NATIONAL OPEN UNIVERSITY OF NIGERIA

SCHOOL OF SCIENCE AND TECHNOLOGY

COURSE CODE: PHY 457

COURSE TITLE: ENVIRONMENTAL PHYSICS

NATIONAL OPEN UNIVERSITY OF NIGERIA

SCHOOL OF SCIENCE AND TECHNOLOGY

COURSE GUIDE

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Course Code	PHY 457
Course Title	ENVIRONMENTAL PHYSICS
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NATIONAL OPEN UNIVERSITY OF NIGERIA

CONTENTS

INTRODUCTION

In your study of elementary physics, you found that energy is very important to us. Without energy, no work can be done. However, in the process of generating energy, environmental problems are created. Some of these problems are global warming, ozone layer depletion, energy crisis, nuclear waste disposal complications and water pollution. In order to solve these environmental problems, we must have a good understanding of the physics of the environment. In PHY 457, we shall strive to understand how environmental problems are created and how the problems can be solved using the principles of physics.

THE COURSE

The course title is Environmental Physics and the course code is PHY 457. This course comprises a total of six units organized into two modules.

Module 1 is composed of three units

Module 2 is composed of three units

In Module 1, you shall learn about different energy resources, the atmosphere, remote sensing and artificial satellites. In this Module, emphasis laid on the impact of each item on the environment. In Unit 1 of Module 1, you shall learn about different sources of energy and their effects on the environment. In Unit 2 of Module 1, you shall be introduced to the basic properties of the terrestrial atmosphere. Unit 3 discusses the basic concepts involved in the operation of artificial satellites and the remote sensing techniques.

Module 2 is devoted to satellite orbits, global weather and climatic patterns. In Unit 1 of Module 2, you shall be introduced to the steps involved in determining the orbit of a satellite within the framework of a two-body system. You shall see how a many-body problem may (approximately) be reduced to a two-body problem.

In Unit 2 of Module 2, you shall be introduced to the basic mathematical tools needed to solve a many-body problem. You shall also learn how a satellite may transfer from one orbit to another. In Unit 3 of Module 2, you shall learn some of the most serious environmental problems facing the Earth. In particular, you shall examine possible methods of solving these problems using the principles and techniques of physics.

COURSE AIMS AND OBJECTIVES

The aim of PHY 457 is to provide you with a deep and broad knowledge of the environmental problems that affect the Earth and the possible ways of solving them. In this course, you shall learn the sources of air pollution, water pollution, chemical pollution and thermal pollution. We shall also examine the role of artificial satellites in identifying and solving environmental problems by means of identifying and solving environmental problems by means of remote sensing techniques. In order to do this effectively, a good knowledge of the density and temperature profiles of the atmosphere is essential. For this reason, adequate attention is also paid to the structure and composition of the terrestrial atmosphere.

One of the principal aims of this course is to show you how the energy cycle leads to the changing weather conditions associated with the Earth. You shall also learn how *cost-benefit* analysis serves to expose the *risk* and the *benefit* associated with the formulation and execution of policies relating to environmental issues. After working diligently through this course, you should be able to provide a satisfactory answer to each of the following questions: How does energy-generation mechanism affect the environment? What roles does the terrestrial atmosphere play in ensuring that the environment remains conducive to life? Why does remote-sensing scientist rely extensively on artificial satellites? How are satellites placed in orbit? How does space weather affect global weather and climatic conditions?

WORKING THROUGH THE COURSE

You will find that PHY 457 is presented in a very simple manner. Some parts of the course are purely descriptive. Others are either semi-quantitative or purely quantitative. In all cases, the earlier parts of the course leads you step-by-step to the later parts of the course. You are advised to pay attention to all parts of the course. You should try to re-work all the examples on your own before proceeding to do the assignments. You should also ensure that you participate actively in all the tutorial sessions. In that way, you shall be able to derive *maximum benefit* from your course mates, your course facilitators and your study units.

COURSE MATERIALS

Before the commencement of this course, you shall be provided with your Course Guide, your Study Units, and a list of recommended textbooks. These course materials are very important. You should make all necessary efforts to ensure that you gain access to an adequate number of relevant textbooks. You should also do your best to study the materials very well.

STUDY UNITS

The 3 study units contained in Module 1 are arranged in the following order:

Unit 1: Energy Resources and Environmental Pollution

- Physics and the Environment
- Air Pollution
- Thermal Pollution
- Fossil Fuel Steam Plant
- Nuclear Energy
- Hydroelectric Power Plant
- Tidal Power Plant
- Wind Power Plant
- Solar Energy

Unit 2: Structure and Composition of the Atmosphere

- Properties of the Atmosphere
- Vertical Diminution of Density with Height

- The Isothermal Atmosphere
- The Adiabatic Atmosphere

Unit 3: Artificial Satellite and Remote Sensing

- The Orbit of the Earth
- Satellites in Circular Orbits
- Remote Sensing

The 3 Study Units Contained in Module 2 are arranged as follows:

Unit 1: Satellite Orbits in the Two-Body System

- Concept of a Two-Body problem
- Solution to the Two Body problem
- The Angular Momentum Integral
- Mathematical Form of Kepler's Second Law
- Energy Conservation Equation of the System
- Satellite Orbit in a Two-Body System

Unit 2: Satellite Orbit in a Many-Body Problem

- The Many-Body Problem
- Equations of Motion in a Many-Body Problem
- The Ten Known Integrals of the Motion
- The Force Function
- Moment of Inertia and Force Function
- Transfer Orbits

Unit 3: Global Weather and Climatic Patterns

- The Five Distinct Components of the Earth
- The Energy Cycle

- The Terrestrial Atmosphere
- Weather and Land form
- Human activity and the Environment
- Greenhouse Effect
- Ozone Layer Depletion
- Environmental Management

TEXTBOOKS

J.D. Cutnell & K.W. Johnson (1997). *Physics*. John Wiley & Sons Inc., New York.

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ASSESSMENT

The National Open University of Nigeria assesses your performance by means of the following instruments:

(a) Tutor Marked Assignment (TMA);

(b) End of Course Examination.

TUTOR MARKED ASSIGNMENT

Tutor Marked Assignment (TMA) represents the traditional Continuous Assessment. It accounts for 30% of your total score. In this course, you are provided with six TMAs. You should try to do all the 6 TMAs. Please, note that you will not be allowed to sit for the End of Course Examination without completing at least 5 out of the 6 TMAs. Your course facilitator shall provide the TMAs. You are required to complete each TMA and return same to your course facilitator within the stipulated period of time. You must carry out the Tutor Marked Assignments.

END OF COURSE EXAMINATION

The End of Course Examination carries 70% of your total score. The date, time and venue of the examination shall be made known to you in advance. This may or may not coincide with the Semester examination of the National Open University of Nigeria. You must sit for the End of Course Examination.

SUMMARY

This course is split into two modules. Each of the two modules has three units. Each unit is carefully designed to ensure that you understanding its *message* with relative ease. Abstract concepts are systemically translated into concrete applications following a simple *logical* sequence.

Upon satisfactory completion of this course, you should be able to discuss the sources of environmental pollution and possible solutions in the light of the principles and techniques of Physics. You should also be able to explain the effect of the terrestrial atmosphere on the weather conditions of the Earth.

It is advisable for you to study each of the 6 units contained in this course carefully. It is also highly recommended that you read as many of the recommended textbooks as possible. In this way, you shall be able to derive the maximum benefit from this course.

I wish you the best in your effort to gain knowledge.

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NATIONAL OPEN UNIVERSITY OF NIGERIA

MODULE 1

UNIT 1: ENERGY RESOURCES AND ENVIRONMENTAL POLLUTION

CONTENTS **1**

1. INTRODUCTION

The Sun is the ultimate source of the energy that sustains life on the Earth. In this units, you will be introduced to the principal techniques by which energy is transformed from one form to a more useful form. You will also learn how each of these energy resources impacts on the environment.

2. OBJECTIVES

At the end of this unit, you shall able to describe

- (i) the source of air, thermal and chemical pollution
- (ii) seven energy-generating mechanisms.
- (iii) the adverse effects of energy resources on the environment.

3 BASIC CONCEPTS

3.1 Physics and the Environment

Environmental Physics is the application of the laws and techniques of physics in order to provide a description of the natural environment. Today, there are many environmental problems facing mankind. Some of these problems are global warming, ozone layer depletion, energy crisis, nuclear and electronic waste disposal complications. Others are air, soil, water and thermal pollution. These problems and their solutions can be understood from the perspective of physics. This explains why Environmental Physics is a very important discipline. The ultimate aim of the Environmental Physicist is to understand these issues and provide solutions using the principles and techniques of physics.

3.2 Air Pollution

Air is polluted when fossil fuel is burned. Fossil fuels include coal, oil, gas and fire-wood. Air pollution can result from burning the fossil fuel in cars, in industrial furnaces for smelting, and in electric generating plants. The situation is worse if the burning takes place so quickly that the combination is incomplete. That is why the internal combustion engines of automobiles are especially hostile to the environment. Because of the incomplete nature of the combustion in automobile engines, more noxious gases are produced. Even when the combustion is complete, the situation still remains bad since the carbon-dioxide (CO_2) released into the atmosphere absorbs some of the natural infrared radiations emitted by the warm Earth (and prevents them from escaping). The resulting buildup of atmospheric CO_2 and the consequent heating of the atmosphere is known as the greenhouse effect.

Scientists have projected that the greenhouse effect is capable of raising the average temperature of the atmosphere by several Celsius degrees within this century. Such rise in the temperature of the atmosphere is of serious concern due to the fact that it can cause a shift in rainfall patterns and melt polar ice caps. The enormous amounts of water resulting from the melted polar ice caps will be dumped into the oceans thereby raising the sea-level and flooding the low-land areas. There is, therefore, an urgent need for humans to limit the rate at which fossil fuel is burned.

3.3 Thermal Pollution

Every heat engine exhausts heat to the environment. Automobiles and power plants are good examples of heat engines. Electricity-generating power plants use a heat engine to convert thermal energy into electrical energy. The exhausted heat is generally absorbed by a coolant such as water.

The basic idea behind a heat engine is that thermal energy can be transformed into mechanical energy only when heat is allowed to flow from a high-temperature reservoir to a low-temperature reservoir. In the process, some of heat can then be used to do mechanical work. The efficiency η of the heat engine is given by

$$\eta = 1 - \frac{Q_L}{Q_H},$$

where Q_H is the heat input at the high temperature T_H, and Q_L is the heat condemned at a low temperature T_L. For an ideal Carnot engine, the heat Q is proportional to the corresponding temperature T. Thus, the efficiency η of the ideal Carnot engine is given by

$$\eta = 1 - \frac{T_L}{T_H}.$$

This means that the efficiency of the heat engine can be increased by reducing the value of T_L. Hence, for greater efficiency, a large quantity of water must flow as coolant through a power plant in order to keep the temperature T_L as low as possible. Of course, this water has to come from the environment. The water (which may come from a nearby river, a lake, a sea or an ocean) is ultimately recycled back to where it came from. As more and more heat is transferred to the water, its temperature continues to rise. The resulting warm water holds less oxygen. As a result, this process can cause significant damage to aquatic life in the environment.

Sometimes, large cooling towers are used to exhaust heat at an electric–generating plant. The cooling towers discharge the heat into the atmosphere. The heated air resulting from this process can adversely affect the weather of the region. Thus, this method gives rise to serious environmental pollution. In what follows, you will be introduced to some of the energy resources and their negative effects on the environment.

3.4 Fossil-Fuel Steam Plants

A fossil-fuel steam plant burns coal, oil or natural gas in order to boil water and produce high pressure steam that turns the turbine. These steam plants are very useful to man. However, they also create serious environmental problems. The product of their combustion creates air pollution; the heat condemned at the lower temperature reservoir creates thermal pollution; the extraction of the minerals creates environmental disasters and oil spills create environmental nightmares. These problems have the capacity to destroy every possible means of livelihood in the affected places.

3.5 Nuclear Energy

Nuclear energy may be released through the process of nuclear fusion or through the process of nuclear fission. Nuclear fusion involves the coming together of two or more light nuclei (like hydrogen nuclei) to form a more massive nucleus. Nuclear fission involves the splitting of a massive nucleus (like uranium or plutonium nucleus) to form smaller nuclei. In both cases, huge amounts of nuclear energy are released. The nuclear fusion process has two major difficulties. The first is the difficulty encountered in generating the extremely high temperatures needed to initiate the thermonuclear reaction. The second is the problem of containment. For this reason, all the present-day nuclear power plants make use of the fission process to generate energy. A nuclear power plant may, therefore, be regarded as a steam engine that makes use of uranium as its fuel.

Under normal conditions, a nuclear power plant produces practically no air pollution. It does not release any carbon-dioxide (CO_2) into the environment. Thus, neither the fusion process nor the fission process contributes to the greenhouse effect. However, nuclear accidents can deal a devastating blow to the environment. For instance, the 1979 nuclear accident at Three Mile Island, and the 1986 nuclear disaster at Chernobyl readily come to mind. More recently, the 2011 catastrophic meltdown of some of the Japanese reactors at Fukushima is still fresh in mind. In all, three reactors experienced total meltdown in Japan. Today, the Japanese authorities are still struggling to gain control of that nuclear crisis.

3.6 Geothermal Energy

In some places, the underground water makes contact with the hot interior part of the Earth. This causes the temperature of the water to rise significantly. When this happens, the hot water may come to the surface as hot springs or steam vents. These natural vents can be used to produce the steam needed to drive a turbine. Sometimes, scientists and engineers drill holes from the surface of the Earth to the steam beds trapped below the ground. Cold water may also be heated by causing it to make contact with the hot dry rock.

Geothermal energy is largely clean. It produces practically no air pollution. However, any non-steam emission from a geothermal plant may still be harmful to the environment. But, the more serious problem is the thermal pollution resulting from the spent water. The mineral content of the spent water may be hostile to the environment. In particular, such minerals, may be corrosive.

3.7 Hydroelectric Power Plant

In hydroelectric power stations, falling water is used to turn the turbines of the electric generators directly. For this reason, hydroelectric power plants do not need heat engines. Usually, the turbines are located at the base of a dam. Hydroelectric power plants are almost 100% efficient. However, the reservoirs behind the dams inundate lands that may otherwise be used for agricultural (as well as other) purposes. As a result, the environment also suffers.

3.8 Tidal Power Plants

The adverse effect of tidal power on the environment is minimal. Yet, the abrupt changes in water level, as the tidal power plant operates between high-tide and low-tide sessions, can have a negative effect on wildlife.

3.9 Wind Power Plants

Windmills of various shapes and sizes may be used to turn turbines in order to generate electrical energy. Generally, windmills are regarded as a clean source of energy. However, a large array is required to meet a significant portion of a country's energy needs. Such large arrays might affect the weather and damage the beauty of the affected place.

3.10 Solar Energy

Solar energy can be harnessed by means of active solar heating, passive solar heating and photovoltaic cells. Like heat engines, the efficiency of this cells lies somewhere between 30% and 40%. Solar energy is extremely useful to man. For instance, photovoltaic cells might be placed at rooftops for home use. Solar energy is, to a large extent, considered ‘‘clean’’. However, the chemical pollution resulting from large scale manufacture of solar cells impacts adversely on the environment.

