# NATIONAL OPEN UNIVERSITY OF NIGERIA

## SCHOOL OF SCIENCE AND TECHNOLOGY

## COURSE CODE:PHY 454:

## COURSE TITLE: ASTROPHYSICS

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Course Writer: DR ADELOYE

Physics Department

University of Lagos

Course Editing Team: Eng. Esuruoso A.

Igbogbo, Ikorodu Lagos.

Eng. Tanimola O.

School of Engineering

Lagos State Polytechnic

and

Dr. Ajibola S.

National Open University of Nigeria

National Open University of Nigeria

Headquarters

14/16 Ahmadu Bello Way

Victoria Island

Lagos

Abuja Annex

245 Samuel Adesujo Ademulegun Street

Central Business District

Opposite Arewa Suites

Abuja

e-mail: centralinfo@nou.edu.ng

URL www.nou.edu.ng

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PHY 454 ASTROPYHSICS

COURSE GUIDE

NATIONAL OPEN UNIVERSITY OF NIGERIA

**Contents**

## INTRODUCTION

During your course of study of the prerequisite to this course, you were acquainted with certain basic concepts which will enable you to easily grasp the topics treated in this course.

This course should not be entirely alien to you if you take cognizance of the world around you; right from infancy, when you learnt through nursery rhymes that stars twinkle through your growing up years when you discovered that satellites far up in the sky were responsible for communications and satellite telecasts. Certainly you know that man has been on the moon and that planets are up there somewhere in the sky.

While the content and scope of prerequisite courses are expected to have developed in you an enquiring attitude towards the day to day astrophysics which influence you directly and indirectly on a daily basis, in PHY 454 we shall strive to build upon the gains of the prerequisite through overlaying the more advanced concepts upon the foundation already built; and also shall immerse ourselves in the specialised domain of Astrophysics and the numerous applications derivable thereof.

We shall of course commence with the underlying concepts, principles, rules and laws behind the physics of Astronomy, discuss galaxies and other exotic extraterrestrial entities. We shall arrive at the boundlessness of space and time –which appear to stretch infinitely in whatsoever direction we choose to proceed.

We shall personally ponder and wonder at such questions as “what is the origin of the universe” and “what will ultimately be the end of the Universe” – questions which have been asked since the dawn of time – and perhaps – might find answers within your own generation; - perhaps through you.

## THE COURSE

### PHY 454 Astrophysics

This course, PHY 454 – Astrophysics comprises a total of six Units arranged into three modules as listed below:

Module 1 is composed of 2 Units

Module 2 is composed of 2 Units

Module 3 is composed of 2 Units

Module 1 is devoted entirely to basic Astrophysical measurements of distance and time as well as stellar magnitudes. In unit 1, we shall commence with the Big Bang and proceed through the genesis of the various elements and their relationship with the composition of the universe. Having acquainted ourselves with the vastness of the universe, we shall proceed to study the different methods of measuring objects at extremities of distance; objects such as stars and Galaxies.

In Unit 2 we shall learn about stellar magnitudes, brightness and luminosities through which we shall begin to classify heavenly bodies

Module 2 which comprises two units will take us through stellar dynamics and those factors which establish stellar equilibrium. Here we shall discover some important equations and theorems which find invaluable application in Astrophysics. In addition to stellar dynamics, Unit 3 will discuss the material from which stars are made – "star stuff" if you wish; the physical state of this material and the energy transport systems attributable to stars.

Unit4 is the exclusive preserve of the origin, evolution and structure of the solar system – our solar system. We shall take a closer look at our sun as well as all the planets and planet-like bodies of the solar system. We shall not leave out the moons and their possible origins from this unit.

In the final Module, we shall similarly study the origin, evolution and structure of the galaxies. We shall classify galaxies and study galaxy populations as well as take a critical look at inter and intra galactic medium and its composition – what is "dark matter"? "is intergalactic space truly empty"? "what role has gravitational fluctuations played in the creation of galaxies"? We shall be able to answer all these and many more questions by the time we have studied Units 5 and 6.

**COURSE AIMS AND OBJECTIVES**

The aim of PHY 454 is to provide you a broad based understanding of Astrophysics through explaining to you in simple terms how the universe was created from a cosmic singularity known as the Big Bang. From the unimaginably violent explosion of this "cosmic egg" galaxies were subsequently derived– which comprise billions of stars – some of which host planetary systems, of which our own solar system is one of such.

This course strives to describe the Universe, Galaxies, our Solar System and Planetary system both qualitatively and quantitatively. Several rules and theories are presented and many principles advanced to support scientific Astronomical observations and inferences.

In view of the enormous size of quantities involved in Astrophysics, a major aim of this course is to acquaint you with the units of measurements such as the light year and the parsec which when well understood are effective shorthand.

After you have worked diligently through this course, you should have been provided a unique platform upon which you should be able to proffer answers to some of the fundamental questions which have plagued humankind right from the dawn of time: questions such as “what is the origin of the universe and how will it end”? “How long will our Sun remain in the sky”? “Is Jupiter really a stillborn star”? “Where do the Asteroids come from”.

## WORKING THROUGH THE COURSE

While the presentation style of this course PHY 454 is easily digestible, it is recommended that you should not take the content herein for granted, or assume that you already know them. Astrophysics demands that you spend quality time to read, as well as to relate what you have read to your surroundings and environment. The content of this course is quite comprehensive and is presented in clear language with numerous illustrations that you can easily relate to.

This presentation is both qualitative in its descriptiveness, while none the less; its quantitative complement takes the subject from mere esoteric to serious and objective science. Thus it is deliberate, and is to ensure that your attention in the course content is sustained as well as your retention ability to enable you work similar problems to those treated.

You should take full advantage of the tutorial sessions. They represent an invaluable opportunity for you to “rub minds” with your peers – and this represents a valuable feedback channel as you have the opportunity of comparing and personally scoring your progress with your course mates.

## COURSE MATERIAL

Prior to the commencement of this course you will be provided course material which will comprise your Course Guide as well as your Study

Units. You will also be provided a list of recommended textbooks which you are expected to acquire and which shall be an invaluable asset to complement your course material. While they are strongly recommended, these textbooks are not mandatory.

## STUDY UNITS

Now let us take a look at the study units which are contained in this course below. First you will observe that there are three modules which comprise two units each. Secondly you will see that the organisation of the content represents a logical flow from module 1 through to module 3 by which time you must have been reasonable acquainted with the subject matter of this course material.. Let's take a closer look at the scope of this book ; shall we?

### MODULE 1

**Unit 1 Distance and Time Measurements**

The Origin of the Elements and the Composition of the Universe
The composition of earth
The Method of Trigonometric Parallaxes
Stellar Parallax and Parallax Formula
Brightness of a Star

**Unit 2 Stellar Magnitudes**

The Magnitude Scale
Apparent Magnitude
Absolute Magnitude
Distance Modulus
The Influence of Wavelength on Brightness
Brightness – Luminosity Relationship
Measuring Apparent Brightness
Flux Photometry
Luminosity of Stars
Comparing Luminosities and Brightness

### MODULE 2

**Unit 3 The Origin, Evolution And Structure Of Stars**

Dynamics of Stars
Hydrostatic Equilibrium
Equation of Mass Concentration
The Dynamical Timescale

Construction of a Solar Model
Solar Constant
Intensity in the Solar System
Composition of the Solar Radiation
Surface Illumination
Climate Effects
Effects of Solar Radiation

**TEXTBOOKS**

It is recommended that you acquire the most recent editions of the recommended textbooks for your further reading.


- Barrow, J. D, The Origin of the Universe, Science Masters Series, Basic Books, a Division of Harper Collins Publishers , 1994, 150 pages.

- Dalrymple, G. B., The Age of the Earth, Stanford University Press, 1991,474 pages.

- Ferris, T. , The Whole Shebang - A State of the Universe(s) Report, Simon & Schuster, 1997

- Michael A. Zeilik, Stephen A. Gregory (1998). *Introductory Astronomy & Astrophysics* (4th ed.). Saunders College Publishing

- C.D. Murray & S.F. Dermott (1999). *Solar System Dynamics*. Cambridge University Press. ISBN 0-521-57295-9

- Advanced Astrophysics Cambridge Planetary Science by Neb. Duric


**ASSESSMENT**

It is standard NOUN practice to assess your performance partly through Tutor Marked Assessment which you can refer to as TMA, and partly through the End of Course Examinations.

**TUTOR MARKED ASSIGNMENT**

TMA is basically Continuous Assessment and accounts for 30% of your total score. During the study of this course, you will be given 4 Tutor Marked Assignments and of which you must compulsorily answer three of them to qualify to sit for the end of year examinations. The Tutor Marked Assignments will be provided by your Course Facilitator and upon completing the assignments; you must return them back to your Course Facilitator within the stipulated period of time.

**END OF COURSE EXAMINATION**

You must sit for the End of Course Examination as these accounts for 70% of your score upon completion of this course. You will be notified in advance of the date, time and the venue for the examinations which may, or may not coincide with National Open University of Nigeria semester examination.

## SUMMARY

Each of the four modules of this course has been carefully tailored to stimulate your interest in a specific area of Astrophysics to enable you to understand the content of the course with relative ease and also to facilitate the transition of abstract concepts to the real world which you see around you all the time.

This coursework provides you invaluable insight into the discovery, development and the functioning of comparatively simple concepts which constitute the building blocks for the more elaborate constructs and Astrophysical theories which having been based on scientific observations perhaps have been most catalytic in transforming the way we view the universe in which we live.

You will upon completion of this course be able to discuss the knowledge space contained within with confidence, and will also be able to proffer realistic solutions and answers to everyday questions that arise.

You just make certain that you have enough referential and study material available and at your disposal as this course will change your perception of the world around you in more ways than one.

On this note;

I wish you the very best in your quest for knowledge.

**TABLE OF CONTENTS** **Pages**

## USEFUL PHYSICAL AND ASTRONOMICAL CONSTANTS

| | |
|---|---|
| Gravitational constant $G$ | $6.674\ 3 \times 10^{-11}$ Nm2kg-2 |
| Speed of light in vacuum $c$ | $2.997\ 924\ 58 \times 10^{8}$ m s−1 |
| Planck constant $h$ | $6.626\ 069 \times 10^{-34}$ J/s |
| Radiation density constant $a$ | $7.565\ 78 \times 10^{-16}$ Jm-3 K−4 |
| Stefan-Boltzmann constant σ | $5.670\ 40 \times 10^{-8}$ J $s^{-1}m^{-2}$ $K^{-4}$ |
| Boltzmann constant $k$ | $1.380\ 650 \times 10^{-3}$ J $K^{-1}$ |
| Electron volt eV | $1.602\ 176\ 5 \times 10^{-19}$ J |
| Electron charge e | $1.6 \times 10^{-10}$ J |
| Electron mass $m$e | $9.109\ 382 \times 10^{-31}$ kg |
| Atomic mass unit $m$u | $1.660\ 538\ 8 \times 10^{-27}$k g |
| Proton mass $m$p | $1.672\ 621\ 6 \times 10^{-27}$k g |
| Neutron mass $m$n | 1.674 927 2 × 10−27k g |
| α-Particle mass $m$α | $6.644\ 656\ 2 \times 10^{-27}$k g |
| Solar mass $M$□ | $1.988\ 4 \times 10^{30}$ kg |
| $GM$□ | $1.327\ 124\ 42 \times 10^{17}$ m3 s−2 |
| Solar radius $R$□ | $6.957 \times 10^{8}$ m |
| Solar luminosity $L$□ | $3.839 \times 10^{26}$W |
| Year yr | $3.155\ 76 \times 10^{7}$ s |
| Astronomical unit AU | $1.495\ 978\ 71 \times 10^{11}$ m |
| Parsec pc | $3.085\ 678 \times 10^{16}$ m. |

# MODULE 1

## UNIT 1 DISTANCE AND TIME MEASUREMENTS

### 1.0 INTRODUCTION

In astronomy and cosmology, distance is often represented using the units called "light-years". A light-year is the distance that light travels in one year, and since light travels at approximately $3 \times 10^8$ m/s and there are approximately $3.15 \times 10^7$ seconds in one year, a light-year (distance is the velocity multiplied by the time) is just under $10^{16}$ m, or about 10 trillion kilometres.

The distance from the Sun to Pluto, which is the dimension of our solar system, is about 5.4 light-hours and the distance to the nearest star, Proxima Centauri, is about 4.25 light-years. Our solar system is part of the Milky Way Galaxy, which is a typical size galaxy and is about 100,000 light-years across. Andromeda, our nearest galactic neighbour, is located about 2,140,000 light-years away and the distance to the farthest object we can see is about of 10-20 billion light-years.

The universe is between 10 and 20 billion years old, and the common accepted age is 15 billion years. This is a very long time. To appreciate its depth, we'll map the history of the universe into one of our twelve-month calendar years. Our cosmic year begins with the formation of the universe at midnight. Although a lot goes on in the first few months, our galaxy, the Milky Way, doesn't form until May 1 and our solar system is absent until around September 9. Earth forms on September 14 and life originates in late September or early October. The Eukaryotes, the first cells with nuclei, are flourishing by mid-November but most of what we know about the history of life occurs in December.

December opens with an increase in oxygen in our atmosphere, a by-product of oxygen-producing algae that destroy themselves by overproducing oxygen. The first worms appear in mid-December, and plants begin colonizing the land around December 20. Life enters middle age on Christmas and dinosaurs dominate the next few days. The first primates appear on the December 29th but the first humans don't develop until late evening on the last day of the year. Agriculture is invented with just 40 seconds before midnight, Rome falls with three seconds remaining, and recent history makes up the last second. We are beginning the second year as newcomers to the universe.

## 1.1 THE BIG BANG

Our current, best hypothesis for the origin of the universe is called the Big Bang. The cause of the Big Bang is unknown, and also unknown is what, if anything, went on before it. But we have learned, and are continuing to learn much about how the universe evolved since its birth. The existence of a universe-initiating explosion is supported by several lines of evidence, most notably are:

(i) the observed motions of the galaxies

(ii) the observed background electromagnetic hiss generated during the bang

(iii) evidence from the observed abundance of light chemical elements in the universe.

Everything we know of is a composite of a mere 109 building blocks that we call elements. Atoms of the 92 naturally occurring elements combine to form the myriad of materials that we see and use every day. An atom is a collection of particles called protons, neutrons, and electrons (these particles are composites of still smaller particles, but we'll keep it simple). We can imagine an atom as a dense core of protons and neutrons called the nucleus, surrounded by a cloud of orbiting electrons and since protons and neutrons are much more massive than electrons, most of the mass of an atom resides in the nucleus.

### 1.1.1 THE ORIGIN OF THE ELEMENTS AND THE COMPOSITION OF THE UNIVERSE

All matter in the universe was generated during the Big Bang but much of it has been reworked in the interior of stars.

Our understanding of the processes that formed the solar system comes from our understanding of the physics of rotating bodies and investigation into the chemical composition of the Sun, planets, moons, and asteroids. Although much exciting work remains to be done on many aspects of solar-system evolution, we have a robust hypothesis describing the solar-system formation that provides a stable framework for research**.**

The types of Meteorites include: stony, iron, and the methods for dating the age of rocks and meteorites are based on the spontaneous transition of certain elements to other elements. This is known as the radiometric dating technique. The spontaneous transition is called radioactive decay and we call the isotopes that decay the "parent" and the product of decay a "daughter".

The result of such analyses indicates that meteorites have ages near 4.56 Ga, and hence so does Earth. The oldest rocks are just under 4 Ga; no rocks survived the earliest part of Earth history.

**THE COMPOSITION OF EARTH**

After inferring the original composition of the solar system from the sun and meteorites we can fill in gaps in our knowledge of Earth's composition.

Earth's radius is about 6,371 km and the radius of the core is about 3,486 km (the inner core radius is about 1,217 km (a little more than two-thirds of the radius of the Moon). Earth's mass is approximately $5.973 \times 10^{24}$ kg, and its mean density is 5.515 g/cm$^3$. The typical density of continental rocks is about 2.7 g/cm$^3$. Crust accounts for less than one-half of one percent of the mass of the planet. The mantle accounts for about 84% of Earth's volume but the core contains almost 70% of the planet's mass.

