

Problem Set 03

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Problem 1

$$h(t - \tau) = e^{2(t-\tau)}u(\tau + 4 - t) + e^{-2(t-\tau)}u(t - \tau - 5) \quad (1)$$

$$e^{2(t-\tau)}u(\tau + 4 - t) = \begin{cases} e^{2(t-\tau)} & \text{if } \tau < t - 4 \\ 0 & \text{if } \tau > t - 4 \end{cases} \quad (2)$$

$$e^{-2(t-\tau)}u(t - \tau - 5) = \begin{cases} e^{-2(t-\tau)} & \text{if } \tau < t + 5 \\ 0 & \text{if } \tau > t + 5 \end{cases} \quad (3)$$

$$h(t - \tau) = \begin{cases} e^{-2(t-\tau)} & \text{if } \tau < t + 5 \\ 0 & \text{if } t - 4 < \tau < t + 5 \\ e^{2(t-\tau)} & \text{if } t - 4 < \tau \end{cases} \quad (4)$$

$$\therefore A = t + 5 \text{ and } B = t - 4$$

Problem 2

Part a)

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau) d\tau \quad (1)$$

Case 1: No Overlap

$$y(t) = 0 \text{ for } t < 0 \text{ and } t - \alpha > 1 \text{ or } t > 1 + \alpha \quad (2)$$

Case 2: Partial Overlap

$$y(t) = \int_0^t x(\tau) d\tau \text{ for } t \geq 0 \text{ \& } t - \alpha < 0 \text{ or } 0 \leq t < \alpha \quad (3)$$

Case 3: Full Overlap

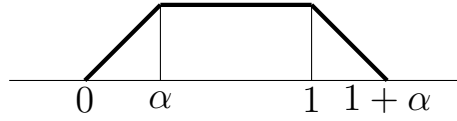
$$y(t) = \int_{t-\alpha}^t 1 d\tau = \alpha \text{ for } t - \alpha \geq 0 \text{ \& } t < 1 \text{ or } \alpha \leq t < 1 \quad (4)$$

Case 4: Partial Overlap

$$y(t) = \int_{t-\alpha}^1 1 d\tau = 1 - \alpha \text{ for } t - \alpha < 1 \text{ \& } t \geq 1 \text{ or } 1 \leq t < \alpha + 1 \quad (5)$$

Finally:

$$y(t) = \begin{cases} t & \text{if } 0 \leq t \leq \alpha \\ \alpha & \text{if } \alpha \leq t < 1 \\ 1 + \alpha - t & \text{if } 1 \leq t < 1 + \alpha \\ 0 & \text{Otherwise} \end{cases} \quad (6)$$



Part b)

Current discontinuities of $\frac{dy}{dt}$ are at $t = 0, \alpha, 1, 1 + \alpha$. To remove one of the discontinuities we can set $\alpha = 1$.

Problem 3

Part a)

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau) d\tau \quad (1)$$

$$= \int_{-\infty}^{\infty} h(\tau)x(t-\tau) d\tau \quad (2)$$

$$= \int_{-\infty}^{\infty} h(\tau)(u(t-3-\tau) - u(t-5-\tau)) d\tau \quad (3)$$

$$= \int_{-\infty}^{\infty} e^{-3\tau}u(\tau)(u(t-3-\tau) - u(t-5-\tau)) d\tau \quad (4)$$

$$= \int_0^{\infty} e^{-3\tau}(u(t-3-\tau) - u(t-5-\tau)) d\tau \quad (5)$$

The above $(u(t-3-\tau) - u(t-5-\tau))$ is only non-zero when $(t-5) < \tau < (t-3)$.

$$y(t) = \int_0^{t-3} e^{-3\tau} d\tau = \frac{1 - e^{-3(t-3)}}{3} \text{ for } 3 < t \leq 5 \quad (6)$$

$$y(t) = \int_{t-5}^{t-3} e^{-3\tau} d\tau = \frac{(1 - e^{-5})e^{-3(t-5)}}{3} \text{ for } t > 5 \quad (7)$$

$$\therefore y(t) = \begin{cases} 0 & \text{if } -\infty < t \leq 3 \\ \frac{1-e^{-3(t-3)}}{3} & \text{if } 3 < t \leq 5 \\ \frac{(1-e^{-5})e^{-3(t-5)}}{3} & \text{if } 5 < t \leq \infty \end{cases} \quad (8)$$

Part b)

Part c)

Problem 4

Problem 5

Problem 6

Problem 7

Problem 8

Problem 9