IMAGE RESTORATION

Degradation/Restoration Process

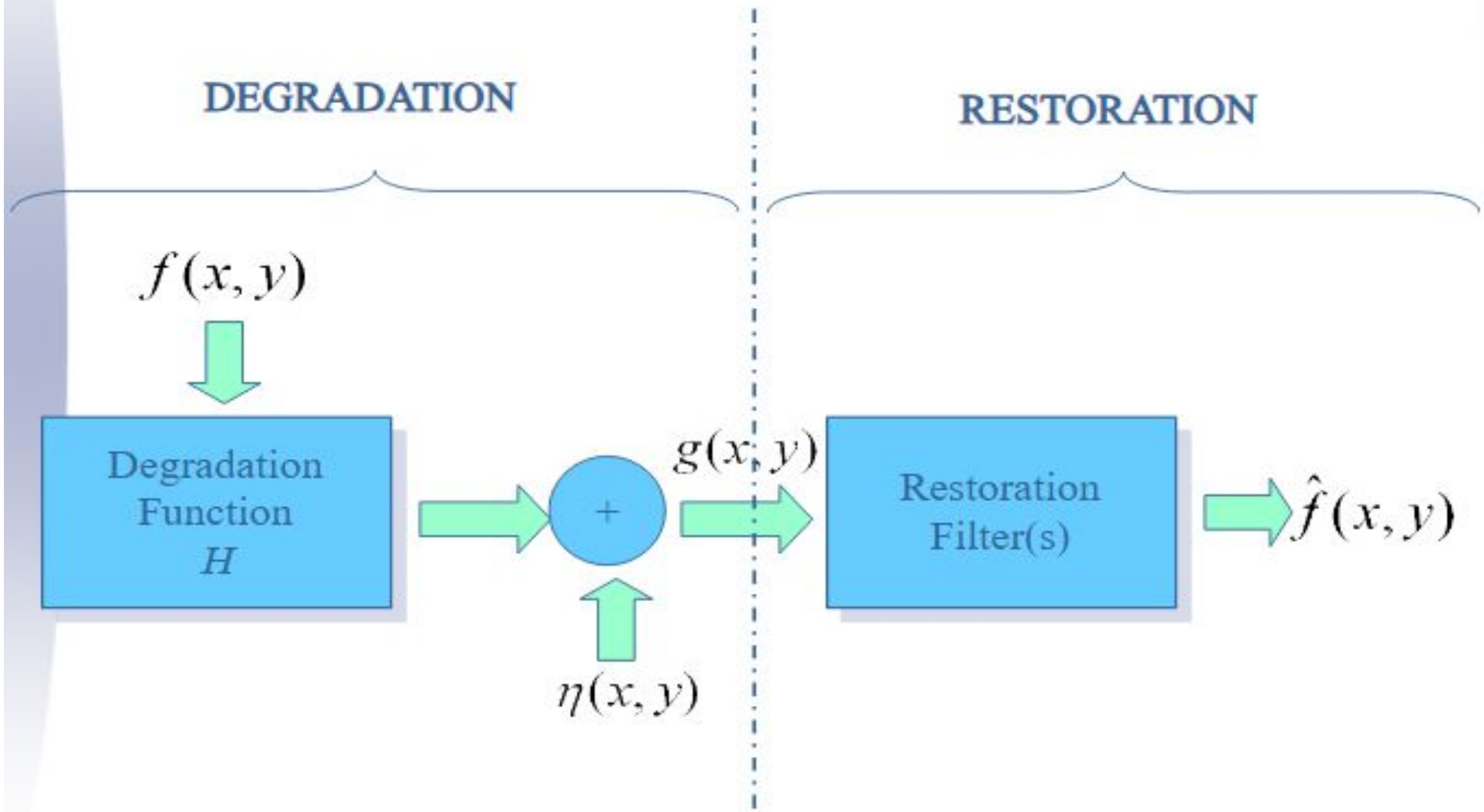


Image Degradation

- The degradation is modeled as a degradation function that, together with an additive noise term, operates on an input image f(x,y) to produce a degraded image g(x,y)
- If H is a linear the the degraded image is given in spatial domain by

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

Where the symbol "*" indicates spatial convolution

Image Restoration

- Given g(x,y), some knowledge about the degradation function H, and some information about the additive noise $\eta(x, y)$
- The objective of the restoration is to obtain an estimate $\hat{f}(x, y)$ of the original image.

