1. f is a function that satisfies the following:
   - f is in $O(n)$,
   - f is in $\Omega(1)$,
   - f is neither in $\theta(1)$ nor in $\theta(n)$.

   Can you give an example of such a function f? Show that the function you name indeed satisfies all of the above.

A:

Function : *f*(*n*)= sqrt(n)

1. If there are existence constant like x = 1 and n1 = 1 , let n > n1 ,

so we will have sqrt (n) <= n. So f = O(n)

2. If there are existence constant like x = 1 and n1 = 1, let n > n1 ,

So we will have sqrt(n) >= 1 . So f = Omega(1)

3. If f is not in theta(1), let x,y >0 , then let x <= sqrt(n) <= y to satisfies bigger enough value of n . But sqrt(n) will be increase infinity. So it does not make sense.

   If f is not in theta(n), let x , y > 0, then x*n <= sqrt(n) <= y*n. But sqrt(n) / n = 1/sqrt(n) => 0. therefore it won't satisfies x*n <= sqrt(n). So f is not in theta(n).

2. For each pair of functions given below, point out the asymptotic relationships that apply: $f = O(g)$, $f = \theta(g)$, and $f = \Omega(g)$.
   - $f(n) = \sqrt{n}$ and $g(n) = log\, n$
   - $f(n) = 10$ and $g(n) = 11$
   - $f(n) = 100 \cdot 3^n$ and $g(n) = 4^n$
   - $f(n) = 4^{n+2}$ and $g(n) = 2^{2n+3}$
   - $f(n) = 7n \cdot log\, n$ and $g(n) = n \cdot log\, 7n$
   - $f(n) = n!$ and $g(n) = (n+1)!$

A :

1.The asymptotic relationship is f = Omega(g)

Because, based on polynomial growth is faster than logarithmic growth, we can calculate the limit of it. So it will be infinity : sqrt(n) / log n

2. The asymptotic relationship is f = theta(g)

Because 10/11 will be a constant and won't be 0.

3. The asymptotic relationship is f = O(g)

Because lim n-> infty 100*3^n / 4^n = 100* (3/4)^n = 0. so f(n)=O(g(n)).

4. The asymptotic relationship is f = theta(g)

Because 4^(n+2) = 2 ^2(n+2) , and g(n) = 2^(2n+3), therefore 2 ^2(n+2) /2^(2n+3) will be a constant number.

5. The asymptotic relationship is f = theta(g)

Because g(n) = n(log 7 + log n), when n can be grow infinity, therefore f(n) / g(n) < 7. Which is a constant number.

6. The asymptotic relationship is f = O(g)

Because g(n) = (n+1) * n!, so the function will be n! / (n+1) ! = 1/n+1 = 0.