NATIONAL OPEN UNIVERSITY OF NIGERIA

SCHOOL OF SCIENCE AND TECHNOLOGY

COURSE CODE: MTH 204:

COURSE TITLE: BASIC MATHEMATICS FOR NURSES

MTH 204: BASIC MATHEMATICS FOR NURSES

COURSE GUIDE

NATIONAL OPEN UNIVERSITY OF NIGERIA

COURSE UNIT

MTH 204 BASIC MATHEMATICS FOR NURSES

CREDIT UNITS = 2

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Content

Num bers - Natural numbers, integers, rationals, Number bases. Operations with Surds, Ratio, proportion and percentages.

Graphs - the C artesian plane, plotting of a graph from a table of values and graphical treatment.

Geometry: Angles and parallel lines construction of loci, Angles, properties of circles, Mensuration: Perimeter or circumference, Surface area and volume.

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2 Introduction.

MTH 204: Basic Mathematics for Nurses is a two (2) unit Credit Course meant for Nurses who are pursing their B.Nsc degree. It is meant to be a foundational course that will expose them to the aspects of application of mathematics in their
Nursing career.

This course is a three module, fourteen units course, which covers the number systems and number bases, indices, logarithms and Surds, ratios and proportions, angle constructions and graphs and the metric system units of measurements with their applications.

This course is meant to remove the phobia that bewails students at the mere mention of mathematics but will expose them to the applications of mathematics in everyday life.
This course guide in a nutshell will tell you all you have to do to be mathematics literate and appreciate the applications of mathematics in your daily activities as a nurse.

2.0 What you will learn

The overall aim of MTH 204 - Basic Mathematics for Nurses as the name implies is to expose you to the concepts of mathematics which are relevant to your profession. Hence during this course, you will learn the number system and number bases, ratios and proportions, graphs and measurements of all sorts - perimeters, areas and volumes. The graphs will expose you to Apical radial measurements which is the first and foremost duty of a nurse to both in and out patients as the case may be.

2.1 Course Aims

The aim of this course is to give you an understanding on the basic concepts necessary for your application of mathematics in your area of study.

This aim will be achieved by:

- Exposing you to the different kinds of numbers that make up the entire number systems.
- Introducing you to base conversion of numbers for use in this computer age.
- Identifying the aspects of your profession where ratios, proportions, angles and areas are used.
- Demonstrating to you, how to draw graphs especially for taking down temperatures, Blood pressure, pulse beats etc. of patients.

2.2. Course Objectives

To achieve the above aims, the overall objectives of this course will be stated here. But each unit has its own set objectives achievable by the end of the unit.

The objectives are as follows:

- To recognise and identify the different kinds of numbers that make up the number system
- Use the laws of indices in calculation and simplification of numbers.
- Manipulate the surds correctly.
- Determine the ratios and proportions and even percentage of contents of drug components.
- Find ratio, solutions and mixtures in proportions of liquid and powder drugs.
- Plot graphs for temperature, blood pressure and pulse of patients
- Interpret the temperature and blood pressure chats correctly.
- Determine the angle of injection correctly
- Apply the tourniquet correctly to collect blood specimen.

3.0 Working Through This Course

To complete this course successfully, you are expected to read the study units carefully and meaningfully and also any mathematics textbook that is relevant to the units and even materials that you can find helpful. Lots of exercises are provided to enable you study effectively. The key to sound knowledge in mathematics is practice.

So solve as many problems as possible on any unit you have learnt. Do not be discouraged when you fail again. Try, try, and try again.

3.1 Course Materials

The major components here are:
(a) The course guide
(b) The study units
(c) Textbooks
(d) Assignments files
(e) Presentation schedule.

3.2 Study Units

There are fourteen study units in this course.
Unit 1: Number Systems - Natural, Integer and Rational

Unit 2: Number bases
Unit 3: Indices
Unit 4: Logarithms
Unit 5: Surds
Unit 6: Ratios, Proportions and Percentages
Unit 7: Graphs
Unit 8: Angles on Parallel lines and Polygons, Circles
Unit 9: Construction of Angles
Unit 10: Loci
Unit 11: Perimeters and Areas of Plane Shapes
Unit 12: Areas of Circles, Sectors and Segments
Unit 13: Surface Areas and Volumes of Cuboid, Cube and Prisms.
Unit 14: Surface Areas and Volumes of Pyramids, Cylinders and Cones.
Revisional Exercises.

3.3 Textbooks

There are lots of Mathematics textbooks in the market and libraries, feel free to purchase or use whichever that is within your reach.

Some of these ones can also be used:

1. Amazigo, J.C (Ed) (1991). Introductory Mathematics I: Algebra, Trigonometry and Complex Numbers. Onitsha: Africana Fep Publishers Ltd.
2. Backhouse, J.K, and Houldsworth, S.P.T. (1991) Pure Mathematics A first Course. England: Longman
3. Talbert, J.F, Godman A and Ogun, G. (1992), Additional Mathematics. London: Longman.
4. Egbe, E, Odili, G.A and Ugbebor, 0.0. (1999). Further Mathematics. Onitsha: African Fep Publishers Ltd.
5. David - Osuagwu, M; Anemelu, C., Onyeozili,1. (2000) New School Mathematics for Senior Secondary Schools. Onitsha: Africana - Fep Publishers Ltd.
6. Indira Gandhi National Open University (IGNO) Teaching Mathematics Teaching Geometry and Trigonometry 4. ES 342.

3.4 Assignment File

The assignment file contains the exercises you are required to do for this course. Each unit has a tutor-marked assignment. You are expected to complete the exercises and submit to your tutors on a date to be announced for grading. The marks obtained will contribute 50% to your final grade in this course.

3.5 Presentation Schedule

The presentation schedule included in your course materials will enable you note the important dates for contact with your tutors, the days of submission of completed assignments and tutorials.

4.0 Assessment

Your assessment will be based on 40% of the total marks scored in the tutor marked assignment and 60 % of the final examination (written) giving you a total of 100%.
Do not lag behind, endeavour to at least score 80% in this course.

4.1 Tutor - Marked Assignm ents (TMA)

There are fourteen assignments in this course you are expected to perform creditably well in at least 8 out of the 14. These marks will contribute 40% to your final grade in the examination in this course.

4.2 Final Examination and Grading

The final examination for MTH104 - Basic Mathematics will be a 2hr written examination that will cover all aspects of the course. This will contribute to the remaining 50% of the marks for your grade in this course.

4.3 Course Marking Scheme

The table below shows the break down of this course grade marks.

Assessment	Marks
Tutor marked The tutor assignment marks based on your	14 assignments to pass at least 8 to contribute 40%. (1-14) will use his/her discretion to distribute the best performances
Examination - written	60% of overall course marks
Total	**100% of course marks**

4.4 Course Overview

Each of the fourteen units will be studied in one week of three hours intensive study. 1
See study units for the units.

5.0 Sum mary

This course Basic Mathematics for nurses intends to expose you to a greater percentage to the relation of some basic concepts in mathematics to the Nursing profession especially in the areas of mixtures of drugs during injection, knowing which angle to place the needles for injection to avoid abscess, what ratios of each drug component to take at a time, Apical - radial measurements. It will help educate you on how to interpret the temperature pulse and blood pressure charts. This is an enjoyable course if your will disabuse your minds on the phobia beclouding mathematics.

Good luck!

MTH 204: BASIC MATHEMATICS FOR NURSES

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NATIONAL OPEN UNIVERSITY OF NIGERIA

MODULE 1

UNIT 1

NUMBERS

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1.0 Introduction

The first idea of the number system was evolved by the primitive man, when he used the method of matching of objects or his flocks with pebbles or sticks. This was called the one-to-one correspondence. This helped him check his flock; when they go out to pasture and when they come back.

As the years progressed, the need for exchange of goods between tribes and people (trade by barter) became dominant. This introduced its attendant problem of either under-estimating or over-estimating the quantities. In a bid to find a remedy to this problem, the idea of numbers came. The number system passed through several civilizations before the Hindu - Arabic numbered (the numbers at present) became the standard.

2.0 Objectives

By the end of this unit, the students should be able to:

• Distinguish between number, numeral and word name

Write with examples, the properties of the Natural, integer and rational numbers
• Draw a diagrammatic representation of the tree of numbers
Write the relationship between the natural numbers, integers and rational numbers.

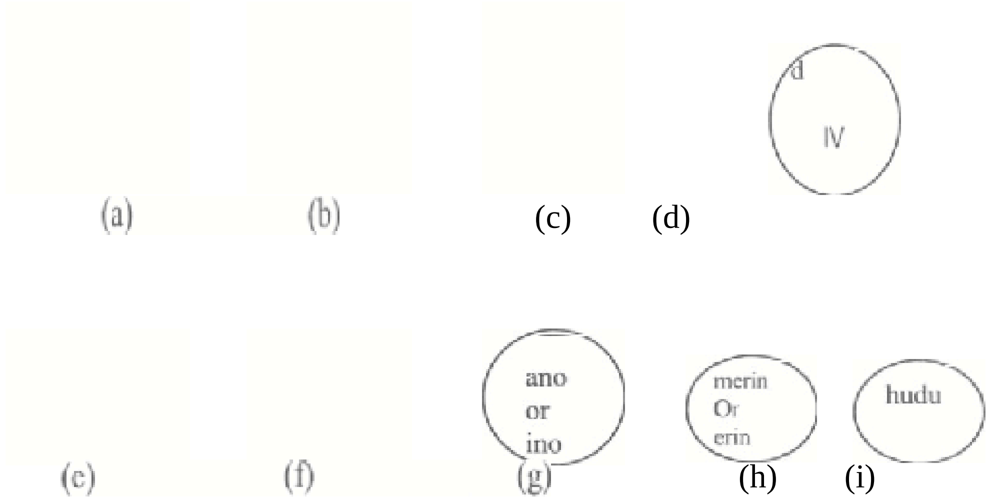
3.0 Main Content

3.1 The Concept of Number

Assuming somebody enters into your house or class and asks you to give him/her four (not in a joking mood), what will be your reaction? The first question that comes to your mind is four of what? This now brings us to the concept of numbers.

Definition: A number is the concept of quantity or the mental concept (not visible but imaginable) of the property of equivalent sets.

The following diagrammatic representations will enable you to understand what a number actually means.



From figure (1), (a), (b), (c) have the property of fourness i.e. each contains four objects or elements. Here the number in each case is if meaning each contains four objects. You cannot get four, what you can have is four pencils, or four bics or four persons or four boxes, etc.

This mental concept of the number is referred to as the cardinal number and it is used to answer the question, "how many"?

(d) and (e) in fig (i) are symbols representing the number four. These symbols are referred to as numerals. And (f), (g), (h) and (i) symbol, so you can see that the numeral and word-name of numbers change from race to race but the numbers does not change. Example the word-name in the number 4 (four) in the three Nigeria major languages (Igbo, Yoruba and Hausa) is as shown in fig (1), diagrams (g), (h) and (i) respectively. Having known what number is, we move to the next topic - Natural numbers.

3.2 Natural Numbers or Counting Numbers

The natural numbers or counting numbers as the name implies show that they are numbers used for counting they are denoted by N.

This was the first conception of numbers by the primitive society, when it began counting of people, animals and their various articles and possessions, through matching of objects with pebbles or sticks. The human hand with five fingers was an invaluable natural tool for counting, also the ten fingers and in some tribes the vegesimal i.e. number based on twenty (these were the first number to be used) For example, the Igbos count in twenties they express 45 as (2 x 20) + 5 pronounced two twenties plus five.

The Yoruba system uses the number 20 which relies heavily as "five from ten from three twenties". In symbolic notations, some of the numbers are written thus

	Igbo version
45 = (3 x 20) – 10 – 5	45 = (2 x 20) + 5
106 = (20 x 6) – 10 - 4	106 = (5 x 20) + 6
300 = 20 x (20 – 5)	300 = (15 x 20)
	525 = (26 x 20) + 5

525 = (200 x 3) – (20 x 4) – 5 Zaslavsky in Sertima (1992:119)

Hence the Natural Numbers are numbers from 1, 2, 3, ... (infinity) spaced at regular intervals of one unit. Properties of Natural Numbers (N)

Addition Property on Natural Numbers: given two numbers x and y

(a) Closure - The sum of any two natural numbers is also a natural number. That is if x and y are two natural numbers, then x + y will equally be a Natural number.

Example 2 and 3 are natural numbers, then 2 + 3 = 5 and 5 is also a natural number. When this happens, it is referred to as closure. Mathematically, it is written

Closure: if x y e (belong to) N (natural number
then .x+ y E N.

(b) Commutative: if x + y = Y + x where x y E N

Example 2 + 3 = 3 + 2 = 5.

(c) Associative: If x y and E N
then (x + y) + = x + (y +)
Example, if 2, 3, and 5 are natural numbers
then (2 + (3) + 5) = 2 + (3 + 5)
5 + 5 = 2 + (8)
10 = 10

(d) Cancellation: If x, y, p E N then
If x +p = y + p, then x = y
Multiplication property on Natural Numbers for all x, y, E N (i.e. x, y, and belong to N

(a) Closure: x x y E N => the product of natural numbers is also a natural number. Example 2 x 3 = 6. 6 is a counting number

(b) Commutative: x x y = y x x,
Example 2 x 3 = 3 x 2 = 6

(c) Associative: X (y x z = y x z, then)x = y,
Example 2 x (3 x 4) = (2 x 3) x 4
= 2 x 12 = 6 x 4
24 = 24

(d) Cancellation: If x x z = y x z, then .x =y.
Distribute laws - the addition and multiplication are subject to this law.
For all x, y, z, E N
D1: x x (y + z) = x x y + x x z
Example (5 + 7) x 2 = 5 x 2 + 7 x 2
12 x 2 = 10 +14

24 = 24

Note these properties are used extensively in dealing with numbers but the natural numbers cannot answer questions such as (2 - 3) i.e. subtract 3 from 2 since it is counting numbers. Minus numbers are not used in counting objects and for this reason, the set of integers was introduced.

3.2.1 Integers Z

The set of natural numbers may or may not be used to solve problems of this nature - given x and y EN i.e. both x and y are Natural numbers.

X + z = y , may or may not have a solution (answer)

For example x .+ z = x has no solution (because there is no counting number, which when added to it other number gives the number). Example 2 +. x = 2 (is not possible in the set of Natural to solve the above equations brought the additional numbers of zero and the negatives of counting numbers to the set of counting numbers to form the set of integers.

Hence the set of integers denoted by Z or I in some text comprise of numbers from - Ve (negative infinity) to zero to positive infinity

. . ., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4.... as 0 ± 1, ± 2, ± 3, . . . so integers are written and this includes all whole numbers from -8 to +8

So in the example 2 + x = 2, the solution of this will give x = 2 - 2 = 0 and 0 is regarded as an integer.

Note: the positive integers are also called Natural numbers.

Properties of integers Z

The properties of integers are the same as those of Natural numbers with the following additions (d) Cancellation law: if x x y = P x y and y is not equal to 0 belong to Z, then x = P for all x , pE Z. Also the following are two properties not found
in the set of Natural numbers N

Addition:

(e) There exists an identity elements o E Z, such that

x + 0 = 0 +. x, = for ever y x E Z i:e

Any number added to zero gives that number

i:e 2 + 0 = 0 + 2 = 2; 10 + 0 = 0 + 10 = 10 etc

(f) For each x E Z, there exists an inverse called the additive inverse

- x E Z,

such that x + (-x) = (-x) = 0 for every x E Z

Note (a) identity element under addition or multiplication is the number which when added multiplied by any number gives that number and the number is zero (0) for addition and one (1) for multiplication

i.e. 5 = 0 = 5 or 1000 = 0 = 1000 and
5 x 1 = 5 or 55 x 1= 55 etc.

(b) Inverse element means the number which when added to another number gives the identity element. Example 5 - (-5) = 0
Here - 5 is the inverse of 5 and
5 is the inverse of -5 because the sum of the two gave zero (0).

Other properties of integers include:

1. Factors or division: An integer x which is not zero i.e. x ± 0 is called a factor or division of another integer y ± 0, if there is a third integer Z such that y = x, Z. This means that when a number divided another number without a remainder the divisor is called a factor. For example at

12 divided by 2 means $\frac{12}{2} = 6$. This means that 12 = 6 x 2, therefore, 2 is a divisor of factor of 12 and 6 is also a factor of 12 etc.

2. Primes and Composite Num bers

(a) Primes: Numbers that do not have any other divisors are called prime numbers. Example 2, 3, 5, 7, etc.
In other words, prime numbers are numbers which have two divisors, itself and 1 (one).

(b) Composites: Numbers that have other divisors besides 1 and itself
That is numbers which can be divided by other numbers, other than themselves and one. Example 4 can divide by l, 2, and 4.
21 can be divided by 1, 3, 7 and 21.

Examples of Composite numbers are 4, 6, 8, 9, 10, 12, 14, etc.

Though the set of integers has both negative and positive numbers, it is still lacking because solutions to problems that involve fractions cannot be done in the set of integers. This then takes us to the next number systems - Rational Numbers.

3.2.2 Rational Num bers

Because of the defects in the set of integers additional numbers i.e. common fractions are added to the set of integers to form the set of rational numbers. Hence, rational numbers denoted by Q are defined as numbers that can be expressed as ration of two integers.

Examples: $1.25 = \frac{125}{100} = \frac{5}{4}$ - $2.3 = \frac{-2.3}{10}$, 5 can be written as $\frac{5}{1}$ i.e. all integers that can be put in fraction form. Now let us go to the properties of rational numbers:

Properties of Rational Numbers

(a) If x and y are two positive rational numbers and x is less than y (x < y) then 1/x is greater than

$$\left(\frac{1}{x} > \frac{1}{y}\right)$$

If 2 and 3 are given then ½ is greater than 1/3 but 2 is less than 3 (i.e. (2 < 3) but ½ > 1/3).

(b) Between any two rational numbers x and y, there exists another rational number z. This means that between any two numbers countless numbers exist. Example between 1/3 and ½, we can have other fractions 1/6, ¼, 1/5 etc.

The list of the number system is not exhaustive but with the aid of the diagrammatic representation, it will be well understood.

Here, we start with the highest number system i.e. the complex number to the least number.

The Tree diagram of the Number System

Complex Numbers = a + bi (where a and b are real numbers but i is imaginary)

Real Numbers Imaginary numbers I = -1

(all numbers except R imaginary numbers)

-4, -7, -64 etc

(e.g. -1, -0.5, p, e, 2.75 etc

e.g. 5/4, - ½ , 5 etc

Rational Numbers Irrational Numbers 1 e.g. 2 , 3, p, e, etc.

Non Integers Z1 Integers Z (0, ±1, ± 2, …)

e.g.

(1/3r d, 5/4, 0.4. 5%)

Fractions

Negative Integers Zero Positive Integers

Improper Fractions decimal form percentage

Natural or counting Numbers N

Prime Numbers Composite

Complex Numbers - This is the highest in the hierarchy of numbers and is made up of two namely Real numbers - numbers that can be represented on a number line and imaginary numbers i i.e. v– 1 the above Number Systems are not discussed in this course.

Irrational numbers are numbers which cannot be expressed as fractions. For example p = 3.141 ... e = 2.71 ..., v2 = 1.44 ..., v3 = 1.71 ... etc. Numbers except the above mentioned are rational numbers.

Exercise 1

1. Distinguish between a number and a numeral
2. With the aid of a diagram, show the relationship between the number systems - Natural Integers and rational numbers and give at least five examples of each.

4.0 Conclusion

In this unit, you have learnt the difference between a number, a numeral and a word name. The different number systems - Natural, Integers and Rational numbers have been discussed. It is believed that these three number systems have a lot to do with your professions as a Nurse. The unit ends with the tree diagram of the entire number system.

5.0 Sum mary

In the number system, it was discovered that different tribes/race have different names for the same number. Hence a number does not change but its symbolic representation (numeral) and pronunciation change (word - name). The natural numbers were also seen to be contained in the Integer in the rational. So they can be represented as:

Rational Numbers

Non-Integers

Integers Z

Negative Integers

Zero

Positive Integers

|

Natural Numbers(N)

6.0 Tutor - Marked Assignm ent.

l. Is the prime Numbers closed under subtraction? (1 mark)

2. State whether each of the following is true or false

a) - 7 E Z

b) 2 E $\sqrt{'1}$

c) 4EZ

d) $\sqrt{-5}$ E 1

e) $\sqrt{3}$ 8 E Z

f) 9/4 $\sqrt{E\ 1}$

g) ½ E Z

h) 11 E R (1 mark each) = 8

7.0 References

Ayres, Frank, JR. (1965). Modern Algebra. Shaun's Outline Series, New York: McGraw Hill Books Company.

Vygodsky, M. (1972). Mathematical Handbook: Elementary Mathematics. Moscow: MR Publishers.

Sertima, Ivan Van (ed) (1992). Blacks in Science: Ancient and Modern. New Brunswick (USA) and London (U.K): Transaction Books.

MODULE 1

UNIT 2

NUMBER BASES

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1.0 Introduction

Unit 1 has thrown a little light in the development of the number systems. The different number bases that were in use by some major tribes in Nigeria. Again, you are of course, perfectly familiar with the base ten number system, since all our money, measuring instrument or scales, and distances are in base ten.

In this unit, the denary scale or base ten has been included in the number base to be discussed as this will lead on to the other systems which have the same type of structure but which use different place values.

2.0 Objectives

By the end of this unit, you should be able to:

- Explain what number base is.
- Express given numbers in their expanded form
- Convert given numbers correctly to a stated base,
- Carr y out operations on given bases.

3.0 Main Content

3.1 Num ber Bases - Concept/m eaning

Any grouping of number (in numeration) system i.e. counting system) is called the number base or often called base.

The international community has accepted grouping in tens as the standard. This grouping in tens is referred to as the decimal system or denary scale or base ten (10).

In base ten, the numbers of numerals is equal to the base. The numerals are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Each numeral is called a digit and each digit has a place value (position of the digit) which is fixed in any given value. The place values are in powers of ten.

Example

(i) In 2435 which is called two thousand, four hundred and thirty-five.

2435 = 2 thousand 4 hundred 3 tens and 5 ones
here 2, 4, 3 and 5 are digits
thousand, hundred, tens and units or ones are place values this means
2000 + 400 +30 +5 = (2x 1000) + (4 x 100) + (3 x 10) + (5 x 1)
In powers of ten, the above can be written as (2 x 103) + (4 x 102) + (3 x 101) + (5 x 10°)

(ii) 468

468 = 4 hundred 6 tens, 8 units/ones
In 468each of 4, 6 and 8 is a digit and the place values are hundreds, tens and units/one
468 = (4 x 100) + (6 x 10) + (8 x 1)
In powers of ten = (4 x 102) + (6 x 101) + (8x 10°)
It is possible to use any number from 2(two) in the number system. Hence, number system with two digits (0 and 1) is called the binary

scale or base 2; that which has five digits 0, 1,2, 3, 4, 5, 6 and 7, the octal or base 8.

Note:

(a) In bases which exceeds the decimal or base ten, letters, are used to represent digits above 9. For example in base 112 the duodecimal scale, the digits are 0, 1,2, 3, 4, 5, 6, 7, 8, 9, 10 and 11 (here 10 and 11 are digits) and can easily be confused with 10 and 11 in base ten which means (9 + 1) and (10 + 1) respectively. To avoid this the following letters are used to denote these digits up to an equivalent denary of 15, so after 9 the letters of the alphabet are used as follows: 10=A, 11 = B, 12 =C, 13 =D, 14 = E and 15 = F.

(b) The binary scale is widely used in all forms of switching applications while base sixteen (hexadecimal) has computer applications.

(c) To avoid confusion between numbers written in other bases and base ten the base of the number system is always written. Example 10 read as one' zero base two meaning one (twos) and in (ones) i.e.

1 x 21 + 0 x 2° (in expanded form) units

or

10eight => one - eights meaning one (eights) and no ones i.e.

1 x 81 + 0 x 8o

121three = one, two, one base three meaning

= 1 (three squared), 2 (threes) and 1 (units)

= 1 x 32 +2 x 31 + 1 x 30

1 D5 sixteen read as one D five (base sixteen) which implies one (sixteen squared), D (sixteen) and five ones. In expanded form it is written as

1 D 5 sixteen = (1 x 162) + (DN61) + (5 x 16°)

Note: numbers written in base ten need not have the base indicated for example, when 2, 15, 126 etc are written, it is understood that they are in base 10.

Now try this:

(a) 57328 (ii) 1764eight (iii) 1011111two

This practice will help you pronounce numbers correctly and also in translating numbers from figures to words and vice versa.
Since the decimal system is the standard or reference system, assuming you encounter a problem of converting numbers from this decimal system to another base, how do you get about it. The answer is provided here, so move on to the next part.

3.2 Conversion from baser ten (denary) to any other base.

Conversion from base ten (denary) to any base say base two, base five, base eight, base twelve etc should be treated here. The same procedure is used for other bases not mentioned here we shall concentrate on bases mentioned above i.e. (2, 5, 8, 12 and 16).

In this conversion, all you need to do is to divide the base ten number repeatedly or successively by the given base noting the remainder at each stage, until a final zero (i.e. last quotient is zero) is obtained.

Example (b) Convert the following denary numbers (base 10) 20 to binary

(base 2) (ü) 64 to quinary (base five), (iii) 173 to octal

(iv) 416 to hexadecimal (base 16)

Solution

Divide 20 successively by 2 writing out the remainder at each stage thus

2 | 120
2 | 10 remainder 0
2 | 5 remainder 0
2 | 2 remainder 1
2 | 1 remainder 0
0 remainder 1

Note: the remainders are often replaced by the plus sign and when 0 is got, you stop and write the answers from the bottom to the top. This is because of place values where the highest digits are written first: Thus (see the direction of the arrow here)

2 | 20
3 | 10 + 0
2 | 5 + 0
2 | 2 + 1
2 | 1 + 0 ↑

0 + 1

20ten = 10100two i.e. the remainders are written in the reverse direction.
Note: take care not to include the last zero as part of the remainder.

(ii) In 64 to base five, the same process is followed thus

5 | 64
5 | 12 + 4
5 | 2 + 2
| 0 + 2

.'. 64ten = 224five

(iii) Covert 173ten to Octal (base 8)

Solution: The successive division is still used

8 173
8 21+5
8 2 +5
0 + 2

.'. 173ten = 255eight

(iv) 416 to hexadecimal (base 16)

Solution: The repeated division is still used

16 | 416
16 | 26 + 0
16 | 1 + 10 (here 10 is a digit)
| 0 + 1

.'. 41610 = 1 (10) 0sixteen but 10 = A in base 16

= I A 0Sixteen

.'. 416 = 1 A 0sixteen

So far, we have changed numbers from denary (basel0) into their equivalent numbers in various bases: binary, quinary, octal and hexadecimal, we now move to the next frame - other bases to denary but before that tr y this exercise.

Exercise 2.1

Convert the following base ten numbers to

(a) Binary (b) quinary (c) Octal (d) hexadecimal
(1) 23 (2) 125 (3) 452.

Have you tried them? That was easy enough. The answers are

(a) binary quinary Octal

23 = 1011 l two, 23 = 43five 23 = 27eight
125 = 1111101 two, 125 = 43five 125 = 175eight
452 =111000100 452 = 3302 452 = 704eight

that of the hexadecimal is

16 | 23
16 | 1 + 7
 | 0 + 1
23 = 17sixteen

16 | 125
16 | 7 + 13 (D)
 | 0 + 7
125 = 7 D sixteen

16 | 125
16 | 28 + 4
16 | 1 + 12 (C)
 | 0 + 1
452 = 1 C 4sixteen

452 = 1 C 4sixteen

3.2.1 Conversion from Other bases to denary

Here there are two ways to do this
(1) By the expanded form and
(2) By the repeated or successive multiplication method.

Now let us take them, one by one:
Example: Convert the following numbers to denary

(a) 1011101two (b) 134five (c) 642eight (d) 1 D 7sixteen

Solution: Using the expanded form method

(a) 1011101 two - write this number in powers of two thus

26 25 24 23 22 21 20
1 0 1 1 1 0 1

= (1 x 26) + (0 +25) + (1 x24) + (1 x 23) + (1 x22) + (0 x 21) + (1 x 20)
= (1 x 64) + (0 x 32) + (1 x 16) + (1 x 8) + (1 x 4) + (0 x 2) + (1 x 1)
= 64 + 0 + 16 + 8 + 4 + 0 + 1

1011101two = 93ten

(b) 134five in expanded powers of five is

(1 x 52) + (3 x 51) + (4 x 50)
= (1 x 25) + (3 x 5) + (4 x 1)
= 25 +15 + 4
= 44

134five, = 44ten,

(c) Weight in powers of eight gives

(6 x 82) + (4 x 81) + (2 x 80)
= (6 x 64) + (4 x 8) + (2 x 1)
= 384 + 32 + 2
= 418

642eight = 418ten

(d) I D 7sixteen in powers of sixteen

= (1 x 162) + (D x 161) + (7 x 160)
= (1 x 256) + (13 x 16) + (7 x 6)
= 256 +208 + 7

= 471 ten

1 D 7sixteen = 471 ten

Note: The expanded form means writing the given numbers in the powers of the given base. In assigning the powers, always start from the units (ones) with the base to power zero therefore the units (ones) will have 20, 50, 80, 160, as in the example and increase the powers accordingly.

