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SCHOOL OF SCIENCE AND TECHNOLOGY

COURSE CODE: PHY 403

COURSE TITLE: ELECTRODYNAMICS III

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PHY 403 ELECTRODYNAMICS III

MODULE 1: PROPAGATION OF PLANE WAVE IN UNBOUNDED ISOTROPIC MEDIA.

UNIT 1: PLANE WAVES IN ISOTROPIC MEDIA

1.0 INTRODUCTION:

PROPAGATION OF PLANE WAVE IN UNBOUNDED ISOTROPIC MEDIA

There are several kind of waves in nature. We have sound, light, heat, electromagnetic waves, mechanical waves e.t.c. These waves are either transverse or longitudinal. The electromagnetic wave we are discussing is a transverse wave. The movement of wave from one point to another with respect to time is referred to as propagation.

Plane waves refer to waves whose wavefront are parallel to each other. A plane electromagnetic wave that is polarised would have its electric field vector oscillating in a specific direction and its Magnetic field (B) oscillating perpendicularly to it. The direction of propagation is then normal to both directions of E and B. If the E and B vectors oscillate randomly {with both perpendicular to each other} the electromagnetic wave is said to be unpolarised or randomly polarised. An electromagnetic wave can be linearly polarised. An electromagnetic wave can be linearly polarised (the E vector oscillating in a straight line) or circularly polarised (the E vector oscillating or a circular path) or elliptically polarised { the E vector oscillating in an elliptical path)

An isotropic medium is a medium having a similar property in all direction. An unbounded isotropic medium is therefore an unconfined medium having simiar properties in all direction. A simple example is free space.

In this module we will look at maxwell's equations in free space and in material medium,wave equations of electric and magnetic field. We will also consider reflection and refraction of electromagnetic waves.

2.0 OBJECTIVES

By the end of this module the student should be able to:

1. define a plane polarised wave
2. express a plane wave with a mathematical expression
3. understand isotropic media
4. understand polarisation of plane waves
5. explain the origin of electromagnetic wave.

3.0 PROPAGATION OF PLANE WAVE IN UNBOUNDED ISOTROPIC MEDIA.

A wave may be regarded as a plane wave far away from its source of radiation. This applies to wavefronts of different shapes. The wavefronts of a plane wave are parallel to each other. It is necessary to mention that a line normal to the wavefronts or planes is called a ray. A ray indicates the direction of propagation.

There are many types of waves such as sound wave, hydromagnetic wave, electromagnetic wave etc. Their properties include (i) transfer of energy from one place to another. (ii) exhibition of diffraction effect and (iii) obeying the principle of superposition.

The wave we are considering here is electromagnetic wave which is generated by accelerated charged particles. In the neighbourhood of an electric charge is electric field, E. As the charge moves (oscillates), both electric field and magnetic field exist in the neighbourhood. An electromagnetic wave is then propagated. Recall that electromagnetic wave exists as a result of variation in electric field with time, producing magnetic field (i.e. at high frequency) and the varying magnetic field producing electric field (faraday's law) which process is repeated continuously.

The electric field, E, is represented by

$$E = u_x E_o \exp\left[jw\left(t - \frac{z}{v} \right) \right] \text{ ------------------------------- (1)}$$

Equation (1) implies that E oscillates along the x-axis while the wave propagates along the z-axis. The velocity of the wave, v = w/k, where w is the angular frequency and k is the wave number. In free space, $v = 2.998 \times 10^8 ms^{-1}$ (to 3 d.p). E_o is the amplitude or peak value of the varying electric field. The magnetic field, B, which oscillates along the y-axis is represented by

$$B = u_y B_o \exp\left[jw\left(t - \frac{z}{v} \right) \right] \text{ ------------------------------- (2)}$$

where B_o is the amplitude or peak value of the magnetic field. A polarized plane wave has its field of oscillation changing with time in a specified direction while for unpolarized plane wave the direction of oscillation of its field change randomly with time. The specified direction of oscillation of the field could be rectilinear, circular or elliptical in which case rectilinear, circular and elliptical polarization result. In equation (1) is represented a linearly polarized plane wave. Circular polarization of the plane wave will be represented by

$$E = u_x E_o \exp\left[jw\left(t - \frac{z}{v} \right) \right] + u_y E_o \exp\left[jw\left(t - \frac{z}{v} + \frac{\pi}{2} \right) \right]$$

This is because for circular polarization the two components of equal amplitude must be perpendicular and have a phase difference of $\pi/2$ between them. If the amplitude of the components are unequal, elliptical polarization results.

Note that the directions of oscillations of electric field and magnetic field and the direction of propagation of electromagnetic wave are mutually perpendicular to each other i.e. the three directions are orthogonal. See **Figure 1**.

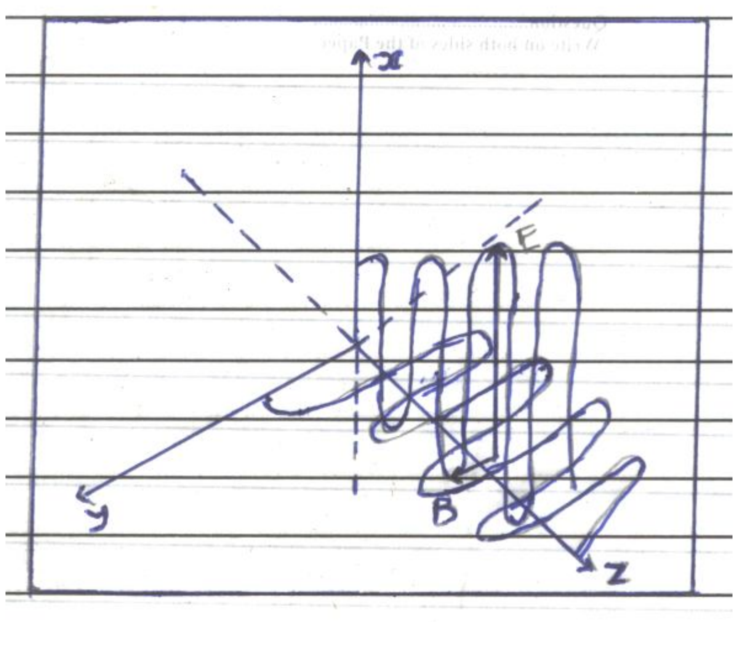

Fig 1

The relationship between the electric field, E, and the magnetic field, B, is obtained from Faraday's law as follows.

Using differential form of Faraday's law i.e.

$$\nabla \times E = -\frac{dB}{dt} \text{ ---------------------------------- (3)}$$

or

$$u_y \frac{dE_x}{dz} - u_z \frac{dE_x}{dy} = -\frac{dB}{dt}$$

Since the simple case of E_x constant in x-y plane is considered, $\frac{d}{dy} = 0$.

The 2nd term on the LHS = 0 since $u_y \frac{dEx}{dy} = 0$, therefore,

$$-u_y j \frac{w}{v} E_o \left[jw\left(t - \frac{z}{v}\right)\right] = -\frac{dB}{dt}$$

Integrating with respect to t gives,

$$B = u_y \frac{1}{v} E_o \exp\left[jw\left(t - \frac{z}{v}\right)\right] \text{ ------------------------- (4)}$$

Comparing equations (2) & (4) shows that the amplitude, B_0, of the magnetic field equals E_0/ v. The wave equations of the of electric and magnetic fields of which equations (1) and (2) are solutions are obtained as follows:

Maxwell's equations in free space in differential form are

$$\nabla . E = {}^{\rho}\!/_{\varepsilon_o} \text{ ------------------------ (i)}$$

where ρ ≡ charge density and ε_o ≡ permittivity of free space.

$$\nabla . B = 0 \text{ ------------------------- (ii)}$$

$$\nabla \times E = -\frac{dB}{dt} \text{ ---------------------- (iii)}$$

$$\nabla \times B = \varepsilon_o \mu_o \frac{dE}{dt} + \mu_o j \text{ ---------------- (iv)}$$

where j ≡ conduction current density and μ_o ≡ permeability of free space. Note that equation (iv) is the Ampere's law modified by the addition of displacement current, $\varepsilon_o \frac{dE}{dt}$ when the electric field varies rapidly.

Outside a region of changing charge and current distribution, the Maxwell's equations given above i.e. equations (i) to (iv) becomes

$$\nabla . E = 0 \text{ --------------------------------- (5)}$$

$$\nabla . B = 0 \text{ ------------------------------- (6)}$$

$$\nabla \times E = -\frac{dB}{dt} \text{ ------------------------- (7)}$$

$$\nabla \times B = \varepsilon_o \mu_o \frac{dE}{dt} \text{ ------------------------ (8)}$$

By taking the curl of (7) i.e. $\nabla \times \nabla \times E = -\frac{d}{dt}(\nabla \times B)$, we have

$$\nabla \times \nabla \times E = \nabla(\nabla . E) - \nabla^2 E$$

Substituting for $\nabla \times \nabla \times E$ we have

$$\nabla(\nabla . E) - \nabla^2 E = -\frac{d}{dt}(\nabla \times B)$$

Substituting for $\nabla . E = 0$ from equation 5 and for $\nabla \times B = \varepsilon_o \mu_o \frac{dE}{dt}$ gives,

$$-\nabla^2 E = -\frac{d}{dt}(\varepsilon_o \mu_o \frac{dE}{dt}) \text{ or } \nabla^2 E = \varepsilon_o \mu_o \frac{d^2 E}{dt^2} \text{ -------------- (9)}$$

or

$$\nabla^2 E = \frac{1}{c^2}\frac{d^2 E}{dt^2} \text{ -- (10)}$$

where $c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$ --- (11)

by taking the curl of (8) i.e. $\nabla \times \nabla \times B = \varepsilon_o \mu_o \frac{d}{dt}(\nabla \times E)$, we have

$$\nabla(\nabla . B) - \nabla^2 B = -\frac{d}{dt}(\nabla \times E)$$

Using equations (6) & (7) we have

$$-\nabla^2 B = -\varepsilon_o \mu_o \frac{d^2 B}{dt^2} \text{ or } \nabla^2 B = \varepsilon_o \mu_o \frac{d^2 B}{dt^2} \text{ ---------------------- } (12)$$

or

$$\nabla^2 B = \frac{1}{c^2} \frac{d^2 B}{dt^2} \qquad \text{--} \qquad (13)$$

Equations (10) & (13) are the wave equations of electric and magnetic fields respectively.

4.0 CONCLUSION:

Plane waves propagating in an unbounded isotropic medium may be polarised or unpolarised. An unpolarised wave is also referred to as randomly polarised wave. A polarised plane wave has its field of oscillation changing with time in a specified direction. E.g linear, circular or elliptical directions. For an unpolarised wave the diection of oscillation of its field change randomly with time.

5.0 SUMMARY

1. A plane wave has wavefronts parallel to each other and for away from any source of radiation, a wave can be said to be plane polarised.
2. A plane wave can be represented mathematically.
3. An isotropic medium is an unconfined medium having similar properties in all directions. An example of an isotropic medium is free space.
4. A wave is polarised if its field of oscillation change with time in a specified direction.
5. Electromagnetic wave is generated by accelerated charged particles.

6.0 TUTOR MARKED ASSIGNMENT

1. What is the relationship between a wave and a ray.
2. Give examples of anisotropic medium (i.e. a medium that is not isotropic)
3. Distinguish between circularly and elliptically polarised waves by stating the conditions to be met to produce each.
4. Give the electromagnetic wave spectrum.

7.0 REFERENCES AND FURTHER READING

1. Kennedy, Davis Electronic Communication System , Fourth ed. McGraw-Hill International Edition 1993.
2. Dunlop, J. and Smith, D.G. Telecommunications Engineering second edition, Chapman and Hall London 1989.
3. Grant, I.S. and Philips, W.A. Electromagnetism ELBS Manchester 1974
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UNIT 2: PROPAGATION OF ELECTROMAGNETIC WAVE IN AN ISOTROPIC MEDIUM

1.0: INTRODUCTION

The student should be familiar with wave equation ie the second order differential equation that relates the displacement of vibration of particles of medium to the spatial and temporal variationm of wave. For electromagnetic waves, rather than the particles of medium,it is the electric field and magnetic fields that vibrate. As such we talk of wave equation for magnetic waves. These wave equations are derived in this unit

2.0: OBJECTIVE

By the end of this unit, the student should be able to:

1. derive wave equations for electric and magnetic field in an isotropic insulating medium.
2. derive wave equations for electric and magnetic field in a conducting medium.

