! This class has been made inactive. No posts will be allowed until an instructor reactivates the class.

question @10 stop following 7 views

Actions

How do I get my Theorem published?

I have come up with an incredible theorem, called Fermat's Giant Theorem (FGT).

FGT states that if you take any prime number, and you write it down twice, the resulting number is composite.

Examples:
1) 2 -> 22 , which is composite.
2) 13 -> 1313 , which is composite.
3) 23 -> 2323 , which is composite.

I would like for this theorem to be included in the MATH 135 textbook, so that future students can enjoy prime numbers even more than they already do.

examples_of_student_posts_from_past_piazza

~ An instructor (Mukto Akash) endorsed this question ~

Edit undo good question | 1 Updated 5 years ago by Mukto Akash and Anonymous Calc

the students' answer, *where students collectively construct a single answer*

Actions

You might have to get approval from Fermat™ himself before using his name in your theorem.

~ An instructor (Mukto Akash) endorsed this answer ~

Edit undo good answer | 1 Updated 5 years ago by Anonymous Atom

the instructors' answer, *where instructors collectively construct a single answer*

Actions

Well, one thing that you might ask yourself is whether or not this result is specific to primes.

Edit good answer | 0 Updated 5 years ago by Mukto Akash

followup discussions *for lingering questions and comments*

Resolved Unresolved @10_f1 Actions

Anonymous Comp 5 years ago
Email Fermat and see what he thinks.

good comment 0

Anonymous Helix 5 years ago

I believe his email is pdfermat@aol.com

good comment 0

Reply to this followup discussion

Resolved Unresolved @10_f2 Actions

Anonymous Gear 5 years ago

you should call it Fermat's Large Theorem instead, FLT has a nice ring to it.

good comment 0

Anonymous Comp 5 years ago Actions

But Fermat's last theorem is FLT...

good comment 0

Anonymous Gear 5 years ago Actions

well if fermat's little theorem is FlT and fermat's last theorem is FLT, it logically follows that fermat's large theorem should be F**LT**, since it's way bigger and bolder than both FLT and FlT.

good comment 0

Anonymous Calc 5 years ago Actions

I think I'll call it Fermat's Second Last Theorem

good comment 0

Reply to this followup discussion

Resolved Unresolved @10_f3 Actions

Anonymous Mouse 5 years ago

Proposition:

```
If an integer is written twice adjacently, the resulting number is composite.
```

Proof:

1. Let p represent the integer.
2. Let n represent p written twice.
3. Clearly, p is either positive, negative, or 0.
4. Suppose p is positive or 0.
5. Clearly, $n = p \cdot 10^{\lceil \log_{10}(p+1) \rceil} + p = p \cdot \left(10^{\lceil \log_{10}(p+1) \rceil} + 1\right)$.
6. Clearly, $10^{\lceil \log_{10}(p+1) \rceil} + 1 \in \mathbb{Z} \wedge 2 \leq 10^{\lceil \log_{10}(p+1) \rceil} + 1 \leq n$.
7. So $10^{\lceil \log_{10}(p+1) \rceil} + 1 \mid n$.
8. Therefore, n is composite.
9. Suppose p is negative.

10. Clearly, $n = -|p| - |p| = 2p$.
11. So $2 \mid n$ and n is composite.
12. In all cases, n is composite.

good comment | 0

Anonymous Scale 5 years ago

Actions

First, 2323 = 23*(101)
and 123123 = 123(1001), and so on.

Thus:

Let $p = 10^k a_k + \cdots + a_0$, and let n represent p written twice, then it follows that $n = 10^{k+1}(10^k a_k + \cdots + a_0) + (10^k a_k + \cdots + a_0) = (10^k a_k + \cdots + a_0)(10^{k+1} + 1)$, which is composite.

Anthony, Your proof is a little off.

good comment | 0

Anonymous Mouse 5 years ago

Actions

Oops! Forgot the 10^\ldots. That should be fixed now.

good comment | 0

Anonymous Scale 5 years ago

Actions

perfect!

good comment | 0

Anonymous Beaker 5 years ago

Actions

Hold on:

Using logarithm rules, can't we simplify $10^{\log_{10}(p+1)}$ as $(p+1)$?