Earth is hot because of three heat sources:

- Heat of formation
- Heat from "freezing" of inner core

- Radioactive heat

The motions deep in Earth are much slower than those common at the surface. The motion in the outer core proceeds at a rate of approximately ten kilometres per hour. This is about six miles per hour which may seem slow, but which subjects the earth's material to enormous pressure and which slow motion is the source of Earth's magnetic field. The mantle is much more sluggish, creeping along at a rate of about ten centimetres per year, about 100,000 times slower than the core. But these slow moving processes have had a tremendous impact on the character of our planet.

**SELF ASSESSMENT 1**

Q1. Define a light year. Give an estimate of the distance between the Sun and Pluto.

Q2. Use the Big Bang theory to explain how the Universe was formed.

### 1.1.2 THE METHOD OF TRIGONOMETRIC PARALLAXES

Nearby stars appear to move with respect to more distant background stars due to the motion of the Earth around the Sun. This apparent motion (it is not "true" motion) is called Stellar Parallax.

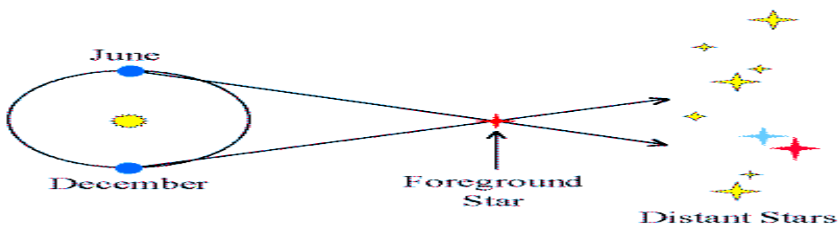



**Figure 1.1** ***Parallax Measurements***

In the picture above, the line of sight to the star in December is different than that in June when the Earth is on the other side of its orbit. As seen from the Earth, the nearby star appears to sweep through the angle shown. Half of this angle is the parallax, **p**. Take note that parallax *decreases* with distance.

As the distance to a star increases, the parallax decreases. This is easy to see in the following two figures below:

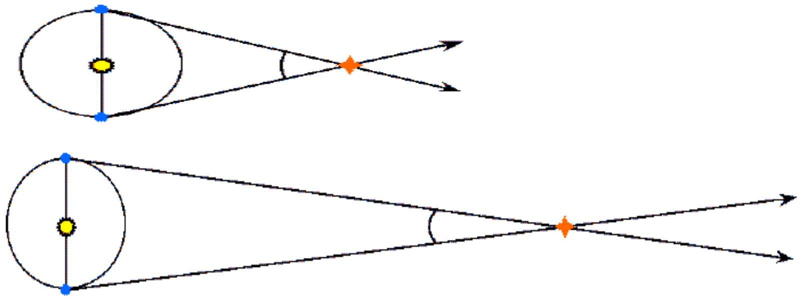


**Figure 1.2** ***Parallax method***

In the upper figure, the star is about 2.5 times nearer than the star in the lower figure, and has a parallax angle which is 2.5 times larger. This gives us a means to measure distances directly by measuring the parallaxes of nearby stars. We call this direct distance technique the Method of Trigonometric Parallaxes.

### 1.1.3 STELLAR PARALLAX AND PARALLAX FORMULA

Since the nearest stars are still very far away, the largest measured parallaxes is very small usually less than an arc second. For example, the nearest star, Proxima Centauri, has a parallax of 0.772-arcsec (the largest parallax observed for any star). We use photography and digital imaging techniques to measure parallaxes today. Interestingly, we measure parallaxes from space to avoid blurring due to the Earth's atmosphere.

We have seen earlier that the smaller the parallax, the larger the distance. We can express this as a simple formula:

$$d = \frac{1}{p} \qquad 1.1$$

Where: **p** = parallax angle in arc seconds

**d** = distance in "Parsecs"

Writing our parallax formula in this way allows us to define a new "natural" unit for distances in astronomy: the Parallax-Second or **Parsec**.

**Parallax Second = Parsec (pc)**

Fundamental unit of distance in Astronomy

A star with a parallax of 1 arc second has a distance of 1 Parsec.
1 parsec (pc) is equivalent to:

206,265 AU
3.26 Light Years(ly)
$3.086 \times 10^{13}$ km

An alternative unit of astronomical distance is the **Light Year** (ly).

1 light year (ly) is equivalent to: 0.31 pc = 63,270 AU

The light year is used primarily by writers of popular science books and science fiction writers. It is rarely used in research astronomy.

The reason for this is that the parsec is directly derived from the quantity that is being measured (the stellar parallax angle), whereas the light-year must be derived from having previously measured the distance in parsecs. In this way, the parsec is a more "natural" unit to use than the light year.

**Examples of Parallax Distances:**

1. The star alpha Centauri has a parallax of p = 0.742-arcsec:
$$d = \frac{1}{p} = \frac{1}{0.742} = 1.35pc$$
2. A star is measured to have a parallax of p = 0.02-arcsec:

$$d = \frac{1}{p} = \frac{1}{0.02} = 50pc$$

### 1.1.4 BRIGHTNESS OF A STAR

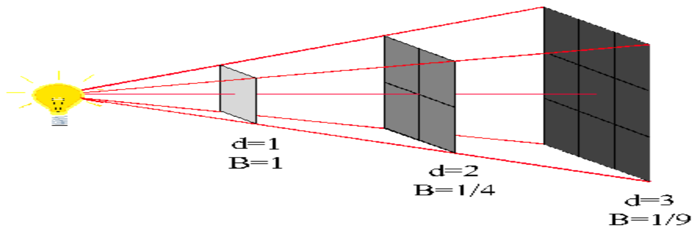
There are two ways to know the brightness of a star quantitatively:

(i) Intrinsic Luminosity – this is a measure of the total energy output of the star. It is distance independent and is a physical property of the star itself

(ii) Apparent Brightness - This measures how bright the star appears to be as seen from a distance it depends on the distance to the star.

**Inverse Square Law of Brightness**

The Apparent Brightness of a source is a consequence of geometry. As light rays emerge from a source, they spread out in area:




**Expressed mathematically**:

$$B = \frac{1}{d} \qquad 1.2$$

**And in words:**

The Apparent Brightness (B) of a source is inversely proportional to the square of its distance (d)

The Implications of the above statements are:

For a light source of a given Luminosity the closer the source is the brighter it becomes that is if you move 2x closer to a light source it will appear $2^2 = 4$ times brighter.

Also the farther is the light source the fainter is appears. This means that moving 2x further away from a light source it will appear $2^2 = 4$ times fainter

**TUTOR MARKED ASSIGNMENT**

1. Define parallax angle. How is it related to the distance between a star and the Sun?

2. The parallax angle for a nearby Star measured from the earth is $0.0015^{o}$. Estimate the distance of the star from the earth in ( i) parsec (ii) light years

3. Calculate the parallax angle in arc sec for a G- class of star if it is at a distance of ( i ) 200pc (ii) 1.5x1013km (iii) 350AU

4. Define Brightness of a star. Also, mention how the brightness can be determined.

**SUGGESTIONS FOR FURTHER READING**

Barrow, J. D, The Origin of the Universe, Science Masters Series, Basic Books, a Division of Harper Collins Publishers , 1994, 150 pages.

Dalrymple, G. B., The Age of the Earth, Stanford University Press, 1991,474 pages.

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# UNIT 2 STELLAR MAGNITUDES

## 2.0 INTRODUCTION

A basic observable quantity for a star is its brightness. Since stars can have a very broad range of brightness, astronomers commonly introduce a logarithmic scale called a *magnitude scale* to classify the brightness**.**

### 2.1.1 THE MAGNITUDE SCALE

The magnitudes $m_1$ and $m_2$ for two stars are related to the corresponding brightness $b_1$ of star $m_1$and $b_2$ of star $m_2$ through the equation

$$m_2 - m_1 = 2.5(\log b_1 - \log b_2) = 2.5\left(\frac{b_1}{b_2}\right) \qquad 2.1$$

Where "log" means the (base-10) logarithm of the corresponding number; that is, the power to which 10 must be raised to give the number. Because this relation is logarithmic, a very large range in stellar brightness corresponds to a much smaller range of magnitudes; this is a major utility of the magnitude scale.

If a star of magnitude 1 is 2.512 × brighter than a star of magnitude 2 and 100 × brighter than a sixth magnitude star how much brighter is it than a star of magnitude 3?

You need to be careful here. It is not simply 2 × 2.512 different. You need to remember that a difference of one magnitude equals $\sqrt[5]{100} = 2.512$. A difference of 2 magnitudes therefore = $2.512^2$ = 6.31 × difference in brightness.

Two objects of different magnitudes therefore vary in brightness by 2.512 raised to the power of their magnitude difference. If we write this as an equation, the ratio of brightness or intensity, $\frac{b_1}{b_2}$ between two objects, A and B, with magnitudes $m_1$ and $m_2$ is given by the following equation:

$$\frac{b_1}{b_2} = (2.512)^{m_2 - m_1} \qquad 2.2$$

Where $b_1$ and $m_1$ are the brightness and magnitude of star 1 and where $b_2$ and $m_2$ are the brightness and magnitude of star 2.

The following table will help keep the magnitude differences and brightness ratios straight. Both the absolute and apparent magnitudes are given on the same scale.

| Magnitude Difference | Brightness Ratio |
|---|---|
| 0.0 | 1.0 |
| 0.2 | 1.2 |
| 1.0 | 2.5 |
| 1.51 | 4.0 |
| 2.0 | 6.3 |
| 2.5 | 10.0 |
| 4.0 | 40.0 |
| 5.0 | 100.0 |
| 7.5 | 1000.0 |

For example the apparent magnitude of the Sun is about -26. The brightest star in the sky, Sirius, has an apparent magnitude of -1.4. The difference in magnitude is about 25; for every five magnitudes the brightness ratio is 100. So Sirius is $10^2$ x $10^2$ x $10^2$ x $10^2$ x $10^2$ = $10^{10}$ times less bright than the sun in apparent magnitude. This result tells us nothing about the luminosities of the Sun and Sirius, only how bright they appear in the sky. In contrast, the absolute magnitude of the sun is approximately +4.8, and Sirius is +1.4. The difference in absolute magnitudes is roughly 4, corresponding to a brightness ratio of approximately 40. This comparison of absolute magnitudes tells us that Sirius is roughly 40 times as luminous as the Sun.

Using the information above, we can work out ratio of the brightness of Sirius-Sun as:

$$\frac{b_1}{b_2} = (2.512)^{m_2-m_1}$$

$$= (2.512)^{4.8-1.4}$$

$$= (2.512)^{3.4}$$

$$= 2$$

A similar comparison can be made to find the luminosities of other stars since brightness and intensity are proportional.

### 2.1.2 APPARENT MAGNITUDE

The preceding equation gives us a way to relate the magnitudes and brightness of two objects, but there are several ways in which we could specify the brightness and this leads to several different magnitudes that astronomers define. One important distinction is between whether we are talking about the apparent brightness of an object, or its "true" brightness. The former is a convolution of the true brightness and the effect of distance on the observed brightness, because the intensity of light from a source decreases as the square of the distance (the inverse square law).

The *apparent magnitude* of an object is the "what you see is what you get" magnitude. It is determined using the apparent brightness as observed, with no consideration given to how distance is influencing the observation. Obviously the apparent magnitude is easy to determine because we only need measure the apparent brightness and convert it to a magnitude with no further thought given to the matter. However, the apparent magnitude is not so useful because it mixes up the intrinsic brightness of the star (which is related to its internal energy production) and the effect of distance (which has nothing to do with the intrinsic structure of the star).

### 2.1.3 ABSOLUTE MAGNITUDE

Clearly, a star that is very bright in our sky could be bright primarily because it is very close to us (the Sun, for example), or because it is rather distant but is intrinsically very bright (Betelgeuse, for example). It is the "true" brightness, with the distance dependence factored out that is of most interest to us as astronomers. Therefore, it is useful to establish a convention whereby we can compare two stars on the same footing, without variations in brightness due to differing distances complicating the issue.

Astronomers define the *absolute magnitude* to be the apparent magnitude that a star would have if it were (in our imagination) placed at a distance of 10 parsecs (which is 32.6 light years) from the Earth.

We can do this if I know the true distance to the star because we can use the inverse square law to determine how its apparent brightness would change if I moved it from its true position to a standard distance of 10 parsecs. There is nothing magical about the standard distance of 10 parsecs. We could as well use any other distance as a standard, but 10 parsecs is the distance astronomers have chosen for this standard. A common convention, and one that we will mostly follow, is to use a lower-case "m" to denote an apparent magnitude and an upper-case "M" to denote an absolute magnitude.

The absolute magnitude, $M$, of a star is defined as the magnitude that star would have if it were at a distance of 10 parsecs from us.

A distance of 10 pc (parsec) is purely arbitrary but now internationally agreed upon by astronomers. The scale for absolute magnitude is the same as that for apparent magnitude, that is a difference of 1 magnitude = 2.512 times difference in brightness. This logarithmic scale is also open-ended and unit less. Again, the lower or more negative the value of M, the brighter the star is. Absolute magnitude is a convenient way of expressing the luminosity of a star. Once the absolute magnitude of a star is known you can also compare it to other stars. Betelgeuse, $M$ = -5.6 is intrinsically more luminous than Sirius with an $M$ = 1.41. Our Sun has an absolute visual magnitude of 4.8.

**SELF ASSESSMENT 1**

Q1. Write down an expression which relates the magnitudes of two stars to their brightness.

Q2. Explain the concept of apparent magnitude of a star.

### 2.1.4 DISTANCE MODULUS

As you may recall from the section on distance measurement, most stars are too distant to have their parallax measured directly. Nonetheless if you know both the apparent and absolute magnitudes for a star you can determine its distance. Let us look again at Sirius and Betelgeuse plus another star called GJ 75.

| Star | Apparent magnitude ( m) | Absolute magnitude ( M ) | Distance Modulus (m-M) |
|---|---|---|---|
| Sirius | 1.44 | 1.41 | - 2.85 |
| Betelgeuse | 0.45 | 5.14 | 5.59 |
| GJ75 | 5.63 | 5.63 | 0.00 |

How far away is GJ 75? It is an unusual star in that its apparent and absolute magnitudes are the same. Why? The reason is that it is actually 10 parsecs distant from us, so by definition its two magnitudes must be the same.

What about Sirius? Its apparent magnitude is lower (therefore brighter) than its absolute magnitude. This means that it is closer than 10 parsecs to us. Betelgeuse's apparent magnitude is higher (therefore dimmer) than its absolute magnitude so it would appear even brighter in the night sky if it were only 10 parsecs distant.

Is there a quick way of checking whether a star is close or not? Looking at the above table we see that if a star is at a distance of 10 parsecs, then $m = M$ or $m - M = 0$.

For Sirius, $m - M = (-1.44) - 1.41 = -2.85$. This value is negative and Sirius is closer than 10 pc. For Betelgeuse, m - M = 0.45 - (-5.14) = 5.59. This value is positive and Betelgeuse is more than 10 pc distance. Astronomers use the difference between apparent and absolute magnitude, the distance modulus, as a way of determining the distance to a star.

- Distance Modulus = $m - M$.
- Distance modulus is negative for stars closer than 10 parsecs.

- Distance modulus is positive for stars further away than 10 parsecs.
- The size of the distance modulus determines the actual value of the distance, so that a star of distance modulus 1.5 is closer than one with a distance modulus of 8.7.

The distance modulus can be used to determine the distance to a star using the equation:

$$m - M = 5\log\left(\frac{d}{10}\right) \tag{2.3}$$

Where *d* is in parsecs. Note that if *d* = 10 pc then *m* and *M* are the same. If we make M the subject matter, we have:

$$M = m - 5\log\left(\frac{d}{10}\right) \tag{2.4}$$

But this is simply a reworking of equation 4.2. You should be comfortable in solving this equation given any two of the three variables. Let us know look at how you can solve some examples.

It should be noted that we can determine the apparent magnitude m of a star simply by measuring how bright it appears to be, but to determine the absolute magnitude M the distance to the star must also be known.

### 2.1.5 THE INFLUENCE OF WAVELENGTH ON BRIGHTNESS

You might think that introducing the apparent and absolute magnitudes would resolve ambiguities about what we mean when we refer to the brightness of a star, but there is a further complication. The brightness of an object (whether apparent or absolute) depends on the wavelength at which we observe it.

