	49.951730	1.73 min
22	Titanium	Ti	47.90	44*	43.959572	47 y
				46	45.952632	
				47	46.951768	
				48	47.947950	
				49	48.947870	
				50	49.944786	
23	Vanadium	V	50.942	50*	49.947164	$\sim6\times10^{15}$ y
				51	50.943961	
24	Chromium	Cr	51.996	50	49.946055	
				52	51.940513	
				53	52.940653	
				54	53.938882	
25	Manganese	Mn	54.9380	50*	49.954215	0.29 s
				55	54.938050	
26	Iron	Fe	55.847	54	53.939616	
				55*	54.938299	2.4 y
				56	55.939395	
				57	56.935398	
				58	57.933282	
					59.933964	$\sim10^5$ y
27	Cobalt	Co	58.9332	59	58.933189	
				60*	59.933813	5.24 y
28	Nickel	Ni	58.71	58	57.935342	
				59*	58.934342	8×10^4 y
				60	59.930787	
				61	60.931056	
				62	61.928342	
				63*	62.929664	92 y
				64	61.927958	
29	Copper	Cu	63.54	63	62.929592	
				65	64.927786	
30	Zinc	Zn	65.37	64	63.929145	
				66	65.926052	
				67	66.927145	
				68	67.924857	
				70	69.925334	
31	Gallium	Ga	69.72	69	68.925574	

Z	Element	Symbol	Chemical Atomic Weight	A	Mass (u)	$T_{1/2}$
				134	133.905815	
				136	135.907221	
55	Cesium	Cs	132.905	133	132.905355	
				134*	133.906823	2.1 y
(55)	(Cesium)			135*	134.905770	2×10^{6} y
				137*	133.906770	30 y
56	Barium	Ba	137.34	130	129.906245	
				132	131.905120	
				133*	132.905879	7.2 y
				134	133.904612	
				135	134.905550	
				136	135.9[illegible]4300	
				137	136.9[illegible]	
				138	137.905000	
57	Lanthanum	La	138.91	137*	136.906040	6×10^{4} y
				138*	137.906910	1.1×10^{11} y
				139	138.906140	
58	Cerium	Ce	140.12	136	135.907100	
				138	137.905830	
				140	139.905392	
				142*	141.909140	5×10^{15} y
59	Praseodymium	Pr	140.907	141	140.907596	
60	Neodymium	Nd	144.24	142	141.907663	
				143	142.909779	
				144*	143.910039	2.1×10^{15} y
				145	144.912538	
				146	145.913086	
				148	147.916869	
				150	149.920960	
61	Promethium	Pm		145*	144.912691	18 y
				146*	145.914632	1600 d
				147*	146.915108	2.6 y
62	Samarium	Sm	150.35	144	143.91[illegible]989	
				146*	145.912992	1.2×10^{8} y
				147*	146.914867	1.08×10^{11} y
				148*	147.914791	1.2×10^{13} y
				149*	148.917180	4×10^{14} y
				150	149.917276	
				151*	150.919919	90 y
				152	151.919756	
				154	153.922282	
63	Europium	Eu	151.96	151	150.919838	
				152*	151.921749	12.4 y
				153	152.921242	
				154*	153.923053	16 y
				155*	154.922930	1.8 y
64	Gadolinium	Gd	157.25	148*	147.918101	85 y
				150*	149.918605	1.8×10^{6} y
				152*	151.919794	1.1×10^{14} y
				154	153.920929	
				155	154.922664	
				156	155.922175	
				157	156.924025	
				158	157.924178	
				160	159.927115	
65	Terbium		158.92[illegible]	159	158.92[illegible]	
66	Dysprosium	Dy	162.50	[illegible]	155.92[illegible]	$2\times10^{[illegible]}$ y

Z	Element	Symbol	Chemical Atomic Weight	A	Mass (u)	$T_{1/2}$
				158	157.924449	
				160	159.925202	
				161	160.926945	
				162	161.926803	
				163	162.928755	
				164	163.929200	
67	Holmium	Ho	164.930	165	164.930421	
				166*	165.932289	1.2×10^{3} y
68	Erbium	Er	167.26	162	161.928740	
				164	163.929287	
				166	165.930907	
				167	166.932060	
				168	167.932383	
				170	169.935560	
69	Thulium	Tm	168.934	169	168.934245	
				171*	170.936530	1.9 y
70	Ytterbium	Yb	173.04	168	167.934160	
				170	169.935020	
				171	170.936430	
				172	171.936360	
				173	172.938060	
				174	173.938740	
				176	175.942680	
71	Lutecium	Lu	174.97	173*	172.938800	1.4 y
				175	174.940640	
				[illegible]	17[illegible]42660	2.2×10^{10} y
72	Hafnium	Hf	178.49	174*	173.940360	2.0×10^{15} y
				176	175.941570	
				177	176.943400	
				178	177.943880	
				179	178.946030	
				180	179.946820	
73	Tantalum	Ta	180.948	180	179.947544	
				181	180.948007	
74	Wolfram	W	183.85	180	179.947000	
	(Tungsten)			182	181.948301	
				183	182.950324	
				184	183.951025	
				186	185.954440	
75	Rhenium	[illegible]	186.2	185	184.953059	
				187*	186.955833	5×10^{10} y
76	Osmium	Os	190.2	184	183.952750	
				186	185.953870	
				187	186.955832	
				188	187.956081	
				189	188.958300	
				190	189.958630	
				192	191.961450	
				194*	193.965229	6.0 y
77	Iridium	Ir	192.2	191	190.960640	
				193	192.963012	
78	Platinum	Pt	195.09	190*	189.959950	7×10^{11} y
				192	191.961150	
				194	193.962725	
				195	194.964813	
				196	195.964967	
				198	197.967895	

