

SuperQ Interview

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Presentation overview

- Problem explained
- Inputs
- Define & formulate problem
- Results
- Further improvements

Problem explained

Given a series of distance between various locations and a depot, as well as the delivery requirement for each location find the path a set of vehicles should take that minimizes the total distance travelled, such that:

- Each location is visited once,
- No vehicle exceeds its capacity before reaching the depot
- Each vehicle starts and stops at depot

Inputs

Distance matrix (from row i to column j)

```
[0, 12, 9, 21, 18, 15, 28, 25, 19, 24, 27, 14, 17, 30, 22, 20, 23, 16, 26, 29, 13],  
[12, 0, 7, 16, 14, 11, 24, 22, 15, 19, 23, 10, 13, 26, 18, 16, 20, 12, 21, 24, 8],  
[9, 7, 0, 15, 12, 10, 21, 19, 14, 17, 20, 9, 11, 24, 16, 14, 18, 10, 19, 21, 6],  
[21, 16, 15, 0, 8, 17, 12, 14, 18, 10, 13, 19, 20, 9, 11, 15, 10, 18, 12, 14, 17],  
[18, 14, 12, 8, 0, 13, 10, 12, 15, 8, 11, 15, 17, 6, 9, 12, 8, 15, 9, 11, 14],  
[15, 11, 10, 17, 13, 0, 19, 17, 12, 16, 19, 12, 14, 20, 13, 11, 15, 11, 18, 19, 10]  
[28, 24, 21, 12, 10, 19, 0, 9, 14, 8, 7, 22, 23, 11, 7, 13, 10, 15, 9, 8, 20],  
[25, 22, 19, 14, 12, 17, 9, 0, 11, 9, 10, 20, 21, 9, 10, 12, 9, 14, 8, 10, 18],  
[19, 15, 14, 18, 15, 12, 14, 11, 0, 13, 16, 13, 15, 17, 12, 10, 13, 11, 15, 16, 12],  
[24, 19, 17, 10, 8, 16, 8, 9, 13, 0, 7, 17, 18, 7, 10, 14, 9, 16, 10, 9, 16],  
[27, 23, 20, 13, 11, 19, 7, 10, 16, 7, 0, 20, 22, 8, 11, 18, 12, 19, 13, 11, 21],  
[14, 10, 9, 19, 15, 12, 22, 20, 13, 17, 20, 0, 10, 22, 15, 13, 17, 11, 18, 20, 9],  
[17, 13, 11, 20, 17, 14, 23, 21, 15, 18, 22, 10, 0, 24, 17, 15, 19, 13, 20, 22, 12]  
[30, 26, 24, 9, 6, 20, 11, 9, 17, 7, 8, 22, 24, 0, 8, 17, 11, 19, 8, 6, 23],  
[22, 18, 16, 11, 9, 13, 7, 10, 12, 10, 11, 15, 17, 8, 0, 11, 9, 14, 10, 8, 19],  
[20, 16, 14, 15, 12, 11, 13, 12, 10, 14, 18, 13, 15, 17, 11, 0, 7, 12, 14, 15, 11],  
[23, 20, 18, 10, 8, 15, 10, 9, 13, 9, 12, 17, 19, 11, 9, 7, 0, 14, 9, 11, 20],  
[16, 12, 10, 18, 15, 11, 15, 14, 11, 16, 19, 11, 13, 19, 14, 12, 14, 0, 17, 19, 10]  
[26, 21, 19, 12, 9, 18, 9, 8, 15, 10, 13, 18, 20, 8, 10, 14, 9, 17, 0, 9, 22],  
[29, 24, 21, 14, 11, 19, 8, 10, 16, 9, 11, 20, 22, 6, 8, 15, 11, 19, 9, 0, 23],  
[13, 8, 6, 17, 14, 10, 20, 18, 12, 16, 21, 9, 12, 23, 19, 11, 20, 10, 22, 23, 0]
```

