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ECE355H1 F - Signal Analysis and Communication

Problem Set 2
Fall 2023

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Notation: In the following, we use $x(t) \longrightarrow y(t)$ to indicate that $y(t)$ is the output of a CT system for an input $x(t)$. Similarly, we use $x(t) \not\longrightarrow y(t)$ to indicate that a CT system with input $x(t)$ does *not* produce an output $y(t)$. The same notation is also used in the case of DT systems.

Problem 1

(Problem 1.16 - Textbook)

Consider a discrete-time system with input $x[n]$ and output $y[n]$. The input-output relationship for this system is

$$y[n] = x[n]x[n-2].$$

a) Is this system memoryless?

b) Determine the output of the system when the input is $A\delta[n]$, where A is any real or complex number.

c) Is the system invertible?

Solution

a) The system is *not* memoryless, since output $y[n]$ at any time n depends on a *past* input value $x[n-2]$, e.g., $y[0]$ depends on $x[-2]$.

b)
$$x[n] = A\delta[n] \longrightarrow y[n] = A\delta[n]A\delta[n-2] = A^2\delta[n]\delta[n-2] = 0, \ \forall n.$$

c) The system is *not* invertible. For example, from part *b*) we know that the two *different* inputs $\delta[n]$ and $2\delta[n]$ will produce the *same* output $y[n] = 0, \ \forall n$.

Problem 2

(Problem 1.17 - Textbook)

Consider a continuous-time system with input $x(t)$ and output $y(t)$ related by

$$y(t) = x(\sin(t)).$$

a) Is this system causal?

b) Is this system linear?

Solution

a) The system is *not* causal, since output values at some times t depend on future inputs values. For example, for any positive integer k, the output at time $t = -\pi k$ is $y(-\pi k) = x(\sin(-\pi k)) = x(0)$.

b) Let
$$x_1(t) \longrightarrow y_1(t) = x_1(\sin(t)) \quad \text{and} \quad x_2(t) \longrightarrow y_2(t) = x_2(\sin(t)).$$
Then, for any scalars α and β,
$$x_3(t) = \alpha x_1(t) + \beta x_2(t) \longrightarrow y_3(t) = x_3(\sin(t)) = \alpha x_1(\sin(t)) + \beta x_2(\sin(t)) = \alpha y_1(t) + \beta y_2(t).$$
Since $\alpha x_1(t) + \beta x_2(t) \longrightarrow \alpha y_1(t) + \beta y_2(t)$, the system is linear.

Problem 3

(Problem 1.27 (a, d, e) - Textbook)

In this chapter, we introduced a number of general properties of systems. In particular, a system may or may not be

1) Memoryless
2) Time invariant
3) Linear
4) Causal
5) Stable

Determine which of these properties hold and which do not hold for each of the following continuous-time systems. Justify your answers. In each example, $y(t)$ denotes the system output and $x(t)$ is the system input.

a) $y(t) = x(t-2) + x(2-t)$

b) $y(t) = \begin{cases} 0, & t < 0 \\ x(t) + x(t-2), & t \geq 0 \end{cases}$

c) $y(t) = \begin{cases} 0, & x(t) < 0 \\ x(t) + x(t-2), & x(t) \geq 0 \end{cases}$

Solution

a) Observe that output values for any time $t < 1$ depend on *future* input values, e.g., $y(0) = x(-2) + x(2)$. Therefore, the system is ***not* causal** (hence, also ***not* memoryless**).

Let
$$x_1(t) \longrightarrow y_1(t) = x_1(t-2) + x_1(2-t) \quad \text{and} \quad x_2(t) \longrightarrow y_2(t) = x_2(t-2) + x_2(2-t).$$

Then, for any scalars α and β,

$$\begin{aligned} x_3(t) = \alpha x_1(t) + \beta x_2(t) \longrightarrow y_3(t) &= x_3(t-2) + x_3(2-t) \\ &= \alpha x_1(t-2) + \beta x_2(t-2) + \alpha x_1(2-t) + \beta x_2(2-t) \\ &= \alpha \left[x_1(t-2) + x_1(2-t)\right] + \beta \left[x_2(t-2) + x_2(2-t)\right] \\ &= \alpha y_1(t) + \beta y_2(t). \end{aligned}$$

Since $\alpha x_1(t) + \beta x_2(t) \longrightarrow \alpha y_1(t) + \beta y_2(t)$, the system is **linear**.

