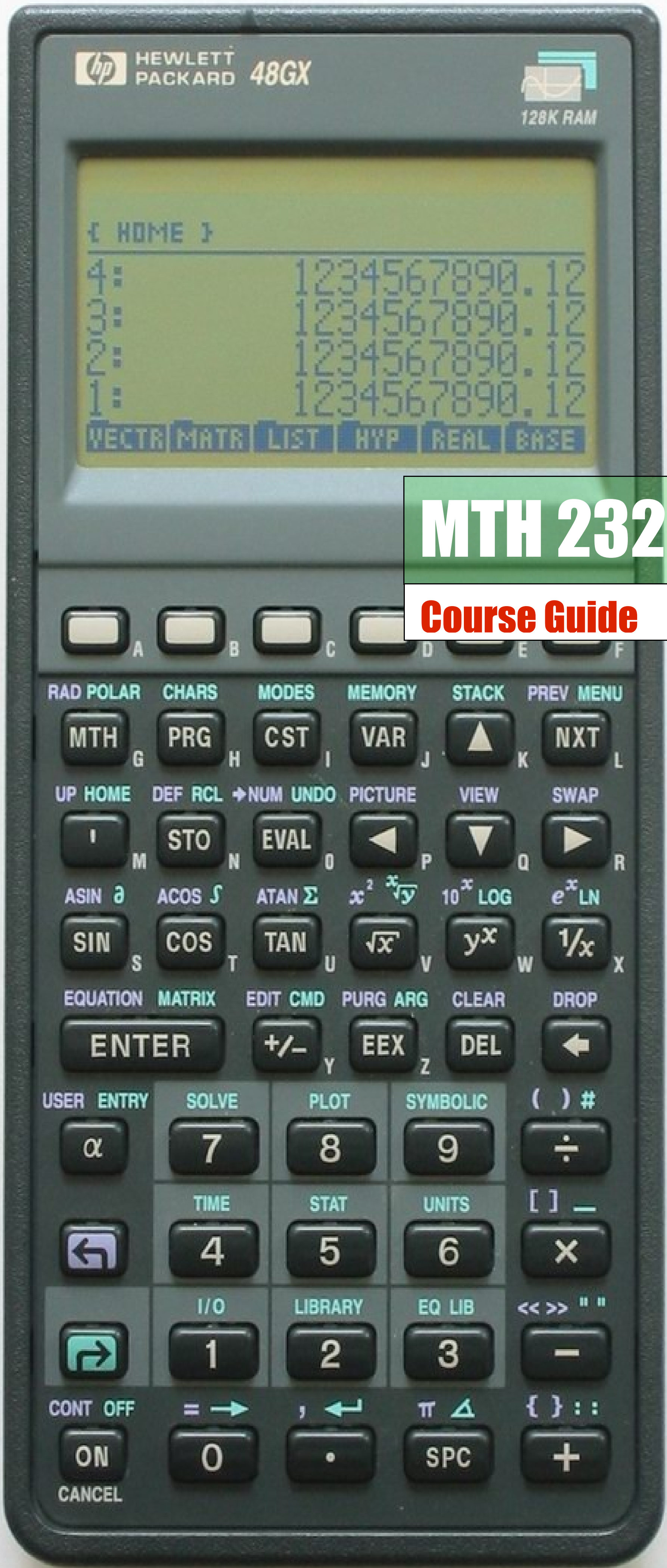

MTH 232

ELEMENTARY DIFFERENTIAL EQUATION

Course Guide

COURSE GUIDE

MTH 232
ELEMENTARY DIFFERENTIAL EQUATION

Course Developer	Ajibola Saheed Oluwatoyin National Open University of Nigeria 14/16 Ahmadu Bello Way Victoria Island, Lagos
Course Writer	Ajibola Saheed Oluwatoyin National Open University of Nigeria 14/16 Ahmadu Bello Way Victoria Island, Lagos
Course Editor	Ajibola Saheed Oluwatoyin National Open University of Nigeria Victoria Island, Lagos.
Programme Leaders	Proffessor Adebanjo National Open University of Nigeria Victoria Island, Lagos