Noise Models

- The principal source of noise in digital images arise during image acquisition (digitization) and/or transmission.
- The performance of imaging sensors is affected by a variety of factors.
- Images are corrupted during transmission due to interference in channel

Spatial Properties of Noise

- With the exception of spatially periodic noise, noise is independent of spatial coordinates, and it is uncorrelated with respect to the image itself.
- We can describe that spatial noise is concerned with the statistical behavior of the gray-level values.

Some Importance Noise

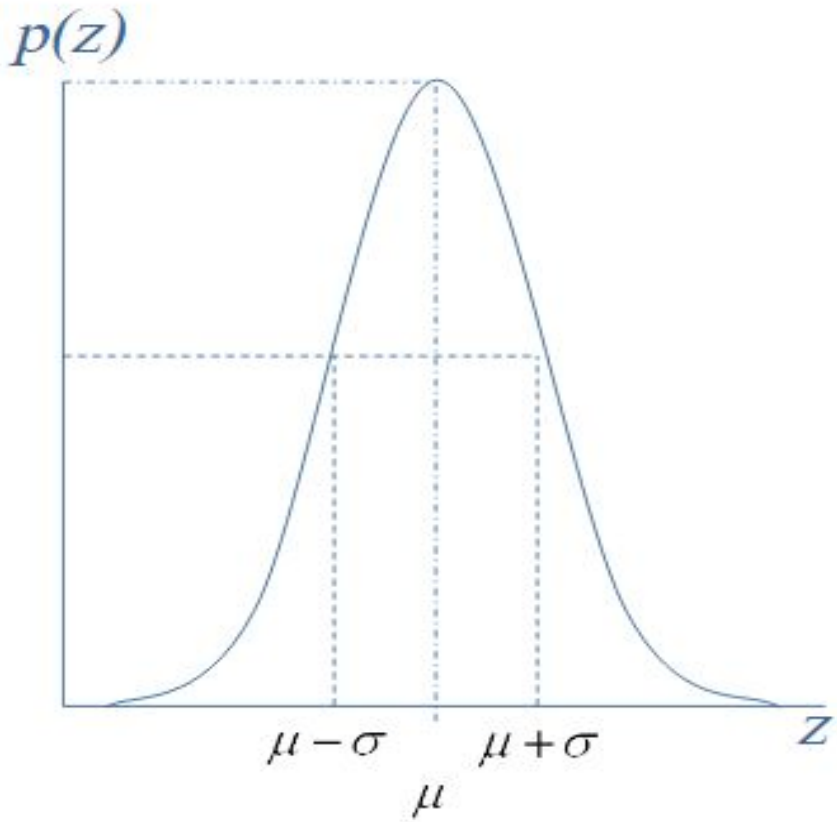
These noises are common found.

- Gaussian noise
- Rayleigh noise
- Erlang (Gamma) noise
- Exponential noise
- Uniform noise
- Impulse (salt-and-pepper) noise

Gaussian noise

- **The PDF of a Gaussian noise is given by**

$$\boxed{p(z) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(z-\mu)^2/2\sigma^2}}$$