Generally, astronomical observations are made with an instrument that is sensitive to a particular range of wavelengths. For example, if we observe with the naked eye, we are sensitive only to the visible part of the spectrum, with the most sensitivity coming in the yellow-green portion of that. On the other hand, if we use normal photographic film to record our observation, it is more sensitive to blue light than to yellow-green light.

Thus, to be precise in discussing brightness or the associated magnitude, we must specify which region of the electromagnetic spectrum our instrument is most sensitive to. We address this issue in the next section.

### 2.1.6 BRIGHTNESS – LUMINOSITY RELATIONSHIP

This relates the Apparent Brightness of a star (or other light source) to its Luminosity (Intrinsic Brightness) through the Inverse Square Law of Brightness and it is given by:

$$B = \frac{L}{4\pi d^2} \quad 2.5$$

Where B is the brightness of the star, L is the Luminosity and d the distance from the source.

At a particular Luminosity, the more distant an object is, the fainter its apparent brightness becomes as the square of the distance. To measure the Luminosity of a star we need to take two measurements in account namely:

- the Apparent Brightness (flux) measured via photometry, and
- the Distance to the star measured in some way

Thus together with the inverse square law of brightness, we can compute the Luminosity.

**MEASURING APPARENT BRIGHTNESS**

The process of measuring the apparent brightness of objects is called Photometry.

Two ways to express apparent brightness are:

1. Stellar Magnitudes
2. Absolute Fluxes (energy per second per area)

**FLUX PHOTOMETRY**

This is a device which is used to measure the photons received from a star using a light-sensitive detector. Some of the detectors are:

- Photographic Plates
- Photoelectric Photometer
- Solid State Detector (e.g., photodiodes or CCDs)

We now use solid-state detectors like CCDs and similar technologies (with very rare exceptions), as these detectors are far more sensitive and stable than any previous technology.

### 2.1.7 LUMINOSITY OF STARS

The absolute magnitude of a star is simply a simple way of describing its luminosity. **Luminosity**, *L*, is a measure of the total amount of energy radiated by a star or other celestial object per second. This is therefore the power output of a star. A star's power output across all wavelengths is called its bolometric luminosity.

Our Sun has a luminosity of $3.84 \times 10^{26}$ W or J.s$^{-1}$ which can be denoted by the symbol $L_{sol}$ (actually the subscript symbol is normally a dot inside a circle - the standard astrological symbol for the Sun). Rather than always use this exact value it is often more convenient to compare another star's luminosity $L_*$ to the Sun's as a fraction or multiple. Thus if a star is twice is luminous as the Sun, $L_*/L_{sol} = 2$. This approach is convenient as the luminosity of stars varies over a huge range from less than $10^{-4}$ to about $10^{6}$ times that of the Sun so an order of magnitude ratio is often sufficient. Luminosity is determined by:

1. **Temperature**: A black body radiates power at a rate related to its temperature - the hotter the black body, the greater its power output per unit surface area. An incandescent or filament light bulb is an everyday example. As it gets hotter it gets brighter and emits more energy from its surface. The relationship between power and temperature is not a simple

linear one though. The power radiated by a black body per unit surface area varies with the **fourth power** of the black body's effective temperature, $T_{eff}$.

So; the power output, $l$ is directly proportional to $T^4$ or $l = \sigma T^4$ for a perfect black where σ is a constant called the *Stefan-Boltzmann constant*. It has value of $\sigma = 5.67 \times 10^{-8}$ W m$^{-2}$ K$^{-4}$ in SI units. As a star is not a perfect black body we can approximate this relationship as:

$$l \approx \sigma T^4 \qquad 2.6$$

This relationship helps account for the huge range of stellar luminosities. A small increase in effective temperature can significantly increase the energy emitted per second from each square metre of a star's surface.

2. **Size (radius)**: If two stars have the same effective temperature but one is larger than the other it has more surface area. The power output per unit surface area is fixed by equation 4.3 so the star with greater surface area must be intrinsically more luminous than the smaller one.

Assuming stars are spherical then surface area is given by:

$$surface\ area = 4\pi R^2 \qquad 2.7$$

Where $R$ is the radius of the star

To calculate the total luminosity of a star we can combine the equations above to give:

$$L \approx 4\pi R^2 \sigma T^4 \qquad 2.8$$

Using equation 4.6 all we need in order to calculate the intrinsic luminosity of a star is its effective temperature and its radius. In practice this equation is not used to determine the luminosity of most stars as only a few hundred stars have had their radii directly measured. If however, the luminosity of a star can be measured or inferred from other means (e.g. by spectroscopic comparison) then we can actually use equation 4.6 to determine the radius of the star.

### 2.1.8 COMPARING LUMINOSITIES AND BRIGHTNESS

Let us imagine we have two stars, A and B that we wish to compare. If we can measure their respective apparent magnitudes, $m_A$ and $m_B$ how will they differ in brightness? The ratio of their brightness (or intensities) $I_A/I_B$ corresponds to their difference in magnitude, $m_B - m_A$ . Remember, as a difference of one magnitude means a brightness ratio of the fifth root of 100 or $100^{1/5}$, a difference of $m_B - m_A$ magnitudes gives a ratio of $(100^{1/5})^{m_B - m_A}$ ∴

$$\frac{I_A}{I_B} = 100^{\frac{(m_B - m_A)}{5}} \qquad 2.9$$

The inverse-square law of light means that the flux, $l$ (or intensity) of a star at a distance $d$ can be related to its luminosity $L$ at a distance $D$ *by* the following relationship:

$$\frac{L}{l} = \left(\frac{d}{D}\right)^2 = \left(\frac{d}{10}\right)^2 \qquad 2.10$$

At distance of 10 parsecs, $D$ is represented by absolute magnitude, $M$ and the flux at distance $d$ is represented by the apparent magnitude, $m$ then the luminosity ratio is given by:

$$m - M = 2.5\log(L/l)$$

$$m - M = 2.5\log(d/10)^2$$

$$m - M = 5\log(d/10)$$

**SELF ASSESSMENT 2**

Q1. Define Luminosity and state its units.

Q2. Mention the factors that determine the luminosity of the stars.

**REFERENCE / FURTHER READINGS**

Ferris, T. , The Whole Shebang - A State of the Universe(s) Report, Simon & Schuster, 1997,

**TUTOR MARKED ASSIGNMENT**

1. How much brighter is Altair with apparent magnitude of + 1.73 than Proxima Centauri with magnitude of 11.09?

2. If Mimosa has an apparent magnitude of 1.25 and 108parsecs distant. What is its absolute magnitude?

3. If a star has an apparent magnitude of 0.45 and an absolute of -5.14. How far away is it?

4. The ratio of the luminosities $I_1$ and $I_2$ of two stars of apparent magnitudes $m_1$ and $m_2$ respectively is given by $\frac{I_1}{I2} = 100^{(m_2-m_1)^{\frac{1}{5}}}$. Show that their apparent magnitudes differ by 5.

5. Estimate the brightness of a star if it has a luminosity of $1.5x\frac{10^4 J}{s}$ and is at a distance of 685000km from the observer.

# MODULE 2

## UNIT 3 THE ORIGIN, EVOLUTION AND STRUCTURE OF STARS

### 3.0 INTRODUCTION

Stars are held together by gravitational attraction exerted on each part of the star by all other parts. Collapse of the star is resisted by internal thermal pressure and gravitational force.

These two forces play the principal role in determining stellar structure as they must be balanced.

### 3.1 DYNAMICS OF STARS

For our stars which are isolated, static and spherically symmetric. There are four basic equations to describe their structures. All physical quantities depend on the distance from the centre of the star alone.

1. Equation of hydrostatic equilibrium: at each radius, forces due to pressure differences balance gravity

2. Conservation of mass

3. Conservation of energy flux equal to local rate of energy release

4. Equation of energy transport relation between the energy flux and the local gradient of temperature.

These basic equations are supplemented with

(i) Equation of state (pressure of a gas as a function of its density and temperature).

(ii) Opacity (how opaque the gas is to the radiation field)

(iii) Core nuclear energy generation rate.

**SELF ASSESSMENT 1**

Q1. Mention two forces on which the structure of the stars depend.

### 3.1.1 HYDROSTATIC EQUILIBRIUM

The equation of hydrostatic support balance between gravity and internal pressure is known as hydrostatic equilibrium.

Mass of element $\partial m = P(r)\partial s\partial r$ 3.1

Where $P(r) =$ density at $r$

$\partial s$ and $\partial r$

Considering the forces acting in radial direction

1. Outward force is equal pressure exerted by stellar material on the lower face:
   $P(r)\partial s$ 3.2
2. Inward force: pressure exerted by stellar material on the upper face, and gravitational attraction of all stellar material lying within r
   $$P(r+dr)\partial s + \frac{Gm(r)}{r^2} \quad \partial m$$ 3.3

   $$P(r+dr)\partial s + \frac{Gm(r)}{r^2} P(r)\partial s\partial r$$ 3.4

The two opposing forces are at work within a star such that gravity pulling inward wants to make the star contract and pressure pushing outwards wants to make the star expand.

Pressure and Gravity work on each other:

- Gravity confines the gas in the star against Pressure expansion.
- Pressure supports the star against Gravitational collapse.

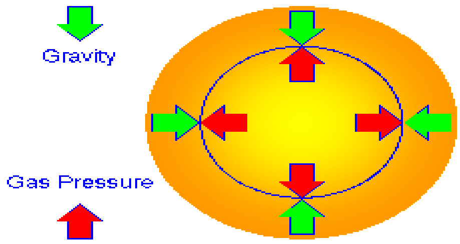



***Figure 3.1 A star in hydrostatic equilibrium due to two forces***

When there is exact balance between the two, we have a condition of Hydrostatic Equilibrium. In this condition, the star neither expands nor contracts. Outer layers press down on the inner layers. The deeper one goes into a star, the greater the pressure.

### 3.1.2 EQUATION OF MASS CONCENTRATION

The mass m(r) contained within a star of radius r is determined by the density of the gas and can be deduced as follows:

If we consider a thin shell (a star) with inside radius r and outer radius r+∂r

$$\partial v = 4\pi r^2 \partial r \qquad 3.5$$

$$\partial m = \partial v \rho(r) \qquad 3.6a$$

$$= 4\pi r^2 \partial r \rho(r) \qquad 3.6b$$

$$\frac{\partial m(r)}{\partial r} = 4\pi r^2 \rho(r) \qquad 3.7$$

This is the equation of mass concentration.

**SELF ASSESSMENT 2**

Q1 For the accuracy of hydrostatic equilibrium, we have assumed that the gravity and pressure forces are balanced – how valid is that?

Consider the case where the outward and inward forces are not equal, these will be a resultant force acting on the element which will give rise to an acceleration a.

$$P(r+dr)\partial s+\frac{Gm(r)}{r^2}P(r)\partial s\partial r-P(r)\partial s=P(r)\partial s\partial ra$$

$$\rightarrow\frac{dP(r)}{dr}\frac{Gm(r)}{r^2}P(r)=P(r)a \qquad 3.8$$

Now acceleration due to gravity is $g=\frac{Gm(r)}{r^2}$

Which is the generalized from of the equation of hydrostatic support. Now suppose there is a resultant force on the element ( $\partial r$ $\neq 0$).

And suppose they sum to a small fraction of gravitational term $(\beta)$

$$\beta P(r)g=\mathrm{P}(r)a \qquad 3.9$$

Hence, there is an inward acceleration of $a=\beta g$

Assuming it begins at rest, the spatial displacement after a time t is

$$d=\frac{1}{2}at^2=\frac{1}{2}\beta gt^2 \qquad 3.10$$

**SELF ASSESSMENT 3**

1. Estimate the timescale for the sun's radius to change by an observable amount (as a function of $\beta$). Assume $\beta$ is small, is the time scale likely? ( $7x10^8 m$; g= $2.5x10^2 ms^{-2}$ ).

2. We know from geologist and fossil records that it is unlikely to have changed its flux output significantly over the last $10^9$ years hence find an upper limit for β. What does this imply about the assumption of hydrostatic equilibrium?

**THE DYNAMICAL TIMESCALE**

If we allowed the star to collapse i.e. set $d=r$ and substitute

$$g=\frac{Gm}{r^2}$$
$$t=\frac{1}{\sqrt{\beta}}\left(\frac{2r^3}{Gm}\right)^{1/2} \qquad 3.11$$

Assuming $\beta = 1$

Then $t_d = \left(\dfrac{2r^3}{Gm}\right)^{1/2}$ 3.12

$t_d$ is known as the dynamical time

**SELF ASSESSMENT 4**

Q1. Compute dynamical time given that:

$r_o = 7x10^8 m$ ; $m_o = 1.99x10^{30} Kg$

Stars are rotating gaseous bodies. To what extent are they flattened at the poles? If so, departures from spherical symmetry must be accommodated for.

Consider mass $\partial m$ near the surface of star of mass m and radius r. The element mass will be acted on by additional inwardly acting force to provide circular motion.

The centripetal force is given by:

$F_c = m\omega^2 r$ 3.13

Where $\omega$= angular velocity of star

There will be no departure from spherical symmetry provided that

$$\frac{m\omega^2 r}{\frac{GMm}{r^2}} << 1 \text{ or } \omega^2 << \frac{Gm}{r^2}$$

Note that RHS of this equation is similar to $t_d$

$t_d = \left(\dfrac{2r^2}{Gm}\right)^{\frac{1}{2}}$ or $\dfrac{Gm}{r^2} = \dfrac{2}{r_d^2}$ 3.14

$$\omega^2 << \frac{2}{t_d^2}$$

And as $\omega = \frac{2\pi}{p}$; where p= rotation period if spherical symmetry is to hold then

$p >> t_d$

For example $t_d$(sun) or $2000s$ and per month? For the majority of stars, departures from spherical symmetry can be ignored. Some stars do rotate rapidly and rotational effects must be included in the structure equations because they can change the output of models.

Before deriving the equations for (3) and (4) we will consider several applications of our current knowledge and will derive mathematical formulae for the following

1. Minimum value for central pressure of a star
2. The Virial theorem
3. Minimum mean temperature of a star
4. State of stellar material

In doing this you will learn important assumptions and approximation that allow the values for minimum central pressure, mean temperature and the physical state of stellar material to be derived.

### 3.1.3 MINIMUM VALUE FOR CENTRAL PRESSURE OF STAR

We have only two of the four equations and no knowledge yet of material composition or physical state of stars but we can deduce a minimum central pressure. Why in principle, do you think this need to be a minimum value? Given what we known, what is this likely to depend upon?

$$\frac{dP(r)}{dr} = -\frac{GM(r)P(r)}{r^2} \qquad 3.15$$

$$\frac{dM(r)}{dr} = 4\pi r^2 P(r) \qquad 3.16$$

Divide these two equations

$$\frac{dP(r)}{dr}\bigg/\frac{dM(r)}{dr} \equiv \frac{dP}{dM} = -\frac{GM}{4\pi r^4} \qquad 3.17$$

If we integrate equation 3.17

it gives $P_m - P_o = \int_0^{m_o} \frac{GM}{4\pi r^4} dM$ 3.18

$$\int_0^{m_o} \frac{GM}{4\pi r^4} dM > \int_0^{m_o} \frac{GM}{4\pi r_o^4} dM = \frac{GM_o^2}{8\pi r_o^4} \qquad 3.19$$

Hence, we have

$$P_m - P_o > \frac{GM_o^2}{8\pi r_o^4} \qquad 3.20$$

We can approximate the pressure at the surface of the star to be zero;

$$P_o > \frac{GM_o^2}{8\pi r_o^4} \qquad 3.21$$

For example, for the sun,

$P_c = 4.5x10^{13} Nm^{-2} = 4.5x10^8$ Atmospheres

## 3.1.4 THE VIRIAL THEOREM

Again let us take the two equations of hydrostatic equilibrium and mass conservation and divide them

$$\frac{dP(r)}{dr}\bigg/\frac{dM(r)}{dr} \equiv \frac{dP}{dM} = -\frac{GM}{4\pi r^4} \qquad 3.22$$

Now multiply both sides by $4\pi r^2$ and we have

$$4\pi r^8 dP = -\frac{GM}{r} dM \qquad 3.23$$

On integrating over the whole star

$$3\int_{P_o}^{P_c} VdP = -\int_0^{M_o} \frac{GM}{r} dM \qquad 3.24$$

Where $V =$ volume contained within radius r

Using integration by parts, integration of the LHS of equation 3. 24 leads to

$$3[PV]_0^c - 3\int_{V_o}^{V_c} PdV = -\int_0^{M_o} \frac{GM}{r} dM \qquad 3.25$$

At the centre, $V_c = 0$ and on the surface $P_s = 0$

Hence we have

$$3\int_0^{V_o} PdV + \int_0^{M_o} \frac{GM}{r} dM = 0 \qquad 3.26$$

Now the right hand term equal total gravitation potential energy of the star or it is the energy released in forming the star from its components dispersed to infinity.