Then $n = p(10^{\log_{10}(p+1)} + 1) = p(p+2)$.

BUT, choosing 23 (which is obviously prime):

n=2323 (by definition), and
n=23(23+2)=23(25)=575 (by formula).

575≠2323.

So something's not right...

I'm not saying your theorem is wrong (I can't come up with any counterexamples), I just think that there's something wrong with the proof.

good comment | 0

Anonymous Scale 5 years ago

Actions

There is nothing wrong with the proof. You are forgetting he is taking the ceiling of the logarithm;

Let $a_j \in \mathbb{N},\ 0 \le a_j \le 9$, not all 0 and $a_k \neq 0$ then,

$$\left\lceil \log_{10}\left(\sum_{i=0}^{k} a_i 10^i\right)\right\rceil = \log_{10} 10^{k+1} = k+1.$$

Proof:

Assuming the hypothesis, let $a = \sum_{i=0}^{k} a_i 10^i$, then

$$10^k < \sum_{i=0}^{k} a_i 10^i < 10^{k+1}$$

so,

$$\log_{10} 10^k = k < \log_{10}\left(\sum_{i=0}^{k} a_i 10^i\right) < \log_{10} 10^{k+1} = k+1 \qquad 1)$$

therefore,

$$\lceil k \rceil = k < \left\lceil \log_{10}\left(\sum_{i=0}^{k} a_i 10^i\right)\right\rceil = k+1 \qquad 2)$$

His proof is the same as mine, just using logarithms. Take $\log_{10} 13$ for example. Well, since $10 < 13 < 100$, then $1 < \log_{10} 13 < 2$ so $\lceil \log_{10} 13 \rceil = \log_{10}(10^2) = 2$. Since the highest power in the decimal expansion of 13 is 10 (as $13 = 10^1 \cdot 1 + 10^0 \cdot 3$), you are multiplying 13 by $10^2 = 100$ and adding 13 to get $13 * (10^2) + 13 = 13(100+1) = 1313$.

Back to the proof. His proof says, if you have a prime, p (this actually isn't prime specific, so it would work with any number), where p is k digits long, then $p = p_k 10^k + \cdots + p_0$, so in my proof I multiplied by 10^{k+1}, and that is exaclty what he does, just using logarithms. So $1) \implies 2)$ and $2) \implies 10^{\lceil \log_{10} p \rceil} = 10^{k+1}$ (in his proof he uses $\log_{10}(p+1)$ which is unnecessary, as you only need to the the logarithm of p) .... All that is really happening here, is let $a \in \mathbb{Z},\ a > 1$, then I say $ab + a$ is composite, which it is, as $ab + a = a(b+1)$.

good comment | 0

Anonymous Beaker 5 years ago

Actions

Oh, sorry, I didn't notice the ceiling symbol there (it was late at night when I reviewed this).

I think that, from first reading it, the answer to why n was what is was claimed to be was not as "clear" as was implied, but I do see now why n is what it is claimed to be.

Aside from that minor detail, I see nothing wrong then. This is indeed an interesting theorem. Good job :)

good comment | 0

Anonymous Scale 5 years ago

Actions

It is indeed interesting, and upon looking at the proof again, the $\lceil\ \rceil$ is kind of hard to see when they are used in exponents, making them small. Also, yes I agree, it can be dangerous to claim something is trivial, or obvious.

good comment | 0

Anonymous Mouse 5 years ago

Actions

Note that the $+1$ is in fact redundant for primes, but also allow you to extend the proof for all positive integers! Consider 1, 100, and 1000, for example.

