UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
DIVISION OF ENGINEERING SCIENCE

ECE355H1 F - Signal Analysis and Communication

Problem Set 2
Fall 2023

Submit by: **September 29, 2023**

Problem 1

(Problem 1.16 - Textbook)

Consider a discrete-time system with input $x[n]$ and output $y[n]$. The input-output relationship for this system is

$$y[n] = x[n]x[n-2].$$

a) Is this system memoryless?

b) Determine the output of the system when the input is $A\delta[n]$, where A is any real or complex number.

c) Is the system invertible?

Problem 2

(Problem 1.17 - Textbook)

Consider a continuous-time system with input $x(t)$ and output $y(t)$ related by

$$y(t) = x(\sin(t)).$$

a) Is this system causal?

b) Is this system linear?

Problem 3

(Problem 1.27 (a, d, e) - Textbook)

In this chapter, we introduced a number of general properties of systems. In particular, a system may or may not be

1) Memoryless

2) Time invariant

3) Linear

4) Causal

5) Stable

Determine which of these properties hold and which do not hold for each of the following continuous-time systems. Justify your answers. In each example, $y(t)$ denotes the system output and $x(t)$ is the system input.

a) $y(t) = x(t-2) + x(2-t)$

b) $y(t) = \begin{cases} 0, & t < 0 \\ x(t) + x(t-2), & t \geq 0 \end{cases}$

c) $y(t) = \begin{cases} 0, & x(t) < 0 \\ x(t) + x(t-2), & x(t) \geq 0 \end{cases}$

Problem 4

(Problem 1.28 (a, d, g) - Textbook)

Determine which of the properties listed in Problem 3 hold and which do not hold for each of the following discrete-time systems. Justify your answers. In each example, $y[n]$ denotes the system output and $x[n]$ is the system input.

a) $y[n] = x[-n]$

b) $y[n] = \mathcal{E}v\{x[n-1]\}$

c) $y[n] = x[4n+1]$

Problem 5

(Problem 1.30 (c, d, k, l) - Textbook)

Determine if each of the following systems is invertible. If it is, construct the inverse system. If it is not, find two input signals to the system that have the same output.

a) $y[n] = nx[n]$

b) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

c) $y[n] = \begin{cases} x[n+1], & n \geq 0 \\ x[n], & n \leq -1 \end{cases}$

d) $y(t) = x(2t)$

Problem 6

(Problem 1.31 - Textbook)

In this problem, we illustrate one of the most important consequences of the properties of linearity and time invariance. Specifically, once we know the response of a linear system or a linear time-invariant

(LTI) system to a single input or the responses to several inputs, we can directly compute the responses to many other input signals. Much of the remainder of this book deals with a thorough exploitation of this fact in order to develop results and techniques for analyzing and synthesizing LTI systems.

a) Consider an LTI system whose response to the signal $x_1(t)$ in Figure 1(a) is the signal $y_1(t)$ illustrated in Figure 1(b). Determine and sketch carefully the response of the system to the input $x_2(t)$ depicted in Figure 1(c).

b) Determine and sketch the response of the system considered in part (a) to the input $x_3(t)$ shown in Figure 1(d).

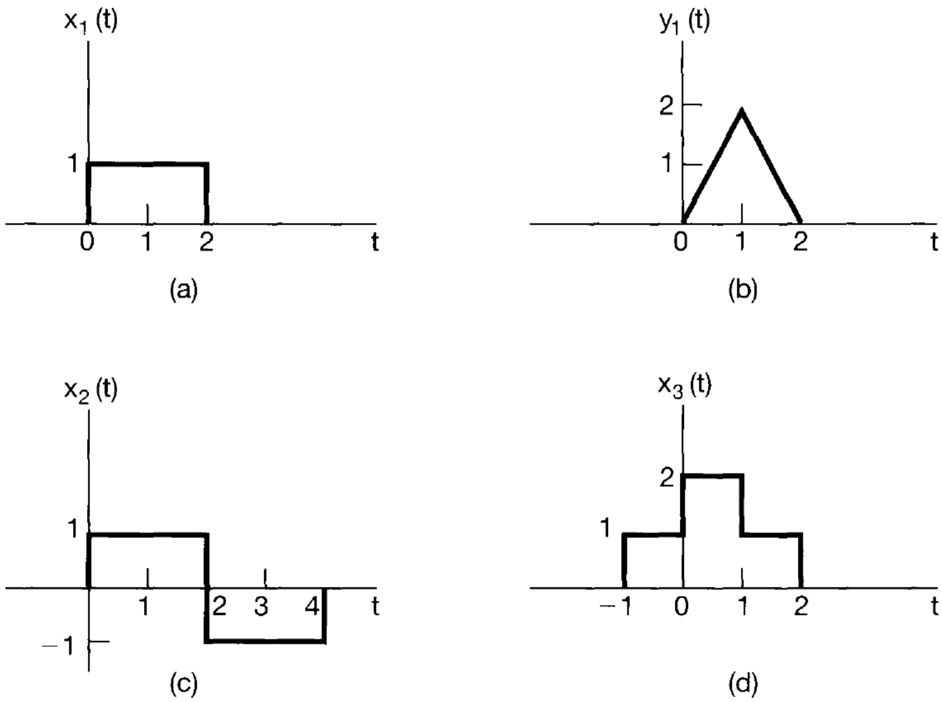

Figure 1: Problem 6

Problem 7

(Problem 2.2 - Textbook)

Consider the signal

$$h[n] = \left(\frac{1}{2}\right)^{n-1} \{u[n+3] - u[n-10]\}.$$

Express A and B in terms of n so that the following equation holds:

$$h[n-k] = \begin{cases} (\frac{1}{2})^{n-k-1}, & A \leq k \leq B \\ 0, & \text{elsewhere} \end{cases}$$

Problem 8

(Problem 2.21 (b, d) - Textbook)

Compute the convolution $y[n] = x[n] * h[n]$ of the following pairs of signals:

a) $\left.\begin{aligned} x[n] &= \alpha^n u[n], \\ h[n] &= \beta^n u[n], \end{aligned}\right\} \alpha \neq \beta$

b) $x[n]$ and $h[n]$ are as in Figure 2.

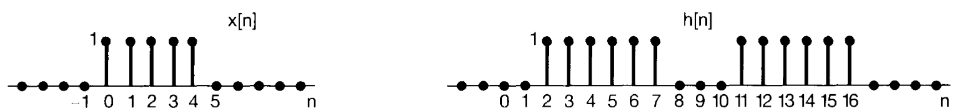

Figure 2: Problem 8

Textbook

Alan V. Oppenheim, Alan S. Willsky, and S. Hamid Nawab, Signals & Systems, 2nd Ed., Prentice-Hall, 1996 (ISBN 0-13-814757-4)