3.0 PROPAGATION OF ELECTROMAGNETIC WAVE IN AN ISOTROPIC MEDIUM

CASE 1

Propagation of electromagnetic wave in an isotropic insulating medium

Suppose the electromagnetic wave travels in an isotropic insulating medium and that the relative permittivity and relative permeability of the medium are ε_r and μ_r respectively.

Equation (9) becomes $\nabla^2 E = \varepsilon\mu \dfrac{d^2 E}{dt^2}$ ------------------------------------ (14)

or $$\nabla^2 E = \frac{1}{v^2}\frac{d^2 E}{dt^2}$$ --- (15)

where ε, the permittivity of the medium is the product of ε_o and ε_r i.e. $\varepsilon = \varepsilon_o\varepsilon_r$ and μ, the permeability of the medium is the product of μ_o and μ_r i.e. $\mu = \mu_o\mu_r$.

Note that $v = \dfrac{1}{\sqrt{\varepsilon\mu}} = \dfrac{1}{\sqrt{\varepsilon_o\varepsilon_r\mu_o\mu_r}}$ -------------------------------------- (16)

i.e the velocity of the wave in the medium. Equation (12) becomes

$$\nabla^2 B = \varepsilon\mu \frac{d^2 B}{dt^2}$$ -- (17)

or

$$\nabla^2 B = \frac{1}{v^2}\frac{d^2 B}{dt^2}$$ -- (18)

Comparing equations (11) and (16) shows that

$$\sqrt{\varepsilon_r\mu_r} = \frac{c}{v}$$

But the refractive index, n = c/v therefore, $n = \sqrt{\varepsilon_r\mu_r}$ -------------------- (19)

n is the refractive index of the medium.

Both relative permittivity and relative permeability are known to vary with frequency for a dispersive medium implying that the refractive index of a dispersive medium varies with the frequency.

Case II : Propagation of electromagnetic wave in a conducting medium

For the propagation of electromagnetic wave in a conducting medium, the modified Ampere's law can be written as

$$\nabla \times H = j_f + \frac{dD}{dt}$$ ---------------------------------- (20)

Where H – the magnetic intensity – equals $B/\mu_o\mu_r$ (μ_r being the relative permeability of the medium) and D – the electric displacement – equals $\varepsilon_o\varepsilon_r E$ (ε_r being the relative permittivity of the medium).

NOTE: that (i) $H = B/\mu_o\mu_r$ in the absence of magnetisation current and $D = \varepsilon_o\varepsilon_r E$ in the absence of polarisation charges otherwise $\underset{-}{H} = \frac{B}{\mu_o\mu_r} - \underset{-}{M}$, where M = magnetisation, a vector quantity and $D = \varepsilon_o\varepsilon_r E - P$ (P = polarisation, a vector quantity). Equation (20) can be written as

$$\nabla \times \frac{B}{\mu_o\mu_r} = j_f + \varepsilon_o\varepsilon_r \frac{dE}{dt} \text{ --------------------------------- (21)}$$

From ohm's law i.e. I = V/R --- (22),

the electric field E = V/l ≡ potential gradient, the resistance $R = \frac{\rho l}{a}$ -------- (23)

where ρ ≡ resistivity, a ≡ area and l ≡ length,

$$I = \frac{Ela}{\rho l} or \frac{I}{a} = \sigma E \text{ ---------------------------------- (24)}$$

by substituting for V from equation (22) and for R from equation (23)

($\sigma = 1/\rho$ = conductivity), I/a = j., thus equation (23) becomes $j = \sigma E$, and equation (21) can be written as

$$\nabla \times \frac{B}{\mu_o\mu_r} = \sigma E + \varepsilon_o\varepsilon_r \frac{dE}{dt} \text{ ----------------------------- (25)}$$

Taking the curl of equation (3),

$$\nabla \times \nabla \times E = -\frac{d}{dt}(\nabla \times B) \text{ ---------------------------------- (26)}$$

Substituting equation (25) into equation (24), we have

$$\nabla \times \nabla \times E = -\mu_o\mu_r\left[\sigma \frac{dE}{dt}\right] + \varepsilon_o\varepsilon_r \frac{d^2E}{dt^2} \text{ ------------------ (27)}$$

The LHS of equation (26) can be written as

$$\nabla \times \nabla \times E = \nabla(\nabla.E) - \nabla^2 E$$

Substituting this into equation (26) and noting that $\nabla.E = 0$ (since the microscopic charge density is zero for a wave propagating through a conductor) gives

$$\nabla^2 E = \mu_o\mu_r\sigma \frac{dE}{dt} + \mu_o\mu_r\varepsilon_o\varepsilon_r \frac{d^2E}{dt^2} \text{ -------------------- (28)}$$

Note that $\nabla^2 E = \frac{d^2 E}{dz^2}$

This is because the electric field, E is constant in x-y plane at a fixed z-coordinate, its amplitude however decreases exponentially with increase in z.

Thus equation (27) becomes,

$$\nabla^2 E = \mu_o \mu_r \sigma \frac{dE}{dt} + \mu_o \mu_r \varepsilon_o \varepsilon_r \frac{d^2 E}{dt^2} \text{ ------------------ (29)}$$

A simple approximation can be made by comparing the coefficients of the two terms in the RHS. To do this effectively d^2E/dt^2 can be put in terms of dE/dt i.e. $\frac{d^2 E}{dt^2} = jw\frac{dE}{dt}$ (since $E = E_o \exp jw\left(t - \frac{z}{v}\right)$) so that equation (28) becomes

$$\frac{d^2 E}{dt^2} = \mu_o \mu_r \sigma \frac{dE}{dt} + jw\mu_o \mu_r \varepsilon_o \varepsilon_r \frac{dE}{dt} \text{ ----------------- (30)}$$

$$= \mu_o \mu_r \frac{dE}{dt}\left[\sigma + jw\varepsilon_o \varepsilon_r\right] \text{ ----------------------------- (31)}$$

The electrical conductivity, $\sigma >> \omega\varepsilon_o\varepsilon_r$. Therefore equation (30) can be written as

$$\frac{d^2 E}{dz^2} = \mu_o \mu_r \sigma \frac{dE}{dt} \text{ --- (32)}$$

The solution of this differential equation (i.e. equation 31) is of the form

$$E = E_o \exp\left[j(wt - \beta z)\right]\exp(-\alpha z) \text{ -------------------- (33)}$$

Substituting (32) into (31) yields

$$\alpha = \beta = \sqrt{\frac{\mu_o \mu_r \sigma\omega}{2}}$$

The reciprocal of $\alpha = \sqrt{\frac{2}{\mu_o \mu_r \sigma\omega}}$ is referred to as skin depth, δ and it measures how rapidly the wave is attenuated. Using $\mu_r \approx 1, \delta \approx \sqrt{\frac{2}{\mu_o \sigma\omega}}$.

When ω is high, δ is very small.

Example: determine δ when frequency is (i) 60Hz (ii) 60MHz

Solution:

(i) $\delta = \sqrt{\frac{2}{4\pi \times 10^{-7} \times 5.9 \times 10^{7} \times 120\pi}} = 0.85cm$

(ii) $\delta = \sqrt{\dfrac{2}{4\pi \times 10^{-7} \times 5.9 \times 10^{7} \times 120\pi \times 10^{6}}} = 8.5 \times 10^{-4}\, cm$

Example (ii) shows that at high frequency, the current is carried in a very thin outer layer of the conductor. This phenomenon of current being carried in a thin outer layer is called SKIN EFFECT.

4.0 CONCLUSION:

The wave equations for electric field is different for an insulating medium and a conducting medium as expected. This is because for an insulating medium conduction current J = σE is zero. Thus the expressions for the electric fields on both media differs.

5.0 SUMMARY

1. The wave quations for electric field and magnetic field have been derived for an insulating medium.
2. The same equationms have also been derived for a conducting medium
3. The difference in these equatons for both media is on the fact that conduction cureent is not in an insulating medium.

6.0 TUTOR MARKED ASSIGNMENT

1. Derive the wave equations for electric and magnetic fields in an insulating medium
2. Derive these equations for a conducting medium
3. Define (i) skin depth (ii) skin effect
4. Calculate the skin depth in copper for a wave of frequency (i) 50Hz (ii) 1MHz given that $\mu_o = 4\pi$ $\mu_r = 1$, $\sigma = 5.9 \times 10^{7}$

7.0 REFERENCES AND FURTHER READING

1. Kennedy, Davis Electronic Communication System , Fourth ed. McGraw-Hill International Edition 1993.
2. Dunlop, J. and Smith, D.G. Telecommunications Engineering second edition, Chapman and Hall London 1989.
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MODULE 2: REFLECTION AND REFRACTION OF ELECTROMAGNETIC WAVES

UNIT 1: BOUNDARY CONDITIONS ON ELECTRIC AND MAGNETIC FIELDS

1.0 INTRODUCTION:

When a wave passes from one medium to another medium of different refractive index, its speed decreases if the second medium has a greater refractive index relative to the first medium and the wave bends towards the normal.

If refractive index of the second medium is less than that of the first speed of the wave increases and it is bent away from the normal. We also know that apart from the refracted waves some of the incident wave is reflected. The electric fields E of the incident, reflected and refracted waves bear a relationship at te interface i.e boundary between the two media. This is also the case for magnetic field,B, the electric displacement,D and the magnetic intensity,H. These relationship are called boundary conditions. For te various parameters.

2.0 OBJECTIVES:

By the end of this unit, the student should be able to

1. State the boundary condition on each of the parameter, E , H , B and D.
2. Know ow to derive these boundary conditions.

3.0 REFLECTION AND REFRACTION OF ELECTROMAGNETIC WAVES

When a plane wave is incident at the boundary between two different media, part of the wave is reflected and the other part is refracted or transmitted. We need to determine the ratio of the intensity of the reflected part to the intensity of the incident wave as well as the ratio of the intensity of the refracted (transmitted) part to the intensity of the incident wave. Note also that intensity is proportional to the rate of energy per unit area i.e. the Poynting vector. These ratios are called coefficients of reflection and transmission respectively. These coefficients can be determined in terms of the refractive indices of the media. First we must make sure that the fields satisfy Maxwell‘s equations at the boundary between the media.

From equation (7) in section 1.0 i.e.

$$\nabla \times E = -\frac{dB}{dt},$$

In integral form this equation becomes

$$\oint_r E.dl = -\frac{d}{dt}\int_s B.ds \text{ ------------------------------------- } (1)$$

Let r be the rectangular loop placed along the surface common to the two media as shown in Figure 1.

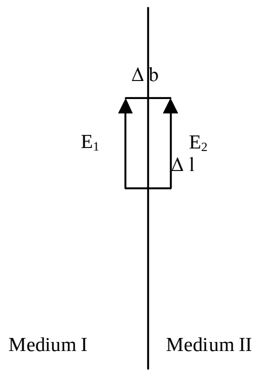

FIG: 1

In the limit that Δb → 0 i.e. along the interface, $\int B.ds = 0$, ds being the surface area and the product of Δl and Δb. Thus the LHS of equation (1) equals zero and

$$\oint E.dl = E_1 l - E_2 l = 0 \text{ or } E_1 = E_2.$$

This implies that the electric field is continuous. This is the boundary condition for the electric field. The boundary condition for magnetic intensity can be determined from the modified Ampere's law (i.e.Maxwell's 4th equation) by considering Figure 2 below.

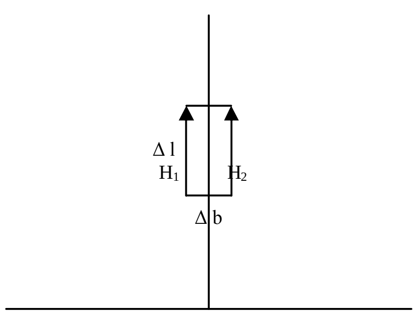

FIG: 2

Equation (20) in section 1.0 can be rendered in integral form as

$$\oint H.dl = \int_s j_f .ds + \frac{d}{dt}\int_s D.ds \text{ ----------------------------------- (2)}$$

For the other medium (i.e. the second medium) of finite conductivity, the first term in the RHS of equation (2) becomes zero. The second term is zero in the limit that $\Delta b \to 0$ i.e. at the interface since ds is the product of dl and db. Thus the equation is

$\oint H.dl = 0$ or $H_1 l - H_2 l = 0$ or $H_1 = H_2$ across the interface. H is continuous across the interface. From equation (1) of Gauss law for electric field in section 1.0 i.e. $\nabla.E = \dfrac{\rho}{\varepsilon_o} or \nabla.D = \rho_f$

where $D = \varepsilon_o E - P$

We can write $\nabla.D = 0$ since there is no free charge inside the cylinder placed perpendicularly to the interface. See **Figure 3** below.