4. CONCLUSION

In this unit, you have been introduced to some of the energy resources such as steam plants, nuclear energy, geothermal energy, hydroelectric power, tidal energy, wind power and solar energy. You have also seen how each energy-generating mechanism impacts negatively on the environment.

5 SUMMARY

Having gone through this unit, you now know that

(i) environmental physicists apply the principles and techniques of physics in order to provide a description of the natural environment.
(ii) the ultimate aim of the environmental physicist is to provide solution to the problem of environmental degradation.
(iii) air is polluted when fossil fuel is burned.
(iv) the heat exhausted by heat engines and cooling towers creates thermal pollution.
(v) nuclear wastes and nuclear accidents are a great threat to the environment.
(vi) geothermal power plants creates thermal pollution resulting from the spent water.
(vii) a large array of windmills might distort the weather pattern and damage the beauty of the environment.
(viii) chemical pollution is produced in the process of manufacturing large quantities of solar cells.

6 Tutor Marked Assignments (TMAs)

Exercise 6.1

1. In the following, answer *true* or *false*.
   a) the ultimate aim of the environmental physicist is to design a heat engine.
   b) every energy-production mechanism has some undesirable side effect.
   c) air is polluted when fossil fuel is burned.
   d) the buildup of atmospheric CO_2 helps in heating up the atmosphere.

2. Which of the following does not pose a direct threat to the environment?
   a) tree planting
   b) nuclear waste
   c) global warming
   d) electronic waste

3. What do the heat engines exhaust to the environment?
   a) water
   b) heat
   c) oxygen
   d) nitrogen

4. When heat is allowed to flow from a high-temperature reservoir to a low-temperature reservoir, thermal energy can be converted into
   a) electrical energy
   b) chemical energy
   c) mechanical energy
   d) nuclear energy

5. Which of the following energy resources poses no threat to the environment?
   a) solar energy
   b) steam power plant
   c) nuclear energy
   d) hydroelectric power plant

6. Air is polluted when
   a) solar cells are placed on rooftops.
   b) nuclear power plant is functioning well
   c) when fossil-fuel is burned
   d) when geothermal power plant is functioning well

7. The buildup of atmospheric carbon dioxide (CO_2) and the consequent heating of the atmosphere is called
   a) global warming
   b) greenhouse effect
   c) ozone layer depletion

d) all of the above

8. As polar ice caps melt, low-land areas are flooded because

a) the resulting water goes into the ocean

b) the sea-level rises

c) water from the oceans overflows into the low-land areas

d) all of the above

9. When operating normally, which of the following power plants exhausts heat to the environment?

a) steam power plant

b) nuclear power plant

c) photovoltaic cells

d) windmills

10. The efficiency h of the ideal Carnot engine is given by

a) $h = 1- \frac{T_L}{T_H}$

b) $h = 1+ \frac{T_L}{T_H}$

c) $h = 1- \frac{T_H}{T_L}$

d) $h = 1+ \frac{T_H}{T_L}$

11. To keep the efficiency of the ideal Carnot engine as high as possible,

a) little water must flow through the power plant

b) no water must flow through the power plant

c) much water must flow through the power plant

d) the power plant must thoroughly lubricated

12. Given that the efficiency of the ideal Carnot engine is h, the heat Q_L condemned at the low-temperature reservoir is given by

a) $Q_L = Q_H(1 + h)$

b) $Q_L = Q_H(1 - h)$

c) $Q_L = Q_H(1 - h)$

d) none of the above

13. What do the cooling towers discharge into the atmosphere?

a) boiling water

b) ice water

c) heat

d) oxygen

14. The 2011 nuclear meltdown in Japan took place in the month of

a) January, 2011

b) February, 2011

c) March, 2011

d) April, 2011

15. The major problem associated with geothermal power plants is the thermal pollution resulting from

a) the spent water

b) the low efficiency of the plant

c) the high efficiency of the plant

d) the non-steam emissions.

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MODULE 1

UNIT 2: STRUCTURE AND COMPOSITION OF THE ATMOSPHERE

CONTENTS **1**

1 INTRODUCTION

The terrestrial atmosphere is a weakly ionized plasma. Indeed, the plasma-neutral-density ratio is no more than 1:100. In response to any incident electromagnetic energy, the plasma particles irradiate in such a manner that propagation of the total electromagnetic energy can be described in terms of characteristic waves. A neutral atmosphere is capable of executing wave motion. The plasma particles respond to such wave motions through collisions. A good example of wave motion in a neural gas is the propagation of sound.

The gravitational force causes the density of the atmosphere to be distributed in a nonuniform manner along the vertical. If the frequency of a wave is sufficiently low, buoyancy forces play an important role in making the propagation of the sound wave anisotropic. The resulting wave is called acoustic gravity wave. In the troposphere, many observed acoustic gravity waves are guided. However, most experimental observations at ionospheric heights tend to support free waves and not guided waves. In this unit, you shall be introduced to the basic concepts relating to the structure and composition of the terrestrial atmosphere. Only the tropospheric regions of the atmosphere shall be considered.

2. OBJECTIVES

At the end of this unit, you shall be able to

(i) describe the vertical diminution of density with height.

(ii) distinguish between an isothermal atmosphere and an adiabatic atmosphere.

(iii) show that the adiabatic atmosphere has a finite height.

(iv) discuss the temperature profile of an adiabatic atmosphere.

3. MAIN CONTENT

3.1 Properties of the Atmosphere

The terrestrial atmosphere is characterized by the following properties: temperature, pressure, density, composition and motion. Conventional meteorology deals with these quantities from the sea-level to a height of about 20 kilometers. In the past, kites, balloons, and aircraft were used to access these regions. However, as experimental techniques improved, the probing of the atmosphere extended in height. Today, rockets and satellites are being used to take measurements at much higher altitudes.

3.2 Vertical Diminution of Density with Height

The density of the atmosphere diminishes with height. This vertical diminution of density with height is one of the most striking features of the terrestrial atmosphere. In practice, it may be assumed that the atmosphere is horizontally stratified. Evidently, gravity is the root cause of this stratification. The hydrostatic equation expresses the balance between the gravitational force at any point and the pressure gradient $\frac{dp}{dz}$ at that point. Quantitatively, the hydrostatic equation may be written as

$$\frac{dp}{dz} = -\rho g \, .$$

This implies that

$$dp = -\rho g dz \tag{1}$$

where p is the pressure, ρ is the mass density, g is the acceleration due to gravity and z is the height.

Now, consider an ideal gas. The equation of state for the ideal gas is

$$pV = nkT^{'},$$

where p is pressure, V is volume, n is number of molecules, $T^{'}$ is absolute temperature and k is Boltzmann constant. You can, however, define the temperature T (in energy units) such that $T = kT^{'}$. In this case, the equation of state reduces to

$$pV = nT$$

or

$$p = \left(\frac{n}{V}\right)T$$

or

$$p = NT, \qquad (2)$$

where $N = \dfrac{n}{V}$ is the number density. If you divide $Eq(1)$ by $Eq(2)$, you get

$$\frac{dp}{p} = -\left(\frac{\rho g}{NT}\right)dz$$

or

$$\frac{dp}{p} = -\left(\frac{1}{H}\right)dz$$

where

$$\frac{1}{H} = \frac{\rho g}{NT}.$$

Of course, by making H the subject of the formula, you get

$$H = \left(\frac{N}{\rho}\right)\left(\frac{T}{g}\right) \qquad (3)$$

or

$$H = \left(\frac{1}{m}\right)\left(\frac{T}{g}\right)$$

where

$$\frac{1}{m} = \frac{N}{\rho} \qquad (4)$$

so that m (which is equal to $\frac{\rho}{N} \equiv \frac{\rho V}{n}$) is the mean molecular mass at a given height. The quantity H is called the *scale height*. By integrating $Eq(1)$ from some reference height $z = z_\circ$ (at a pressure $p = p_0$) to an arbitrary height z (at a pressure p) you get:

$$\int_{p_0}^{p} \frac{dp}{p} = -\int_{z_0}^{z} \frac{dz}{H}$$

$$\left[\log_e p\right]_{p_0}^{p} = -\int_{z_0}^{z} \frac{dz}{H}$$

$$\log_e p - \log_e p_0 = -\int_{z_0}^{z} \frac{dz}{H}$$

$$\log_e\left(\frac{p}{p_0}\right) = -\int_{z_0}^{z} \frac{dz}{H}$$

$$\frac{p}{p_0} = \exp\left(-\int_{z_0}^{z} \frac{dz}{H}\right)$$

$$p = p_0 \exp\left(-\int_{z_0}^{z} \frac{dz}{H}\right). \qquad (5)$$

To obtain the distribution of number density with height, you may use $Eq(5)$ and $Eq(2)$. Equation *(5)* shows how the pressure diminishes with height. Equation *(2)* is just an expression of the ideal gas law. At the reference point z_0, the pressure is p_0 and the number density is N_0 so that $Eq(2)$ yields

$$p_0 = N_0 T_0 .$$

At an arbitrary height z, the ideal gas equation is

$$p = NT .$$

Therefore, by taking ratio, you obtain

$$\frac{p}{p_o} = \frac{NT}{N_o T_o}$$

so that

$$p = p_o\left(\frac{NT}{N_oT_o}\right)$$

or

$$p_0 \exp\left(-\int_{z_0}^{z}\frac{dz}{H}\right) = p_o\left(\frac{NT}{N_oT_o}\right)$$

or

$$N = \left(\frac{N_oT_o}{T}\right)\exp\left(-\int_{z_0}^{z}\frac{dz}{H}\right) \tag{6}$$

where T_0 is the temperature at z_o. From *Eqs (5)* and *(6)* it becomes clear to you that the scale height H is very important in the determination of the pressure and density distributions. In what follows, you shall learn more about two highly idealized atmospheres i.e the isothermal atmosphere and the adiabatic atmosphere.

3.3 The Isothermal Atmosphere

An isothermal atmosphere is one in which the temperature remains constant. This is a highly idealized case. In practice, it is impossible to find a perfectly isothermal atmosphere. However, in the region of the upper thermosphere, the temperature profile shows very little variation with height. A possible explanation for this *isothermality* is the existence of high thermal conductivity which tends to smooth out any temperature gradient. This happens above a height of about 300km. The atmosphere in this region tends to achieve thermal equilibrium, so that eventually the temperature approaches a constant value. At such heights, there is practically no mixing. In consequence, different gases tend to exist in separate portions of the space. The different gases which make up the atmosphere are distributed with partial pressure given by the constituent gases. Suppose that the scale height H_i of the $i\,th$ gas is given by

$$H_i = \frac{T}{m_i g}\ . \tag{7}$$

Consider a limited region of the atmosphere in which the gravity remains practically constant. By making use of Equations (5) and (7), you arrive at the expression

$$p_i = p_{o_i} \exp\left(-\int_{z_0}^{z} \frac{dz}{H_i} \right)$$

which simplifies to

$$p_i = p_{o_i} \exp\left[-\left(\frac{z - z_0}{H_i} \right) \right] . \qquad (8)$$

By extension, the density distribution is found in the following way:

$$p_i = p_{o_i} \left(\frac{N_i T}{N_{0i} T} \right)$$

$$\therefore p_i = p_{oi} \left(\frac{N_i}{N_{0i}} \right).$$

Also,

$$p_i = p_{o_i} \exp\left(-\int_{z_0}^{z} \frac{dz}{H_i} \right).$$

Equating the right-hand sides of the two preceding equations, you obtain:

$$p_{o_i} \exp\left(-\int_{z_0}^{z} \frac{dz}{H_i} \right) = p_{oi} \left(\frac{N_i}{N_{oi}} \right).$$

$$\therefore p_{0i}\, e^{-\int_{z_0}^{z} dz / H_i} = p_{0i} \left(\frac{N_i}{N_{0i}} \right)$$

$$\therefore N_i = N_{0i}\, e^{-\int_{z_0}^{z} dz / H_i}$$

$$\therefore N_i = N_{oi}\, e^{-(z - z_0)/H_i} \qquad (9)$$

Eqs (8) and (9) show that the scale height, in an isothermal atmosphere, is the e-folding distance for pressure and density. A large scale height is associated with a light gas. Thus, it is reasonable to expect that the light gas will predominate at sufficiently high altitudes.

3.4 The Adiabatic Atmosphere

In the lower atmosphere, convective motions are set up when the atmosphere is heated from below. As a result, the gases which make up the atmosphere are thoroughly mixed so that the average mass remains practically unchanged with height. In this region, the thermal conductivity is very low. Because of the slow conduction, the gas expands and contracts in a manner that is nearly adiabatic as it moves from one part to another.

The equation describing an adiabatic process is

$$pV^{\gamma} = k, \tag{9}$$

where k is a constant and γ is the ratio of the principal specific heat capacities. That is,

$$\gamma = \frac{c_p}{c_v},$$

where c_p is the specific heat capacity at constant pressure and c_v is the specific heat capacity at constant volume. You will recall that the volume V is the ratio of the mass m to the density ρ. That is,

$$V = \frac{m}{\rho}.$$

Therefore, the adiabatic equation becomes

$$p\left(\frac{m}{\rho}\right)^{\gamma} = k,$$

$$\therefore p = k\left(\frac{\rho}{m}\right)^{\gamma}$$

$$\therefore p = A\rho^{\gamma} \tag{10}$$

where A is a constant defined by

$$A = \frac{k}{m^{\gamma}}.$$

Differentiating p with respect to ρ, you obtain

$$\frac{dp}{d\rho} = A\gamma\rho^{\gamma-1},$$

so that the change in pressure dp is given by

$$dp = \gamma A\rho^{\gamma-1} d\rho. \qquad (11)$$

It is now a simple matter for you to eliminate p from the hydrostatic equation (1) and the adiabatic equation (11).

This yields

$$\gamma A\rho^{\gamma-1} d\rho = -\rho g dz.$$

or

$$\gamma A\rho^{\gamma-2} d\rho = -g dz$$

Integrating the expression given above, and simplifying the result, you obtain:

$$\gamma A\int_{\rho_o}^{\rho} \rho^{\gamma-2} d\rho = -g\int_{z_o}^{z} dz$$

$$\left(\frac{\gamma A}{\gamma-1}\right)\left[\rho^{\gamma-1}\right]_{\rho_0}^{\rho} = -g\left[z\right]_{z_o}^{z}$$

$$\left(\frac{\gamma A}{\gamma-1}\right)\left(\rho^{\gamma-1} - \rho_0^{\gamma-1}\right) = -g\left(z - z_o\right)$$

$$\rho^{\gamma-1} - \rho_o^{\gamma-1} = -\left(\frac{\gamma-1}{\gamma A}\right) g\left(z - z_o\right)$$

$$\rho^{\gamma-1} = \rho_o^{\gamma-1} - \left(\frac{\gamma-1}{\gamma A}\right) g\left(z - z_o\right)$$

$$\rho = \left[\rho_o^{\gamma-1} - g\left(\frac{\gamma-1}{\gamma A}\right)\left(z - z_o\right)\right]^{\frac{1}{1-\gamma}} \qquad (12)$$

Eq.(12) implies that the mass density ρ decreases continuously with height. A very interesting result is obtained when ρ is set equal to zero. In that case,

$$0 = \rho_0^{\gamma-1} - \left(\frac{\gamma-1}{\gamma A}\right) g\left(z - z_o\right)$$

$$\rho_0^{\gamma-1} = \left(\frac{\gamma-1}{\gamma A}\right) g\left(z - z_o\right)$$

$$z - z_0 = \frac{\gamma A}{g\left(\gamma-1\right)}\left(\rho_0^{\gamma-1}\right)$$

$$z - z_0 = \frac{\gamma A}{g\left(\gamma-1\right)}\left(\rho_0^{\gamma}\right)\rho_0^{-1}$$

$$z - z_0 = \frac{\gamma\left(A\rho_0^{\gamma}\right)\rho_0^{-1}}{g\left(\gamma-1\right)}$$

$$z - z_0 = \frac{\gamma p_0}{\rho_o g\left(\gamma-1\right)} \ , \qquad (13)$$

where p_0 $(\equiv A\rho_o^{\gamma})$ represents the adiabatic equation at $z = z_0$. This result shows that the density of the adiabatic atmosphere vanishes at a certain height given by Eq.(13). Thus, the adiabatic atmosphere must have a finite height. (Note that the isothermal atmosphere does not have a finite height).