**SELF ASSESSMENT**

Q1. Show that this is consistent with work – energy theorem.

$$3\int_0^{V_o} PdV + \Omega = 0$$

### 3.1.5 MEAN TEMPERATURE OF A STAR

We have seen that pressure, P, is an important term in the equation of hydrostatic equilibrium and the viral theorem. We have derived a minimum value for the central pressure $(P_c = 4.5x10^8 atmos)$

What physical processes give rise to this pressure and which are the most important?

(i) Gas pressure $P_g$

(ii) Radiation pressure $P_r$

We shall show that $P_r$ is negligible in the stellar interior and pressure is dominated by $P_g$

To do this we first need to estimate the minimum mean temperature of a star

Consider the $\Omega$ term, which is the gravitational potential energy:

$$-\Omega = \int_0^{M_o} \frac{GM}{r} dM \qquad 3.27$$

We can obtain a lower bound on the RHS by noting: that at all points inside the star $r < r_s$

Hence, $\frac{1}{r} > \frac{1}{r_s}$

$$\Rightarrow \int_0^{M_o} \frac{GM}{r} dM > \int_0^{M_c} \frac{GM}{r} dM = \frac{GM_o}{2r_o} \qquad 3.28$$

Now $dM = PdV$ and the Virial theorem can be written as:

$$-\Omega = 3\int_0^{M_c} PdV = 3\int_0^{M_o} \frac{P}{p} dM \qquad 3.29$$

Now pressure is sum of radiation pressure and gas pressure: $P = P_g + P_r$

Assume, for now, that stars are composed of ideal gas with negligible $P_r$

$$P = nkT = \frac{kpT}{m} \qquad 3.30$$

The equation of state of ideal gas

Where $n =$ Number of particles per $m^3$

$m =$ Amount of mass of particles

$k =$ Boltzmann's constant

Hence we have

$$-\Omega = 3\int_0^{m_o} \frac{P}{p} dm = 3\int_0^{m_o} \frac{kT}{m} dm \qquad 3.31$$

And we may use the inequality derived above to write

$$-\Omega = 3\int_0^{m_o} \frac{kmT}{m} dm > \frac{Gm_o^2}{2r_o}$$

$$\Rightarrow \int_0^{m_o} TdM > \frac{GM_o^2 m}{6kr_o} \qquad 3.32$$

We can think of the LHS as the sum of the temperature of all the mass elements $dM$ which make up the star. The mean temperature of the star $\overline{T}$ is then just the integral divided by the total mass of the star $M_s$

$$\Rightarrow m_o.\overline{T} = \int_0^{m_o} TdM$$

$$\overline{T} > \frac{GM_o m}{6kr_o} \qquad 3.33$$

### 3.1.6 MINIMUM MEAN TEMPERATURE OF THE SUN

As an example for the Sun we have

$$\overline{T} > 4x10^6 \frac{m}{m_H} k \qquad 3.34$$

Where $m_H = 1.67x10^{-27} Kg$

Since hydrogen is the most abundant element in stars and for a fully ionized Hydrogen star $\frac{m}{m_H} = \frac{1}{2}$

(As these are two particles, $n + e$ for each H atom). And for any other element $\frac{m}{m_H}$ is greater.

$$\Rightarrow \overline{T} > 2x10^6 K$$

### 3.1.7 PHYSICAL STATE OF STELLAR MATERIAL

We can also estimate the mean density of the Sun using the following equation:

$$P_{av} = \frac{3M_o}{4\pi r_o^2} = 1.4x10^3 Kgm^{-3} \qquad 3.35$$

Mean density of the sun is only a little higher than water and other ordinary liquids. We know such liquids become gaseous at $T$ much lower than $T_o$. Also the average K.E of particles at $T_o$ s much higher than the ionization potential of Hydrogen. Thus the gas must be highly ionized, i.e. is plasma. It can thus withstand greater compression without deviation from an ideal gas. Note that an ideal gas demands that the distances between the particles are much greater than their sizes, and nuclear dimension is $10^{-13}$ m compared to atomic dimension of about$10^{-10}$ m.

Let us revisit the issue of radiation as gas pressure we assumed that the radiation pressure was negligible. The pressure exerted by photons on the particles in a gas is:

$$P_{rad} = \frac{aT^4}{3} \qquad 3.36$$

Now consider gas pressure and radiation pressure at a typical point in the sun

$$\frac{P}{P_o} = \frac{aT^4}{3} / \frac{kTP}{m} = \frac{maT^3}{3kP} \qquad 3.37$$

Taking $T \sim T_o = 2x10^6 K, P - P_o \approx 1.4x10^3 kgm^{-3}$

And $m = \frac{1.67x10^{-27}}{2} Kg$

Gives $\frac{P_o}{P_a} \sim 10^{-4}$

Hence radiation pressure appears to be negligible at a typical (average) point in the Sun.

In summary, with no knowledge of how energy is generated in stars we have been able to derive a value for the sun's internal temperature and deduce that it is composed of near ideal gas plasma with negligible radiation pressure.

### 3.1.8 MASS DEPENDENCY OF RADIATION ON GAS PRESSURE

However we shall later see that $P_r$ does become significant in higher mass stars. To give a basic idea of this dependency we replace P in the ratio equation above:

$$\frac{P_r}{P_g} = \frac{maT^3}{3k\left(\frac{3M_s}{4\pi r_s^3}\right)} = \frac{4\pi mar_s^3T^3}{gkM_s} \qquad 3.38$$

And from the Virial theorem: $T \propto \frac{M_s}{r_s}$

$$\Rightarrow \frac{P_r}{P_g} \propto m_o^2 \qquad 3.39$$

i.e. $P_r$ becomes more significant in higher mass stars.

### 3.1.9 ENERGY GENERATION IN STARS

So far we have only considered the dynamical properties of the star, and the state of the stellar material. We need to consider the source of the stellar energy.

Let's consider the origin of the energy i.e. the conversion of energy from some form in which it is not immediately available into some form that it can radiate. How much energy does the sun need to generate in order to shine with its measured flux?

$$l_o = 4x10^{25}, \omega = 4x10^{25}\,Js^{-1}$$

The Sun has not changed flux in $10^9\,yr\left(3x10^6\,s\right)$

$\Rightarrow$ Sun has radiated $1.2x10^{10}\,J$

$$E = mc^2$$

$$\Rightarrow m = 10^{26}\,Kg = 10^{-4}\,m_o$$

The sources of this energy are: (i) Cooling or contraction (ii) Chemical (iii) Nuclear reaction

(i) **Cooling and Contraction**

These are closely related. So we consider them together. Suppose the irradiative energy of sun is due to the sun being much hotter when it was formed, and has since been cooling down. We can test how plausible this is. Or is the sun slowly contracting with consequent release of gravitational potential energy, which is converted to radiation?

For an ideal gas, the thermal energy of a particle (where $n_s =$ number of degrees of freedom which is three (3)) is.

Total thermal energy per unit volume $= \dfrac{3knT}{2}$

$N =$ Number of particles per unit volume

Now, according to the Virial theorem:

$$3\int_0^{V_o} PdV + \Omega = 0 \tag{3.40}$$

Assuming that stellar material is an ideal gas (with negligible $P_r$)

$$\Rightarrow P = nkT$$

$$3\int_0^{V_o} nkTdV + \Omega = 0 \tag{3.41}$$

Now let’s define U = integral over volume of the thermal energy per unit volume.

$$\text{Thermal energy per unit volume} = \frac{3knT}{2} \tag{3.42}$$

$$\Rightarrow 2u + \Omega = 0 \tag{3.43}$$

The negative gravitational energy of a star is equal to twice its thermal energy. This means that the time for which the present thermal energy of the sun can supply its radiation and the time for which the past release of gravitational potential energy could have supplied its present rate of radiation differ by only a factor of two. Negative gravitational potential energy of a star is given by:

$$\Omega > G\,\frac{M_s^2}{2r_s}$$

$$\Omega \approx G\,\frac{M_s^2}{2r_s} \tag{3.44}$$

Total release of gravitational potential energy would have been sufficient to provide radiant energy at a rate given by the luminosity of the stars, for a time given by :

$$t_s \approx \frac{GM_o^2}{L_o P_o} \qquad 3.45$$

(ii) **Chemical reaction**

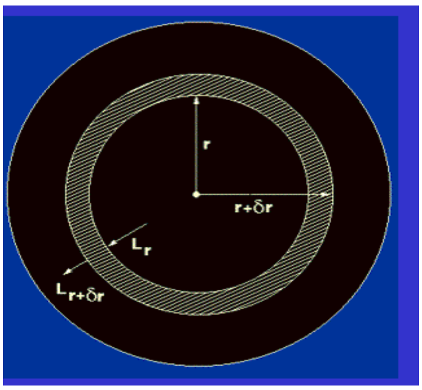
We can quickly rule these out as possible energy sources for the sun. We calculated above that we need to find a process that can produce at least $10^{-4}$ of the rest mass energy of the sun. Chemical reactions such as the combustion of fossil fuels release $\approx 5x10^{-10}$ of the rest mass energy of the fuel.

(iii) **Nuclear reaction**

The only known way of producing sufficiently large amounts of energy to fuel the stars is through nuclear reactions. There are two types of nuclear reaction, fission reaction and fusion reaction. Fission reaction, such as those that occur in nuclear reactors or atomic weapons can release $\approx 5x10^{-4}$ of rest mass energy through fission of heavy nuclei (Uranium or Plutonium).

## 3.1.10 EQUATION OF ENERGY PRODUCTION

The third equation of stellar structure establishes the relationship between energy release and the rate of energy transport.



Consider a spherically symmetric star as shown above in which time variations are unimportant

$L(r) =$ Rate of energy flow across the sphere of radius $r$

$L(r+\partial r) =$ Rate of energy flow across sphere of radius $r+\partial r$

Because the spherical shell is thin:

$$\partial V(r) = 4\pi r^2 \partial r \qquad 3.46$$

And $$\partial m(r) = 4\pi r^2 \rho(r) \partial r \qquad 3.47$$

We can define

$\varepsilon =$ Energy release per unit mass per unit volume ( $(WKg^{-1})$

Hence, energy release in the spherical shell is written as:

$$4\pi r^2 \rho(r) \partial r \varepsilon \qquad 3.48$$

Conservation of energy leads us to

$$L(r+\partial r) = L(r) + 4\pi r^2 \rho(r) \partial r \varepsilon$$
$$\Rightarrow \frac{L(r+\partial r)}{\partial r} - L(r) = 4\pi r^2 \rho(r) \partial r \varepsilon \qquad 3.49$$

And for $\partial r \to 0$

$$\frac{\partial L(r)}{\partial r} = 4\pi r^2 \rho(r) \varepsilon \qquad 3.50$$

This is the equation of energy production in the star. We now have three of the equations of stellar structure; however there are five unknown and these are: $P(r), M(r), L(r), \rho(r), \varepsilon(r)$. In order determine them; we need to consider energy transport in stars.

## 3.1.11 METHODS OF ENERGY TRANSPORT

There are three ways energy can be transported in stars:

(i) Convection: It is the energy transport by mass motions of stellar gas.

(ii) Conduction: Which is energy exchanged during collisions of stellar gas particles.

(iii) Radiation: Energy transport by the emission and absorption of photons.

Conduction and radiation are similar processes because they both involve transfer of energy by direction interaction, either between particles or between photons and particles.

Energy earned by a typical particle $E = \frac{3kT}{2}$ is comparable to energy carried by a typical photon. This is given as $E = \frac{hc}{\lambda}$

But the number density of particles is much greater than that of photons which would imply conduction is more important than radiation in stellar energy transport.

Convection is the mass motion of gas elements. It only occurs when temperature gradient exceeds some critical value. We can derive an expression for convection.

Consider a convective element at distance r from the centre of a star where the element is in equilibrium with its surroundings.

Now let's suppose it rises to $r + \partial r$. It expands, $P(r)$ and $\rho(r)$ are reduced to $P - \partial P$ and $\rho - \partial\rho$.

But these may not be the same as the new surrounding gas conditions. We define those as $P - \Delta P$ and $\rho - \Delta\rho$.

If the gas element is denser than the surroundings at $r + \partial r$ then it will sink (i.e. stable) if it is less dense then it will keep on rising – connectively unstable.

The condition for instability is that

$$P - \partial P < P - \Delta P \tag{3.51}$$

Whether or not this condition is satisfied depends on two things; the rate at which the element expands due to decreasing pressure, and the rate at which the density of the surroundings decreases with height.

Consider these two assumptions:

1. The element rises adiabatically

2. The element rises at a speed much less than the speed of sound

For adiabatic process

$$PV^{\gamma} = \text{Constant}$$

Where $\gamma = \frac{c_p}{c_v}$ is specific heat at constant pressure, divided by specific heat at constant volume?

Given that $V$ is inversely proportional to $P$, we can write

$$\frac{P}{\rho} = \text{Constant}$$

**REFERENCES / FURTHER READINGS**

1. Michael A. Zeilik, Stephen A. Gregory (1998). *Introductory Astronomy & Astrophysics* (4th ed.). Saunders College Publishing

**TUTOR MARKED ASSIGNMENT**

Q1. Derive the equation of hydrostatic support. How will you show that the assumption of hydrostatic equilibrium is valid?

Q2. Deduce the mass conservation equation. Hence, verify with equation(s) the statement that the negative gravitational energy of a star is twice its thermal energy.

Q3. Write down the FOUR differential equations which govern the structure of stars defining each term in the equations.

Q4. State without proof the thermal time scale equation and mention the significance of the equation in the evolution of stellar mass.

Q5. Discuss concisely how energy is generated in stars.

# UNIT 4 THE ORIGIN, EVOLUTION AND STRUCTURE OF THE SOLAR SYSTEM

## 4.0 INTRODUCTION

The formation and evolution of the Solar system is estimated to have begun in about 4.568 billion years ago with the gravitational collapse of a small part of a giant molecular cloud. Most of the collapsing mass collected in the centre, forming the Sun, while the rest flattened into a proto-planetary disk out of which the planets, moons, asteroids, and other small Solar System bodies formed.

## 4.1 EVOLUTION OF THE SOLAR SYSTEM

The Solar System has evolved considerably since its initial formation. Many moons have formed from circling discs of gas and dust around their parent planets, while other moons are believed to have formed independently and later been captured by their planets.

Still others, as the Earth's Moon, may be the result of giant collisions. Collisions between bodies have occurred continually up to the present day and have been central to the evolution of the Solar System.

The positions of the planets have often shifted, and planets have switched places. This planetary migration now is believed to have been responsible for much of the Solar System's early evolution.

In roughly 5 billion years, it has been estimated that the Sun will cool and expand outward to many times its current diameter (becoming a red giant), before casting off its outer layers as a planetary nebula, and leaving behind a stellar corpse known as a white dwarf.

In the far distant future, the gravity of passing stars gradually will whittle away at the Sun's retinue of planets. Some planets will be destroyed, others ejected into interstellar space. Ultimately, over the course of trillions of years, it is likely that the Sun will be left with none of the original bodies in orbit around it.

### 4.1.1 SUN'S INTERIOR

Stars are formed from clouds of gas and collapse under self-gravity. The collapse is stopped by internal pressure in the core of the star. During the collapse, the potential energy of in-falling hydrogen atoms is converted to kinetic energy, heating the core. As the temperature goes up, the pressure goes up to stop the collapse. The heat from the collapse is sufficient for the Sun to shine, but only for a timescale of 15 million years (called the Kelvin-Helmholtz time). Since the Sun is 5 billion years old, then it must be producing its own energy rather than shining on leftover energy from stellar formation (like Jupiter).