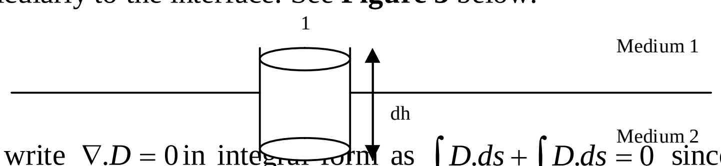

Fig 3:

We can write $\nabla.D = 0$ in integral form as $\int_1 D.ds + \int_2 D.ds = 0$ since the total flux of electric displacement has contribution from top and bottom surfaces of the cylinder as $dh \to 0$ at the interface. The flux of D is normal to the surfaces 1 and 2 of the cylinder. Thus we can write

$$D_\perp ds_1 = D_\perp ds_2$$

implying that $D_\perp$ is continuous.

The last boundary condition is determined from Gauss law for magnetic field i.e. $\nabla.B = 0$ which in integral form becomes $\int B.ds = 0$. Using the same cylinder in Figure 3, the contributions of the flux of magnetic field is from the top and bottom surfaces of the cylinder so that we can write

$$\int_1 B.ds + \int_2 B.ds = 0$$

$B_\perp ds_1 + B_\perp ds_2 = 0$ where $B_\perp$ is continuous.

From the four boundary conditions i.e.

$E_{1along} = E_{2along}$

$H_{1along} = H_{2along}$

$D_{1across} = D_{2across}$, and

$B_{1across} = B_{2across}$.

4.0 CONCLUSION

The boundary conditions on the electric field, E and magnetic intensity,H are somewhat similar. Also those on magnetic field, B and electric displacement, D ae similar. This is because for E and H we use line integrals while for D and B we use surface integral.

5.0 SUMMARY

The boundary conditions of electromagnetic wave are

1. $E_{tangential}$ is continous i.e $E_{tangential}$ (medium 1) = ($E_{tangential}$ (medium 2)
2. $H_{tangential}$ is continous i.e $H_{tangential}$ (medium 1) = $H_{tangential}$ (medium 2)
3. $D_{perpendicular}$ is continous i.e $D_{perpendicular}$ (medium 1) = $D_{perpendicular}$ (medium 2)
4. $B_{perpendicular}$ is continous i.e $B_{perpendicular}$ (medium 1) = $D_{perpendicular}$ (medium 2)

6.0 TUTOR MARKED ASSIGNMENT

1. The boundary conditions on E and H are similar with a slight difference. Explain.
2. Why are D and B's boundary conditions similar but different from those of E and H.
3. The refractive index of water for waves of frequency MHz is about 9. Calculate the reflection and transmission coefficient.

7.0 REFERENCES AND FURTHER READING

1. Kennedy, Davis Electronic Communication System , Fourth ed. McGraw-Hill International Edition 1993.
2. Dunlop, J. and Smith, D.G. Telecommunications Engineering second edition, Chapman and Hall London 1989.
3. Grant, I.S. and Philips, W.A. Electromagnetism ELBS Manchester 1974
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UNIT 2: REFLECTION AND REFRACTION (TRANSMISSION) COEFFICIENTS OF ELEECTROMAGNETIC WAVES

1.0 INTRODUCTION

In this unit the coefficients of reflection (ratio of reflected to incident wave) and transmission (ratio of transmitted to incident wave) are determined. Calculations of percentages reflected and transmitted wave in glass and water, for example , are made.

2.0 OBJECTIVES:

By the end of this unit, the student should be able to

1. define (a) reflection coefficient, R

   (b) transmission coefficient, T
2. obtain the expressions of R and T in terms of poynting vector
3. calculate the percentages of R and T for any medium relative to free space given its refractive index.

3.0 REFLECTION AND REFRACTION (TRANSMISSION) COEFFICIENTS OF ELEECTROMAGNETIC WAVES

Noting that in medium 1 we have only the incident and reflected waves i.e. two waves while in medium 2 we have only refracted or transmitted wave i.e. one wave, applying the 1st boundary condition we can write

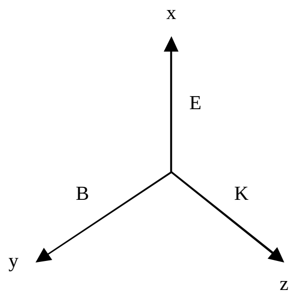
$$E_i + E_r = E_t \text{ -- } (1)$$

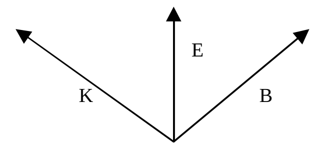
The second boundary condition gives

$$H_i - H_r = H_t \text{ --- } (2)$$

where the substricts i, r and t represent incident, refracted and transmitted.

Note that the reflected magnetic intensity, H_r, points in the direction opposite the direction of H_i. This can be visualised by making the middle finger of the left hand point in the direction of propagation (z-direction) while forefinger and the thumb represents E and B respectively (i.e. in x and y directions). By pointing the middle finger in the negative z-direction, the thumb representing magnetic field is observed to be pointing downward i.e. opposite its former direction.





using

$$H_i = \frac{B_i}{\mu_o} = \frac{E_i}{c\mu_o} \quad \text{--} \ (3)$$

since B = E/c from equation (6) in section 1.0 and H_i is in free space. Substituting for $c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$ in (3) gives

$$H_i = \frac{(\varepsilon_o)^{\frac{1}{2}} E_i}{(\mu_o)^{\frac{1}{2}}}$$

Similarly H_r (also in free space) = $\frac{(\varepsilon_o)^{\frac{1}{2}} E_r}{(\mu_o)^{\frac{1}{2}}}$

H_t is however in a medium of relative permittivity, ε_r and relative permeability, μ_r.

Thus $H_t = \frac{(\varepsilon_o \varepsilon_r)^{\frac{1}{2}} E_t}{(\mu_o \mu_r)^{\frac{1}{2}}}$

Equation (2) can then be written as

$$\frac{(\varepsilon_o)^{\frac{1}{2}} E_i}{\mu_o^{\frac{1}{2}}} - \frac{(\varepsilon_o)^{\frac{1}{2}} E_r}{(\mu_o)^{\frac{1}{2}}} = \frac{(\varepsilon_o \varepsilon_r)^{\frac{1}{2}} E_t}{(\mu_o \mu_r)^{\frac{1}{2}}}$$

or

$$E_i - E_r = \frac{(\varepsilon_r)^{\frac{1}{2}} E_t}{(\mu_r)^{\frac{1}{2}}} \quad \text{--------------------------------------} \ (4)$$

$\mu_r \approx 1$ for most dielectrics so that equation (4) becomes

$$E_i - E_r = (\varepsilon_r)^{\frac{1}{2}} E_t \text{------------------------} \ (5)$$

Using the amplitude of the electric field of the incident, reflected and transmitted waves and substituting for $n = (\varepsilon_r)^{\frac{1}{2}}$, equation (5) can be written as

$$E_{oi} - E_{or} = nE_{ot} \quad \text{---------------------------------------} \ (6)$$

Also using the amplitude of the electric fields in equation (1) we have

$$E_{oi} + E_{or} = E_{ot} \text{ -- (7)}$$

By combining equations (5) and (6) to eliminate E_{ot} gives

$$\frac{E_{or}}{E_{ot}} = \frac{1-n}{1+n} \text{ -- (8)}$$

Equations (5) and (6) can also be combined to eliminate E_{or}. The result is

$$\frac{E_{ot}}{E_{oi}} = \frac{2}{1+n} \text{ -- (9)}$$

The ratio of the reflected energy of the wave to that of the incident energy is referred to as the REFLECTION COEFFICIENT, R. Also the ratio of the transmitted energy of the wave to that of incident energy is referred to as the TRANSMITTED COEFFICIENT, T. These ratios or coefficients are determined from the time average of the Poynting vector over one cycle for the incident wave, the reflected wave and the transmitted wave since Poynting vector is energy per unit time per unit area.

We can use the combination of the energy of static electric and magnetic fields to approximate the energy of an electromagnetic wave. Using a capacitor, the energy of static electric field is obtained as follows:

Work done in transferring a charge from one plate of the capacitor to the other plate is

$$dW = Vdq = \left(\frac{q}{c}\right)dq \qquad \text{since } V = q/c$$

Integrating we have

$$W = \int_0^q \frac{q}{c}dq = \frac{1}{2}\frac{q^2}{c} \qquad \text{or} \quad W = \frac{1}{2}CV^2$$

For a parallel plate capacitor $C = \frac{\varepsilon_o A}{d}$,

the energy density of the electric field, U_E, i.e. energy per unit volume,

$$U_E = \frac{\varepsilon_o A}{Ad.d}V^2 = \frac{1}{2}\varepsilon_o\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_o E^2 \text{ ------------ (10)}$$

(volume = Ad, E = V/d and $\varepsilon_o E = D$)

$$= \frac{1}{2}D.E \text{ -- (11)}$$

The energy density in a magnetic field is obtained as follows: let a solenoid have a cross sectional area. The volume Al of the solenoid would store magnetic energy. The energy density is the energy stored divided by Al where A is the cross sectional area of the solenoid and l its length.

The magnetic energy dU, stored $= Vidt$ or $\frac{dU}{dt} = Vi$ or $\frac{dU}{dt} = Li\frac{di}{dt}$

Since $V = \frac{Ldi}{dt}$ where L is the self inductance

Integrating,

$$\int dU = \int Lidi = \frac{1}{2}Li^2$$

The energy density of the magnetic field will then be,

$$U_B = \frac{1}{2}\frac{Li^2}{Al}$$

Substituting for $L = \mu_o n^2 lA$ and $i = \frac{B}{\mu_o n}$

where n ≡ number of turns of solenoid.

$$U_B = \frac{1}{2}\frac{B}{\mu_o} \text{ -- } (12)$$

Substituting for $H = \frac{B}{\mu_o}$, $U = \frac{1}{2}B.H$

and the total magnetic energy is

$$U_B = \frac{1}{2}\int B.Hdv \text{ --- } (13)$$

Equations (10) and (12) can be combined to give the energy of the electromagnetic wave assuming that it doesn’t change for varying fields. Thus

$$U_{em} = \frac{1}{2}\int (E.D + B.H)dv$$

For a plane wave travelling along z-axis see Figure --- at speed c in a box of cross sectional area, A and thickness dx, the sun of equations (1) and (3) give the energy density of the electromagnetic wave i.e.

$$U_{em} = U_E + U_B = \frac{1}{2}\varepsilon_o E^2 + \frac{1}{2}\frac{B^2}{\mu_o}$$

The energy U_{em} stored in the box = $(U_E + U_B)Adx$ i.e.

$$dU_{em} = \left(\frac{1}{2}\varepsilon_o E^2 + \frac{B^2}{\mu_o}\right)Adx \text{ --------------------------- } (14)$$

Substituting the relation B = E/c into equation (14), we have

$$dU_{em}\left[\frac{1}{2}\varepsilon_o(E)(cB)+\frac{B}{\mu_o}\left(\frac{E}{c}\right)\right]Adx=\left[\frac{1}{2}EB\left(\varepsilon_o c+\frac{1}{\mu_o c}\right)\right]Adx=\left[\frac{1}{2}EB\left(\frac{\varepsilon_o\mu_o c^2+1}{\mu_o c}\right)\right]Adx$$

------ (15)

so that equation (14) is

$$dU_{em}=\frac{EBAdx}{\mu_o c}$$

The rate of transfer of this energy per unit area, i.e. the poynting vector, N,

$$N=\frac{dU_{em}}{dtA}=\frac{EBdx}{\mu_o cdt} \quad \text{------------------------------------- (16)}$$