Example 1

Assuming that $p_o = 1.01\times10^5 Nm^{-2}$, $\rho_o = 1.23 kgm^{-3}$, $\gamma = 1.4$ and $g = 9.80 ms^{-2}$, estimate the limit of the adiabatic atmosphere.

Solution 1

Set $z_o = 0$, at the surface of the Earth, so that Eq.(13) reduces to

$$z = \frac{p_o\gamma}{\rho_o g\left(\gamma-1\right)} \ .$$

Substituting the given values, you find that

$$z = \frac{1.01 \times 10^5 \times 1.4}{1.23 \times 9.80 \times (1.4 - 1)}$$

$$z = 0.293 \times 10^5$$

$$z = 29.3 \times 10^3$$

$$z = 29.3 km$$

Thus, the upper limit of the adiabatic atmosphere is about 29 kilometers above the ground.

Notice that the troposphere falls entirely within the adiabatic atmosphere. It is only what happens within the troposphere that significantly affects the weather conditions of the Earth. This explains why greater attention shall now be given to the adiabatic atmosphere.

3.4.1 Temperature Profile of an Adiabatic Atmosphere

You can derive an expression for the temperature profile of an adiabatic atmosphere in the following way. Eq.(2) shows that

$$p = NT$$

and Eq.(10) shows that

$$p = A\rho^{\gamma} \text{ .}$$

From these two equations, you find that

$$NT = A\rho^{\gamma}$$

or,

$$T = \left(\frac{A}{N}\right)\rho^{\gamma} \text{ .}$$

But, the mass density ρ is defined as follows:

$$\rho = \frac{\text{Number of particles} \times \text{Mass of individual particles}}{\text{Volume}}$$

or,

$$\rho = \frac{nm}{V}$$

$$\therefore V = \frac{nm}{\rho}$$

Also, by definition of number density N, you get the equation

$$N = \frac{n}{V}$$

$$\therefore V = \frac{n}{N}$$

Therefore, equating the two expressions for volume, you obtain

$$\frac{nm}{\rho} = \frac{n}{N}$$

$$\frac{m}{\rho} = \frac{1}{N}$$

$$\therefore N = \frac{\rho}{m}$$

Hence, the temperature profile T of the adiabatic atmosphere is given by

$$T = \left[\frac{A}{(\rho / m)}\right]\rho^{\gamma}$$

$$T = Am\rho^{\gamma}\rho^{-1}$$

$$\therefore T = Am\rho^{\gamma-1}.$$

Using Eq.(12), you obtain

$$T = Am\left[\rho_o^{\gamma-1} - \frac{g(\gamma-1)}{\gamma A}(z - z_o)\right]^{(\gamma-1)/(\gamma-1)}$$

$$T = Am\rho_o^{\gamma-1} - \frac{mg(\gamma-1)}{\gamma}(z - z_o) \qquad (14)$$

Thus, the temperature gradient becomes

$$\frac{dT}{dz} = -\frac{mg(\gamma-1)}{\gamma}. \qquad (15)$$

The temperature gradient is also called the lapse rate,

4. CONCLUSION

In this unit, you have been introduced to the basic properties of the terrestrial atmosphere. You have seen that the density of the atmosphere diminishes with height. You have also studied the differences between an isothermal atmosphere and an adiabatic atmosphere. You have found out that, in principle, the isothermal atmosphere has no upper limit. You have also seen that the upper limit of the adiabatic atmosphere is at a height of about 29 kilometers above the ground.

5. SUMMARY

Having gone through this unit, you now know that

(i) the terrestrial atmosphere is characterized by temperature, pressure, density, composition and motion.

(ii) the atmosphere is a weakly ionized plasma.

(iii) a neutral atmosphere is capable of executing wave motion of its own.

(iv) gravitation causes the density to become less and less as one goes higher and higher above the ground.

(v) many observed acoustic waves, within the troposphere, are guided.

(vi) the scale height H is given by $H = T/mg$, where T is the absolute temperature, m is mass and g is acceleration due to gravity.

(vii) the pressure p is distributed in accordance with the relation

$$p = p_o \exp\left(-\int_{z_o}^{z} \frac{dz}{H}\right),$$

where symbols have their usual meanings.

(viii) the number density N is distributed in accordance with the relation

$$N = \left(\frac{N_o T_o}{T}\right) \exp\left(-\int_{z_o}^{z} \frac{dz}{H}\right),$$

where symbols have their usual meanings

(ix) the isothermal and the adiabatic atmospheres are highly idealized concepts.

(x) at higher altitudes (of about 300 kilometers), the atmosphere may be considered (to a good approximation) as an isothermal atmosphere.

(xi) there is practically no *mixing* within the isothermal atmosphere so that gases tend to exist in separate colonies.

(xii) for an isothermal atmosphere, the pressure p_i is given by

$$p_i = p_{oi}\, e^{-(z-z_o)}/H_i$$

where H_i is the scale height of the $i\,th$ gas.

(xiii) the temperature gradient (or the lapse rate) of an adiabatic atmosphere is given by

$$\frac{dT}{dz} = -mg\frac{(\gamma-1)}{\gamma}$$

6. TUTOR MARKED ASSIGNMENTS (TMAs)

1 In the following, answer *true* or *false:*

a) The atmosphere of the Earth is a strongly ionized plasma.
b) The terrestrial atmosphere is a weakly ionized plasma.
c) The density of the atmosphere increases with height.
d) A neutral atmosphere cannot execute wave motion.

2 The plasma particles respond to any wave motion in the atmosphere through

a) Interference.
b) diffraction.
c) collision.
d) reflection.

3 Along the vertical, the density is distributed in a non-uniform manner due to

a) electromagnetic radiation.
b) acoustic waves.
c) gravity.

d) ionization.

4 In the atmosphere, the plasma-neutral density ratio is about

a) 100:1

b) 10:1

c) 1:10

d) 1:100

5. The terrestrial atmosphere is characterized by

a) temperature and pressure only.

b) density only.

c) composition and motion only.

d) all of the above.

6. Atmospheric pressure is distributed according to the relation

$$p = p_o \exp\left(-\int \frac{dz}{H}\right),$$

where H is

a) the temperature coefficient.

b) the pressure coefficient.

c) the scale height.

d) the number density.

7. At heights of about 300 kilometres, the atmosphere may, to a good approximation, be considered as

a) an adiabatic atmosphere.

b) an isothermal atmosphere.

c) a real atmosphere.

d) an ideal atmosphere.

8. There is practically no mixing within the

a) real atmosphere.

b) ideal atmosphere.

c) adiabatic atmosphere.

d) isothermal atmosphere.

9. The upper limit of an adiabatic atmosphere is about

a) 9 kilometres.

b) 19 kilometres.

c) 29 kilometres.

d) 39 kilometres.

10. The pressure distribution given by

$$p_i = p_{oi}\, e^{-(z-z_o)/H}$$

applies only to

a) the adiabatic atmosphere.

b) the isothermal atmosphere.

c) the real atmosphere.

d) the ideal atmosphere.

11. The *lapse rate* of the atmosphere represents

a) the pressure gradient.

b) the potential gradient.

c) the temperature gradient.

d) the velocity gradient.

12. The lapse rate of the terrestrial atmosphere is numerically equal to

a) $-\dfrac{mg(g-1)}{g}$.

b) $\dfrac{mg(g-1)}{g}$.

c) $-\dfrac{mg(1-g)}{g}$.

d) $\dfrac{mg(1-g)}{g}\times$

13. For an adiabatic atmosphere $p = Ar^{g}$, where

a) $A = \dfrac{k}{m^{-g}}$.

b) $A = \dfrac{-k}{m^{-g}}$.

c) $A = km^{-g}$.

d) $A = -km^{-g}$.

14. The temperature profile *T* of an adiabatic atmosphere is given by

a) $T = Amr^{1-g}$.

b) $T = -Amr^{1-g}$.

c) $T = Amr^{g-1}$.

d) $T = -Amr^{g-1}$.

15. In practice, the troposphere lapses at an average rate of about

a) 4.5^{0} Kelvin per kilometer.

b) $6.\ 5^{0}$ Kelvin per kilometer.

c) $8.\ 5^{0}$ Kelvin per kilometer.

d) $10.\ 5^{0}$ Kelvin per kilometer.

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MODULE 1

UNIT 3: ARTIFICIAL SATELLITES AND REMOTE SENSING

CONTENTS **1**

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1. INTRODUCTION

Artificial satellites play a significant role in the Geographic Positioning System (GPS) and Remote Sensing. In particular, any three suitably arranged synchronous satellites may be used to specify the location of an object on the surface of the Earth. In this unit, you shall be introduced to the basic concepts involved in satellite motion. You shall also be introduced to the basic ideas of remote sensing.

2. OBJECTIVES

At the end of this unit, you shall be able to

(i) describe the Earth in relation to the solar system.

(ii) discuss the significance of the centre-of-mass of the Earth-Moon system.

(iii) define the Gaussian constant of gravitation.

(iv) perform simple calculations involving idealized synchronous satellites in circular orbits.

3. MAIN CONTENT

3.1 The Orbit of the Earth

The solar system contains the following objects:

a. the Sun
b. the eight planets (and the *dwarfs* like Pluto)
c. the satellites
d. the comets
e. the meteors
f. the interplanetary medium.

To understand and compare the magnitudes of the different forces acting upon an artificial satellite, it is necessary that you understand the Earth and its environment. In what follows, you shall be introduced to the concept of the Earth as a planet. A brief description of the structure of the Earth and its atmosphere shall also be given. From there, you shall proceed to learn more about the orbit of a satellite under the action of the major forces involved.
The Earth is one of the eight planets in the solar system. These eight planets are Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune. (Pluto is a dwarf). The orbit of the Earth lies between the orbits of Venus and Mars. To a good approximation, the orbit of the Earth is an ellipse of small eccentricity. The elements of this orbit suffer changes due to the presence of the Moon and the other planets. The Moon is a natural satellite of the Earth. Indeed, it is the centre-of-mass of the Earth-Moon system that revolves about the Sun. The centre-of-mass of the Earth-Moon system lies about 1600 kilometres below the surface of the Earth.

The orbit of the Earth is very important. For instance, astrophysicists find it convenient to use data connected with the orbit of the Earth to define the units of time and distance. You will recall that Kepler's third law, for a planet of mass m_2 revolving above the Sun of mass m_1, states that

$$k^2 \left(m_1 + m_2\right) T^2 = 4\pi^2 a^3 .$$

Taking the solar mass, the mean solar day, and the Earth's mean distance from the Sun as the units of mass, time and distance, respectively, the precise statement of this law becomes

$$k^2 \left(1 + m_2\right) T^2 = 4\pi^2 a^3 .$$

Here, k^2 is written for the gravitational constant G; m_2, T and a are in the units defined above. The quantity k is called the Gaussian constant of gravitation.

Example 1

Estimate the value of a (the mean radius of the Earth's orbit) given that $T \approx 365.256$ mean solar days, $m_2 \approx \frac{1}{354710}$ solar masses and $k \approx 0.01721$.

Solution 1

Mass of the Sun $m_1 = 1$

Mass of the Earth $m_2 \approx \frac{1}{354710}$

Period of revolution of the Earth $T \approx 365.256$

Gaussian constant of gravitation $k \approx 0.01721$.

Now,

$$k^2 (1+m_2) T^2 = 4\pi^2 a^3$$

$$a^3 = \frac{k^2 (1+m_2) T^2}{4\pi^2}$$

$$a = \left[\frac{k^2 (1+m_2) T^2}{4\pi^2} \right]^{1/3}.$$

$\therefore a \approx 1.0000$ astronomical unit.

3.2 Satellites in Circular Orbits

3.2.1 The Relationship between Radius and Orbital Speed

There are many satellites orbiting about the Earth. Some of these satellites move in approximately circular orbits. Such satellites are kept in their circular orbits by the centripetal force provided by gravity. To remain in circular orbit, the satellite must have a centripetal force F given by

$$F = \frac{mv^2}{r} \qquad (1)$$

Here, $m =$ mass of the satellite

$v =$ speed of the satellite

$r =$ radius of the orbit of the satellite

$M_E =$ mass of the Earth.

Since it is gravity alone that provides this centripetal force, it follows that F must be equal to the gravitational force of attraction between the Earth and the satellite. This implies that

$$F = \frac{GM_E m}{r^2}. \qquad (2)$$

When the right-hand sides of Equations (1) and (2) are equated, you find that

$$\frac{mv^2}{r} = \frac{GM_E m}{r^2}. \qquad (3)$$

Solving for v, you get

$$v = \sqrt{\frac{GM_E}{r}}. \qquad (4)$$

If the satellite is to remain in a circular orbit of radius r, its speed must be equal to the value of v in Equation (4) above.

Example 3.1

The Hubble Space Telescope (HST) is a satellite orbiting at a height of 596km above the surface of the Earth. What must the orbital speed of HST be if it is to remain in a circular orbit?

Solution 3.1

The velocity of the satellite is given by

$$v = \sqrt{\frac{GM_E}{r}}$$

$$v = \sqrt{\frac{\left(6 \cdot 67 \times 10^{-11}\right)\left(5.98 \times 10^{24}\right)}{6.98 \times 10^{6}}}$$

$$\therefore v = 7.56 \times 10^{3}\, m/s$$

Thus, the HST must be moving with a velocity of about $7{,}560\,km/s \cdot$ (Note that the orbital radius r is the distance from the centre of the Earth to the satellite. Note also that v is independent of the mass of the satellite).

3.2.2 The Period of the Satellite

The period T of a satellite is the time required for the satellite to move round its orbit. It is the time taken for the satellite to complete one orbital revolution. In all circular motions, the linear velocity v and the angular velocity ω are related by the expression

$$v = r\omega$$

or,

$$v = r\left(\frac{2\pi}{T}\right) \cdot$$

If you substitute v from Equation (4), you get

$$\sqrt{\frac{GM_E}{r}} = \frac{2\pi r}{T}.$$

Solving this expression for T, you obtain

$$T = \left(\frac{4\pi^2 r^2}{GM_E}\right)^{1/2}$$

$$T = 2\pi\left(\frac{r^3}{GM_E}\right)^{1/2}$$

$$\therefore\ T = \frac{2\pi r^{3/2}}{\sqrt{GM_E}}.$$

The equation given above represents Kepler's third law of planetary motion.

