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ATTITUDE TOWARDS MATHEMATICS: EMOTIONS, EXPECTATIONS AND VALUES

ABSTRACT. This article suggests a new framework for analysing attitude and changes in attitude. After reviewing relevant findings in the field of psychology of emotions, the author will provide a new conceptualisation for attitude. Four different evaluative processes are identified as aspects of attitude: emotions aroused in the situation, emotions associated with the stimuli, expected consequences, and relating the situation to personal values. The usefulness of this analytical framework will be illustrated with an exemplary case study. An ethnographic case study of Rita, a lower secondary school student, will be analysed. The case study will describe her negative attitude towards mathematics, and then examine how negative emotions developed during problem solving situations. Within half a year her attitude towards mathematics changed dramatically to more positive.

KEY WORDS: attitude, case study, change, emotion, expectation, value

1. INTRODUCTION

1.1. *Earlier findings*

The everyday notion of attitude refers to someone's basic liking or disliking of a familiar target. A general overview of the development of attitudes throughout school years is documented through surveys and meta-analyses. These studies have shown that, for example, girls tend to have more negative attitudes towards mathematics than boys (Frost et al., 1994; Leder, 1995), and that attitudes tend to become more negative as pupils move from elementary to secondary school (McLeod, 1994). The general attitude of the class towards mathematics is related to the quality of the teaching and to the social-psychological climate of the class (Haladyna et al., 1983).

Ma and Kishor (1997) synthesised 113 survey studies of the relationship between attitude towards mathematics and achievement in mathematics. The causal direction of the relationship was from attitude to the achievement. Although the correlations were weak in the overall sample, they were stronger throughout grades 7 to 12, and in studies that had done separate analysis of male and female subjects.

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The effort to promote positive attitudes has been somewhat successful on the individual level. For example, mathematics anxiety can be reduced through systematic desensitisation (Hembree, 1990). On the whole class level the efforts to reform teaching to promote desired attitudes have generally been unsuccessful (McLeod, 1994). However, recent evidence suggests that collaborative approaches can promote positive attitudes among students (e.g. Boaler, 1997a, b, 1998; Ridlon, 1999).

A lot of research has been done on attitude towards mathematics, but theoretically the concept needs to be developed. Several authors (e.g. Di Martino and Zan, 2001; Ruffell, Mason and Allen, 1998) have pointed out that attitude is an ambiguous construct, it is often used without proper definition, and it needs to be developed theoretically. The most obvious problem with attitudes is the discrepancy between espoused and enacted attitudes. Moreover, attitude measures need substantial refining (Ma and Kishor, 1997). Di Martino and Zan (2001) distinguish two basic approaches to defining attitude towards mathematics:

1. A 'simple' definition describes it as the degree of affect associated with mathematics; i.e. attitude is the emotional disposition toward mathematics. This kind of definition ignores the cognitive element in attitude. However, even those who use this kind of definition, often rely on paper and pencil test, which makes it hard to distinguish emotional disposition from beliefs.
2. A three-component definition distinguishes emotional response, beliefs, and behaviour as components of attitude.[1] This second approach seems incompatible with the widely accepted view (e.g. McLeod, 1992, DeBellis and Goldin, 1997) of attitude, emotions and beliefs as belonging to the affective domain.

1.2. *Aim of this paper*

This paper will develop a new framework for analysing attitude. A foundation will be built from the background of psychology of emotions. The observable category 'student's attitude towards mathematics' will be separated into four different evaluative processes: 1) the emotions the student experiences during mathematics related activities; 2) the emotions that the student automatically associates with the concept 'mathematics'; 3) evaluations of situations that the student expects to follow as a consequence of doing mathematics; and 4) the value of mathematics-related goals in the student's global goal structure.

The development of a new framework will be followed by a case study that will illustrate the analytical power of the proposed framework. All

four elements of the framework are clearly present and without them a full understanding of the case would be hard to achieve. The case also provides some insight into the development of attitude.

2. Theoretical background

2.1. *Foundation*

The present approach to the processes underlying attitudes relates to the work by DeBellis and Goldin. For them, affects are a representational system that is parallel to, and crucial for, cognition. Their work is concerned with the significance of affective factors in students' problem solving behaviour: how global affect influences subjective experience. There is, of course, influence in both directions, and in this paper the main focus is different: how does the subjective experience influence the global affect? They suggest four facets of affective states: emotional states, attitudes, beliefs, and values/morals/ethics (DeBellis and Goldin, 1997). The present paper will use similar distinction of different aspects of mental states that underlie attitudes, although terminology and underlying theoretical foundation will be different.

Mandler's constructivist approach (e.g. 1989) sees emotions as initiated by a gut reaction to a discrepancy of an expected schema, which is followed by a cognitive analysis. According to Mandler, emotion is always expressing some aspect of value. Although Mandler's theory has been important in the field of mathematics education, it is based on an overly simplistic theory of emotions and it does not properly incorporate the influence of less intensive emotional states. Because of similar incompatibility with the theoretical foundations the vast literature on mathematical beliefs is also peripheral to the present article.

To reconceptualize attitude, we shall use emotion and cognition as two central concepts. Emotion and cognition are seen as two complementary aspects of mind. These two have some phenomenological differences that make it reasonable to separate them. Cognition is neuron-based information processing, whereas emotions include other physiological reactions, too. However, this splitting of mind into emotion and cognition is only an analytical tool, and the interaction between the two is so intense, that neither can be fully understood in separation from the other. Rather, emotion and cognition are seen as two sides of the same coin.