But $\frac{dx}{dt}=c$ and equation (16) becomes

$$N=\frac{EB}{\mu_o}=EH \quad \text{-- (17)}$$

since $\frac{B}{\mu_o}=H$. Note that the average energy crossing unit area per second = $\frac{E_oH_o}{2}$ i.e. $N=\frac{E_oH_o}{2}$ -- (18)

since N is in the direction of propagation and the directions of E and H are orthogonal we can write,

$$N=E\times H \quad \text{--- (19)}$$

By the definitions of the reflection coefficient, R and the transmission coefficient, T given earlier,

$$R=\frac{(E_r\times H_r)ave}{(E_i\times H_i)ave}=\frac{E_{or}{}^2.c\mu_o}{E_{oi}{}^2.c\mu_o}=\frac{E_{or}{}^2}{E_{oi}{}^2}$$ (from equation (18) and since $H_o=\frac{E_o}{c\mu_o}$). Using equation (7)

$$R=\left(\frac{1-n}{1+n}\right)^{\frac{1}{2}} \quad \text{--- (20)}$$

$$T = \frac{(E_t \times H_t)ave}{(E_i \times H_i)ave} = \frac{E_{ot}H_{ot}}{E_{oi}H_{oi}} = \frac{(E_{ot})\left(\frac{E_{ot}}{\mu_o \mu_r V}\right)}{E_{oi}\left(\frac{E_{oi}}{\mu_o c}\right)} = \frac{E_{ot}^{\ 2}}{E_{oi}^{\ 2}}\left(\frac{c}{\mu_r V}\right) = \frac{E_{ot}^{\ 2}}{E_{oi}^{\ 2}}\left[\frac{(\varepsilon_o \varepsilon_r \mu_o \mu_r)^{\frac{1}{2}}}{\mu_r (\varepsilon_o \mu_o)^{\frac{1}{2}}}\right] = \frac{E_{ot}^{\ 2}.n}{E_{oi}^{\ 2}}$$

-------------------- (21)

Where $\sqrt{\varepsilon_r} = n$ and $\sqrt{\mu_r} \approx 1$

Using equations (8) and (21)

$$T = \frac{4n}{(1+n)^2}$$ --- (22)

Example: Given that the refractive index, n, of water for waves of frequency 100MHz is 9 ($n = \sqrt{\varepsilon_r}$ varies with frequency). Calculate the reflection and transmission coefficients of the medium. From equation (20),

$$R = \left(\frac{1-9}{1+9}\right)^2 = \frac{64}{100} = 0.64$$

using equation (22)

$$T = \left(\frac{4n}{(1+9)^2}\right) = \frac{(4)(9)}{10^2} = 0.36$$

4.0 CONCLUSION

The higher the value of refractive index the larger the fraction of energy reflected and the smaller the fraction transmitted. This is because a longer relative permittivity (which gives large refractive index) the oscillating dipole moment of each molecules of the medium resulting in a large radiated field. The forward wave in this field interferes destructively with the original wave within the medium and gives rise to small transmission.

5.0 SUMMARY

1. Reflection coefficient is the ratio of reflected energy to incident energy while transmission coefficient is the ratio of transmitted energy to incident energy.

2. The reflection coefficient and transmission coefficient in terms of poyting vector, respectively are

$$R = \frac{(E_r \times H_r)ave}{(E_i \times H_i)ave}$$

$$T = \frac{(E_t \times H_t)ave}{(E_i \times H_i)ave}$$

3. From $$R = \left(\frac{1-n}{1+n}\right)^2$$

$$T = \frac{4n}{(1+n)^2}$$

the reflection coefficient and the transmission coefficient of any medium relative to free space can be determined.

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6.0 TUTOR MARKED ASSIGNMENT

1. Define reflection coefficient and transmission coefficient
2. Calculate the reflection and transmission coefficient given that the reflection index of a medium is 7

7.0 REFERENCES AND FURTHER READING

1. Kennedy, Davis Electronic Communication System , Fourth ed. McGraw-Hill International Edition 1993.
2. Dunlop, J. and Smith, D.G. Telecommunications Engineering second edition, Chapman and Hall London 1989.
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MODULE 3: TRANSMISSION LINES.

UNIT 1: TRANSMISSION LINES.

1.0 INTRODUCTION:

A transmission line is the medium for transmitting electromagnetic energy from one point to another. For electromagnetic signals with high frequencies, the energy has to be well guided so that the energy will not be radiated away. At high frequencies, of the order of kilohertz and above, electromagnetic energy is transmitted via parallel wire transmission lines, coaxial cables, waveguides and resonant cavities. The first two will be discussed in this unit as the type of waves transmitted by them is different from those transmitted by the last two.

2.0 OBJECTIVES

By the end this module the student should be able to

1. give a simple sketch of transmission line in any number of cascade.
2. derive the wave equations of voltages and currents of a transmission line.
3. determine the expression for the (i) characteristic impedance (ii) speed of propagation of signals in transmission lines.

3.0 TRANSMISSION LINES.

A transmission line is a system of material boundaries which forms a continuous path that can direct transmission of electromagnetic energy from a power station to other stations or for transmitting energy from one point to another. A transmission line is uniform if there is no change in its cross sectional geometry. The wavelengths on a transmission line are compatible with its size at gigahertz frequencies and its capacitances and inductances are very small.

The change in potential difference per unit length in Figure 1 i.e.

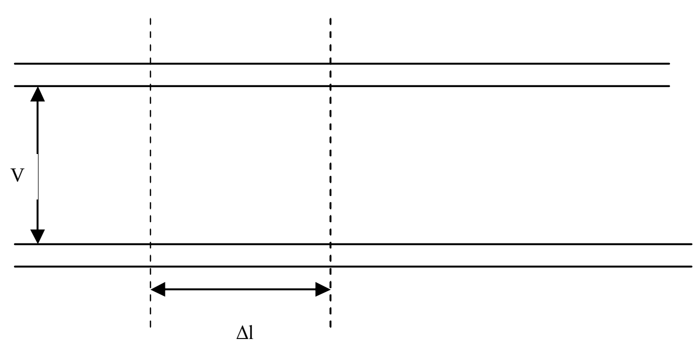

Fig 1:

$$\frac{\Delta V}{\Delta L} = \frac{-LdI}{dt} = -j(wL + R)I \text{ -------------------------------- (1)}$$

Or in the limit of ΔL becoming dl

$$\frac{dV}{dl} = -(jwL + R)I$$

Where L is self inductance due to magnetic field around the conductor V and R, is resistance of the conductor. The change in current flowing in the line is given by

$$\frac{dI}{dl} = \frac{cdV}{dt} - (jwc + G)V \text{ ------------------------------------- (2)}$$

where C is capacitance per unit length (formed because of the finite distance between the conductors of the line) and G is conductance per unit length (whose existence is due to the dielectric losses of the dielectric medium in between the conductors).

If we assume that conductors have zero resistance and that they are separated by a perfect insulator (in which case the transmission line becomes lossless), equations (1) and (2) which are the basic equations of transmission line becomes,

$$dV/dl = -L\frac{dI}{dt} \text{ -------------------------------------- (3)}$$

$$\frac{dI}{dl} = -C\frac{dV}{dt} \text{ -------------------------------------- (4)}$$

Differentiating equations (3) and (4) with l and t respectively we have

$$\frac{d^2V}{dl^2} = -L\frac{d^2I}{dldt} \text{ ------------------------------------ (5)}$$

$$\frac{d^2I}{dtdx} = -C\frac{d^2V}{dt^2} \text{ ------------------------------------- (6)}$$

Equations (5) and (6) can be combined to obtain

$$\frac{1}{LC}\frac{d^2V}{dl^2} = \frac{d^2V}{dt^2} \text{ -------------------------------------- (7)}$$

$$\frac{1}{LC}\frac{d^2I}{dl^2} = \frac{d^2I}{dt^2} \text{ ------------------------------------- (8)}$$

Equations (7) and (8) are familiar wave equations of voltage and current respectively implying that both voltage and current propagate as waves along transmission lines. The velocity of propagation is $V = \frac{1}{\sqrt{LC}}$ from equations (7) and (8). It can be shown that the characteristic impedance of a transmission line, $z_o = \sqrt{LC}$. Try to obtain the net impedance of the circuit in **Figure 2** which is three cascades of a transmission line.

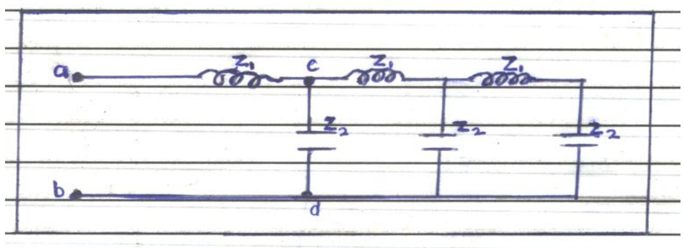

Fig 2

Hint: let z_o equal the net impedance between a and b. The impedance to the right of c and d also equals z_o.

PARALLEL WIRE AND COAXIAL CABLE TRANSMISSION LINE

Two major examples of transmission lines are (i) the parallel wire and (ii) the coaxial cable.

To obtain the propagation velocity, V and the characteristics impedance of any transmission line the inductance per unit length, L and the capacitance per unit length must be determined.

(a) **PARALLEL WIRE TRANSMISSION LINES**: Recall that the electric field E of a conductor is given by Gauss law for electric as,

$$E = \frac{\lambda}{2\pi r\varepsilon_o\varepsilon_r} = \frac{q}{2\pi r\varepsilon_o\varepsilon_r l}$$

where λ (linear charge density) = q/l, for a conductor in a medium of relative permittivity (dielectric constant), ε_r. For the two wires, each of radius x and separated by distance y.

$$\underline{E} = \underline{E}_1 + \underline{E}_2 = \frac{q}{2\pi r\varepsilon_o\varepsilon_r l} + \frac{q}{2\pi r\varepsilon_o\varepsilon_r l} = \frac{q}{\pi\varepsilon_o\varepsilon_r l}$$

The capacitance of the pair is obtained from

$$C = \frac{q}{V} = \frac{q}{\int_x^{2y} Edr} = \frac{q}{\left(\cancel{q} \Big/ \left(\pi\varepsilon_o\varepsilon_r l \int_x^{2y} \frac{dr}{k} \right) \right)}$$

giving

$\frac{C}{l} = \frac{\pi\varepsilon_o\varepsilon_r}{\ln\left(\frac{2y}{x}\right)}$ i.e. capacitance per unit length. The inductance per unit length is obtained as follows: the magnetic flux of the two conductors (whose shape are approximately cylindrical),

$$\phi_B = 2l\int_x^{2y-x} \frac{\mu_o\mu_r Idr}{2\pi r} \approx \frac{\mu_o\mu_r I \ln\left(\frac{2y}{x}\right)}{\pi}$$

But self inductance, $L = \frac{\phi_B}{i} = \frac{\mu_o\mu_r \ln\left(\frac{2y}{x}\right)}{\pi}$

Thus the velocity of propagation, $V = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\varepsilon_o\varepsilon_r\mu_o\mu_r}}$ as expected. The characteristics impedance, $z_o = \sqrt{\frac{L}{C}} = \left(\frac{\mu_o\mu_r}{\pi^2\varepsilon_o\varepsilon_r}\right)^{\frac{1}{2}} \ln\left(\frac{2y}{x}\right)$

(b) COAXIAL CABLE TRANSMISSION LINE: The capacitance for coaxial cable is obtained following the procedure used for the parallel wire except that the $\int Edr$ is from a to x and b to y, a being the radius of the central conductor and b, the distance between the centres of both conductors. See **Figure 3**. The capacitance per unit length is thus

$$\frac{C}{l} = \frac{2\pi\varepsilon_o\varepsilon_r}{\ln\frac{y}{x}}$$

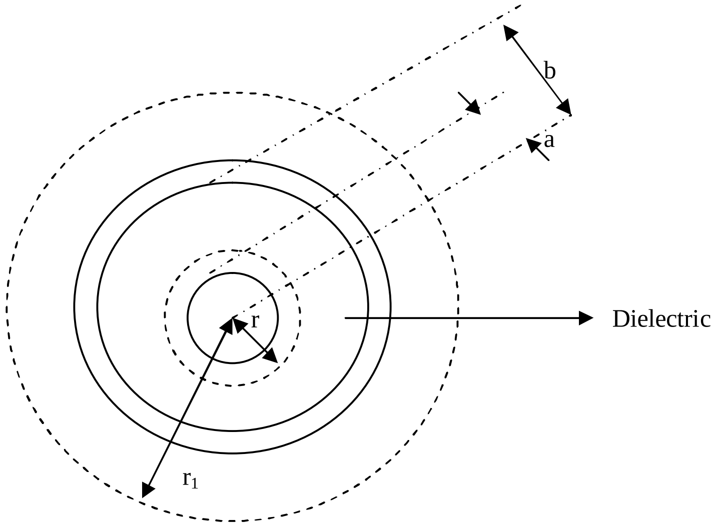

Fig 3: Cross section of a co-axial cable

The inductance of a coaxial cable is obtained as follows: the flux of magnetic field, ϕ_B, through a closed circuit formed by joining the conductors at the end of a section of cable of length, l, is $l\int_x^y \frac{\mu_o \mu_r I}{2\pi r} dr = \frac{\mu_o \mu_r}{2\pi} Il \ln\left(\frac{y}{x}\right)$

Recall that self inductance, $L = \frac{\phi_B}{I} = L = \frac{\mu_o \mu_r}{2\pi} \ln\left(\frac{y}{x}\right)$

The velocity of propagation, $V = \frac{1}{\sqrt{LC}}$

Substituting, $V = \frac{1}{\sqrt{\varepsilon_o \varepsilon_r \mu_o \mu_r}}$, and the characteristics impedance,

$$z_o = \sqrt{\frac{L}{C}\left(\frac{\mu_o \mu_r}{4\pi^2 \varepsilon_o \varepsilon_r}\right)^{\frac{1}{2}} \ln \frac{y}{x}}$$

EQUIVALENT CIRCUIT.