Now move to the second method:

(b) Successive multiplication method

The steps involved in this method are as follows:

(1) Space out the digits to be converted
(2) Multiple the first digit by the base of the number to be converted
(3) Add the product got from this multiplication to the second digit.
(4) Multiply their sum by their base
(5) Add the third digit to this new product, repeat steps (4) and (5) until you get to the last digit. This last sum is the answer.

Example: Convert (a) 1011101two (b) 134five, (c) 642eight (d) 1 D7sixteen to denary using the successive multiplication.

Solution:

(a) Applying the steps

1 0 1 1 1 0 1 (spaced out)
x 2 (multiply the first digit 1 by 2)
2 + 0 = 2 (Add to second digit 0 i.e. 2 + 0 = 2)
x 2 (multiply (0 + 2) x 2 = 4)
4+1 (Add 1 to 4 = 5
x 2 (multiply 5 x 2 = 10 etc.
10+1 = 11
x 2
22 + 1 = 23
x 2
46 + 0 = 46
x 2
92 + 1 = 93

Since the last digit 1 has been added, the answer is that last sum
1011101 two = 93ten. The same answers as in example (1 a)

(b) 134five again following the steps, gives

1 3 4
x 5 (multiply 1 x 5 = 5)
5 +3 = 8 (Add3to5i.e.3+5=8)
x 5 (multiply 8 x 5 = 40)
40 + 4 = 44 (add 4 to 40 = 44)

134five = 44ten same answer as (I b)

(c) 642eight still the same processes is repeated

6 4 2
x 8
48 + 4 = 52
x 8
416
+ 2
418

642 eight = 418 ten

(d) 1 D 7 sixteen. First convert the letter D to its original meaning, so Do = 13

1 D 7 Sixteen = 1 13 7
x 16
16 x 13 = 29
x 16

464 + 7 = 471 ten

I D 7 Sixteen = 471ten

Note: the repeated or successive multiplication method appears simpler but care must be taken not to add when you are to multiply and vice versa. It is always better to use the expanded form when dealing with large numbers.
Tr y this little exercise to know if you have been following by the successive method: Covert 2 AB twelve to base ten is that not easy. Now we do it together once more.
2 AB twelve to base 10. First remember that A and B represent 10 and 11 respectively {because they are still digits in base 12 (duodecimal)}

2 AB twelve = 2 10 11
x 12
24 +10 = 34
x 12
408 + 11 = 419

2 AB twelve becomes in powers of 12

= (2 x 122) + (10 x 121) + (11 x 120)
= (2 x 144) + (10 x 12) + (11 x 1)
= 288 + 120 + 11
= 419

2 ABtwe1ve = 419 ten

Now have fun with the following exercises.

Exercise 2.2

Convert the following number to numbers in base ten

(a) 10101two (b) 143 five (c) 717 eight

Ans: (a) 21ten (b) 48ten (c) 463ten

(d) A 39 twelve (e) 5BCsixteen Ans 146 ten

Now move on to the next section.

3.2.2 Conversion from One base to another

From the previous sections, you have learnt conversions from (1) denary to other bases (ii) Other bases to denary. Nothing has been said of conversion between bases.

In this section, we shall discuss this different. It follows the same pattern.

What is done in conversion from one base to another, for example numbers from binary (base 2) to base five (quinary) is first convert the binary number to denary then convert the denary to quinary.

The chain is this
Binary → denary → quinary. That from given number base → denary → desired base.

Example (1) convert 101101 two to quinary (base five)

Solution: Convert 101 l01two - This is easy.
101101 two = 45ten (whichever method you use)
then convert 45ten to quinary

5	45
5	9 + 0
5	1 + 4
	0 + 1

l0110two = 45ten = 140five,

101101 two = 140 five

Example 2: 245Seven to binary

Solution: First convert 245Seven to base ten and then reconvert back to base two

245seven = (2 x 72) + (4 x 71) + (5 x 7°)

= (2 x 49) + (4 x 7)+ (5 x 1)
= 98 + 28 + 5

= 131ten

245seven = 131 ten. Then convert 131ten to binary

2 | 131
2 | 65 + 1
2 | 32 + 1
2 | 16 + 0
2 | 8 + 0
2 | 4 + 0
2 | 2 + 0
2 | 1 + 0
0 + 1

245seven = 131ten = 1000001 1 two

Example 2: Find N if 222four + 203four = N eight

Solution: First add the numbers in base four then convert the sum to base eight

$$\begin{array}{r} 222 \\ +\ 203 \\ \hline 1031\ \text{four} \end{array}$$

So 1031 four = 72ten. Convert 77ten to base eight.
Thus: 77ten to base eight gives

8 | 77
8 | 7 +5
8 | 1 + 1
0+1

N = 115eight .

Now try the following. Exercise 2.3

Convert (1) Find A if 1222four + 2003four = A eight
Answer A eight = 355eight

(2) If 241six = Xnine Find X Answer X = 117nine

3.3 Operations With Number Bases

The basic operations - addition, subtraction, multiplication and division in various bases other than the decimal scale (base ten) will be treated.

3.3.1 Addition and Subtraction

The two operations will be treated here. They are handled the same way as in the denary system but you should always bear in mind the place values of the base you are operation in.

ADDITION

Simplify the following:

1. (a) 100two + 1 ltwo (b) 1110two + 10two
2. (a) 213five + 411five (b) 4133five + 2314five
3. (a) 3 Btwelve + 2 Btwelve (b) 716eight + 145eight

Solution: Put the numbers in tabular form, for easy transfer to the proper place value

1 (a) 1 0 0two (b) 1 1 1 0two
+ 1 l two + 1 0 two
1 1 1two 1 0 0 0 0two.

Note in binary the highest digit is l, so in this operation no sum will exceed l, once get a number greater than 1, immediately convert to base 2 that is why 1 + 1 was written l0two. Always start from the right hand (unit column) and move to the other powers. Now let us see other examples.

2 (a) 213five
+ 411five
1124five. Here 3 + 1 = 4, 1 + 1 = 2 but 2 + 4 = 6 and in base 5, 6 = 11 i.e. 1 bundle of five and 1

the sum of 213five, and 411five i.e. 1124five

(b) 4113five
+2314five. What will be the number under the unit column? Can you do that?

114326five,

What is 3 + 4 in base five? That's right, 3 + 4 = 7 but in base five it 7 = 12five, so write 2 in the unit column and transfer 1 to the next. Now continue. Are you comfortable? Move to the next example.

3 (a) 3 Btwelve + 2 Btwelve. Here the temptation will be to add 3B + 2B as though it was addition of algebra. Remember that 3Btwelve and 2Btwelve in base twelve means (3 11) i.e. both are separate digits. So before carrying out the operation first change these number to base ten, thus

3B = (3 x 121) + (11 x 120)
= 36 + 11 = 47ten
2B = (2 x 121) + (11 x 120)
= 24 +11=35ten

then add the numbers in base 10 and convert back to base twelve, thus

47 + 35 = 82ten and 82ten by successive division gives

12 | 82
12 | 6 +10
(A)
0 + 6

3 Btwelve + 2 Btwelve = 6 Atwelve

Note: When adding numbers above base ten remember to first bring them to base ten, carry out the operation then convert back to the given base as in the above example.

One m ore example

(b) 7 1 6eight
+1 4 5eight
1 0 6 3eight

As a check to know if your answers are correct, convert each of the numbers to base ten, add and convert back to the given base. Do this as practice.

Subtraction:

Simplify the following

1 (a) 1 1 0 ltwo (b) 1 1 1 0two

-1 1 1two   -1 0 1 two

2 (a) 2 2 4 1 five (b) 3 3 1 2 five (c) 1 6 6twelve

-1 0 2 4five   - 2 3 4 3five,   - 2 9twelve

In subtraction, before you carry out the operation check if the digit to be subtracted is greater than the digit above it. Whenever this is the came add the base to the digit above and then subtract the digit below. Having said this, now simplify the above.

Solution:

1 (a) 1 1 0 1 two   In the twos column 0 is above and is less than 1 below it, so take 1 -1 1 I two   from the 22 column and call it 2 and add to 0 i.e. (0 + 2 = 2) then 1 1 0   subtract 1. Do the same for the others until there are no digits to be subtracted.

Again continue the solution of others

1 (b) 1 1 I 0two

- 1 0 1two

101 two

2 (a) 2 2 4 1 five Here 4 is greater   than 1, so 1 is renamed as 1 + 5 = 6 (5 gotten from - 1 0 2 4five, the 1 in the fives column) then 6 - 4 = 2.

1 2 1 2five

Then go ahead and do the others.

(b) 3   3   1 2five,

-2   3   4   3five

4 1   4 five,

(c) 1 6 6twelve   The same process, pick 1 from the twelve column (121) add to 6 in -2 9twelve the unit column (12 + 6 = 18) then subtract 9. What is left in the 12, 1 3 9 twelve   column is 5 then subtract 2 i.e. 5 - 2 = 3 and since 1 has nothing under it bring it down.

166twelve - 29twelve = 139twelve

Now take your time, practice first converting the numbers to base ten before subtraction. Any difference in the answer? Feel free to do it in whichever way you understand best.

3.3.2 Multiplication and Division

The operation are the same as that of working in base ten.

Multiplication:
This follows the same pattern as multiplication of numbers in the denary scale. The only difference is remembering to rename the products according to the base you are working with.

Example: 1 Find the product of 1110two and 101 two

Solution: The long multiplication method i.e. used

26 25 24 23 22 21 20 - Powers of 2
1 1 1 0two After multiplying

x 1 0 1 two remember when adding
the
1 1 1 0 corresponding digits to
treat as
0 0 0 0 in addition.
1 1 1 0

1 0 0 0 1 1 0two

Example 2: Multiply 342five, by 42five,

Solution: again the same method of long division is used. The powers of five are to remind you of place values.

54 53 52 5' 5°

3 4 2 Here 2 x 4 = 8 but 8 = 13five
x 4 2 write 13 under 51 and take 1 to 52

1 2 3 4 etc.
3 0 2 3

3 2 0 1 4five

As a check, covert 342five and 42five, to base ten 342five = 97ten, and 42five, = 22ten, Now multiply 97ten x 22ten = 2134ten then convert 2134ten to base five.

```
5 | 2134
5 | 426 + 4
5 | 85 + 1
5 | 17 + 0
5 | 3 + 2
  |   0 + 3
```

342five x 42five = 32014five, the essence of the check is to internalize learning.

Example 3: Multiply 134eight x 122eight

Solution:

```
  1 3 4eight
  x 1 1 2eight
  ----------
  2 7 0
  1 3 4
  1 3 4
  -----
  1 5 2 3 0eight
```

Check 134eight = 92ten

112eight = 74ten

134 = eight x 112eight = 92ten x 74ten = 6808ten

Now 6808ten to Octal

```
8 | 6808
8 | 851 + 0
8 | 106+3
8 | 13 +2
8 | 1+5
  |   0+1
```

134eight x l l2eight = 15230eight.

Division

In division of other bases the same process as in the denary system, the difference between them is when the first digit of the dividend (the number being divided) is less than that of the divisor (the number dividing), the digit of the dividend is converted to the next place value.

Example: Simplify the following

(a) 1010two ÷ l0two (b) 1034five = 22five

Solution:

101
10 1010two

(a) 10 Using the long division method
10
10

This is easy. Now move to the next example.

(b) 1034five ÷ 22five

22 here the first 2 digits of the dividend
22 1034five to be divided i.e. (10) i less than the
divisor 22, then

44
44
44

Convert 10 to the next place value and divide by 22.
To avoid confusion, it is always advisable to convert both numbers to denary, divide and reconvert back to the given base.

Here is an example
1034five, = 144ten and 22five, = 12ten
144ten ÷ 12ten = 12ten,
and 12ten, to base five gives

5 12
5 2
0 + 2

1034five ÷ 22five = 22five.

You can also check your answers using the knowledge that multiplication is the inverse of division or vice versa - here the quotient is multiplied by the divisor.

So 22five x 22five
= 22
x 22
   44
 44
1034five

Now move to other examples but before then, try the following

Exercise 2.4

Simplify the following

(a) 1101011 two + 1111110two (b) 10111000two - 100011 two
(c) 1522eight ÷ 31eight (d) (4001five - 2304five) + 234five
(e) 121three x 22three.

More examples on bases

1. If 202x = l0100two. Find the value of

Solution: The simpliest way is to express both numbers to denary, thus

(2 x x2)+(0 x. x1)+(2x x °) = (1 x 24)+(0 x 23)+(1 x 22)+(0 x 2')+(0 x 2°)
2x2 + 2 = 20

Now you can solve this equation.

2 + 1 = 10 dividing through by 2
2 = 9
= ± 3. The answers is x = 3 because number bases are not negatives. Check 2023 = (2 x 32) + (0 x 31) + (2 x 3°)
= (2 x 9) + (2 x 1)
= 18 + 2 = 20ten.

2. If 41025six = 21125six + six Find the value of y.

Solution: Solve like in every other equation

41035six - 2112six = six

- 4103
- 2112

1551six

=1551

You can check by the usual methods.

3. If 35y + 62y = 125y and the value of y

Solution: Just express each of the terms to base ten then solve the equation that follows.

53y = 5y + 3 (by know you can do *so-these conversion off by heart.
62y = 6y +2
125y= y2 + 2y + 5
53y + 62y = 125y
(5y +3) + (6y + 2) = y2 + 2y + 5 gives the equation and simplifying gives
5y +3 + 6y +2 = y2 + 2y + 5
lly + 5 = y2 + 2y + 5
Colleting like terms and simplifying further
y2 - 9y = 0
Factorising y (y - 9) = 0 i.e. either y = 0 or y = 9. The base of a number is never zero.
y = 9. You can check
If this answer is correct by substituting back to the given equation.

4.0 Conclusion

In this unit, you have learnt definition of the number base and to convert from one base to another. You have also learnt operations with bases and how to solve some equations involving bases. This unit ends with a tutor - marked assignment. Work through it in your own time and do not hurr y over it.

5.0 Sum mary

In this unit, you have learnt the following number systems and operations in them.

I Number System` Place Values Symbols or digits

(a) Denary or Decimal System or base 10	Powers of 10 is 103 102, 10', 10° etc.	0, 1, 2, 3, 4, 5, 6, 7, 8 and 9
(b) Binary System base 2	Powers of 2 is ... 24. 23. 22, 21,20.	0 and 1 used in snitch applications (on off)
(c) Quinary - base 5	Power of 5,.... 53, 52, 51 , 5°...	0, 1, 2, 3, and 4.
(d) Octal - base 8	Powers of 16 ...162, 161, 16° etc	0, 1. 2, 3, 4, 5, 6 and 7
(e) Hexadecimal - Base 16	Powers of 16 ...162, 161, 16°, etc	0. 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E and F.

Base 16 has computer applications and this is why you have to know its conversions at the tips of your hands.

2. Conversion from any base to deanery (base 10) either (1) Use the expanded form - expressing in powers of the base or. (ii) the successive or repeated multiplication method, following the steps (i) Space out the digits
   (ii) Multiply the highest digit by the base
   (iii) Add the next digit
   (iv) Multiply sum by the base
   (v) Add the next digit, repeat steps (iv) and
   (vi) Until you get to the last digit, this last sum becomes the answer.

3. Conversion from denary to any base
   Use repeated or successive method, that is divide repeatedly by the base, noting the remainders at each stage, continue dividing until a final zero quotient is obtained. Write the remainders in the reverse order i.e. from bottom to top (do not include the final zero).

4. Conversion from one base (other than the deanery) to another base (a lot deanery) first, convert to deanery, then reconvert back to the desired base.

HAPPY R EADING.

6.0 Tutor - Marked Assignm ent 2

1. Express the following as denary numbers

(a) 101101_{two} (b) 21201_{three}

2. Write the following in the hexadecimal form

(a) $2 \times 16^2 + 1 \times 16^1 + 4 \times 16^0$

(b) $A \times 16^1 + 4 \times 16^2 + B \times 16^1 + C \times 16^0$

(c) $3 \times 16^1 + D \times 16^0$

3. If 9 + 13five, = 283ten. Find 9.

4. Solve for if all the numbers are in binary, 11 = 101 + 100

5. Find the number bases .x and y from which the following simultaneous equations hold.

$25_x - 23_y = 6_{10}$ --------- (1)
$34_x + 32_y = 36_{10}$ ---------- (2)

===> (4 marks to each question)
4 x 5 = 20marks

7.0. References and Other Resources

David - Osuagwu, M; Anemelu, C and Onyeozili, I. (2000), New School Mathematics for Senior Secondary Schools, Onitsha: African - Fep Ltd.

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UNIT 3

INDICES

Table of Contents

1.0 Introduction

In the previous unit on Number bases, numbers were expressed in powers in terms of their place values. You are familiar with the denary scale (base 10) and the powers starting from the units (ones) up to at least the fourth power are 10°, 101, 102, 103, 104. In other bases like the binary, the powers are 2°, 21, 22, 23, 24 and in base 8 (Octal), the powers are 8°, 81, 82, 83, 84. Now have you ever asked the questions about the numbers at the top of these bases? What do they mean? The answer to this question will be found in this unit indices.

In this unit you will learn what indices are, the law of indices and its applications.

2.0 Objective

By the end of this unit, you should be able to:

- Explain what indices are
- State and explain with examples the laws of indices
- Give meaning to a0, a-1 and am/n
- Apply the law of indices to solve problems. Correctly.

3.0 Main Contents

3.1 Indices, Exponents or Powers

In calculations especially in the multiplication of numbers, certain factors are repeated. In such cases, where repeated factors of the same base occur, for example 2 x 2 x 2 x 2 x 2 x 2, it is written as 26. This simply means that 2 has multiplied itself 6 times. Also 8 x 8 x 8 x 8 can be written as 84, showing that 8 has multiplied itself four times. If you look back to our familiar base 10, 103 means 10 x 10 x 10 = 1000.

Hence the number of times, a number .X multiplies itself referred to as its index or exponent or power. That is the index or exponent or power tells how many times the number of a number is taken as a factor (repeats itself). Thus indices the plural form of index is the shorthand way of expressing numbers in their powers.

Thus in the examples above, in 26, 6 is the index, in 84, 4 is the index while in 103, 3 is the index.

So: In 26, 2 is the base and 6 is the index or exponent.
84, 8 is the base and 4 is the index or power.
103, 10 is the base and 3 is the index or power.

In general, the product of n factors, of a number , where x is a positive number is written as n, where x is the base and n is the index or power or exponent.
Again in 2 + 2 + 2 + 2 + 2 +2 is written as 6 x 2 = 3 x 6, this shows how many times 2 is added to itself. Here 6 is called the coefficient of 2. a general, *x + x + x + x … + x (n times) is written as n x i.e. adding x up to n number of* times. N is called the coefficient of x The diagram below illustrate this

Index or power or exponent

Coefficient base

This diagram can be taken to mean x7 + x7 + x7 + X7 + X7 + X7 + X7 + X7 where each of the x7 = x x x x x x x x x x x x x Now back to indices.
Express the following numbers in index notations
(a) 4 (b) 27 (c) 625

Here expressing numbers in index notations means finding out the factors of such a given number which multiplies itself a number of times to get the given number.

(a) 4 = 2 x 2 and in index notation 4 is written as 22

4 = 2 x 2 = 22

(b) 27 = 3 x 3 x 3 and in index notation is written as 33

27 = 33

(c) 625 = 25 x 25 = 252 but note also that 25 = 52,

625 = 252 = 52 x 52 = 5 x 5 x 5 x5 = 54 that is 25 in index notation is 54.

Tr y these (a) 64 (b) 243 (c) 512.

Now having known what indices are and how to express numbers in index notations. It is time to move on to the laws which govern indices.

3.2 Laws of Indices

The following are laws of Indices. There are omitted but enough examples shall be given for clarity.
The Laws: If m and n are positive numbers and a is not equal to zero (a 0), the following laws hold.

1 . am x an = am + n

Examples (i) 35 x 34 = (3 x 3 x 3 x 3 x 3) x (3 x 3 x 3 x 3) (from defining of indices

= 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3

= 39

35 x 34 = 39. Observe that 5 + 4 = 9, which means that 35 x 34 = 39 = 35 + 4

Example (ii) 27 x 26 = (2 x 2 x 2 x 2 x 2 x 2 x 2) x (2 x 2 x 2 x 2 x 2 x2)

= 2 x 2 x 2 x 2 x 2 x2 x 2 x 2 x 2 x 2 x 2 x 2 x 2

= 213

27 x 26 = 213 = 27 + 6, again 7 + 6 = 13, therefore 27 x 26 = 213 = 27+6

Hence from these examples the multiplication of numbers on the same bases with indices is the same as adding the indices of the numbers.

2. am an = am - n (provided m > n)

Examples (1) 37 35 this means 3 $\frac{7}{35}$

$$\frac{3^7}{3^5} = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3}$$

and by simplification gives $\frac{3^7}{3^5} = 3^2$. Here you known that 7 - 5 = 2.

So $3^7 \div 3^5$ can be written as $= 3^2 = 3^{7-5}$

Example (ii) $5^6 \div 5^2 = \frac{5^6}{5^2} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5} = 5^4$ which is the same as 5^{6-2}.

$5^6 \div 5^2 = \frac{5^6}{5^2} = 5^4 = 5^{6-2}$

Note again dividing numbers on the same base means subtracting the index of the divisor from the index of the dividend. In the two examples, the divisors are the denominators 3^5 and 5^2 respectively while the numerators are 3^7 and 5^6 respectively.

3. $(a^m)^n = a^{mn}$

Example

(i) $(3^2)^5 = 3^2 \times 3^2 \times 3^2 \times 3^2 \times 3^2$ (definition of indices)

$= 3^{2+2+2+2+2}$ (law 1)

$= 3^{10}$

$(3^2)^5 = 3^{10} = 3^{2 \times 5}$

Example

(ii) $(7^4)^3 = 7^4 \times 7^4 \times 7^4$ (definition of indices)

$= 7^{4+4+4}$ (law 1)

$(7^4)^3 = 7^{12} = 7^{4 \times 3}$

Hence when a number has been expressed in index form and again is raised to another power, the powers are multiplied.

These three laws above are called the basic laws of indices which comes from the definition of indices. These laws lead to a good number of important results that shall be discussed soon.

Fractional, Zero and Negative Indices

4. In the basic laws, there is a condition that must be greater than n, this is to avoid answers involving fractions or zero or negative indices. From these basic laws then, meanings will now be given to then starting with the fractional indices will be considered.

The meaning of $a^{1/m}$

What can you say about $5^{½}$ since you have known that 5^2 means 5 x 5.
Now by applying the basic laws

$5^{½} + 5^{½} = 5^{½ + ½}$ (law 1)
$= 5^1$. This means that
$(5^{½})^2 = 5^1$. (law 3)

$5^{½} = \sqrt{5}$ where $5^{½}$ is called the square root of 5.

Again

$5^{1/3} \times 5^{1/3} \times 5^{1/3} = 5^{1/3 + 1/3 + 1/3}$ (law 1)
$= 5^1$

That is, $(5^{1/3})^3 = 5^1$ (law 3).

$5^{1/3} = \sqrt[3]{5}$. Here $5^{1/3}$ is called the cube root of 5.

So $a^{1/m} \times a^{1/m} \times \ldots a^{1/m} = a$ and $(a^{1/m})^m = a$

$a^{1/m} = \sqrt[n]{a}$ called the mth root or a

Example (i) $8^{1/3} = \sqrt[3]{8}$

$= \sqrt[3]{2^3} = 2$. This means that 2 x 2 x2 equals 8.

Example (11) $4^{½} = \sqrt{4}$

$= \sqrt[2]{2^2} = 2$. This also means that 2 x 2 = 4. Note in square roots the 2 is omitted in the root.

This fractional index $a^{1/m}$ can be extended to fractional indices of the form $a^{m/n}$.

Example: find $8^{2/3}$. From the above example $8^{1/3} = \sqrt[3]{8} = 2$.
This then means that $8^{2/3} = 8^{1/3} \times 8^{1/3}$ (law 1)
$= (8^{1/3})^2$ (law 3)
which is equal to the square of the cube root of 8.

i.e. $8^{2/3}$ can be written as $(8^{1/3})^2$ and also as $\left(\sqrt[3]{8}\right)^2$

hence $a^{m/n}$ can be written as $\left(\sqrt[n]{a}\right)^m$ or as $\sqrt[n]{a^m}$

Therefore, whenever the index is in fractional form to simplify the number, the numerator of the index is the power while the

denominator is the root Example: Find the value of (i) $8^{4/3}$ (ii) $81^{3/4}$

Solution

$= 8^{4/3} = \sqrt[3]{8^4}$ (here 4 is the numerator of the index 4/3 and 3 is the

$= \sqrt[3]{(2^3)^4}$ (law 3)

$= 2^4$

Alternatively $8^{4/3} = (2^3)^{4/3}$ and by law (3) $2^{3 \times 4/3} = 2^4$

(ii) $81^{3/4}$ can be expressed as $(3^4)^{3/4}$

$81^{3/4} = (3^4)^{3/4}$

$= 3^{4 \times 3/4}$

$= 3^3$

Alternatively $81^{3/4} = \sqrt[4]{81^3}$

$= \sqrt[4]{(3^4)^3}$

$= \sqrt[4]{3^{12}} = 3^{12/4} = 3^3$

The same answers were obtained in both examples so in problems like these always use whichever method that appeals to you.

5. What does $a^0 = 1$ means? This is the very questions you would have asked in the number bases. In the expansion method, the unit (ones) column were interpreted as say 10°, 20, 80 etc and given the value 1, why is it so?

Recall that $8^1 \div 8^1 = \frac{8^1}{8^1} = 8^{1-1}$

$= 8^0$ (by law 2)

but $8^1 \div 8^1 = 1$ (when a number divide itself the answer is 1)

$\therefore 8^1 \div 8^1 = 8^{1-1} = 8^0 = 1$

In the same way $a^1 \div a^1 = a^0 = 1$

Alternatively, this can be explained using law (1)

$a^0 \times a = a^{0+1}$ (by law 1)

$= a^1$

dividing both sides by a gives

$$\frac{a^0 \times a}{a} = \frac{a^{0+1}}{a}$$

$a^0 = \frac{a^1}{a}$ (but a^1 = a and a number dividing itself gives 1)

$a^0 = 1$

So for all numbers a except 0 (i.e. a $\neq$ 0) because $0^0 = \frac{0}{0}$ is meaningless, $a^0 = 1$. Examples $10^0 = 1$, $5^0 = 1$, $15^0 = 1$, $8^0 = 1$, $2^0 = 1$ etc. And from this the units (ones) get the value of 1 in the expanded form of the powers of the base. Now to the next result.

6 a^{-m}. To find what meaning that should be given to this number a^{-m} we do the following. To find what meaning to be given to 5^{-1}, divide 5^0 by 5^1, using the second law

$$5^0 \div 5^1 = \frac{5^0}{5^1} = 5^{0-1}$$

$$= 5^{-1}$$

but $5^0 = 1$, therefore

$$5^0 \div 5^1 = \frac{5^0}{5^1} = \frac{1}{5^1}$$

$$= 5^{-1}$$

$\frac{1}{5^1} = 5^{-1}$ so going back to a^{-m} and applying the same rule

$$a^{-m} \times a^m = a^{-m+m}$$

$$= a^0 = 1$$

dividing by a^m

$$\frac{a^{-m} \times a^m}{a^m} = \frac{1}{a^m}$$

$a^{-m} = \frac{1}{a^m}$ and here a^{-m} is called the reciprocal of a^m

Following from this definition

(a) $a^m \times a^{-n} = a^m \times \frac{1}{a^n}$ (both m and n are integers)

$= a^m \times a^n$

$= a^{m+n}$ (law 1)

(b) If $a^m \div a^{-n}$ then $a^m \div a^{-n}$ by law (2) gives

$a^{m-(-n)} = a^{m+n}$

(c) $(a^m)^{-n} = a^{-mn}$ (laws 3)

$= \frac{1}{a^{mn}}$

Examples (i) $5^{-4} = 5^0 \div 5^4 = \frac{1}{5^4}$. Alternatively

$5^0 \div 5^{-4} = 5^0 \div \frac{1}{5^4} = 5^4$

5^{-4} is the reciprocal of 5^4

(ii) $2^{-1} = \frac{1}{2}$

(iii) $7^{-6} = \frac{1}{7^6}$

(iv) $5^3 \div 5^{-2} = 5^3 \div \frac{1}{5^2}$

$= 5^3 \times 5^2$ by law (1)

$= 5^{3+2} = 5^5$ by law (1)

by law (2) $5^3 \div 5^{-2}$ can be written as $5^{3-(-2)} = 5^{3+2} = 5^5$ This brings us the relationship between the laws.