good comment | 0

Anonymous Scale 5 years ago

Actions

ah yes, I overlooked that. The statement,

Let $a_j \in \mathbb{N},\ 0 \leq a_j \leq 9,$ not all 0 and $a_k \neq 0$ then,

$$\lceil \log_{10}\left(\sum_{i=0}^k a_i 10^i\right) \rceil = \log_{10} 10^{k+1} = k+1.$$

utterly excludes that possibility by forcing at least one of a_j to be nonzero. I didn't even think of the case where $n = 10^k$, my apologies.

good comment | 0

i **Mukto Akash** 5 years ago

Actions

Excuse my interruption, but since y'all are so keen on using $\LaTeX$ here, you can also use the `\left` and `\right` modifiers for symbols other than parentheses. So to get a "better looking" version of Christian's statement above, try

```
\left\lceil \log_{10} \left(\sum_{i=0}^k a_i 10^i\right) \right\rceil = \log_{10}
10^{k+1} = k+1
```

which yields

$$\left\lceil \log_{10} \left(\sum_{i=0}^k a_i 10^i\right) \right\rceil = \log_{10} 10^{k+1} = k+1.$$

You can also use it for other things, like |, [, { or ||.

good comment | 0

Anonymous Scale 5 years ago

Actions

Thanks, that is much nicer looking.

good comment | 0

i **Mukto Akash** 5 years ago

Actions

I have been really enjoying this particular thread.

What do you guys have so far with respect to the proposition?

I see a statement of the proposition, and some attempts to prove it. I want to now bring you attention back to the first instructors' response (from Jack): does this proposition fail if you are not looking at primes? Or is it a more general result?

Finally, you are proving something is composite, which invokes the concept of divisibility, which then naturally invokes the idea of congruence. Can you find a way to use this concept to give a proof?

good comment | 0

Anonymous Mouse 5 years ago

Actions

I extended my proof to apply to all integers, **including negative numbers!**

Consider -3:

$2 \mid -3-3$, so the value is composite.

good comment | 0

Anonymous Scale 5 years ago

Actions

I don't quite get what you are saying; Our proofs both say that p may be positive, negative, prime or composite.. Although you are doing something wonky with the negatives. I must be missing something.. $p<0$ \implies $n=-|p|-|p|$, I dont follow. The way I see it, if $p<0 implies n=-p\cdot 10^{k+1}-p$ So for -3 I think it should be $n=-33$, not $-3-3$.

good comment | 0

Anonymous Scale 5 years ago

Actions

something weird is happening with the latex, it keeps adding """ everywhere, and when i try to fix it, it gets even weirder. So I hope you can make out what Im trying to say in the last post

good comment | 0

Anonymous Mouse 5 years ago

Actions

I don't see any "'s, but let me clarify a little bit.

A negative number would include the sign as well as the magnitude.

So we would write "-3" (negative 3) as "-3-3" (negative 3 minus 3) - we wrote the number twice, including the sign. So the unary minus became a binary subtraction. Another way to describe the operation I'm thinking of is:

```
write_twice = function(n)
    return eval(tostring(n) .. tostring(n))
end
```

run code snippet Visit Manage Class to disable runnable code snippets ×

good comment | 0

Anonymous Scale 5 years ago

Actions

nah, you cant see it but if you were able to edit my post you would see p<0 implies="" n="-p \cdot 10^{k+1}-p" latex=""> which is just really weird.. anyways, I understand what you are doing now. The beginning part of my post "I don't quite get what you are saying;" was meant for Mukto, as his post left me a little perplexed. And in response to Muktos first question, I personally haven't done anything else concerning this prop. Have you, Anthony? I mean, the only thing I can think of other than, "yes this number is always composite" is $p|n$ and $10^{\lceil log_{10}(p+1)\rceil}+1|n$, which are both obvious consequences of proving the proposition true.

good comment | 0

Actions

Mukto Akash 5 years ago

Anthony, it may be a bit too far fetched to try this proposition for negative numbers. Mostly because if you are interpreting $-3-3$ as -6, then it begs the question whether 33 is supposed to be interpreted as the number thirty-three or six $(+3+3)$? The concept of digits is primarily used for positive numbers, and when thinking about the digits of negative numbers, you truly have to be careful about the negative sign. Do you treat the negative sign as simply another digit of the number? Or is it an overall artefact of recognizing negative numbers as the ``additive inverse'' of positive numbers? In short, things seem to be getting messy here if you start considering negative numbers. Focus on the positive integers for now.