Note that $GM_E = 4\pi^2 r^3$, provided that T is chosen to be one day i.e. provided that $T = 1$. In this case, the satellite remains in a *parking orbit* above the Earth.

3.2.3 The Orbital Radius of Synchronous Satellites

Synchronous satellites are very important in the field of communications. Usually, the synchronous satellites are put in a circular orbit in the plane of the equator. The period T is chosen to be one day so that the Earth-based observer finds the satellites at fixed positions in the sky. As a result, the satellites can serve as "stationary" relay stations for communication signals sent up from the Earth's surface. That is how the Digital Satellite System works. (The cable television works in a different way).

All synchronous satellites move in their orbits with the same orbital speed v.
Thus, they must be placed at the same height above the surface of the Earth. Their orbital radius r is given by

$$\sqrt{\frac{GM_E}{r}} = \frac{2\pi r}{T}.$$

Solving this expression for the orbital radius r, you find that

$$r = \left[\frac{T\sqrt{GM_E}}{2\pi}\right]^{2/3}$$

$$\therefore r = \left[\frac{T^2\left(GM_E\right)}{4\pi^2}\right]^{1/3}.$$

Example 3.2

At what height above the surface of the Earth must all synchronous satellites be placed in orbit?

Solution 3.2

The period of a synchronous satellite is one day which corresponds to 24 hours or $8.64\times10^4 s\cdot$ (Strictly, the *sidereal day* corresponding to 23 hours 56 minutes should be used here).

$$r^{3/2} = \frac{T\sqrt{GM_E}}{2\pi}$$

$$r^{3/2} = \frac{\left(8.64\times10^4\right)\sqrt{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)}}{2\pi}$$

$$\therefore r = 4\cdot23\times10^7 m.$$

Now,

$$r = R + H \quad ,$$

where $R(\equiv 6400km)$ is the radius of the Earth and H is the height above the surface of the Earth.

$$\therefore H = r - R$$

$$= 4.23\times10^7 - 0.64\times10^7$$

$$= \underline{\underline{3.59\times10^7 m}}$$

Thus, all synchronous satellites must be placed at a height of $3.59\times10^7 m$ (whatever the masses may be).

Example 3.3

All synchronous satellites are put into orbit whose radius $r = 4.23\times10^7 m.$ The orbit is in the plane of the equator. The arc length s that separates two adjacent synchronous satellites is $7.4\times10^5 m.$ Find the angular separation of the satellites in degrees.

Solution 3.3

The length of the arc $s = r\theta$, where θ is the angular separation of the two adjacent satellites. You may therefore write

$$s = r\theta.$$

Substituting values, you get

$$7.4\times10^5 = 4.23\times10^7\theta$$

$$\theta = \frac{7.4\times10^5}{4.23\times10^7\theta}$$

$$\therefore \theta = 0.01745 \text{ radians}$$

But, 2π radians are equivalent to 360^0.

1 radian is equivalent to $\frac{360}{2\pi}$ degrees

$$\therefore 0.01745 \text{ radians } = \frac{360}{2\pi}\times\frac{0.01745}{1} \text{ degrees}$$

$$\therefore \theta = 1^o$$

The angular separation between the two adjacent synchronous satellites is $1^o \cdot$

3.3 Remote Sensing

3.3.1 Preliminary Remarks

Remote sensing is an extremely useful technique. It enables man to acquire information about an object without making physical contact with that object. Remote sensing often involves the use of aerial sensor techniques to detect and classify objects on different parts of the Earth. In what follows, you will be introduced to this rapidly expanding subject called remote sensing.

As a discipline, remote sensing started as soon as humans developed flying objects. Initially, man learnt how to fly kites, unmanned balloons and rockets. As time went on manned balloons and aircraft were developed. Today, artificial satellites have made it possible for the discipline of remote sensing to assume a global dimension. Remote sensing makes it possible to collect data on hostile objects and inaccessible areas. For this reason, remote sensing techniques may be employed in making systematic aerial photographs for military surveillance and reconnaissance purposes. Also, space probes to other planets have provided man with the opportunity to conduct remote sensing studies in extraterrestrial environment.

In general, remote sensing is all about collection of data about an object at a distance. Humans and other animals accomplish this task in many different ways such as seeing, smelling and hearing. Scientists may also use specialized instruments and remote sensing techniques to monitor what goes on in the Earth’s atmosphere, hydrosphere, cryosphere, biosphere and lithosphere. They conduct remote sensing studies of the environment with the aid of mechanical devices called remote sensors. Such devices are often placed far above the object of interest using balloons, helicopters, air planes and satellites. Many of these sensors record information about an object by measuring how the object transmits electromagnetic energy from reflecting and radiating surfaces. These gadgets, known as remote sensors, have greatly improved the ability of humans to receive and record information about an object of interest without making any physical contact with that object.

Broadly speaking, remote sensing may be classified into two, namely, passive remote sensing and active remote sensing. Passive sensors detect natural radiations emitted or reflected by the object or area of interest. Active sensors emit radiations in order to scan the desired object or the surrounding area being observed. In this case, the sensor detects and measures the radiation that is reflected or backscattered from the target. Examples of active remote sensors include RADAR and LiDAR. Scientists use RADAR and LiDAR techniques to establish the location, height, speed and direction of the object of interest.

3.3.2 Data Collection

Orbital platforms collect and transmit data from suitable regions of the electromagnetic spectrum. By means of this multispectral data collection technique, scientists are able to reconstruct an accurate image of the object being observed. Satellite, aircraft, spacecraft, buoy, ship and helicopter images provide data that may be analyzed in order to paint a comprehensive picture of things like vegetation rates, erosion, pollution, forestry, weather, and land use. The data collected from different parts of the electromagnetic spectrum are normally used to provide researchers with enough information to monitor trends such as El Niño (in the pacific) and other long and short term phenomena.

3.3.3 Processing of Remote Sensing Data

The data collected by remote sensing techniques must be carefully and skillfully analyzed. The quantity of any remote sensing data is measured in terms of its resolution. There are many types of resolution associated with remote sensing data. Scientists are usually interested in the spatial resolution, spectral resolution, radiometric resolution, and temporal resolution of remote sensing data.

The wavelength width (of the different frequency bands recorded) provides a measure of the spectral resolution. It is related to the number of frequency bands recorded by a particular orbital platform. For instance, the current LANDSAT collection involves **seventy** bands ranging from a spectral resolution of 0.07 to 2.1 μm. Many of these bands are in the infrared region of the electromagnetic spectrum. Another example is the HYPERION sensor on EARTH OBSERVING-1. This sensor resolves 220 bands from 0.4 to 2.5 μm with a spectral resolution of 0.10 μm per band.

The number of different intensities of radiation which a particular sensor is able to distinguish gives a measure of its radiometric resolution. One of the factors affecting radiometric resolution is **instrument noise.** Temporal resolution is measured by the frequency of flyovers by the satellite. Temporal resolution is relevant when time-series studies are required. It is also important to scientists engaged in studies requiring an averaged or mosaic image. Thus, temporal resolution is useful in monitoring deforestation and, to some extent, desertification. Again, if an object of interest is obscured by cloud cover during the time of data collection, it becomes necessary to repeat the process of data collection when he cloud cover has gone away. In this case, temporal resolution becomes relevant too.

One of the major challenges faced by scientists who utilize remote sensing data is the huge volume of such data. Sensors are capable of transmitting extremely large amounts of data to ground-based stations for analysis.

In 1986, NASA defined several processing "levels" aimed at facilitating the discussion of data processing; and minimizing the difficulty resulting from the huge volume of the remote sensing data. The first level involves reconstructed, unprocessed instrument and payload data at full resolution. A higher level involves data that have been processed to sensor units. As the level increases, the volume of the data decreases. Thus, level 4 data set tends to be less voluminous than level 3 data set; level 3 data set tends to be less voluminous than level 2 data set; level 2 data set tends to be less voluminous than level 1 data set; level 1 data set tends to be less voluminous than level 0 data set. The highest level on this scheme is level 4. It gives a model output derived from the analyses of lower level data. A computer software known as **remote sensing application** may be used to process and analyze remote sensing data.

4. Conclusion

In this unit, you have been introduced to the basic concepts involved in the motion of artificial satellites. You have seen that the centre-of-mass of the Earth-Moon system is very important. You have also found out that all synchronous satellites must have the same speed (given by $v = \sqrt{\frac{GM_E}{r}}$) if they are to remain in circular orbits. These satellites may be used to collect data applicable to Geographic Positioning System (GPS) and remote sensing. Finally, you have seen how remote sensing operations may be conducted.

5. SUMMARY

Having gone through the unit, you now know that

(i) the Earth is one of the eight planets in the solar system.

(ii) it is the centre-of-mass of the Earth-Moon system that revolves about the Sun.

(iii) the Gaussian constant k is related to the Universal constant of gravitation *G.*

(iv) any satellite in circular orbit is kept in orbit by a centripetal force provided by gravity.

(v) the speed v of all satellites, in circular orbit, must be given by

$$v = \sqrt{\frac{GM_E}{r}}.$$

(vi) the Hubble Space Telescope (HST) is a satellite orbiting at a height of about 596 kilometers above the surface of the Earth.

(vii) the Hubble Space Telescope must be moving with a velocity of about 7560 km s^{-1}.

(viii) the orbital radius r is the distance from the centre of the Earth to the satellite.

(ix) the period T of a satellite is the time taken for the satellite to move round its orbits (once).

(x) the period T of a satellite is given by

$$T = \frac{2\pi r^{3/2}}{GM_E}.$$

(xi) the velocity of a satellite is independent of its mass.

(xii) synchronous satellites are used extensively in Geographic Positioning Systems (GPS).

(xiii) synchronous satellites are used extensively in Digital Satellite Systems (as opposed to the Cable Television).

(xiv) the period of a synchronous satellite is one sidereal day (23 hours, 56 minutes).

(xv) all synchronous satellites must be placed in orbit at a height of $3.55\times10^{7}\,m$.

(xvi) remote sensing enables humans to acquire information about an object without making physical contact with that object.

(xvii) remote sensing may be classified into two:

a) active remote sensing and
b) passive remote sensing.

(xviii) the quantity of any remote sensing data is measured in terms of its resolution.

(xix) scientists are usually interested in the spatial resolution, spectral resolution, radiometric resolution and temporal resolution of remote sensing data.

(xx) a computer software known as *remote sensing application* may be used to process and analyze remote sensing data.

6 TUTOR MARKED ASSIGNMENTS (TMAs)

Exercise 6.1

1. Which of the following bodies is not a planet?

a) Earth
b) Mercury
c) Pluto
d) Uranus

2. The Moon is a natural satellite of

a) the Earth.

b) the Sun.

c) Venus.

d) Mars.

3. The orbit of the Earth lies between the orbits of

   a) Mercury and Venus.

   b) Venus and Mars.

   c) Mars and Jupitar.

   d) Jupitar and Uranus.

4. Astrophysicists find it convenient to use the mean solar day as the unit of

   a) mass.

   b) length.

   c) time.

   d) current.

5. Astrophysicists find it convenient to use the mass of the Sun as the unit of

   a) mass.

   b) length .

   c) time.

   d) current.

6. Astrophysicists find it convenient to use the Earth's mean distance from the Sun as the unit of

   a) mass .

   b) length .

   c) time.

   d) current.

7. The velocity of a satellite in circular orbit is inversely proportional to

a) r.

b) r^2.

c) $r^{3/2}$.

d) $r^{1/2}$.

8. The period of a satellite in circular orbit is directly proportional to

a) r.

b) r^2.

c) $r^{3/2}$.

d) $r^{1/2}$.

9. The period of synchronous satellites is usually chosen to be

a) one second.

b) one hour.

c) one day.

d) one month.

10. As a discipline, remote sensing started as soon as humans were able to cause something to

a) walk.

b) swim.

c) fly.

d) transmit.

11. Remote sensing may be classified into two, namely

a) satellite and non-satellite remote sensing.

b) satellite and active remote sensing.

c) active and passive remote sensing.

d) passive and satellite remote sensing.

12. Which of the following sensors emit radiation in order to scan the desired object?

a) Active sensors
b) Satellite sensors
c) Non-satellite sensors
d) Passive sensors

13. Which of the following sensors detect natural radiation emitted by the object of interest?

a) active sensors
b) passive sensors
c) satellite sensors
d) non-satellite sensors

14. In analyzing remote sensing data, scientists are usually interested in the

a) spatial resolution only.
b) spectral resolution only.
c) radiometric and temporal resolutions only.
d) all of the above.

15. The wavelength width (of the different frequency bands recorded) provides a measure of the

a) spectral resolution
b) spatial resolution
c) temporal resolution
d) radiometric resolution

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MODULE 2

UNIT 1: SATELLITE ORBITS IN THE TWO-BODY SYSTEM

CONTENTS **1**

1. INTRODUCTION

In your study of the Earth and its Artificial Satellites, you defined a number of quantities like the velocity of the satellite and its period. The definitions hold so long as the orbit is circular and a number of conditions are met. In this unit, you will be introduced to the steps involved in defining the actual orbit of a satellite within the framework of a two-body system.

2. OBJECTIVES

At the end of this unit, you shall be able to

(i) give a precise statement of the two-body problem.

(ii) solve the equation of motion in a two-body system.

(iii) reduce a many-body problem to several two-body systems.

(iv) relate the orbit of a satellite to the conic sections.

3. MAIN CONTENT

3.1 Concept of the Two-Body Problem

Suppose that the masses, positions and velocities of two massive particles moving under their mutual gravitational force are known at any time. Is it possible to calculate the positions and velocities of the two particles at any other time? This is the question which the two-body problem seeks to answer.

In practice, there are many investigations that involve more than two bodies. For instance, consider a hypothetical satellite placed in orbit between the Earth and Mars. The Earth, the Sun and Mars exert forces on the satellite. Thus, four bodies (Earth, Sun, Mars and the satellite itself) are involved. This is no longer a two-body problem. It is a four-body problem. Fortunately, it is often permissible to split a many-body problem into several two-body problems. This method makes it possible for approximate values of the parameters involved to be calculated with relative ease. For instance, the four-body problem mentioned above can be split into three different two-body problems consisting of (i) a two-body problem involving the Earth and the satellite only (ii) a two-body problem involving the Sun and the satellite only (iii) a two-body problem involving Mars and the satellite only.

When the satellite is under the predominant attraction of the Earth, the two-body problem involving the Earth and the satellite becomes approximately valid. A similar assertion holds for each of the remaining two-body problems. However, in the carry-over region where the Earth and the Sun attract the satellite with the same order of magnitude of force, a three-body treatment becomes necessary. A similar assertion also holds in the carry-over region where Mars and the Sun attract the satellite with about the same force. Fortunately, these carry-over regions constitute only a small fraction of the total journey of the satellite in its orbit. Consequently, much computation time is saved using this method. Thus, a thorough knowledge of the two-body problem and its applications is an invaluable tool in your study of satellite orbits.