The rule $a^m \div a^n = a^{m-n}$ can now be applied not only when $m > n$ but also when $m < n$.

7. $(ab)^m = a^m \times b^m$. This results from a combination of laws (3) and (1)

Example (i) 6^5 bylaw (1) is $6 \times 6 \times 6 \times 6 \times 6 = 7776$ and 6 has factors 2 and 3

$6^5 = (2 \times 3)^5$

$= 2^5 \times 3^5$

$= 32 \times 243$

$7776 = 7776$

$6^5 = (2 \times 3)^5 = 2^5 \times 3^5 = 7776$

Example (2) $(5 \times 2)^3 = 5^3 \times 2^3$

$10^3 = 125 \times 8$

$1000 = 1000$

Having gone through the laws of indices, you will now learn their applications.

3.3 Application of the Laws of Indices

In this section, several examples will be worked on the application of the laws of indices.

1. Simplify the following

(a) 4 2y3 x 2 3a y7b

(b) a3/2 c7/2 x a5/2 c-9/2

(c) (5 y - 4)- 3

Solution:

(a) 4 2 y3 x 2 3a y7b

In this type of problem, multiply their Therefore it becomes

(4 x 2)(2+3a y3+7b)

8 2 + 3a y3+7b

(b) a3/2 c7/2 x a5/2 c-9/2

Here treat this type by applying law (1), i.e. adding their indices

a3/2 c7/2 x a5/2 c-9/2 = a3/2 + 5/2 c7/2-9/2

= a8/2 c-2/2

= a4 c-1

= a 4 = a 4

C1 c

(c) (5 y -4)-3 . Here apply law (3), multiply the indices the index outside this (5 y-4)-3 = 5 x (-3) y- 4 (-3)

= -15 y12

(d) 7 3y ÷ 4 2

Apply law (2)

7 3y = 7 3-2 y.

4 2 4 Here the subtraction after the index of and y because the divisor is in .

7 3y = 7xy or 7 xy

4 2 4 4

2. Evaluate [2° + 4– ½]2

Solution: You might be wondering, how to tackle this problem but remembering BODMAS, life becomes easy.

Now first simplify the terms inside the bracket thus:

20 = 1 and 4- ½ = 1

4½

= 4

√4

Simplifying the terms inside the bracket and squaring gives

[1 + ½]2 = [3/2] 2 = 9/4 so easy. Try and identify the laws applied here.

3. If 8 /2 = 23/8 x 43/4 find

Solution: Since the terms are on different bases first bring them to the same base -2 in this case.

8 /2 = 23/8 x 23/2

= 23/8 + 3/2 (Law 1)

23x/2 = 215/8

Equate their index, since the bases are equal

3x = 15

2 8

= 15 x 2

3 x 8 or cross multiply 3 x 8 = 15 x 2

24 = 30 and simplifying

= 30 = 5

24 4

4. Simplify 27½ x 3 ½ 9

Solution: This is similar to question 93), so treat the same way thus:

Change to the same base

2 ½ = (33) ½ = 33/2

9 = 32

27 ½ x 3 3/2 = 3 3/2 x 3 3/2

9 32

= 33/2+ 3/2 -2

= 33-2
= 31 = 3

5. If 3 x 91+ x find .

Solution: Convert to the same base

3 x 32(1+x) = 33(-x) (applying law (1)

= 32 + 2x+ 1 = 3-3 x and simplifying

32x + 1 = 3-3 x

2x + 1 = -3x (equating indices)

5x = -3 (solving for x)

x = - 3/5

Not tr y these exercises:

Exercise 3.1

1. Simplify 4 -½ x 24 ÷ 15 -2 25 2

2. Simplify 6 2n+1 x 9 n x 4 2n
18n x 2n x 122n

3. If 4x = 2 ½ x 8. find x

4. If 92x + 1 = 81 x-2 find x
3x

5. Simplify and express with positive indices

(a) x -2 y 3 Z -4 *x* *9*
6 x 3 y -3 Z4

(b) 3 4 abc
4 a3b-3c

Solutions to Exercise 3.1 to check your progress.

1. 1 x 44 ÷ 1 = ½ x 24 x 15 2
4 ½ 15 2 5 2
25 2

= 5 x 24 x 225
2 4

$$= \frac{5 \times 16 \times 225}{2 \times 4} = 2250$$

2. $\frac{(2 \times 3)^{2n+1} \times (3^2)^n \times (2^2)^{2n}}{(3^2 \times 2^n)^n \times (3 \times 2^2)^{2n}} = \frac{2^{2n+1} \times 3^{2n} \times 2^{4n}}{3^{2n} \times 2^n \times 3^{2n} \times 2^{4n}}$

(32 x 2n)n x (3 x 22)2n 32n x 2n x 32n x 24n

$= \frac{2^{6n+1} \times 3^{4n+1}}{3^{4n} \times 6^{6n}}$

34n x 66n

$= \frac{2^{6n} \times 2 \times 3^{4n} \times 3 \times 6}{3^{6n} \times 2^{6n}} = 6$

36n x 26n

3. $2^{2x} = 2 \times \sqrt{2^3}$

$2^{2x} = 2^{1/2} \times 2^3 = 2^{3+1/2}$

$2x = 3\tfrac{1}{2} = 7/2$

$x = 7/4$

4. $3^{4x+2} = \frac{3^{4x-8}}{3^3}$

$3^{4x+2} = 3^{4x-8} = 3^{3x-8}$

$4x + 2 = 3x - 8$

$x = \underline{-10}$

5(a). $\frac{y^3}{6 \times 2Z^4} \times \frac{x^3}{x\ 3Z^4}\ 9y = \frac{9y^6}{6 \times 5Z^8} = \frac{3y^2}{2x\ 5Z^8}$

5(b) $\frac{(abc^4)^{1/3}}{(a^3b^{-3}C)^{1/4}} = \frac{a^{1/3}b^{1/3}C^{4/3}}{a^{3/4}b^{-3/4}C^{1/4}}$

= a1/3 – ¾ b1/3 + ¾ C-4/3 – ¼

= a4-9/12 b4+9/12 C-16-3/12 = a-5/12 b13/12 C-18/12

$= \frac{b^{13/12}}{a^{5/12}C^{19/12}} = \sqrt[12]{\frac{b^{13}}{a^5C^{19}}}$

Do not stop only at these exercise pick up any mathematics textbook that deals with indices and solve as many problems as possible.

4.0 Conclusion

In this unit, you have learnt that an index is that number indicating the number of times a number repeats itself. And that there are three basic laws from which other rules derive their meanings. Then you will be happy to tackle problems relating to indices or powers in given problems.

5.0 Sum mary:

In this unit, you have learnt the following basic laws of indices and the other deductions made there from.

1. $a^5 = a \times a \times a \times a \times a$
2. $a^m \times a^n = a^{m+n}$
3. $a^m \div a^n = a^{m-n}$
4. $(a^m)^n = a^{mn}$
5. $a^0 = 1$
6. $a^{-1} = 1/a$
7. $a^{m/n} = \sqrt[n]{a^m}$

You have also learnt from these laws that (i) to multiply two indicial problems on the same base, simply add their individual index (indices) (ii) to divide two indicial problems subtract the index of the divisor from the dividend.

Tr y and solve as many problems as possible on this topic from any mathematics textbook you can lay your hands on. Some of these are listed in sections 7.0.

6.0 Tutor – Marked Assignm ent 3

1. Simplify without tables or calculators

(a) $25x\sqrt{\frac{4}{16}} \quad \left(\frac{x}{-}\right)^{-3} x \; 2$

(b) (36)-3/2

(c) (5¼) ½

2. Simplify $\frac{4 \times 2^{n+1} - 2^{n+2}}{2n+1 - 2n}$

3. Simplify 10 n/3 x 15 n/2 x 6 n/6 ÷ 45 n/3.

4. Solve for n: $\frac{2^{n+1} \quad \pi\, 2}{3(22n+1) - 4\,(22n-1)} = 1$

7.0. References and Other Resources

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UNIT 4

LOGARITHMS

Table of Contents

1.0 Introduction

Human beings encounter the basic operations - addition, subtraction, multiplication and division in their everyday life. At times these operations involve multi digit numbers which, when it concerns multiplication and division are time-consuming than those of addition and subtraction.

The constant need for these operations in connection with the development of sea navigation which gave rise to astronomical observations and calculations like the distance of the sum from the earth or from a particular planet or the observations of the chemists of the size of an atom of hydrogen led to the computation of numbers by means of logarithms at the turn of the 17th century by astronomers.

In this unit, you will learn about logarithms and the laws which govern them. You will also learn how to use the logarithms tables effectively.

2.0 Objectives

By the end of this unit, you should be able to

- Define logarithms
- Explain in clear terms the relationship between indices and logarithms
- State and prove the laws of logarithms
- Apply the laws of logarithms to solve problems accurately.

3.0 Main Content

3.1 Logarithm s

Though the introduction of calculators has greatly reduced the use of logarithm tables to facilitate lengthy calculations. It is still necessary to have a sound knowledge of the theory and uses of logarithms because of its application in the physical sciences.

Having just studied indices, now is the appropriate time to introduce logarithms because of logarithms is an index. You will recall that in index form we can write 64 = 26 where 2 is the base and 6 is the exponent or index. This index 6 can also be defined as the logarithm of the number 64 to base 2, abbreviated as 1og2 64 = 6. Similar ly, 34 = 81 can be expressed as

Log3 81 = 4

Exercise 4.1

Now what are the bases and logarithms of the following:

(a) 43 = 64 (b) 1 = 20 (c) 9 = 32. This is meant to be an oral exercise.

In general, if a number N = ax, then x is the logarithm of N to base a that log N = x. Note the base number of any logarithm is always written in a small letter and also at the tail end of the letter g in log (short form of logarithm).

Exercise 4.2

Now express in logarithmic notation (oral)

(a) 27 = 33 (b) 64 = 163/2 (c) a3 = b and also express in index notation the following (oral)

(a) log2 32 = 5 (b) log39 = 2 (c) 2 = log525

Now move on the properties of logarithms. Having seen the two sides; index notation and logarithmic notation. It is time now to give a working definition of logarithm and this will enable you score yourself on the oral exercises above.

3.2 Definition of logarithm and properties

(i) The logarithm of a number n to a given base x where x is the positive number not equal to 1 is the index (or power or exponent) to which x must be raised to get N.

(ii) The logarithm of a number N to a base x is the exponent y indicating the power to which x must be raised to obtain N.
In notation form log x N = y and symbolically.

Log x N = y is equivalent to x y = N

Examples
(i) Log2 64 = 6 since 26 = 64
(ii) Log16 64 = 3/2 since 163/2 = 64
(iii) Log16 (½) = -1/4 since 16– ¼ = ½

Having gone through the above examples evaluate the following:

(a) 1og464 (b) log82 (c) log2/34/9 (d) 1og273 (e) 1og 41 (f) log a a2.

That's easy enough. The answers are based on the definition of logarithm and writing in index notation.
Now from the definition of a logarithm follows the identity that logx N = N

3.2.1 Properties or Rules of Logarithms

You have discovered that a logarithm is an index. Therefore, the rules of indices and logarithms are closely related. Thus, the laws of indices here will be written using logarithms.

Let a > 0 and a 0 or 1 (meaning that a is a positive number not equal to zero or one) and M > 0, N > 0 (both M and N are positive numbers) the following are the rules.

1. Log (MN) = Log a M + LogaN
Proof: Let LogaM = x => M = a x (definition)
Log a N = y => N = ay (definition)

The product of MN = ax x ay = ax +y (law of indices) in logarithmic notation

Log a MN = x + y = Log a M + LogaN and substituting for x and y

Log a MN = x + y = Log a M + Log a N

The rule simply states that the logarithm of a product is equal to the sum of the logarithms of the factors

Example: Given that Log102 = 0.3010, Log103 = 0.4771 and Log107 = 0.8451

(1) Evaluate: (a) Log10 6 (b) Log1042 without tables or calculators

Solution: Log10 6 = Log10 (2 x 3) because 2 and 3 are the factors of 6.

Log10 6 = Log (2 x 3) = Log10 2 + Log10 3

Substituting the values of Log102 and Log103 in gives

Log106 = Log10 2 + Log10 3
= 0.3010 + 0.4771
= 0.7781. (Check with your calculators now)

Log106 = Log102 + Log103 = 0.7781

Note here that Log102 + Log103 is not equal to Logio5 because 2 x 3 not equal to 5. Get this clearly.

Now to the second example.

(b) Log1042 = Log (6 x 7)
= Logl0 (2 x 3 x 7). This has to be used because the values given are in 2, 3 and 7

Log10 42 = Log10 (2 x 3 x 7)
= Log10 2 + Log103 + Log107 again, substituting the values of Log102, Log103 and Log107 in above gives

Log1042 = 0.3010 + 0.4771 +0.8451
= 1.6232

Note that the logarithm of a sum is not equal to the sum of the logarithm i.e. Log a M + Log a N Log a (M + N)

2. Log (M ÷ N) = Log a $\left(\frac{M}{N}\right)$ = Log a M – Log a N

Proof Let LogaM = => M = ax (by definition)

LogaN = y ==> N = ay (by definition)

M ÷ N = ax ÷ ay

$\frac{M}{N} = \frac{a^x}{a^y}$ = ax-y (law of indices)

In logarithmic notation

$\text{Log}_a \left(\frac{M}{N}\right) = x - y$ and substituting the values of x and y above gives

$\text{Log}_a \left(\frac{M}{N}\right) = x - y = \text{Log}_a M - \text{Log}_a N$

$\text{Log}_a (M \div N) = \text{Log}_a \left(\frac{M}{N}\right) = \text{Log}_a M - \text{Log}_a N$

Using the values given in for Log102, Log103 and Log107, Evaluate

(i) $\text{Log}_{10} \left(\frac{7}{2}\right)$ (ii) $\text{Log}_{10} \left(\frac{14}{5}\right)$ without tables or calculators

Solution: (i) $\text{Log}_{10} \left(\frac{7}{2}\right) = \log_{10} 7 - \log_{10} 2$

Substituting for Log10 2 = 0.3010 and Log10 7 we obtain

$\text{Log}_{10} \left(\frac{7}{2}\right) = 0.8451 - 0.3010$

$= 0.5441$

$\text{Log}_{10} \left(\frac{7}{2}\right) = \text{Log}_{10} 7 - \text{Log}_{10} 2 = 0.5441$

(ii) $\text{Log}_{10} \left(\frac{14}{3}\right) = \text{Log}_{10}14 - \text{Log}_{10}3$

$= \text{Log}_{10} (7 \times 2) - \text{Log}_{10}3$

Substituting the given values for Log102, Log103 and Log107 in , we obtain

$\text{Log}_{10} \left(\frac{14}{3}\right) = \text{Log}_{10} 7 + \text{Log}_{10} 2 - \text{Log}_{10} 3$

$= 0.8451 + 0.3010 - 0.4771$

$= 1.1461 - 0.4771$

$= 0.6690$

Here the logarithm of the quotient of two numbers is equal to the logarithm of the dividend minus the logarithm of the divisor.

3. Loga MP = P Loga M

Proof. Let Loga M = x => M = ax (by definition) raising M to power P, we have

MP = (ax) P = ax P (law of indices)

In logarithmic notation

Loga MP = P and substituting the value of gives

Loga MP = P LogaM

Example: Evaluate (i) Log10 8, (ü) Log109 using without tables or calculators

Were Log108 can be expressed as Log1023 since 8 = 23

Log108 = Log1023 and by this property

= 3 Log102, then substituting Log10 2 = 0.3010,

gives, 3 x 0.03010

Log108 = Log1023 = 3 Log102

= 3 x 0.3010

= 0.9030

(ii) Log109 = Log1032 = 2Log103 and substituting for Log103 = 0.4771, gives

Log109 = Log1032

= 2Log103

= 2 x 0.4771

= 0.9542

So the logarithms of the power of a number is equal to the exponent times the logarithms of that number.

We shall revisit these examples as we move along. But for the meantime cross-check these answers concerning the properties with your calculators to see that properties 1 - 3 proved above are true for all numbers.

Note: (1) The number a which is the base must not be equal to 1 (unity) because numbers not equal to unity will not have logarithms and again any number will be the logarithm of one.

The following properties are also deduced from the relationship between indices and logarithms.

$a^0 = 1$ and by the definition of logarithm

4. $\text{Log}_a 1 = 0$. This means that the logarithm of 1 to any base is zero.
   $\text{Log}_5 1 = 0 \Rightarrow 5^0 = 1$, $\text{Log}_a 1 = 0 \Rightarrow 8^0 = 1$
   $\text{Log}_2 1 = 0 \Rightarrow 2^0 = 1$ etc.

5. From $a^1 = a$ we have $\text{Log}_a a = 1$. Thus means that the logarithm of a number to the same base is 1.

   But $\text{Log}_0 1$ is not accepted and 0^0 i.e. $\frac{0}{0}$ is meaningless. So $\text{Log}_{10} 10 = 1$,

   $\text{Log}_2 2 = 1$ etc.

6. From $b = \sqrt[c]{a}$ which in index notation is $b = a^{1/c}$

   we have $\text{Log}_a b = \dfrac{1}{\log_b a}$

   To prove let $\text{Log}_b a = c$, then by the definition of logarithm $a = b^c$ and $b = a^{1/c}$

   $\text{Loga } b = \text{Log}_a a^{1/c} = 1/c$,
   then substituting for c, it then becomes

   $\text{Log}_a b = \dfrac{1}{\text{Log}_b a}$

   Example $\text{Log}_3 5 = \dfrac{1}{\log_5 3}$

7. $\text{Log}_a b = \dfrac{\text{Log}_x b}{\text{Log}_x a}$. This is called the transformation rule and is used to convert numbers from one base to another. Recall change of bases in Unit 2.

   Example $\text{Log}_2 9$. This cannot be solved by tables or calculators because the bases in use in these calculating devices is 10. Both can then be transformed to base 10 thus

   $\text{Log}_2 9 = \dfrac{\log_{10} 9}{\log_{10} 2}$

   The following exercises are meant for you.

Exercise: 4.3

1. Express in terms of Log10 x, Log10y and Log10

(a). Log10 y (b) Log10 x (c) Log10 $\left(\frac{xy^3}{z}\right)$ $\left(\frac{}{}\right)$ z

(d) Log10 $\left(\frac{1}{y}\right)$

2. Express as single Logarithms
   (a) Log 2 + Log 3 (b) Log 18 - Log 9
   (c) 1 + Log x - ½ log y (d) 3Log x - ½ Log y + 1

3. Simplify
   (a) 1/3 Log 64 (b) 3 Log3 - Log 27 (c) $\frac{\text{Log } 8}{\text{Log } 2}$

   (d) $\frac{\text{Log}49}{\text{Log}343}$

From the definition of logarithms also the following identity was obtained $a^{\text{Log}_a N} = N$

Example

(i) $2^{\text{Log}_2 8} = 8$ that is $2^3 = 8$ because the exponent $\text{Log}_2 8 = 3$

(ii) $2^{\log_2 64} = 64$ since $2^6 = 64$ and

(iii) $3^{\log_3 81} = 81$. This is got by simple substitution from the definition of logarithm. You are still seeing different ways of the same thing.

Now move on to the next section 3.2 Common Logarithms

3.3 Common Logarithms

The idea of constructing of table of logarithms to the base 10 belongs to the Scotman Napier and the Englishman Briggs. After the death of Napier, Briggs continued the work and published it completely in 1624 (Nlgodsky 1972:225). That is why the base 10 (common) logarithm is called Briggsian, written as Iog10x or Log x, For practical purposes the most convenient logarithms are logarithms to base 10 i.e. common logarithms. In theoretical investigations however, the other type of logarithms - the Natural logarithms or logarithm to base e, where e = 2.771828183 ... is used. Sometimes logarithms taken to base e are called Naperian logarithms.

Natural logarithms are usually denoted by in x instead of log x ..
Now move on to the use of logarithm tables. Here we are concerned with logarithm to base 10 i.e. common logarithm.

3.3.1 Use of Logarithms Tables

Here the common logarithms will be made reference to. This is because they are simple to use and numbers in base ten can easily be expressed in standard form.

In the use of common logarithms table, the following relations are important

Log1010 = 1   10 = 101
Log10100 = 2   100 = 102
Log101000 = 3   1000 = 103 etc.

Also

Log100.1 = -1
Log100.01 = - 2
Log100.001 = -3 etc

Using the above information to find the logarithm of any number, first express the number in standard form, to determine the size of the number.
Every logarithms has two parts:

(a) The characteristic - this is the whole number or integral part got from the value by which the given number is expressed in standard form and

(b) The mantissa which is the decimal part is got from the logarithm table. So to find the logarithms of a number from the tables
  (i) Find its characteristics by expressing the number in standard form. A simple rule is that the characteristic is always 1 less than the number of digits in the whole number part.
  (ii) Then find the mantissa from the logarithm table.

Examples in finding the value of log375. First express 375 in standard form i.e. 3.75 x 102

2 is the characteristics, then the mantissa is got by look for 37 under 5 from the tables. As a check since 375 his between 100 and 1000, its logarithm will also be between log100 and log1000 i.e. between 2 and 3.

Note: (a) For positive numbers greater than 1

(i) The characteristic is positive and is one less than the number of digits in the whole number part before the decimal points in the given number

(ii) The mantissa is always positive

(b) For positive numbers less than 1

(i) The characteristic is negative and is one more than the number of zeros immediately following the decimal point

(ii) The mantissa is always positive

Example Log 375 = 2 + (. .)

Log 0.02664 = -2 + ( ), written as 2 called bar 2 because the minus sign affects only the characteristics.

3.4 Application of Logarithm s

Worked Examples.

1. Simplify without tables or calculators

(a) $\text{Log}32 + \text{Log}\ \frac{10}{9}$

Solution by property (1)

$\text{Log}32 + \text{Log}\ \frac{10}{9} = \text{Log}\ (32 \times \frac{10}{9})$

= Log 10 (but log => log10)

$\text{Log}_{10}10 = 1$

$\text{Log}32 + \log \frac{10}{9} = 1$

(b) $\text{Log}\ \sqrt[3]{5} = \log 5^{1/3}$ by property (3), it is = $\frac{1}{3}\log 5$

$\text{Log}\ \sqrt{5} = \log 5^{1/2}$ ½ log 5

Simplifying

$= \frac{1}{3} \times \frac{2}{1} = \frac{2}{3}$

2. Find the value of K. given that

Log K - Log (K-2)= Log 5

Solution: by property (2)

Log K - log (K -2) = Log $\left(\frac{K}{K-2}\right)$ = Log 5 and simplifying since both sides contain logarithm, we have

$\frac{K}{K-2} = 5$, cross multiply

K = 5(K -2)

K = 5K - 10

Collection of like terms

5K – K = 10

4K= 10

$K = \frac{10}{4}$

K = 5/2 = 2.5

Check:

Substitute back K = 2.5 in the original equation.

Log K - Log (K - 2) = Log 2 .5 - Log (2.5 - 2)

= Log 2.5 - Log (0.5) by (property (2))

= Log $\left(\frac{2.5}{0.5}\right)$ = Log $\left(\frac{25}{5}\right)$

= Log 5 = RHS.

It is always good to check your answers at the end of each solution. 3. Simplify without tables or calculators

$$\frac{\text{Log}\sqrt{216} - \text{Log}\sqrt{125} - \log 8}{2(\log 3 - \log 5)}$$

Solution: Applying the rules of logarithm

Log $\sqrt{216}$ - $\sqrt{\text{Log}}$ 125 $\sqrt{\ }$ - log 8

$= \text{Log} \dfrac{216}{125 \times 8}$ (law (2))

and 2(Log 3 - Log5) = $2\text{Log}\left(\dfrac{3}{5}\right)$

$$\frac{\text{Log}\sqrt{216} - \text{Log}\sqrt{125} - \text{Log}\sqrt{8}}{2(\text{Log } 3 - \text{Log } 5)}$$

$$= \frac{\text{Log}\sqrt{\dfrac{216}{125 \times 8}}}{2\text{ Log}\left(\dfrac{3}{5}\right)}$$

$$= \frac{\text{Log}\sqrt{\dfrac{63}{53 \times 23}}}{2\text{Log }(3/5)} = \frac{\text{Log}\left(\dfrac{6}{5 \times 2}\right)^{3/2}}{2\text{ Log }(3/5)}$$

by law (3) $= \dfrac{3/2 \text{ Log }(3/5)}{2 \text{ Log }(3/5)}$

Simplifying $= \dfrac{\frac{3}{2}}{2}$

$$\frac{\text{Log}\sqrt{216} - \text{Log}\sqrt{125} - \text{Log}\sqrt{8}}{2(\text{Log}3 - \text{Log}5)} = \frac{3}{4}$$

4. Given that Log 2 = 0.69, Log 3 = 1.10 and log 7 = 1.90 all to a fixed base. Find Log 10.5 to the same base with using tables.

Solution: first change the number to improper fraction i.e. $10.5 = \dfrac{105}{10}$

Log 10.5 = Log $\dfrac{105}{10}$

= Log 105 - Log10 (law (2))

= Log (7 x 3 x 5) - Log (2 x 5) [since the value are given in these prime numbers.

= Log 7 + Log 3 + log 5 - (log 2 + Log 5) (law (1))

Simplifying then = Log 7 + Log 3 + Log 5 -Log2 – Log 5

= Log 7 + Log 3 - Log2.

Substituting the values of Log7, Log3 and Log2

= 1.90 +1.10 - 0.69

$= 3.0 - 069$

$\text{Log } 10.5 = 2.31$

5. Simplify $\text{Log}_7\ 49^x - \text{Log}_{10}\ 0.01$

Solution:

$\text{Log}_7\ 49^x - \text{Log}_{10}\ 0.01$, first change 0.01 to fractions i.e. $0.01 = \frac{1}{100}$ so

the expression is

$\text{Log}_7\ 49^x - \text{Log}_{10}\ \frac{1}{100}$

$= \text{Log}_7\ 49^x - \text{Log}_{10}100^{-1}$ (by the law of indices $1/100 = 100^{-1}$)

$= x\ \text{Log}_7\ 49 - (-1)\ \text{Log}_{10}100$

$= x\ \text{Log}_7\ 7^2 + \text{Log}_{10}10^2$ (by law (3))

$= 2\ x\ \text{Log}_7\ 7 + 2\ \text{Log}_{10}\ 10$ (but $\text{Log}_a\ a = 1$)

So this will be $= 2x + 2$

$\text{Log}_7\ 49^x - \text{Log}_{10}\ 0.01 = \underline{2x + 2}$

6. Find without tables or calculator the value of $\text{Log}_{10}\ \frac{1}{40}$

given that Log;o4 = 0.6021

Solution:

$\text{Log}\ \frac{1}{40} = \text{Log } 1 - \text{Log } 40$ (by property (2))

OR $= \text{Log } 40^{-1}$ (by Property (3))

$= -1\ \text{Log } 40$ (by Property (3))

$= -1\ \text{Log } (4 \times 10)$

$= -1\ [\text{Log } 4 + \text{Log } 10]$ by Property (1)

$=-1\ [0.6021 + 1]$ substituting

$= -1\ [0.6021\ ]$

$\text{Log}\ \frac{1}{40} = -1\ .6021$

Now try these exercises

Exercise 4.4

l. Simplify without tables or calculators

(a) $\text{Log}_4\ 3.2 + \text{Log}_4\ 20$ (b) $\frac{\text{Log } 81}{\text{Log } 1/3}$

2. Express as single logarithms

(a) 3Log2 + 2Log 3 - 2Log6 3.

3. Simplify

(a) $\frac{\text{Log } 125}{\text{Log } 5}$ (b) 5 Log 2 – Log 32 (c) - Log2 ½

4. Show that $\text{Log}_a (a + b)^2 = 2 + \text{Log}_a \left[1 + \frac{2b}{a} + \frac{b^2}{a^2}\right]$

Take time and do the exercise.

Solutions for Exercise 3.3

1. (a) Log 4 (3.2 x 20) = Log4 64 = Log 4 43 = 3

(b) $\frac{\text{Log } 3^4}{\text{Log } 3^{-1}} = \frac{4 \text{ Log3}}{-1\text{Log3}} = -4$

2. Log23 + Log3 2 - Log6 2 = Log $\left(\frac{8 \times 9}{36}\right)$ = Log2

3. (a) $\frac{\text{Log5 }^3}{\text{Log51}} = \frac{3\text{Log5}}{1\text{Log5}} = 3$

(b) Log $\left(\frac{2^5}{32}\right)$ = Log $\left(\frac{32}{32}\right)$ = Log 1 = 0

(c) Log2 ½ = Log2 (2-1)-1 = Log2 2= 1

4. Log a (a +b)2 = Log a [a2 + 2ab + b2]

$2 + \text{Log}_a \left(1 + \frac{2b}{a} + \frac{b^2}{a^2}\right)$

$\text{Log}_a a^2 + \text{Log}_a \left(1 + \frac{2b}{a} + \frac{b^2}{a^2}\right)$

$= \text{Log}_a a^2 \left(1 + \frac{2b}{a} + \frac{b^2}{a^2}\right)$

$= \text{Log}_a [a^2 + 2ab + b^2]$

4.0 Conclusion

In this unit, you have learnt the definition of logarithm, the relationship between logarithms and indices. You have learnt the properties of logarithms and its relationship with the laws of indices and also how to apply these properties to problems. You have also known why the logarithm to base 10 is called the Briggsian base or common logarithm.

