UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
DIVISION OF ENGINEERING SCIENCE

ECE355H1 F - Signal Analysis and Communication

Problem Set 3
Fall 2023

Submit by: **October 6, 2023**

Problem 1

(Problem 2.9 - Textbook)

Let

$$h(t) = e^{2t}u(-t+4) + e^{-2t}u(t-5)$$

Determine A and B such that

$$h(t-\tau) = \begin{cases} e^{-2(t-\tau)}, & \tau < A \\ 0, & A < \tau < B \\ e^{2(t-\tau)}, & B < \tau \end{cases}$$

Problem 2

(Problem 2.10 - Textbook)

Suppose that

$$x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

and $h(t) = x(t/\alpha)$, where $0 < \alpha \le 1$.

a) Determine and sketch $y(t) = x(t) * h(t)$

b) If $dy(t)/dt$ contains only three discontinuities, what is the value of α?

Problem 3

(Problem 2.11 - Textbook)

Let

$$x(t) = u(t-3) - u(t-5) \text{ and } h(t) = e^{-3t}u(t)$$

a) Compute $y(t) = x(t) * h(t)$.

b) Compute $g(t) = (dx(t)/dt) * h(t)$.

c) How is $g(t)$ related to $y(t)$?

Problem 4

(Problem 2.16 - Textbook)

For each of the following statements, determine whether it is true or false:

a) If $x[n] = 0$ for $n < N_1$ and $h[n] = 0$ for $n < N_2$, then $x[n] * h[n] = 0$ for $n < N_1 + N_2$.

b) If $y[n] = x[n] * h[n]$, then $y[n-1] = x[n-1] * h[n-1]$.

c) If $y(t) = x(t) * h(t)$, then $y(-t) = x(-t) * h(-t)$.

d) If $x(t) = 0$ for $t > T_1$ and $h(t) = 0$ for $t > T_2$, then $x(t) * h(t) = 0$ for $t > T_1 + T_2$.

Problem 5

(Problem 2.22 (b, d) - Textbook)

For each of the following pairs of waveforms, use the convolution integral to find the response $y(t)$ of the LTI system with impulse response $h(t)$ to the input $x(t)$. Sketch your results.

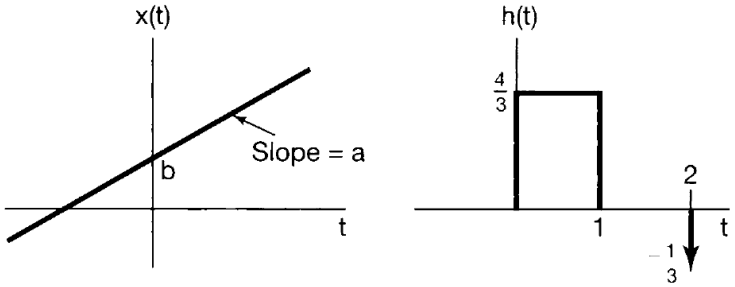

Figure 1: Problem 5 (b)

a) $x(t) = u(t) - 2u(t-2) + u(t-5)$
$h(t) = e^{2t}u(1-t)$

b) $x(t)$ and $h(t)$ are as in Figure 1.

Problem 6

(Problem 2.23 - Textbook)

Let $h(t)$ be the triangular pulse shown in Figure 2(a), and let $x(t)$ be the impulse train depicted in Figure 2(b). That is,