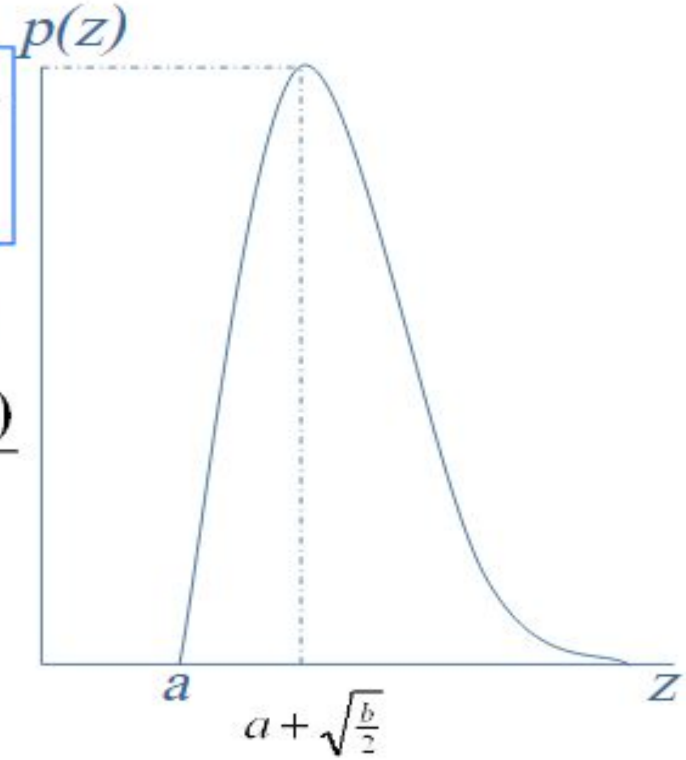
Rayleigh noise

- **The PDF of a Rayleigh noise is given by**

$$\boxed{p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \ge a \\ 0 & \text{for } z < a \end{cases}}$$

The mean and variance are given

$$\mu = a + \sqrt{\pi b/4} \text{ and } \sigma^2 = \frac{b(4-\pi)}{4}$$



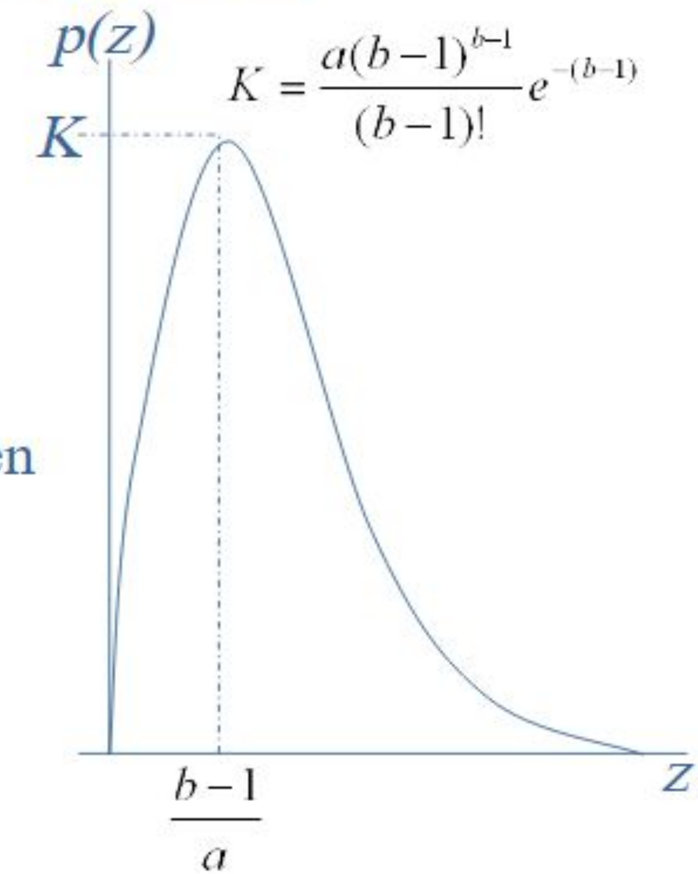
Erlang (Gamma) noise

- **The PDF of a Erlang noise is given by**

$$\boxed{p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & for\ z \ge 0 \\ 0 & for\ z < 0 \end{cases}}$$

The mean and variance are given

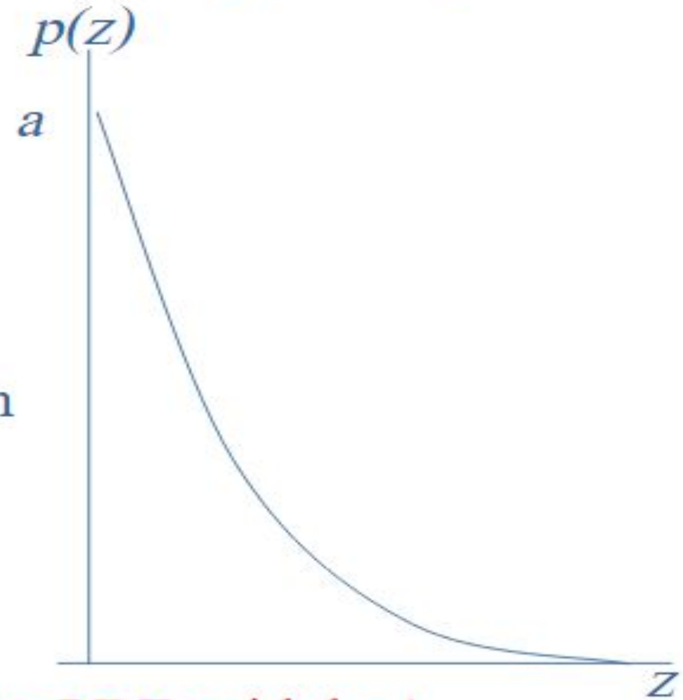
$$\mu = \frac{b}{a} \quad \text{and} \quad \sigma^2 = \frac{b}{a^2}$$