3.2 Solution to the Two-Body Problem

Consider two bodies of masses m_1 and m_2 separated by a linear displacement $\vec{r}$ (Fig.1). Newton's second law of motion states that

$$\vec{F} = \frac{d\left(m\vec{v}\right)}{dt} \tag{1}$$

or,

$$\vec{F} = \frac{md^2\vec{r}}{dt^2} \tag{2}$$

or,

$$\frac{md^2\vec{r}}{dt^2} = \sum_{i=1}^{k} \vec{F}_i \tag{3}$$

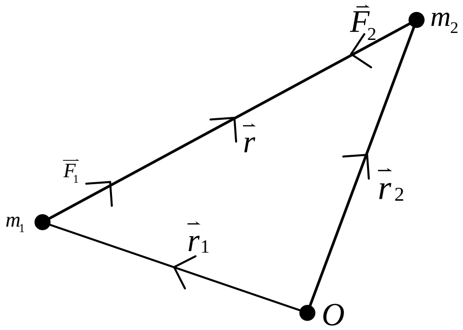

Fig. 1: *Two bodies of masses* m_1 *and* m_2 *separated by a displacement* $\vec{r}$

The force of attraction, $\vec{F}_1$, on m_1 is directed along the vector $\vec{r}\cdot$ The force of attraction, F_2, on m_2 is in the opposite direction.

By Newton's third law,

$$\vec{F}_1 = -\vec{F}_2 \cdot \qquad (4)$$

Also, by Newton's law of gravitation,

$$\vec{F}_1 = G\frac{m_1 m_2}{r^2}\ \frac{\vec{r}}{r}\cdot \qquad (5a)$$

Again,

$$\vec{F}_2 = -G\frac{m_1 m_2}{r^2}\ \frac{\vec{r}}{r}\cdot \qquad (5b)$$

on account of Equation (4).

Let vectors $\vec{r}_1$ and $\vec{r}_2$ be directed from some fixed reference point O to the particles of masses m_1 and m_2, respectively.

Now,

$$\vec{F} = m\frac{d^2\vec{r}}{dt^2}$$

so that

$$\vec{F}_1 = m_1 \frac{d^2\vec{r}_1}{dt^2}$$

and

$$\vec{F}_2 = m_2 \frac{d^2\vec{r}_2}{dt^2}$$

Equation (1) can then be rewritten as

$$m_1 \frac{d^2\vec{r}_1}{dt^2} = \frac{Gm_1m_2}{r^2}\frac{\vec{r}}{r} \cdot \qquad (6)$$

Equation (2) can also be rewritten as

$$m_2 \frac{d^2\vec{r}_2}{dt^2} = -G\frac{m_1m_2}{r^2}\frac{\vec{r}}{r} \qquad (7)$$

Adding (6) and (7), you obtain

$$m_1 \frac{d^2\vec{r}_2}{dt^2} + m_2 \frac{d^2\vec{r}_2}{dt^2} = 0\,.$$

Integrating once, you obtain the first integral:

$$m_1 \frac{d\vec{r}_1}{dt} + m_2 \frac{d\vec{r}_2}{dt} = \vec{a} \cdot \qquad (8)$$

Integrating again, you obtain the second integral:

$$m_1\vec{r}_1 + m_2\vec{r}_2 = \vec{a}t + \vec{b}\,, \qquad (9)$$

where $\vec{a}$ and $\vec{b}$ are constant vectors.

But, by definition, the centre-of-mass C of the system of masses m_1 and m_2 is given by

$$M\vec{R} = m_1\vec{r}_1 + m_2\vec{r}_2$$

where,

$$M = m_1 + m_2$$

and $\vec{R}$ is the position vector of C.

Hence, by Eq.(8), you get

$$M\frac{d\vec{R}}{dt}=\overline{a}\cdot \qquad (A)$$

Also, by Eq.(9), you get

$$M\vec{R}=\vec{a}\,t+\vec{b} \qquad (B).$$

These two relations *(A)* and *(B)* show that the centre of mass of the system moves with constant velocity (that is, with constant speed in a straight line).

3.3 The Angular Momentum Integral

Equations (6) and (7) may be written as

$$\frac{d^2\vec{r}_1}{dt^2}=Gm_2\frac{\vec{r}}{r^3} \qquad (10)$$

and

$$\frac{d^2\vec{r}_2}{dt^2}=-Gm_1\frac{\vec{r}}{r^3}\ , \qquad (11)$$

respectively.

Equation (10) minus Equation (11) gives

$$\frac{d^2\left(\vec{r}_1-\vec{r}_2\right)}{dt^2}=G\left(m_1+m_2\right)\frac{\vec{r}}{r^3}\ .$$

But, from Fig. 1, it is obvious that

$$\vec{r}_1-\vec{r}_2=-\vec{r}\ .$$

Hence,

$$\frac{d^2\vec{r}}{dt^2}+\frac{\mu\vec{r}}{r^3}=0 \qquad (12)$$

where,

$$\mu = G(m_1 + m_2)\cdot$$

Taking the vector product of $\vec{r}$ with Equation (12), you obtain

$$\vec{r} \times \frac{d^2\vec{r}}{dt^2} + \frac{\mu\vec{r}}{r^3} \times \vec{r} \ = 0$$

i.e.

$$\vec{r} \times \frac{d^2\vec{r}}{dt^2} = 0 \quad \text{(since } \vec{r} \times \vec{r} = 0\text{)}$$

Integrating, you have

$$\vec{r} \times \frac{d\vec{r}}{dt} = \vec{h} \ , \qquad (13)$$

where $\vec{h}$ is a constant vector.

Equation (13) is the angular momentum integral. Now, since $\vec{h}$ is constant, it must point in one direction for all values of t. Thus, the motion of one body about the other must lie in a plane defined by the direction of $\vec{h}$.

3.4 Mathematical form of Kepler's Second Law

If polar coordinates r and θ are taken in this plane (as in Figure 2), the velocity component along the radius vector joining m_1 to m_2 is $\dot{r}$. Also, the perpendicular component of the radius vector is $r\dot{\theta}$, where the dot replaces d/dt.

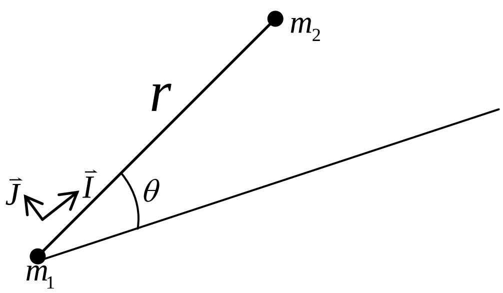

Fig 2: *Polar coordinates*.

By defining $\vec{I}$ and $\vec{J}$ as the unit vectors along and perpendicular to the radius vector, the resultant velocity $\dot{\vec{r}}$ becomes

$$\dot{\vec{r}} = \vec{I}\dot{r} + \vec{J}r\dot{\theta} \qquad (14)$$

Hence, using Equations (13) and (14), you can easily verify each of the following steps:

$$\vec{r} \times \frac{d\vec{r}}{dt} = \vec{h}$$

$$\vec{r} \times \dot{\vec{r}} = \vec{h}$$

$$\vec{r} \times \left(\vec{I}\dot{r} + \vec{J}r\dot{\theta}\right) = \vec{h}$$

$$\vec{I}r \times \left(\vec{I}\dot{r} + \vec{J}r\dot{\theta}\right) = \vec{h}$$

$$\left(\vec{I}r \times \vec{I}\right)\dot{r} + \left(\vec{I}r \times \vec{J}\right)r\dot{\theta} = \vec{h}$$

$$0 \quad + \left(\overrightarrow{K}r\right)r\dot{\theta} = \vec{h}$$

$$\therefore \overrightarrow{K}r^2\dot{\theta} = \vec{h}$$

(Note that $\vec{I} \times \vec{J} = \overrightarrow{K}$, where $\overrightarrow{K}$ is a unit vector perpendicular to the plane of the orbit).

You may therefore write

$$r^2\dot{\theta} = h, \qquad (15)$$

where the constant h is twice the rate at which the radius vector describes the area. This is the mathematical form of Kepler's second law. For a circular orbit, the rate of change of area may, of course, be estimated as follows:

$$A = \pi r^2$$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

$$= 2\pi r \frac{dr}{dt}.$$

$$= 2\pi r\dot{r}$$

3.5 Energy Conservation Equation of the System

If the scalar product of $\dot{\vec{r}}$ with Equation (12) is now taken, you obtain

$$\dot{\vec{r}}.\frac{d^2\vec{r}}{dt^2} + \mu\frac{\dot{\vec{r}}.\vec{r}}{r^3} = 0,$$

i.e

$$\dot{\vec{r}}.\ddot{\vec{r}} + \frac{\mu}{r^3}\dot{\vec{r}}.\vec{r} = 0\,.$$

This may be integrated to give

$$\frac{1}{2}\dot{\vec{r}}.\dot{\vec{r}} - \frac{\mu}{r} = C\,,$$

where C is a constant v.

That is,

$$\frac{1}{2}v^2 - \frac{\mu}{r} = C\,, \qquad (16)$$

where v is the velocity.

Equation (16) is the energy conservation equation of the system. The quantity C is not the total energy of the system. However, the quantity $\frac{1}{2}v^2$ is related to the kinetic energy while the quantity $-\mu/r$ is related to the potential energy of the system.

3.6 Satellite Obits in a Two-Body System

Referring to Figure 2 again, and remembering that the components of the acceleration along and perpendicular to the radius vector are

$$\ddot{r} - r\dot{\theta}^2 \text{ and } \frac{1}{2}\frac{d}{dt}\left(r^2\dot{\theta}\right)$$

respectively. Equation (12) may be written as

$$\bar{I}\left(\ddot{r}-r\dot{\theta}^2\right)+\bar{J}\left[\frac{1}{r}\frac{d}{dt}\left(r^2\dot{\theta}\right)\right]+\frac{\mu}{r^3}\bar{I}r=0\cdot$$

Equating coefficients of the vectors, you obtain

$$\ddot{r}-r\dot{\theta}^2=-\frac{\mu}{r^2} \tag{17}$$

and

$$\frac{1}{r}\frac{d}{dt}\left(r\dot{\theta}^2\right)=0\ . \tag{18}$$

The integration of Equation (18) gives the angular momentum integral

$$r^2\dot{\theta}=h. \tag{19}$$

Substituting $u=1/r$ and eliminating the time t between Equations (17) and (19), you obtain the equation

$$\frac{d^2u}{dt^2}+u=\frac{\mu}{h^2}\cdot$$

The general solution of this equation is

$$u=\frac{\mu}{h^2}+A\cos\left(\theta-\omega\right), \tag{20}$$

where A and ω are the two constants of integration. Reintroducing $r,$ Equation (20) becomes

$$\frac{1}{r}=\frac{\mu}{h^2}+A\cos\left(\theta-\omega\right),$$

or,

$$r=\frac{1}{\frac{\mu}{h^2}+A\cos\left(\theta-\omega\right)}$$

or,

$$r=\frac{h^2/\mu}{1-\left(Ah^2/\mu\right)\cos\left(\theta-\omega\right)}\cdot$$

But, in general, the polar equation of a conic section may be written as

$$r = \frac{p}{1 + e\cos(\theta - \omega)} \tag{21}$$

so that

$$p = h^2/\mu$$

and

$$e = Ah^2/\mu.$$

Thus, the solution of the two-body problem is a conic section. It includes Kepler's first law as a special case. In particular, the orbit of a satellite about the Earth is classified by the value of the eccentricity e. Four cases readily come to mind:

1. For $0 < e < 1$, the orbit is an ellipse.
2. For $e = 1$, the orbit is a parabola.
3. For $e > 1$, the orbit is a hyperbola.
4. For $e = 0$, the orbit is a circle.

4. CONCLUSION

In this unit, you have been introduced to the concept of the two-body problem and the behaviour of satellite orbits in a two-body system. You have seen that it is often permissible to reduce a many-body problem to several two-body problems. That enables one to calculate approximate values of the parameters involved with relative ease. You have also seen that the solution of the two-body problem shows that the orbit of a satellite is a conic section that includes Kepler's first law as a special case.

5. SUMMARY

Having gone through this unit, you now know that

(i) a two-body problem involves the behaviour of two massive particles moving under the influence of their mutual gravitational fields.

(ii) the positions and velocities of the two massive particles can be calculated at any other time provided that their masses, positions and velocities are known at any given time.

(iii) strictly, satellite orbits are a many-body problem. However, it is often permissible to reduce a many-body problem to several two-body systems. This approach enables one to calculate approximate values of the required parameters with relative ease.

(iv) a satellite placed in orbit between the Earth and Mars is really a four-body problem involving the Earth, the Sun, Mars and the satellite itself.

(v) the position vector $\vec{R}$ of the centre-of-mass of a two-body system is given by $M\vec{R} = m_1\vec{r}_1 + m_2\vec{r}_2$.

(vi) the centre-of-mass of a two-body system moves with constant velocity (i.e with constant speed in a straight line).

(vii) in a two-body system, the motion of one body about the other must lie in a plane defined by the direction of the angular momentum integral $\vec{h}$.

(viii) the cross product $\vec{I} \times \vec{J} = \vec{K},$ where $\vec{I}, \vec{J}$ are unit vectors along and perpendicular to the radius vector, and $\vec{K},$ is a unit vector perpendicular to the plane of the orbit.

(ix) in the energy conservation equation, the quantity C is not the total energy. However, the quantity $\frac{1}{2}v^2$ is related to the kinetic energy while the quantity $-\frac{\mu}{r}$ is related to the potential energy of the system.

(x) the solution of the two-body problem indicates that the satellite orbit is a conic section.

6. TUTOR MARKED ASSIGNMENTS (TMAs)

Exercise 6.1

1. In the following, answer *true* or *false*.

a) A satellite in orbit between the Earth and Mars is strictly a two-body problem.

b) A three-body problem can be resolved into several two-body systems.

c) Newton's law of gravitation states that

$$\vec{F} = G\frac{m_1 m_2}{r^3}\vec{r}.$$

d) Newton's second law of motion states that

$$\vec{F} = m\frac{d^2\vec{r}}{dt^3}.$$

2. The position vector $\vec{R}$ of the centre-of-mass of a two-body system is given by

a) $M\,\ddot{\vec{R}} = m_1\,\ddot{\vec{r}}_1 + m_2\,\ddot{\vec{r}}_2$

b) $M\,\dot{\vec{R}} = m_1\,\dot{\vec{r}}_1 + m_2\,\dot{\vec{r}}_2$

c) $M\,\vec{R} = m_1\,\vec{r}_1 + m_2\,\vec{r}_2$

d) $M\,\vec{R} = m_1\,\dot{\vec{r}}_1 + m_2\,\dot{\vec{r}}_2$

3. The position vector $\vec{R}$ of the centre-of-mass of a two-body system is given by $M\,\vec{R} = \vec{a}t + \vec{b}$, where

a) $\vec{a}$ is a variable vector and $\vec{b}$ is a variable vector.

b) $\vec{a}$ and $\vec{b}$ are constant vectors.

c) $\vec{a}$ is a constant vector and $\vec{b}$ is a variable vector.

d) $\vec{a}$ and $\vec{b}$ are variable vectors.

4. The centre-of-mass of a two-body system moves with

a) constant speed in a circular path.

b) constant speed in a straight line.

c) variable speed in a circular path.

d) variable speed in a straight line.

5. In the quantity $-\frac{\mu}{r}$, which is related to the potential energy of the system, the parameter μ is given by

a) $\mu = G\frac{m_1 m_2}{r}$.

b) $\mu = G\frac{(m_1 + m_2)}{r}$

c) $\mu = G\frac{(m_1 - m_2)}{r}$

d) $\mu = G(m_1 + m_2)$.