Demand vector (column j)

```
[0, 8, 12, 7, 15, 9, 11, 6, 14, 8, 10, 7, 13, 5, 9, 12, 6, 8, 11, 10, 9],
```

- Number of vehicles: 2
- Vehicle capacity: 100
- 20 locations + 1 depot

Define & formulate problem

- $N = \{0, 1, \dots, n - 1\}$: set of all locations
- Depot is node 0
- $C = N \setminus \{0\}$: customer nodes
- $V = \{1, \dots, K\}$: set of vehicles
- d_{ij} : distance from node i to node j (distance_matrix)
- q_j : demand at node j (demand_vector), with $q_0 = 0$
- Q : vehicle capacity (vehicle_capacity)

Define & formulate problem

Routing variables

$$x_{ijv} = \begin{cases} 1 & \text{if vehicle } v \text{ travels directly from } i \text{ to } j \\ 0 & \text{otherwise} \end{cases}$$

defined for all $i, j \in N, i \neq j, v \in V$

Load tracking variables

$$L_{jv} \geq 0$$

| Continuous variable representing cumulative load on vehicle v upon arrival at node j .

Define & formulate problem

Objective Function

Minimize total distance traveled by all vehicles:

$$\min \sum_{v \in V} \sum_{i \in N} \sum_{\substack{j \in N \\ j \neq i}} d_{ij} x_{ijv}$$

Define & formulate problem

1. Each customer must be visited exactly once

Every non-depot location must be arrived at exactly once by some vehicle:

$$\sum_{v \in V} \sum_{\substack{i \in N \\ i \neq j}} x_{ijv} = 1 \quad \forall j \in C$$

2. Each customer must be departed exactly once

$$\sum_{v \in V} \sum_{\substack{j \in N \\ j \neq i}} x_{ijv} = 1 \quad \forall i \in C$$

3. Flow conservation for each vehicle

For each vehicle and each customer node, the number of arrivals must equal the number of departures:

$$\sum_{\substack{j \in N \\ j \neq i}} x_{ijv} = \sum_{\substack{j \in N \\ j \neq i}} x_{jiv} \quad \forall i \in C, \forall v \in V$$

Define & formulate problem

4. Each vehicle starts at the depot exactly once

$$\sum_{j \in C} x_{0jv} = 1 \quad \forall v \in V$$

5. Each vehicle ends at the depot exactly once

$$\sum_{j \in C} x_{j0v} = 1 \quad \forall v \in V$$

Define & formulate problem

6. Vehicles start empty at the depot

$$L_{0v} = 0 \quad \forall v \in V$$

7. Load propagation along traveled edges

If a vehicle travels from i to j , its load at j must equal its load at i plus the demand at j .

This is modeled using an indicator constraint in the code:

$$x_{ijv} = 1 \Rightarrow L_{jv} = L_{iv} + q_j \quad \forall v \in V, i \neq 0, j \neq 0, i \neq j$$

This enforces the cumulative load behavior without requiring a big- M formulation.

8. Minimum load requirement when a node is visited

If vehicle v visits node j , its load at that node must be at least the demand there:

$$L_{jv} \geq q_j \sum_{\substack{i \in N \\ i \neq j}} x_{ijv} \quad \forall j \in C, \forall v \in V$$

Define & formulate problem

9. Vehicle capacity limit

At no point may a vehicle exceed its capacity:

$$L_{jv} \leq Q \quad \forall j \in C, \forall v \in V$$

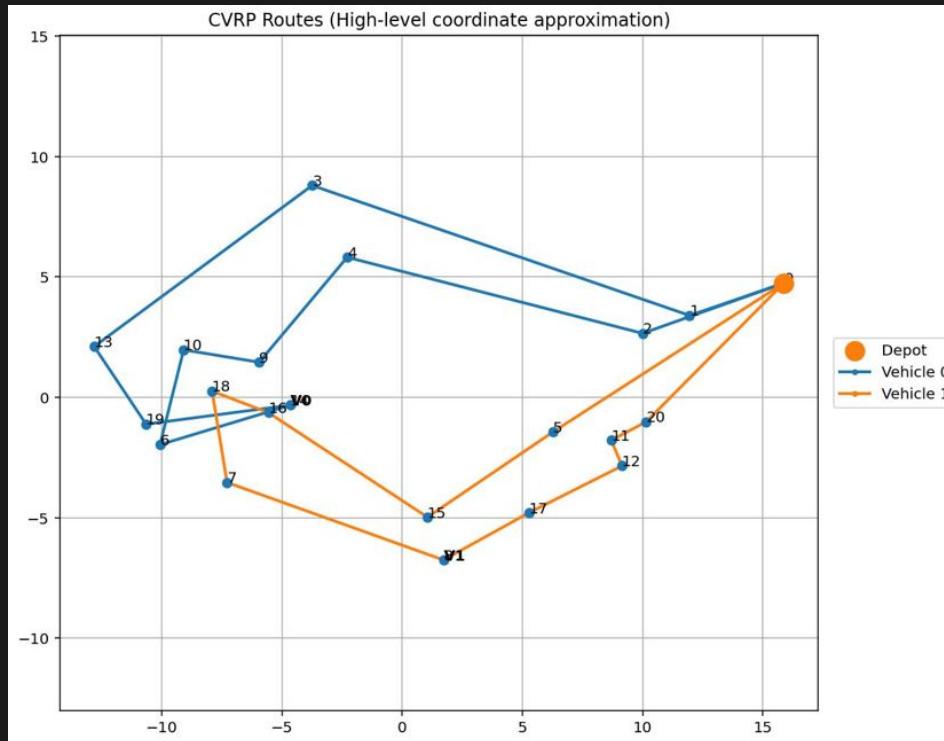
Results

Gap convergence value: **8%**

Minimum distance travelled for entire fleet: **218**

V0: 0 -> 1 -> 3 -> 13 -> 19 -> 14 -> 6 -> 10 -> 9 -> 4 -> 2 -> 0

V1: 0 -> 20 -> 11 -> 12 -> 17 -> 8 -> 7 -> 18 -> 16 -> 15 -> 5 -> 0



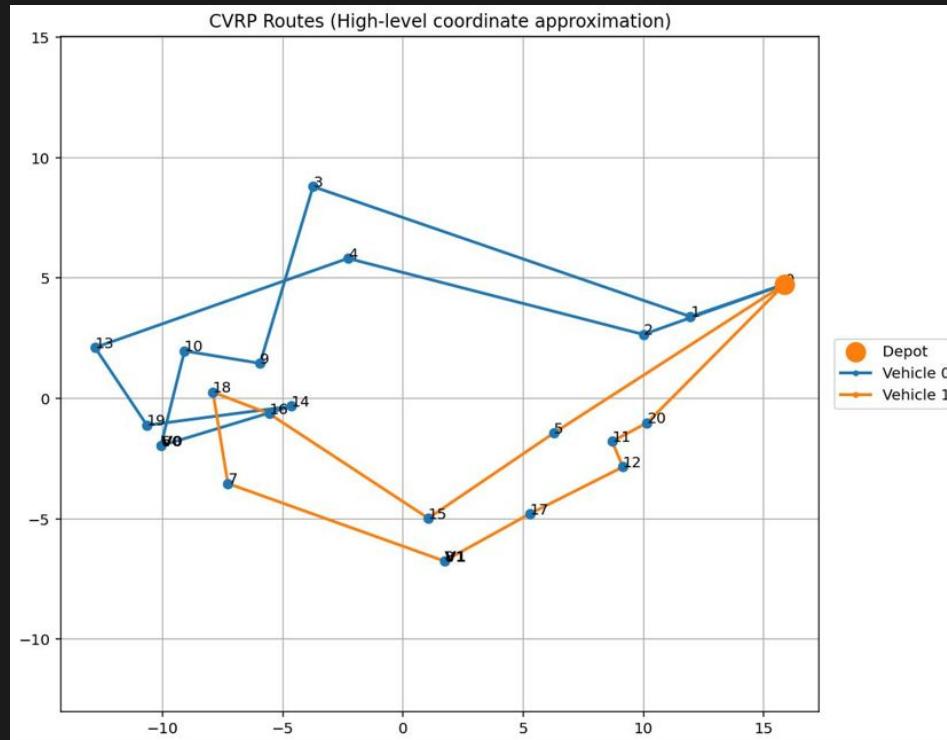
Results

Gap convergence value: 7%

Minimum distance travelled for entire fleet: 217

V0: 0 -> 1 -> 3 -> 9 -> 10 -> 6 -> 14 -> 19 -> 13 -> 4 -> 2 -> 0