Z	Element	Symbol	Chemical Atomic Weight	A	Mass (u)	$T_{1/2}$
79	Gold	Au	196.967	197	196.966541	
80	Mercury	Hg	200.59	196	195.965820	
				198	197.966756	
				199	198.968279	
				200	199.968327	
				201	200.970308	
				202	201.970642	
				204	203.973495	
81	Thallium	Tl	204.19	203	202.972353	
				204*	203.973865	3.75 y
				205	204.974442	
(81)	(Thallium)	Ra E"		206*	205.976104	4.3 min
		Ac C"		207*	206.977450	4.78 min
		Th C"		208*	207.982013	3.1 min
		Ra C"		210*	209.990054	1.3 min
82	Lead	Pb	207.19	202*	201.927997	3×10^5 y
				204*	203.973044	1.4×10^{17} y
				205*	204.974480	3×10^7 y
				206	205.974468	
				207	206.975903	
				208	207.976650	
		Ra D		210*	209.984187	22 y
		Ac B		211*	210.988742	36.1 min
		Th B		212*	211.991905	10.64 h
		Ra B		214*	213.999764	26.8 min
83	Bismuth	Bi	209.980	207*	206.978438	30 y
				208*	207.979731	3.7×10^5 y
				209	208.980394	
		Ra E		210*	209.984121	5.1 d
		Th C		211*	210.987300	2.15 min
				212*	211.991876	60.6 min
		Ra C		214*	213.998686	19.7 min
				215*	215.001830	8 min
84	Polonium	Po		209*	208.982426	103 y
		RaF'		210*	209.982876	138.4 d
		Ac C'		211*	210.986657	0.52 s
		Th C'		212*	211.989629	0.30 μs
		Ra C'		214*	213.995201	164 μs
		Ac A		215*	214.999423	0.0018 s
		Th A		216*	216.001790	0.15 s
		Ra A		218*	218.008930	3.05 min
85	Astatine	At		215*	214.998663	100 μs
				218*	218.008607	1.3 s
				219*	219.011290	0.9 min
86	Radon	Rn				
		An		219*	219.009481	4.0 s
		Tn		220*	220.011401	56 s
		Rn		222*	222.017531	3.823 d
87	Francium	Fr				
		Ac K		223*	223.019736	22 min
88	Radium	Ra	226.0			
		Ac X	232.08	223*	223.018501	11.4 d
		Th X		224*	224.020218	3.64 d
		Ra		226*	226.025360	1620 y
		$MsTh_1$		228*	228.031139	5.7 y
89	Actinium	Ac		227*	227.027753	21.2 y

Z	Element	Symbol	Chemical Atomic Weight	A	Mass (u)	$T_{1/2}$
		RdAc		227*	227.0[illegible]	18.17 d
		RdTh		228*	228.028750	1.91y
				229*	229.031652	7300 y
		Io		230*	230.033087	76000 y
		UY		231*	231.036291	25.6 h
		Th		232*	232.038124	1.39×10^{10} y
		UX_1	231.0359	234*	234.043583	24.1 d
91	Protoac-tinium	Pa	238.03	231*	231.035877	32480 y
		UZ		234*	234.043298	6.66 h
92	Uranium	U		230*	230.033937	20.8 d
(92)	(Uranium)			231*	231.036264	4.3 d
				232*	232.037168	72 y
				233*	233.039522	1.62×10^5 y
				234*	234.040904	2.48×10^5 y
		Ac U		235*	235.043915	7.13×10^8 y
				236*	236.045637	2.39×10^7 y
		UI		238*	238.048608	4.51×10^9 y
93	Neptunium	Np	237.0480	235*	235.044049	410 d
				236*	236.046624	5000 y
				237*	237.048056	2.14×10^6 y
94	Plutonium	Pu	239.0522	236*	236.046071	2.85 y
				238*	238.049511	89 y
				239*	239.052146	24360 y
				240*	240.053882	6700 y
				241*	241.056737	13 y
				242*	242.058725	3.79×10^5 y
				244*	244.064100	7.6×10^7 y