6. In the expression $\vec{h} = \vec{r} \times \frac{d\vec{r}}{dt}$, the quantity $\vec{h}$ is

a) a constant.

b) a variable.

c) always zero.

d) undefined.

7. By defining $\vec{I}$ and $\vec{J}$, respectively, as unit vectors along and perpendicular to the radius vector $\vec{r}$, the third unit vector $\vec{K}$ must be

a) parallel to $\vec{I}$ always.

b) parallel to $\vec{J}$ always.

c) equal to zero always.

d) perpendicular to the plane containing $\vec{I}$ and $\vec{J}$ always.

8. In the expression $h = r^2\dot{\theta}$, the quantity $r\dot{\theta}$ is

a) angular displacement.

b) angular velocity.

c) linear displacement.

d) linear velocity.

9. Mathematically, Kepler's second law may be stated as follows:

a) $r^2\dot{\theta} =$ constant

b) $\dot{\vec{r}} = \vec{I}\,\dot{r} + \vec{J}r\dot{\theta}$

c) $\dfrac{d^2\vec{r}}{dt^2} = GM_2\dfrac{\vec{r}}{r^3}$.

d) $M\vec{R} = \vec{a} + \vec{b}$.

10. In the expression $\frac{1}{2}v^2 - \frac{\mu}{r} = C,$ the quantity $\frac{1}{2}v^2$ represents

a) the kinetic energy.

b) the potential energy.

c) the kinetic energy per unit mass.

d) the potential energy per unit mass.

11. Which of the following expressions represents the angular momentum integral?

a) $\dfrac{d^2u}{dt^2} + u = \dfrac{\mu}{h^2}$

b) $\vec{r}_1 - \vec{r}_2 = -\vec{r}$

c) $\mu = G\left(m_1 + m_2\right)$

d) $r^2\dot{\theta} = h.$

12. Which of the following expressions is the general solution of the two-body equation, $\dfrac{d^2u}{dt^2} + u = \dfrac{\mu}{h^2}$?

a) $u = \dfrac{1}{r}$

b) $u = \frac{\mu}{h^2} + A\cos(\theta - \omega)$

c) $u =$ constant

d) $u = 0$.

13. If $\vec{I}$, $\vec{J}$ and $\vec{K}$ are mutually perpendicular unit vectors and $\vec{I} \times \vec{J} = \vec{K}$, then,

a) $\vec{J} \times \vec{I} = \vec{K}$

b) $\vec{J} \times \vec{I} = -\vec{K}$

c) $\vec{J} \cdot \vec{I} = \vec{K}$

d) $\vec{J} \cdot \vec{I} = -\vec{K}$

14. The solution of the equation of motion of a two-body system is

a) a linear expression.

b) a quadratic expression.

c) a conic section.

d) a constant.

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MODULE 2

UNIT 2: SATELLITE ORBITS IN THE MANY-BODY PROBLEM

1. INTRODUCTION

In your study of satellite orbits in a two-body system, you learnt that satellite motion is really a many-body problem. You also saw that approximate values of the relevant parameters can be obtained by resolving the many-body problem into several two-body systems. In this unit, you will be introduced to the basic mathematical tools needed to treat a many-body system.

2. OBJECTIVES

At the end of this unit, you shall be able to

(i) give a precise statement of the many-body problem.

(ii) discuss the equation of motion in a many-body problem.

(iii) deduce the ten known integrals of the motion.

(iv) discuss the significance of the force function u.

(v) describe how a satellite may transfer from one orbit to another.

3. MAIN CONTENT

3.1 The Many-Body Problem

The many-body problem may be described as follows: Given at any time the positions and velocities of three or more massive particles, moving under their mutual gravitational fields, the masses also being known, calculate their positions and velocities for any other time. The first precise formulation of the many-body problem was given by Sir Isaac Newton.

The many-body problem is very complicated especially when the shapes and the internal constitutions of the bodies have to be taken into account. The point-mass many-body problem has inspired many scientists. Even the three-body problem is much more complex than the two-body problem. In the three-body problem, each body is subjected to complicated variable gravitational fields due to its attraction by the other two. As a result, close encounters with each of the other two bodies may be brought about. Each near-collision gives rise to a completely new type of orbit.

It would require a general formulation of unimaginable complexity to describe all the consequences of all possible close encounters. However, several general and useful statements can easily be made concerning the many-body problem.

Such statements are embodied in the ten known integrals of the motion. These integrals of the motion were known even in Euler's lifetime (1707-1783). Since then, no further integrals of the motion have ever been discovered. In addition, particular solutions were found by Lagrange. Such solutions exist only when certain relationships hold among the initial conditions. The Earth, the Moon, and a space vehicle in the Earth-Moon space constitute an approximate example of a three-body problem.

3.2 Equations of Motion in the Many-Body Problem

Consider n massive particles of masses m_i, $i = 1, 2, \cdots, n \cdot$ Suppose that their radius vectors from an unaccelerated reference point O are $\vec{R}_i$, while their mutual radius vectors are given by $\vec{r}_{ij}$, where

$$\vec{r}_{ij} = \vec{R}_j - \vec{R}_i \,. \qquad (1)$$

Then, Newton's law of motion becomes

$$F = m_i \ddot{\vec{R}}_i$$

while Newton's law of gravitation becomes

$$F = G \sum_{j=1}^{n} \frac{m_i m_j}{r_{ij}^{\;3}} \vec{r}_{ij}, \quad j \neq i, i = 1, 2, \cdots, n$$

where G is the universal constant of gravitation. When these two laws are combined, you obtain

$$m_i \ddot{R}_i = G \sum_{j=1}^{n} \frac{m_i m_j}{r_{ij}^{\;3}} \vec{r}_{ij}; \quad j \neq i, i = 1, 2, \cdots, n. \qquad (2)$$

Note that $\vec{r}_{ij}$ implies that the vector between m_i and m_j is directed from m_i to m_j so that

$$\vec{r}_{ij} = -\vec{r}_{ji} \qquad (3)$$

Equation (2) represents the required set of equations of motion in a many-body problem.

3.3 The Ten Known Integrals of the Motion

The ten known integrals of the motion include six integrals of the centre-of-mass, three integrals of area and one integral of energy.

3.3.1 Integrals of the Centre-of-Mass

By summing (2) and using (3), you obtain

$$\sum_{j=1}^{n} m_i \ddot{\vec{R}}_i = 0.$$

Integrating, you obtain

$$\sum_{j=1}^{n} m_i \dot{\vec{R}}_i = \vec{a} \qquad (4)$$

where $\vec{a}$ is a constant vector.

Integrating once more, you obtain

$$\sum_{j=1}^{n} m_i \vec{R}_i = \vec{a}t + \vec{b}\,. \qquad (5)$$

By definition, the centre-of-mass of the system has a radius vector $\vec{R}$ given by

$$M\vec{R} = \sum_{i=1}^{n} m_i \vec{R}_i$$

where

$$M = \sum_{i=1}^{n} m_i\,.$$

Thus, (4) becomes

$$M\dot{\vec{R}}_i = \vec{a}$$

and (5) becomes

$$M\vec{R} = \vec{a}t + \vec{b}$$

Hence, the radius vector $\vec{R}$ of the center-of-mass is given by

$$\vec{R} = \left(\vec{a}t + \vec{b}\right)/M \qquad (6)$$

while the velocity of the center of mass is given by

$$\dot{\vec{R}} = \vec{a}/M \,. \tag{7}$$

Relation (6) shows that the center-of-mass of the system moves through space in a straight line. Relation (7) shows that the center-of-mass of the system moves through space with a constant speed. This brings you to the conclusion that the center-of-mass of the system moves through space with a constant velocity (i.e. with a constant speed in straight line).

The constant of integration in (6) is $\vec{b}$, while the constant of integration in (7) is $\vec{a}$. If (6) and (7) are resolved with respect to a set of three unaccelerated rectangular axes through O, you obtain the six constants of integration, namely, a_x, a_y, a_z, b_x, b_y and b_z.

3.3.2 The Three Integrals of Area

Taking the vector product of $\vec{R}_i$ and $\ddot{\vec{R}}_i$ for each of the set (2) and summing, you obtain

$$\sum_{i=1}^{n} m_i \vec{R}_i \times \ddot{\vec{R}}_i = G\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{m_i m_j}{r_{ij}^3}\vec{R}_i \times \vec{r}_{ij}, \qquad j \neq i_i \,. \tag{8}$$

But,

$$\vec{r}_{ij} = \vec{R}_j - \vec{R}_i$$

so that

$$\vec{R}_i \times \vec{r}_{ij} = \vec{R}_i \times \left(\vec{R}_j - \vec{R}_i\right) \equiv \vec{R}_i \times \vec{R}_j \qquad \left(\sin ce\ \vec{R}_i \times \vec{R}_i = 0\right)$$

Hence, the right-hand side of (8) reduces in pairs to zero, giving

$$\sum_{j=1}^{n} m_i \vec{R}_i \times \ddot{\vec{R}}_i = 0.$$

Integrating the preceding equation, you obtain

$$\sum_{j=1}^{n} m_i \vec{R}_i \times \dot{\vec{R}}_i = \vec{C} \,. \tag{9}$$

Equation (9) states that the sum of the angular momenta (i.e. moments of momenta) of the masses in the system is a constant. This constant vector $\vec{C}$ defines a plane known as *the invariable plane of Laplace.* In the planetary system, the invariable plane of Laplace is inclined at about 1.5^0 to the plane of the ecliptic. This plane lies between the orbital planes of Jupiter and Saturn (the two most massive planets in the solar system).

If relation (9) is resolved with respect to the set of unaccelerated rectangular axes through O, the three *integrals of area* are obtained, namely,

$$\sum_{i=1}^{n} m_i \left(x_i \dot{y}_i - y_i \dot{x}_i \right) = C_1 ,$$

$$\sum_{i=1}^{n} m_i \left(y_i \dot{z} - z_i \dot{y}_i \right) = C_2 ,$$

$$\sum_{i=1}^{n} m_i \left(z_i \dot{x}_i - x_i \dot{z}_i \right) = C_3 ,$$

where,

$$C^2 = C_1^2 + C_2^2 + C_3^2 ,$$

giving three more constants of integration, C_1, C_2, C_3. Thus, the sums of the angular momenta of the n masses about each of the reference axes are constants.

So far, you have seen how the nine integrals of the motion were deduced. The associated constants of integration are nine, namely, $a_x, a_y, a_z, b_x, b_y, b_z, C_1, C_2$ and C_3. Next, you shall examine the tenth integral of the motion, namely, the *energy integral.*

3.3.3 The Integral of Energy

To obtain the tenth and final constant, you take the scalar product of $\dot{\vec{R}}_i$ with equation (2) in i and sum over all i. This leads to the expression

$$\sum_{i=1}^{n} m_i \,\dot{\vec{R}}_i \cdot \ddot{\vec{R}}_i = G\sum_{i=1}^{n} \sum_{j=1}^{n} \frac{m_i m_j}{r_{ij}^3} \cdot \dot{\vec{R}}_i \cdot \vec{r}_{ij}, \qquad j \neq i \;. \qquad (10)$$

Now,

$$\dot{\vec{R}}_i \cdot \vec{r}_{ij} = \dot{\vec{R}}_i \cdot \left(\vec{R}_j - \vec{R}_i\right), \qquad (11)$$

while

$$\dot{\vec{R}}_j \cdot \vec{r}_{ji} = \dot{\vec{R}}_j \cdot \left(\vec{R}_i - \vec{R}_j\right). \qquad (12)$$

Adding (11) and (12), you obtain

$$\begin{aligned}
\dot{\vec{R}}_i \cdot \vec{r}_{ij} + \dot{\vec{R}}_j \cdot \vec{r}_{ji} &= \dot{\vec{R}}_i \cdot \left(\vec{R}_j - \vec{R}_i\right) + \dot{\vec{R}}_j \cdot \left(\vec{R}_i - \vec{R}_j\right) \\
&= \dot{\vec{R}}_i \cdot \left(\vec{R}_j - \vec{R}_i\right) - \dot{\vec{R}}_j \cdot \left(\vec{R}_j - \vec{R}_i\right) \\
&= \left(\dot{\vec{R}}_i - \dot{\vec{R}}_j\right) \cdot \left(\vec{R}_j - \vec{R}_i\right) \\
&= -\left(\dot{\vec{R}}_j - \dot{\vec{R}}_i\right) \cdot \left(\vec{R}_j - \vec{R}_i\right).
\end{aligned}$$

Using equation (1), you find that equation (10) integrates to give

$$\frac{1}{2}\sum_{i=1}^{n} m_i \,\dot{\vec{R}}_i \cdot \dot{\vec{R}}_i - \frac{1}{2}G\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{m_i m_j}{r_{ij}} = E, \qquad j \neq i\,. \qquad (13)$$

Now, the velocity of the $i\,th$ mass is v_i, where

$$v_i^2 = \dot{\vec{R}}_i \cdot \dot{\vec{R}}_i \cdot$$

Also, if you put

$$u = \frac{1}{2}G\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{m_i m_j}{r_{ij}},$$

equation (13) becomes

$$T - u = E, \qquad (14)$$

where

$$T = \frac{1}{2}\sum_{i=1}^{n} m_i v_i^2 \cdot$$

Here, T is the kinetic energy of the system while $-u$ is its potential energy.

Hence, equation (13) states that the total energy of the system of n particles is a constant. This is the tenth (and final) constant of integration. Systems of constant total energy are called *conservative systems.*

No further integrals of the motion have ever been found. Indeed Bruns and $Poncar\hat{e}$ proved that apart from the energy integral, the integrals of area and the centre-of-mass integrals, no other integrals of the many-body problem exist that give rise to equations involving only algebraic or integral functions of the coordinates and velocities of the bodies, valid for all values of the masses and which satisfy the equations of motion.

3.4 The Force Function

Consider more closely the function u defined by

$$u = \tfrac{1}{2} G \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{m_i m_j}{r_{ij}}, \ i \neq j \cdot$$

This is a symmetrical function of all the masses and their mutual distances apart. Neither time nor the particles' radius vectors from the origin enter u explicitly. This is significant because, if u did not possess these properties, it would have been practically impossible to derive the ten integrals of the motion. The first nine integrals result from the property that u is invariant with respect to rotations of the axes or transformations of the origin. The energy integral arises because u does not contain the time explicitly (though it is, of course, a function of time through the $\bar{r}_{ij}$).

If you introduce the unit vectors $\hat{i}, \hat{j},$ and $\hat{k}$ along the axes Ox, Oy *and* Oz, the gradient of u is given by

$$\nabla u = \quad grad\ u = \hat{i}\frac{\partial u}{\partial x} + \hat{j}\frac{\partial u}{\partial y} + \hat{k}\frac{\partial u}{\partial z} \cdot$$

The symbol "∇" denotes the gradient operator (pronounced "nabla" of "del"), where

$$\nabla \equiv grad \equiv \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\cdot$$

Then, for the particle of mass m_i , it is seen that, since

$$\ddot{\bar{R}}_i = \hat{i}\ddot{x}_i + \hat{j}\ddot{y}_i + \hat{k}\ddot{z}_i, \qquad (14)$$

the following relation must hold:

$$m_i \ddot{\bar{R}}_i = grad\ u\ ,$$

where

$$grad\ u = \hat{i}\frac{\partial u}{\partial x} + \hat{j}\frac{\partial u}{\partial y} + \hat{k}\frac{\partial u}{\partial z}\cdot$$

Hence, equating coefficients of the unit vectors, you get

$$\left.\begin{aligned} m_i\ddot{x}_i &= \frac{\partial u}{\partial x_i} \\ m_i\ddot{y}_i &= \frac{\partial u}{\partial y_i} \\ m_i\ddot{z}_i &= \frac{\partial u}{\partial z_i} \end{aligned}\right\} \qquad (15)$$

Equations (15) are the equations of motion of the particle of mass m_i in rectangular coordinates. Consequently, u is called the force function since the partial derivatives of u with respect to coordinates give the components of the force acting on the particle.