The structure of the Sun is determined by some relations or physical concepts. These include:

( 1.) Hydrostatic equilibrium - the fact that pressure balances the self-gravity

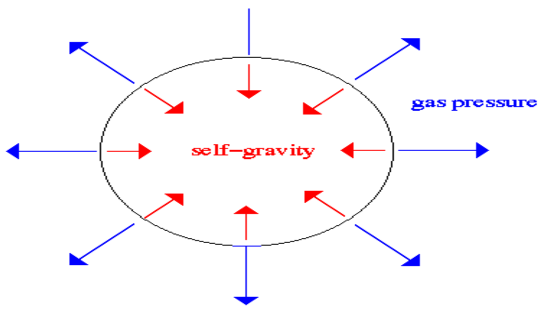



***Figure 4.1 The Sun In Hydrostatic Equilibrium***

(2.) Thermal equilibrium - the amount of energy generated equals the amount radiated away

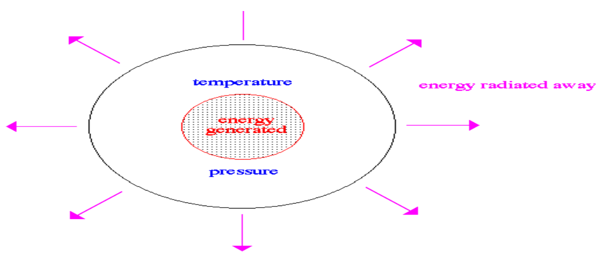



***Figure 4.2 Thermal Equilibrium in the Sun***

( 3.) opacity - the resistance of the solar envelope to the flow of photons i.e how fast the energy is released

We can observe the chemical composition of the earth’s crust, and we know from geophysical data that the crust is much less dense than the earth as a whole. The crust is thus not representative of the mantle and core.

To gain additional information, there is need to consider the formation of the earth and solar system. Here the geology/astronomy overlap area.

**SELF ASSESSMENT**

Q1. How did the solar system evolve?

Q2. Mention the three factors on which the structure of the Sun depends.

### 4.1.2 MODEL OF THE SOLAR INTERIOR: THE SUN

The composition of the Sun is found from spectroscopy studies. It consists of about 70% hydrogen, 28% helium and about 2% everything else. The sun represents 99.9% of the total mass of the solar system. From orbital calculations its average density is 1.4 gm/cm3.

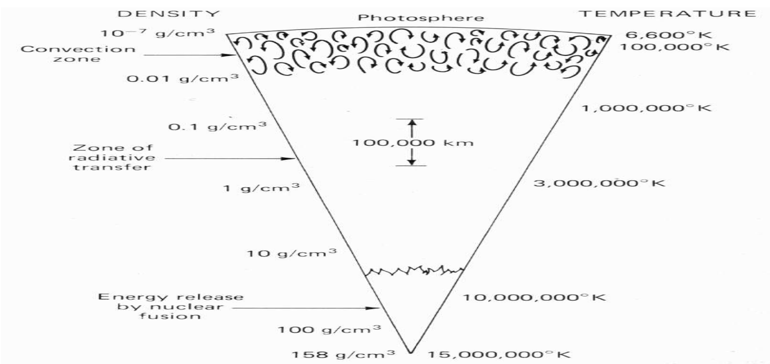



***Figure 4.2: The composition of the Sun Interior***

The sun undergoes nuclear reaction to produce energy which is received on the earth surface. The nuclear fusion reaction is called a "proton-proton" or "hydrogen-burning" reaction - multiple stages as shown in the figure 4.3 below.

(1) Two hydrogen nuclei (protons) combine to form a deuteron –a hydrogen isotope of atomic weight 2. To do this they emit a positron (positive electron) making one proton into a neutron.

(2) A second proton combines with deuterium to form helium-3.

(3) Two helium-3 nuclei fuse into helium-4 releasing energy 2 protons.

This reaction gives off lots of energy which is the energy we receive from the sun.

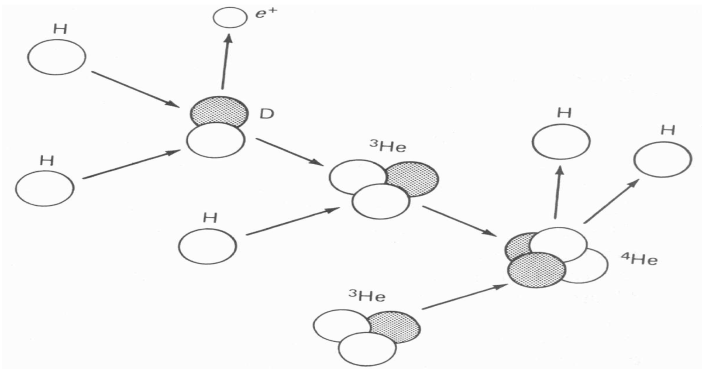



***Figure 4.3: Formation of the Sun due to nuclear reaction***

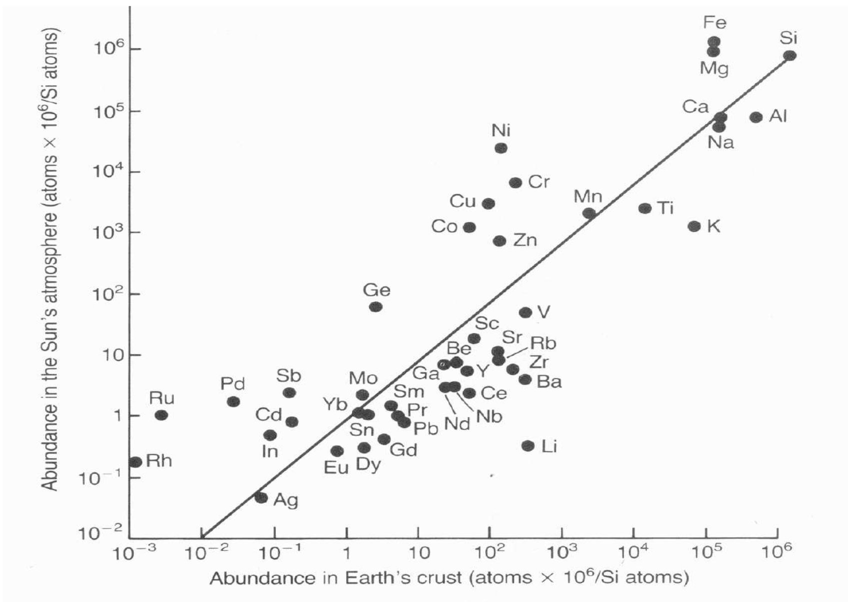



***Figure 4.4: The composition of the Sun***

**SELF ASSESSMENT**

Q1. Give an estimate of the various composition of the Sun.

Q2. Draw an annotated diagram of the internal structure of the Sun indicating the relevant composition.

### 4.1.3 THE ORIGIN, EVOLUTION AND STRUCTURE OF THE PLANETS

The widely accepted model known as the nebular hypothesis, was first developed in the 18th century by Emanuel Swedenborg, Immanuel Kant, and Pierre-Simon Laplace. Its subsequent development has interwoven a variety of scientific disciplines including astronomy, physics, geology, and planetary science.

Since the dawn of the space age in the 1950s and the discovery of extra solar planets in the 1990s, the models have been both challenged and refined to account for new observations. There are so many theories concerning the formation of the solar system. However two which are most prominent shall be mentioned:

**Aristarchus's concept**:
The first step toward a theory of Solar System formation and evolution was the general acceptance of heliocentrism, which placed the Sun at the centre of the system and the Earth in orbit around it. This conception had gestated for millennia, but was widely accepted only by the end of the 17th century.

**Nebular hypothesis**:
In this theory, the whole Solar System starts as a large cloud of gas that contracts under self-gravity. Conservation of angular momentum requires that a rotating disk form with a large concentration at the centre (the proto-Sun). Within the disk, planets form.

The current standard theory for Solar System formation, the nebular hypothesis, has fallen into and out of favour since its formulation by Emanuel Swedenborg, Immanuel Kant, and Pierre-Simon Laplace in the 18th century. The most significant criticism of the hypothesis was its apparent inability to explain the

Sun's relative lack of angular momentum when compared to the planets. However, since the early 1980s studies of young stars have shown them to be surrounded by cool discs of dust and gas, exactly as the nebular hypothesis predicts, which has led to its re-acceptance.

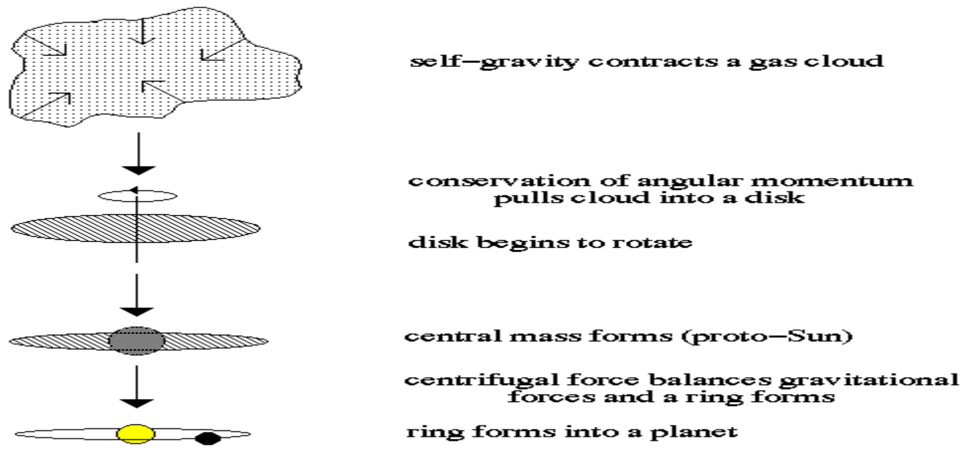



***Figure 4.5: Formation of Planets round the Sun***

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Understanding of how the Sun will continue to evolve required an understanding of the source of its power. Arthur Stanley Eddington's confirmation of Albert Einstein's theory of relativity led to his realisation that the Sun's energy comes from nuclear fusion reactions in its core.

In 1935, Eddington went further and suggested that other elements also might form within stars. Fred Hoyle elaborated on this premise by arguing that evolved stars called red giants created many elements heavier than hydrogen and helium in their cores. When a red giant finally casts off its outer layers, these elements would then be recycled to form other star systems.

### 4.1.4 PRE-SOLAR NEBULA

The nebular hypothesis maintains that the Solar System formed from the gravitational collapse of a fragment of a giant molecular cloud. The cloud itself had a size of about 20 pc while the fragments were roughly 1 pc (three and a quarter light-years) across.

The further collapse of the fragments led to the formation of dense cores 0.01–0.1 pc (2,000–20,000 AU) in size. One of these collapsing fragments (known as the *pre-solar nebula*) would form what became the Solar System. The composition of this region with a mass just over that of the Sun was about the same as that of the Sun today, with hydrogen, along with helium and trace amounts of lithium produced by Big Bang nucleosynthesis, forming about 98% of its mass. The remaining 2% of the mass consisted of heavier elements that were created by nucleosynthesis in earlier generations of stars. Late in the life of these stars, they ejected heavier elements into the interstellar medium.

Studies of ancient meteorites reveal traces of stable daughter nuclei of short-lived isotopes, such as iron-60, that only form in exploding, short-lived stars. This indicates that one or more supernovae occurred near the Sun while it was forming. A shock wave from a supernova may have triggered the formation of the Sun by creating regions of over-density within the cloud, causing these regions to collapse. Because only massive, short-lived stars produce supernovae, the Sun must have formed in a large star-forming region that produced massive stars, possibly similar to the Orion Nebula. Studies of the structure of the Kuiper belt and of anomalous materials within it suggest that the Sun formed within a cluster of stars with a diameter of between 6.5 and 19.5 light-years and a collective mass equivalent to 3,000 Suns. Several simulations of our young Sun interacting with close-passing stars over the first 100 million years of its life produce anomalous orbits observed in the outer Solar System, such as detached objects.

Because of the conservation of angular momentum, the nebula spun faster as it collapsed. As the material within the nebula condensed, the atoms within it began to collide with increasing frequency, converting their kinetic energy into heat. The centre, where most of the mass collected, became increasingly hotter than the surrounding disc. Over about 100,000 years, the competing forces of gravity, gas pressure, magnetic fields, and rotation caused the contracting nebula to flatten into a spinning protoplanetary disc with a diameter of approximately 200 AU and form a hot, dense protostar (a star in which hydrogen fusion has not yet begun) at the centre.

At this point in its evolution, the Sun is believed to have been a T Tauri star. Studies of T Tauri stars show that they are often accompanied by discs of pre-planetary matter with masses of 0.001–0.1 solar masses. These discs extend to several hundred AU. The Hubble Space Telescope has observed protoplanetary discs of up to 1000 AU in diameter in star-forming regions such as the Orion Nebula—and are rather cool, reaching only one thousand Kelvin at their hottest.

Within 50 million years, the temperature and pressure at the core of the Sun became so great that its hydrogen began to fuse, creating an internal source of energy that countered gravitational contraction until hydrostatic equilibrium was achieved. This marked the Sun's entry into the prime phase of its life, known as the main sequence. Main sequence stars derive energy from the fusion of hydrogen into helium in their cores. The Sun remains a main sequence star today.

**SELF ASSESSMENT**

Q1. With reference to the source of the Sun’s power, account for its evolution.

Q2. Explain using the shock wave concept of Supernova, how the Sun is formed.

### 4.1.5 FORMATION OF PLANETS

The various planets are thought to have formed from the *solar nebula*, the disc-shaped cloud of gas and dust left over from the Sun's formation. The currently accepted method by which the planets formed is known as accretion, in which the planets began as dust grains in orbit around the central protostar. Through direct contact, these grains formed into clumps up to 200 metres in diameter, which in turn collided to form larger bodies (planetesimals) of about 10 km in size. These gradually increased through further collisions, growing at the rate of centimetres per year over the course of the next few million years.

## TERRESTRIAL PLANETS

The inner Solar System, the region of the Solar System inside 4 AU, was too warm for volatile molecules like water and methane to condense, so the planetesimals that formed there could only form from compounds with high melting points, such as metals (like iron, nickel, and aluminium) and rocky silicates. These rocky bodies would become the terrestrial planets (Mercury, Venus, Earth, and Mars). These compounds are quite rare in the universe, comprising only 0.6% of the mass of the nebula, so the terrestrial planets could not grow very large.

When the terrestrial planets were forming, they remained immersed in a disk of gas and dust. The gas was partially supported by pressure and so did not orbit the Sun as rapidly as the planets. The resulting drag caused a transfer of angular momentum, and as a result the planets gradually migrated to new orbits. Models show that temperature variations in the disk governed this rate of migration, but the net trend was for the inner planets to migrate inward as the disk dissipated, leaving the planets in their current orbits.

## JOVIAN PLANETS

The gas giants (Jupiter, Saturn, Uranus, and Neptune) formed further out, beyond the frost line, the point between the orbits of Mars and Jupiter where the material is cool enough for volatile icy compounds to remain solid. The ices that formed the Jovian planets were more abundant than the metals and silicates that formed the terrestrial planets, allowing the Jovian planets to grow massive enough to capture hydrogen and helium, the lightest and most abundant elements. Planetesimals beyond the frost line accumulated up to four Earth masses within about 3 million years.

Today, the four gas giants comprise just fewer than 99% of all the mass orbiting the Sun. Theorists believe it is no accident that Jupiter lays just beyond the frost line. Because the frost line accumulated large amounts of water via evaporation from in-falling icy material, it created a region of lower pressure that increased the speed of orbiting dust particles and halted their motion toward the Sun. In effect, the frost line acted as a barrier that caused material to accumulate rapidly at about 5 AU from the Sun. This excess material coalesced into a large embryo of about 10 Earth masses, which then began to grow rapidly by swallowing hydrogen from the surrounding disc, reaching 150 Earth masses in only another 1000 years and finally topping out at 318 Earth masses. Saturn may owe its substantially lower mass simply to having formed a few million years after Jupiter, when there was less gas available to consume.

T Tauri stars like the young Sun have far stronger stellar winds than more stable, older stars. Uranus and Neptune are believed to have formed after Jupiter and Saturn did, when the strong solar wind had blown away much of the disc material. As a result, the planets accumulated little hydrogen and helium—not more than 1 Earth mass each. Uranus and Neptune are sometimes referred to as failed cores.