An output time-shifted by any $\tau \in \mathbb{R}$ is

$$\begin{aligned} y(t-\tau) = \left. y(s) \right|_{s=t-\tau} &= \left. \{x(s-2) + x(2-s)\} \right|_{s=t-\tau} = x((t-\tau)-2) + x(2-(t-\tau)) \\ &= x(t-2-\tau) + x(2-t+\tau). \end{aligned}$$

An input time-shifted by the same τ, $\hat{x}(t) = x(t-\tau)$, will produce an output

$$\begin{aligned} \hat{y}(t) = \hat{x}(t-2) + \hat{x}(2-t) &= \left. \{\hat{x}(s) + \hat{x}(-s)\} \right|_{s=t-2} = \left. \{x(s-\tau) + x(-s-\tau)\} \right|_{s=t-2} \\ &= x(t-2-\tau) + x(2-t-\tau). \end{aligned}$$

Since $y(t-\tau) \neq \hat{y}(t)$, i.e., $x(t-\tau) \not\longrightarrow y(t-\tau)$, the system is ***not* time-invariant**.

Suppose that $|x(t)| \leq A < \infty$ for all times t, then

$$|y(t)| = |x(t-2) + x(2-t)| \leq |x(t-2)| + |x(2-t)| \leq 2A < \infty, \ \forall t,$$

so the system is **stable**.

b) Note that the system can be expressed using the unit step function[1]

$$y(t) = \left[x(t) + x(t-2)\right] u(t).$$

Output values may depend on past input values, e.g., $y(1) = \left[x(1) + x(-1)\right] u(1) = x(1) + x(-1)$, so the system is ***not* memoryless**. Since output values do not depend on future input values, the system is **causal**.

Let

$$x_1(t) \longrightarrow y_1(t) = \left[x_1(t) + x_1(t-2)\right] u(t) \quad \text{and} \quad x_2(t) \longrightarrow y_2(t) = \left[x_2(t) + x_2(t-2)\right] u(t).$$

Then, for any scalars α and β,

$$\begin{aligned} x_3(t) = \alpha x_1(t) + \beta x_2(t) \longrightarrow y_3(t) &= \left[x_3(t) + x_3(t-2)\right] u(t) \\ &= \left[\alpha x_1(t) + \beta x_2(t) + \alpha x_1(t-2) + \beta x_2(t-2)\right] u(t) \\ &= \alpha \left[x_1(t) + x_1(t-2)\right] u(t) + \beta \left[x_2(t) + x_2(t-2)\right] u(t) \\ &= \alpha y_1(t) + \beta y_2(t). \end{aligned}$$

Since $\alpha x_1(t) + \beta x_2(t) \longrightarrow \alpha y_1(t) + \beta y_2(t)$, the system is **linear**.

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[1] Note that we use here an alternative definition of the unit step function, $u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$. That is, while in class the value of $u(t)$ at $t = 0$ was undefined, here we assume for convenience that $u(t) = 1$ at $t = 0$.

An output time-shifted by any $\tau \in \mathbb{R}$ is

$$y(t-\tau) = y(s)\big|_{s=t-\tau} = \{[x(s) + x(s-2)]\, u(s)\}\big|_{s=t-\tau} = [x(t-\tau) + x(t-\tau-2)]\, u(t-\tau).$$

An input time-shifted by the same τ, $\hat{x}(t) = x(t-\tau)$, will produce an output

$$\hat{y}(t) = [\hat{x}(t) + \hat{x}(t-2)]\, u(t) = [x(t-\tau) + x(t-\tau-2)]\, u(t).$$

Since $y(t-\tau) \neq \hat{y}(t)$, i.e., $x(t-\tau) \not\longrightarrow y(t-\tau)$, the system is ***not*** **time-invariant**.