3.5 Moment of Inertia and Force Function

Suppose that I is a function defined by

$$I = \sum_{i=1}^{n} m_i \bar{R}_i^2 \cdot$$

If you differentiate I twice with respect to time, you obtain

$$\ddot{I} = 2\sum_{i=1}^{n} m_i \dot{\bar{R}}_i^{\,2} + 2\sum_{i=1}^{n} m_i \bar{R}_i \cdot \ddot{\bar{R}}_i$$

or

$$\ddot{I} = 4T + 2\sum_{i=1}^{n} \vec{R}_i \cdot \nabla u \quad , \qquad (16)$$

where,

$$\nabla u = m_i \ddot{\vec{R}}_i \; .$$

Equation(16) relates the second time derivative of the moment of inertia I to the kinetic energy through the force function.

3.6 Transfer between Orbits

It is often desirable for a satellite to transfer from one orbit to another. Consider a satellite in an orbit about a massive spherical body. In the absence of perturbations by other masses, the satellite moves in a central force field. The orbit of the satellite remains a conic section as long as the motors are not fired. However, if the motors are fired, there will be changes in the satellite orbit. Such changes will, in general, affect all the six elements of the conic section. As a result, the satellite goes into a completely new orbit (with six new elements of the conic section).

4. CONCLUSION

In this unit, you have been introduced to the concept of the *many-body* problem. You have seen how the equations of motion in a many-body problem may be used to deduce the ten known integrals of the motion. You have also seen how a satellite may transfer from one orbit to another.

5. SUMMARY

Having gone through this unit, you now know that

(i) the behaviour of satellites in orbit is strictly a many-body problem.

(ii) a many-body problem involves three or more massive particles moving under the influence of their mutual gravitational forces.

(iii) in a many-body system, each close encounter gives rise to a completely new type of orbit.

(iv) several useful statements concerning the many-body problem are embodied in the ten known integrals of the motion.

(v) particular solutions of the many-body equations of motion exist only when certain relationships hold among the initial conditions.

(vi) if the radius vector from an unccelerated reference point O is $\vec{R}_i$, then the mutual radius vectors $\vec{r}_{ij}$ of the massive particles of masses $m_i\left(i=1,2,\cdots,n\right)$ is given by $\vec{r}_{ij}=\vec{R}_j-\vec{R}_i\cdot$

(vii) the Earth, the Moon and a satellite in the Earth-Moon space constitute a three-body problem.

(viii) the centre-of-mass of a many-body system has a radius vector $\vec{R}$ given by $M\vec{R}=\sum_{i=1}^{n}m_i\vec{R}_1$, where $M=\sum_{i=1}^{n}m_i$.

(ix) the radius vector $\vec{R}$ of the centre-of-mass of a many-body system is given by $\vec{R}=\vec{a}t+\vec{b},$ where $\vec{a}$ and $\vec{b}$ are constant vectors.

(x) the velocity of the centre-of-mass is given by $\dot{\vec{R}}=\vec{a}/M$.

(xi) the centre-of-mass of a many-body system moves through space with a constant speed in a straight line (i.e. with a constant velocity).

(xii) the first six integrals (of the centre-of-mass) are a_x,a_y,a_2,b_x,b_y *and* b_z.

(xiii) the next three integrals (of the area) are C_1,C_2 and C_3 where

$$C_1=\sum_{i=1}^{n}m_i\left(x_i\dot{y}_i-y_i\dot{x}_i\right)$$

$$C_2=\sum_{i=1}^{n}m_i\left(y_i\dot{z}_i-z_i\dot{y}_i\right)$$

and

$$C_3=\sum_{i=1}^{n}m_i\left(z_i\dot{x}_i-x_i\dot{z}_i\right),$$

where

$$C^2 = C_1^2 + C_2^2 + C_3^2 \cdot$$

(xiv) the last integral (of energy) is given by

$$T - u = E$$

where T is the kinetic energy and $-u$ is the potential energy of the system. Of course,

$$T = \sum_{i=1}^{n} m_i v_i^2.$$

and

$$u = \frac{1}{2} G \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{m_i m_j}{r_{ij}} \cdot$$

(xv) Bruns and Poincare′ proved that (apart from the six integrals of the centre-of-mass, the three integrals of area and the integral of energy) no other integrals of the many-body problem exist that give rise to equations involving only algebraic or integral functions of the coordinates and velocities of the masses and which satisfy the equations of motion.

(xvi) the force function u is defined by

$$u = \frac{1}{2} G \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{m_i m_j}{r_{ij}}, \quad i \neq j.$$

(xvii) the force function is a symmetrical function of all the masses and their mutual distances apart.

(xviii) the six integrals of the centre-of-mass and the three integrals of area arise because u is invariant with respect to rotations of the axes or transformations of the origin.

(xix) the energy integral results from the fact that u does not contain time explicitly (though, it is an *implicit* function of time through the radius vectors, $\vec{r}_{ij}$).

(xx) the gradient of u is given by $grad\, u \equiv \nabla u = \hat{i}\frac{\partial u}{\partial x} + \hat{j}\frac{\partial u}{\partial y} + \hat{k}\frac{\partial u}{\partial z}$, where $\hat{i}, \hat{j}, \hat{k}$ are mutually perpendicular unit vectors specifying a particle of mass m_i.

(xxi) the force function u is related to the force $\vec{F}$ according to the expression $\vec{F} = m_i \ddot{\vec{R}}_i = \nabla u.$

(xxii) the force function u is related to the second derivative of the moment of inertia I of the particle of mass m_i according to the expression $\ddot{I} = 4T + 2\sum_{i=1}^{n} \vec{R}_i \cdot \nabla u,$ where $\nabla u = m\ \dot{\vec{R}}_i \cdot$

6. TUTOR MARKED ASSIGNMENTS (TMAs)

Exercise 6.1

1. The first precise formulation of the many-body problem was given by
   a) Faraday.
   b) Maxwell.
   c) Newton.
   d) Einstein.

2. In a three-body problem, each near-collision with the other two gives rise to
   a) a completely new orbit.
   b) a completely new velocity.
   c) a completely new energy.
   d) all of the above.

3. How many are all the known integrals of the motion?
   a) 1
   b) 3

c) 6

d) 10

4. In the many-body system, Newton's law of gravitation may be stated as follows:

a) $\vec{F} = m_i \dot{\vec{R}}_i$

b) $\vec{F} = G\sum_{i=1}^{n}\sum_{j=1}^{n} m_i m_j \frac{\vec{r}_{ij}}{r_{ij}^3}\ , j \neq i, i = 1, 2, \cdots, n$

c) $F = G\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{m_i m_j}{r_{ij}^2} \cdot \vec{r}_{ij}\ , j \neq i, i = 1, 2, \cdots, n$

d) none of the above

5. The radius vector $\vec{r}$ is defined as follows:

a) $\vec{r}_{ij} = \vec{R}_i - \vec{R}_j$

b) $\vec{r}_{ij} = \vec{R}_j - \vec{R}_i$

c) $\vec{r}_{ij} = \vec{R}_j + \vec{R}_i$

d) $\vec{r}_{ij} = \vec{R}_j \times \vec{R}_i$

6. Particular solutions of the many-body problem were found by

a) Newton.

b) Lagrange.

c) Hamilton.

d) Poincare.

7. In the many-body problem, the position vector $\vec{R}$ of the centre-of-mass of the system is given by

a) $\vec{R} = \sum_{i=1}^{n} m_i \vec{R}_i$

b) $\vec{R} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{R}_i$,

c) $\vec{R} = 0$

d) $\vec{R} = \sum_{i=1}^{n} m_i$

8. The radius vectors $\vec{r}_{ij}$ and $\vec{r}_{ji}$ are related by the expression

a) $\vec{r}_{ij} = \vec{r}_{ji}$

b) $\vec{r}_{ij} = -\vec{r}_{ji}$

c) $\vec{r}_{ij} = k^2 \vec{r}_{ji}$, $k =$ positive constant.

d) $\vec{r}_{ij} = k^2 \vec{r}_{ji}$, $k =$ negative constant.

9. Which of the following expression is equivalent to the equation

$$\int\left(\sum_{i=1}^{n} m_i \ddot{\vec{R}}_i\right)dt = \int(zero)\,dt\,?$$

a) $\sum_{i=1}^{n} m_i \ddot{\vec{R}}_i t = \vec{a},$ $\vec{a} =$ constant vector.

b) $\sum_{i=1}^{n} m_i \ddot{\vec{R}}_i\, t = 0.$

c) $\sum_{i=1}^{n} m_i \dot{\vec{R}}_i = \vec{a},$ $\vec{a} =$ constant vector.

d) $\sum_{i=1}^{n} m_i \dot{\vec{R}}_i = 0\cdot$

10. The expression $\dot{\vec{R}}_j \cdot \vec{r}_{ji}$ is equivalent to

a) $\dot{\vec{R}}_i \cdot \left(\vec{R}_j - \vec{R}_i\right).$

b) $\dot{\vec{R}}_j \cdot \left(\vec{R}_i - \vec{R}_j\right).$

c) $\dot{\vec{R}}_i \cdot \left(\vec{R}_j + \vec{R}_i\right).$

d) $\dot{\vec{R}}_j \cdot \left(\vec{R}_i + \vec{R}_j\right).$

11. The expression $\dot{\vec{R}}_i \cdot r_{ij}$ is equivalent to

a) $\dot{\vec{R}}_i \cdot \left(\vec{R}_j - \vec{R}_i\right).$

b) $\dot{\vec{R}}_j \cdot \left(\vec{R}_i - \vec{R}_j\right).$

c) $\dot{\vec{R}}_i \cdot \left(\vec{R}_j + \vec{R}_i\right).$

d) $\dot{\vec{R}}_j \cdot \left(\vec{R}_i + \vec{R}_j\right).$

12. A system of constant total energy is called

a) a constant system.

b) a closed circuit system.

c) a conservative system.

d) a liberal system.

13. The function $u,$ defined by $u = G\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{m_i m_j}{r_{ij}},\ i \neq 1,$ is called

a) the potential function.

b) the force function.

c) the work function.

d) the energy function.

14. The function $u,$ defined in question 13 above, is

a) a complex function of $m\, and\, r$.

b) a linear function of $m\, and\, r$.

c) a symmetrical function of $m\, and\, r$.

d) an asymmetrical function of $m\, and\, r$.

15. If the moment of inertia I of a many-body system is defined by

$I = \sum_{i=1}^{n} m_i \vec{R}_i^2$, then

a) $\ddot{I} = \sum_{i=1}^{n} m_i \dot{\vec{R}}_i^2$

b) $\ddot{I} = 2\sum_{i=1}^{n} m_i \dot{\vec{R}}_i^2 + 2\sum_{i=1}^{n} m_i \dot{\vec{R}}_i \cdot \dot{\vec{R}}_i$

c) $\ddot{I} = 4T + 2\sum_{i=1}^{n} m_i R_i$

d) $\ddot{I}$ is none of the above.

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MODULE 2

UNIT 3: GLOBAL WEATHER AND CLIMATIC PATTERNS

CONTENTS

1 INTRODUCTION

The environment of the Earth is gradually decaying. The planet is warming. There is an urgent need for man to save the environment from the adverse effects of certain human activities. In this unit, you shall be introduced to some of the most serious environmental problems facing the Earth. You shall also examine possible methods of reducing these problems.

2. OBJECTIVES

At the end of this unit, you shall be able to

- (i) describe the five distinct components that make up the Earth.
- (ii) describe two mechanisms by which wind is produced.
- (iii) discuss the effect of weathering on the environment.
- (iv) discuss the impact of human activity on the environment.
- (v) suggest possible methods of reducing environmental pollution.

3. MAIN CONTENT

3.1 The Five Distinct Components of the Earth

The planet Earth is a system with five distinct components: the atmosphere, the hydrosphere, the cryosphere, the lithosphere and the biosphere. The atmosphere is a gaseous envelop which surrounds the Earth. This gaseous envelop contains clouds, suspended particles and gas. It protects life from the harshness of space. The atmosphere extends beyond a height of 100km above the ground.

The hydrosphere is the liquid water component of the Earth's system. It consists of the oceans and other large bodies of water. The hydrosphere covers about 71% of the surface of the Earth. It contains most of the planet's water. The cryosphere is the solid water component of the Earth's system. This component is made up of ice, and includes glaciers and polar caps. A large proportion of the Earth's fresh water is found in the cryosphere.

The solid component of the Earth's system is called the lithosphere. The lithosphere includes the soil and the rocks. Humans and land animals live upon the lithosphere. Plants make use of the nutrients fixed in the soil. Animals, plants and decaying organic matter make up the biosphere. In what follows, you will be introduced to the weather and climatic patterns and how they affect the different components of the Earth.

3.2 The Energy Cycle

The Sun is the source of almost all the energy on Earth. It is a giant ball of gas. It contains about 99.9% of the entire mass of the solar system. The energy from the Sun is responsible for the Earth's atmospheric motions. Earth's surface absorbs and reflects solar energy. Solar energy is also absorbed and reflected by atmospheric gases and clouds. Some of the incoming energy is reflected back into space. However, a reasonable amount of the energy makes its way to the surface of the Earth.

Most of the solar radiation reaching the surface of the Earth is used to cause water to evaporate. The resulting water vapour condenses into liquid water droplets, or deposits back into ice crystals, to form clouds. These clouds absorb and reflect some incoming radiation from the Sun. They also absorb radiation emitted from the surface of the Earth. As a result of this process, rainfall and snowfall occur. The cycle repeats itself over and over again. This gives rise to the changing weather and climatic patterns associated with the Earth.

Of all the incoming radiation, only about 3% is absorbed by the clouds; 16% is absorbed by the atmosphere; 51% is absorbed by the land and the oceans; 24% is reflected by the clouds and the Earth's surface; and 6% is reflected by the atmosphere. Of all the energy radiated back into space, 64% comes from clouds and gases; and about 6% of the energy is radiated directly into space from the Earth. The difference between the incoming energy and the outgoing energy gives rise to the stable temperature of the Earth. Thus, the atmosphere acts like a blanket. It warms the Earth and maintains a balance between the amount of solar radiation absorbed, and the quantity of heat reflected back into space.

3.3 The Terrestrial Atmosphere

3.3.1 Composition and Motion

The atmosphere has no definite ceiling. However, more than 99.99% of its mass lies below an altitude of about 100 kilometers. It is the atmosphere that makes the Earth habitable. It provides protection against the harmful radiations coming from the Sun. The atmosphere warms the Earth and stabilizes its temperature. In terms of temperature, the atmosphere is composed of five layers. In order of increasing height, these five layers are: the troposphere (0-10km), the stratosphere (10-50km), the mesosphere (50-80km), the thermosphere (80-500km), and the exosphere (above 500km).