In Figure 1 above is illustrated two infinitely long lines carrying equal and opposite current. The part between point X and Y is a subsection that consists of impedance z =R+ jωL and admittance y= G+ jωC . The resistance, R and inductance, L are due to the length and diameter of the conductors while the capacitance exists by virtue of the close separation of the conductors. The conductors being separated by an imperfect insulator or dielectric necessitate leakage of current. This represents the shunt conductance which together with shunt capacitance gives the admittance. The 2 conductors can thus be represented by Figure 2 in which case the resistance , R and conductance, G are negligible. Each subsection is an equivalent circuit and the infinitely loss line is regarded as a cascade of infinite number of such circuits. For example in Figure 2 is shown a transmission line of three cascade.

4.0 CONCLUSION

The type of waves propagated in parallel wire transmission lines and coaxial cables are the Transverse Electric and Magnetic (TEM) waves. In these waves bothe the electric and magnetic fields are transverse to the direction of propagation. As such they are different from wave propagated in transmission lines such as waveguides which are discussed next.

5.0 SUMMARY:

1. The transmission line for three cascade is given in the text.
2. Voltage and current are transmitted as wave, the equation of which are given in the Text
3 From the inductance and capacitance per unit length of both the parallel wire transmission line and the coaxial cable, the characteristic impedance and speed of propagation of both transmission lines are determined in the text.

6.0. TUTOR MARKED ASSIGNMENT

1. Sketch a transmission line in 5 cascade
2. Derive the voltage and current wave equation.
3 Determine the characteristic impedance and speed of propagation of (i) coaxial cable (ii) parallel wire transmission line.

7.0 REFERENCES AND FURTHER READING

1. Kennedy, Davis Electronic Communication System , Fourth ed. McGraw-Hill International Edition 1993.
2. Dunlop, J. and Smith, D.G. Telecommunications Engineering second edition, Chapman and Hall London 1989.
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MODULE 4: WAVEGUIDES

UNIT 1: A **PAIR OF PARALLEL CONDUCTING PLANES**

1.0 INTRODUCTION

For the sake of simplicity, a pair of parallel conducting planes is discussed first before considering a wave guide. Note that a waveguide is a system of two pairs of parallel conducting planes forming a rectangular hollow box.

The only difference between a pair of conducting planes on one hand and parallel wire transmission lines and coaxial cable on the other hand is that conducting planes are not as lossy as the other two and therefore more efficient. The same TEM waves are however propagated in the three of them.

2.0 OBJECTIVES

By the end of this unit, the student should be able to

1. obtain the equation relating the wave number and the guide wave number.
2. determine cut-off frequencies of a pair of parallel conducting planes.
3. obtain the expressions of electric and magnetic field components of TE waves.

3.0 WAVEGUIDES

Waveguides are used for the same purposes as transmission lines i.e. to transmit electromagnetic energy. However, waveguides are used at microwave frequencies and they are not as lossy as transmission lines which are used at lower frequencies. There are two types of waveguides, the metal tubes of any cross section and the dielectric rods. Wave travels on the inside of the first type and on the outside of the other. The second type is outside the scope of this course. Also, we limit our discussion to rectangular wave guides.

RECTANGULAR WAVEGUIDE.

Rectangular waveguide is a hollow infinite rectangular pipe of highly conducting material. A rectangular waveguide is made up of two pairs of planes. For the sake of simplicity a pair of planes will be considered first.

TRANSMISSION OF WAVES IN A PAIR OF PARALLEL CONDUCTING PLANES

Let the separation between the pair of planes be the distance, d, assuming the planes are perfectly conducting and of infinite length, the characteristics of the electric and magnetic fields that can travel down between the planes have to be determined. It is necessary for the fields to obey Maxwell's equations in the free space in between the plates. The following are the boundary conditions on the planes.

(i) parallel or tangential component of electric field and the perpendicular or normal component of the magnetic field must be zero at every point on the planes. This is because there is no electromagnetic field inside perfect conductors.

(ii) perpendicular or normal component of electric field and parallel or tangential component of magnetic field need not equal zero. This is because there can be charges on the surfaces of the plane, and surface currents in them.

Obeying Maxwell's equations implies the satisfaction of electric field wave equations and the magnetic field wave equations i.e.

$$\nabla^2 E = \varepsilon_O \mu_O \frac{d^2 E}{dt^2} \text{ --(1)}$$

$$\nabla^2 B = \varepsilon_O \mu_O \frac{d^2 B}{dt^2} \text{ --(2)}$$

A linearly polarized plane wave whose electric and magnetic fields, respectively are

$$E = e_x E_O \exp[jw(t - kz/w)] \text{ --(3)}$$

$$B = -e_y \frac{E_O}{c} \exp[jw(t - kz/w)] \text{ -------------------------------------(4)}$$

is not only a solution to equations (1) & (2) but also satisfies boundary conditions (i) & (ii) stated above.

A linearly polarized wave whose electric and magnetic fields are expressed above is a transverse electric and magnetic (TEM) wave. This is because both fields are not only perpendicular to each other but are also transverse to the direction of propagation such that the three are mutually perpendicular i.e. orthogonal. Though TEM waves can travel in a pair of planes just as they could in parallel wire and coaxial cable transmission lines, they cannot, however, be propagated in wave guide which is a hollow conductor. This is because the perfectly conducting walls of the waveguide will short circuit the electric field, electric potential not being able to exist across perfect conductors.

In **Figure 1** is shown a pair of parallel planes separated by distance, a, in the x – axis.

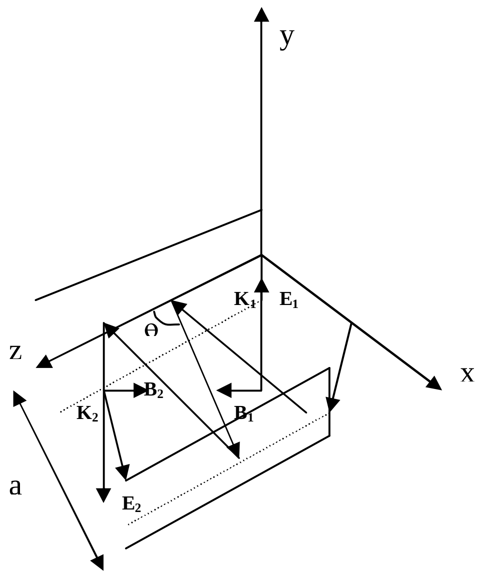

Fig 1:

The superposition of the incident and reflected waves whose electric fields and magnetic fields are represented by E_1 and B_1, and E_2 and B_2 respectively in Figure 1is also a solution to equations (1) and (2).

If E_1 oscillates along y – direction, its expression becomes

$$E_1 = u_y E_o \exp[jw(t - \frac{k_1.r}{w})]$$

$$= u_y E_o \exp[jw(t - r/w)] \text{ --- (5)}$$

Where $v_1 = \frac{-u_x \sin\theta}{V} + \frac{u_z \sin\theta}{V}$

Such that

$$\frac{1}{V_1}.r = \frac{-x\sin\theta}{v} + \frac{z\cos\theta}{v}$$

And equation (5) becomes,

$$E_1 = u_y E_o \exp[jw(t - \frac{z\cos\theta}{v} + \frac{x\sin\theta}{v})] \text{ ---------------------------- (6)}$$

on reflection the wave has velocity

$$\frac{1}{V_2} = \frac{u_x \sin\theta}{v} + \frac{u_z \cos\theta}{v}$$

The electric field, E_2, of the reflected wave is given by equation (7)

$$E_2 = -u_y E_o \exp[jw(wt - \frac{z\cos\theta}{v} - \frac{x\sin\theta}{v})]$$

Superposing E_1 and E_2 to obtain the resultant field E gives

$$E = u_y 2j\sin(\frac{wx}{v}\sin\theta)\exp[jw(t - \frac{z\cos\theta}{v})] \text{ ------------------------- (7)}$$

The planes in **Figure 1** are placed at x = 0 and x = a since by our boundary conditions E = 0 at the walls i.e. first at x = 0; substituting x = 0 in equation (7), E automatically becomes zero showing that the boundary condition is satisfied.

If θ is chosen such that $\sin\left(\frac{wx}{v}\sin\theta\right) = 0$ then E = 0 at x = a. This satisfies the boundary conditions on the second plane. The condition that

$$\sin\left(\frac{wx}{v}\sin\theta\right) = 0$$

Implies that $\frac{wx}{v}\sin\theta = n\pi$ or $\sin\theta = \frac{n\pi v}{wx}$ ------------------------------- (8)

putting $\sin\theta \leq 1$ and x = a, we have,

$$\frac{nv\pi}{wa} \leq 1$$

suppose $\frac{v\pi}{wa} \geq 1$, when n = 1, $\frac{\lambda}{2a} \geq 1$ or $f \leq \frac{c}{2a}$.

f is the cut- off frequency when n = 1. For n > 1, $f_n = nc/2a$.