The present view of cognition will be outlined only briefly. Cognition is seen in a connectionist perspective, emerging from neural activity in the brain. Its three basic processes are pattern recognition, categorisation,

and association. More complex cognitive phenomena (e.g. memory and consciousness) emerge from these processes (see, for example, Bereiter and Scardamalia, 1996). Emotions need significantly more elaboration for two reasons. Firstly, there is less agreement on this field over the theories of emotion or even the definitions for emotion. Therefore the present standpoint can not be explained with such brevity, as is the case with cognition. Secondly, emotions are more central to attitudes than cognition is.

2.2. *Emotions*

Researchers have used different approaches to study emotions and although there is no final agreement upon what emotions are, there is large agreement on certain aspects. First and foremost emotions are seen in connection to personal goals. Emotions are also seen to involve a physiological reaction, as a distinction from non-emotional cognition. Thirdly, emotions are also seen to be functional, i.e. they have an important role in human coping and adaptation (e.g. Buck, 1999; Lazarus, 1991; Power and Dalgleish, 1997; Mandler, 1989).

In this paper emotions are seen as always present in human existence. However, only when the intensity of emotions is high enough, they may become observable by self and others. Emotions have three mutually independent readouts: adaptive-homeostatic arousal responses (e.g. releasing adrenaline in blood), expressive displays (e.g. smiling), and subjective experience (e.g. feeling sad) (Buck, 1999).

Regarding the number of emotions, this paper adopts the basic approach that there are only a few basic emotions: happiness, sadness, fear, anger, disgust, and interest. The more complex emotions are based on these (Buck, 1999; Power and Dalgleish, 1997).

There are at least two fundamentally different ways that stimulus may change emotional state (Power and Dalgleish, 1997). One way is the cognitive (possibly unconscious) analysis of the situation with respect to one's goals. Another route to change emotional state can be learned via classical conditioning. This learned automatic associative route between the stimulus and the emotion has an important role with respect to the simple definition of attitude as an emotional disposition.

Emotions are not only consequences of cognitive processing; they also affect cognitive processing in several ways: emotions bias attention and memory and activate action tendencies, (summary by Williams et al., 1988; in Power and Dalgleish, 1997, p. 73). Students are aware of their emotions and may reflect on and control them. In this paper these cognitive processes are called emotional cognition. Furthermore, emotions that are related to cognitive goals are called cognitive emotions. Cognitive goals may be ex-

plicit, as when one wants to remember a fact or a procedure, or vague, like 'to understand' a topic. Approaching these goals or lack of progress often induce such emotions as frustration or pride. (see Hannula, 2001 b, for further elaboration on emotional cognition and cognitive emotions).

2.3. *Attitude in cognitive-emotional terms*

Let us now explore the different emotional-cognitive processes that produce an expression of an evaluation of mathematics. Emotions are the most fundamental process, which underlies every expression of evaluation in one way or another. While a student is engaged in a mathematical activity, there is a continuous unconscious evaluation of the situation with respect to personal goals. This evaluation is represented as an emotion: proceeding towards goals induces positive emotions while obstacles that block the progress may induce anger, fear, sadness or other unpleasant emotions. Buck (1999) has separated three readouts for emotions: the adaptive-homeostatic arousal response can be measured (e.g. Isoda and Nakagoshi, 2000); the facial expression can be observed (e.g. DeBellis and Goldin, 1997); and the student can verbally express her subjective experience (affect). Hence all three readouts can produce some expression of an evaluation of mathematics.

The psychological process is slightly altered, when the student is not actually engaged in mathematical activity. For example, in case of a questionnaire the first reaction is usually emotional and based on associations. These automatic associations are a product of the student's previous experiences with mathematics. This second process falls under the simple definition of attitude as an emotional disposition.

If the student considers the response to the questionnaire any longer, an additional cognitive process sets constraints to the evaluation. In this case student may imagine a mathematical situation, and expect consequences that would involve some emotions. This expectancy is the third kind of evaluation that the student may express.

Finally, in all cases we have discussed so far, we have ignored the role of other, related goals. For example, a student may have a goal to become a veterinarian, and in order to get into the university she needs a good grade in mathematics. The student has an understanding of the personal value of mathematics. This fourth kind of evaluation is always based on a cognitive analysis (which often is unconscious) of the role of mathematics in relationship with other goals. The issue of goal systems is further elaborated in Hannula (in print).

These four evaluations are fundamentally different from each other. The first one is purely situational and it requires no prior experience of

the entity to be evaluated. When people encounter novel situations they have to rely on this evaluation. The second one, on the other hand, solely depends on previous experiences. It is the fastest of these four, and reflects the kind of evaluations one gets in typical questionnaire studies. The third evaluation is most cognitive of the four. This kind of evaluation is typically activated when the situation is partially familiar, yet has novel elements. An example of this kind of evaluation is when one has to express one's opinion about alternative settings of which one has no experience (e.g. smaller classes, single-sex setting, ability grouping). The fourth evaluation is a holistic one, it is based on the evaluation of one's whole life and the value one gives to different goals in it. This kind of evaluation is often the best explainer for student's course selections. On one hand this evaluation is based on a comparative evaluation of different goals. On the other hand it is based on expectations of how different choices lead towards different goals.

Together, these four evaluations produce attitude. Attitude is not seen as a unitary psychological construct, but as a category of behaviour that is produced by different evaluative processes. Students may express liking or disliking of mathematics because of emotions, expectations or values.

All these evaluative processes are strongly influenced by the social setting that the student is in and by the student's cognitive interpretations of the situation. However, I shall not elaborate on those stages here, but focus on what happens in the student's mind after he or she has made a cognitive interpretation of the situation. From the point of view of this paper it makes no difference whether the teacher is hostile to the student or the student only believes so.