The terrestrial atmosphere contains gases, clouds and other airborne particles called aerosols. Nitrogen is the main gas in atmosphere. It comprises up to 78% of the atmospheric gas. About 21% of the gas is oxygen. Other gases make up the remaining 1%.

In the troposphere, the gas temperature decreases with height at the rate of about 6.5^0 K per kilometre. This decrease in temperature promotes vertical atmospheric motions. For this reason, the weather is largely determined by what goes on within the troposphere.

3.3.2 Pressure-gradient Winds

At any point on the Earth, the atmospheric pressure is caused by the weight of the column of air above that point. On a global scale, there is a well defined pattern showing the areas of ''high'' pressure and the areas of ''low'' pressure. This pressure systems are closely related to the weather conditions experienced on the ground. High pressures are usually associated with fine weather; low pressures give rise to unsettled conditions.

The atmosphere tends to restore equilibrium. To achieve this, air moves to the low-pressure areas from the surrounding areas of high pressure. This movement of air from the areas of high pressure to the areas of low pressure creates wind. Low-pressure cells are always associated with rising air. This air associated with low-pressure systems assists in the production of cloud and precipitation.

3.3.3 Thermal-gradient Winds

The differential manner in which solar energy heats up the Earth produces wind too. The resulting changes in the airflow pattern gives rise to varying weather conditions at different latitudes. The tropics, receives intense heat throughout the year. This situation sets up convection currents. As a result, warm air rises and a low-pressure belt is created round the equator. Eventually, the rising air meets parts of the troposphere beyond which it can no longer rise. The air gradually cools and sinks back to the surface of the Earth (at about latitude 30^0 north and latitude 30^0 south) and flows back to the equator. This cell is called Hadley cell, in honour of the English Scientist, George Hadley, who first described them in 1753.

Some of the air from the Hadley cell continues to move toward the poles. This system (which rises at about 60^0 north and 60^0 south) is called the Ferrel cell. It is named in honour of William Ferrel who first described them in 1856. Cold air at the poles sinks and travels towards the equator. Upon meeting the Ferrel cell, this polar cell rises. The interaction between these atmospheric motions gives rise to (i) the strong, high-altitude, westerly winds known as the jet stream (ii) the windless area at the equator called the duldrum (iii) the warm, moist winds (blowing from the west) called the westerlies and (iv) the cold easterly winds (blowing from the north pole) called the polar easterlies.

The combination of local temperature contrasts and the shape and size of physical barriers produces a wide range of local winds. These physical barriers include mountain ranges, escarpments and valleys. Local winds produced in this way can be fascinating. They can sometimes be destructive too. Finally, the contrast between *cold* ocean temperatures and *warm* land temperatures produces regular sea breezes along the coast.

3.4 Weather and Landform

Earth's rocks may be broken down by environmental agents (such as wind, water and living organisms) through a process known as weathering. Some weathering processes are physical (involving the actions of frost and water). Other physical processes may involve changes in temperature and pressure. These weathering processes may break down the rocks without altering their chemical composition. This type of weathering process is known as physical weathering.

Sometimes, the chemical composition of the rock is altered when water carrying dissolved materials interacts with the minerals of the rock. As a result of the change in chemical composition, the rock may crumble. This type of weathering is known as chemical weathering. The weathering process is hostile to the environment. It prepares the way for erosion, as the sediments it creates may then be removed by other agents such as wind, water (rain, rivers, streams and oceans) or ice (snow and glaciers). As these agents transport the removed sediments, their abrasive action gradually erodes the landscape.

3.5 Human Activity and the Environment

Trees convert carbon dioxide into the oxygen which supports life. Most of these trees are found in forests. These forests are becoming increasingly threatened, partly because of population growth and partly because of some other human activities. Large-scale industries also play a disturbing role in promoting deforestation. Trees have been felled to make way for buildings and farms. Livestock are accelerating the pace of desertification throughout the semiarid lands. The combined effect of all these factors impacts adversely on local, regional and global climatic patterns. This situation leads to a dramatic increase in the likelihood of disrupting the delicate balance of plant and animal habitat.

3.6 Greenhouse Effect

Some natural gases like ozone, water vapour and carbon dioxide have an interesting and useful property: they absorb infrared radiation from the Sun and reradiate it to the Earth (thereby warming our planet). This process provides the warm condition necessary to support life on Earth. Regrettably, some of the activities of humans are producing much more greenhouse gases than the Earth really needs. This situation has become worse in this post-industrial era in which we live. There is evidence that the atmosphere today has an increased concentration of greenhouse gases like methane, carbon dioxide and nitrous oxide. As a result, the natural greenhouse effect has become enhanced. The Earth is becoming warmer than necessary, and the environment is at the receiving end.

3.7 Ozone Layer Depletion

Ozone is a gas that absorbs ultraviolet (UV) radiation from the Sun. It plays an extremely important role in shielding the Earth from the harmful effects of the ultraviolet radiation. When humans are exposed to high levels of UV radiation, they develop skin cancer. Other forms of life are also threatened by high doses of UV radiation.

For over 30 years, scientists have noticed that human activity is increasingly depleting this life-protecting umbrella called the *ozone layer*. Often, man-made chemicals rise above the troposphere into the stratosphere where they help in depleting the ozone layer. These man-made chemicals are known as CFCs (chlorofluorocarbons). In 1987, an international treaty (the Montreal Protocol) aimed at eliminating certain CFCs from industrial production was signed. Within five years, the use of the most harmful CFCs fell by 40%. However, much more needs to be done in order to completely eliminate the CFCs in the atmosphere.

3.8 Global Warming

It is clear that the Earth is heating up. What is not clear is the extent to which human activity contributes to this process of global warming. Direct temperature measurements and analyses indicate that the Earth has been warming steadily for over one hundred years. In particular, it is estimated that the Earth has warmed by 0.5^{o}C in the past one hundred years alone. The warmest global year on record is 1998.

Indirect observations also support the fact that the Earth is warming. For instance, glaciers are retreating more than at any time on record. Snow lines are creeping higher up the mountains, and sea levels are rising. The rising temperature causes ice to melt. The sea levels rise because water from the melting ice is dumped into the seas and the oceans. Consequently, the waters expand and flood the low-land areas. All these lead to a sad situation in which the environment suffers terribly.

3.9 Environmental Management

3.9.1 Environmental Modeling

Environmental modeling is an important discipline. It provides the tool needed by the policy makers and the project implementation agencies in order to manage environmental issues on the basis of the best available information.

Environmental modeling is applicable to areas of environmental management, land cover change analysis and prediction, land planning, habitat impact analysis and climate trend analysis. Environmental modeling serves as a bridge between the theoretical knowledge of environmental issues and the practical need for policy makers to take decisions in the best interest of the environment. Often, a complete mathematical description of important environmental models, together with the underlying assumptions and accompanying analysis, serves as an invaluable resource material for the policy makers and those who execute projects that impact on the environment.

The scope of environmental modeling is very wide. Specialized tools are needed to carry out an effective environmental modeling exercise. In the following paragraphs, three environmental modeling software (GMS, SMS and WMS) are briefly discussed.

The groundwater modeling and site analysis system (i.e. GMS software) is useful in hydrogeologic modeling. GMS offers efficient and effective tools for groundwater simulation including site characterization, model development, calibration, post-processing and visualization. It supports both finite-difference and finite-element modeling in 2D and 3D.

The surface water modeling system (i.e. SMS software) is useful in hydrodynamic modeling of rivers, bays, lakes, harbours and coastal regions. The numeric models supported by the SMS software readily generate data applicable to surface water modeling. It is a comprehensive environment for 1D, 2D and 3D hydrodynamic modeling.

The watershed analysis system (i.e. the WMS software) is useful in hydrologic and hydraulic modeling of watersheds and rivers. It is a comprehensive graphical modeling environment for all phases of watershed hydrology and hydraulics. The WMS can be used to automate modeling processes such as automated basin delineation and geometric parameter calculations.

3.9.2 Environmental Cost-Benefit Analysis

The origin of cost-benefit analysis (CBA) dates back to the 19^{th} century when some European countries embarked upon infrastructural appraisal and project evaluation exercise. Today, cost-benefit analysis has become an important instrument in the hands of policy makers and project managers. The special challenges posed by environmental problems and environmental policy formulation underlines the need for analysts and decision makers to remain up-to-date on information derivable from cost-benefit analyses.

Cost-benefit analysis ensures that public funds are efficiently utilized in major public investments. Environmental cost-benefit analysis seeks to quantity the risks involved and the benefits derivable from a particular environmental policy and project implementation. It is a fusion of the New Economics of welfare (reconstructed on the basis ordinal utility only) and the practical decision-making process. Environmental cost-benefit analysis is the major appraisal technique for public policy on (and public investment in) projects that impact upon the environment.

Cost-benefit analysis is built on the following *two* essential theoretical foundations:

1. Cost is defined as a decrease in human well-being.
2. Benefit is defined as an increase in human well-being

If the social benefits of a policy exceeds the total cost, then the project qualifies on cost-benefit grounds.

To carry out an effective cost-benefit analysis, the two aggregation rules stated below must be applied:

1. The aggregate benefit across different social groups involves the sum of the willingness to pay (or the willingness to accept compensation) for losses, regardless of the circumstances of both the beneficiaries and the losers.
2. Higher weight should be attached to benefits and costs accruing to disadvantaged or low-income groups.

In order to conduct a well executed cost-benefit analysis, one must follow a logical sequence of steps. For instance, relevant questions must be asked; *standing* (i.e those whose costs and benefits are to count) must be determined; time horizon (over which costs and benefits are counted) must be established; the risk (probabilistic outcome) and the uncertainty (when the probabilities are not known) must also be taken into account.

In the developed nations, detailed official guidelines on how to conduct cost-benefit analysis exist. Environmental cost-benefit analysis is widely used in the United States of America and within the European Union. The United States Environmental Protection Agency (U.S. EPA) has its own guidelines for preparing economic analyses for regulation. The Organization for Economic Cooperation and Development (OECD) has its own guidelines. In the United Kingdom, regulatory impact analysis (RIA) is mandatory for regulation. Regrettably, cost-benefit analysis results are rarely utilized by policy makers in the developing nations.

4. CONCLUSION

In this unit, you were introduced to the five distinct components that make up the Earth: the atmosphere, the hydrosphere, the cryosphere, the lithosphere and the biosphere. You have seen how the energy cycle leads to the changing weather conditions associated with the Earth. The movement of air from the high-pressure areas to the low-pressure areas produces winds due to the pressure gradient. Furthermore, the differential heating of different parts of the Earth creates a temperature gradient which produces winds. These winds play a significant role in shaping the climatic patterns that characterize different regions of the Earth. You have also seen how human activity has produced an enhanced greenhouse effect, and contributed to ozone layer depletion and global warming. A systematic collection and analysis of data enables scientists to forecast weather and climatic patterns. Finally, environmental cost-benefit analysis exposes the *risk* and *benefit* associated with policy formulation and implementation. It guides those who make policies on environmental issues, those who execute projects that impact on the environment, and those at the receiving end of the entire process.

5. SUMMARY

Having gone through this unit, you now know that

(i) the Earth is made up of five distinct components: the atmosphere, the hydrosphere, the cryosphere, the lithosphere and the biosphere.

(ii) the atmosphere is a gaseous envelop that surrounds the Earth.

(iii) the hydrosphere is the liquid water component of the Earth.

(iv) the cryosphere is the solid water component of the Earth.

(v) the lithosphere is the solid component of the Earth.

(vi) the biosphere is made up of three parts: animals, plants and decaying organic matter.

(vii) the Sun is the source of nearly all the energy on the Earth.

(viii) most of the solar energy reaching the Earth is used to cause water to evaporate.

(ix) more than 99.99% of the mass of the atmosphere lies below an altitude of 100 kilometres.

(x) the atmosphere provides protection against the harmful radiations coming from the Sun.

(xi) in the troposphere, the temperature decreases with height at the rate of about $6.5^{o}K$ per kilometre.

(xii) the movement of air from high-pressure areas to low-pressure areas gives rise to winds.

(xiii) the differential heating of the surface of the Earth sets up a temperature gradient that produces wind.

(xiv) the rocks on the Earth may be broken down by environmental agents through a process known as weathering.

(xv) there are three types of weathering, namely,

a) physical weathering
b) chemical weathering
c) biological weathering.

(xvi) weathering has many adverse effects on the environment.

(xvii) Felling of trees and large-scale farms impact negatively on the environment.

(xviii) the ozone layer is depleting.

(xix) human activity is creating an enhanced greenhouse effect that is harmful to the environment.

(xx) the Earth is becoming warmer.

(xxi) the warmest global year on record is 1998.

6. TUTOR MARKED ASSIGNMENTS (TMAs)

Exercise 6.1

1. In the following, answer *true* or *false*.

a) the lithosphere is the solid component of the Earth.
b) the hydrosphere is the solid water component of the Earth.
c) the cryosphere is the liquid water component of the Earth.
d) the atmosphere is the gaseous envelop that surrounds the Earth.

2. Nearly all the energy on the Earth comes from

a) the Moon.
b) the satellites.
c) the Sun.
d) the oceans.

3. Most of the solar energy reaching the Earth is used to

a) increase atmospheric motion.
b) cause water to evaporate.
c) change atmospheric gas into plasma.
d) deplete the ozone layer.

4. Which of the following components of the Earth provides protection against harmful radiations from the Sun?.

a) the hydrosphere
b) the biosphere
c) the lithosphere
d) the atmosphere.

4 Which of the following is not a type of a weathering process

a) physiological weathering process
b) biological weathering process
c) chemical weathering process
d) physical weathering process

5 When air moves from a high-pressure region to a low-pressure region

a) ice is produced.
b) wind is produced.
c) rain is produced.
d) lightning is produce.

6 Earth’s rocks are broken down by

a) Wind.
b) Water.
c) living organisms.

d) all of the above.

8. Which of the following radiations (from the Sun) is absorbed by the ozone layer?

a) ultraviolent radiation

b) visible rays

c) radio waves

d) infrared radiation

7 Which of the following radiations (from the Sun) causes skin cancer?

a) ultraviolent radiation

b) visible rays

c) radio waves

d) infrared radiation

8 Scientists have noticed that the ozone layer

a) is increasing.

b) is decreasing.

c) remains the same.

d) has vanished completely.

9 In what follows, answer *true* or *false.*

a) The Earth is heating up.

b) The Earth is cooling down.

c) The temperature of the Earth remains constant.

d) The warmest global year on record is 1998.

10. Chlorofluorocarbons(CFCs) rise above the troposphere into the stratosphere where they help to

a) deplete the ozone layer.

b) cool the stratosphere.

c) increase the ozone layer.

d) produce rainfall.

10 In the troposphere, the gas temperature decreases at the rate of about

a) 36^0 K per kilometre.

b) 26^0 K per kilometre.

c) 16^0 K per kilometre.

d) 6.5^0 K per kilometre.

11 In the following, answer *true* of *false.*

a) On a global scale there are well defined areas of ‘‘high’’ and areas of ‘‘low’’ pressures.

b) High pressure is associated with unsettled weather.

c) Low pressure is associated with fine weather.

d) Low pressure cells are associated with rising air.

12 About seventy-eight percent of atmospheric gas is

a) oxygen.

b) carbon dioxide.

c) nitrogen.

d) ozone.

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