UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
DIVISION OF ENGINEERING SCIENCE

ECE355H1 F - Signal Analysis and Communication

Problem Set 5
Fall 2023

Submit by: **October 20, 2023**

Problem 1

(Problem 3.4 - Textbook)

Use the Fourier series analysis equation to calculate the coefficients a_k for the continuous-time periodic signal

$$x(t) = \begin{cases} 1.5, & 0 \leq t < 1 \\ -1.5, & 1 \leq t < 2 \end{cases}$$

with fundamental frequency $\omega_0 = \pi$.

Problem 2

(Problem 3.13 - Textbook)

Consider a continuous-time LTI system whose frequency response is

$$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t}\, dt = \frac{\sin(4\,\omega)}{\omega}$$

If the input to this system is a periodic signal

$$x(t) = \begin{cases} 1, & 0 \leq t < 4 \\ -1, & 4 \leq t < 8 \end{cases}$$

with period $T = 8$, determine the corresponding system output $y(t)$.

Problem 3

(Problem 3.20 - Textbook)

Consider a causal LTI system implemented as the RLC circuit shown in Figure 1. In this circuit, $x(t)$ is the input voltage. The voltage $y(t)$ across the capacitor is considered the system output.

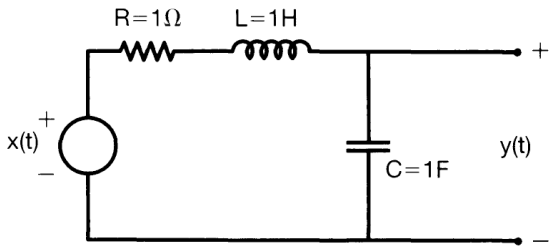

Figure 1: Problem 3

a) Find the differential equation relating $x(t)$ and $y(t)$.

b) Determine the frequency response of this system by considering the output of the system to inputs of the form $x(t) = e^{j\omega t}$.

c) Determine the output $y(t)$ if $x(t) = \sin(t)$.

Problem 4

(Problem 3.22 (a (d, e), c) - Textbook)

Determine the Fourier series representations for the following signals:

a) Each $x(t)$ illustrated in Figure 2a and Figure 2b.

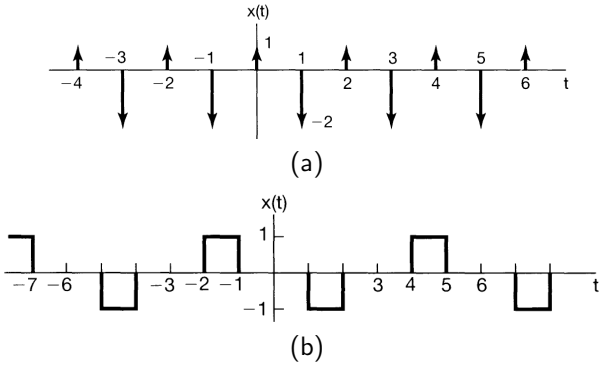

Figure 2: Problem 4

b) $x(t)$ periodic with period 4 and

$$x(t) = \begin{cases} \sin \pi t, & 0 \leq t \leq 2 \\ 0, & 2 < t \leq 4 \end{cases}$$

Problem 5

(Problem 3.23 (a, c) - Textbook)

In each of the following, we specify the Fourier series coefficients of a continuous time signal that is periodic with period 4. Determine the signal $x(t)$ in each case.

a) $a_k = \begin{cases} 0, & k = 0 \\ (j)^k \frac{\sin\, k\pi/4}{k\pi}, & \text{otherwise} \end{cases}$

b) $a_k = \begin{cases} jk, & |k| < 3 \\ 0, & \text{otherwise} \end{cases}$

Textbook

Alan V. Oppenheim, Alan S. Willsky, and S. Hamid Nawab, Signals & Systems, 2nd Ed., Prentice-Hall, 1996 (ISBN 0-13-814757-4)