3. METHODOLOGY

This report is part of a research project focused on the development of Finnish lower secondary school pupils' beliefs about, and attitudes towards mathematics (grades 7 to 9) (Hannula et al., 1996). Findings have been reported (for example in Hannula, 1998b, c, d, e, 2000). A type of action research was adapted to the project, in which the author acted for two years as a teacher-researcher: one class was taught mathematics in an attempt at implementing gender-inclusive pedagogy (see e.g. Solar, 1995). The pupils were observed at school. Moreover, the pupils, their parents, and their other teachers were interviewed. With repeated interviews and continuous observation it was possible to see some changes in attitudes, beliefs, and behaviour, and to recognise some plausible reasons behind these developments.

I have worked within a qualitative paradigm and I have been inspired by enactivist methodology, which encourages the use of multiple theoretical perspectives and a rich variety of data (Hannula, 1998a; Reid 1996). The focus of this paper is the development of theory, and the case of Rita is presented as an example of using the analytical framework that has been presented. However, this one case study can be read independently from the interpretations, as an inspiring narrative. Though this is my narrative, I have not created it from void. Rather, it has been extracted from both the episodes that I observed during classes, and from the stories that my informants told me in their interviews. I have selected the episodes to report and chosen the framework with which to analyse it. Other approaches to the case of Rita have been reported (Hannula, 1998b, d, e, 2000c).

A large variety of data on Rita was available to me as her teacher and her form master. I have interviewed her seven times during four years, and I have also interviewed her mother and her primary school teacher. Furthermore, I have recorded observations of her in my field notes and several lessons were video-recorded. This paper relies mainly on three first interviews and field notes. In my field notes pupils' exact words may be inaccurate, because I often had to wait until the next break before I could write them down.

4. The Data and Interpretations

The data tell a story of one student, Rita, whose attitude towards mathematics changed dramatically within a few months. I have divided the story into three 'acts'. Each act will consist of a collection of ethnographic data (mainly interview transcripts) followed by an interpretation. The first act will describe the attitude and beliefs Rita had at elementary school and at the beginning of lower secondary school. The second act describes the emotional processes of Rita, while she and her two peers solved three problems. The third act will describe the new attitude that Rita gained some time later. In the transcript omitted text has been marked with "...", unclear words with "(–)" and interrupted flow of speech with a "<". Furthermore, comments about context or tone of voice are written in curly brackets.

4.1. *Act I*

4.1.1. *"I don't remember anything, it was so stupid"*

Rita had three different teachers during her primary school years. In an interview the person who had been teaching her for the last three years gave the following description of Rita.

> She was hustling and talking all the time. . . . But Rita was easy in the sense that when you spoke with her, she always made an effort. She was a bit like Pippi Longstocking – a good heart, but somewhat of an anarchist.

I strongly agreed with this description. I thought that Rita was a pleasant pupil with a strong will. Earlier, in primary school, Rita had experienced problems with mathematics. Her previous teacher described those problems as follows.

> She was so anxious about mathematics. But then in the sixth grade she had some rather wonderful experiences of success. . . . Rita was possessed by some kind of belief that she can not understand mathematics, or that she somehow is not talented enough for it. . . . Rita if anyone, needs to have self-confidence pumped into her. . . . Rita did< I think that she had a very positive attitude towards mathematics, so that she worked, worked, worked, and bit her nails and pondered, and said things like "I can't"; "I don't learn"; "this is awful".

I conducted my first interviews with students after 4 months of teaching. So far, I had evaluated Rita's achievement in mathematics as satisfactory. The whole class was interviewed in groups of three or four (Grade 7, December). Rita was interviewed together with two other girls from her class. Their comments have mostly been omitted. As you will see, Rita's comments were not too flattering for me as her teacher.

Rita:	Mmmm. It mathematics was nicer in elementary school than it is in secondary school . . . I don't remember anything {about mathematics in elementary school} but at least we did . . . And things like that. I don't remember anything, it was so stupid.
Markku:	What was the most dull?
Rita:	The, the the the the word problems . . . I don't understand them ever.

Maria continued talking about why she prefers word problems to routine tasks. Then Rita suddenly changed the subject.

Rita: You don't need math in life. I think. Because I do know enough math to manage when I go to buy a shirt, or need to know the time or those types of things. . . . I can't explain, but in a way like now when we have really strange things in math. All that we have had in elementary school, all the fractions and those, and those you do need, but not these, these other things. These, I can't explain, the things that come, for example in ninth and in high school. You don't much need those, there in life.

Later, she complained about the book, the noise in the class, that some of the lessons are in the dining hall and not in an ordinary classroom, and that most lessons are just before lunch, when hunger makes it difficult to concentrate. However, not everything had been bad. When I asked if the pupils could recall a memorable lesson, Maria told me about a math game that we had played a couple of times. Rita remarked that it had been fun.

Also, my field notes revealed that Rita had been learning, and that she eagerly let me know that.

Rita: I have been teaching this to Nea. I couldn't do it myself, but then we understood it together. (Field notes, grade 7, October 4.)

Rita: I don't understand this! {in a short while} These are really easy! (Field notes, grade 7, November, 22.)

4.1.2. *Interpretation of act I*

So far, I have tried to be as objective as possible and only describe. However, I will once more remind that as a researcher I have selected and edited the data. Let me now give my interpretation of what these data reveal about the beliefs and attitudes Rita had.

