Problem Set 03

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Problem 1

$$h(t-\tau) = e^{2(t-\tau)}u(\tau+4-t) + e^{-2(t-\tau)}u(t-\tau-5) \tag{1}$$

$$e^{2(t-\tau)}u(\tau+4-t) = \begin{cases} e^{2(t-\tau)} & \text{if } \tau < t-4 \\ 0 & \text{if } \tau > t-4 \end{cases} \tag{2}$$

$$e^{-2(t-\tau)}u(t-\tau-5) = \begin{cases} e^{-2(t-\tau)} & \text{if } \tau < t+5 \\ 0 & \text{if } \tau > t+5 \end{cases} \tag{3}$$

$$h(t-\tau) = \begin{cases} e^{-2(t-\tau)} & \text{if } \tau < t+5 \\ 0 & \text{if } t-4 < \tau < t+5 \\ e^{2(t-\tau)} & \text{if } t-4 < \tau \end{cases} \tag{4}$$

$$\therefore A = t+5 \text{ and } B = t-4$$

Problem 2

Part a)

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau \tag{1}$$

Case 1: No Overlap

$$y(t) = 0 \text{ for } t < 0 \text{ and } t-\alpha > 1 \text{ or } t > 1+\alpha \tag{2}$$

Case 2: Partial Overlap

$$y(t) = \int_0^t x(\tau)\,d\tau \text{ for } t \geq 0 \;\&\; t-\alpha < 0 \text{ or } 0 \leq t < \alpha \tag{3}$$

Case 3: Full Overlap

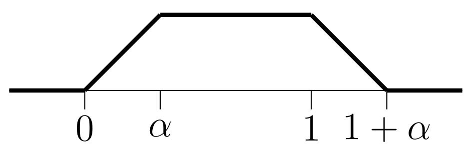
$$y(t) = \int_{t-\alpha}^{t} 1\, d\tau = \alpha \text{ for } t - \alpha \geq 0 \;\&\; t < 1 \text{ or } \alpha \leq t < 1 \tag{4}$$

Case 4: Partial Overlap

$$y(t) = \int_{t-\alpha}^{1} 1\, d\tau = 1 - \alpha \text{ for } t - \alpha < 1 \;\&\; t \geq 1 \text{ or } 1 \leq t < \alpha + 1 \tag{5}$$

Finally:

$$y(t) = \begin{cases} t & \text{if } 0 \leq t \leq \alpha \\ \alpha & \text{if } \alpha \leq t < 1 \\ 1 + \alpha - t & \text{if } 1 \leq t < 1 + \alpha \\ 0 & \text{Otherwise} \end{cases} \tag{6}$$



Part b)

Current discontinuities of $\frac{dy}{dt}$ are at $t = 0, \alpha, 1, 1 + \alpha$. To remove one of the discontinuities we can set $\alpha = 1$.

Problem 3

Part a)

$$
\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau && (1)\\
&= \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau && (2)\\
&= \int_{-\infty}^{\infty} h(\tau)(u(t-3-\tau)-u(t-5-\tau))\,d\tau && (3)\\
&= \int_{-\infty}^{\infty} e^{-3\tau}u(\tau)(u(t-3-\tau)-u(t-5-\tau))\,d\tau && (4)\\
&= \int_{0}^{\infty} e^{-3\tau}(u(t-3-\tau)-u(t-5-\tau))\,d\tau && (5)
\end{aligned}
$$

The above $(u(t-3-\tau)-u(t-5-\tau))$ term is only non-zero when $(t-5) < \tau < (t-3)$.

$$
\begin{aligned}
y(t) &= \int_{0}^{t-3} e^{-3\tau}\,d\tau = \frac{1-e^{-3(t-3)}}{3} \text{ for } 3 < t \le 5 && (6)\\
y(t) &= \int_{t-5}^{t-3} e^{-3\tau}\,d\tau = \frac{(1-e^{-5})e^{-3(t-5)}}{3} \text{ for } t > 5 && (7)
\end{aligned}
$$

$$
\therefore y(t) = \begin{cases} 0 & \text{if } -\infty < t \le 3 \\ \frac{1-e^{-3(t-3)}}{3} & \text{if } 3 < t \le 5 \\ \frac{(1-e^{-5})e^{-3(t-5)}}{3} & \text{if } 5 < t \le \infty \end{cases} \qquad (8)
$$

Part b)

$$
\begin{aligned}
\frac{dx}{dt} &= \delta(t-3)-\delta(t-5) && (9)\\
g(t) &= \frac{dx}{dt} * h(t) = e^{-3(t-3)}u(t-3) - e^{-3(t-5)}u(t-5) && (10)
\end{aligned}
$$

$$g(t) = \begin{cases} 0 & \text{if } -\infty < t \leq 3 \\ e^{-3(t-3)} & \text{if } 3 < t \leq 5 \\ (e^{-6} - 1)e^{-3(t-5)} & \text{if } 5 < t \leq \infty \end{cases} \tag{11}$$

Part c)

If we compute the derivative of $y(t)$ we get:

$$y(t) = \begin{cases} 0 & \text{if } -\infty < t \leq 3 \\ e^{-3(t-3)} & \text{if } 3 < t \leq 5 \\ (e^{-6} - 1)e^{-3(t-5)} & \text{if } 5 < t \leq \infty \end{cases} \tag{12}$$

$$\therefore\ y(t) = g(t)$$