Christian, sorry, I did not read your proof carefully. That is a good (and complete) proof, and works for all non-negative integers. Good work. Try to now extrapolate this result to see what information you may obtain about the factors of numbers of this type.

good comment | 0

Actions

Anonymous Scale 5 years ago

There is, actually one thing that i noticed which I am having a bit trouble proving involving 10^n+1.

good comment | 0

Actions

Anonymous Poet 5 years ago

Consider $n=1$.

11 is prime, and therefore, FGT does not work.

Consider $n=0$.

$00=0$ is not composite, and therefore, FGT does not work.

Consider $n=-1$.

$-1-1=-2$ and -11 are both primes, and therefore, FGT does not work.

Thus, FGT only works with integer n, where $|n|>1$.

good comment | 0

Actions

Anonymous Scale 5 years ago

As Mukto said, we should only really be considering positive integers, so no 0 either. Other than that, I find it a little funny that non of us realized 1 does not work.)

good comment | 0

Actions

Anonymous Calc 2 5 years ago

wasn't the original assumption that p is prime? Since 1 is not prime...

good comment | 0

Actions

Anonymous Scale 5 years ago

It was, but since this is not prime specific, it makes little sense to only consider the case when p is prime, unless you are trying to see if there is anything different about when p is prime.

good comment | 0

Actions

Mukto Akash 5 years ago

Good catch Kurt Choi (regarding $n = 1$).

So the proposition should be changed to all integers $n \geq 2$.

Getting back to Christian regarding $10^n + 1$. Clearly, when n is odd, $11 \mid (10^n + 1)$.

What about when n is even? Well, $10^2 + 1 = 101$ is a prime, but $10^4 + 1 = 10001 = 73 \times 137$ is not a prime, so it's hard to say what will happen past this point.

One thing I noticed (and was able to prove using strong induction) is that $(101) \mid (10^{4k+2} + 1)$ for all $k \geq 0$. Try to see if you can get this (my proof seems non-trivial, may be you can come up with something easier).

But what about numbers of the form $10^{4k} + 1$? Again, I can partition these into two and get that $(10^4 + 1) \mid (10^{8k+4} + 1)$.

Seems like a similar trend would follow and we can try to then break down numbers of the form $10^{8k} + 1$ into two categories: $10^{16k} + 1$ and $10^{16k+8} + 1$.

Overall, I am inclined to hypothesize that $10^{2k} + 1$ is not prime for $k \geq 2$. Now, writing a well-constructed proof of this may take some time. Any takers?

good comment | 0

Anonymous Scale 5 years ago

Actions

The primality of $10^n + 1$ is actually what was on my mind for a little while. It can be shown that if $10^n + 1$ is prime, n must be 2^k for some $k \in \mathbb{N}$, so we can refine your conjecture to $10^{2^k} + 1$ is not prime for all $k > 2$. That being said, after looking around online (I really didn't want to leave my computer checking countless numbers), the next smallest prime after 101 would have to be above $10^{16777216} + 1$, so I believe your claim is fairly strong. Time to get out the coffee.

</p>

good comment | 0

Anonymous Scale 5 years ago

Actions

I was able to come up with a proof for divisibility by 101 using normal induction, in case you are interested (the proof of the general case is very similar).

The base case is trivial, so let $k = n$, and assume $10^{4n+2} + 1 = 101y$ for $y \in \mathbb{Z}$ (induction hypothesis) so then for $k = n + 1$, we get;

$$10^{4(n+1)+2} + 1 = 10^{4n+2}10^4 + 1 = 10^4(10^{4n+2} + 1) - (10^4 - 1)$$

and so by the I.H.

$$10^{4(n+1)+2} + 1 = 10^4(101y) - (101)(99)$$

I was originally worried about your claim, because I thought i showed $10^{4k+2} + 1 \equiv 2 \pmod{101}$ for all $k \in \mathbb{Z}$, and then I realized that I wasn't paying much attention, as I had shown $10^{4k+2} + 1 \equiv 2 \pmod{11}$ instead. whoops!