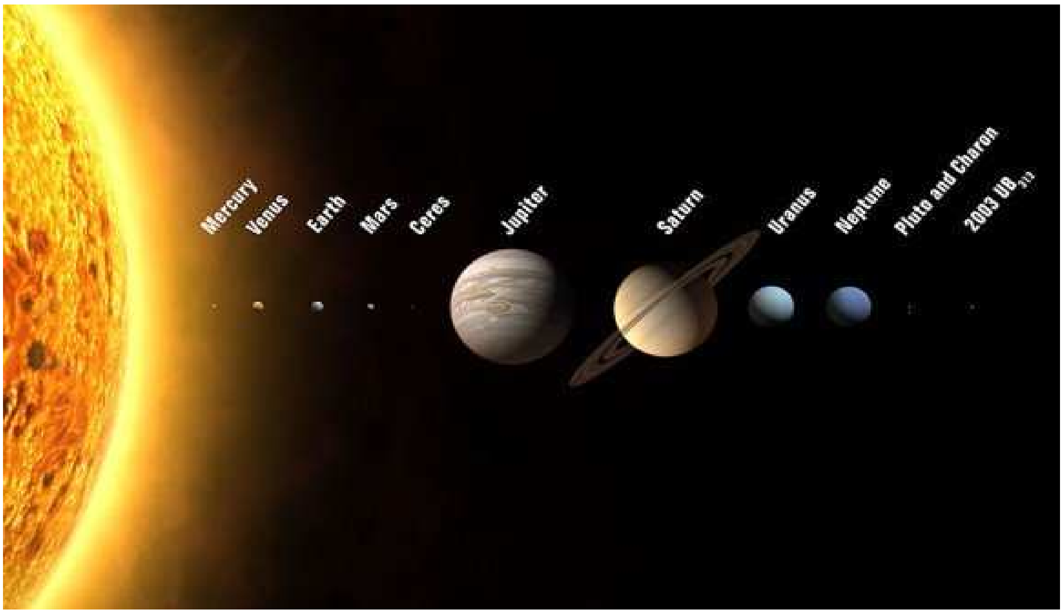


***Figure 4.6: Diagram of the Solar System (image credit: NASA)***

**Sun**

The Sun The central star in the Solar System

**Planets**

| | |
|---|---|
| Mercury | The first planet in the Solar System which is also the smallest planet in the Solar System. Mercury takes just 88 days to complete an orbit around the Sun. |
| Venus | The second planet from the Sun. In many ways, Venus is a twin to our own Earth. It has nearly the same size and mass as Earth, but the thick atmosphere on Venus makes surface temperatures hot enough to melt lead. Venus is also unusual because it rotates in reverse to all the other planets. |
| Earth | Our home planet, the third planet from the Sun. Earth is the only planet in the Solar System known to support life. This is because we are at just the right distance from the Sun so that our planet doesn’t get too hot or too cold. We also have one moon – the Moon. |
| Mars | Mars is the fourth planet from the Sun and is much smaller and colder than the Earth. Temperatures on Mars can rise to 20-degrees C, but dip down to -140-degrees C in the northern winters. Mars is thought to be the best candidate for life elsewhere in the Solar System. Mars has two small, asteroid-shaped moons: Phobos and Deimos. |
| Ceres | Ceres is the first dwarf planet in the Solar System, and the largest member of the asteroid belt. |
| Jupiter | Jupiter is the 5th planet from the Sun, and the largest planet in the Solar System. Jupiter has as much mass as 2.5 times all the rest of the planets combined – almost all of this mass is hydrogen and helium; although, scientists think it has a solid core. Jupiter has at least 63 moons. |
| Saturn | Saturn is the 6th planet from the Sun, and is well known for its beautiful system of icy rings. Saturn is almost as large as Jupiter, but it has a fraction of Jupiter’s mass, so it has a very low density. |

| | |
|---|---|
| | Saturn would float if you could find a tub of water large enough. Saturn has 60 moons at last count. |
| Uranus | Uranus is the 7th planet from the Sun, and the first planet discovered in modern times; although, it's just possible to see with the unaided eye. Uranus has a total of 27 named moons. |
| Neptune | Neptune is the 8th and final planet in the Solar System. Neptune was only discovered in 1846. It has a total of 13 known moons. |
| Pluto | Pluto isn't a planet any more. Now it's just a dwarf planet. Pluto has one large moon, called Charon, and then two smaller moons. |
| Eris | The next dwarf planet in the Solar System is Eris, which was only discovered back in 2003. In fact, it was because of Eris that astronomers decided to reclassify Pluto as a dwarf planet. |

**Moons**

Moons have come to exist around most planets and many other Solar System bodies. These natural satellites originated by one of three possible mechanisms:

(i) co-formation from a circum-planetary disc (only in the cases of the gas giants)

(ii) formation from impact debris (given a large enough impact at a shallow angle)

(iii) Capture of a passing object.

Jupiter and Saturn have a number of large moons, such as Io, Europa, Ganymede and Titan, which may have originated from discs around each giant planet in much the same way that the planets formed from the disc around the Sun. This origin is indicated by the large sizes of the moons and their proximity to the planet. These attributes are impossible to achieve via capture, while the gaseous nature of the primaries make formation from collision debris impossibility. The outer moons of the gas giants tend to be small and have eccentric orbits with arbitrary inclinations. These are the characteristics expected of captured bodies.

## Moon ringing system

The evolution of moon systems is driven by tidal forces. A moon will raise a tidal bulge in the object it orbits (the primary) due to the differential gravitational force across diameter of the primary. If a moon is revolving in the same direction as the planet's rotation and the planet is rotating faster than the orbital period of the moon, the bulge will constantly be pulled ahead of the moon. In this situation, angular momentum is transferred from the rotation of the primary to the revolution of the satellite. The moon gains energy and gradually spirals outward, while the primary rotates more slowly over time.

The Earth and its Moon are one example of this configuration. Today, the Moon is tidally locked to the Earth; one of its revolutions around the Earth (currently about 29 days) is equal to one of its rotations about its axis, so it always shows one face to the Earth. The Moon will continue to recede from Earth, and Earth's spin will continue to slow gradually. In about 50 billion years, if they survive the Sun's expansion, the Earth and Moon will become tidally locked to each other each will be caught up in what is called a "spin–orbit resonance" in which the Moon will circle the Earth in about 47 days and both Moon and Earth will rotate around their axes in the same time, each only visible from one hemisphere of the other.

## TUTOR MARKED ASSIGNMENTS

Q1. Mention two theories which explain the evolution of the solar system

Q2. How is the Solar system formed at about four billion years ago?

Q3. Give two differences between Terrestrial and Jovian planets

Q4. With diagram, describe briefly how nuclear reaction occurs in the Sun.

Q5. Advance reasons for the structure of the Sun.

Q6. Predict with reasons, what is likely to be the fate of the solar system in some billion of year’s from now.

## FURTHER READINGS

1. C.D. Murray & S.F. Dermott (1999). *Solar System Dynamics*. Cambridge University Press. ISBN 0-521-57295-9

2. Michael A. Zeilik, Stephen A. Gregory (1998). *Introductory Astronomy & Astrophysics* (4th edition.). Saunders College Publishing

**MODULE 3**

## UNIT 5 THE ORIGIN, EVOLUTION AND STRUCTURE OF THE GALAXIES

### 5.0 INTRODUCTION

With the discovery of the nature of galaxies, the first hypothesis developed to explain their existence was one of gravitational collapse in the primordial gas. As the forming galaxies grew smaller, the gas tended to fall into a flat plane, with fragmentation into stars occurring during both the collapse phase and continuing after formation of the final disk.

### 5.1 ORIGIN AND EVOLUTION OF GALAXIES

The formation of a galaxy was completed when the mass distribution came into equilibrium between motions and gravity. The differentiation between the types of galaxies was thought to have been the result of initial conditions. If lots of angular momentum were present, a disk galaxy would be produced. If initially there is little angular momentum, all matter would become stars during the collapse phase, resulting in an elliptical galaxy.

Observational and theoretical work in more recent times has shown that galaxy formation is a much more complicated process. First, the efficiency of star formation is low. As a result, elliptical galaxies cannot be produced as was once thought; galaxy formation produces disk galaxies with significant interstellar material left over. Second, interactions between galaxies over the history of the universe can be significant. Galaxies do merge, and they cannibalize smaller companions. Violent interactions between disk galaxies appear to randomize motions and also to efficiently convert colliding interstellar gas into stars, leaving behind gas-free elliptical galaxies.

Galaxies that may have grown in size, but avoided major disruptive encounters, appear to have evolved into the spectrum of spiral galaxies that exist today. Also, gentle encounters between two gassy disk galaxies are possible, and these encounters leave their fundamental stellar distributions unchanged but result in the gas being swept out, thus producing the relatively rare, flat, gas-free galaxies.

It is now hypothesized that the early era of galaxies was much more turbulent than today's universe. The process of producing equilibrium galaxies was associated with the growth of massive, non-stellar black holes in the nuclei. The liberation of tremendous energies during their formative stages is observed as quasars, but quasars died when galaxies achieved their equilibrium structures and ended mass in-fall into the centres. When new mass falls into the centres of galaxies, the central black hole phenomena can be re-ignited, explaining the active galactic nuclei of the present day.

**SELF ASSESSMENT**

Q1. Define a Galaxy.

Q2. How do they produce the largest amount of energy know in the history of Astronomy?

## 5.2 ACTIVE GALACTIC NUCLEI

Our vision of the cosmic world and in particular of the whole Universe has been changing dramatically in the last century. As we will see much later, galaxies were repeatedly the main protagonist in the scene of these changes. It is about 80 years since E. Hubble established the nature of galaxies as gigantic self-bound stellar systems and used their kinematics to show that the Universe as a whole is expanding uniformly at the present time.

Galaxies are the building blocks of the Universe. By looking inside galaxies we find that there are the arena where stars form, evolve and collapse in constant interaction with the interstellar medium (ISM), a complex mixture of gas and plasma, dust, radiation, cosmic rays, and magnetic fields. The centre of a significant fraction of galaxies harbors super massive black holes. When they are fed with infalling material, the accretion disks around them release, mainly through powerful plasma jets, the largest amounts of energy known in astronomical objects. This phenomenon of Active Galactic Nuclei (AGN) was much more frequent in the past than in the present, being the high-red shift quasars (QSO's) the most powerful incarnation of the AGN phenomenon. But the most astonishing surprise of galaxies comes from the fact that luminous matter (stars, gas, AGN's, etc.) is only a tiny fraction about 1 - 5% of all the mass measured in galaxies and the giant halos around them.

Thus, exploring and understanding galaxies is of paramount interest to cosmology, high-energy and particle physics, gravitation theories, and, of course, astronomy and astrophysics. As astronomical objects, among other questions, we would like to know how they take shape and evolve, what is the origin of their diversity and scaling laws, why they cluster in space as observed, following a sponge-like structure, what is the dark component that predominates in their masses. By answering to these questions we would able also to use galaxies as a true link between the observed universe and the properties of the early universe, and as physical laboratories for testing fundamental theories.

**SELF ASSESSMENT**

Q1. What is the dark component of galaxies made of?

## 5.3 GALAXY PROPERTIES AND CORRELATIONS

During several decades galaxies were considered basically as self-gravitating stellar systems so that the study of their physics was a domain of Galactic Dynamics. Galaxies are mainly conglomerates of hundreds of millions to trillions of stars *supported against gravity either by rotation or by random motions*. In the former case, the system has the shape of a *flattened disk,* where most of the material is on circular orbits at radii that are the minimal ones allowed by the specific angular momentum of the material. Besides, disks are dynamically fragile systems, unstable to perturbations. Thus, the mass distribution along the disks is the result of the specific angular momentum distribution of the material from which the disks form, and of the posterior dynamical (internal and external) processes. In the latter case, however, the shape of the galactic system is a concentrated *spheroid/ellipsoid*, with mostly (disordered) radial orbits. The spheroid is dynamically hot, and stable to perturbations**.**

**SELF ASSESSMENT**

Q1. Are the properties of the stellar populations in the disk and spheroid systems different?

## 5.3.1 STELLAR POPULATIONS

In the 1940's, W. Baade discovered that according to the ages, metallicities, kinematics and spatial distribution of the stars in our Galaxy is separable into two groups namely:

(i) Population I stars, which populate the plane of the disk; their ages do not go beyond 10 Giga years and a fraction of them in fact are young; being less than $10^6$ yr.

(ii) Population II stars, which are located in the spheroidal component of the Galaxy (stellar halo and partially in the bulge), where velocity dispersion (random motion) is higher than rotation velocity (ordered motion); they are old stars greater than 10 Giga years with very low metallicities, on the average lower by two orders of magnitude than Population I stars. In between Population's I and II there are several stellar subsystems.

Stellar populations are true fossils of the galaxy assembling process. The differences between them are evidenced in the formation and evolution of the galaxy components. The Population II stars, being old, of low metallicity, and dominated by random motions (dynamically hot), had to form early in the assembling history of galaxies and through rather violent processes. There is a large range of ages of Population I stars, but on average younger than the Population II stars. This indicates a slow star formation process that continues even today in the disk plane.

The common astronomical wisdom says that *spheroids form early in a violent collapse (monolithic or major merger), while disks assemble by continuous in fall of cosmic gas rich in angular momentum.*

### 5.3.2 INTERSTELLAR MEDIUM (ISM)

Galaxies are not only conglomerates of stars. The study of galaxies is incomplete if it does not take into account the ISM, which for late-type galaxies accounts for more mass than that of stars. Besides, it is expected that in the deep past, galaxies were gas-dominated and with the passing of time the cold gas was being transformed into stars.

The complex structure of the ISM is related to:

(i) Its peculiar thermo-dynamical properties (in particular the heating and cooling processes)
(ii) its hydro-dynamical and magnetic properties which imply development of turbulence
(iii) the different energy input sources.

The star formation unities (molecular clouds) appear to form during large-scale compression of the diffuse ISM driven by supernovae (SN), magneto-rotational instability, or disk gravitational instability. At the same time, the energy input by stars influences the hydro-dynamical conditions of the ISM: the star formation results self-regulated by a delicate energy (turbulent) balance.

## 5.4 CLASSIFICATION OF GALAXIES

Galaxies are true "ecosystems" where stars form, evolve and collapse in constant interaction with the complex ISM. Following a pedagogical analogy with biological sciences, we may say that the study of galaxies proceeded through taxonomical and anatomical approaches.

### 5.4.1 TAXONOMY

As it happens in any science, as soon as galaxies were discovered, the next step was to attempt to classify these news objects. This endeavour was taken on by E. Hubble. The showiest characteristics of galaxies are the bright shapes produced by their stars, in particular those most luminous. Hubble noticed that by their external look (morphology), galaxies can be divided into three principal types namely:

(i) Ellipticals (E, from round to flattened elliptical shapes)

(ii) Spirals (S, characterized by spiral arms emanating from their central regions where a spheroidal structure called bulge is present)

(iii) Irregulars (Irr, clumpy without any defined shape)

In fact, the last two classes of galaxies are disk-dominated, rotating structures. Spirals are subdivided into Sa, Sb, Sc types according to the size of the bulge in relation to the disk, the openness of the winding of the spiral arms, and the degree of resolution of the arms into stars (in between the arms there are also stars but less luminous than in the arms). Roughly 40% of S galaxies present an extended rectangular structure (called bar) further from the bulge; these are the barred Spirals (SB), where the bar is evidence of disk gravitational instability.

### 5.4.2 ANATOMY

The morphological classification of galaxies is based on their external aspect and it implies somewhat subjective criteria. Besides, the "showy" features that characterize this classification may change with the colour band: in blue bands, which trace young luminous stellar populations, the arms, bar and other features may look different to what it is seen in infrared bands, which trace less massive, older stellar populations. It is interesting to explore deeper into the internal physical properties of galaxies and see whether these properties correlate along the Hubble sequence. Fortunately, this seems to be the case in general so that, in spite of the complexity of galaxies, some clear and sequential trends in their properties encourage us to think about regularity and the possibility to find driving parameters and factors beyond this complexity.

## 5.5 COSMIC STRUCTURE FORMATION

In the previous section we have learn that galaxy formation and evolution are definitively related to cosmological conditions. Cosmology provides the theoretical framework for the initial and boundary conditions of the cosmic structure formation models. At the same time, the confrontation of model predictions with astronomical observations became the most powerful test bed for cosmology. As a result of this fruitful convergence between cosmology and astronomy, there emerged the current paradigmatic scenario of cosmic structure formation and evolution of the Universe called A Cold Dark Matter (CDM). The CDM scenario integrates namely:

(i) cosmological theories (Big Bang and Inflation)
(ii) physical models (standard and extensions of the particle physics models),
(iii) Astrophysical models (gravitational cosmic structure growth, hierarchical clustering, astrophysics)
(iv) Phenomenology.