Exponential noise

- **The PDF of a Exponential noise is given by**

$$\boxed{p(z) = \begin{cases} ae^{-az} & for\ z \geq 0 \\ 0 & for\ z < 0 \end{cases}}$$



The mean and variance are given

$$\mu = \frac{1}{a} \quad \text{and} \quad \sigma^2 = \frac{1}{a^2}$$

Note: It is a special case of Erlang PDF, with b=1.

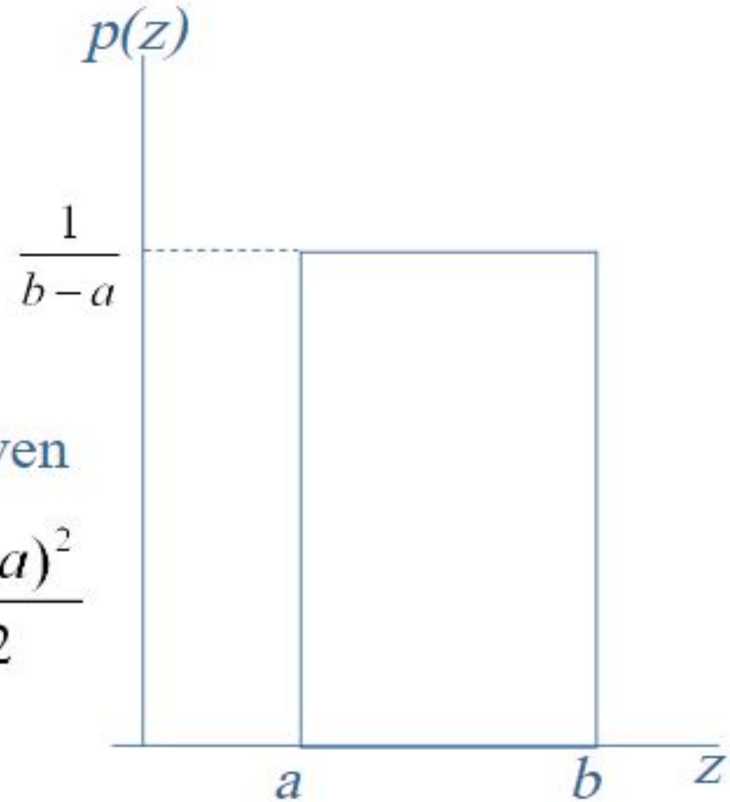
Uniform noise

- **The PDF of a Uniform noise is given by**

$$\boxed{p(z)=\begin{cases}\dfrac{1}{b-a} & if\ a\le z\le b\\ 0 & otherwise\end{cases}}$$