5.0 Sum mary

In this Unit, you have learnt the following properties of logarithms in relation to the laws of indices.

1. Log a M + Log a N = Log a MN
2. Log a M - Log a N = Log a $\left(\frac{M}{N}\right)$
3. pLog M = LogaMP

6.0 Tutor - Marked Assignm ent 4

1. Simplify without tables or calculators
   (a) Log32 +Log 10/9
   (b) Log 2 + Log 50
   (c) $\frac{\text{Log64}}{\text{Log64}}$

2. Evaluate
   (a) Log273
   (b) Log2/3 4/9
   (c) Show that Loga b x Logb c Logc a = 1

3. Find the characteristics of the logarithm of the following numbers
   (a) 32 (b) 148.57 (c) 1327 (d) 28.96

4. Find the mantissa of the logarithm of each of the following numbers
   (a) 1.78 (b) 987.3 (c) 1039 (d) 47.2

5. Find x , if Log10 (3x - 1) – Log10 2 = 3.

7.0 References and Other Resources

Amazigo, J.C. (ed) (1991). Introductory Mathematics I: Algebra, Trigonometry and Complex Numbers. Onitsha: Africana-Fep Publishers Ltd.

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UNIT 5

SURDS

Table of Contents

1.0 Introduction

Many a time we come across numbers whose exact square roots cannot be found. Such numbers are referred to as surds. Hence surds are part of irrational numbers but cannot be taken to be irrational numbers.

Example of surds are v2, v3, v17 etc, but irrationals numbers are p, e, v2, v5, v17, v3 etc. therefore, surds are contained in the set of irrational numbers.

In this unit you shall study the rules for manipulating surds, rationalising the denominator and finding the square root of numbers using surds.

2.0 Objectives

By end of this unit, you should be able to
• Explain in clearly what surd is
• Manipulate surds correctly
• Find the square root of given numbers using surds.

3.0 Main Content

3.1 Surds

You will recall from unit 1, that rational numbers were defined as numbers that can be expressed in the form a/b, (b 0) i.e. numbers which can be expressed as fractions.

Also irrational numbers were described as numbers which cannot be expressed as fractions, for example v2, v5, p, e etc. then what are surds?

Surds are numbers such as v2, v3, v5, v7 etc which do not have exact roots. Having defined surds, we now do the following:

Find in each of the following whether the two expressions are equal or not

(a) v16 x v9, v16 x 9 (b) v16 + v9 and v16 + 9

(c) v9 - v4, v9 - 4 (d) v $\frac{9}{v4}$. $\sqrt{\frac{9}{4}}$

Having done the above exercises, what did you discover?

From the above exercise it can be shown that (a) va- v b = vab and
(b) $\frac{va}{vb}$ = $\sqrt{\frac{a}{b}}$, where a and b are positive numbers.

These facts are then used as a means of simplifying surds.

For example: Simplifying

(i) v32 = v2x4x4 = v42 x 2 = v42 x v2 = 4v2

(ii) v216 = v6 x 6 x 6 = v62 x 6 = v62 x v6 = 6v6

(iii) v108 = v36 x 3 = v36 x v3 = 6v3

In this type of problem, always break up the numbers into factor, one of which is a perfect square. This takes us to multiplication of surds

Multiplication of Surds

In the multiplication of Surds, the first thing you should do is (i) to simplify where it is possible

(ii) Group the whole numbers together, surds together and multiply
(iii) Then simplify again if it is necessary.

Examples

(i) Simplify vl2 x v8 xv15

Solution: In treating multiplication of surds, find if they have common factors to enable you get the perfect squares thus

v12 x v8 x v15 = v4 x 3 x v4 x 2 x v3 x 5 since va x vb = vab

=> v12 x v8 x v15 = v4 x 3 x 4 x 2 x 3 x 5

= v42 x 32 x 2 x 5

= v42 x v32 x v2 x 5

= 4 x 3 v2 x 5

= 12v10

Alternatively: multiply out and then simplify

v12 x v8 x vl 5 = v12 x 8 x 15

= v1440

= v122 x 10

= 12v10

2) Simplify 2v7 x 3v5 x 2v3

Solution: This is surds that have coefficients
First multiply the coefficients then the surds thus
2v7 x 3v5 x 2v3
= 2 x3 x2 v7x5x3
= 12v105
It cannot be simplified further then this becomes the answer
2v7 x 3v5 x2v3 = 12v105.

Then we move to the next operation that is division, but before doing this, there are certain rules in surds similar to that in the previous sections on indices, and logarithms

3.2 Rules for Manipulating Surds

From the Previous exercise, it was discovered that for the operations of addition and subtraction of surds, you do the following

1 (a) avb + cvb = (a + c) v b. Here you only add the coefficients as if dealing with 3a+2a=5a

Example Simplify 3v7 + 5v7
Solution = (3 + 5) v7 = 8v7
But if the addition is of the sort

(b) avb + bvc Leave as it is.
Example 9v6 + 2v3. Notice here that v6 and v3 are not the same. So do not add. The answer remains the same.

2. Subtraction: The same with addition that is for avb - cvb = (a - c) vb
Example 5v7 - 3v7 = (5 – 3) v7
= 2v7

3. Multiplication:

vab = va x vb and the procedure for further multiplications have been stated above with other examples.

4. Division:

The division of surds has to do with a surd being the denominator. To avoid this we talk of rationalising the denominator.
(a) Express the division as a fraction
(b) Simplify the fraction
(c) If the denominator contains a surd, then the numerator and the denominator by the irrational part of the denominator
(d) Simplify further if possible:
Now we move on to rationalising the denominators

3.3 Rationalising the Denominator

Rationalising the denominator means making the denominator a whole or rational number.

Example: Simplify (i) 5 (ii) 7 v18
v5 4v12

In the above, the denominators are one digit, so multiply the numerator and the denominator by the surd in the denominator thus the solutions are

(i) 5 = 5 x v 5 = 5v5
v5 v5 x v5 v25

= 5 v5
5

= v 5

(ii) 7 v18 / 4v12 = 7v18 x v12 / 4v12 x v12

= 7 v18 x 12 / 4vl2 x 12

= 7 v9 x 3 x 4 x 3 / 4v12 x 12 = 7v9 x 9 x 4 / 4v122

= 7 x 3 x 3 x 2 / 4 x 12

= 7 x 3 / 8 = 21 / 8

= 2 5/8

Again the denominator may contain surds like

(a) va + vb . To rationalize here, look for a similar surd which when multiplied together gives a rational number. Here the difference of two squares come to mind

a2 – b2 = (a - b)(a + b), so to rationalize va + vb multiply by va - vb

i. e. (va + vb) (va - vb) = (va)2 - vab + vab - vb)2

= a - b

and if the denominator is va + vb multiply by va + vb.

Example: Simplify (i) 12 / v3 + v5 (ii) 18 / v24 - v6

Solution:

(i) 12 / v3 + v5 . This denominator here is in the form va + vb so we multiply both numerator and denominator by va - vb

12 / v3 + v5 x v3 - v 5 / v3 - v5 = 12(v3 - v 5) / (v3 + v5)(v3 - v5)

Expanding and simplifying, gives the following

$$= \frac{12(\sqrt{3} - \sqrt{5})}{(\sqrt{3})^2 - \sqrt{3 \times 5} + \sqrt{3 \times 5} - (\sqrt{5})^2}$$

$$= 12\left(\frac{\sqrt{3} - \sqrt{5}}{3 - 5}\right)$$

$$= 12\left(\frac{\sqrt{3} - \sqrt{5}}{-2}\right)$$

$$= -6(\sqrt{3} - \sqrt{5})$$

$$= 6(\sqrt{5} - \sqrt{3})$$

(ii) $\frac{18}{\sqrt{24} - \sqrt{6}}$, rationalising the denominator, multiply numerator and denominator by $\sqrt{24} + \sqrt{6}$

Thus $\frac{18}{\sqrt{24} - \sqrt{6}} \times \frac{\sqrt{24} + \sqrt{6}}{\sqrt{24} + \sqrt{6}}$

Expanding $= \frac{18(\sqrt{24} + \sqrt{6})}{(\sqrt{24})^2 - \sqrt{24 \times 6} + \sqrt{24 \times 6} - (\sqrt{6})^2}$

Simplifying $= \frac{18(2\sqrt{6} + \sqrt{6})}{24 - 6}$, because $\sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6}$

$= \frac{18(3\sqrt{6})}{18}$, recall that $2\sqrt{6} + \sqrt{6} = (2+1)\sqrt{6} = 3\sqrt{6}$

$= 3\sqrt{6}$

$$\frac{18}{\sqrt{24} - \sqrt{6}} = 3\sqrt{6}$$

Note that here when the sign between the two digits is plus (+), we rationalise by just putting minus between the two digits i.e. if $\sqrt{a} + \sqrt{b}$ multiply with $\sqrt{a} - \sqrt{b}$ and if $\sqrt{a} - \sqrt{b}$ multiply with $\sqrt{a} + \sqrt{b}$.
Hence we call them conjugates i.e. $\sqrt{a}$ and $-\sqrt{a}$, $\sqrt{a} + \sqrt{b}$ and $\sqrt{a} - b$,

x + y vb and x - y vb , xva + yvb and xva - yvb are called conjugate surds. Now having given the conjugates of surds, we now proceed to simplify problems.

Example

1. Simplify $\frac{\sqrt{2}}{\sqrt{3} - \sqrt{2}} - \frac{3 - \sqrt{2}}{\sqrt{3} - \sqrt{2}}$

Solution: Here is a combination of all that has been said of surds. First rationalise the denominators thus

$\frac{\sqrt{2}}{\sqrt{3} - \sqrt{2}} - \frac{3 - \sqrt{2}}{\sqrt{3} - \sqrt{2}} = \frac{\sqrt{2}(\sqrt{3} + \sqrt{2})}{(\sqrt{3})^2 - \sqrt{3} \times \sqrt{2} + \sqrt{3} \times \sqrt{2} - (\sqrt{2})^2}$

expanding the numerator and simplifying

$\frac{(\sqrt{3} \times \sqrt{2}) + (\sqrt{2} \times \sqrt{2})}{3 - 2} = \frac{\sqrt{6} + 2}{1}$

taking the other v $\frac{3 - \sqrt{2}}{\sqrt{3} + \sqrt{2}} \times \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}}$

$= \frac{(\sqrt{3} - \sqrt{2})(\sqrt{3} - \sqrt{2})}{(\sqrt{3})^2 - \sqrt{3} \times \sqrt{2} + \sqrt{3} \times \sqrt{2} - (\sqrt{2})^2}$

expanding = v $\frac{3 \times \sqrt{3} - 3 \times \sqrt{2} - 3 \times (\sqrt{2} + \sqrt{2})^2}{3 - \sqrt{6} + \sqrt{6} - 2}$

$= 3 - \frac{\sqrt{6} - \sqrt{6} + 2}{3 - 2} = \frac{5 - 2\sqrt{6}}{1}$

Hence combining the two expressions

$\frac{\sqrt{2}}{\sqrt{3} - \sqrt{2}} - \frac{3 - \sqrt{2}}{\sqrt{3} + \sqrt{2}} = \frac{\sqrt{6} + 2}{1} - \frac{5 - 2\sqrt{6}}{1}$

Simplif ying

$(\sqrt{6} + 2) = \sqrt{6} + 2\sqrt{6} + 2 - 5$

$= 3\sqrt{6} - 3$

$= 3(\sqrt{6} - 1)$

Study the above solution carefully and identify the rules applied in the steps. If not clear, go to the section on rules for manipulating surds. You will be happy you did.

2. Simplify $\frac{2\sqrt{2} - 3}{3 + \sqrt{8}}$. You can do this.

Solution:

$$\frac{2\sqrt{2} - 3}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}}$$

$$= \frac{(2\sqrt{2} - 3)(3 - \sqrt{8})}{\mathbf{(3 + \sqrt{8})(3 - \sqrt{8})}}$$

expand the numerator by multiplying term by term

$(2\sqrt{2} - 3)(3 - \sqrt{8}) = (2\sqrt{2})(3 - \sqrt{8}) - 3(3 - \sqrt{8})$

$= 6\sqrt{2} - 2\sqrt{2} \times 8 - 9 + 3\sqrt{8}$

Simplifying $= 6\sqrt{2} - 2\sqrt{16} - 9 + \sqrt{4} \times 2$
$= 6\sqrt{2} - 2 \times 4 - 9 + 6\sqrt{2}$
$= 12\sqrt{2} - 8 - 9 = 12\sqrt{2} - 17$

Denominator $(3 + \sqrt{8})(3 - \sqrt{8}) = 3^2 + 3\sqrt{8} - 3\sqrt{8} - (\sqrt{8})^2$
$= 9 - 8 = 1$

Combining numerator and denominator gives

$$\frac{12\sqrt{2} - 17}{1} = 12\sqrt{2} - 17$$

$$\frac{2\sqrt{2} - 3}{3 + \sqrt{8}} = 12\sqrt{2} - 17$$

Alternatively you can just rationalise the denominator only, that is

$\frac{\sqrt{2}}{\sqrt{3} - \sqrt{2}}$ $- \sqrt{}$ $\frac{3 - \sqrt{2}}{\sqrt{3} + \sqrt{2}\quad 3^2 - 3\sqrt{8} + 3\sqrt{8} - 8} = \frac{(2\sqrt{2} - 3)(3 - \sqrt{8})}{}$

$= (2\sqrt{2} - 3)(3 - \sqrt{8})$

This is also the answer but when you are asked to simplify, the first method is acceptable.

3.4 Find Square Roots of Num bers Using Surds.

The essence of surd is to use it in finding the square root of numbers without logarithm tables or calculators.

Examples: Show that (va - vb)2 = a + b - 2 vab . Hence find the square root of

(i) (9 + 4v5) and (ii) 18 -12v2

Solution:

(i) (va - vb)2 = (va - vb)(vb - vb). Expanding gives

(va)2 - vab - vab + (vb)2

= a - 2vab + b. You will know that these surds are similar to the expansion of (a - b)2

(va - vb)2 = a + b - 2vab.

In applying it in finding the square root of the given numbers, we treat as follows

(ii) The square root of 9 + 4v5 is expressed as v9 + 4 v5,

Let v9+4 v5 = a + vb Here va + vb is used because
Square both sides we are finding the root of
(9 + 4v5)2 = (va + vb)2 v9 + 4v5
(9 + 4v5 = a + b + 2vab
Equating corresponding terms
9 = a + b ----- (1)
4v5 = 2vab ----- (2)

Note the number without the root signs go together and those with the roots signs equated together

Solving the two equations simultaneous, I hope you have not forgotten how we solve simultaneous equations. If yes, do not worry follow this solution.

9 = a + b ----- (1)

4v5 = 2vab ----- (2)

From (2) 2v5 = vab dividing through by 2

(2v5)2 = (vab)2 Squaring both sides

4 x 5 => 20 = ab

Making a the subject it becomes

$a = \frac{20}{b}$

Now substitute for $a = \frac{20}{b}$ in (1)

$9 = \frac{20}{b} + b$

Clear the fraction by multiplying every term by b

9b = 20 + b2 now we have a quadratic in b, so we factorise if possible

b2 - 9b + 20 = 0 = (b - 4) (b - 5) = 0
b = 4 or 5

So substituting for the value of b, one by one when b = 4, $a = \frac{20}{b} = \frac{20}{4} = 5$

and when b = 5, $a = \frac{20}{5} = 4$

the square root of 9 + 4v5 = v4 + v5
or = v5 + v4

v9 + 4v5 = 2+ v5

(ii) Let v18 - 12v2 = va - vb
Squaring both sides

(v18 -12v2 = (va - vb)2

18 - 12v2 = a + b - 2vab , recall the initial condition. Here since the sign between 18 and 12v2 is minus, also choose two digits with a minus sign in between.

Going by the same process as in a + 4v5 equate coefficients
18 = a + b ---- (1)
12v2 = 2vab ---- (2)

From (2) 6v2 = vab and squaring and simplifying 72 = ab

$a = \frac{72}{b}$

Substituting in (1) for $a = \frac{72}{b}$ and clearing the fraction

18b + 72 + b2

b2 - 18b + 72 = 0 (quadratic in b)

Factorising (b - 12)(b - 6) = 0

b = 12 or 6

when b = 12, a = 72 = 6

when b = 6, $a = \frac{72}{6} = 12$

the square root of 18 - 12v2 is either

v6 - v12 or v12 - v6

by your knowledge of estimation, you should be able to know the right answer. In the case of addition as in the former example, there is no problem

v18 - 12v2 = v12 - v6
= 2v3 - v6

though the two answers are correct. If in doubt leave it as it is except where the examiners insists on the correct option

the square root of 18 - 12v2 = 2v3 - v6
= v 6 -v 12

Exercise 5.3

1. Simplify 3v12 x 2v18 x v27

2. Expand (v28 + 2v3)(v3 + 5v7)

3. Rationalise the denominator

(a) 5 / (v5 – 2) (b) 2 (v5 - v7 2) / (3v7 - 2v3)

4. Show that (va - vb)2 = a + b + 2vab . Hence find the square root of 17 + 12v2

Answers to Exercise 5.3

1. v12x9 x v18x4 x v27

= v12x9x18x4x27 = v4x3x9x9x2x9x3x4

= v4x9x9x9x9x2 = v22x92x92x22x2

= v22v22 x v92 x v92 x v2 = 2x2x9x9 v2

= 324v2.

2. (v28 + 2v3)(v3 + 5v7) = (v28)(v3 + 5v7)+ (2v3)(5v7)

(2 x 3) + v28 x 3 + 5v28 x 7 + 10v3 x 7

6 + v84 + 5v196 + 10v21+ v4x21 + 5v4x49+10v21

6 + 2v21+5 x 2 x 7 + 10v21

= 76 + (10 +2)v21 + 76 + 12v21

3a. 5 / (v5 - 2) x (v5 + 2) / (v5 + 2) = (5 v5 + 10) / (5 - 4) = 5 v5 + 10

(b). 2 (v5 - v7 2) / (3v7 - 2v3) x (3v7 + 2v3) / (3v7 + 2v3) = (6 v35 - 21 v14 + 4 v15 - 14 v6) / (9 x 7 - 4 x 3)

= 6 (v35 - 21 v14 + 4 v15 - 14 v6) / (63 - 12)

= / 51

4.0 Conclusion

In this unit, you have been exposed to the different methods of rationalising the denominators of surds and how to find the square root of numbers using the knowledge of surds.

5.0 Summary

In this unit, you have learnt the following properties of surds

1. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

3. To rationalise the denominator means making the denominator a whole number by multiplying the numerator and denominator by the conjugate surds i.e.

(a) $\frac{a}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$

(b) $\frac{a}{\ } = \frac{a(\sqrt{b} + \sqrt{c})}{\ }$

(c) $\frac{a}{\sqrt{b} + \sqrt{c}} = \frac{a(\sqrt{b} - \sqrt{c})}{b - c}$

(d) $\frac{\sqrt{a} - \sqrt{b}}{x + y\sqrt{b}} = \frac{(\sqrt{a} - \sqrt{b})(x - y\sqrt{b})}{(x + y\sqrt{b})(x - y\sqrt{b})}$

(e) $\frac{\sqrt{a} + \sqrt{b}}{x\sqrt{a} - y\sqrt{b}} = \frac{(\sqrt{a} + \sqrt{b})(x\sqrt{a} + y\sqrt{b})}{(x\sqrt{a} - y\sqrt{b})(x\sqrt{a} + y\sqrt{b})}$

The operations in (d) and (e) can be reversed and the same processes followed

6.0 Tutor - Marked Assignm ents

1. Expand and simplify (1) $4\sqrt{2}\,(2\sqrt{2} + 3\sqrt{3})$

2. $(2\sqrt{30} - 9)^2$

3. Rationalise the denominator

   (a) $\sqrt{2} + \frac{1}{\sqrt{2}}$

   (b) $4 \quad \frac{\sqrt{2} + 2}{2 + \sqrt{2}}$

4. Find the square root of $22 - 12\sqrt{2}$

5. Verif y that $5\sqrt{\frac{5}{24}} = \sqrt{5\frac{5}{24}}$

6. Find a, if $a\sqrt{\frac{3}{a}} = \sqrt{3\frac{3}{a}}$

7.0 References and Other Resources

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UNIT 6

RATIONS, PROPORTIONS AND PERCENTAGES

Table of Contents

1.0 Introduction

In real life situations we are faced with a lot of problems of comparing like and unlike quantities, such as age, height, weight, number, solutions of drugs etc.

Because of lots of comparison problems in the nursing profession, the following concepts, rations, proportions and percentages become necessary.

In this unit, you shall learn, the concept of ratio, proportion and percentages. You shall also learn the relationships between these three concepts.

2.0 Objectives

By the end of this unit, you should be able to:

- Explain the meaning of ratio, proportion and percentage.
- Distinguish between ratio, proportion and percentage
- Establish the relationship between ratio, proportion and percentage
- Perform computations involving ratio, proportion and percentage correctly.

3.0 Main Content

3.1 Rations

The term ratio is used to compare two or more quantities. Hence ratio is expressed as a comparison of two numbers. For example, we can state that the ratio of men to women in a church is 1: 2 which means that there are twice as many women as there are men.

Ratio is used to compare quantities of the same kind. But atimes, ratio may involve non homogenous quantities (quantities of different kinds) like we speak of the ratio of a weight to a height or systolic pressure to age.

The ratio of one quantity x to another quantity y is denoted by the expression *x : y read as x is to y which is equal to the quotient or fraction* $\frac{x}{y}$.

Example

(i) the ratio solutions express concentrations in terms of ratios, hence 1:1000 means the number of grams per volume of solution. That is, expressing 1 in grammes and 1000 in millimeters, also written as 1/1000.

(ii) Again the Dextrose – nitrogen ratio represents the ratio between dextrose and nitrogen in the urine etc.

Note (a) when expressing quantities in ratios, they must be in the same units. The following are also important in dealing with ratios.

(b) The order of the quantities: if the quantities are expressed in the form *x is to y i.e. x : y it means x/y but if expressed as y: x it means y/x.* so you see that x : y is not the same as y: x.
(c) Two ratios x : y is meaningful only if neither is equal to zero i.e. x : y is defined if and only if x 0 and y 0.
(d) Two rations can be multiplied or divided by the same number without changing their values. In fact ratios are treated as fractions.
(e) Ratio involves two or more quantities and it is better to express ratio quantities in their lowest term where possible.

Examples:

Compare the relative sizes of the two quantities given below by finding the ratio of the first quantity to the second

(a) 9m to 12m (b) 8g to 1 ½ kg (c) 15mins to 3hrs
(d) 1cm 6mm to 1m 8mm

Before attempting these problems, you should be conversant with the Measurement units so that it will be easy for you to express both quantities in the same units before simplifying the ratios.

Solution

(a) 9m: 12m - they are both in metres, so reduce to the simpliest form by dividing both by 3
9m : 12m = 3:4
So the ratio is ¾ or 3: 4

(b) 8g to l ½ kg. To find the ratio, first convert l ½ kg to gramme. Thus lkg = 1000g, so l ½ kg is equal to 1500g.
Hence the ratio is 8:1500. Dividing by 4 gives 2:375
8g to l ½ kg = 2: 375
Now move on to the next problem

(c) 15mins to 3hrs. What do you say about this/ now convert hours to minutes or minutes to hours as you desire
lhrs = 60mins then 3hrs = 3 x 60 = 180mins. So 15mins to 3hrs is 15:180 and simplifying or reducing to the lowest term gives 1:12
15mins to 3hrs => 1:12
Now do this. So easy is it not. We do it together.

(d) lcm 6mm to lm 8mm. If you do not look carefully, you will not discover the mix up. So here the best thing is to convert all to millimeters. If you have forgotten, consult your metric system table at the back of any exercise book or preliminary pages of science and mathematics textbooks you will be glad you did. Now move on.
l cm = l 0mm and 1 m = l 00mm
l cm 6mm = 16mm and 1 m 8mm = 1008mm
1cm 6mm to 8mm = 16:1008
= <u>1:63</u>

Now try these

Express the following in the form x/y, leaving your answers in the simplest forms

1(a) 210g to 560g (b) 340g to 2kg (c) 1.25cm to 0.15m
(d) 6hrs to 1 day
The above x/y is another form of expressing ratios.
Got all. Good. Now, move on to more examples

More Examples

1. Three student Nurses got a bursary award to be shared in the ratio 2:3:5. If the total amount for the bursary away is N150, 000. = How much will each get.

Solution: This problem is slightly different from the others. All that is to be done is to

(i) Calculate the total ratio i.e. 2 + 3 + 5 = 10
(ii) Find the ratio of each of the students to the total i.e. 2:10, 3:10 and 5:10
(iii) Then find how much each received by expressing each ratio as a fraction and multiply by the amount. This gives their individual shares

Thus $N\left(\frac{2}{10} \times 150,000.\right) = N\ 30,000$ =

$N\left(\frac{3}{10} \times 150,000.\right) = N\ 45,000.$ = and

$N\left(\frac{5}{10} \times 150,000.\right) = N\ 75,000.$ =

the ratio 2:3:5 = 30000:45000:75000.
Do the following exercises

Exercise 5.1

1. A well known Hospital charges N1000. = for 1 day for an in-patient and N25200. = for 1 week at festive periods. Find the ratio of the daily charge to the charge at festive period.

2. A lump of meat weighing 100kg is to be shared in the ratio of 8:7:5 among 3 nursing attendants. What did the nurse attendant with 7 shares get?

3. (a) Increase 21 in the ratio 2:1 Hint: express the ratio
(b) Decrease 40 in the ratio 3:4 in fraction and treat
(c) Decrease 36 in the ratio 5:12 accordingly

4. In what ratio must the first number be increased or decreased to become the second number
(a) 24 to 32 (b) 72 to 68 (c) 525 to 750
(d) 15 to 20 (e) 490 to 140

Answers to Exercise 5.1 to check your progress

1. 5:12 (divide N $\frac{25200}{7}$ and the ratio is now in days i.e. 1000:3600)

2. 7 gets 7 x $\frac{1000}{20}$ = 350kg.

3. (a) 21 is increased to 42   2 x 21 = $\frac{42}{1}$

(b) 40 is decreased to 30 (¾ x 40 = 30)
(c) 36 is decreased to 15 (15/12 x 36 =15)

4. (a) 24 to 32 in the ratio 4:3. Here reduce the numbers to their lowest term and if the first is smaller than the second the ratio is reversed. If not the ratio remains the same.
So 24 to 32, divide through by 8 gives 3 to 4
Since 24 is smaller than 32, the ratio is reversed to 4 to 3
24 to 32 is increased by 4:3

(b) 72 to 68 is decreased by 12:13
(c) 525 to 150 is decreased by 7:2
(d) 15 to 20 is increased by 4:3
(e) 490 to 140 is decreased by 7:2

Notice that questions 3 and 4 are similar

3.2 Proportions

Two equal rations form a proportion. Hence proportion is the equality of ratios between two pairs of quantities. Example is a school library contains 10,000 books of which 8000 are Unisar materials and another school has 12000 books of which 9600 are Unisar materials, then the ratio of the Unisar materials to the total books in both libraries is the same as 8000:10000 =

0.81; 9600:1200 = 0.8:1, you say that we have proportion, written as follows 8000:10000 = 9600:12000. in other words, we say that 8000 is to 10000 as 96000 is to 12000.

8000 and 12000 are called extremes of the proportion and 10000 and 9600 are the means of the proportion. Hence the product of the means equals the product of the extremes. Here 8000 x 12000 = 10000 x 9600 = 96000,000.

To find one of the extremes, divide the product of the means by the given extreme

The same applies in finding one of the mean

So if x : y = c : d

Then $x = \frac{y \times c}{d}$ and $y = \frac{xc}{d}$

This property is always used in finding the missing term of a proportion when the other three values are given.

Example 12 : y = 6:5 (y is the missing term and we are given the other three terms 12, 6 and 5). Applying the above rule

$$y = \frac{12 \times 5}{6} = 10$$

Examples of a practical application of proportion:

In a mixture that contains two or more elements that are not bound to each other, they exist in variable proportions. Example the constituents of air such as oxygen, nitrogen an carbon dioxide are not bound together (i.e. not bound to each other but the proportion of these elements in a sample of air would var y in different environments.