For example when the pair of planes is separated by 2cm,

$$f_{01} = \frac{3\times10^8}{(2)(2)\times10^{-2}} =$$

from equation (8) i.e., $\sin\theta = \frac{nv\pi}{wx}$

$$\cos\theta = \left[1 - \left(\frac{nv\pi}{wx}\right)^2\right]^{\frac{1}{2}}$$

Substituting $\sin\theta$ & $\cos\theta$ into equation (7) gives

$$E = u_y 2jE_o \sin\left(\frac{n\pi x}{a}\right)\exp\left[jw\left(t - \frac{z}{V_g}\right)\right] \text{ ------------------------- (9)}$$

Where $$\frac{1}{V_g} = \left(\frac{1}{V^2} - \frac{n^2\pi^2}{w^2b^2}\right)^{\frac{1}{2}} \text{ --- (10)}$$

Using the fact that $\frac{1}{V} = \frac{k}{w}$

$$\frac{1}{V_g} = \frac{k_g}{w}$$

and $$k_g = \left(k^2 - \frac{n^2\pi^2}{a^2}\right)^{\frac{1}{2}} \text{ -- (11)}$$

Electric field components for TE waves are

$$E_y = 2jE_o \sin\left(\frac{n\pi x}{a}\right)\exp\left[jw\left(t - \frac{z}{v_g}\right)\right]$$

$$E_x = 0 \text{ and } E_z = 0$$

By the differential form of Faraday's law i.e. $\nabla \times E = \frac{dB}{dt}$ we can obtain the components of the magnetic fields by using

$$\begin{vmatrix} u_x & u_y & u_z \\ d/dx & d/dy & d/dz \\ 0 & E_y & 0 \end{vmatrix} = -\frac{dB}{dt}$$

we have

$$u_x\left(-\frac{dE_y}{dz}\right) + u_z\left(\frac{dE_y}{dx}\right) = -\frac{dB}{dt}$$

or

$$\frac{dB}{dt} = \frac{jw}{v_g}.2jE_o \sin\left(\frac{n\pi x}{a}\right)\exp\left[jw\left(t - \frac{z}{v_g}\right)\right]u_x - \frac{n\pi}{a}.2jE_o \cos\left(\frac{n\pi x}{a}\right)\exp\left[jw\left(t - \frac{z}{v_g}\right)\right]u_z$$

Integrating gives

$$B_x = \frac{2j}{v_g}E_o \sin\left(\frac{n\pi x}{a}\right)\exp\left[jw\left(t - \frac{z}{v_g}\right)\right]$$

$$B_y = 0$$

$$B_z = \frac{-2n\pi}{wa} E_o \cos\left(\frac{n\pi x}{a}\right) \exp\left[jw\left(t - \frac{z}{v_g}\right)\right]$$

$$= -2\left(\frac{1}{v^2} - \frac{1}{v_g{}^2}\right)^{\frac{1}{2}} E_o \cos\left(\frac{n\pi x}{a}\right) \exp\left[jw\left(t - \frac{z}{v_g}\right)\right]$$

where

$$\frac{1}{v^2} - \frac{1}{v_g{}^2} = \frac{n^2\pi^2}{w^2a^2} \text{ from equation (10)}$$

Note that we can also add two waves having their magnetic field in the y-direction only such that $B_1 = e_y B_o \exp\left[jw\left(t - \frac{z}{v_g}\right)\right]$ and $B_2 = -e_y B_o \exp\left[jw\left(t - \frac{z}{v_g}\right)\right]$

Superposing the two fields gives a set of equations in which the resultant B has only y-component, $B_{y;}$ B_x and B_z being equal to zero. The electric field, E, will then have a component in the z-direction. Such a wave is called a transverse magnetic (TM) wave.

4.0 CONCLUSION

The pair of parallel conducting planes just like parallel wire transmission line and coaxial cables transmit TEM waves. TE and TM waves are also propagated in the pair of parallel conducting planes but not in the other two.

5.0 SUMMARY

1. The equation relating the wave number, K amd the guide wave number is derived in the text.
2. The cut-off frequency is obtained from $f_n \le C\sqrt{\left(\frac{n}{2a}\right)^2}$
3. The expressions of electric and magnetic field components are obtained in the text.

6.0. TUTOR MARKED ASSIGNMENT

1. A pair of parallel conducting plates are separated by 2.5cm. Find the guide wavelength given that a 3cm wave propagates in the pair.
2. Calculate the cut-off frequency of the wave in question I.
3. Determine the electric and magnetic field components of wave propagating in a pair of parallel conducting plates.

7.0 REFERENCES AND FURTHER READING

1. Kennedy, Davis Electronic Communication System , Fourth ed. McGraw-Hill International Edition 1993.
2. Dunlop, J. and Smith, D.G. Telecommunications Engineering second edition, Chapman and Hall London 1989.
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UNIT 2: WAVEGUIDE

1.0 INTRODUCTION

Two pairs of parallel conducting planes form a rectangular waveguide. There two types of waveguides- the metal tubes of any cross section and the dielectric rods. Wave travels on the inside of the first type and on the outside of the other. The second type is outside the scope of this course. Also,we limit our discussion to rectangular waveguides.

2.0 OBJECTIVES

By the end of the unit, the student should be able to

1. determine guide wavelength
2. obtain the equation relating wave number and the guide wave number.
3. determine the expressions for the electric and magnetic fields.

3.0 WAVEGUIDE

Adding another pair of perfectly conducting parallel planes to the pair already treated such that these new planes are located at y = 0 and y = b, we obtain the waveguide shown in **Figure 2**. The waveguide is a hollow rectangular metal pipe. Only the TE and TM waves can propagate in the waveguide. By the presence of the new pair of parallel plates separated along y – axis, the electric field is expected to also vary in the y – direction in addition to its variation in the x – direction such that,

$$E \propto Kf(y)\sin\left(\frac{n\pi x}{a}\right)\exp\left[jw\left(t-\frac{z}{v}\right)\right]$$

or

$$E = Kf(y)\sin\left(\frac{n\pi x}{a}\right)\exp\left[jw\left(t-\frac{z}{v}\right)\right] \text{ ----------------------------- (12)}$$

For the wave to be able to propagate in the waveguide, equation (12) must satisfy equation (1), the electric field wave equation as well as Maxwell's equations. Taking the 1st Maxwell's equation i.e. $\nabla.E = 0$ or

$$\frac{dE_x}{dx}+\frac{dE_y}{dy}+\frac{dE_z}{dz}=0 \text{ -- (13)}$$

$E_z = 0$ since the wave cannot have component in the direction of propagation. Therefore $\frac{dE_z}{dz}=0$ thus equation (13) becomes

$$\frac{dE_x}{dx}+\frac{dE_y}{dy}=0 \text{ or } \frac{dE_x}{dx}=-\frac{dE_y}{dy} \text{ -- (14)}$$

$$\frac{dE_x}{dx}=-K\frac{df(y)}{dy}\sin\left(\frac{m\pi x}{a}\right)\exp\left[jw\left(t-\frac{z}{v}\right)\right]$$

By integration E_x becomes

$$E_x=\frac{ca}{m\pi}\frac{df(y)}{dy}\cos\left(\frac{m\pi x}{a}\right)\exp\left[jw\left(t-\frac{z}{v}\right)\right] \text{ ----------------------------- (15)}$$

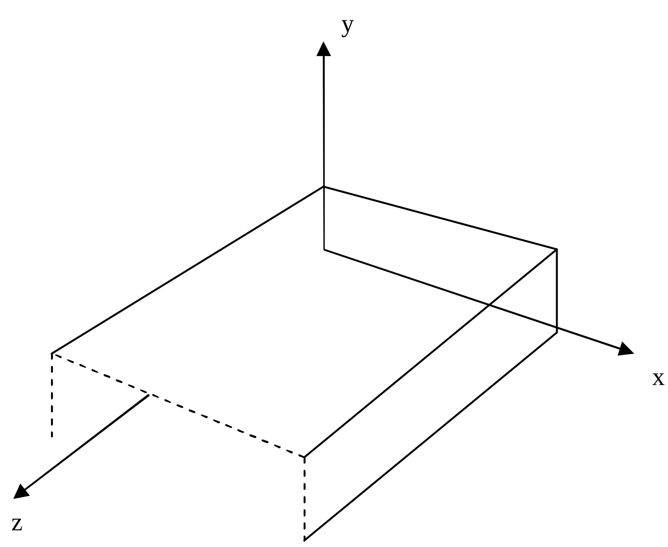

Fig 2:

We can determine f (y) by making E_x satisfy the boundary conditions that it be zero at the walls y = 0 and y = b and satisfy the wave equation and making sure tat the wave equation is satisfied by E_x.

For $E_x = 0$ at y = 0, $\frac{a}{m\pi}\frac{df(y)}{dy} = \sin ky = \sin n\pi$

or,

$$\frac{df(y)}{dy} = \frac{m\pi}{a}\sin ky = \frac{m\pi}{a}\sin m\pi$$

putting y = b, nπ = kb or k = nπ/b, and,

$$\frac{df(y)}{dy} = \frac{m\pi}{a}\sin\left(\frac{n\pi y}{b}\right)$$

Integrating $\frac{df(y)}{dy}$ we have

$$f(y) = -\frac{m\pi}{a}\cos\left(\frac{n\pi y}{b}\right)$$

For a pair of parallel plates we had,

$$k_g{}^2 = k^2 - \frac{n^2\pi^2}{b^2}$$

For two pairs of parallel plates,

$$k_g{}^2 = k^2 - \left(\frac{n^2\pi^2}{b^2} + \left(\frac{m^2\pi^2}{a^2}\right)\right)$$

or

$$k^2 - k_g{}^2 = \frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2} \quad \text{--------------------------------- (16).}$$

which is the waveguide equation.

The guide wave number, k_g, from which the cut-off wave length is determined for a particular mode is obtained from the dimensions (width and length) of the wave guide and free space wave number. We can write equation (16) as

$$k_C{}^2 = k^2 - k_g{}^2 \quad \text{-- (17)}$$

where

$$k_C{}^2 = \frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2} \quad \text{--------------------------------------- (18)}$$

or

$$\frac{4\pi^2}{\lambda_c{}^2} = \frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2} \quad \text{-- (19)}$$

Thus equation (17) can be written as

$$\frac{4\pi^2}{\lambda_c{}^2} = \frac{4\pi^2}{\lambda^2} - \frac{4\pi^2}{\lambda_g{}^2} \text{ or } \frac{1}{\lambda_c{}^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_g{}^2}$$

Substituting for f(y) into equations (12) and (15) we obtained the components of the electric field as shown below.

$$E_x = \left(\frac{kn\pi}{b}\right)\sin\left(\frac{n\pi y}{b}\right)\cos\left(\frac{m\pi x}{a}\right)\exp\left[jw\left(t-\frac{z}{v}\right)\right]$$

$$E_y = -\left(\frac{km\pi}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin\left(\frac{m\pi x}{a}\right)\exp\left[jw\left(t-\frac{z}{v}\right)\right]$$

$$E_z = 0$$

That $E_z = 0$ is actually the reason the wave is referred to as TE since it is transverse to the direction of propagation i.e. z-direction having no component in that direction. The expressions for the magnetic fields are obtained from the differential form of Faraday's law i.e. $\nabla \times E = -\frac{dB}{dt}$ as follows:

$$B_x = -\frac{k_g Km\pi}{wa}\cos\left(\frac{n\pi y}{b}\right)\sin\left(\frac{m\pi x}{a}\right)\exp\left[jw\left(t-\frac{z}{v}\right)\right]$$

$$B_y = -\frac{k_g Kn\pi}{wb}\sin\left(\frac{n\pi y}{b}\right)\cos\left(\frac{m\pi x}{a}\right)\exp\left[jw\left(t-\frac{z}{v}\right)\right]$$

$$B_z = jw\left(\frac{1}{v^2} - \frac{1}{v_g{}^2}\right)\cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{m\pi x}{a}\right)\exp\left[jw\left(t-\frac{z}{v}\right)\right]$$

Note that the direction of propagation is the z-direction. Thus the energy of the wave is in the z-direction. The rate of energy flow across unit area is referred to as POYNTING VECTOR, N. it can be shown that N is the vector product of the electric field and magnetic field of the wave i.e.

$$\underline{N} = \underline{E} \times \underline{B} \quad \text{--- (20)}$$

Expanding equation (4), we have,

$$N = \begin{vmatrix} u_x & u_y & u_z \\ E_x & E_y & E_z \\ B_x & B_y & B_z \end{vmatrix} = u_x\left(E_y B_z - E_z B_y\right) + u_y\left(E_z B_x - E_x B_z\right) + u_z\left(E_x B_y - E_y B_x\right)$$

Thus,

$N_x = E_y B_z - E_z B_y = E_y B_z$ since $E_z = 0$.

$N_y = E_z B_x - E_x B_z = -E_x B_z$ since $E_z = 0$, and

$N_z = E_x B_y - E_y B_x$ -- (21)

Since the direction of energy is the z-direction as mentioned above, $N_x = N_y = 0$ i.e. $E_y B_z = E_x B_z = 0$ implying that E_y and B_z are out of phase and E_x and B_z are out of phase. The energy produced by these products merely flows into and out of the planes. The cut-off frequency of TE waves is obtained from equation (3) and $f_c = c/\lambda_c$ to give

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \text{ --- (22)}$$

Note that only waves of frequency greater than f_c can be propagated in the guide of given a×b. Thus the wave guide behaves like a high-pass filter. In other words only modes whose cut-off frequencies are lower than radiation frequency can be propagated.