Suppose that $|x(t)| \leq A < \infty$ for all times t, then

$$|y(t)| = |[x(t) + x(t-2)]\, u(t)| = |x(t) + x(t-2)|\, \underbrace{|u(t)|}_{\leq 1} \leq |x(t)| + |x(t-2)| \leq 2A < \infty, \ \forall t,$$

so the system is **stable**.

c) Output values never depend on future input values, so the system is **causal**. Unless the input is such that $x(t) < 0$ for *all* times t (in which case $y(t) = 0$ for all times t), output values will depend on past input values (e.g., if for some time $t = t_0$ we have $x(t_0) > 0$, then $y(t_0) = x(t_0) + x(t_0 - 2)$), so the system is ***not*** **memoryless**.

Consider an input $x(t) = 2 + \sin\left(\frac{\pi}{2}t\right)$. Since $x(t) > 0$ for all times t,

$$x(t) = 2 + \sin\left(\frac{\pi}{2}t\right) \longrightarrow y(t) = x(t) + x(t-2) = 2 + \sin\left(\frac{\pi}{2}t\right) + 2 + \underbrace{\sin\left(\frac{\pi}{2}(t-2)\right)}_{= -\sin\left(\frac{\pi}{2}t\right)} = 4.$$

Scaling $x(t)$ by a scalar $\alpha = -1$, we get the input $\hat{x}(t) = -x(t)$. Since $\hat{x}(t) < 0$ for all times t,

$$\hat{x}(t) = -x(t) \longrightarrow \hat{y}(t) = 0, \ \forall t.$$

Therefore, since $x(t) \longrightarrow y(t)$ but $-x(t) \not\longrightarrow -y(t)$, the system is ***not*** **linear**.

An output time-shifted by any $\tau \in \mathbb{R}$ is

$$\begin{aligned} y(t-\tau) = y(s)\big|_{s=t-\tau} &= \left.\begin{cases} 0, & x(s) < 0 \\ x(s) + x(s-2), & x(s) \geq 0 \end{cases}\right|_{s=t-\tau} \\ &= \begin{cases} 0, & x(t-\tau) < 0 \\ x(t-\tau) + x(t-\tau-2), & x(t-\tau) \geq 0. \end{cases} \end{aligned}$$

An input time-shifted by the same τ, $\hat{x}(t) = x(t-\tau)$, will produce an output

$$\hat{y}(t) = \begin{cases} 0, & \hat{x}(t) < 0 \\ \hat{x}(t) + \hat{x}(t-2), & \hat{x}(t) \geq 0 \end{cases} = \begin{cases} 0, & x(t-\tau) < 0 \\ x(t-\tau) + x(t-\tau-2), & x(t-\tau) \geq 0. \end{cases}$$

Since $y(t-\tau) = \hat{y}(t)$, i.e., $x(t-\tau) \longrightarrow y(t-\tau)$, the system is **time-invariant**.

Suppose that $|x(t)| \leq A < \infty$ for all times t, then

$$\begin{aligned} |y(t)| &= \begin{cases} 0, & x(t) < 0 \\ |x(t) + x(t-2)|, & x(t) \geq 0 \end{cases} \leq \begin{cases} 0, & x(t) < 0 \\ |x(t)| + |x(t-2)|, & x(t) \geq 0 \end{cases} \\ &\leq \begin{cases} 0, & x(t) < 0 \\ 2A, & x(t) \geq 0 \end{cases} \leq 2A < \infty, \ \forall t, \end{aligned}$$

so the system is **stable**.

Problem 4

(Problem 1.28 (a, d, g) - Textbook)

Determine which of the properties listed in Problem 3 hold and which do not hold for each of the following discrete-time systems. Justify your answers. In each example, $y[n]$ denotes the system output and $x[n]$ is the system input.

a) $y[n] = x[-n]$

b) $y[n] = \mathcal{E}v\{x[n-1]\}$

c) $y[n] = x[4n+1]$

Solution

a) Output values for all $n < 0$ clearly depend on future input values, e.g., $y[-1] = y[1]$, so the system is ***not* causal** (hence, also ***not* memoryless**).