In elementary school Rita had had many difficulties with mathematics. Although she had had some experiences of success this did not change her expectancies for mathematics lessons. In her opinion, the word problems were the dullest thing because she never understood them. This indicates that her primary reason for not liking mathematics were the unpleasant cognitive emotions she expected to experience. As a consequence of her expectation that she could not learn mathematics, she did not like it either. However, her teacher had said that Rita had had a very positive attitude. With this statement she referred to Rita's hard work and effort. In the light of the other evidence I would suggest that this reflected the values-aspect of Rita's attitude: it was important to work hard.

In secondary school mathematics was even worse. However, it was not all bad, and sometimes it was even fun. However, she did not spontaneously bring up these more positive experiences in the interview. My interpretation is that although her situational emotions in mathematics class

had been both positive and negative, the concept 'mathematics' was in her mind strongly associated with a negative emotion – either directly, or via an expectation of failure. After all, in the long run she obviously had had an overwhelming majority of negative experiences over positive ones. On the level of values, the goal of learning mathematics in school had a low ranking in her personal goal structure. Note how swiftly she changed the topic from word problems to how she would not need mathematics in life.

4.2. *Act II: "Look, you don't need this, for example, in your life"*

In the same interview as above (Grade 7, December) I presented the group of girls with three tasks to solve together. I handed out the written tasks one at a time. In the following section I shall describe an overview of the process in which I concentrate on Rita's contribution. After the description of each task will follow my interpretation of that episode. After all three tasks I will draw some conclusions from the whole problem-solving phase of this interview.

4.2.1. *Task 1*

The first task was this:

> Salla is working on an abstract painting. She has divided an area with straight lines into parts. She would like to paint the picture with as few colours as possible. The parts that are side by side, may not be of the same colour, but those touching only in the corners may. How many colours will Salla need?

Below the text was a picture that could be coloured with three colours. When I handed out the task, Rita was sitting in a position where she could not see it properly.

Rita: What's written there? . . . What's abstract?

Maria and I explained the meaning of the word 'abstract'. Maria and Lisa began solving the task together with a 'trial and error' method. They checked one area at a time ticking them with letters and mumbling "*blue, red, blue, blue*". Rita's utterances during the solution of this task are reported below with some contextual information.

{Beginning the task}		{Time: 0.00}
Rita?:	(–) yellow. . . .	{0.43}
Rita:	I don't like this task at all. . . .	{1.15}
Rita:	Yhm. Yes. . . . Then is this yellow, 'cos that (–). . . .	{2.03}
Rita:	Why is it blue then?	{2.18}
Lisa	{to Maria}: Yes, probably it would work with three colours.	
Maria	{to Lisa}: Three colours.	
Rita:	Hey! Because that one is yellow. Mm?	{2.20}
Maria	{puzzled}: What did you say?	
Rita:	What reason do you have to make that one blue?	{2.30}
Lisa:	Which one?	
Maria:	Well, may not touch except with a corner.	
Lisa:	Ah, this piece. It may not be red, because these are red. And it could be yellow, of course.	

After this, the three girls continued together to make sure that there were no mistakes and that two colours were not enough.

In another interview (grade 8, December), Rita described her emotions in this problem solving session. She said that she had been feeling *"pissed off"* because she had been *"made an outsider"*. Although a full year had passed, her feelings were still intense enough to alter the tone of her voice, which became somewhat sad. Even when she had been taken in, she had not felt welcome. She felt that the others had been pretending; "*they thought something different inside their heads*".

4.2.2. *Interpretation of task 1*

Right from the beginning of the problem solving session Rita had difficulties. She was not automatically taken into the group. She also needed help in understanding the written text. Soon Maria and Lisa began solving the task together, with effective co-operation. At the beginning of the solving process, Rita was very close to becoming frustrated. After two minutes, the first sign of understanding became apparent. When she tried to break in, the other two ignored her, at first, but she was persistent. When Rita's efforts were noticed, the others did not know what to say first. Lisa and Maria were so intensively solving the problem that it took a while before they realised what Rita was asking. Then, Rita was taken in, and she got an answer to her question. After Rita made her way in she continued to participate in checking the answer.

Rita's situational emotions during the first task can be interpreted with the obstacles on her way towards a goal. On one level she had a cognitive goal of understanding the solution of the task. On another level she had

a social goal for her interactions with her peers. Her first emotional expression was that she did not like the task. This indicates that she did not expect to reach her goals. However, her question, and her persistence to receive an answer indicate interest towards the solution. When the others did not immediately give an answer, her tone of voice indicated some anger and later she told that she had been feeling "*pissed off*". Hence, she saw the others as preventing her from reaching her goal or goals (which may have been true). However, after the minor conflict, she did achieve both of her (tentative) goals, and her overall evaluation of the first task was not negative, as we shall see later.

4.2.3. *Task 2*

This task was given immediately after task 1.

> Estimate in five minutes how many letters the author Aleksis Kivi used to write his novel, 'The Seven Brothers'.

The book was handed out to the pupils. Student's reacted immediately.

Lisa:	This is exactly (–) estimation. {tone of dislike}
Maria:	I know, I know! Hey! Lets count the letters on one page, about and then check . . . Let's see how many pages there are.
Lisa:	(–) 367, but you see
Rita:	May I write 367 here?
Markku:	Yes, you may.
Lisa:	Here there are always (–) letters
Rita:	Well, it doesn't matter (–). Here too. See, here are those texts, too.

They continued together and arrived at an estimation of 300 000.

Lisa:	It seemed like there would be a lot more.
Rita	{to Lisa}: 300 000 is a lot.
Lisa:	Well it is quite a lot, but I thought that there would be even more.
Rita:	Think that there are some< is it 5 million people in Finland?