V1: 0 -> 20 -> 11 -> 12 -> 17 -> 8 -> 7 -> 18 -> 16 -> 15 -> 5 -> 0



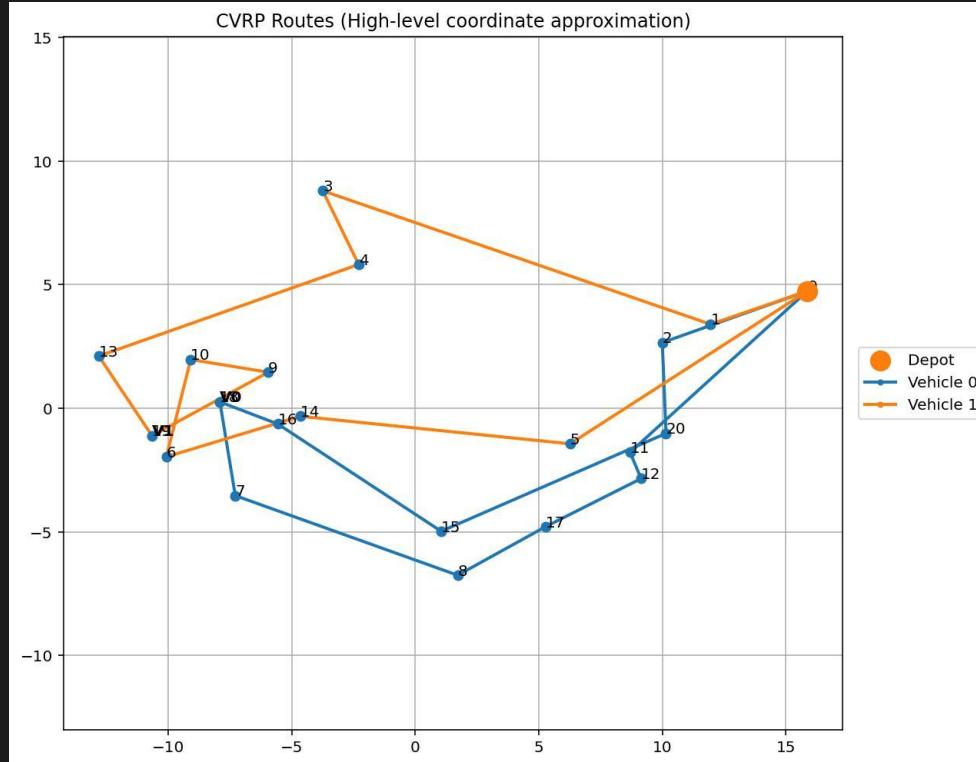
Results

Gap convergence value: **6%**

Minimum distance travelled for entire fleet: **215**

V0: 0 -> 2 -> 20 -> 15 -> 16 -> 18 -> 7 -> 8 -> 17 -> 12 -> 11 -> 0

V1: 0 -> 1 -> 3 -> 4 -> 13 -> 19 -> 9 -> 10 -> 6 -> 14 -> 5 -> 0



Further improvements

- If fleet expands, relax constraint that every vehicle is required
- Lower gap value by allowing more runtime or using better solver
- Implement time/schedule constraints