Let
$$x_1[n] \longrightarrow y_1[n] = x_1[-n] \quad \text{and} \quad x_2[n] \longrightarrow y_2[n] = x_2[-n].$$
Then, for any scalars α and β,
$$x_3[n] = \alpha x_1[n] + \beta x_2[n] \longrightarrow y_3[n] = x_3[-n] = \alpha x_1[-n] + \beta x_2[-n] = \alpha y_1[n] + \beta y_2[n].$$
Since $\alpha x_1[n] + \beta x_2[n] \longrightarrow \alpha y_1[n] + \beta y_2[n]$, the system is **linear**.

An output time-shifted by any integer $n_0 \in \mathbb{Z}$ is
$$y[n-n_0] = y[m]|_{m=n-n_0} = x[-m]|_{m=n-n_0} = x[-(n-n_0)] = x[-n+n_0].$$
An input time-shifted by the same n_0, $\hat{x}[n] = x[n-n_0]$, will produce an output
$$\hat{y}[n] = \hat{x}[-n] = \hat{x}[m]|_{m=-n} = x[m-n_0]|_{m=-n} = x[-n-n_0].$$
Since $y[n-n_0] \neq \hat{y}[n]$, i.e., $x[n-n_0] \not\longrightarrow y[n-n_0]$, the system is ***not* time-invariant**.

Suppose that $|x[n]| \leq A < \infty$ for all (integer) times n, then
$$|y[n]| = |x[-n]| \leq A < \infty, \ \forall n,$$
so the system is **stable**.

b) Recalling the definition of $\mathcal{E}v\{\cdot\}$, i.e., $\mathcal{E}v\{w[n]\} = \frac{1}{2}(w[n] + w[-n])$, the system can be expressed as $y[n] = \frac{1}{2}(x[n-1] + x[-n+1])$, since
$$\begin{aligned} y[n] &= \mathcal{E}v\{x[n-1]\} = \mathcal{E}v\{x[m]\}|_{m=n-1} = \frac{1}{2}(x[m] + x[-m])|_{m=n-1} = \frac{1}{2}(x[n-1] + x[-(n-1)]) \\ &= \frac{1}{2}(x[n-1] + x[-n+1]). \end{aligned}$$

Output values for any $n \leq 0$ depend on *future* input values, e.g., $y[0] = \frac{1}{2}(x[-1] + x[1])$, so the system is ***not* causal** (hence, also ***not* memoryless**).

Let

$$x_1[n] \longrightarrow y_1[n] = \mathscr{E}v\{x_1[n-1]\} = \frac{1}{2}\left(x_1[n-1] + x_1[-n+1]\right)$$
$$x_2[n] \longrightarrow y_2[n] = \mathscr{E}v\{x_2[n-1]\} = \frac{1}{2}\left(x_2[n-1] + x_2[-n+1]\right).$$

Then, for any scalars α and β,

$$\begin{aligned} x_3[n] = \alpha x_1[n] + \beta x_2[n] \longrightarrow y_3[n] &= \mathscr{E}v\{x_3[n-1]\} = \frac{1}{2}\left(x_3[n-1] + x_3[-n+1]\right) \\ &= \frac{1}{2}\left(\alpha x_1[n-1] + \beta x_2[n-1] + \alpha x_1[-n+1] + \beta x_2[-n+1]\right) \\ &= \alpha\frac{1}{2}\left(x_1[n-1] + x_1[-n+1]\right) + \beta\frac{1}{2}\left(x_2[n-1] + x_2[-n+1]\right) \\ &= \alpha y_1[n] + \beta y_2[n]. \end{aligned}$$

Since $\alpha x_1[n] + \beta x_2[n] \longrightarrow \alpha y_1[n] + \beta y_2[n]$, the system is **linear**.