Oilier examples will be treated later.

Again, a proportion in which the means are equal is termed a continued proportion; for example 18:16 = 6:2. The mean ter m of a continued proportion is the geometric mean of the extreme terms. In the example 18:6 = 6:2, 6 which is the mean term is got by 6 = v18 x 2 where 18 and 2 are the extreme terms, hope you remember that. If not go back to proportion.

Practical Application of Proportion

The values of two different quantitative can be interdependence (depending on one another). Thus the area of a square depends on the length of its side and the length of the side of a square depends on the area of a square. Hence, the term two mutually dependent quantitative proportional if the ratio of their values remains constant.

Example, the weight of a liquid is proportional to its volume; 2 cartons of cough syrup weigh 1.6kg, 5 cartons of the same cough syrup weigh 4kg, cartons weigh 5.6kg.

The ratio of their weights to their volume is $\frac{1.6}{2} = 0.8$, $\frac{4}{5} = 0.8$ and $\frac{5.6}{7} = 0.8$

This constant ratio of proportion quantitative is called the constant of proportionality or proportional factor.

There are two types of proportions to be treated here namely (a) direct proportion and (b) inverse proportion.

Direct proportion implies that when one quantity increase, the second quantity also increases by the same number of times. Problems involving direct proportion can be solved using the method of ratio.

Example if 8 tins of coffee cost ₦ 2720 = what is the cost of 11 tins

Solution:
Cost of 8 tins of coffee = ₦ 2720

Cost of 11 tins of coffee = ₦ $2720 \times \frac{11}{8}$

$= ₦\ (340 \times 11)$
$= ₦\ 3740$

Alternatively, it can be stated thus

Cost of 8 tins of coffee = ₦ 2720
Cost of 11 tins of coffee = $\frac{₦2720}{8}$

Cost of 11 tins of coffee = $\frac{₦\ 2720}{8} \times 11$

$= ₦\ 3740$

But in case you are in doubt as to what the numerator or denominator of the multiplying factor should be, you ask yourself whether the required answer should be larger or smaller than the given (original) quantity. If the answer is larger.

In the above example if the quantities are expressed in the ratio of 8:11, we multiply by 11/8 but if the answer is smaller multiply by 8/11. Recall ratio examples on the section on ratio.

To avoid confusion you can use the alternative method where you decide what one is and proceed.

Example 2

Find the cost of 11 bottles of cough syrup if the cost of 2 dozen of it is N 2520. =

Solution: 2 dozen = 24 bottles = N 2520. =

$$1 \text{ bottle} = N \frac{2520}{24}$$

$$11 \text{ bottles} = N \frac{2520}{24} \times 1$$

$$= N1155.$$

You see the solution becomes easy, instead of battling with what the multiplying factor should be. Now move on to the other type of proportion.

Inverse proportion: implies that the increase in one quantity will lead to a decrease in the same quantity by the same proportion.

Example: if 5 nurses can carry out an assignment in 16 days, how long will it take 8 nurses to do the same job.

Solution: All things being equal, the more the number of people doing a job, the less time it will take, so

5 men = 16 days
1 man = 16 x 5 (greater time)

$$8 \text{ men} = \frac{16 \times 5}{8} = 10 \text{ days}$$

Hence as the number of nurses increased, the time to do the job is decreased.

Example 2.

A quantity of food lasts 5 men for one month (30 days)
For how long will it last 6 men, if their rate of eating is the same?

Solution:

5 men = 30 days
1 man = 30 x 5
6 men = $\frac{30 \times 5}{6}$
= 25 days

Proportional Parts

When we say that certain values are proportional to specified numbers, we mean that the ratio of the values is equal to the ratio of the specified numbers. For example, if ₦240 is divided into three parts which are in the ratio of 7:5:4, all we say is, here lets take the three value of ₦240 to be A, B, and C. this can be expressed as

A: B: C = 7:5:4

Example: divided ₦240 into three parts which are in the ratios of 7:5:4

Solution: Find the sum of the total number (7 + 5 + 4) because it looks as if the money has been shared placing 7 parts in a, 5parts in B and 4 parts in C.
total number of parts = 7 + 5 + 4 = 16
The amount got by each part is

A = ₦ $\left(\frac{7}{16} \times 240\right)$ = ₦ 105. =

B = ₦ $\left(\frac{5}{16} \times 240\right)$ = ₦ 75. =

C = ₦ $\left(\frac{4}{16} \times 240\right)$ = ₦ 60.=

As a check, you can sum up the money got by the three parts and see if you will obtain the original money shared i.e. ₦ (105 + 75 + 60) = ₦ 240. =

Now check your progress

Exercise 5.2

1. The rent of a house for one 1 year is N54,000 and the hospital wants to just make use of it for 7 months. How much will they pay?
2. A lady earns N10,260 in a year. How much does she earn in 4 months.
3. A nurse walks to his/her friend's house at 3 ½ km/h in 9 minutes. How long will it take him/her to walk at 5km/h?
4. A sum of money can pay the wages of 50 workers a year. For how many months will it pay the wages of 30 workers; at the same rate?
5. The weights of three boys are in the ratios. 6:3:2. The sum of their weights is 165kg. Find the weight of each.

Answers to Exercise 5.2

1. N $\frac{5400}{12} \times 7$ = N31,500. =

2. N 10260 x $\frac{1}{3}$ = 13680.

3. $\frac{9 \times 7}{2 \times 5}$ = 6.3 mins

4. $\frac{50\text{Yrs}}{30}$ = 1 2/3 Yrs

5. 90, 45, 30.

3.3 Percentages

The expression ‘per cent’ means a hundredth part symbolically, 170 stands for $\frac{1}{100}$ =
0.01 and 72% = $\frac{72}{100}$ 2 and 100% stands for 1 etc. the symbol of percentage is %

To find the percentage of a given number, multiply the number by 100.

Example (i) The percentage of 2 is 200 and the percentage of 0.0357 is 35.7% i.e. 0.0357 x 100
To change a percent to a number, divide the percent by 100.

Example: (a) 15.3% = $\frac{15.3}{100}$ = 0.153

(b) 3.5% = $\frac{3.5}{100}$ = 0.035

(c) 40% = $\frac{40}{100}$ = 0.4

Percent as ratio: you should have mastered the sections or ratios and proportion before attempting problems here.

Example: In a class of 60 students, 42 were present. What is the percent attendance?

Solution: the ratio of those present = $\frac{42}{60}$ this will be expressed as a percent by making the denominator 100

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$$\frac{42}{60} \times \frac{100}{100} = \frac{(42 \times 100)}{60} \times \frac{1}{100}$$

$$= \frac{70}{100} = 70\%$$

the percentage present is 70% but the percent present is $\frac{70}{100}$ (present is a hundredth) so it is a fraction.

Here we discuss three principal problems involving percentage

(a) Find the indicated percent of a given number.
Multiply the number by the percent and divide by 100 as in the above example of expressing percent as a ratio. This means that the given number is multiplied by the fraction expressing the given percent.

(b) Find a number on the basis of a given percent
The given quantity is divided by the percent and then multiplied by 100. That is the given is divided by the fraction expressing the given percent.

Example: In processing sugar beets, 12.5% of the weight of the beets is granulated sugar.
What quantity of beets has to be processed to produce 3000 centners of granulated sugar?

Solution: Divide 3000 by 12.5%

$$= 3000 \quad 25 \; \overline{} \; \frac{}{200}$$

$$\frac{200}{25}$$

= 24000 centners.

Alternatively divide 3000 by 0.125

$$= \frac{3000}{0.125} = 2400 \text{ centners}$$

(c) Find what percent one number is of another

Multiply the first number by 100 and divide by the second number
Example: A new plant for manufacturing tissue papers made it possible to increase the output of tissue papers for 1200 to 2300 rolls of tissue paper. What was the increase in tissue paper output in percent?

Solution: first find the difference between the original output and the present output i.e. 2300 -1200 = 1100.
Then multiply this difference by 100 = 1100 x 100 and divide by the original output i.e. $\frac{1100 \times 100}{120} = 91.67\%$

Applications of Percent

1. Profit and Loss

Profit = selling price - cost price
Loss = cost price - selling price.

$$\text{Profit percent} = \frac{\text{Profit}}{\text{Cost price}} \times 100$$

$$\text{Loss percent} = \frac{\text{Loss}}{\text{Cost price}} \times 100$$

2. Commission - The price paid to a middleman or a mediator in a business and this is associated with a percent, called commission rate.

Commission = rate x sale value of the good sold.

Example: A salesman sells goods worth ₦25000. = and charges a commission at the rate of 6%. Find the amount of his commission

Solution: Commission = rate x sale value of good sold the rate is 6% or $\frac{6}{100}$,
while the price of the good sold is ₦ 25000

$$\text{His commission} = ₦\left(\frac{6}{1000} \times 25000\right)$$

$$= ₦\ 1500$$

3. Discount - This is the concession or rebate given to customer who had purchased a large amount of good. There is also a percent associated with discount and is called discount rate.

Example: A BP equipment is sold for ₦17,500 less a discount of 10%. Find the cash price of the BP equipment

Solution: Selling price = ₦ 17500. =
Discount rate = 10%

$$\text{Amount of discount} = ₦\left(\frac{10}{100} \times 17500\right) = ₦\ 1750.\ =$$

Cash price = selling price - discount
= ₦ (17500 -1750) ₦ 15750. =
Cash price = ₦ 15,750.

Alternatively,
Discount= 10%
The marked price (price that is it meant to be sold) = 100%
Actual sale price = 100% - 90% = 90%
Hence, the sale price = 90% of marked price
= ₦ 90 x 17500. =
= ₦ 15,750

4. Brokerage - The commission an agent gets from both the buyer and the seller. Assuming the brokerage is 2%, this means that the agent will collect 2% from both the buyer and seller making it 4%.

Example: A person sell a (Mazda) car for N 80000. = through a broker (agent). If the brokerage is 2%. Find the net selling price and the seller and the total brokerage of the broker.

Solution: The selling price = N 8,0000 =
Rate of brokerage = 2%

Amount of brokerage from the seller = N $\left(\frac{2}{100} \times 80000\right)$

= N 1600. =

The net selling price of the motor car = actual selling price minus brokerage

= N 80,000 - N 1600 = N 78400.=

then the total brokerage received by the broker

= N (2 x 1600)
= N 3200. =

Do the following exercises, to see if you followed the lectures. Do not hurry over any problem, if not clear go to the text, only through this means that you will understand what is taught.

Exercise 5.3

1. What percent of 700 is 77?
2. If 5% of a number is 15. Find the number.
3. A pharmacist bought a drug for N 375 and sold it for N 420. =. Find his or her profit or loss percent.
4. A salesman sells goods worth N 5,500. He receives N165 as commission. Find the rate of his commission.
5. A broker finalises a deal of a piece of land for N 125000 at 1% brokerage. Find the total amount of brokerage received by him in this deal.

Answers to Exercise 5.3

1. 77 $\left(\frac{\times 100}{100}\right)$ = 11%

2. 100 $\left(\frac{\times 15}{5}\right)$ = 300

3. Profit % = $\frac{S.P - C.P}{CP} \times 100 = \frac{420 - 375}{375} \times 100$

$= \frac{45}{375} \times 100 = 12\%$

4. Commission rate = $\frac{165}{5500} \times 100 = 3\%$

5. Total amount of brokerage = 2% of N 125000

$\left(N \frac{2}{100} \times \frac{125000}{1} \right)$

$= \underline{N\ 2500}$

4.0 Conclusion

In this unit, you have learnt ratio, proportion and percentage and have also learnt that equal ratios means proportion and percent is another form of ratio, thereby seeing the relationship between the three concepts and their applications.

5.0 Sum mary

In this unit, you have learnt that

(i) Ratio is used in the comparison of two or more quantities of the same kind. Also a : b mean a/b while b: a => b/a so a: b b : a.

(ii) Proportion => equality of ratios between two pairs of quantities

(iii) Percent => a hundredth that is a fraction whose denominator is 100 and that percent also ratios of a special kind.

(iv) Direct proportion => the two quantities increase or decrease by the same amount.

(v) Indirect proportion =>, as one quality is decreasing or increasing the other quantity is increasing or decreasing at the same rate. That is

A increases =:> B decreases and
A decreases =:> B increases.

(vi) Applications of percent to profit and loss, discount, commission and brokerage are also learnt.

$$\text{Profit percent} = \frac{\text{Selling Price - Cost Price}}{\text{Cost Price}} \times 100$$

$$\text{Loss percent} = \frac{\text{Cost Price - Selling Price}}{\text{Cost price}} \times 100$$

Discount = discount rate x marked price

Commission = rate x sale value of good sold.

Brokerage = rate of brokerage x sale value of property

Total brokerage = 2(rate of brokerage x sale value of property)

6.0 Tutor - Marked Assignm ent.

1. In what ratios are the quantities 4m 20cm, 14m and 2m 80cm?
2. Three nurses provide N1400, N 3000 and N 2000 and opened among them in the same proportion. If the interest in the first month is N 1582, how much does each receive?
3. Fatima pays N 496.00 for a nursing textbook. She is told her discount is 20%. What is the list price of the textbook?
4. A saleslady's commission rate is 7.5%. She sells drugs worth N 3700. How much commission does she receive?
5. A broker received N 1650 as brokerage in a deal for a house if the rate of brokerage is 1.5%, find the selling price of the house

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UNIT 7

GRAPHS

Table of Contents

1.0 Introduction

Graphs are pictorial representation of the functional relations between two variables. Note a variable is a quantity which takes various values in a given problem and functional relation. Hence when the relation between x and y is drawn on paper, it is called the graph of the function.

There is an extensive use of graphs in the nursing profession. Like in the temperature charts, Blood pressure, Apical -radial measurements. Etc.

In this unit, you will learn about coordinates of points in the Cartesian plane, plotting of points and choice of scales, drawing of graphs.

2.0 Objectives

By the end of this unit, you should be able to:
- Indicate the coordinates of given points in the Cartesian plane
- Choice scales and plot points of given functions
- Make table of values and draw graphs
- Solve given problems using graphs.

3.0 Main Content

3.1 Rectangular Coordinates and Cartesian Plane

In drawing graphs, two mutually rectangular straight lines XX1 and YY1 (XX1 and YY1 are in relation to one another) from a rectangular coordinate system. These straight lines XX1 and YY1 are called coordinate axes. The XX1 is drawn horizontally i.e. the same as the lines on the exercise books and is called the axis of abscissas (x - axis) while the YY1 is drawn vertically i.e. at right angle to the XX1 and is called the axis of ordinates (y - axis), their point of intersection) 0 is called the origin of coordinates. Example.

Fig. 6.1
Rectangular Coordinates

In fig 6.1, 0 is the origin and to the right of 0 in the direction of X is the positive x - axis marked X or the abscissa while to the left of 0 is the negative *x -axis marked X1 and above 0 is the positive is the positive y - axis marked* Y and below 0 is the negative y-axis marked Y1. The numbers or quantities on both the abscissa (x - axis) and the ordinate (y-axis) are referred to as the rectangular coordinates or simply coordinates of a point in the plane and this plane is referred to as the Cartesian plane named after Descartes the great mathematician and philosopher points O, A, B, C, D are marked respectively. O is the origin and are represented by (0,0) which means that it has no value along the x and y axis i.e. it has no value along the abscissa and ordinate and therefore the value (0,0) indicates the coordinates of the point 0. Consider the point A in fig 6.1: to get to a, travel 3 units in the direction of 0x and then 2 units in the direction of 0y. That is

The x -coordinate (or abscissa) of A is + 3

The y coordinate (or ordinate) of A is + 2. Then we say that the coordinates of A are (3,2) or that A is the point (3, 2)

Note the x - coordinate or abscissa is always given first so A, (3, 2) and Az (2, 3) are not the same. Consider the points B, C and D in fig 6.1. B has coordinate (-l, 3) showing that it is 1 unit in the direction of 0 x 1 (negative axis) and 3 unit in the 0y direction. The other points are C (-3, -2) and D (4, -3).

Now give the coordinates of the points E, F, G and H marked in fig 6.1

Answers to check your progress.

The coordinates are E (2, -2),
F (1, 2)
G (-1, 0) note: G his on the negative x-axis at the point

Where x = Y. So in the x -axis, y has no values and on the y-axis, x has no values. This also applies to H, on the x axis H has no values and on the y-axis H has a value of 3, so it is written as H = (0, 3). i.e. H = (0, 3).

3.2 Graphical Representation of functions.

Plotting of Points and Choosing of Scales

Every point in the plane is associated with one number pair: x, y. every pair of real numbers x, y is associated with one point as in fig 6.1. When points are marked as in fig 6.1, we say we are plotting the points. Hence to find a given functional relation graphically, marks say 1, 2, 3, 4, 5, ... of the variable x and construct the ordinates y1, y2, y3, y4, y5, ... which are the corresponding values of the variable y hence we obtain a number of points like A1, (1, y1,) A2 (2 y2), A3(3, y3) etc from the given functional relation.

Choice of Scales:

The choice of scales for both the axis of abscissas (x – axis 0 and the ordinates (y-axis) depends on the range of values got from the table of values. Atimes the scales are given.

Always indicate the scale in the top right hand corner of the graph.

Now the next stage is graphical representation of functions.

In order to draw the graphs of either the straight line y = m + c or any graph smoothly and accurately, the following techniques are useful.

(1) Construct a table of values of y against (since y is dependent on the value of)
(2) Choose suitable scales for both the abscissa and ordinate (y -axis). If no scales are given, taking into account the range of values for x and y in the table of values constructed.
(3) Always indicate the scale on the top right - hand corner of your graph sheet.
(4) Plot the tabulated points or values (remember the)C coordinates comes before the y coordinates)
(5) Join the plotted points for a straight line, any three points is okay [to draw a straight line containing the other points]. But if the graph involves an equation of the form a 2 + b + = 0 (which is a quadratic equation) use either a broomstick or free hand to obtain a smooth curve.
(6) Title your graph - indicate the function whose graph is plotted always.

Example: (i) Draw the graph of y = 2 +1 from = 0 to = 10.
(ii) Draw the graph of y = 3 + 1 from = -3 to = 3

Solution

(i) First make a table of values for the equation

y = 2 + 1 from = 0 to ,=10.

Here the values of has been given you then find the corresponding values for y, thus: when = 0
Substitute for = 0 in y = 2 + 1 i.e. y = 2 x 0 + 1 = 1
When = 2, y = 2 x 2 + 1 = 4 + 1 = 5 etc. the values are then arranged in a table called the table of values. Alternatively, treat the terms individually and combine later as shown in the table of values

Table of values for y = 2x +1

x	0	1	2	3	4	5	6	7	8	9	10
2x	0	2	4	6	8	10	12	14	16	18	20
+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1
y=2 +1	1	3	5	7	9	11	13	15	17	19	21

Notice that + 1 was added throughout because 1 is a constant, its value does nor change.
Now move to choice of scale

Choice of Scales

On the axis of abscissa (- axis)
Let 1 cm = 1 unit. Because the values of are all positive and the range of values is from 0 to 10. It can be accommodated on the horizontal axis.

On the ordinate axis (y – axis) the range of values are between 1 and 21 1 cm to 1 unit will be two spread out and there is the need to leave a little space above and below the graph. So we can then say

Let 1 cm represent 2 units.
Note if you do not know what 1 cm is from the graph sheet, they are five of the tiny squares.
Now see the graph sheet attached.

For the graphs of (i) y = 2 + 1 see fig 6.2
(ii) y = 3 + 1 see fig 6.3 on both sides of the graph sheet.

Table of values for y = 3 + 1 from = -3 to = +3

Table of values

	-3	-2	-1	0	1	2	3
3	-9	-6	-3	0	3	6	9
	+1	+l	+1	+1	+1	+1	+1 +1
y=3 +l	-8	-5	-2	+1	4	7	10

Here the ranges of values for are given and that of y ranges from - 8 to 10.

Choice of Scale: on the axis of abscissa (- axis)
Let 2cm represent 1 unit
On the ordinate axis (y -axis)
Let 2cm represent 2 units.

Scale on the abscissa (- axis): 1 cm to 1 unit

On the ordinate axis (y -axis0 1 cm to 2 units.

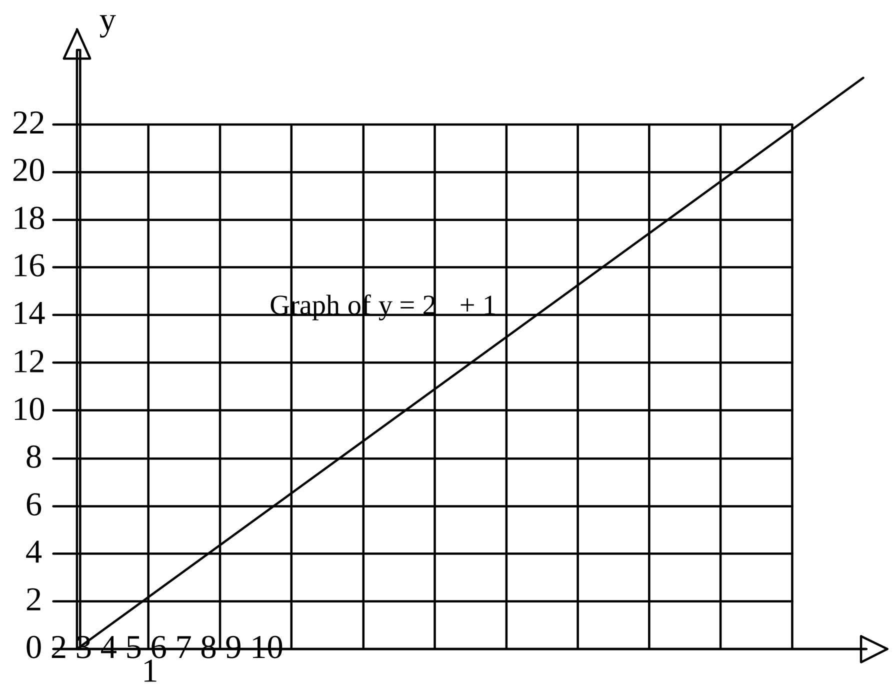


Fig 6.2

Scales: On the abscissa (- axis). Let 2cm represent 1 unit
On the ordinate   (y – axis) 2cm represent 2 units.

Fig 6.3

Extract from: Basic Nursing Skill: A Self - instructional Approach. By Quiring Rubeck (1977). California: Wadsworth Publishers Company.

5 - Apical - Radial Measurement

Graphic Sheet

Oral - Temperature recorded with a circle
Rectal - Temperature recorded with the letter R above
Axillary - Temperature recorded with the letter A above

Fig 6.4

The above graph is a typical type of graph you are expected to draw in your profession.

Exercise 7.

Now practice these graphs.

1. y = - 8 from = -1 to = 11.
2. y =7x - 9 from = -1 to = 3
3. 2y + 3 - 2 = 0 from = -5 to = -1. Here first make y the subject of the formulas by shifting the other terms to the right hand side of the equation.

   Thus 2y = - 3 + 2

   $y = \frac{-3}{2} + 2$. Now make up the table of values for the equation,

   choose your scales and plot the coordinates, then join. Try plotting only three points and see if the other coordinates fall into the straight line drawn

3.3 Application - Graphical Treatments

Having known how to draw the graphs. We now move on to the uses of graph; hence the graphical treatments. Through the already plotted graph you can read off or read meaning into the situation of things.

For example, from fig 604 the Apical - Radial measurement graph sheet. You can interpret the patient's temperature and pulse. The temperature rose from 120° F to 130° F within 4hours but dropped from 130° to 120° in the next 12 hours. At the same time the pulse maintained a bit balance between the 8 and 4 18 hours and rose from 4 - 8 (16 hours). The blood pressure (both the systolic and diastolic was also indicated. These are the information about the patient the doctor would like to know to enable him/her diagonise and treat accordingly.

Now move on to graphical treatment of the solutions of simultaneous linear equations simultaneous equations means solving the two equations together. In graphical treatment, all you need to do is to plot the graphs of the two equations on the same graph sheet and find their point of intersection, if any. The coordinates of the point of intersection are the solution to the two equations.

Here is an example:

Solve the following equations graphically.
5y - 5 + 3 = 0 from = -4 to = 1
2y + 3 – 2 = 0 from = -4 to =1

Solution:

First make up the table of values but before then make y the subject of formula.

5y - 5 + 3 = 0 => 5y = 5 - 3

$$y = \frac{5x}{5} - \frac{3}{5}$$

Table of values for y = - 3/5 from = -4 to .

-4 -3 -2 -1	0 1					
- - - -	-	-	-			
y = – -4	- 3	-2 -1	-			
y = - -2	3/5 -18	/5 -13/5	-8/5 -	3/5 2/5		

2y + 3x – 2 = 0 => 2y = -3x + 2

$$y = \frac{-3}{2}\chi + 1$$

Table of values for y = - 3/2 + 1 from x = - 4 to x = 1

x -4 -3 -2 -1	*0 1*					
-3/2x 6 9/2 3	3/2 0 -	3/2				
+1 +1 +1 +1	+1 +1	+1				
y = -3/2x + 1	7 11/2	4 5/2	1 - ½			

Choice of scale:

On the axis of abscissa (x -axis)
Let 2cm represent 1 unit
On the ordinate axis (y -axis)

Let 2cm represent 2 units

Now plot the coordinates of both equations i.e. y –   and y = - 3/2 x + 1 on the same graph sheet we can do this together.

From the graph the intersection of the two straight lines are 10.64, 0.04). This is got by drawing a straight line from the point of intersection to the abscissa (x - axis) and ordinate (y -axis)

(2) Draw the graph of the temperatures of Lokoja taken every hour during the day time on one day using the following table of values.

Use your graph to find temperature at (a) 10.30am (b) 4.30pm

Time	7am	8am	9am	10am	11am	12am	1pm	2pm	3pm	4pm	5pm	6pm	7pm
Temperature	25oC	25.9	26.8	27.5	28.1	28.6	29.1	27.8	27	26.4	25.8	25.4	25.2

Use your graph to find temperature at

(a) 10.30am (b) 4.30pm (c) 2.30pm
(d) 9.15 (e) 6.45pm

Solution:

The table of values was given. The next step is to choose the scales.

Choice of Scale:
On the time axis: let l cm represent 1 unit.
On the temperature axis: let 2cm represent 1 unit (since the values ranged from 25° to 29.1o.
After drawing the graph; to answer the questions, draw a vertical line from the point where the time is located to the graph. Wherever it touches the graph, draw a horizontal line to the temperature axis. Then read off the value.

From your graph, the answers to the above questions are
(a) 27.8°C (b) 26°C (c) 27.3°C (d) 26.9°C (e) 25.3°C

See graph.

Temperature - time graph

4.0 Conclusion:

In this unit, you have learnt how to find the coordinates of points in a Cartesian plane, choose scales and make up table of values. You have learnt also how to draw graphs and solve problems using graphs.

5.0 Sum mary

In this unit, you have learnt that to draw graphs the following techniques are necessary, to obtain accurate and smooth lines.

1. Construct a table of values from the function
2. Choose suitable scales for both the abscissa and ordinates axes. If not given to take into account the range of values for x and y.
3. Always indicate the scale used on the right hand corner of the graph.
4. Plot the tabulated values
5. Join the plotted points and for a straight line graph, any three points is okay.
6. Title the graph.

6.0 Tutor - marked Assignments

1. The temperature of a patient in hospital from 6am to 6p.m measured at hourly intervals. From your graphs, find the most likely temperature of the patient at

   (a) 7.30am (b) 12.30p.m

   Time (hrs) 6am 7am 8am 9am 10am 11 am 12noon

   Temp (°C) 37.8 38.0 38.3 38.6 39.1 39.2 39.0

   Time (hrs) lpm 2pm 3pm 4pm 5pm 6pm

   Temp (°C) 38.9 38.7 138.8 138.7 38.2 138.0

2. The temperature of a pan of water on a stove measured at 2 minute intervals, in degrees Celsius. Find the temperature at

   (a) 3 minutes
   (b) 13 minutes from the start. At what time approximately do you think the heat supply was turned off?

Period of time after start (min)	0 2	4 6 8	10 12	14 16	18 20						
Temp (oC) to nearest degree	20	28 38	50 62	74 84	92 96	93 88					

3. Solve graphically the following equations

   y = 2x + 2 ------- (1)
   y = 3x + 2 ------- (2) for values of x from x = -2 to x = 4

7.0 References and Other Resources

Quiring, Rubeck (1977): Basic Nursing Skills: A Self- instructional Approach. California Wadsworth Publisher Company Inc.

Vygodsky, M. (1972). Mathematical Handbook: Elementary Mathematics. Moscow: MIR Publishers.

UNIT 8

ANGLES ON PARALLEL LINES AND POLYGONS CIRCLES

Table of Contents

1.0 Introduction:

Our knowledge of geometry is organised in such a way that more difficult concepts are made clearer by simple concepts.