$$x(t) = \sum_{k=-\infty}^{+\infty} \delta(t - kT).$$

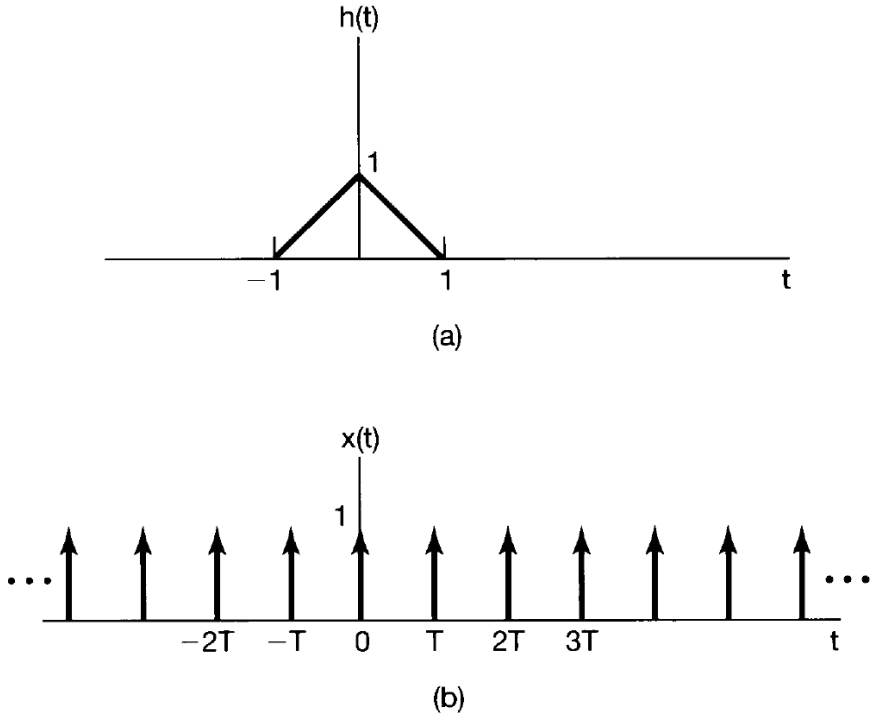

Figure 2: Problem 6

Determine and sketch $y(t) = x(t) * h(t)$ for the following values of T:

a) $T = 4$

b) $T = 2$

c) $T = 3/2$

d) $T = 1$

Problem 7

(Problem 2.24 - Textbook)

Consider the cascade interconnection of three causal LTI systems, illustrated in Figure 3(a). The impulse response $h_2[n]$ is

$$h_2[n] = u[n] - u[n-2],$$

and the overall impulse response is as shown in Figure 3(b).

a) Find the impulse response $h_1[n]$.

b) Find the response of the overall system to the input

$$x[n] = \delta[n] - \delta[n-1].$$

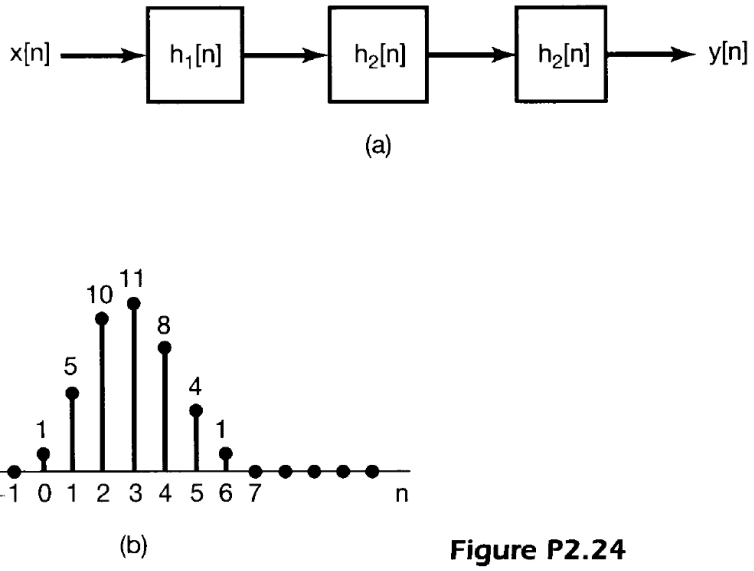

Figure 3: Problem 7

Problem 8

(Problem 2.25 - Textbook)

Let the signal

$$y[n] = x[n] * h[n],$$

where

$$x[n] = 3^n u[-n-1] + \left(\frac{1}{3}\right)^n u[n]$$

and

$$h[n] = \left(\frac{1}{4}\right)^n u[n+3].$$

a) Determine $y[n]$ *without* utilizing the distributive property of convolution.

b) Determine $y[n]$ *utilizing* the distributive property of convolution.

Problem 9

(Problem 2.27 - Textbook)

We define the area under a continuous-time signal $v(t)$ as

$$A_v = \int_{-\infty}^{+\infty} v(t)dt,$$

Show that if $y(t) = x(t) * h(t)$, then

$$A_y = A_x \, A_h$$

Textbook

Alan V. Oppenheim, Alan S. Willsky, and S. Hamid Nawab, Signals & Systems, 2nd Ed., Prentice-Hall, 1996 (ISBN 0-13-814757-4)