An output time-shifted by any integer $n_0 \in \mathbb{Z}$ is

$$\begin{aligned} y[n-n_0] = y[m]\big|_{m=n-n_0} &= \frac{1}{2}\left(x[m-1] + x[-m+1]\right)\big|_{m=n-n_0} \\ &= \frac{1}{2}\left(x[(n-n_0)-1] + x[-(n-n_0)+1]\right) \\ &= \frac{1}{2}\left(x[n-1-n_0] + x[-n+1+n_0]\right). \end{aligned}$$

An input time-shifted by the same n_0, $\hat{x}[n] = x[n-n_0]$, will produce an output

$$\begin{aligned} \hat{y}[n] &= \frac{1}{2}\left(\hat{x}[n-1] + \hat{x}[-n+1]\right) = \frac{1}{2}\left(\hat{x}[m] + \hat{x}[-m]\right)\big|_{m=n-1} \\ &= \frac{1}{2}\left(x[m-n_0] + x[-m-n_0]\right)\big|_{m=n-1} \\ &= \frac{1}{2}\left(x[n-1-n_0] + x[-n+1-n_0]\right). \end{aligned}$$

Since $y[n-n_0] \neq \hat{y}[n]$, i.e., $x[n-n_0] \not\longrightarrow y[n-n_0]$, the system is ***not* time-invariant**.

Suppose that $|x[n]| \leq A < \infty$ for all (integer) times n, then

$$|y[n]| = \left|\frac{1}{2}\left(x[n-1] + x[-n+1]\right)\right| \leq \frac{1}{2}\left(|x[n-1]| + |x[-n+1]|\right) \leq \frac{1}{2}(A+A) = A < \infty,\ \forall n,$$

so the system is **stable**.

c) Output values (at all times n) depend on *future* input values, e.g., $y[0] = x[1]$, so the system is ***not* causal** (hence, also ***not* memoryless**).

Let

$$x_1[n] \longrightarrow y_1[n] = x_1[4n+1] \quad \text{and} \quad x_2[n] \longrightarrow y_2[n] = x_2[4n+1].$$

Then, for any scalars α and β,

$$x_3[n] = \alpha x_1[n] + \beta x_2[n] \longrightarrow y_3[n] = x_3[4n+1] = \alpha x_1[4n+1] + \beta x_2[4n+1] = \alpha y_1[n] + \beta y_2[n].$$

Since $\alpha x_1[n] + \beta x_2[n] \longrightarrow \alpha y_1[n] + \beta y_2[n]$, the system is **linear**.

An output time-shifted by any integer $n_0 \in \mathbb{Z}$ is

$$y[n-n_0] = y[m]|_{m=n-n_0} = x[4m+1]|_{m=n-n_0} = x[4(n-n_0)+1] = x[4n+1-4n_0].$$

An input time-shifted by the same n_0, $\hat{x}[n] = x[n-n_0]$, will produce an output

$$\hat{y}[n] = \hat{x}[4n+1] = \hat{x}[m]|_{m=4n+1} = x[m-n_0]|_{m=4n+1} = x[4n+1-n_0].$$

Since $y[n-n_0] \neq \hat{y}[n]$, i.e., $x[n-n_0] \not\to y[n-n_0]$, the system is ***not* time-invariant**.

Suppose that $|x[n]| \leq A < \infty$ for all (integer) times n, then

$$|y[n]| = |x[4n+1]| \leq A < \infty, \ \forall n,$$

so the system is **stable**.

Problem 5

(Problem 1.30 (c, d, k, l) - Textbook)

Determine if each of the following systems is invertible. If it is, construct the inverse system. If it is not, find two input signals to the system that have the same output.

a) $y[n] = nx[n]$

b) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

c) $y[n] = \begin{cases} x[n+1], & n \geq 0 \\ x[n], & n \leq -1 \end{cases}$

d) $y(t) = x(2t)$

Solution

a & c) Consider any non-zero scalars A and B, such that $A \neq B$. Then, the two (different) input signals $A\delta[n]$ and $B\delta[n]$ produce the same output $y[n] = 0, \ \forall n$. Therefore, the system is *not* invertible.

b) The inverse system is the differentiation operator, i.e., $\frac{d}{dt}y(t) = \frac{d}{dt}\int_{-\infty}^{t} x(\tau)\, d\tau = x(t)$.

d) The inverse system corresponds to a time-scaling by a factor of $\frac{1}{2}$, i.e., $y\left(\frac{t}{2}\right) = x\left(2\frac{t}{2}\right) = x(t)$.