In this unit, you will learn about the angle on parallel lines, and polygons, properties of parallelogram and circles. But before treating the contents of the unit, the first sections of unit shall deal on definitions to enable you understand and appreciate the unit.

As nurses to be, you might be wondering the place of angles in your area of specialisation. B y the time you come to the end of this unit, your fears must have been allayed because you might have been able angle to identify the application of the knowledge of angles in your day-to-day activities as a nurse.

2.0 Objectives

By the end of this unit, you should be able to:
- Define angles
- Explain the different types of angles
- Define parallel lines and transversal
- Identify the different angles on parallel lines
- Apply the concept of angles in practical problems
- State the properties of parallelograms
- Find the interior and exterior angles of a polygon
- Describe the circle in relation with its parts.

3.0 Main Content

3.1 Angles - Definition and Types

Definition (1) An angle is formed by two rays (see fig. 7.1) 0A and 0B coming from a single point 0.

B

A

Fig. 7.1

(ii) An angle is formed when two straight lines meet at a point.

The rays (straight lines having a starting point but extends indefinitely) are called the arms or sides of the angle. Symbolically, an angle is represented by “ $\angle$ ”

Angles are measured in two systems namely (1) the radians and (ii) the degrees. They differ in their unit of measurement. The unit of measurement in the degree is one degree = 1/360 of one complete rotation, round the circle or exemplified by the hand of a clock moving from 0° hours to 12 hours which constitute 360°. A degree consists of 60 minutes denoted by 60' and a minute consists of 60' seconds denoted by 60".

In radian measure
2p radians is equivalent to 360°

p radians is equivalent to 180°. This relationship is always used in the conversion of angles from one unit of measurement to the other.

3.2 Types of Angles

Angles are named either accordingly to their sizes (the measurement) or according to shapes

(1) An angle of 90° (that one-quarter of a complete revolution) is called a right angle and is denoted by "rt. "

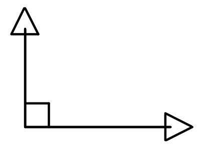
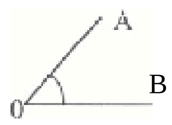


Fig 7.2. Right angle

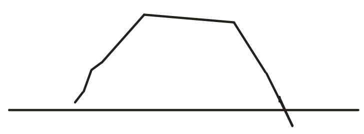
(2) An acute angle is an angle whose size is less than 90° OR an angle that is less than 90° is called an acute angle.



(3) An angle greater than 90° but less than 180° is called an obtuse angle.

Fig 7.4: Obtuse angle

(4) 180° is half of a complete revolution is referred to an angle on a straight line.



(5) An angle greater than 180° but less than 360 (one complete revolution) is called a reflex angle.

Note in angular measurements, clockwise rotation indicates negative angles or negative values of angles, while counter clockwise rotation of the rays correspond to positive angles

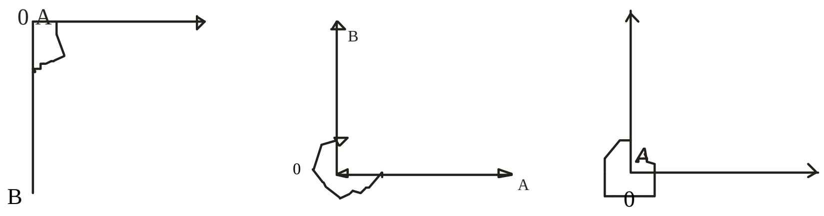


Fig. 7.5 (a) (b) (c)

In Fig. 7.5, if a ray OA moves to join (coincide) with OB see Fig. 7.5 (a), the < AOB = + 90°
In fig. 7.5 (b), AOB = - 90°. These angles are
In Fig. 7.5 (c), AOB = - 270° the same but the only different is in the direction of measurement where the plus sign shows measurement in the counterclockwise direction and the minus sign shows measurements in the clockwise directions.

(6) Adjacent angles

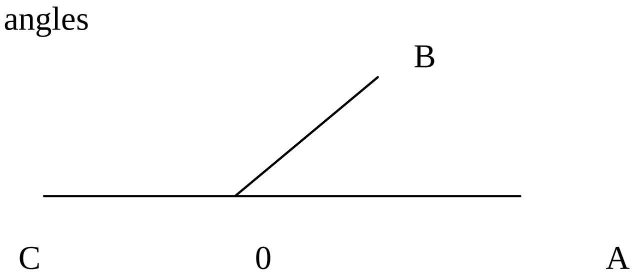


Fig. 7.6 (a) Adjacent angles

A pair of angles AOB and COB, in Fig 7.6 with common vertex 0 and common side OB are called adjacent angles. The sum of two adjacent angles on a straight line is 180°. There are other types of adjacent angles (see Fig. 7.6 (b)

Fig. 7.6 (b)

AOB and BOA are adjacent angles.

(7) Vertically opposite angles (or vertical angles) C

Fig. 7.7

are angles which have a common vertex and the sides of one are the extended sides of the other. In fig 7.7 FOC and DOE, also COE and DOF are vertically opposite angles

Note vertically opposite angles are equal

i.e. FOC = DOE and

COE = DOF. The four angles are called angles at a point.

(1) Now take your protractors and verify this claim

(2) Go to any hospital near your house and find out, the angle they use in giving injections?

(8) Bisector of an angle: This is a straight line that divides an angle into two equal parts. This will be shown clearly in unit 8.

bisector

A

Fig. 7.8

In Fig. 7.7 the bisectors are OA and) B.

Now move to angles in parallel line

3.3 Angles on Parallel Lines

What do you mean by parallel lines. This takes us to the definition of parallel lines.
Some define parallel lines as lines that do not meet. But parallel lines are better described as lines that lie on the same plane and maintain equal distance apart. Examples are the lines on your exercise books; the ones on the railway track, opposite edges of the exercise books etc. find more examples

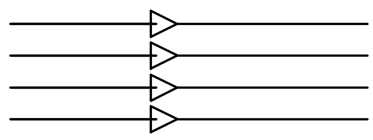
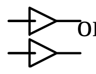


Fig 7.9 Parallel lines

Parallel lines are denoted by "  or //c or // "

A line which cuts across other straight lines is called a transversal.

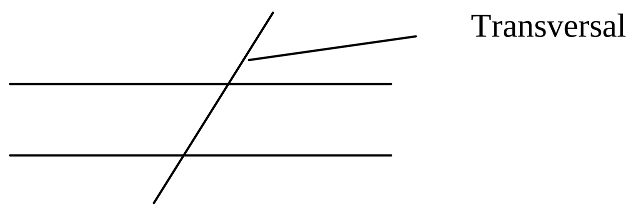


Fig 7.10 transversal

When a transversal cuts two parallel lines, angles in fact eight angles are formed.

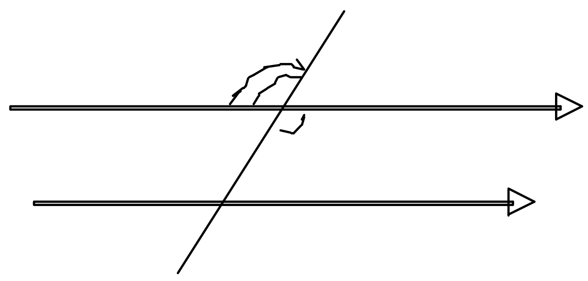


Fig 7.11. The transversal EF cuts two parallel lines AB and CD at H and G respectively.

Can you now identify the angles treated in section 3.1?

Looking at fig. 7.11,you will see that there are four adjacent angles CGF, FGD, AHE and EHB (You can name them yourselves). From the previous section also, you can identify and name the vertically opposite angles - AHE and EBH, CGF and FGD. You can from section 3.1 that vertically opposite angles are equal. Have you verified that? Please do.

Exercise 8

Now bring out your protractor and measure all the eight angles. What did you discover?

Here letters will now be used to identify the angles for easy referencing see fig. 7.12.

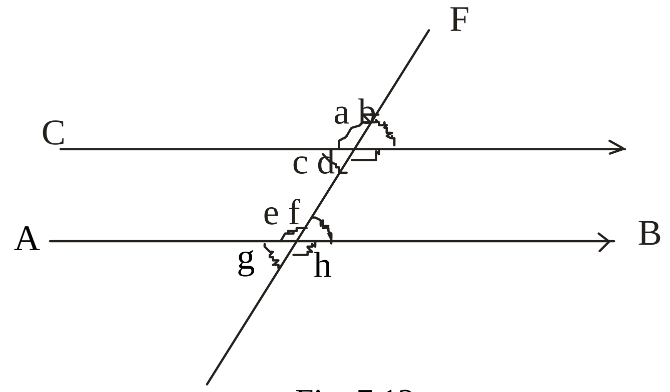


Fig. 7.12.

Angles on parallel lines

1. Corresponding angles are angles on the same side and maintaining the same position on the same side of the transversal.
   In Fig. 7.12.

   Angles a and e
   b and f
   c and g
   d and h
   are all corresponding angles

   They are also equal if they are both acute or both are obtuse angles. If not their sum is equal to 180°

2. Alternate angles are angles at alternate positions and at alternate sides of the transversal.

In Fig 7.12

Angles c and f
d e

are alternate interior angles. They are also equal. While angles a and h; b and g are alternate exterior angles and are equal.

3. Interior angles on the same of the transversal.

In Fig. 7.12, angles d and f

c and e are interior angles. The sum of d + f = 180° and the sum of c + e = 180°. Therefore we conclude that the sum of the interior angles on the same side of the transversal are supplementary

Note, if you have taken time to measure the angles in Fig. 7.11 or the ones you have drawn, you will be able to find out for yourselves these properties and many more.

4. Exterior angles on the same sides of the transversal are angles b and h; a and g. their sum is equal to 180° i.e. b + h = 180°; a + g = 180°. Again they are supplementary angles.

Now try this exercise.

Using the letters in Fig. 7.12. Identify the following angles

(i) adjacent angles. (ii) Vertically opposite angles.
(iii) Angles at a point

You might use the answers below as a check.

(i) Adjacent angles on a straight line are angles a and b; c and d, a and c, b and d, e and f, e and g, g and h and f and h. are there others.
(ii) Vertically opposite angles are angles b and c, a and d, e and h, f and g.
(iii) Angles at a Point are angles a, b, c, d and e, f, g, h.

Applications

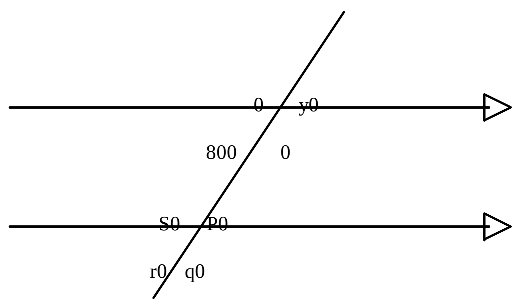
1. In Fig. 7.13 below, which angles correspond to f

(a) r (b) p (c) b (d) s and justify

Solution

a) Q correspond to r (corresponding s)
b) g corresponds to p (“)
c) t corresponds to b (“)
d) c corresponds to s (“)

2. Find the values of the marked angles and give reasons for your answers.



Solution:

+ 80° = 180° (adjacent Ls on a straight line)
= 180 - 80 =100°
y = 80° (vertically opposite angle to 80°)
= 100° (vertically opposite angle to ° = 100°)
S = 100° (corresponding angle to ° = 100°)
P = 80° (corresponding angle to y° = 80°)
q° = 100° (vertically opposite angle to S = 100°)

r = 80° (corresponding angles to 80°)
You shall observe that there are other relationships between angles that can also be used o justify the claims. For example

80° + S = 180°{interior   s on same side of transversal}
S = 180 - 80 = 100o
P° = r° (vertically opposite angles) etc. now find the other angles using other relationships not used above.

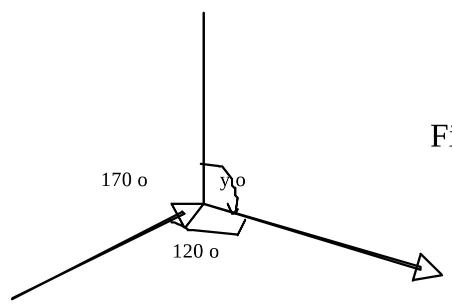
(3)

Find the marked angles in this figure. And justify your answers.

Solution:

b + 85° = 180° (adjacent   s on a straight line)
b = 180° - 85 = 95°
° = 85° (alternate angles)
c° = x° = 85° (corresponding angles)
a = b = 95° (corresponding angles)

4.



Find y.

Solution:

y + 170° + 120° = 360° (angles at a point)
y + 290 = 360°

$y = 360 - 290° = 70°$
$y = \underline{70}$

Having known the angles on parallel lines move on to the next section.

3.3.1 Properties of Parallelograms

Having treated parallel lines and angles on parallel lines. We now consider the parallelogram from practical activity.

Now cut out a parallelogram from a piece of paper. You know what a parallelogram or `parm' as it is often times called. If not, a parallelogram is a quadrilateral (i.e. four sided figure) with opposite sides parallel? The other properties are what you shall find out very soon.

$$\left.\begin{array}{l} AB = CD \\ BC = AD \end{array}\right\} \text{Opposite and side of parallelogram}$$

Fig 7.14 parallelogram

i) With your protractor, measure the angles. Write down your observations as you get along.
ii) Draw the diagonal [i.e. either join A to C or B to D] what can you say of the shape, when it is divided into two?
iii) Draw the remaining diagonal. Any comments?

Now we do it together and report our observations. Also practise alone at your spare time.

In any parallelogram here Fig. 7.14

(a) The diagonals of a parallelogram bisect each other. (AO = OC and BO = OD)
(b) The opposite angles of a parallelogram are equal (A = C and B = D)
(c) The sum of squares of the diagonals is equal to the sum of squares of the four sides.

AC2 + BD2 = AB2 + BC2 + CD2 + DA2 = 2(AB2 + BC2)

(d) AF is the altitude (perpendicular height).

The area of a parallelogram is equal to the product of the base (any of its sides can be called base but in Fig. 7.14, let the base be DC] and the altitude (AF)

AREA DC x AF

Distinguishing features of parallelograms

A quadrilateral ABCD is a parallelogram if and only if

(1) The opposite sides are equal AB = CD, BC = DA
(2) Two opposite sides are equal and parallel AB = CD, AB//CD
(3) The diagonals bisect each other
(4) Opposite angles are equal (A = C; B = D)

If one of the angles of a parallelogram is a right angle then all angles are right angles. And such a parallelogram is either (a) a rectangle if the sides serve as altitude

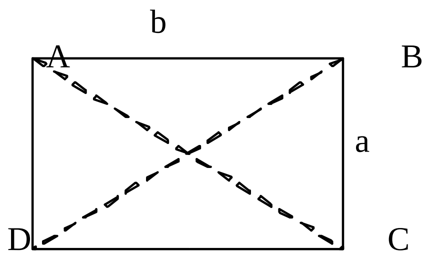


Fig 7.15

i) The area of a rectangle is equal to the product of its sides.
ii) The diagonals of a rectangle are equal. In fig. 7.15 AC = BD.
iii) In a rectangle, the square of a diagonal is equal to the sum of the squares of the sides: AC2 = AD2 + DC2

(b)

If a parallelogram has all sides equal, it is called a rhombus. In a rhombus, the diagonals are mutually perpendicular.

(AC -ar BD) and bisect the angles, so that DCA = BCA and DBC = ABD

The area of a rhombus is equal to half the product of its diagonals. (If AC = d1 and BD = d2 in Fig 7.16)

Area of rhombus = ½ AC x BD = ½ d1 d2.

(c). A square is a parallelogram with right angles and equal sides

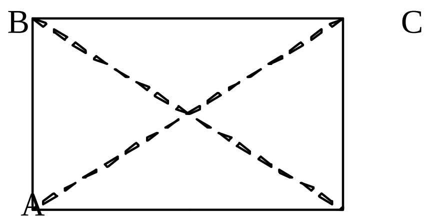


Fig 7.17

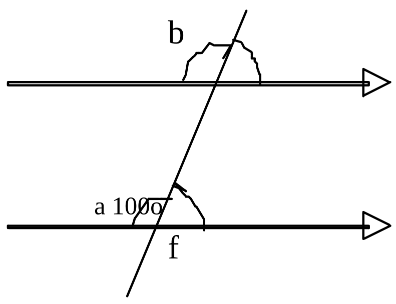
It is a particular type of rectangle and also a special type of rhombus. Now find out which of the properties of these shapes rectangle and rhombus that makes a square special?

Now continue with these exercises below

Exercise 7.1

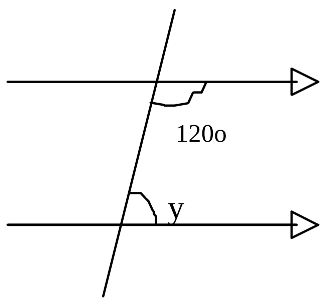
Find the marked angles in the following diagrams and justify your answers.

l.

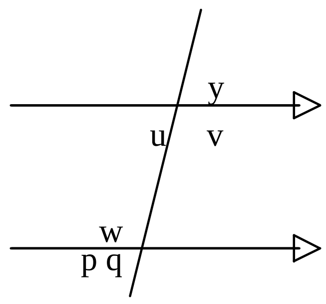


(2)

(3)



(4)



In question No (4). Write out all the following pairs of (u) vertically opposite (u) alternate interior (w) alternate exterior (w) corresponding angles.

Answers to exercise 6.1

1. = 100° (corresponding s)
   a + 100 = 180° (adjacent angles on a straight line)
   a = 180 - 100 = 80°
   b = 80o (corresponding to a)
   f = 80o (vertically opposite to a)

2. = 480° (alternate s)

3. y + 120° = 180° (interior s on same side of transversal
   y = 180 - 120 = 60°

4(i). Vertically opposite angles; & v, u and y, w and q, , and p

(ii). Alternate interior angles: u and , v and w.

(iii). Alternate exterior angles; y and p and x and q

(iv) Corresponding angles x and w, y and Z, u and p, v and q

It seems you are making some progress good. We continue to the next section - polygons

3.4 Polygon Angles in a Polygon.

A closed plane figure whose sides are all straight lines is called a polygon. Examples of polygons - triangles, quadrilateral pentagon (5 sides), hexagon (6 sides), heptagon (7 sides) octagon (8 sides) etc.

Angles in a polygon (Regular Polygon: All sides equal)

Fig. 7.18

Looking at the marked angles of the triangle in fig. 7.18. There are two types namely the interior angles (those inside the triangle i.e. a, (3and 0) and the exterior angles (those outside the triangle i.e. a, b and c).

Re call that the interior angles of a triangle a triangle sum up to 180° i.e. (a + ß +   ) - 180°. We shall come to this later.

For the exterior angles: Take a thread or string go right round the out of triangle ABC, starting from A move to B down to C. you will see that the path you have traced is a circle and the angles in a circle is 360° (angles at a point)

Therefore, the sum of the exterior angles of a triangle i.e. a + b + c = 360°. Now take up a four sides polygon - the quadrilateral

Fig 7. 19

Inter angles - the diagonal divides the quadrilateral into 2 triangles. Therefore, the sum of the interior angles of the quadrilateral is 2 x 180 = 360° Exterior angles: Repeat the activities as that of the triangle. You will also find that the path replaced is a circle and so, the sum of the exterior angles of a quadrilateral is 360°

So, for the Exterior angles of a polygon the path traced from one end of it to the other no matter the number of sides of the polygon is a circle.

There, we conclude that the sum of the exterior angles of a polygon is 360°

The Interior angles of a polygon
Each interior angle and an exterior angle adjacent to it equals 180° = 2right s (2rt.   s).

Here if the figure (polygon) has n sides, the sum of both interior and exterior angles equals 2n right angles. Now if we subtract the sum of the exterior angles (360o = 4rts   s) of any polygon.

The sum of the interior angles of a polygon will then be (2n - 4)rt .   s.
Each interior angle = $\left(\frac{2n - 4}{n}\right)$ s and each ext $= \frac{360}{n}$ where n = number of sides

Examples

1. A quadrilateral the sum of the interior angles

Using (2n - 4) rt s where n = 4 is
(2 x 4 - 4)rt s
= (8 - 4)rt s
= 4 x 90 = 360° same answer as above (i.e. using the fact that quadrilateral = 2 x)

2. A pentagon (5 sides)

The sum of the interior angles = (2 x 5 - 4) rt. s
= (10 - 4)rt s
= 6rt s
= 6 x 90
= 540°
You can verify this by joining all the vertices of the polygon to a point p (centre) inside the polygon. Count the number of triangles and multiply by 180 or 2rt angles then subtract 360° 4rt s for H the sum of the angles at P

3.5 Circles - Properties

A circle is the locus of the point at a constant distance from a given fixed points.

Parts of a Circle

1. Radius: A line segment from The centre to any point on the circumference. It is usually denoted by r. in fig 7.20 OP =OA=OB=OC=r

A

C

Fig 7.20

2. Diameter: The straight line joining any two points on the circumference but passes through the centre 0 of the circle. In fig. 7.20. BP and AC are diameters. It is usually denoted with D and it is equal to twice the radius i.e. D = 2r.

3. Secant: A straight line passing through any two points on the circumference. In fig 7.21 the lines UV and XY are called secants

Fig 7.21

4. Chord: A chord is a straight line that joins any two points on the circumference.
It is part of a secant. In fig 7.21, AB, CD are chords

5. Segment - A segment of a circle is the area between a chord and the arc subtended by the chord. A chord divided a circle into two parts called segments. In fig 7.21 the marked areas are segments. There are two types of segments, minor segment - the unshaded part and major segment the shaded part which contains the centre of the circle: A semicircle is half of a circle and
it is a special form of segment why?

Fig. 7.21

6. Arc: An arc is part of a circle. In fig 7.21 the curve AB is an arc.

7. Sector: A sector of a circle is the area, enclosed by two radii (plural of radius) and an arc. In fig 7.22 the shaded area is a sector. The unshaded area is also sector.

B

Sector

Fig 7.22

8. Circum ference - The circumference of a circle is the distance round the circle.

4.0 Conclusion

In this unit, you have learnt the different types of angles - acute, obtuse, reflex, vertically opposite, adjacent angles, and alternate angles. You have also learnt the properties of the parallel lines when cut by a transversal in terms of the angles formed. You have also learnt the sum of the interior and exterior angles of a regular polygon and the properties of a circle.

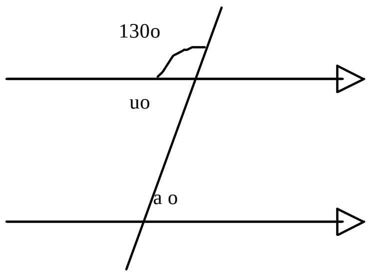
5.0 Sum mary:

In this unit you can summarise your studies thus that:

1. Angles are measured in the counter clockwise direction either in degrees or in radians.
2. Acute angles are < 90°; Obtuse >90o and reflex >180o
3. The sum in adjacent angles on a straight equals 180°
4. Vertically opposite angles are equal, alternate angles are equal.
5. When two parallel lines are cut by a transversal
   - Corresponding angles are equal
   - Alternate angles are equal
   - The sum of the interior or exterior angles on the same side of the transversal equals 180°
6. Parallelograms are quadrilaterals with pair of opposite sides equal and parallel

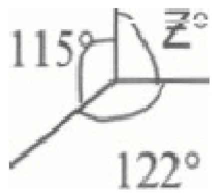
6.0 Tutor-Marked Assignm ent 8

1. Find the values of the angles marked in the following figures. Justify your answers.

(a) 

(b)

(c)

(d) 

(e)

2. Find the size of each interior angles of the following

   (a) a regular pentagon (b) a regular 15 -sided polygon

3. If a regular polygon has an exterior angles of 10° find the number of sides of this polygon.

4. The angles of a pentagon are °, (+20°), (-15°), 2 °, $\left(\frac{3x}{2} - 30^{o}\right)$

   find x.

7.0 References and Other Resources

Vygodsky, M. (1972). Mathematical Handbook: Elementary Mathematics. Moscow: MIR Publishers.

David - Osuagwum M; Anemelu, C; Onyeozili, 1 (2000) New School Mathematics for Senior Secondary Schools. Onitsha: Africana - Fep Publishers Ltd.

UNIT 9

CONSTRUCTION OF ANGLES

Table of Contents

1.0 Introduction

Construction as the name implies means to use certain instruments (here the mathematical set) in performing certain activities. To construct is different from to draw in the sense that the instruments used in construction are specified but in drawing any instrument that you deem necessary can be used.

In this unit, all constructions will be done by a pair of compasses and a ruler. Here also you will learn the bisection of a straight line and angle, construction of common angles of 90°, 60°, 45°, and 30°. Also the construction of straight lines perpendicular or parallel to a given straight line from a given point on or outside the given straight line.

2.0 Objectives:

By the end of this unit, you should be able to:

- Bisect given straight lines.
- Construct a perpendicular bisector from a point on and outside a given straight line
- Construct a straight line perpendicular to a given straight line
- Construct or copy given angles
- Bisect given angles.

3.0 Main Content

3.1 Geometric Constructions

The following precautions are to be taken to ensure a good construction

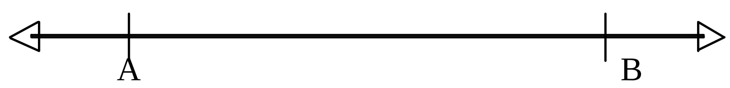
1. Use good pair of compasses that is the pair of compasses whose joint are not loose.
2. Use well sharpened and hard pencil to avoid double lines preferably use HB pencils.
3. Great care should be taken in drawing the line segments or lines as the case may be
4. Leave all construction marks as it is.
5. In case of erasure, erase completely, leaving no trace of the pencil marks.

3.2 Bisector of a Straight Line

A straight line has no definite length.

In this section, whenever straight lines are mentioned, line segments will be used in its place.
The bisect means to divide into two equal parts. This can be done excellently with the use of a ruler and pair of compasses. Now move on to the bisection of a straight line.

The following steps are to be followed
1) With a ruler and pair of dividers, mark any length of the straight line AB (line segment AB) on ~~AB~~ paper. The marks on will be clearly shown. Example



2) With any convenient radius (opening your pair of compasses to any width) greater than half of AB and with A and B as centres, make arcs (curves) above and below the line. AB. See fig 8.1 below

3) These arcs or curves will meet at the point C and D see figure 8.1

4) Join the points C and D. The line CD is the desired line and it touches AB at E. See fig 8.1 and the point E is the bisector

5) As a check, use your pair of dividers and rulers to measure AE and EB. What is your observation? Good move on. Also measure angles AEC and B EC. Are they equal? What are their values? If 90°, then the line CD is called the perpendicular bisector of the line segment AB.

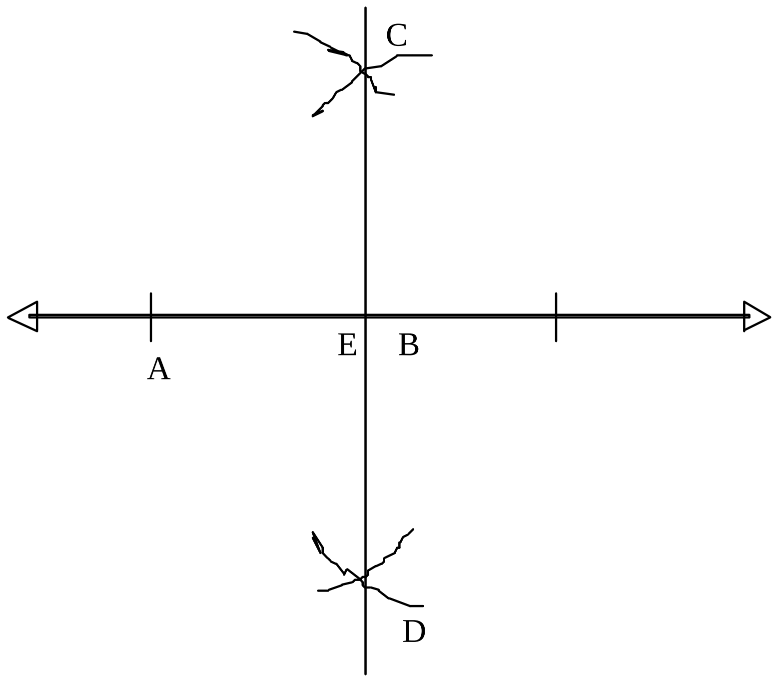


Fig. 8.1
Perpendicular bisector of a line AB

Note: Practise it along with me. In case of any problem, go through the steps.