4.2.4. *Interpretation of task 2*

This time Lisa expressed dislike of the task, and Maria became excited. Although Rita was not as good in mathematics as the others were, and she was feeling that the others did not really want her, she was an active contributor from the beginning. Rita first assumed the role of a scribe. She was also active on an intellectual level defending the chosen strategy and the result. The significance of this task is that it shows that Rita would like to have an active role in this group, and that her attitude will not explain her behaviour in any straightforward manner.

4.2.5. *Task 3*

This last task was given to students with an instruction that they could work on it as long as they wished.

> Addition, subtraction, multiplication and division are operations. Let's define a new operation # in the following way:
> When a and b are numbers, then a#b = (a+b)·(a-b).
> An example: 2#3 = (2+3)·(2–3)= 5·(–1)=–5
>
> a) Do the following calculations:
> 2# (–3) =
> (–2)#3 =
> (–2)#(–3) =
>
> b) Addition is a commutative operation. For example, 2+3=3+2.
> Is the defined operation # commutative?

There were 16 seconds of silence after they had read the task. Then Lisa and Maria began – obviously puzzled – and Rita joined in after a few lines. After a minute of puzzled re-reading of the task, Maria and Lisa began to grasp the idea of the task.

Maria:	I'm beginning to understand, or . . . first 2+3 was the 5 . . . Then 2–3 is –1. 5 times –1
Lisa:	So we ought to start 2–(–3)

After this point, Maria and Lisa worked on the task for some time before Rita gave her next comment – incorrect and ignored.

Maria:	–1 · (–6) = –6	
Lisa:	No, but. But there are two minuses.	
Maria:	–1 · (–6)	
Rita:	That will be minus (–)	
Lisa:	(–) *then* {talks over Rita}	
Maria:	–1 times · –6	
Lisa:	Did we (somehow) calculate this wrong? Will it be +6?	{time}
Rita	{to Markku}: Will it be +6?	{0.00}

I pointed out the error they made, and they corrected it. From here on Rita started to give up. She commented once in a while, as the other two continued with the task.

Rita	{tired voice}: (That is) a nice (task). . . .	{0.50}
Rita	{yawning}: What would have been the right answer? . . .	{1.15}
Rita	{offers chewing gum}: You want some? . . . {checks if she has more} Let me see. {Gives a piece to Lisa.} . . .	
Rita	{to Markku}: I don't understand a piffle of what they are even trying to do there. . . .	{1.42}
Rita	{parodying Lisa and Maria}: Minus minus five minus minus six hundred. Look, you don't need this, for example, in your life. . . . These are exactly the kind (I mean).	

When they had finished I asked the girls how they had liked the tasks. Rita thought that the first two were OK, but as for the third task. . .

Rita:	That was really stupid. You don't need those things in life.

4.2.6. *Interpretation of task 3*

Obviously none of them knew what to do in the beginning. During this puzzled period in the beginning, Rita uttered some thoughts. After this things proceeded fairly similarly to task 1. As soon as Lisa and Maria grasped the idea, Rita was left outside. After a while Rita tried to contribute, but her comment was ignored. Soon after Rita became bored, or frustrated {0.50, 1.15}. She still wanted to maintain the connection with the others, even if not within the task {1.24}. She once more asked for help by explicitly stating that she did not understand {1.42}. When she was left alone without help, she stopped trying to be included and took distance. She first ridiculed the other girls' work and finally assumed the position that this kind of mathematics is not needed in life {2.40}. However, that did not seem to be the reason that she gave up trying. *First,* she told the group that she did not understand. *Then,* she taunted the others. And *finally,* she claimed that this kind of mathematics is not needed.

It will be hard to convey the role that Rita's attitude had in this episode. What I see in this episode, is the emergence (development) of an attitude in Rita towards the third task as she is working on it with her peers. Looking back at her earlier behaviour, we can assume that her original goal was to actively participate in the solution process. As she did not understand the third task her emotions towards the task changed. Her task-related emotions were intertwined with her emotions towards being rejected by her peers. First, she expressed sorrow or frustration. This emotion was then followed by contempt (expressed by ridiculing others). The emotional state activated a value position towards this task: you don't need it. This change was functional, because it helped her to keep her self-respect in a situation that threatened it.

4.2.7. *Interpretation of act II*

Maria and Lisa understood each other well. They made a good team, within which Rita remained a bit of an outsider all the time. The detailed descriptions of the three tasks show great variety in the emotions and behaviour Rita went through. During the solution process of the first task she expressed her dislike towards the task and towards the situation (situational emotions). However, she was persistent in her efforts to participate in solving the task (positive value), and she later evaluated the task to be OK (association). The third task gave a nice example of the emergence of a negative attitude in a situation.

4.3. *Act III*

4.3.1. *"I think that now, mathematics is quite nice, sort of"*

After this interview Rita had some experiences that seem to have had an impact on her attitude towards mathematics. She did well in the next mathematics test. She was very happy about it and spontaneously commented upon it several times. She told me that this grade was the best that she had ever received on a mathematics test. (Field notes: grade 7, February)

In May, at the end of our first year, students filled out a questionnaire I had designed. One of the items dealt with the way the students thought about mathematics then as compared to the previous year. Pupils' views of mathematics had primarily changed during their first year of secondary school. Rita was one of the two students who had chosen the most extreme alternative.

Students were interviewed again, individually. Rita was asked to explain how she then thought about mathematics differently than before. (Grade 7, May)

Rita:	Yes. Sort of, now sometimes, mathematics has been a bit more fun, because I've been understanding it a bit more. I have always had a six {a rather low grade} or something in math, so I've been a bit more along now.
Markku:	Why so?
Rita:	I don't know, maybe I have started to like it more. I think that now mathematics is quite nice, sort of. In elementary school I didn't like it at all.
Markku:	Can you< Do you find anything about the secondary level that would explain why math is more fun on the secondary than on the elementary level?
Rita:	I don't really know. Maybe if I have learned more sort of tho< things sort of, so it is easier to do it. I don't know.