The mean and variance are given

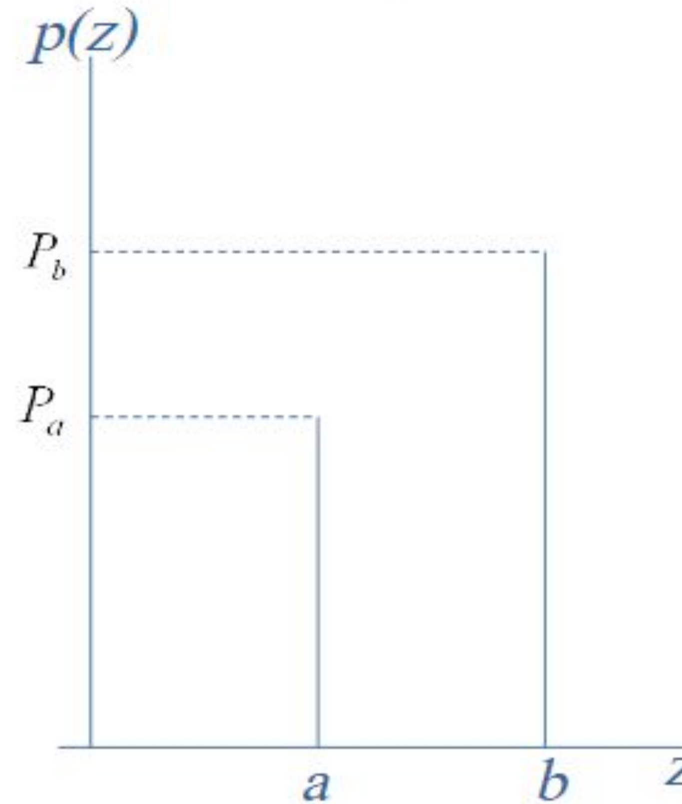
$$\mu=\frac{a+b}{2} \quad \text{and} \quad \sigma^2=\frac{(b-a)^2}{12}$$



Impulse (salt-and-pepper) noise

- **The PDF of a (bipolar) impulse noise is given by**

$$p(z) = \begin{cases} P_a & for\ z = a \\ P_b & for\ z = b \\ 0 & otherwise \end{cases}$$



Restoration in the Presence of Noise

- When the only degradation present in an image is noise

$$g(x, y) = f(x, y) + \eta(x, y)$$

- The noise is unknown, so subtracting them from g(x,y) is not a realistic option.
- In fact, enhancement and restoration become almost indistinguishable disciplines in this particular case.

Mean Filters

- This is the simply methods to reduce noise in spatial domain.
  - Arithmetic mean filter
  - Geometric mean filter
  - Harmonic mean filter
  - Contraharmonic mean filter
- Let S_{xy} represent the set of coordinates in a rectangular subimage window of size mxn, centered at point (x,y).

Arithmetic mean filter

- **Compute the average value of the corrupted image g(x,y) in the aread defined by S $_{x,y}$.**
- **The value of the restored image $\hat{f}$ at any point (x,y)**

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{x,y}} g(s,t)$$

Note: Using a convolution mask in which all coefficients have value 1/mn. Noise is reduced as a result of blurring.

Geometric mean filter

- Using a geometric mean filter is given by the expression

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

Harmonic mean filter

- The harmonic mean filter operation is given by the expression

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

Contraharmonic mean filter

- **The contraharmonic mean filter operation is given by the expression**

$$\hat{f}(x,y)=\frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Where Q is called the order of the filter. This filter is well suited for reducing or virtually eliminating the effects of salt-and-pepper noise.

Order-Statistics Filters

- Order-Statictics filters are spatial filters whose response is based on ordering (ranking) the pixels contained in the image area encompassed by the filter
  - Median filter
  - Max and Min filter
  - Midpoint filter
  - Alpha-trimmed mean filter

Median filter

- Process is replaces the value of a pixel by the median of the gray levels in region S_{xy} of that pixel:

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

Max and Min filter

- **Using the 100 th percentile results in the so-called max filter, given by**

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$

This filter is useful for finding the brightest points in an image. Since pepper noise has very low values, it is reduced by this filter as a result of the max selection processing the subimage area S_{xy}.

- **The 0th percentile filter is min filter:**

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

This filter is useful for finding the darkest points in an image. Also, it reduces salt noise as a result of the min operation.

Midpoint filter

- **The midpoint filter simply computes the midpoint between the maximum and minimum values in the area encompassed by the filter:**

$$f(x,y)=\frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\}+\min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

Note: This filter works best for randomly distributed noise, like Gaussian or uniform noise.

Alpha-trimmed mean filter

- **Suppose that we delete the d/2 lowest and the d/2 highest gray-level values of g(s,t) in the area S_{xy}.**
- **Let $g_r(s,t)$ represent the remaining mn-d pixels. And averaging these remain pixels is denoted as:**

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

Where the value of d can range from 0 to mn-1. When d=0, It is arithmetic mean filter and d=(mn-1)/2 is a median filter. It is useful for the multiple types of noise such as the combination of salt-and-pepper and Gaussian noise.