Problem 6

(Problem 1.31 - Textbook)

In this problem, we illustrate one of the most important consequences of the properties of linearity and time invariance. Specifically, once we know the response of a linear system or a linear time-invariant (LTI) system to a single input or the responses to several inputs, we can directly compute the responses to many other input signals. Much of the remainder of this book deals with a thorough exploitation of this fact in order to develop results and techniques for analyzing and synthesizing LTI systems.

a) Consider an LTI system whose response to the signal $x_1(t)$ in Figure 1(a) is the signal $y_1(t)$ illustrated in Figure 1(b). Determine and sketch carefully the response of the system to the input $x_2(t)$ depicted in Figure 1(c).

b) Determine and sketch the response of the system considered in part (a) to the input $x_3(t)$ shown in Figure 1(d).

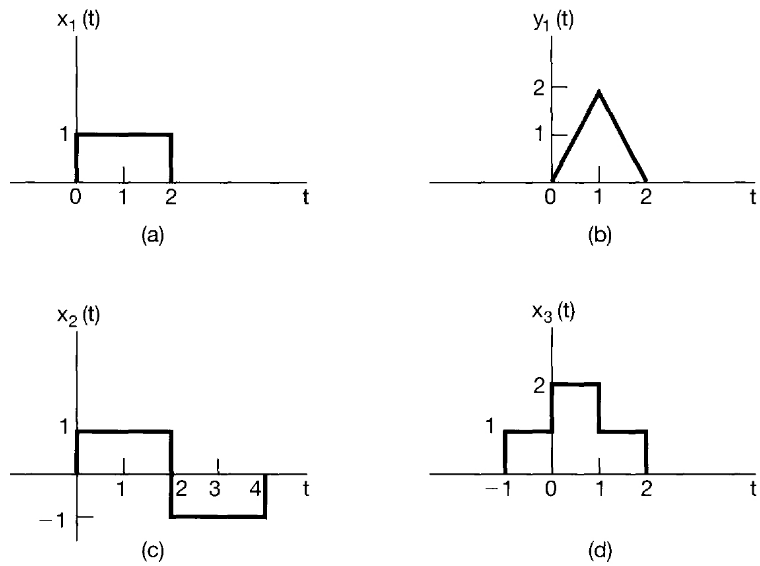
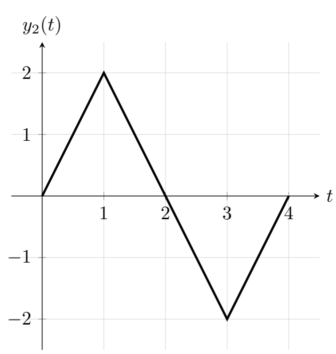


Figure 1: Problem 6

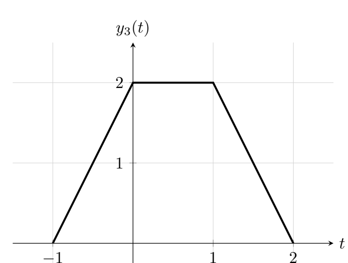
Solution

a) From Figures 1(a) and 1(b), we see that $x_2(t) = x_1(t) - x_1(t-2)$. Since the system is LTI, input $x_2(t)$ will produce the output $y_2(t) = y_1(t) - y_1(t-2)$, as shown in Figure 2a.

b) From Figures 1(a) and 1(d), we see that $x_3(t) = x_1(t) + x_1(t+1)$. Since the system is LTI, input $x_3(t)$ will produce the output $y_3(t) = y_1(t) + y_1(t+1)$, as shown in Figure 2b.



(a) $y_2(t) = y_1(t) - y_1(t-2)$



(b) $y_3(t) = y_1(t) + y_1(t+1)$

Figure 2: Solutions of Problem 6.