In this second interview Rita had told about growing as a person and taking more responsibility. She also told that she was hoping to get a good profession, and that next year she would need to spend less time with friends and more time studying. Her career aspirations, however, had remained the same as they had been in December.

The next fall, following the summer holidays, Rita was active in mathematics class and eager to learn. Some of the quotes from my field notes provide examples:

> Rita checked on the grade that she had received the previous year and said: "This year I will raise my math grade to an eight." I gave the class a collaborative task. Rita was the driving force in her group and they were the fastest to complete the task. (Field notes: Grade 8, October).

> Rita was very active today.
>
> Tina: Rita, you haven't been active in the class before.
>
> Rita: Yes I have, haven't I?
>
> Markku: You have been participating, but I don't recall you being this active last year.
>
> Rita: Good. Then I will get a nine. (Field notes: Grade 8, September).

Her energy in mathematics class had not been tempered by the fact that the content (powers) was not the kind of mathematics that she would be likely to "*need there in life*". She still had a fear of word problems (Field notes: Grade 8, October 22.). However, she was less sceptical about the usefulness of mathematics: "*I no longer say, so strictly, that you don't need it outside the school*," (Interview: Grade 8, January).

For Rita, having a positive attitude towards mathematics was almost equivalent to understanding it. When one of Rita's friends argued that she did not need powers Rita replied – not by claiming the need – but the easiness: "*These powers are really easy*," (Field notes: grade 8, September 24). At another occasion, when she had not understood a new topic she complained "*now mathematics is becoming stupid again*" and inquired wishfully "*We don't need these anywhere, do we*?" (Field notes: Grade 8, October). However, these problems did not lead to rejection, as we shall see, when Rita later had problems with irrational numbers.

> {Rita is leafing through her exercise-book} "At that time I still did understand." (Field notes, grade 8, November)
>
> "Even now I haven't understood anything about irrational numbers." (Interview, grade 8, December)
>
> "I cannot yet solve those, but it would be nice if I could. I don't understand, if I was asked what an irrational number is, so I would just stare." (Interview,

grade 8, January)

After this interview I spent some minutes teaching this topic to Rita; she understood it quite easily.

At this stage, the future seemed to worry her more and more.

> I will hardly even get into any high school ... I think that it is really, you know, ridiculous, ... that at the age of fifteen a person should sort of know what they want and where they want to go. For instance, I really didn't realise in elementary school that one sort of needs to read and even now I haven't learned to study awfully hard. (Interview, grade 8, December)

During the ninth grade I was no longer their teacher, but I was frequently observing the class. Rita continued to be active in mathematics during the ninth grade, but encountered more difficulties than the year before. She did select the more advanced mathematics course for upper secondary school, but was uncertain whether she would go through it. Therefore, she kept open the opportunity to change over to a less advanced mathematics course if she would find it too difficult. She used that opportunity, too. However, her attribution for her failure was that because of her sports activities she had not had enough time to study mathematics. She stated explicitly that in her opinion the more advanced mathematics was not too difficult for her (Interview, grade 10, May).

4.3.2. *Interpretation of act III*

The attitude of Rita – which is expressed by evaluation of mathematics – changed, regardless which of the four evaluations (emotion, expectation, association, values) we consider. Her emotions in the mathematics class still included both pleasant and unpleasant ones, but the balance had clearly changed towards the positive. Also the classical measure of attitude, her verbal expression had become favourable towards mathematics. Furthermore, success in mathematics had become an important sub-goal in her increasingly important pursuit of a good career.

How can we then explain this change? Why did it happen? What brought it about? Our first answer comes from Rita. Mathematics was "*more fun*" because she had "*been understanding more*". In another interview she remarked that "*that must be the nicest thing exactly that one understands the topic*". These statements refer to her cognitive emotions in the class. She more often than before achieved her cognitive goals and therefore her emotional experiences in the class were more pleasurable. This was also reflected in her expectations for mathematics class. Her tentative reason for understanding more was that she had maybe "*started to like it more*" and that for her "*mathematics is quite nice*" now. She also told me that she had been "*more interested*", and that she liked to "*think things over*".

At first glance, her reasoning seems circular: she liked mathematics more because she understood it, and she understood it because she liked it more. However, if we make a distinction between emotions and expectations we can make more sense of her descriptions. She expected to feel good when going to the mathematics class and her initial emotion towards new tasks was interest. In elementary school the initial emotion had been at least occasionally anxiety. Naturally, interest supported learning whilst anxiety had been hindering it.

Understanding mathematics was obviously a positive experience for Rita. Her emotional expectations had become more positive. Furthermore she had come to realize that understanding and having success in mathematics had become accessible for her (expectations). Success in mathematics was also serving her other personal goals so she made it an important sub-goal for her.

Despite all the positive development, Rita was still uncertain about her mathematical skills. However, she no longer had to deny the necessity of problematic topics but, instead, she asked for help until she understood. She now believed that she could learn even those things with which she had difficulties.

5. Discussion

There are three conclusions I wish to draw here. The most important conclusion is that the proposed framework of emotions, associations, expectations, and values is useful in describing attitudes and their changes in detail. The second conclusion is that attitudes sometimes can change dramatically, in a relatively short time. Thirdly, the negative attitude towards mathematics can be a successful defence strategy of a positive self-concept. Finally, we shall discuss the trustworthiness of the presented interpretations and some implications.