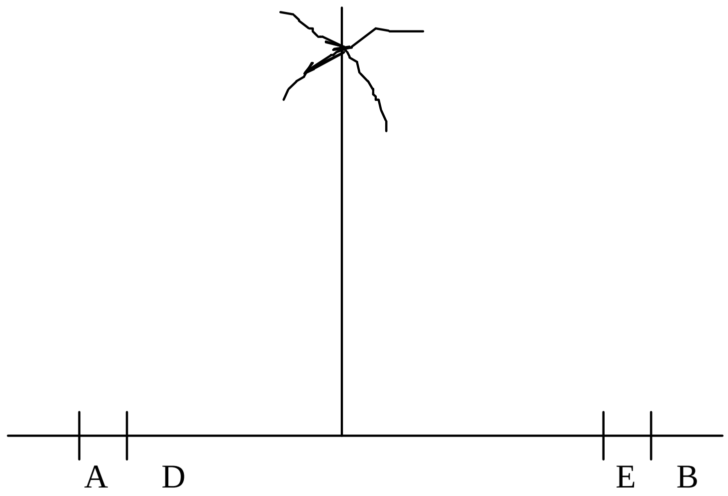
3.2.1 Perpendicular Line to a given Straight Line are of two types

(a) from a point on the line and
(b) from a point outside the line.

(a) From a points on the given line AB steps

(1) Draw a straight line AB with point C on it (see fig 8.2)
(2) With C as centre and any convenient radius draw arcs to cut AB at D and E respectively (see fig 8.2)
(3) With D and E as centres and with any radius greater than half of DE draw arcs to intersect at F (see fig 8.2)
(4) Join CF. This is the required line

I hope you joined in the construction. Learn to practice alone. Again you will observe that this was similar to the first. Do the same checking by measuring the angles on both sides it measure LACF and LBCF are they equal. Are their values 90°? What do you conclude?



(Fig 8.2)

A perpendicular to a given straight line from a given point on the given line.

(b) A perpendicular from a given straight line from a given point outside the given line.

Steps:

1. Draw a line AB with a point X outside AB see fig 8.3 below.
2. With X as centre and with any convenient radius, draw arcs to cut AB at M and N. (see fig 8.3)
3. With M and N as centres and any convenient radius greater than half XY, draw arcs to cut at Y. (see fig 8.3)
4. Join XY. The line XY is the desired line. By now you should mastered how to construct perpendicular because the steps here are similar to those of figs 8.1 and 8.2 respectively.

Fig 8.3, : A perpendicular to a given straight line from a given point outside the given line

3.2.2 A line Parallel to a given Straight line through a given point not on the given line.

Steps:

1) Draw the line AB and let C be the point outside AB
2) With C as centre and any convenient radius a circle to cut AB at D and E respectively. (See fig 8.4)
3) With D or E as centre (in fig 8.4) D is chosen) and the same radius make an arc to cut DE at F
4) With F as centre and the same radius again, draw an arc to cut the circle at G.
5) Join CG. This is the required line

Fig 8.4: Parallel line to a given straight line

There are other methods of constructing a parallel line to a given straight line. You check any mathematics textbook for these.

3.3 Bisector of Angles

Here again bisection of angle is similar to that of a straight line. The only difference is that an angle is involved.

To bisect any angle, the following are the steps:

1. Draw any ABC or measure the angle if the value is given.
2. With the point B as centre (since the angle is at this point) and any convenient radius draw arcs to cut line AB and BC (the arms of the angle at D and E respectively
3. With D and E as centres and with radius equal to that in (2) draw arcs to meet at F. (See fig 8.5)
4. Join BF. FBA = FBC. Check with your protractor.

3.3.1 Construction of Angle 90° and 45°

You know that 45° is $\frac{90^\circ}{2}$. The construction of angle 90° is the same what has been discussed in section 3.1.2 fig 8.2. So to construct 45° means to construct 90° and then bisect. Again you are combining all you have learnt from the start of this unit.

Now construct 90°

Steps for constructing 90° and 45°

1. Draw a line segment AB
2. With A and B as centres and any convenient radius draw arcs to cut AB at C and D
3. With C and D as centres and any radius greater than CD, draw arcs to meet at F
4. From F drop a line to meet AB at G. See fig 8.6.   FGB =   FGA = 90°
5. Bisect either FGB or FGA to get 45°

Using the Steps in section 3.2. From fig 8.6
 ZGA =   ZGA = 45°
Note to construct 22 ½° which is half of 45° bisect 45° and construct other angles which are
(i)   half of 22 ½ °, bisect 22 ½ ° to get 11 ¼ °
(ii) to construct 135°, add 45° to 90° in fig 8.6   BGZ = 135°

Now try the construction of 221/2° and at the end use your protractor to measure the angle.

Have a nice day.

So from fig 8.6,   BGF or   FGB = 90° and   ZGF or   FGZ and or   ZAG are both 45° each.

3.3.2 Angle 60°

There are several ways of constructing angle 60°.

(1) Draw a line segment AB
(2) With the end points of A and B as centres or simply put with A and B as centres and radius equal to $\overline{AB}$, draw two arcs to meet each other at C and D (above and below the line AB)
(3) Join CD to cut AB at E. See fig 8.7
(4) Then join AC. The required angle CAE = 60°

Fig. 8.7

Alternatively

1. Draw a line AB
2. With A or B as center and any Convenient radius, draw an are to cut AB at D.
3. With D as center and the same radius draw an arc to cut the first arc at E.
4. Join EA. Then EAB = 60°

This often referred to as constructing a line at 60° to a given line at a given point on the line. See Fig 8.8

3.3.3 Angle 30°

Angle 30° is half of angle 60. Therefore simply bisect angle 60°. Fig 8.9 (b) is for you to access your progress.

Fig 8.9 (b)

(iii) Third form of the construction of angles 60° and 30° respectively.

Fig. 8.8 (b)

Fig. 8.9 (b)

Exercise 8.1

With ruler and a pair of compasses only construct the following angles

(1) 150° (2) 22112° (3)120°

4.0 Conclusion:

In this unit, you have learnt how to use your pair of compasses and ruler to construct perpendicular bisectors to a given straight line from either a given point on the line or a given point outside the line. You have also learnt how to construct and bisect angles. This will go a long way in helping you in your career where a lot depends on angles for example needle angle and angle of injection.

5.0 Sum mary

In this unit, you have learnt that to construct means to use a ruler and a pair of compasses. You have learnt that to bisect means to divide into two equal parts, hence to bisect angle 60° gives angle 30° and to construct angle 30° means to construct angle 60° and bisect. The same applies to angles 90° and 45°.

Then to construct an obtuse angle of (1) multiple of angle 45°, simply construct angle 90° bisect one side of it to get angle 45°, add this to the other part of angle 90° (90 + 45= 135°)

(ii) To construct 120°, construct angle 60°, but let the first are cut the line at two points then with the two points as center, repeat the other processes. This means that you have made a semicircle with the given line as diameter and have bisected the angle into three equal parts add any of the two and you will get angle 120°

Steps:

(i) With A as center draw a semicircle AB cut AB at C and F respectively
(ii) With C and F as centres and the same radius, draw arcs to cut the semi-circle at D and E respectively
(iii) Join EA or DA
(iv) LEAB or DAF is the required angle 120°.

6.0 Tutor-Marked Assignm ent

With you ruler and a pair of compasses only, construct the following angles

(a) 15° (b)   22 ½ ° (c)135° (5 marks each) = 15marks

(b) and (c)

Angle 22 ½ ° and 135°

Correct construction indicating pencil marks score 10 points for each (total 30pts)

7.0 References and Other Resources

David- Osugwu M, Anemelu, C and Onyeozili I (2000) New School Mathematics for Senior Secondary Schools. Onitsha: Africana-Fep Pub. Ltd.

Vygodsky, M. (1972). Mathematical Handbook: Elementary Mathematics. Moscow: MIR Publishers

UNIT 10

LOCI – PRACTICAL

Table of Contents

1.0 Introduction

The locus (plural loci) of a point is the path traced by the point of a pencil or pen or any Instrument you are using in drawing on paper obeying certain rules.

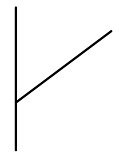
Hence the definition of locus of points as the totality of the points leaving a given property, satisfying some stated conditions. (Vygodsky, 1972)

Again the locus of a point is defined as the path traced by that point under certain conditions.

From the above definitions, you will discover that all the shapes, in geometry are loci of points satisfying certain conditions.

Example of Loci

(1) For instance, if an object falls off from your hand, the gravitational force will make it fall straight to the ground. This straight fall is the locus of the weight that falls off form your hand.



Straight line

(2) If you fix a pin or needle to a tight string, which is fixed to, a board,
the path the pin or needle traces is the locus and it is a circle.
Therefore, the circle is defined as the locus of the point the pin or
needle trace as it moves along the tight string on the board.

Circle

(3) As children like to thrown stones in the air. The path the stone traces
from the child's hand to the ground is almost in the shape of a
semicircle.
This path is the locus. It looks like this

In mathematics, this locus is called a parabola.

In this unit you shall learn the locus of moving points and how to construct loci of moving points.

2.0 Objectives

By the end of this unit, you should be able to

- Describe the locus of moving points
- Construct locus of some given points.

3.0 Main Content

3.1 Locus of Moving Points

This should be done practically. Try these activities and trace the path of each and answer the following questions

1. What is the locus of:

(a). A point on the blade of a fan?
(b) The height attained by a stone thrown vertically upwards into the air?
(c) The tip of the hour hand of your wristwatch?

In each of the activities, you have to do it yourself. Observe carefully and then answer the questions. Once you have carried out these activities, move in to the next section on how to construct loci of moving points.

3.2 Loci of Moving Points

In loci, certain terms like a "fixed point", "equal distance" or constant distance form" are used. These terms describe the special conditions under which the paths of the points are to be traced.

Examples

1. The locus of a point at a constant distance from a given fixed point is a circle. Not here this fixed point is the centre of the circle and the constant distance is the radius. Hence in drawing a circle, all you need to do is maintain a distance from a fixed point.
2. The loci of a point at a constant distance from a given line are parallel lines. This is easy to see. The lines on the pages of the exercise book maintain an equal distance and hence they are all parallel lines.
3. The locus of a point equidistance (equal distance from two fixed points is the perpendicular bisector of the line joining those points this will be constructed.
4. The locus of a point equidistant is maintaining an equal distance form two fixed intersecting straight lines are the bisectors of the angles formed by these two lines.

Now practical demonstrating of the locus discussed above
Recall the constructions, you have learnt form unit 8. They will be applied here.
You will practice along and at the end you are expected to write out the steps followed and your observations.

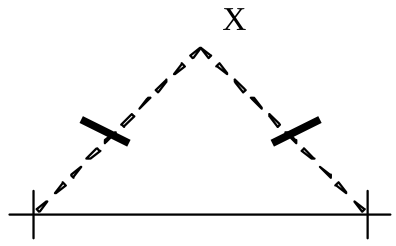
Construction of Loci of moving points
In this construction, first make a sketch of what you are going to construct before doing the construction proper.

Example 1:

Two points A and B on a straight line. A third point C is outside the straight line. Now find the straight line through C which will maintain an equal distance from A and B.

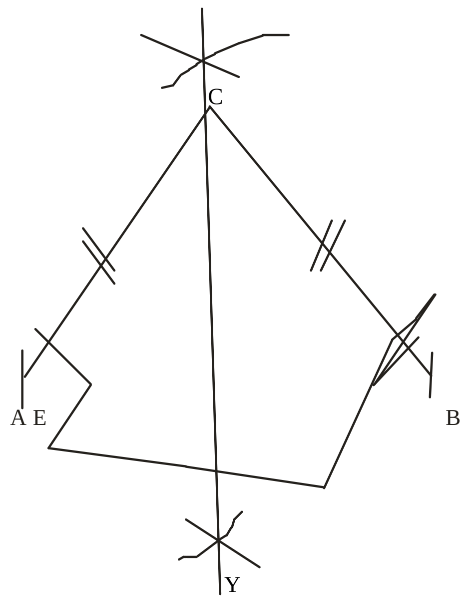
Solution: Here is a clear case of the application of the construction you did in unit 8, Fig 8.3 and 8.1.

Now do the construction. First make a sketch



From the sketch, we are looking for the perpendicular bisector of the straight line joining the two points A and B. Hence the step of the construction in unit 8, fig 8.3 is followed.

Now let us do it together.



The locus of the point equidistant form A and B. Measure AC and BC. What did you observe?

2. Two straight lines AB and CD meet at a point E find a point P such that the distance from P to AB and CD is always equal.

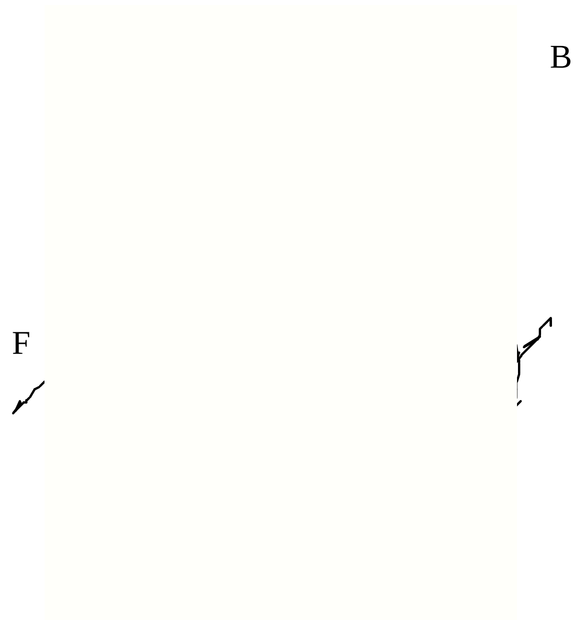
Question: Now, what do you think about the sketch of this problem. How would it look? Make the sketch, and then relate it to any of the constructions in unit 8.

Solution: Sketch

C

Having drawn the sketch, this a case of finding the point equidistant from the two intersecting straight lines. So the point P will lie on the bisector of the angles formed by the two straight lines.

Then bisect CEA and AED respectively. Recall the construction of bisector of angles in unit 8, fig 8.5 and treat accordingly.



The point P will lie on either of the two straight line FG and QR which are the bisectors of the angles between the two given lines AB and CD (see the diagram above).

This unit is an activity -oriented unit
Now bring out your mathematical sets and do the following:

Draw an angle XYZ equal to 45°. On each of the straight lines YX and YZ, mark points 2cm apart from Y. At each mark draw a perpendicular using a set- square. Join up the points at which corresponding perpendicular lines from each of the straight lines YX and YZ meet.

Write out your discoveries either by measurements or observations about the possible positions of the path traced by the moving point.

Hint: If you used your construction materials well, the points will line on the perpendicular bisector of the given angle. This serves as your guide to know, if you are working in the right direction.

4.0 Conclusion:

In this Unit on loci, you have learnt the relationship between loci and construction treated in Unit 8. That loci are practical applications of some aspects of construction. This unit is very vital in your profession where a lot of arrangements and positioning are involved.

5.0 Sum mary:

In this unit, you have learnt that

1. The locus of a point at a constant distance from a fixed point in a plane is a circle

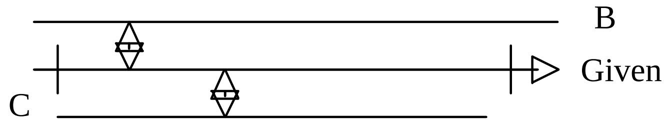
the constant distance is the radius, while the fixed point is the centre of the circle

.

2. The locus of a point in a plane which makes an angle of 90° with two given points is a circle with the line joining the two points as its diameter.

Z

3. The locus of a point at a constant distance from a given indefinite straight line are two parallel indefinite lines



4. The locus of a point from two fixed points is the perpendicular bisector of the line joining the two points

5. The locus of the weight (bob) of a pendulum is a arc of a circle

0

6. The locus of appoint equidistant from two fixed intersecting straight lines are bisectors of the angles formed by those straight lines.

6.0 Tutor-Marked Assignm ent

1. What is the locus of the follow moving points
   (a) A point on the blade of a fan
   (b) A stone thrown vertically upward into the air?
   (c) The weight of bob of a pendulum.
   (d) The tip of the hour-hand on the face of a clock
   (e) A point on the head of a child sitting on a rotating a merry- go-round.

2. Draw two lines PQ and RQ which meet at Q to give angle 60° (PQR = 60°). On each line draw marks of 0.5cm, l cm, 1.5cm, 2.0cm and 2.5cm. from Q. At each mark draw a perpendicular using a set-square. Join up the points at which the corresponding perpendiculars from each of the line meet. The resulting line is the locus of the points equidistant from QP and QR. What is its relationship with PQR.

3. Mark two points X and Y l0cm apart on a piece of paper. Set your compasses at 7cm and draw convenient arcs with X and Y as centres to intersect at two points above and below XY respectively.
   These two points are at equal distance from A and Y
   Repeat this procedure three times with compasses set at 6cm, 6.5cm and 8cm respectively to obtain several pairs of points all equidistant from X and Y. Join up these points. What is the locus of points from these two fixed points X and Y.

7.0 Reference and Other Resources

Vygodsky, M. (1972). Mathematical Handbook: Elementary Mathematics. Moscow: MIR.

David- Osugwu M, Anemelu, C and Onyeozili I (2000) New School Mathematics for Senior Secondary Schools. Onitsha: Africana-Fep Publishers.

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UNIT 11

PERIMETERS AND AREAS OF PLANE SHAPES

Table of Contents

1.0 Introduction

Mensuration was introduced when man had the need to measure land, milk, water, cloth etc. Mensuration had to deal with lengths, areas of surfaces and volume of solids.

In this unit, you shall study the perimeters and areas of plane shapes and the length of arc of circle. Also you shall also learn the units of measurements.

2.0 Objectives

By the end of this unit, you should be able to

- Explain the concepts of perimeter and areas of plane shapes
- Converts units from one form to the other
- Find the perimeters and areas of given shapes
- Find the length of arcs of given circles.

3.0 Main Content

3.1 Units of Measurements

The need for uniformity of the measuring scales brought about the metric system. This is the universally accepted unit of measure. As nurses the need for the knowledge of this scale should not be over emphasized, as drugs and some of the hospital equipment imported are still in the old system. Thus, the metric system of measurement is

Linear measurement (length)
1 kilometre (Km) = 1000 metres (m)
1 metre (m) = 10 decimetre (dm) = 100cm 1 decimetr e (dm) = 10 centimetres
1 centimetre (cm) = 10 millimetres (mm)

Areas (Square)
1 sq. km2 = 1,000,000 sq.metres (m2
1 m2 = 100 dm2 = 10,000cm2
1 hectare (ha) =100 ares = 10,000m2

Volume (Cubic)
1 m3 = 1000 dm3 = 1,000,00cm3
1 dm3 = 1000cm3
1 litre (1) = 1 dm3
1 hectolitre (hl) = 100 litres (1)

Metric Weight
1 ton (metric) = 100 kg
1 centner = 100kg
1 kg = 1000g 1 g = l 000mg.

3.2 Perim eter of Plane Shapes

Perimeter is the distance round an object. For example in fig 11.1. The perimeter of

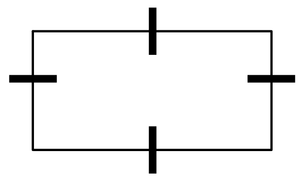
A

ABC is the sum of AB + BC + CA i.e.

c + a + b. Hence Perimeter is defined as the sum of the lengths of all the sides of a closed geometric figure (polygon).

We now find the perimeters of the following plane figures.

1. Square



a units

Fig. 11.2

The perimeter of a square ABCD = 4a units. That it is 4 times the length of one of its sides.

2.

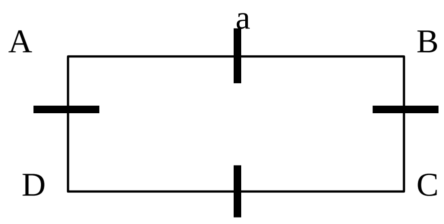


Fig. 11.3

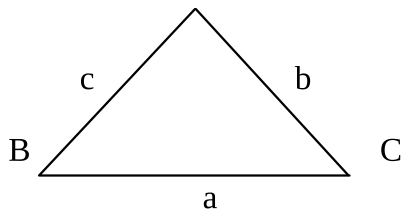
Perimeter of a rectangle ABCD = AB +BC + CD + DA but AB = DC = a and Be =AD=b

Perimeter of rectangle = 2a + 2b

= 2(a + b)

This means that the perimeter is twice the sum of the length and width.

3. Triangle: The perimeter of and triangle ABC is a + b + c, that is the sum of all the sides.

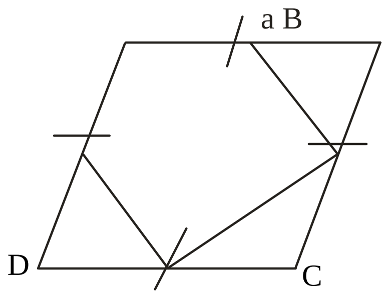


For an equilateral triangle, in which all the sides are equal, the perimeter is three times the length of one of its sides.

By the property of the parallelogram, you have studied in unit 8, the opposite sides are equal, so the perimeter of a parallelogram is the same as that of a rectangle

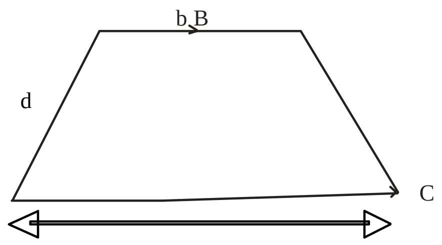
Perimeter of Parallelogram = 2(a + b)

5. Rhombus



Perimeter = 4a unit is also equivalent to that of a square.

6. Trapezium

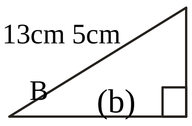


The perimeter is AB + BC + CD + DA
i.e. b + c + a + d

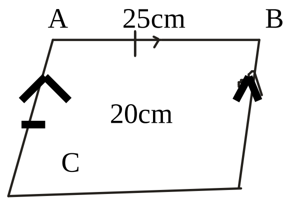
For any polygon the perimeter is equal to the sum of the length of its sides.

Example: Find the perimeters of the following plane shapes

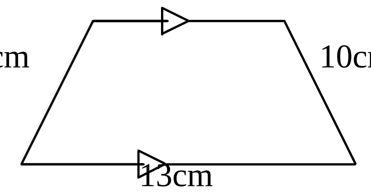
1(a)



(b)



(c)



Solution:

1(a) Perimeter of any plane shape is the sum of the length of all its sides

(a) Perimeter of ABC = (5 + 12 + 13) cm = 30 cm

(b) The figure here represents a parallelogram therefore the values of the sides not given can be found form the diagram.

AB = CD = 25CM (opposite sides of a parallelogram BC = AD = 20cm (opposite sides of a parallelogram

.'. Perimeter of Parm ABCD = 2(25 + 20) cm

= 2 (45) cm = 90 cm

(c) Perimeter of ABCD = (5 + 10 + 13 + 7) cm = 35 cm

Note: Remember to indicate the units of measurements.

3.3 Areas of Plane Shapes

The area of any shape of figure is the surface occupied by that shape or figure. For example in fig 11.4, the area rectangle ABCD is the shaded region.

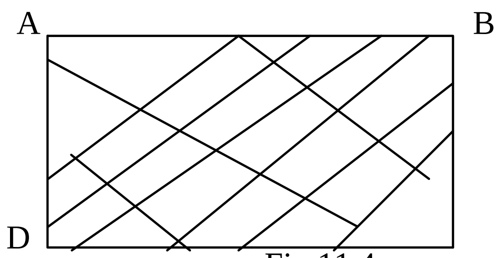


Fig 11.4

So the area of an object i.e. polygon is taken to be an enclosed region bounded by either straight lines and/or curves or arcs.

We shall not go into the proofs of the formula for finding the areas of plane shapes. But we shall apply these formulae to solution to problems.

Areas of Plane Shapes

a Triangle

Height (h)

(i) When the height of the triangle is given area of AABC
= ½ x base x height
= ½ x BD x AD
= ½ bh sq. units

(ii) When two sides and an included angle are given A

a

Area of AABC = ½ AB x BC Sin
= ½ c x a Sin
= ½ ac Sin sq. units

(iii) When the three sides are give, we use the Hero's formula

$= s(s-a)\sqrt{(s-b)(s-c)}$

where s $= \frac{a + b + c}{2}$ i.e.

S = ½ the perimeter of the triangle

Area of ABC $= \sqrt{s(s-a)(s-b)(s-c)}$ sq. units

Example

Find the area of the triangle below.

(a) (b)

12cm B 12cm C

Solution

(a) Since the triangle, ahs a base and height we use the area of triangle
$= \frac{1}{2}$ base x height

Here the base = 12cm and the height = 5cm. .'. Area of AABC ='/2 x 12 x 5 = 30Cm2

(b) Area of triangle ABC; here the sides are given so we apply Hero's formula.

First we find the perimeter of the triangle
i.e. 13 + 12 + 5 = 30

Divide by 2 to get S i.e. $S = \frac{a + b + c}{2} = \frac{13 + 12 + 5}{2}$

$S = \frac{30}{2} = 15$

Now we find the difference of each of the sides and S (i.e. half perimeter)

S – a = s – 12 = 15 – 12 = 3
S – b = s – 13 = 15 – 13 = 2
S – c = s – 5 = 15 – 5 = 10

Area of ABC $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15 \times 3 \times 2 \times 10}$

$= \sqrt{900} = 30cm^2$

b. Parallelogram

(i) Area of Parm ABCD when the base and height is given is

Area of Parm ABCD = CD x AF

= base x height

= ah sq. units.

(ii) When the sides and an included angle 0 is given, the area is

Area of Parm ABCD

= DC x BC Sin

= b x a sin

= ab sin sq. units

Example

Find the area of Parm ABCD below

Solution

Area of Parm ABCD = DC x BC Sin

Note DC = AB = 18cm

.'. Area of Parm ABCD = 18 x 12 x Sin 120°

= 18 x 12 x Sin (180 - 120°)

the sine of an obtuse angle is equal to the sine of its supplement.

.'. Area of Parm ABCD = 18 x 12 Sin 60°

$= 18 \times 12 \times \frac{3}{\sqrt{2}}$

$= \underline{187.06 cm^2}$

C)

Area of Trapezium ABCD = ½ sum of parallel sides x height
= ½ (AB + DC) x AE
= ½ (al + b2) x h sq. units.

Example:

Find the Area of the following trapezium

Solution

Area of trapezium = ½ (AB + DC) x AT sq. units =
= ½ (11.2+14) x 12cm2
= ½ x 25.2 x 12
= 151.2 cm2

d)

C

Area of Rhombus ABCD, if the base and height is given is base x height when the length of the diagonals are given the area of rhombus

ABCD = $\frac{\text{diagonal}(d1) \times \text{diagonal}(d2)}{2}$ = ½ d1 x d2 sq. units.

Example

Find the area of the following rhombus

Solution

Area of Rhombus ABCD = ½ x AC x DB

= ½ x 10 x 13

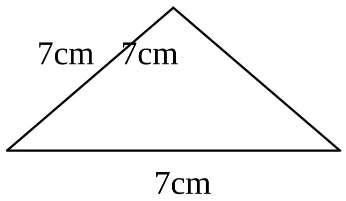
= 65cm2

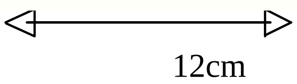
Having known the formulae for finding the areas of these named plane figures. Now do the following exercises

Exercise 11.1

Find the area of the following

a)





c)

The answers are to serve as K check on your progress

a. Area of = $\sqrt{s(s - a)(s - b)(s - c)}$

S= $\frac{7+7+7}{2}$ 21 = $\frac{21}{2}$ = 10.5; s – a = s – b = s – c = 10.5-7 = 3.5

Area of = $\sqrt{10.5 \text{ x } 3.5 \text{ x } 3.5 \text{ x } 3.5}$ = 10.5 x (3.5)3

= 21.22cm2

b. Area of trapezium = ½ (sum of //1 sides) x height
= ½ (12 + 5) x 6
= ½ x 17 x 6
= 51 cm2

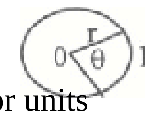
c. Area of Rhombus = ½ x d1 x d2
= ½ x 12 x 8
= 48cm2.

Was it easy? Where you able to recognise the shapes. Good. Always try to know the shape of the figure before you apply its formulae. We now move to the next section.

3.4 Length of Arc of Circle

The formula for the length of arc, would not be derived rather a detailed explanation would be given.

The length of an arc of a circle (1)



is given by $\frac{\quad}{360}$ x 2pr units

(Where is in degrees) and = angle subtended by the arc at the center of the circle

r = radius
1 = length of arc. Here if the value of n is not given, do not assume any value.

(iii) The length of arc of a circle (1), when the angle is in radius is given by

$\frac{\quad}{2p}$ x 2pr = r units.

Where = angle subtended at the center in radians
R = radius of the circle.