Anyways, here is another;

$10^2 \equiv -1 \pmod{101} \implies (10^2)^{2k+1} \equiv 10^{4k+2} \equiv (-1)^{2k+1} \equiv -1 \pmod{101} \implies 10^{4k+2} + 1 \equiv 0$ (mo

good comment | 0

Anonymous Atom 2 5 years ago

Actions

Here's the general proof for what Mukto was talking about with weak induction.

Proposition: For all $n \geq 0$ and for all $k \geq 0$, $(10^{2^n} + 1) \Big| \left(10^{2^{n+1}k+2^n} + 1\right)$.

Proof:

Let n be any non-negative integer and we will use induction on k.

For the base case $k = 0$, $10^{2^{n+1}(0)+2^n} + 1 = 10^{2^n} + 1$.

So suppose for all $m \geq 0$, $(10^{2^n} + 1) \Big| \left(10^{2^{n+1}m+2^n} + 1\right)$.

Then by assumption $(10^{2^n} + 1) \Big| \left[\left(10^{2^{n+1}m+2^n} + 1\right) + \left(10^{2^n(2m+1)+2^n} - 10^{2^{n+1}m+2^n}\right)(10^{2^n} + 1)\right]$.

And that thing on the right is equal to $10^{2^{n+1}(m+1)+2^n} + 1$. So this completes the induction and the proof.

good comment | 0

Anonymous Scale 5 years ago

Actions

AH you beat me to it. Very nice.

good comment | 0

i **Mukto Akash** 5 years ago

Actions

Great job Shaotong (except for the obvious fallacy in how the induction hypothesis is being stated).

So, as I was saying: this takes care of numbers of the form $10^{2^n(k)+2^{n-1}} + 1$. Now, you still need to write a carefully written proof (most likely using the well-ordering principle) to take care of numbers of the form $10^{2^n k} + 1$. Try it out!

good comment | 0

i **Mukto Akash** 5 years ago

Actions

Also, on a more challenging note (just on a hunch, may be it will not work?): it seems that you may now recursively build an algorithm to compute all the prime factors of $10^{2n} + 1$ for any $n \geq 1$. That could be interesting!

good comment | 0

Anonymous Scale 5 years ago

Actions

This is becoming really interesting. I just wanted to add, that all numbers of the form $10^{2^n} + 1$ are coprime to one another, so we must, again, refine our search to all numbers where $n = 2m$ for some odd m. Here is the proof that they are all coprime:

First, we need a lemma.

define $A_n = 10^{2^n} + 1$, then $A_{n+k} - 1 = (A_n - 1)^{2^k}$.

Proof:

By defn, $A_{n+k} - 1 = 10^{2^{n+k}} = 10^{2^n 2^k} = (10^{2^n})^{2^k} = (A_n - 1)^{2^k}$.

Now assume there exists a prime p that divides both A_m and A_n. As A_n and A_m are both odd, then p must be odd. We can also assume, without loss of generality that $m > n$. This implies that $m = n + k$, for some positive integer k.
By assumption, we get

$$A_n - 1 \equiv -1 \pmod{p},$$
$$A_m - 1 \equiv -1 \pmod{p}$$

and so by the lemma,

$A_m - 1 \equiv A_{n+k} - 1 \equiv (A_n - 1)^{2^k} \equiv (-1)^{2^k} \equiv 1 \equiv -1 \pmod{p}$, so $p|2$, but as p must be odd, then this is a contradiction, and thus for all $m, n \in \mathbb{N},\ \gcd(A_n, A_m) = 1$