NATIONAL OPEN UNIVERSITY OF NIGERIA

National Open University of Nigeria
Headquarters
14/16 Ahmadu Bello Way
Victoria Island
Lagos

Abuja Office
No. 5 Dar es Salaam Street
Off Aminu Kano Crescent
Wuse II, Abuja
Nigeria

e-mail: centralinfo@nou.edu.ng
URL: www.nou.edu.ng

Published by:
National Open University of Nigeria 2008

First Printed 2008

ISBN: 978-058-424-2

CONTENTS **PAGES**

Introduction

The subject of differential equations constitute a part of mathematics that plays an important role in understanding the physical sciences. In fact, it is the source of most of the ideas and theories which constitute higher analysis. In physics, engineering, chemistry and many other disciplines it has become necessary to build a mathematical model to represent certain problems. These mathematical models often evolve the search for an unknown function that satisfies an equation in which derivatives of the unknown functions play an important role. Such equations are called differential equation. The primary purpose of differential equation is to serve as a tool for studying change in the physical world.

You may recall if $y = f(x)$ is a given function then its derivative dy/dx can be interpreted as the rate of change of y with respect to x. Sir Isaac Newton observed that certain important laws of natural sciences can be phrased in terms of equation involving rates of change.

In the 16^{th} century, scientists like Isaac Newton, Leibniz, Bernoulli etc were engaged in solving differential equations. Many of the methods which other developed are in use today. In the 18^{th} century, scientists like Euler, Bernoulli, Lagrange and others contributed generously to the development of the subject. The pioneering work that led to the development of ordinary differential equations as a branch of modern mathematics is due to Cauchy, Riemain, Picard, Poincare, Lyapunou, Birkhoff and others.

The Course

As a 2-credit unit course, 9 study units grouped into 4 modules of 3 units – Module 1. 2 units – module 2. while 1 unit in module 3 and 3 units – module 4.

This course guide gives a brief summary of the total contents contained in the course material. Introduction to the Nature of differential equation; Basic concept, solution of a differential equation, family of curves and differential equation and differential equations arising from physical situation. Methods of solving equation of first order and first degree was thoroughly dealt with.

Separation of variable, homogenous and Exact equations and integrating factors are however discussed. We also discussed linear differential equations and differential equation of first order but not of first degree. Family of course orthogonal and oblique trajectories, and application to mechanics and Electricity respectively.

Lastly, higher order linear differential equation. Method of undetermined coefficients and method of variation of parameters were also looked into.

Course Aim/Objectives

On the completion of this course, you are expected to:

- Know basic concepts and definitions related to differential equations.
- Express some of the problems of physical and engineering interest in terms of differential equation
- Distinguish between the order and degree of a differential equation;
- Define the solution of an ordinary differential equation
- Identify an initial value problem .
- State and use the conditions for existence and uniqueness of first order ordinary differential equations.
- Derive differential equations for some physical problem

Working Through The Course

This Course involves that you would be required to spend lot of time to read. The context of this material is very dense and require you spending great time to study it. This accounts for the great effort put into its development in the attempt to make it very readable and comprehensible. Nevertheless, the effort required of you is still tremendous. I would advice that you avail yourself the opportunity of attending the tutorial facilitation where you would have the opportunity of comparing knowledge with your peers.

The Course Material

You will be provided with the following materials;
Course guide
Study units

In addition, the course comes with a list of recommended text books, which though are not compulsory for you to acquire or indeed read, are necessary as supplements to the course materials.

Study Units

The following are the study units contained in this course. The units are arranged into four (4) identifiable but readable modules.
Module 1

Unit 1 Introduction to the Nature of Differential equations

This unit is sub-divided into Basic concept

- Solution of a differential equation, family of courses and differential equations and
- Differential equations arising from physical situations.

Unit 2 Methods of solving Equation of First Order and First Degree

The following topics are discussed: Separation of variable. Homogeneous equation, Exact equation and integrating factor.