### 5.5.1 GRAVITATIONAL EVOLUTION OF FLUCTUATIONS

The CDM scenario assumes the gravitational instability paradigm: the cosmic structures in the Universe were formed as a consequence of the growth of primordial tiny fluctuations (for example seeded in the inflationary epochs) by gravitational instability in an expanding frame. The fluctuation or perturbation is characterized by its density contrast,

$$\delta \equiv \frac{\delta\rho}{\rho} = \frac{\rho - \bar{\rho}}{\bar{\rho}} \qquad (5.1)$$

Where $\bar{\rho}$ is the average density of the Universe, and $\rho$ is the perturbation density. At early epochs, $\delta$ << 1 for perturbation of all scales, otherwise the homogeneity condition in the Big Bang theory is not obeyed. When $\delta$ << 1, the perturbation is in the *linear* regime and its physical size grows with the expansion proportional to $a(t)$. The perturbation analysis in the linear approximation shows whether a given perturbation is stable ($\delta$ ~ const or even $\rightarrow$ 0) or unstable ($\delta$ grows). In the latter case, when $\delta$ $\rightarrow$ 1, the linear approximation is not anymore valid, and the perturbation "separates" from the expansion, collapses, and becomes a self-gravitating structure. The gravitational evolution in the *non-linear regime* is complex for realistic cases and is studied with numerical N-body simulations.

### 5.5.2 DARK MATTER

In astronomy and cosmology, *dark matter* is matter that neither emits nor scatters light or other electromagnetic radiation, and so cannot be directly detected via optical or radio astronomy. Dark matter is believed to constitute 83% of the matter in the universe and 23% of the mass-energy.

Dark matter was postulated by Fritz Zwicky in 1934 to account for evidence of "missing mass" in the orbital velocities of galaxies in clusters. Subsequently, other observations have indicated the presence of dark matter in the universe; these observations include the rotational speeds of galaxies, gravitational lensing of background objects by galaxy clusters such as the Bullet Cluster, and the temperature distribution of hot gas in galaxies and clusters of galaxies. Though the existence of dark matter is generally accepted by the mainstream scientific community, some alternative theories to explain the anomalies that dark matter is intended to solve have been proposed

Dark matter's existence is inferred from gravitational effects on visible matter and gravitational lensing of background radiation, and was originally hypothesized to account for discrepancies between calculations of the mass of galaxies, clusters of galaxies and the entire universe made through dynamical and general relativistic means, and calculations based on the mass of the visible "luminous" matter these objects contain: stars and the gas and dust of the interstellar and intergalactic medium. The most widely accepted explanation for these phenomena is that dark matter exists and that it is most likely composed of heavy particles that interact only through the weak force and gravity; however, alternate explanations have been proposed, and there is not yet sufficient experimental evidence to determine which is correct. Many experiments to detect proposed dark matter particles through non-gravitational means are underway.

According to observations of structures larger than solar systems, as well as galaxies, Big Bang dark matter accounts for 23% of the mass-energy density of the observable universe. In comparison, ordinary matter accounts for only 4.6% of the mass-energy density of the observable universe, with the remainder being attributable to dark energy. From these figures, dark matter constitutes 83%, (23/(23+4.6)), of the matter in the universe, whereas ordinary matter makes up only 17%.

Dark matter plays a central role in state-of-the-art modelling of structure formation and galaxy evolution, and has measurable effects on the anisotropies observed in the cosmic microwave background. All these lines of evidence suggest that galaxies, clusters of galaxies, and the universe as a whole contain far more matter than that which interacts with electromagnetic radiation. The largest part of dark matter, which by definition does not interact with electromagnetic radiation, is not only "dark" but also by definition, utterly transparent.

As important as dark matter is believed to be in the cosmos, direct evidence of its existence and concrete understanding of its nature have both remained elusive. Though the theory of dark matter remains the most widely accepted theory to explain the anomalies in observed galactic rotation, some alternative theoretical approaches have been developed which broadly fall into the categories of modified gravitational laws, and quantum gravitational laws.

### 5.5.3 BARYONIC AND NON BARYONIC DARK MATTER

A small proportion of dark matter may be baryonic dark matter: astronomical bodies, such as massive compact halo objects, that are composed of ordinary matter but which emit little or no electromagnetic radiation. The vast majority of dark matter in the universe is believed to be nonbaryonic, and thus not formed out of atoms. It is also believed that it does not interact with ordinary matter via electromagnetic forces; in particular, dark matter particles do not carry any electric charge.

The nonbaryonic dark matter includes neutrinos, and possibly hypothetical entities such as axions, or supersymmetric particles. Unlike baryonic dark matter, nonbaryonic dark matter does not contribute to the formation of the elements in the early universe ("Big Bang nucleosynthesis") and so its presence is revealed only via its gravitational attraction. In addition, if the particles of which it is composed are supersymmetric, they can undergo annihilation interactions with themselves resulting in observable by-products such as photons and neutrinos ("indirect detection").

Nonbaryonic dark matter is classified in terms of the mass of the particle(s) that is assumed to make it up, and/or the typical velocity dispersion of those particles (since more massive particles move more slowly). There are three prominent hypotheses on nonbaryonic dark matter, called Hot Dark Matter (HDM), Warm Dark Matter (WDM), and Cold Dark Matter (CDM); some combination of these is also possible. The most widely discussed models for nonbaryonic dark matter are based on the Cold Dark Matter hypothesis, and the corresponding particle is most commonly assumed to be a neutralino. Hot dark matter might consist of (massive) neutrinos. Cold dark matter would lead to a "bottom-up" formation of structure in the universe while hot dark matter would result in a "top-down" formation scenario.

One possibility is that cold dark matter could consist of primordial black holes in the range of $10^{14}$ kg to $10^{23}$ kg. Being within the range of an asteroid's mass, they would be small enough to pass through objects like stars, with minimal impact on the star itself. These black holes may have formed shortly after the big bang when the energy density was great enough to form black holes directly from density variations, instead of from star collapse. In vast numbers they could account for the missing mass necessary to explain star motions in galaxies and gravitational lensing effects.

### 5.5.4 EVIDENCE OF DARK MATTER

The first person to provide evidence and infer the presence of dark matter was Swiss astrophysicist Fritz Zwicky in 1933. He applied the virial theorem to the Coma cluster of galaxies and obtained evidence of unseen mass. Zwicky estimated the cluster's total mass based on the motions of galaxies near its edge and compared that estimate to one based on the number of galaxies and total brightness of the cluster. He found that there was about 400 times more estimated mass than was visually observable. The gravity of the visible galaxies in the cluster would be far too small for such fast orbits, so something extra was required. This is known as the "missing mass problem". Based on these conclusions, Zwicky inferred that there must be some non-visible form of matter which would provide enough of the mass and gravity to hold the cluster together.

Much of the evidence for dark matter comes from the study of the motions of galaxies and many of these appear to be fairly uniform, so by the virial theorem the total kinetic energy should be half the total gravitational binding energy of the galaxies. Experimentally, however, the total kinetic energy is found to be much greater: in particular, assuming the gravitational mass is due to only the visible matter of the galaxy; stars far from the centre of galaxies have much higher velocities than predicted by the virial theorem.

Galactic rotation curves, which illustrate the velocity of rotation versus the distance from the galactic centre, cannot be explained by only the visible matter. Assuming that the visible material makes up only a small part of the cluster is the most straightforward way of accounting for this. Galaxies show signs of being composed largely of a roughly spherically symmetric, centrally concentrated halo of dark matter with the visible matter concentrated in a disc at the centre. Low surface brightness dwarf galaxies are important sources of information for studying dark matter, as they have an uncommonly low ratio of visible matter to dark matter, and have few bright stars at the centre which would otherwise impair observations of the rotation curve of outlying stars.

Gravitational lensing observations of galaxy clusters allow direct estimates of the gravitational mass based on its effect on light from background galaxies, since large collections of matter (dark or otherwise) will gravitationally deflect light. In clusters such as Abell 1689, lensing observations confirm the presence of considerably more mass than is indicated by the clusters' light alone. In the Bullet Cluster, lensing observations show that much of the lensing mass is separated from the X-ray-emitting baryonic mass.

**SELF ASSESSMENT**

Q1. The universe is believed to be full of “dark matter”. Advance with four reasons the above statement.

Q2. Give two differences between Hot dark matter and Cold matter.

**FURTHER READINGS**

Advanced Astrophysics Cambridge Planetary Science by Neb. Duric

**TUTOR MARKED ASSIGNMENT**

Q1. Mention the compositions of the interior of Galaxies.

Q2. Distinguish between Population I stars and Population II stars

Q3. Advance reason(s) for the interstellar medium in the evolution of Galaxies

Q4. Mention and explain briefly the two classes of Galaxies.

# UNIT 6 STELLAR ATMOSPHERE

## 6.0 INTRODUCTION

The atmosphere of a star is defined as those layers sufficiently close to the surface such that some photons can escape from the star's surface. In other words; the atmosphere is as deep as one can see into the star.

## 6.1 CLASSIFICATION OF STELLAR ATMOSPHERE

The primary layers in a stellar atmosphere are:

(i) Photosphere: Here, energy transfer is dominated by radiation processes. The Sun's photosphere is 400 km thick, and has temperature greater than 5800K.

(ii) Chromospheres: Non-irradiative energy dissipation heats up the gas to above the irradiative equilibrium temperature; however, the density is sufficiently high that most of the dissipated energy can be radiated away and the resultant temperature is not too high. The Sun's chromosphere is $10^4$ km thick and has temperature equal to $1.5 \times 10^4$ K.

(iii) Corona: Significant non-irradiative energy transport and dissipation; the density is too low for the dissipated energy to be efficiently radiated away, so the corona is very hot.

(iv) Stellar wind: The outermost layer of a stellar atmosphere is so tenuous that it is not longer gravitationally bound; most stars, including our Sun, lose mass through a stellar wind.

## 6.2 MODEL OF THE STELLAR ATMOSPHERE

The theory of stellar atmospheres involves constructing models of the photosphere that take into account the effective temperature, gravity and elemental abundances.

In modelling the following assumptions are used for stellar atmosphere:

(i) Atmosphere is spherically symmetric

(ii) Elements mixture is homogeneous with depth

(iii) Atmosphere is in hydrostatic equilibrium

(iv) Atmosphere is time – independent

(v) The mass of the atmosphere is small compared with the total stellar mass

(vi) There are no sources or sink of energy

(vii) Energy transport takes place by radiation and convection (no heat conduction)

### 6.2.1 CONSTRUCTION OF A MODEL STELLAR ATMOSPHERE

To construct a more realistic model atmosphere, it is necessary to include the assumption of hydrostatic equilibrium given as:

$$\frac{dP}{dZ} = -\rho g \qquad 6.1$$

Which is written as: $$\frac{dP}{d\tau_v} = \frac{dP_g}{d\tau_s} + \frac{dP_r}{d\tau_s} = \frac{\rho g}{k v_s} \qquad 6.2$$

Where $v_s$ is a fixed standard frequency and $\tau_v = \tau_s$ is the optical depth at that frequency (i.e. evaluated for $\mu = 1$) and $P_g$ and $P_r$ are the gas and radiation pressures respectively.

If $P_r \ll P_g$ (valid provided the star is not too hot, or too opaque, and has a high surface gravity) then

$$\frac{dP_g}{d\tau_s} \cong \frac{\rho g}{k v_s} \qquad 6.3$$

From the equation of state of an ideal gas $P_g = NKT$

Where N is the number density of gas particles, $\mu$ is the mean molecular weight and $m_B$ is the atomic mass of hydrogen.

From the above, we can equate the above equations as:

$$P_g = NKT = \frac{\rho_g kT}{\mu m_B} \qquad 6.4$$

Substituting this in above, we have

$$\frac{dP}{d\tau_v} = \frac{\mu m_B \, g P_g}{k\tau k_v} \qquad 6.5$$

$$\left(\frac{lnP_g}{d\tau_v} = \frac{\mu m_{Bg}}{k\tau k v_s}\right) \qquad 6.6$$

This can now be used to prescribed variations in $P_g(v_s)$ for a given distribution of temperature

**SELF ASSESSMENT**

Q1. Define the atmosphere of stars. Mention the four classes of layers in the atmosphere of Stars?

## 6.3 STANDARD SOLAR MODEL

The Standard Solar Model (SSM) refers to a mathematical treatment of the Sun as a spherical ball of gas (in varying states of ionisation, with the hydrogen in the deep interior being completely ionised plasma). This model, which is technically spherical symmetric quasi-static model of a star, has stellar structure described by several differential equations derived from basic physical principles. The model is constrained by boundary conditions, namely the luminosity, radius, age and composition of the Sun, which are well determined.

The composition in the photosphere of the modern-day Sun, by mass, is 74.9% hydrogen and 23.8% helium. All heavier elements, called *metals* in astronomy, account for less than 2 percent of the mass. The SSM is used to test the validity of stellar evolution theory. In fact, the only way to determine the two free parameters of the stellar evolution model, the helium abundance and the mixing length parameter (used to model convection in the Sun), are to adjust the SSM to "fit" the observed Sun.

The SSM serves two purposes:

(i) it provides estimates for the helium abundance and mixing length parameter by forcing the stellar model to have the correct luminosity and radius at the Sun's age,

(ii) it provides a way to evaluate more complex models with additional physics, such as rotation, magnetic fields and diffusion or improvements to the treatment of convection, such as modelling turbulence, and convective overshooting.

Like the Standard Model of particle physics and the standard cosmology model, the SSM changes over time in response to relevant new theoretical or experimental physics discoveries.

As mentioned above, the Sun has a irradiative core and a convective outer envelope. In the core, the luminosity due to nuclear reactions is transmitted to outer layers principally by radiation. However, in the outer layers the temperature gradient is so great that radiation cannot transport enough energy. As a result, thermal convection occurs as thermal columns carry hot material to the surface (photosphere) of the Sun. Once the material cools off at the surface, it plunges back downward to the base of the convection zone, to receive more heat from the top of the irradiative zone**.**

### 6.3.1 CONSTRUCTION OF A SOLAR MODEL

In constructing a solar model, as described in stellar structure, one needs to consider the following parameters: the density $\rho(r)$, temperature T(r), total pressure (matter plus radiation) P(r), luminosity l(r) and energy generation rate per unit mass ε(r) in a spherical shell of a thickness *dr* at a distance r from the centre of the star. The irradiative transport of energy is described by the irradiative temperature gradient equation:

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3\kappa\rho l}{64\pi r^2\sigma T^3}, \qquad (6.7)$$

Where κ is the opacity of the matter, σ is the Stefan-Boltzmann constant, and the Boltzmann constant is set to one.

Convection is described using mixing length theory and the corresponding temperature gradient equation (for adiabatic convection) is:

$$\frac{\mathrm{d}T}{\mathrm{d}r} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{\mathrm{d}P}{\mathrm{d}r}, \qquad (6.8)$$

Where $\gamma = c_p / c_v$ is the adiabatic index, the ratio of specific heats in the gas. (For a fully ionized ideal gas, $\gamma = 5/3$.)

Near the base of the Sun's convection zone, the convection is adiabatic, but near he surface of the Sun, convection is non – adiabatic.

**SELF ASSESSMENT**

Q1. Define the Standard Solar model. Can you mention any similarity between this model and the stellar atmospheric model?

## 6.4 SOLAR RADIATION

Solar radiation describes the visible and near-visible (ultraviolet and near-infrared) radiation emitted from the sun. The different regions are described by their wavelength range within the broad band range of 0.20 to 4.0 µm (microns). Terrestrial radiation is a term used to describe infrared radiation emitted from the atmosphere.

The following is a list of the components of solar and terrestrial radiation and their approximate wavelength ranges:

| | |
|---|---|
| Ultraviolet | 0.20-0.39 µm |
| Visible | 0.39-0.78 µm |
| Near-Infrared | 0.78-4.00 µm |
| Infrared | 4.00-100.00 µm |

Approximately 99% of solar, or short-wave, radiation at the earth's surface is contained in the region from 0.3 to 3.0 µm while most of terrestrial, or long-wave, radiation is contained in the region from 3.5 to 50 um.