Transverse Magnetic (TM) waves in which there is no magnetic field in the direction of propagation are also propagated in the guide. They must, however, satisfy the boundary conditions of the guide as well as the wave equation. For such waves the electric field has component in the direction of propagation. The TM modes would have cut-off frequencies at the same values as the TE waves given by the equation (6). We must however note that unlike the TE there can't be TM_{on} or TM_{mo}. This is because the field lines of magnetic field in the waveguide is two-dimensional there being no magnetic charges or mono poles and as such the magnetic field has to vary with both coordinates other than in the direction of propagation. Thus the lowest TM modes that can exist is the TM_{11} mode. The electric field, on the other side is one-dimensional in the guide. As such its lines of force go from one plane to another. Thus the lowest TE mode is either TE_{01} or TE_{10} depending on which of the width, a, or the height, b, is longer. For instance the cut-off frequencies for

TE_{01} i.e. $f_{01} = \frac{c}{2}\sqrt{\left(\frac{1}{b}\right)^2} = \frac{c}{2b}$

While that of TE_{10} i.e. $f_{10} = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2} = \frac{c}{2a}$

from equation (22).

The lowest TE mode is the fundamental and important mode in the rectangular waveguide. Take for example a waveguide of dimensions a = 3
cm and b= 1.5cm in which a wave whose wavelength in free space is 5cm corresponding to a frequency of 6×10^9Hz i.e. 6GHz, the cut-off frequencies of the following modes are determined and compared.

f_{01} corresponding to TE_{01} mode = 6.7×10^9Hz

f_{10} corresponding to TE_{10} mode = 5×10^9Hz

f_{02} corresponding to TE_{02} mode = 1.33×10^{10}Hz

f_{20} corresponding to TE_{20} mode = 1.01×10^{10}Hz

f_{11} corresponding to TE_{11} mode = 1.12×10^{10}Hz

It is only f_{10} that is lower than the frequency of the propagated wave. As such it is only f_{10} that is allowed in the guide. Note that f_{10} is the fundamental and most important mode.

The velocity with which wave crests and/or troughs travel through a medium is known as phase velocity, v_p. From the fact that the velocity is the quotient of angular frequency and wave number i.e. v = w/k, the phase velocity in a wave guide is v_p = w/k_g (k_g being guide wave number). k_g can be determined from the combination of equations (17) and (18) from which we can obtain v_p. In the example above v_p = 5.5×10^8ms^{-1} and v_p > c (the velocity of light)

The group velocity, v_g is the velocity of a group of waves while phase velocity is the velocity of the individual waves within the group. Waves within the group are observed to travel faster than the group itself, thus $v_p > v_g$. It is, however, the group velocity that is normally measured since wave velocities are measured by the arrival of the disturbances and it is the velocity by which the energy of the wave is propagated. $v_g < c$ and as such there is no contradiction of Einstein's. Note that the product of phase velocity and group velocity equals c^2. Thus v_g = c/v_p = 1.64ms^{-1} Equation (20) can be used to determine the average power a waveguide operate on by finding the average of the poynting vector i.e. the right hand side (RHS) of equation (21). This has to be integrated over the cross-sectional area of the guide since the poynting vector is defined as rate of energy per unit area.

In the example above the poynting vector of the guide in the direction of propagation i.e. N_z for the TE_{10} mode $= E_y H_x = \frac{k_g}{w\mu_o} E_y{}^2 \sin^2\left(\frac{\pi x}{a}\right)$ by combining the expressions of E_y and B_x above and noting that $B = H_x/\mu_o$. Note also that for TE_{10} mode, m = 1 while n = 0.Substituting for m and n in the expressions of E_x and B_y above shows $E_x = B_y = 0$ such that $N_z = E_yB_x$ implying that the first term of the left hand side of equation (21) is zero. The average power of operation of the guide for TE_{10} mode is thus given by $\frac{1}{2}\frac{k_g}{\mu_o w}E_{10}{}^2\frac{ab}{2}$, k_g, a, b and w are obtainable from the wavelength and guide dimension known; the electric field, E, would be given and the average power can then be obtained. Integrating this expression over the area of the guide, we obtain

$$\frac{1}{2}\frac{k_g}{w\mu_o}E_{oy}{}^2 \sin^2\left(\frac{n\pi}{a}\right)$$

4.0 CONCLUSION

The TEM wave which can be propagated in the pair of parallel conducting planes, the parallel wire transmission line and the coaxial cable cannot be propagated in a wave guide. This is because in a waveguide either the electric field or the magnetic has a component in the direction of propagation in which case we have transverse magnetic (TM) or transverse electric (TE) waves respectively.

The average power of operation of a waveguide can be obtained from the poynting vector, which is defined as rate of energy flow across unit area, in the direction of propagation.

5.0 SUMMARY

1. The guide wavelength is obtainable from the guide wave number.
2. The equation relating the wave number and guide wave number is derived in the text
3. The expression for the electric and magnetic field of the guide are obtained in the text

6.0 TUTOR MARKED ASSIGNMENT

1. The dimension of a wave guide is given as 2cm by 3cm.
   Calculate (i) the ground wavelength (ii) the guide wave number.

7.0 REFERENCES AND FURTHER READING

1. Kennedy, Davis Electronic Communication System , Fourth ed. McGraw-Hill International Edition 1993.
2. Dunlop, J. and Smith, D.G. Telecommunications Engineering second edition, Chapman and Hall London 1989.
3. Grant, I.S. and Philips, W.A. Electromagnetism ELBS Manchester 1974
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MODULE 5: RESONANT CIRCUIT, RADIATION FROM AN OSCILLATING DIPOLE AND RADIATION FROM MOVING CHARGES

UNIT 1: RESONANT CIRCUIT

1.0 INTRODUCTION

There are many occurrences of resonance in everyday life. Sometimes in a building, when the door of a room is jammed forcefully, the doors of other rooms are observed to respond by vibrating. This response is known as resonance. In this case the second door is said to resonate with the first door, the frequencies of both doors being the same.

2.0 OBJECTIVE:

By the end of this unit, the student should be able to

1. define resonance
2. explain resonance
3. write the simple differential equation of a resonant circuit.

3.0 RESONANT CIRCUIT

Resonance is a phenomenon in which a vibrating system responds with maximum amplitude to an alternating driving (external) force. Thus any electrical circuit that exhibit resonance is a resonant circuit. **Figure 1** shows a resonant circuit (RLC circuit).

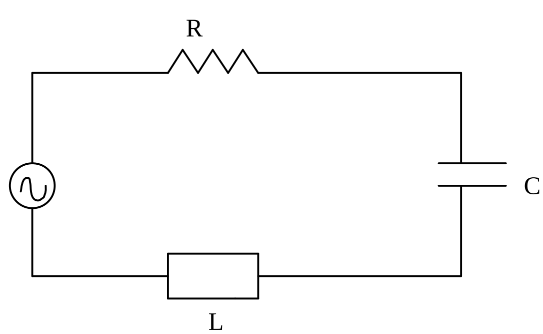


Fig 1:

Resonance occurs when the frequency of the external or driving force is exactly equal to the natural frequency of the undamped system. The natural frequency of the undamped system is determined from the circuit above in the absence of the resistance which causes damping. The equation of motion for such a free oscillatory circuit when no external force is applied is

$$L\frac{di}{dt}+\frac{q}{c}=0 \text{ or } L\frac{d^2q}{dt^2}+\frac{q}{c}=0 \text{ --------------------------- (1)}$$

The solution of this differential equation is

$$q=q_o \sin w_o t \text{ --- (2)}$$

Where q is the charge at any point in time, t and q_o is the maximum value of charge while w, is the angular frequency of oscillation. By differentiating equation (2) twice, we obtain

$$\frac{d^2q}{dt^2}=-w^2q_o \sin wt$$

This equal to $\frac{d^2q}{dt^2}=-w^2q$ or $\frac{d^2}{dt^2}+w^2q=0$ ---------------------------- (3)

Comparing equation (3) and (1) indicates

$$w^2=\frac{1}{LC} \text{ or } w=\frac{1}{\sqrt{LC}} \text{ -------------------------------- (4)}$$

Implying that the natural angular frequency, w, equals $\frac{1}{\sqrt{LC}}$

The impedance, Z, of the RLC circuit in Figure --- is

$$Z=\sqrt{R^2+\left(X_L+X_C\right)^2} \text{ ----------------------------------- (5)}$$

Where X_L, the inductive reactance (positive reactance) = wL and X_C, the capacitance reactance (negative reactance) = 1/wc. Substituting equation (5) becomes

$$Z=\sqrt{R^2+\left[(wL)-\left(\frac{1}{wc}\right)\right]^2} \text{ --------------------------------- (6)}$$

When $wL=\frac{1}{wc}$ --- (7)

the current flowing will be high from $I_{rms}=\frac{E_{rms}}{Z}$ and resonance occurs. The circuit is then said to be tuned to the resonant frequency. From equation (7)

$$w=\frac{1}{\sqrt{LC}} \text{ --- (8)}$$

when resonance occurs (compare equations (4) and (8)). The tuning of the radio set is a demonstration of resonance in which the knob of the receiver-set is used to vary either the inductance or the capacitance such that the natural frequency of the set is equal to the frequency of a desired signal. In this way different radio signals of choice can be received.

Example:

The aerial circuit of a radio set is equipped with a tuning coil of inductance 1.8mH. What tuning capacitor must be used with this to tune to the BBC long wave station (200KHz)

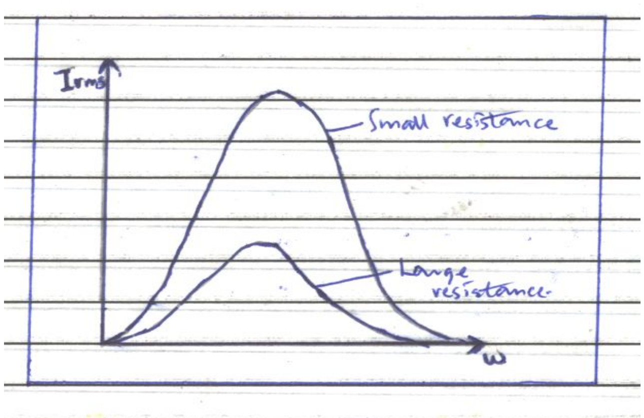
SOLUTION:

From equation (4) or (8)

$$C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 \left(200\times10^3\right)\left(1.8\times10^{-3}\right)}$$

$$= 3.5\times10^{-10} = 350\times10^{-12} = 350\,pF$$

In Figure 2 is illustrated the plots of I_{rms} versus angular frequency.



4.0 CONCLUSION

The simplest resonant circuit consists of an inductor and a capacitor. It responds with maximum amplitude to an alternating driving (external force). In other words, it resonates. A radio set is a complex resonant circuit which resonate when tuned to external radio signals picked by its aerial.

5.0 SUMMARY

1. Resonance is defined as a phenomenon in which a vibrating system responds with maximum amplitude to an alternating driving (external) force.
2. The exploration of this phenomenon is contained in the text

3. A simple differential equation of a resonant circuit is

$$L\frac{di}{dt}+\frac{q}{c}=0$$

The symbols are defined in the text.

6.0 TUTOR MARKED ASSIGNMENT

1. (a) Define (b) explain the phenomenon of resonance.
2. Show that the resonant equation can be written as

$$L\frac{d^2q}{dt^2}+\frac{q}{c}=0$$

3. Show that the natural angular frequency,W, of resonant circuit is

$$w=\frac{1}{\sqrt{LC}}$$

7.0 REFERENCES AND FURTHER READING

1. Kennedy, Davis Electronic Communication System , Fourth ed. McGraw-Hill International Edition 1993.
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UNIT 2: RADIATION FROM AN OSCILLATING DIPOLE

1.0 INTRODUCTION:

The electromagnetic wave spectrum consists of radio waves {104- 109 Hz}, micro waves {10^8-10^{12}Hz} infra-red {10^{12}-10^{14}Hz} visible light {less than 10^{15}Hz},utraviolent {10^{15}-10^{17}Hz}, X-rays {10^{15}-10^{25}Hz} and Gamma rays {10^{17}-10^{32}Hz}.Each component of this spectrum has its source and method of generation. In this unit the method of generating radio wave i.e by an oscillating dipole is discussed.

2.0 OBJECTIVE

By the end of this unit, the student should be able to

1. understand that oscillating dipoles are aerials (antenna)
2. describe how radiations emanate from oscillating dipole
3. derive the electric field and magnetic field componenets of a dipole.