The detailed analysis of this one case illustrates how the provided framework can be used. Different aspects of attitude and the change of attitude were carefully described. This case illustrates phenomena that are attitude-related but can not be described with the concept of attitude: In the beginning Rita had an 'attitude' that was negative and positive at the same time. We may also understand the change of expectations and emotions through the use of this new framework.

Attitudes are considered to be relatively stable, as soon as they have been formed. This story gives us hope. A drastic change, almost akin to falling in love with mathematics, was a simple process (see Hannula, 1998c for another case study of a change of attitude). It took only half a

year and no special treatment was needed. But this scenario is not that simple. Rita's success in mathematics did not change very much. Furthermore, she had classmates whose attitudes followed the more common line towards disliking mathematics. There is no simple recipe for teachers behind this story. Understanding was the key concept for Rita, and in the light of other research (e.g. Boaler, 1997a, 1997b, 1998) that part is most likely generalizable. But, why did she understand, while some others did not? Was the attitude change due to the test on which she performed well? Was it simply a question of becoming more serious about school? The test might have been the trigger in her development.

Rita's behaviour in class and during the collaborative tasks provides us an example of a negative attitude as a part of a functional coping strategy. Student behaviour in class is often counterproductive if only learning goals are considered. However, the behaviour may be functional in the light of social goals (status, power) (Hannula, 2001a). Specifically, in the case of Rita, her behaviour seemed to be functional in allowing her to keep a positive self-image.

In order to evaluate the research undertaken and the results presented, I shall refer to the criteria of trustworthiness of qualitative research (Guba and Lincoln, 1994, 114). For space limitations and readability I have not been able to provide as many direct quotations as I would have wanted. Hopefully the data are sufficient for the reader to evaluate the interpretations I have made and also to create own competing interpretations. In this article only one case study is presented as an example of the use of the analytical framework. Elsewhere (Hannula, forthcoming), I have analysed two other students' attitudes with the same framework. Thus the framework is at least somewhat transferable and it appears general enough to be transferable to any age, topic or culture. However, that needs to be tested separately for each case.

An obvious implication for research methodology is that to get a refined picture of attitude one needs to define explicitly which aspects of attitude are under examination. For example, questionnaires can measure associations ("What comes to your mind from the word mathematics?") expectations ("Are you looking forward to going into a mathematics class?"), or values ("Would you study mathematics, if it were optional?"). In order to study different aspects more accurately, appropriate methods should be applied. For example, reaction times between stimulus and response could be used as a measure for associations between mathematics and different emotions.

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NOTES

1. The three component definition is well established within social psychology. For example Eagly and Chaiken (1993, p. 1) define attitude as "a psychological tendency that is expressed by evaluating a particular entity with some degree of favour or disfavour." In this definition, evaluation includes cognitive, behavioural and affective evaluations.

REFERENCES

Bereiter, C. and Scardamalia, M.: 1996, 'Rethinking learning', in D.R. Olson and N. Torrance (eds.), *The Handbook of Education and Human Development: New Models of Learning, Teaching and Schooling*, Blackwell, London, pp. 485–513.

Boaler, J.: 1997a, *Experiencing School Mathematics. Teaching Styles, Sex, and Setting*, UK: Open University Press.

Boaler, J.: 1997b, 'Reclaiming school mathematics: the girls fight back', *Gender and Education* 9, 285–305.

Boaler, J.: 1998, 'Open and closed mathematics: student experiences and understandings', *Journal for Research in Mathematics Education* 29, 41–62.

Buck, R.: 1999, 'The Biological Affects: A Typology', *Psychological Review* 106(2), 301–336.

DeBellis, V.A. and Goldin, G.A.: 1997, 'The affective domain in mathematical problem solving', in E. Pehkonen (ed.), *Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2, University of Helsinki, Lahti, Finland, pp. 209–216.

Di Martino, P. and Zan, R.: 2001, 'Attitude toward mathematics: some theoretical issues', in M. van den Heuvel-Panhuizen (ed.), *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 3, Freudenthal Institute, University of Utrecht, Utrecht, the Neatherlands, pp. 209–216.

Eagly, A.H. and Chaiken, S.: 1993, *The Psychology of Attitudes*, Harcourt Brace College Publishers, London.

Frost, L.A., Hyde, J.S. and Fennema, E.: 1994, 'Gender, mathematics performance, and mathematics related attitudes and affect: a meta-analytic synthesis', *International Journal of Educational Research* 21, 373–385.

Guba, E.G. and Lincoln, Y.S.: 1994, 'Competing paradigms in qualitative research', in N.K. Denzin and Y.S. Lincoln (eds.), *Handbook of Qualitative Research*, Sage, Thousand Oaks, California, pp. 105–117.

Haladyna, T., Shaughnessy, J. and Shaughnessy, J.M.: 1983, 'A causal analysis of attitude toward mathematics', *Journal for Research in Mathematics Education* 14, 19–29.

Hannula, M.S., Malmivuori, M.-L. and Pehkonen, E.: 1996, 'Research project: development of mathematical beliefs', in E. Pehkonen (ed.), *Current State of Research on Mathematical Beliefs; Proceedings of the MAVI-3 workshop; August 23–26, 1996*, Research report 170, Department of Teacher Education, University of Helsinki, pp. 39–48.

Hannula, M.S.: 1998a, 'Teacher as an enactivist researcher', in M. Hannula (ed.), *Current State of Research on Mathematical Beliefs: Proceedings of the MAVI-5 Workshop*. Research report 184, Department of Teacher Education, University of Helsinki, pp. 23–29.