Module 2

Unit 1 Linear Differential Equations

This unit features: Classification of first order differential Equation. General solution of linear non-homogenous equation (method of undetermined coefficients, variable of parameters). Properties of the solution of linear homogeneous differential equation. Equation reducible to linear equations. And applications of linear differential equations

Unit 2 Differential Equation of first order But not of first Degree

This unit takes us through; Equations which can be factorized. Equations which cannot be factorized (Equation solvable for y and for x, equation in which independent variable or dependent variable is absent, equations homogeneous in x and y and equation of the first degree in x and y – clairant's, Riccati's equation and Bernoullis' equations).

Module 3

Unit 1 Family of Curves Orthogonal and Oblique Trajectories (Application to Mechanics and Electricity)

This unit entales, Orthagonal Trajectories.

- Application to Orthogonal Trajectiories
- Approximate solution (Direction fields; iteration) i.e. method of Direct fields and Picards iteration method.

Text Books

The following text books are recommended for further reading.

Advanced Egineering Mathematics by KREYSZIC.

Genralised functions by R.F. Hoskires

Engneering mathematics by K. A SROUD

Qualitative theory of ordinary Differential Equations by Fred Brauer and John A. Nohel

Engneering Mathematics by P. d. s verma

Generalized Functions in Mathematical physics by V.S Viadimirov.

Mathematical method for science students by G. Stephenson

Assessment

There are two components of assessment for this course. The tutor marked Assignment (TMAS) and the end of the course examination.

Tutor Marked Assignments (Tmas)

The (TMAS) is the continuous assessmet component of your course. It accounts for 30% of the total score. You will be given 4 (TMAS) to answer. Three of these must be answered before you are allowed to sit for the end of course examination.

The (TMAS) would be given to you by your facilitator and returned after you have done the assignments.

End Of Course Examination

This examination concludes the assessment for the course. It constitutes 70% of the whole course. You will be informed of the time for the examination. It may or may not coincide with the university semester examinations.

Summary

So far – the course we have covered the following points: definition of differential equation, ordinary differential equation as partial differential equations with there derivative. Order and degree of differential equation. Linear and non-linear ordinary differential equation.

However, an equation $\frac{dy}{dx}$ = f(x, y) called separable equation if f(x, y) = X(x) Y(x). Hence, we can write a(y) $\frac{dy}{dx}$ + b(x) = 0 for some a(y) and b(x). Integrating w.r.t.x and equating it to a constant, we get its solution.

A real-valued function h(x, y) of two variables x and y is called homogeneous function of degree n, if $h(\lambda x, \lambda y)\lambda^n$ h(x, y), where n is a real number and λ is any constant.

A homogeneous differential equation reduces to separable equation by the substitution y = ux where v = v(x).

An exact differential equation is formed by equating an exact differential to zero.
The differential equation
A(x, y) dy + b(x, y) dx = 0

More other. The general form of the linear equation of the first order is $\frac{dy}{dx}$ x p(x)y = Q(x) where P(x) and Q(x) are continuous real-valued functions on some internal.

When Q(x) = 0 is called homogenous linear differential equation of order one.

When Q(x) = 0 is called non-homogenous linear differential equation of order one.

I. F. for this equation is $e^{\int p(x)\ dx}$ and $\int Q(x)\ e^{\int p(x)\ dx}$ as the particular solution of the equation

Equation of the type
f(y) $\frac{dy}{dx}$ x p(x) f(y) = Q(x) reduces to linear equations by the substitution f(y) = v.

Clairant's equation is an equation of first order and of any degree if it can be put in the form
y = xp x f(p) the solution is y = c x + f(c)

Reccati's equation is an equation of the form $\frac{dy}{dx}$ = a(x) x b(x)y + c(x) y^2 where a(x), b(x) and c(x) are given functions an internal I of R. the general solution of this equation can be obtained if we know a particular solution y, of the equation and then we determine a function V defined by relation y = y, + $\frac{1}{y}$, which has y as the solution.

Lastly let y_1 and y_2 be the linearly independent solutions of the reduced equation of a non-homogeneous second order linear differential equation with constants or variable coefficients. Then on substituting $y = y_1\, u_1 (x) + y_2\, u_2 (x)$ and imposing the necessary conditions, the particular integral of the given equation was found.