3.0 RADIATION FROM AN OSCILLATING DIPOLE

Recall that a dipole is a system of two equal but opposite charges separated by a short distance. The product of the distance, x and the magnitude of one of the charges, q is the dipole moment, P. We can then write

$$P = qx$$

Radio waves are generated by oscillating electrons in conductors and dipole antennae. An example of such a dipole is that of two brass rods with a small gap between is connected to a high voltage supply, say 5kv. An oscillatory discharge between the polished ends of the rods is produced. This sparkling is due to the ionization of the air in the gap between the ends of the rods as a result of the magnitude of the electric field. Suppose the gap between the ends of the brass rods is 1.5mm, the electric field would be the quotient of the potential difference and the separation of the rods i.e. V/x which becomes 5kv × $(1.5mm)^{-1}$ = $3.3MVm^{-1}$ on substitution.

The two brass rods describe an antenna or aerial. Antennas are often in the form of a metal wire or rod in which electrons surge back and forth periodically. At a certain time, one end of the wire or rod will be negative and the other end positive. Half a cycle later polarity of the ends is exactly reversed. This is referred to as an oscillating electric dipole since its dipole moment changes in a periodic way with time

The oscillating dipole also radiates electromagnetic field in the radio wave frequency range. The radiation field of an oscillating electric dipole is obtained as follows. Let the charge of the spark vary with time as

$$q = q_A \sin wt$$

where q_o is the amplitude of the oscillating charge and w is the angular frequency of the oscillations. Let the gap between the brass rods be small compared to the wavelength of the radiation produced. The current set up can be written as

$$I = wq_A \cos wt = I_A \cos wt$$

hence $I_A = wq_A$

Note that the oscillating charge is equal to an oscillating dipole moment. Let the gap between the rods be of magnitude x in length, then dipole moment, P is

$$P = qx = P_A \sin wt$$

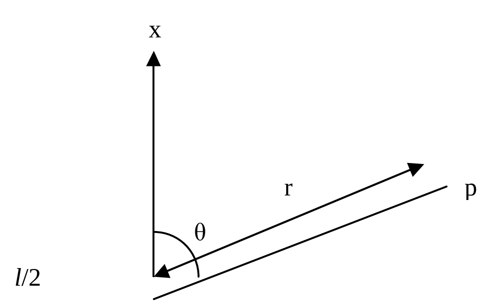
where $P_A = q_A x$

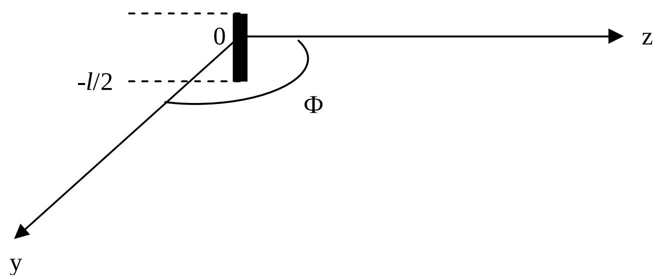
The simple oscillating dipole is called a Hertzian dipole whose importance is in the fact that many radiating systems are thought to be made up of large number of such dipoles. In order to find the total power radiated by the Hertzian dipole, we proceed as follows:

The power density is given by the Poynting vector (The poynting vector, N, is defined as energy crossing unit area per unit time; see section 2.0). The Poynting vector, $N = \underline{E} \times \underline{B}$. We can then write P = E × B (since power density, P ≡ N).

We need to find the expressions of the electric (E) and magnetic fields (B) of the radiation. Consider a very simple antenna i.e. a short filament in which current I = $I_o$sinwt is flowing. Magnetic field is induced in the neighbourhood of the filament. An electric field is associated with the varying magnetic field. The radiation is thus electromagnetic. The various components of the electromagnetic field is given as follows:

In the **Figure** below is a filament of length, *l*. Components of electromagnetic field at a point P, a distance r, from the centre is to be determined.





In this problem in which we have to relate the fields to the changing current producing them, it is easier to work with the retarded potential. Note that the field at P will be produced by the current flowing through the filament at time t – r/c, where r/c is the time taken for the field to travel from the filament to P at the velocity of light.

The retarded potential A, is related to the current in the filament by

$$A = \frac{1}{4\pi}\int_V \frac{i[t-\left(r/c\right)]}{r}dv$$

Since the current is in the x-axis only we can write

$$A_x = \frac{1}{4\pi}\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{I \sin w\left[t-\left(r/c\right)\right]}{r}dx$$

$$= \frac{Il}{4\pi r}\sin w\left[t-\left(r/c\right)\right] \text{ --------------------------- (1)}$$

It can be shown that the retarded potential A is related to the magnetic field B, by

$$\underline{B} = \nabla \times \underline{A} \text{ ------------------------------ (2)}$$

Using Spherical coordinate, by equation (1)

$$A_r = A_x \cos\theta = \frac{Il}{4\pi r}\sin w\left(t-(r/c\right)\cos\theta$$

$$A_\theta = -A_x \sin\theta = \frac{-Il}{4\pi r}\sin w\left(t-(r/c\right)\sin\theta$$

$$A_\phi = 0$$

Using equation (2) we can find the spherical components of the magnetic fields:

$$B_r = 0, B_\theta = 0$$

$$B_\phi = \frac{Il}{4\pi}\sin\left[\frac{w}{cr}\cos w\left(t-r/c\right)+\frac{1}{r^2}\sin w\left(t-r/c\right)\right]$$

The spherical components of the electric fields are obtained from the differential form of Faraday's law i.e. the third Maxwell's equation $i.e. \nabla \times E = \frac{-dB}{dt}$

$$E_r = 0$$

$$E_\theta = \frac{-wIl\sin\theta}{4\pi\varepsilon_o c^2}\frac{\sin\left[w\left(t - r/c\right)\right]}{r}$$

$$E_\phi = 0$$

4.0 CONCLUSION

Many radiating systems are made up of the simple oscillating dipole which is called a Hertzian dipole. These are the common aerials (antenna). The electric and magnetic fields from which the power density is obtained have been determined.

5.0 SUMMARY

1.Oscillating dipoles are aerial, either transmitting or receiving aerials

2. Radiations from an oscillating dipole is fully explained in the text

3 Electric and magnetic fields of an oscillating dipole are derived in the text.

6.0 TUTORED MARKED ASSIGNMENT

1. Define, (i) a dipole,(ii) Hertaian dipole.
2. Explain how radiations emanate from oscillating dipole.
3. Derive the electric and magnetic field components of an oscillating dipole.

UNIT 3: RADIATION FROM MOVING CHARGES.

1.0 INTRODUCTION

The method generation of radiation from moving charges is described in this unit.The devices used to cause the oscillation of charges are magnetrons and Klystrons. These devices employ resonant cavities which are discussed below.

2.0 OBJECTIVES

By the end of this unit the student should be able to;

1. Explain how radiations are generated by moving charges.
2. Derive the electric and magnetic field components of a resonant cavity, a device used in oscillation of charges.

3.0 RADIATION FROM MOVING CHARGES

Moving charges are known as electric currents. Thus a wire in which charges flow is an electric current-carrying-wire. Around such wires or conductors, radiation is generated. In the last section, the radiation from oscillating dipole was treated. This radiation consists of radio waves. When electrons are made to oscillate at frequencies of 10GHz in a tuned cavity, microwaves (with wavelength of a few centimetres) are produced. The devices used to cause the oscillations of charge which produce microwaves are magnetrons or klystrons. These devices use resonant cavities. Simply defined, resonant cavity is a piece of waveguide closed off at both ends. Whereas wave is propagated in a longitudinal direction in the waveguide, in the resonant cavity standing waves exist and oscillations can take place if the cavity resonator – as it is also called – is suitably excited. The simplest cavity whose resonant frequencies can be calculated is a rectangular cavity.

In the rectangular cavity resonator, the electric field can be represented by

$$E = E_O\, f(x, y, z)\exp(jwt)$$

Where w is angular frequency of the wave function f(x,y,z) gives the variation of the electric field in the three directions. Note that the electric field is no longer constant in the z-direction i.e. it now varies in that direction.

Following the example of the waveguide in section 4.3, the components of the electric field of TE standing waves in the cavity which satisfy the boundary conditions on the walls and satisfy the equation $\nabla . E = \mathrm{O}$ are

$$E_X = -\left(\frac{cn\pi}{b}\right)\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\sin\left(\frac{l\pi z}{d}\right)\exp(jwt)$$

$$E_y = \left(\frac{cm\pi}{a}\right)\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin\left(\frac{l\pi z}{d}\right)\exp(jwt)$$

$$E_z = 0$$

The associated magnetic field components are obtainable from the differential form of Faraday's law i.e. $\nabla . E = -\dfrac{db}{dt}$ or by using the pattern of the magnetic field component of the rectangular waveguide. The magnetic field components are

$$B_x = -j\left(\frac{kml\pi^2}{wad}\right)\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{l\pi z}{d}\right)\exp(jwt)$$

$$B_y = \frac{-jknl\pi^2}{wbd}\sin\left(\frac{n\pi y}{b}\right)\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{l\pi z}{d}\right)\exp(jwt)$$

$$B_z = \frac{jk\pi^2}{w}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin\left(\frac{l\pi z}{d}\right)\exp(jwt)$$

These electric field and magnetic field components would be acceptable solutions to Maxwell's equations inside the cavity if they satisfy the wave equations for the electric and magnetic fields. Substituting the electric field and magnetic components into the appropriate wave equations shows that the fields are acceptable. If the integers m, n, *l* and the wave number k are related by

$$\frac{\pi^2 m^2}{a^2} + \frac{\pi^2 n^2}{b^2} + \frac{\pi^2 l^2}{d^2} = k^2$$

(compare equation [16] in section 4.3) from which the resonant frequencies (depending on the integers m, n, *l*) of the cavity can be derived as follows. We can write

$$\frac{4\pi^2}{\lambda^2} = \frac{\pi^2 m^2}{a^2} + \frac{\pi^2 n^2}{b^2} + \frac{\pi^2 l^2}{d^2}$$

and using $\lambda = c/f$, we have

$$\frac{4f^2}{c^2} = \frac{m^2}{a^2} + \frac{n^2}{b^2} + \frac{l^2}{d^2}$$

or

$$f = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{l}{d}\right)^2}$$

Note that the lowest frequency – in which a lowest mode is excited – corresponds to any two of m, n and *l* equals to one and the third integer associated with the shortest side of the cavity, equal to zero i.e. TE_{011}, TE_{101}, or TE_{110} as the case may be. Another set of possible solutions for the electromagnetic fields in the cavity are expressions of electric and magnetic fields corresponding to Transverse Magnetic (TM) standing waves in which the electric field has a z-component while the z-component of the magnetic field is zero. TM_{mnl} waves occur at the same frequencies as the TE_{mnl} waves for the same cavity resonator. The sum of the resonating TE and TM waves gives the particular resonance of the standing wave.

Cavity resonator can be used as a frequency meters apart from its use for the generation of microwaves. Measurement of frequency is made possible by the use of a plunger attached to one end face which then allows the resonant length of the cavity to vary.

EXAMPLE:

The dimensions of a rectangular cavity are a = 2cm, b = 2.5cm and d = 3cm. determine the number of resonances possible within a frequency range of $f = 5 \times 10^9$Hz and $f = 10^{10}$Hz.

SOLUTION:

Resonance occurs in the cavity resonator for frequencies in between the limits of frequencies given. Resonance will occur for

(i) TE_{011} whose $f_{011} = 6.3\times10^9$Hz

(ii) TE_{012} whose $f_{012} = 9 \times10^9$Hz

(iii) TE_{111} whose $f_{111} = 9.88 \times10^9$Hz

4.0 CONCLUSION

Oscillation of charge is made possible by devices such as magnetrons and Klystrons.These devices use resonant cavities. A resonant cavity described in the text is a single waveguide closed off at both open ends. The components of its electric and magnetic fields have been derive.

5.0 SUMMARY

1. Generation of radiations by moving(oscillating)charges is described in the text.
2. Electric and magnetric field components have been derived in the text.

6.0 TUTORED MARKED ASSIGNMENT

1.What is the relationship between magnetron and a resonant cavity.

2. Obtain the electric and magnetic fields of a resonant cavity.

3. Given that the dimensions of a resonant cavity is 1cm by 1.5cm by 2cm.What is the upper limit of a frequency range over which we can have 3 region arces given that the lower limit is $f=10^{10}$Hz.

7.0 REFERENCES AND FURTHER READING

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