Example: Find the length of the arc of a circle of radius 7cm, which subtends an angle of 60° at the center of the circle. Take p = 22/7

Solution:

Length of arc = $\frac{\quad}{360}$ x 2pr units

Substituting for the values of = 60°, p $= \frac{22}{7}$ and r = 7cm into the formula

Length of arc = $\frac{60}{360} \times 2 \times \frac{22}{7} \times 7$

$= \frac{1}{6} \times 2 \times 22$

$= \frac{22}{3}$ cm

Example 2

Find the angle subtended at the centre of a circle radius 6.2cm by an arc of length

12cm. (Take p $\frac{22}{7}$)

Solution

Length of arc $= \frac{\quad}{360}$ x 2pr

Here we are given the length of arc, r and 7r. So we are looking for the angle . First make the subject of formula and then substitute the other values thus

Let length of arc = 1

$\therefore$ $1 = \frac{\quad}{360}$ x 2pr

= 3601= x 2pr

360 $\frac{1}{2pr}$ =

Substituting for r = 6.2cm, l = 12cm and 22/7

$$= \frac{360 \times 12}{2 \times \frac{22}{7} \times 6.2}$$

Simplifying:

$$= \frac{360 \times 12 \times 7}{2 \times 22 \times 6.2}$$

$$= \frac{30240}{272.8}$$

$$= 110.85°$$

Exercise 11.2

1. Find the length of the arc of the following circle of radius 7cm which subtends an angle of 300° at the centre of the circle. (Take p = 3.142)
2. Find the angle subtended at the centre of a circle, radius 6.2cm by an arc of length 18.5cm (Take p = 22/7)
3. Find how far the tip of the minute hand which is 1.5cm long of a watch travels in 10minutes.

Answers to Check your Progress

1. 36.7cm
2. 170° 531
3. 1.571cm

4.0 Conclusion

In this unit, you have learnt the Metric Unit of measurement, perimeters and areas of plane shapes and the length of an arc of a circle. You have also learnt the application of the formulae to solving problems.

5.0 Sum mary

In this unit, you have learnt that

1. Perimeter is the sum of the lengths of the sides of a geometric figure (polygon)
2. Area is the surface occupied by the figure or object.
   Area of Triangle = ½ base x height (when base and height is known)

= ½ ab sin when two sides and an included angle is given when the three sides are known.

$= \sqrt{s(s - a)(s - b)(s - c)}$

Area of Parallelogram = ½ base x height (when base and height are given)
= ab sin (when two sides and an included angle is given.)

Area of Trapezium = ½ (sum of parallel sides) x perpendicular height

Area of Rhombus = ½ x product of the two diagonals (when they are known)
= base x height (when they are known)

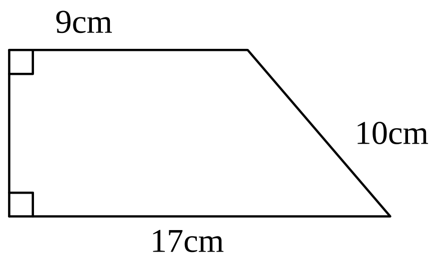
Length of arc of a circle = $\frac{}{360}$ x 2pr, where

= angle subtended at the centre by the arc and is in degrees.

r = radius of a circle.

6.0 Tutor Marked Assignm ent 11

1. Calculate the area of the trapezium in the figure below



2. An arc subtends on angle of 105° at the centre of a circle of radius 6cm. Find the length of arc Take p = 22/7
3. Find the angle subtended at the centre of the circle, radius 3.5cm by an arc of length 8.3cm Take p = 3.142
4. Find the radius of the circle; given the following (Take p = 22/7) An arc of length = 8.8cm and angle subtended at the centre = 144°

7.0 References and Other Resources

David- Osugwu M, Anemelu, C and Onyeozili I (2000) New School Mathematics for Senior Secondary Schools. Onitsha: Africana-Fep Publishers. Ltd.

Egbe, E, Odili, G.A and Ugbebor 0.0 (1999). Further Mathematics. Onitsha: Africana-Fep Publishers. Ltd.`

UNIT 12

PERIMETER AND AREA OF CIRCLE, SECTOR AND SEGMENT.

Table of Contents

1.0 Introduction

In unit 8, you studied the circle and its properties. Here we shall extend our discussion on circle, to the relationship of the parts of the circle to the entire circle.

In this unit, you shall learn the perimeters of circles, sectors and segments and their areas.

2.0 Objectives:

By the end of this unit, you should be able to:

- Distinguish between the circle and the circumference of a circle
- Find the perimeters of sectors and segments correctly.
- Find the areas of circle, sectors and segments correctly

3.0 Main Content

3.1 Perim eters and Area of Circle

You will recall that in unit 8, the circumference of a circle was seen as the distance round the circle. In which case the circumference is the perimeter of the circle. In fig 12.1 the circumference is the path traced by a moving point maintaining an equal distance from a fixed point called Locus of the point as discussed in unit 10.

Fig. 12.1

Circumference of circle = p x D units
Where D = diameter
OR Circumference of a circle = 2p units
Where r is the radius

Note the diameter equals twice the radius. A circle then is the shaded region in fig 12.1. That is why we say the area of a circle meaning the surface occupied and not the area of circumference. The formulae for finding the circumference of a circle and its area are given below

Circumference of circle = 2pr units
= pD units

Area of a circle = pr2 sq. units. The value of p is always given but where it is not leave your answers in p.

Example: Find the circumference and Area of a circle of radius 10cm.
Take p = 3.142

Solution:

Circumference of circle = 2pr units because from the question, you know the value of the radius. If the value of the diameter is given you apply (pD)

.'. Circumference of circle = 2pr units
Substituting for p = 3.142 and r = 10

Circumference = 2 x 3.142 x 10

= 62.84cm

Area of circle = pr^2 sq. units.

Substituting for p = 3.142 x 10^2

Area of circle = 3.142 x 10^2

= 3.142 x 100

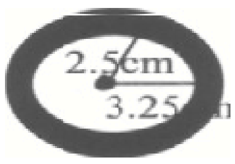
= 314.2cm^2

Example 2:

In a hollow shaft with outside diameter of 6.5cm and inner diameter of 5cm. Calculate the cross-sectional area of the shaft. Take p = 3.142

Solution:

First make a sketch of the shaft it looks like two circles in one but the cross-sectional area is the shaded region in fig 12.2 i.e. the difference between the two circle which is called an annulus (i.e. two circles of different radii but the same centre.)



So the Area of the cross-section of the shaft equals the difference between the area of the outer circle of diameter 6.5cm and the inner circle of diameter 5cm.

Area of Cross-section of shaft = Area of outer circle - Area of inner circle

= $pR^2 - pr^2$

Where R = Radius of outer circle and r = radius of inner circle

Area of Cross-section = $p \times \left(\frac{6.5}{2}\right)^2 - p \times \left(\frac{5}{2}\right)^2$

= $p \times (3.25)^2 - p \times (2.5)^2$

= p x 10.5625 - p 6.25

= p x (10.5625 - 6.25)

= p x 4.3125

Substituting the value for p

Area of cross-section of shaft = 3.142 x 4.1325

= 13.55cm2

Now try these exercises

Exercise 12.1

1. Find the circumference and area of the following circles of radius (a) 12cm (b) 18.5cm. Take p = 22/7
2. Given that the area of a circle = 250m2. Find the radius of the circle. Take p = 3.142.
3. A circular garden of radius 25cm has a circular flowerbed of radius 11 cm at its centre. Find the area of the grass space. Take p = 22/7
4. The circumference of a large circle is 44cm and the circumference of a smaller circle 31 3/7cm. Find the area of animus between these two circles. (p = 22/7).

Tr y the above exercises at your convenient time.

Answers to check your progress

1.(a) Circumference = 2p units

$= 2 \times \frac{22}{7} \times 12 = \frac{528}{7} = 75.4$cm

Area of circle = pr2 $= \frac{22 \times 144}{7} = \frac{3168}{7} = 452.58$cm2

(b) Circumference= 116.29cm

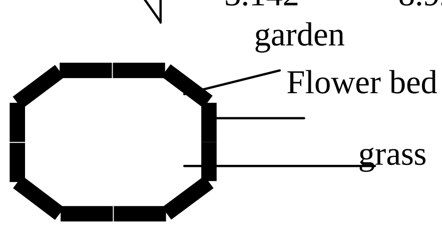
Area of circle $= \frac{7529.5}{7} = 1075.64$cm2

2. Area = pr2

$r = \sqrt{\frac{A}{p}}$

$r = \sqrt{\frac{250}{3.142}} = 8.92$cm

3.



Annulus is two circles of different radii but same centre.

Area of Annulus = pR2 - pr2

= p x 252 - p x p2

$= \frac{22}{7}(25 - 11)(25 + 11)$

= 22 x 2 x 36 =1584m2

4. Circumference = 2pr

$r = \frac{\text{circumference}}{2p}$

$R = \frac{44}{2 \times 22/7} = \frac{44 \times 7}{2 \times 22} = 7cm$

$r = \frac{31\ 3/7}{2 \times 22/7} = \frac{220}{7} \times \frac{77}{44} = 5cm$

Area of annulus = p x 72 - p x 52

= p (72 - 52)

= p (7 + 5)(7 - 5)

$= \frac{528}{7} = 75.43cm2$

Having known how to find the circumference and area of a circle, then move on to the next section.

3.2 Perim eter and Area of a Sector of a Circle

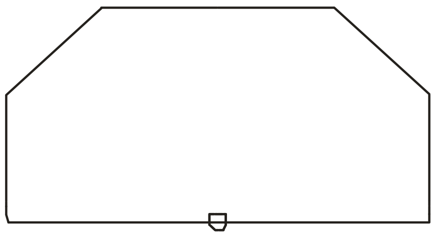
Fig 12.3

From unit 8, a sector was defined as the region between two radii and an arc. There are two sectors, the major sector i.e. the unshaded and the minor i.e. the shaded see fig 12.3.

In calculations, both the major and minor are the same. The only difference is on the angle each subtends at the centre of the circle.

Major arc subtend reflex angles while minor are subtend obtuse or acute angles.

Perimeters and Areas
A semicircle = half of a circle
Perimeter of a semicircle = $\frac{\text{circumference}}{2}$ + diameter



Perimeter of a semicircle = $\frac{2pr}{2}$ + 2r

= pr + 2r units

Area of a semicircle = half area of circle = $\frac{pr2}{2}$ sq. units

Perimeter of a sector (major or minor)

= Length of arc plus the two radii

$\frac{\theta}{360} 2\pi + 2r$

If length of arc is represented by 1 then
Perimeter of sector =1 + 2r units.

Area of sector of a circle = $\frac{}{360}$ x pr2 sq. units.

Examples:

1. A sector is cut out of a circle of radius 21 cm. Find (i) the length of arc (ii) the perimeter and (iii) the area of this sector, if the sector subtends an angle of 120°. Take p = $\frac{22}{7}$

Solution:

Length of arc = $\frac{}{360}$ x 2pr

$= \frac{120}{360} \times 2 \times \frac{22}{7} \times 21 = \frac{10880}{2520} = 44cm$

Perimeter of sector = length of arc + 2 radii

= 44 + 2(21)

= 44 + 42

= 86cm

Area of sector = $\frac{\ }{360} \times pr2$

$= \frac{120}{360} \times \frac{22}{7} \times (21)2$

$= \frac{120}{360} \times \frac{22}{7} \times 441$

= 462cm2

2. Find the perimeter and area of a semicircle of radius 12.3cm. Take p = 3.142.

Solution:

Note that since it is a semicircle, the angle it subtends at the centre is known. still remember? Angle in a semicircle = 180°

Perimeter of semicircle = pr +2r

= (3.142 x 12.3) + 2(12.3)

= 38.65+24.6

= 63.25cm

Area of a semicircle = pr2

$= \frac{3.142 \times (12.3)2}{2}$

$= \frac{475.353118}{2}$

= 237.68cm2

Exercise 12.2

1. An a circle of radius 24cm, two radii form an angle of 135° at the centre. For the smaller sector cut off by the two radii, calculate

(a) its perimeter

(b) its area. Take p = 22 7

2. A chord AB of a circle of radius 15cm subtends an angle of 80° at the centre.

Calculate:
(a) the length of the arc AB $\frac{22}{7}$

3. Find the perimeter and radius of major sector of radius 6cm which subtends an angle of 300o at the centre (Take p = 22/7)

Answers to Exercises 12.3

1. Perimeter of Sector =1 + 2r where 1 is length of arc

$$L = \frac{\quad}{360} \times 2p$$

$$= \frac{135}{360} \times 2 \times \frac{22}{7} \times 24$$

$$= 56.57cm$$

(a) .'. Perimeter of sector = 56.57 + 2(24)
= 56.57 + 48
= 104.57cm

(b) Area of sector = $\frac{\quad}{360} \times pr2$

$$= \frac{135}{360} \times \frac{22}{7} \times (24)2$$

$$= \frac{135}{360} \times \frac{22}{7} \times \frac{576}{1}$$

= 678.86cm2

2. (a) Length of arc AB = $\frac{80}{360} \times 2 \times \frac{22}{7} \times 15$

$$= \frac{52800}{2520}$$

$$= 20.95cm$$

(b) Area of sector = $\frac{80}{360} \times \frac{22}{7} \times 152$

$= \frac{80}{360} \times \frac{22}{7} \times 225 = \frac{396000}{2520}$

Area of sector $= \frac{396000}{2520}$

$= 157.14cm2$

3. Perimeter of major sector = L + 2r

Length of major arc $= \frac{300}{360} \times \frac{22}{7} \times 6$

$= \frac{79200}{2520}$

$= 31.43cm$

Perimeter of major sector = 31.43 + 2(6)

$= 31.43 + 12$

$= 43.43cm$

Area of major sector $= \frac{300}{360} \times \frac{22}{7} \times 36$

$= \frac{300}{360} \times \frac{22}{7} \times 36$

$= 94.29$ cm2

Note, if the angle subtended by the minor arc is given as 0 and you are asked to find the length of arc of the major sector, all you do is to subtract 0 from 360 i.e. (360 -   ) is the angle subtended at the centre by the major arc.

Now try the following exercises

Find the perimeters and areas of the following major sectors

Take p = 22/7

(a) 39cm (ii) 80.14cm2
(a) 29cm (ii) 48.02cm2
(a) 41.9cm (ii) 167.62cm2

3.3 Perim eters and Area of Segment of a Circle

Fig 12.4

A segment of a circle is the region enclosed by an arc and a chord. There are two segments. Namely the minor segment shaded in fig 12.4 and the major segment i.e. the unshaded region which contains the centre of a circle 0.

Perimeter of segment = length of arc plus the length of the chord which cut off the segment

Length of arc, you know the formula from the previous sections i.e. 3.1 and 3.2. Now we determine the formula for finding the length of a chord of a circle.

B

Fig 12.5

To find the length of chord AB

(1) bisect ADB = °. Then BOC = /2
(2) using trigonometric ratios: AC/r = Sin /2

AC = r Sin /2 but AC = CB
AB = AC + CB = r Sin /2 + r Sin /2 = 2r Sin /2 units Remember that AB is a chord.
.'. Length of chord AB = 2r Sin /2 .
The perimeter of a segment is
= Length of arc + length of chord.

Area of segment: form fig 12.5, the area of a segment is the difference between the area of the sector AOB and the triangle AOB.

Area of segment of circle = Area of sector - Area of triangle

Examples:

1. The length of an arc PQ of a circle of radius 9.4cm is 18.4cm. Find the perimeter and area of the minor segments cut off by the chord PQ. If the chord subtends an angle of 60o at the centre Take p = 22/7

Solution:

Sketch the figure to enable you have idea of what is being look for.

Length of chord PQ = 2r sin /2

= 60o, 2r /2 = 60/2 = 30o

Length of chord PQ = 2 x 9.4 x sin 30o

= 18.8 x ½

= 9.4cm

Perimeter of minor segment = length of arc + length of chord

= 18.4 + 9.4

= 27.8cm

Area of minor segment = Area of sector – Area of triangle

$= \frac{\quad}{360} \text{ x pr2 - ½ r2 sin}$

$= \frac{60}{360} \text{ x } \frac{22}{7} \text{ x (9.4)2 - } \frac{1}{2} \text{ x (9.4)2sin 60}$

$= \frac{1}{6} \text{ x } \frac{22}{7} \text{ x 88.36 - } \frac{1}{2} \text{ x 88.36 x 0.8660}$

= 46.28 – 38.26

= 8.02cm2

2. A chord AB which circle with centre 0 and radius 4cm. AOB = 120o. Calculate the perimeter and area of the major segment of this circle. (Take p = 22/7)

Solution:

Length of chord AB
= 2r Sin /2

= 8 Sin 60°
= 2 x 4 Sin 120/2

= 8 sin 60o
$= 8 \frac{v3}{2} = 4v3cm$

= 6.92cm. (2 decimal place)

Length of major arc = $\frac{(360 - \quad)}{360}$x 2pr. why.

$= \frac{360 - 120}{360} \times 2pr$ The major arc contains the centre see unit 12 section 3.2.

$= 240 \times \frac{2}{360} \times 22 \times \frac{4}{7}$

= 16.76cm (2 decimal places)

.'. Perimeter of major segment = length of major arc + length of chord
= 16.76 + 6.92
= 23.68cm

Alternatively, to find the length of major arc when minor arc can be found is the difference between the circumference and minor arc i.e.

Length of major arc = circumference - length of minor arc

Circumference = 27pr = 2 x $\frac{22}{7}$ x 4 = $\frac{176}{7}$ = 25.14cm

7

Length of minor arc = $\frac{120}{360}$ x 2 x $\frac{22}{7}$ x 4 = $\frac{176}{21}$ = 8.38cm

Length of major arc = circumference - length of minor arc

= 25.14 - 8.38
= 16.76cm same as above.

the perimeter of major segment = (25.14 - 8.38) + 6.92
= 16.76 + 6.92
= 23.68cm

Area of major segment = Area of major sector - Area of

$$\left(\frac{}{360} \times pr2 - ½ r2 \sin\right) \text{ sq. units}$$

$$= \frac{r^2}{2}\left(\frac{p}{180} - \sin\right) \text{ sq. units}$$

Remember that the angle subtends by the major segment is (360 - 0) where 0 is the angle subtended by the minor segment.
360 - = 360 - 120 = 240°

$$\text{Area of major segment} = \frac{4^2}{4}\left(\frac{240 \times 22}{180 \times 7} - \text{Sin } 240\right) \text{ sq. units.}$$

$$= \frac{16}{2}\left(\frac{5280}{1260} - (-0.8660\right)$$

= 8[4.1905 + 0.86601]
= 8 x 5.0565
= 40.45cm2

Alternatively

Area of major segment = Area of circle - Area of minor segment

$$pr2 - \frac{r^2}{2}\left(\frac{p}{180} - \sin\right) \text{ sq. units}$$

Where is the angle subtended by the minor arc
In the example above,

$$\text{Area of major segment} = \frac{22}{7} \times 4^2 - \frac{4^2}{2}\left(\frac{22}{7} \times \frac{120}{180} - \text{Sin } 120\right)$$

$$= \frac{22}{7} \times 16 - \frac{16}{2}\left[\frac{22}{7} \times \frac{2}{3} - 0.8660\right)$$

= 50.29 - 9.8339
= 40.45cm2 (2 decimal places)

Which is the same result as above.

Exercise 12.3

1. An arc MN of a circle, radius 4.8cm subtends an angle of 150° at the centre 0 of a circle, find

(a) The perimeter of the minor segment
(b) Perimeter of major segment
(c) The area of the minor segment cut off by the chord MN (Take p 3.142)

2. A chord XY of a circle with centre 0 and radius 8cm subtends an angle of 120°. Calculate the perimeter of

(i) the minor segment
(ii) the major segment (Take p = 22/7)

3. Find the area of the major segment of a circle, centre 0, radius 9.6cm, cut off by a chord XY subtending an angle of 54° at 0.

The answers below are to enable you check how far you have understood the lecture.

1. (a) Perimeter of minor segment $= \frac{158}{360} \times 2 \times 3.142 \times 4.8 + 2\text{Sin } 79°$

$= 13.24 + 1.9632$

$= 15.20$ (2 decimal places)

(b) Perimeter of major segment

= (Circumference - length of minor segment) + length of chord
= (2 x 3.142 x 4.8 - 13.24) + 1.9632
= (30.1632 - 13.24) + 1.9632
= 16.9232 + 1.9632
= 18.89cm (2 decimal places)

(c) Area of minor segment = Area of sector - Area of

$$\frac{r^2}{2}\left(\frac{p}{180} - \sin\right) \text{ sq. units}$$

$$= \frac{(4.8)^2}{2}\left[\frac{3.142 \times 158}{180} - \text{Sin}158\right]$$

$= 11.52[2.758 - 0.3746]$

$= 27.46cm^2$

2. Perimeter of minor segment = $\frac{120}{360} \times 2 \times \frac{22 \times 8}{7} + 2 \times 8 \text{ Sin } 120$

$= 16.762 + 15.59$

$= 32.35cm$

Perimeter of major segment = (circumference - minor arc) + length of chord

$= \left\{2 \times \frac{22 \times 8}{7} - 16.762\right\} + 15.59$

$= (50.29 - 16.762) + 15.59$

$= 33.52 + 15.59$

$= 49.11$ cm (2 decimal places)

3. $283.37cm^2$

4.0 Conclusion

In this unit you have learnt the perimeters and areas of circle, sectors and segments. You have also learnt to find the length of a chord. With the area of the annulus you have learnt in this unit, you will be able to determine the areas of similar shapes in your environment.

5.0 Sum mary

In this unit, you have learnt to determine the perimeters and areas of circles, segments and sectors by the following formulae

a. Circumference = 27pr units

b. Area of circle = pr2 sq. units

c. Perimeter of semicircle = pr + 2r units

d. Area of semicircle = pr2 sq. units.

e. Perimeter of sector of circle = length of arc + 2 radii

= $\frac{}{360}$ x 2pp + 2r units.

f. Area of sector = pr2 x $\frac{}{360}$

.

g. Length of chord of a circle = 2r Sin /2

h. Perimeter of minor segment = length of minor arc + length of chord.

i. Area of minor segment = Area of sector - area of triangle =

$$\frac{r^2}{2}\left(\frac{p}{180} - \sin\right) \text{ sq. units}$$

j. Perimeter of major segment = (circumference of circle - length of minor arc) + length of chord)

$$= 2pr \left(\frac{\quad}{360} \text{ x } 2pr + 2r \text{ Sin } /2\right) \text{ units}$$

OR

$$= 360 - \left(\frac{\quad}{360} \text{ x } 2pr + 2r \sin /2\right) \text{ sq. units}$$

k. Area of major segment = Area of circle - area of minor segment

$$pr2 - \frac{r}{2}\left(\frac{p}{180} - \sin\right) \text{ sq. units}$$

Alternatively

Area of major segment

= Area of major sector - Area of triangle

$$\frac{r^2}{2}\left(\frac{(360 - \ )p}{180} - \sin\right)$$

Where (360 -) is a reflex angle? The angle of the major sector or major segment must be from the given value of (obtuse or acute).
The above formulae will enable you solve problems without stress.

6.0 Tutor - Marked Assignm ent 12

1. The area of a sector POQ of a circle centre 0 and radius 9cm is 92cm2. Calculate the angle 0 subtended at the centre by arc PQ. (Take n = 3.142)

2. Find the perimeter of the following minor sectors (Taken = 22/7)

(a) (b)

3. Find the radius of the circle which subtends an angles of 330° at the centre of the circle and has an arc length of 24cm. Take p= 3.142.
4. A chord PQ of a circle, radius 7cm, subtends an angle of 120° at the centre. Find the ratio of the areas of the segments (minor and major) into which the chord divides the circle. Take p = 3.142
5. A chord PQ of a circle, radius 9.4cm is 12.8cm. Find
   (a) the angle subtended by the chord at the centre of the circle
   (b) the area of the minor segment cut off by this chord
   (c) the area of the major segment cut off by this chord. (Take p = 3.142)

7.0 References and Other Resources

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UNIT 13

SURFACE AREAS AND VOLUMES OF CUBOID, CUBE AND PRISM

Table of Contents

1.0 Introduction

In dealing with two dimensional figures like the rectangle, parallelogram, trapezium, square, rhombus etc we referred to the space occupied by these figures as areas.

In three dimension solids i.e. objects that have length, breadth and height like packets of sugar, boxes, blocks, tins of milk etc we refer to the surface (because some of their parts exists in space) occupied by them as Surface Area. These Solids are made up of the same or different types of plane shapes as we shall see in the course of this lecture.

In this unit, you shall learn how to calculate the total surface areas of Cuboid, cube and prism and their volumes.

2.0 Objectives

By the end of this unit, you should be able to:

• Identify the type and number of faces, of these Solids - Cuboid, cube and Prism have.
• Calculate the total surface areas and volumes of these Solids correctly.

3.0 Main Content

3.1 Total Surface Area and Volume of Cuboid

A Cuboid is in the form of a box or block.

A F

C D

Fig. 13.1 Cuboid

Look around you, pick up a box of match or sugar or chalk or bic. Try and open it through the edges (straight lines joining the sides). What did you observe? In relation with plane shapes, what are these faces?

Any solid of the shape of a Cuboid has six faces (if there is as cover). Therefore a Cuboid is a solid with six faces and each face is either a rectangle or square, see Fig 13.2.

Fig 13.2: Net of a Cuboid.

To find the total surface area of any three dimensional solid, it is always good, to draw the net of the solid and then find the atrea of each face.

Hence in the total surface area of the Cuboid, all you need to do is to find the areas of each of the plane shapes ie. Rectangles and square which make up the solid and add up.

In Fig 13.1. the total surface area is determined by finding and summing up the areas of the four rectangular faces and the two end faces (six rectangular faces) namely the areas of (1) ABGF (ii) ABCH (iii) BGCD (iv) CHED (v) DEFG (vi) AHEF.

Example: Find the total surface area of the wood block shown below.

Solution:

Since it is a wooden box, it is in the form of a Cuboid.

Therefore the total surface area of Cuboid is the sum of all the areas of the rectangular faces that make up the solid.

Here draw the net to enable you know which area being sought for to avoid duplication.

5cm

7cm

4cm

Area of A = 4 x 5cm2 = 20cm2
Area of B = 7 x 5cm2 = 35cm2
Area of C = 7 x 4cm2 = 28cm2
Area of D = 7 x 5cm2 = 35cm2
Area of E = 7 x 4cm2 = 28cm2
Area of A = 4 x 5cm2 = 20cm2

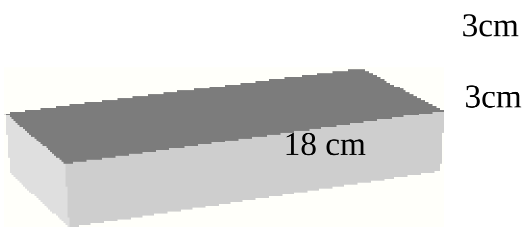
Total surface area = Areas of (A + B + C + D + E + F)
= 20+35+28+35+28+20
= 2(20 + 35 + 28)
= 2 x 83
= 166cm2

From this example, you will observe that assuming that AD = breadth, DC = Height and CE = Length, that each appeared twice, hence we have 2LW + 2LH + 2HW
.'. Total surface Area of a Cuboid = 2LW + 2LH + 2HW if closed i.e. if the lid or cover is not removed.

Total surface Area of a Cuboid
.'. = 2(LB + HB + LH) sq. units.
Where   L = Length
              B = Breadth
              H = Height

Volume of a Cuboid   =   Length x Breadth x Height.
                     =   LHB cubic units.

Example: Find the total surface area and volume of the solid below.



Solution: First identify the solid, then apply the formula since of a Cuboid the total surface area
= 2LB + 2LH + 2HB
Substituting for L = 18cm, B = 5cm and H = 3cm
The total surface area

= 2(18 x 5) + 2(18 x 3) + 2(5 x 3)
= 2(90+54+15)
=   2 (159)
= 318cm2

Volume of a Cuboid = LHB cubic units
Substituting the values for L =18, B = 5 and H = 3
Volume of Cuboid = 18 x 5 x 3
= 270Cm2

Take note of the units of measurement, when dealing with when dealing with volumes, it is cubic units.

Exercise 13.1

.

1. A Cuboid is l0cm long, 8cm wide and 7cm high. Find
   (a) the total surface area of the Cuboid
   (b) the Volume of the Cuboid

2. Find the total surface of a 12.5cm by 10.5cm by 8.5cm box and also find the volume of this box.

3. What is the volume of wood in a plank which is 6m long, 2cm thick and 25cm wide.

The answers below will serve as an incentive for you to make progress.

1(a) Total surface area = 2[(10 x 8) + (10 x 7) + (8 x 7)]
= 2[80+70+56]= 2 x 206= 412cm2

(b) Volume = 10 x 8 x 7 = 560cm2

2. Total surface = 2[(12.5 x 10.5) + (10.5 x 8.5) + (12.5 x 8.5)]
= 2[131.25 + 89.25 + 106.25]
= 2 x 326.75 = 653.5 cm2
Volume = 12.5 x 10.5 x 8.5 = 1115.625cm2

3. Convert to the same unit cm, before finding the volumes