Hannula, M.S.: 1998b, ' That was really stupid. You don't need such in life" ', in G. Törner (ed.), *Current State of Research on Mathematical Beliefs VI; Proceedings of the MAVI Workshop, University of Duisburg, March 6.–9., 1998*, Schriftenraiche des Fachbereichs Mathematik, SM-DU-404, Gerhard-Mercator-Universität Duisburg Gesamthochschule, Germany, pp. 27–32.

Hannula, M.S.: 1998c, 'Changes of beliefs and attitudes', in E. Pehkonen and G. Törner (eds.), *The State-of-Art in Mathematics-Related Belief Research; Results of the MAVI Activities. Research report 184*, Department of teacher education. University of Helsinki, pp. 198–222.

Hannula, M.S.: 1998d, 'The case of Rita: "Maybe I started to like math more." ' in A. Olivier and K. Newstead (eds.), *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education*. Vol 3, University of Stellenbosch, South Africa, pp. 33–40.

Hannula, M.S.: 1998e, 'That must have been the nicest thing exactly that one understands the topic!' in T. Breiteg and G. Brekke (eds.), *Theory into practice in Mathematics Education; Proceedings of Norma 98 the Second Nordic Conference on Mathematics Education*, Agder College, Norway, Research Series No. 13, pp. 147–152.

Hannula, M.S.: 2000, 'Mathematics, emotions, and math attitude – two case studies' in K.Hag, I. Holden and P. van Marion (eds.), *Handling bak ordene; Artikler om jenter og matematik* [Actions behind words; Articles about girls and mathematics], Norway, Trondheim: Norges teknisk-naturvitenskaplige universitetet, pp. 75–92.

Hannula, M.S.: 2001a, 'Intimacy and self-defence in problem solving', in E. Pehkonen (ed.), *Problem Solving Around the World; Proceedings of the Topic Study Group 11 (Problem solving in mathematics education) at the ICME-9 meeting August 2000 in Japan*, University of Turku, Faculty of Education, Report series C:14, pp. 67–73.

Hannula, M.S.: 2001b, 'The metalevel of emotion-cognition interaction', in M. Ahtee, O. Björkqvist, E. Pehkonen and V. Vatanen (eds.), *Research on Mathematics and Science Education. From Beliefs to Cognition, from Problem Solving to Understanding*, Institute for Educational Research, University of Jyväskylä, pp. 55–65.

Hannula, M.S.: In print, 'A case study of two students' belief systems and goal systems in a conflict over teaching methods', Proceedings of Norma 2001 conference, 8 p.

Hannula, M.S.: Forthcoming. ' "So I changed my attitude"; A case study of attitude and its development', A presentation held at the annual meeting of 'Matematiikan ja luonnontieteen opetuksen tutkimusseura' [the Finnish society for research on mathematics and science education] in Tampere 28.9.–29.9.2001.

Hembree, R.: 1990, 'The nature, effects, and relief of mathematics anxiety', *Journal for Research in Mathematics Education* 21, 33–46.

Isoda, M. and Nakagoshi, A.: 2000, 'A case study of student emotional change using changing heart rate in problem posing and solving Japanese classroom in mathematics', in T. Nakahara and M. Koyama (eds.) *Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education* Vol. 3, Hiroshima, Japan, pp. 87–94.

Lazarus, R.S.: 1991, *Emotion and Adaptation*, Oxford University Press, New York, Oxford.

Leder, G.: 1995, 'Equity inside the mathematics classroom: Fact or artifact?', in W.G. Secada, E. Fennema and L.B. Adajian (eds.), *New Directions for Equity in Mathematics Education*, Cambridge University Press.

Ma, X. and Kishor, N.: 1997, 'Assessing the relationship between attitude toward mathematics and achievement in mathematics: A meta-analyses', *Journal for Research in Mathematics Education* 28(1), 26–47.

Mandler, G.: 1989, 'Affect and learning: Causes and consequences of emotional interactions', in D.B. McLeod and V.M. Adams (eds.), *Affect and Mathematical Problem Solving: A New Perspective*, Springer-Verlag, New York, pp. 3–19.

McLeod, D.B.: 1992, 'Research on affect in mathematics education: a reconceptualization', in D.A. Grouws (ed.), *Handbook of Research on Mathematics Learning and Teaching*, MacMillan, New York, pp. 575–596.

McLeod, D.B.: 1994, 'Research on affect and mathematics learning in the JRME: 1970 to the present', *Journal for Research in Mathematics Education* 24, 637–647.

Power, M. and Dalgleish, T.: 1997, *Cognition and Emotion; From order to disorder*, Psychology Press, UK.

Reid, D.: 1996, 'Enactivism as a methodology', in L. Puig and A. Gutièrrez (eds.), *Proceedings of the 20th Conference of the International group for the Psychology of Mathematics Education*, Vol. 4., pp. 203–210.

Ridlon, C.L.: 1999, 'How a problem centered curriculum enhanched the learning of low acheivers', in F. Hitt and M. Santos (eds.), *Proceedings of the Twenty First Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Vol 2., ERIC Clearinghouse for Science, Mathematics, and Environmental Education, Columbus, OH, pp. 582–587.

Ruffell, M., Mason, J. and Allen, B.: 1998, 'Studying attitude to mathematics', *Educational Studies in Mathematics* 35, 1–18.

Solar, C.: 1995, 'An inclusive pedagogy in mathematics education', *Educational Studies in Mathematics* 28, 311–333.

Williams, J.M.G., Watts, F.N., MacLeod, C. and Mathews, A.: 1988, *Cognitive Psychology and Emotional Disorders*, Wiley, Chichester, UK.

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