good comment | 0

Anonymous Scale 5 years ago

Actions

The general result is $a^{2^n} + b^{2^n} | a^{2^n k} + b^{2^n k}$ only if k is not a power of 2. Now this is a direct result from showing that $(x + y)|(x^k + y^k)$ if k is odd, as once that has been shown, you can set $x = a^{2^n}$ and $y = b^{2^n}$ which yields the desired result. This would be a good exercise for anyone who wishes to show this (hint: it is just like the above divisibility proofs done by Shaotong and myself). Moreover, since all A_n are co-prime to one another, plus the result in this post shows that $(10^{2^n} + 1)|(10^{2^n k} + 1) \iff k = 2m + 1$.
About the algorithm, I am curious, as we have seen, $10^n + 1$ is prime, iff n is a power of 2, and if it is ever prime again, the number itself must be incomprehensibly large, this begs the question, though we might very well be able to come up with an algorithm to compute the factors of these numbers, will it fail if the number is prime?

good comment | 0

Anonymous Atom 2 5 years ago

Actions

I have fixed my proof above.

My proof already covers numbers of the form $10^{2^n k} + 1$ for k is not a power of 2 since $2^{a+1}b + 2^a = 2^a(2b + 1)$ can be any positive integer for $a, b \geq 0$. Like Christian said previously, we only need to show that $10^{2^n} + 1$ is composite for $n \geq 2$.

good comment | 0

Anonymous Scale 5 years ago

Actions

I agree Shaotong, however yours says its true for all m such that m is odd. We also needed to rule out any even numbers. I am pretty sure the proof that these numbers are always composite for n>= 2 will be non-trivial, as these types of numbers may be looked at as being somewhat akin to Fermat numbers. Despite the similarities, however, this type of number may yield many different properties.

good comment | 0

Anonymous Atom 2 5 years ago

Actions

Ok I changed some variable names. Perhaps it will reduce confusion. But I'm not sure which m you are talking about.

good comment | 0

Anonymous Scale 5 years ago

Actions

i was just using a generic m. you said your proof covered $10^{2^n k}+1$ because you proved $10^{2^n}+1|(10^{2^n k+2^n}+1)$, where $10^{2^{n+1}k+2^n}=10^{2^n(2k+1)}$. I just added that that has to be the case, meaning that we can't have $10^{2^n}+1|10^{2^n 2k}+1$. I was agreeing with you, but i was just emphasizing the fact this is "odd" specific.

good comment | 0

Anonymous Atom 2 5 years ago

Actions

Ok I see. As long as k is not a power of 2 my proof shows that it is composite. Since even if we have something like $10^{2^n 2k}+1$ we still have $10^{2^{n+1}}+1 \mid 10^{2^{n+1}k}+1=10^{2^n 2k}+1$.

good comment | 0

Anonymous Scale 5 years ago

Actions

totally misread your comment, Shaotong. You are correct :) A corollary would be $10^{2^{n+1}k}+1 \equiv 2 \pmod{10^{2^n}+1}$, which implies $(10^{2^n}+1)|(10^{2^{n+1}k}-1)$. Which isn't helpful (and it's obvious that it is composite, as a^n-b^n is always factorable), but it is a consequence of what we are doing.

P.S. to anyone who might be trying to read my post, I edit it 80000 times before its readable, just in case someone is trying to read it and it keeps changing.

good comment | 0

Anonymous Scale 5 years ago

Actions

I was able to show:

$$10^{2^n k}+1=(10^{2^n}+1)\sum_{i=1}^{k}(-1)^{i+1}10^{2^n(k-i)}$$

and ,

$$10^{2^{n+1}k}+1=(10^{2^n}+1)\left(\sum_{i=1}^{2k}(-1)^{i+1}10^{2^n(2k-i)}\right)+2.$$

This might help with an algorithm, and for trying to prove these numbers are composite for when $k=2^m$, it may help to consider the expansion for even k.

Here is the general statement.

$$a^{2^n k}+b^{2^n k}=(a^{2^n}+b^{2^n})\sum_{i=1}^{k}(-1)^{i+1}a^{2^n(k-i)}b^{2^n(i-1)}$$

good comment | 0

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