Problem 7

(Problem 2.2 - Textbook)

Consider the signal

$$h[n] = \left(\frac{1}{2}\right)^{n-1} \{u[n+3] - u[n-10]\}.$$

Express A and B in terms of n so that the following equation holds:

$$h[n-k] = \begin{cases} (\frac{1}{2})^{n-k-1}, & A \le k \le B \\ 0, & \text{elsewhere} \end{cases}$$

Solution

Since $u[k+3] - u[k-10] = \begin{cases} 1, & -3 \le k \le 9 \\ 0, & \text{elsewhere} \end{cases}$, we can express $h[k]$ as $h[k] = \begin{cases} \left(\frac{1}{2}\right)^{k-1}, & -3 \le k \le 9 \\ 0, & \text{elsewhere} \end{cases}$.

Therefore, $h[-k] = \begin{cases} \left(\frac{1}{2}\right)^{-k-1}, & -9 \le k \le 3 \\ 0, & \text{elsewhere} \end{cases}$, and

$$h[n-k] = \begin{cases} (\frac{1}{2})^{n-k-1}, & n-9 \le k \le n+3 \\ 0, & \text{elsewhere} \end{cases}$$

i.e., $A = n - 9$ and $B = n + 3$.

Problem 8

(Problem 2.21 (b, d) - Textbook)

Compute the convolution $y[n] = x[n] * h[n]$ of the following pairs of signals:

a) $\left.\begin{array}{l} x[n] = \alpha^n u[n], \\ h[n] = \beta^n u[n], \end{array}\right\} \alpha \neq \beta$

b) $x[n]$ and $h[n]$ are as in Figure 3.

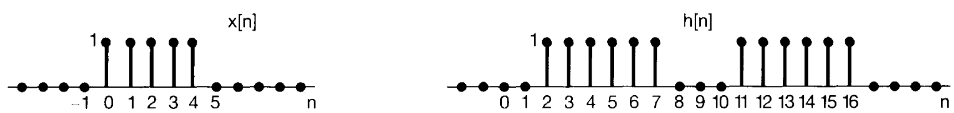


Figure 3: Problem 8

Solution

a)

$$
\begin{aligned}
y[n] &= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\
&= \sum_{k=-\infty}^{\infty} \alpha^k u[k] \beta^{n-k} u[n-k] \\
&\overset{(a)}{=} \begin{cases} \sum_{k=0}^{n} \alpha^k \beta^{n-k}, & n \geq 0 \\ 0, & n < 0 \end{cases} \\
&= u[n]\beta^n \sum_{k=0}^{n} \left(\frac{\alpha}{\beta}\right)^k \\
&\overset{(b)}{=} u[n]\beta^n \frac{1-\left(\frac{\alpha}{\beta}\right)^{n+1}}{1-\frac{\alpha}{\beta}} \\
&= \left(\frac{\beta^{n+1}-\alpha^{n+1}}{\beta-\alpha}\right) u[n],
\end{aligned}
$$

where (a) follows by observing that $u[k]u[n-k] = \begin{cases} 1, & \text{if } 0 \leq k \leq n \\ 0, & \text{otherwise} \end{cases}$, and (b) by recalling the summation of a geometric series $\sum_{k=0}^{n} r^k = \frac{1-r^{n+1}}{1-r}$ (if $r \neq 1$).

b) Observing in Figure 3 the values of n for which $x[n]$ and $h[n]$ equal 0 or 1, the convolution evaluates to the following

$$
y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=0}^{4} h[n-k] = \begin{cases} 0, & n \leq 1 \text{ or } n \geq 21 \\ 1, & n = 2, 20 \\ 2, & n = 3, 10, 11, 12, 19 \\ 3, & n = 4, 9, 13, 18 \\ 4, & n = 5, 8, 14, 17 \\ 5, & n = 6, 7, 15, 16. \end{cases}
$$

Textbook

Alan V. Oppenheim, Alan S. Willsky, and S. Hamid Nawab, Signals & Systems, 2nd Ed., Prentice-Hall, 1996 (ISBN 0-13-814757-4)