Outside the earth's atmosphere, solar radiation has an intensity of approximately 1370 $W/m^2$. This is the value at mean earth-sun distance at the top of the atmosphere and is referred to as the Solar Constant. On the surface of the earth on a clear day, at noon, the direct beam radiation will be approximately 1000$W/m^2$ for many locations.

The availability of energy is affected by location (including latitude and elevation), season, and time of day. All of which can be readily determined. However, the biggest factors affecting the available energy are cloud cover and other meteorological conditions which vary with location and time.

To calculate the amount of sunlight reaching the ground, both the elliptical orbit of the Earth and the attenuation by the Earth's atmosphere have to be taken into account. The extraterrestrial solar illuminance ($E_{\text{ext}}$), corrected for the elliptical orbit by using the day number of the year (dn), is given by:

$$E_{\text{ext}} = E_{\text{sc}} \cdot \left(1 + 0.033412 \cdot \cos\left(2\pi \frac{\text{dn} - 3}{365}\right)\right), \qquad 6.9$$

Where dn =1 on January 1; dn = 2 on January 2; dn= 32 on February 1, etc.

In this formula dn-3 is used, because in modern times Earth's perihelion, the closest approach to the Sun and therefore the maximum $E_{\text{ext}}$ occur around January 3 each year. The value of 0.033412 is determined knowing that the ratio between the perihelion (0.98328989 AU) squared and the aphelion (1.01671033 AU) squared should be approximately 0.935338.

The solar illuminance constant ($E_{\text{sc}}$), is equal to $1.28\times10^5$Lx. The direct normal illuminance ($E_{\text{dn}}$), corrected for the attenuating effects of the atmosphere is given by:

$$E_{\text{dn}} = E_{\text{ext}}\, e^{-cm}, \qquad 6.10$$

Where $c$ is the atmospheric extinction coefficient and $m$ is the relative optical air mass.

**SELF ASSESSMENT**

Q1. Define solar radiation.

Q2. Mention stating the range of wavelengths the various components of solar radiation.

### 6.4.1 SOLAR CONSTANT

The solar constant is a measure of flux density. It is the amount of incoming solar electromagnetic radiation per unit area that would be incident on a plane perpendicular to the rays, at a distance of one astronomical unit (AU) (roughly the mean distance from the Sun to the Earth). The "solar constant" includes all types of solar radiation, not just the visible light. Its value was thought to be an average of approximately 1.366 $kW/m^2$. This value varies slightly with solar activity, but recent recalibrations of the relevant satellite observations indicate a value closer to 1.361 $kW/m^2$ is more realistic.

### 6.4.2 INTENSITY IN THE SOLAR SYSTEM

Different bodies of the Solar System receive light of intensity inversely proportional to the square of their distance from Sun.

The actual brightness of sunlight that would be observed at the surface depends also on the presence and composition of an atmosphere. For example Venus' thick atmosphere reflects more than 60% of the solar light it receives. The actual illumination of the surface is about 14,000 lux, comparable to that on Earth in the daytime with overcast clouds. This explains why sunlight on Mars would be more or less like daylight on Earth wearing sunglasses and can be seen in the pictures taken by the unmanned Rover planetary probes. Thus it would give a perception and "feel" very much like Earth daylight.

A rough table comparing the amount of solar radiation received by each planet in the Solar System is as follows:

| Planet | Perihelion - Aphelion distance (AU) | Solar radiation maximum and minimum ($W/m^2$) |
|---|---|---|
| Mercury | 0.3075 – 0.4667 | 14,446 – 6,272 |
| Venus | 0.7184 – 0.7282 | 2,647 – 2,576 |
| Earth | 0.9833 – 1.017 | 1,413 – 1,321 |
| Mars | 1.382 – 1.666 | 715 – 492 |
| Jupiter | 4.950 – 5.458 | 55.8 – 45.9 |
| Saturn | 9.048 – 10.12 | 16.7 – 13.4 |
| Uranus | 18.38 – 20.08 | 4.04 – 3.39 |
| Neptune | 29.77 – 30.44 | 1.54 – 1.47 |

For comparison purposes, sunlight on Saturn is slightly brighter than Earth sunlight at the average sunset or sunrise. Even on Pluto the sunlight would still be bright enough to almost match the average living room. To see sunlight as dim as full moonlight on the Earth, a distance of about 500 AU (~69 light-hours) is needed.

### 6.4.3 COMPOSITION OF THE SOLAR RADIATION

The spectrum of the Sun's solar radiation is close to that of a black body with a temperature of about 5,800 K. The Sun emits EM radiation across most of the electromagnetic spectrum as shown below. Although the Sun produces Gamma rays as a result of the nuclear fusion process, these super high energy photons are converted to lower energy photons before they reach the Sun's surface and are emitted out into space, so the Sun doesn't give off any gamma rays to speak of.

The Sun does, however, emit X-rays, ultraviolet, visible light, infrared, and even Radio waves. When ultraviolet radiation is not absorbed by the atmosphere or other protective coating, it can cause damage to the skin known as sunburn or trigger an adaptive change in human skin pigmentation.

The spectrum of electromagnetic radiation striking the Earth's atmosphere spans a range of 100 nm to about 1 mm. This can be divided into five regions in increasing order of wavelengths

- **Ultraviolet C** or (UVC) range, which spans a range of 100 to 280 nm. The term *ultraviolet* refers to the fact that the radiation is at higher frequency than violet light (and, hence also invisible to the human eye). Owing to absorption by the atmosphere very little reaches the Earth's surface (Lithosphere). This spectrum of radiation has germicidal properties, and is used in germicidal lamps.
- **Ultraviolet B** or (UVB) range spans 280 to 315 nm. It is also greatly absorbed by the atmosphere, and along with UVC is responsible for the photochemical reaction leading to the production of the ozone layer.
- **Ultraviolet A** or (UVA) spans 315 to 400 nm. It has been traditionally held as less damaging to the DNA, and hence used in tanning and PUVA therapy for psoriasis.
- **Visible range** or **light** spans 380 to 780 nm. As the name suggests, it is this range that is visible to the naked eye.
- **Infrared** range that spans 700 nm to $10^6$ nm (1 mm). It is responsible for an important part of the electromagnetic radiation that reaches the Earth.

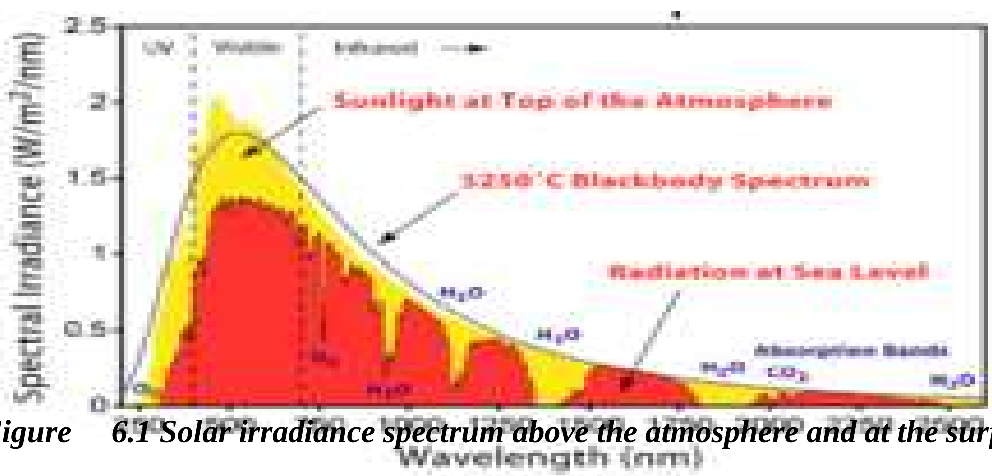


***Figure 6.1 Solar irradiance spectrum above the atmosphere and at the surface***

### 6.4.4 SURFACE ILLUMINATION

The spectrum of surface illumination depends upon solar elevation due to atmospheric effects, with the blue spectral component from atmospheric scatter dominating during twilight before and after sunrise and sunset, respectively, and red dominating during sunrise and sunset. These effects are apparent in natural light photography where the principal source of illumination is sunlight as mediated by the atmosphere. The preferential absorption of sunlight by ozone over long horizon paths gives the zenith sky its blueness when the sun is near the horizon.

### 6.4.5 CLIMATE EFFECTS

On Earth, solar radiation is obvious as daylight when the sun is above the horizon. This is during daytime, and also in summer near the poles at night, but not at all in winter near the poles. When the direct radiation is not blocked by clouds, it is experienced as *sunshine*, combining the perception of bright white light (sunlight in the strict sense) and warming. The warming on the body, the ground and other objects depends on the absorption (electromagnetic radiation) of the electromagnetic radiation in the form of heat.

The amount of radiation intercepted by a planetary body varies inversely with the square of the distance between the star and the planet. The Earth's orbit and obliquity change with time (over thousands of years), sometimes forming a nearly perfect circle, and at other times stretching out to an orbital eccentricity of 5% (currently 1.67%). The total insolation remains almost constant due to Kepler's second law, given as:

$$\frac{2A}{r^2}dt = d\theta. \tag{6.11}$$

Where *A* is an invariant representing the "real velocity". This means that the integration over the orbital period (also invariant) is a constant.

$$\int_0^T \frac{2A}{r^2}dt = \int_0^{2\pi} d\theta = \text{constant} \tag{6.12}$$

If we assume the solar radiation power $P$ as a constant over time and the solar irradiation given by the inverse-square law, we can obtain the average insolation as a constant.

The seasonal and latitudinal distribution and intensity of solar radiation received at the Earth's surface varies. For example, at latitudes of 65 degrees the change in solar energy in summer and winter can vary by more than 25% as a result of the Earth's orbital variation. Because changes in winter and summer tend to offset, the change in the annual average insolation at any given location is near zero, but the redistribution of energy between summer and winter does strongly affect the intensity of seasonal cycles. Such changes associated with the redistribution of solar energy are considered a likely cause for the coming and going of recent ice ages.

### 6.4.6 EFFECTS OF SOLAR RADIATION

Sunlight, or solar radiation, is all the electromagnetic radiation given off by the Sun. Here on Earth, our atmosphere filters the Sun's light, protecting us from harmful radiation and changing the colour of sunlight.

First, let's look at where this radiation comes from. As you probably know, the intense temperature and pressures at the core of the Sun is where the magic of solar fusion happens. Protons are converted into helium atoms at a rate of 600 million tons per second. Since the output of this process has lower energy than the protons that began, the fusion gives off a tremendous amount of energy in the form of gamma rays. These gamma rays are absorbed by particles in the Sun, and then re-emitted. Over the course of 200,000 years, photons of light make their journey through the irradiative zone of the Sun and out into space.

The surface of the Sun we can see is called the photosphere, and it's the point at which light from the Sun can finally escape into space. Through their long journey through the Sun, however, the photons have lost energy, and become other wavelengths of light. That's a good thing; otherwise, we'd just have gamma rays streaming from the Sun.

Solar radiation isn't any one kind of light. It's actually a mixture of different wavelengths. The heat we feel from the sun is infrared, and ranges in wavelength from 1400 nm to 1 mm. visible light than ranges from 400 to 700 nm. Out in space, the Sun's light appears white, but here on Earth we see it as more yellow because the atmosphere deflects blue and violet photons more easily. We are also hit by ultraviolet radiation; fortunately, much of this is absorbed by the Earth's atmosphere as it is quite dangerous to life.

All life on Earth depends on solar radiation. It is the primary source of energy to the Earth, and drives the planet's weather and ocean circulation. Without this source of energy, the Earth would freeze, and only its internal geothermic heat would stop it from freezing into a solid rock.

**Our Relationship with the Sun**

As human beings, we tend to have a love-hate relationship with the sun – on one hand, sunlight keeps us warm, creates food and shelter for us via plant life, and gives us light. On the other hand, as greenhouse gases trap more heat and the ozone layer allows more dangerous UV radiation in, the sun's rays can be distinctly dangerous. UV rays cause skin cancer in humans and animals, but can contrastingly improve other skin conditions like psoriasis. We need the sun biologically, as well; as it causes our bodies to produce vital vitamin D.

Solar radiation and sunlight make it possible for the Earth to house life. The negative aspects of our relationship with the sun are primarily the result of human irresponsibility: we develop skin cancer when we ignore our bodies' signals to avoid sunlight, and we struggle with global warming because we've ignored the environmental concerns of our actions. When we don't give solar radiation the respect it deserves, we are literally playing with fire.

Measuring the sun's radiation output has a variety of useful purposes: it allows for the development of solar energy devices, makes it possible for doctors to issue advice about sun exposure, and permits scientists to predict rates of future global warming on a grander scale.

**Measurement Techniques**

The measurement of solar radiation is based on a rate of kilowatts per square meter, represented as W/m2. This is the measurement standard for scientific data, designed for generating a direct estimate of solar energy – basically, how much sunlight is hitting a particular part of the Earth at any given time. Measurements taken for the purpose of energy production via solar panels and other photovoltaic equipment may be calculated in kilowatt-hours per square meter, or kWh/m2, designed to represent the amount of energy being generated by that sunlight.

Several different tools can be used to measure kilowatts and kilowatt-hours per square meter. These include:

(i) Pyranometer which is a device used to measure global solar radiation. It is comprised of a thermopile sensor with a black coating, which absorbs all solar radiation, and a glass dome, which limits the spectral response of the thermopile

(ii) Pyrheliometer which measures direct radiation. A Pyrheliometer works similarly, but is designed with a solar tracker to keep the device directly aimed at the sun for the duration of the measurement being taken.

**Black Body Radiation**

*A Black Body is a*ny object that is a perfect emitter and a perfect absorber of radiation. Object does not have to appear "black" before it can be called a black body. The sun and earth's surfaces behave approximately like black bodies. So, let's define some basic black-body radiation laws

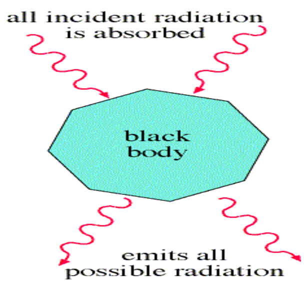


***Figure 6.4: Black body radiation***

**Stefan-Boltzmann Law**

The Stefan-Boltzmann law relates the total amount of radiation emitted by an object to its temperature:

$$E = \sigma T^4$$

Where:

E = total amount of radiation emitted by an object per square meter (Watts $m^{-2}$)

$\sigma$ is a Constant called the Stefan-Boltzmann constant

Stefan-Boltzmann constant = $5.67 \times 10^{-8}$ Watts $m^{-2}$ $K^{-4}$

T = the temperature of the object in K

Consider the Earth and the Sun for instance:

For the Sun: T = 6000 K so E = $5.67 \times 10^{-8}$ Watts $m^{-2}$ $K^{-4}$ $(6000\ K)^4 = 7.3 \times 10^7$ Watts $m^{-2}$

Question: Is this a lot of radiation? Compare to a 100 Watt light bulb.....

Also for the Earth: T = 288K so E = $5.67 \times 10^{-8}$ Watts $m^{-2}$ $K^{-4}$ $(288\ K)^4 = 390$ Watts $m^{-2}$

Question: If you double the temperature of an object, how much more radiation will it emit?

Answer: It will emit 16 times more radiation energy.

**Weins Law:**

This law relates wavelength of solar radiation to its surface temperature**.** Most objects emit radiation at many wavelengths. However, there is one wavelength where an object emits the largest amount of radiation given as Wien's wavelength and written as $\lambda_{*}$:

$$\lambda_{max} = 2897\ m / T(K)$$

**SELF ASSESSMENT**

Q1: At what wavelength does the sun emit most of its radiation if the surface temperature is 6000K?

Q2: At what wavelength does the earth emit most of its radiation?

**TUTOR MARKED ASSIGNMENTS**

Q1. Define the following terms:

(i) Solar Radiation (ii) Solar Constant (iii) Black Body

Q2. Why is the measurement of solar radiation important?

Q3. What are the boundary conditions which are used to constrain the Standard solar model?

Q4. State two functions of standard solar model.

Q5. Itemize the steps needed in creating a solar model.

Q6. Mention stating the range of wavelengths the components of solar radiation. Also, list five factors which govern the availability of solar radiation energy in a given day.

Q7. Calculate the amount of Sunlight that reached the Earth on February 2nd, 2010. Take $E_{sc} = 1.28x10^5 Lx$

Q8. Name two devices which can be used to measure the solar radiation of the Sun. Also, point out the difference between them.

Q9. Why is the measurement of solar radiation important?