UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
DIVISION OF ENGINEERING SCIENCE

ECE355H1 F - Signal Analysis and Communication

Problem Set 6
Fall 2023

Submit by: **October 27, 2023**

Problem 1

(Problem 3.24 - Textbook)

Let

$$x(t) = \begin{cases} t, & 0 \leq t < 1 \\ 2 - t, & 1 \leq t < 2 \end{cases}$$

be a periodic signal with fundamental period $T = 2$ and Fourier coefficients a_k.

a) Determine the value of a_0.

b) Determine the Fourier series representation of $\frac{dx(t)}{dt}$.

c) Use the result of part (b) and the differentiation property of the continuous-time Fourier series to help determine the Fourier series coefficients of $x(t)$.

Problem 2

(Problem 3.25 - Textbook)

Consider the following three continuous-time signals with a fundamental period of $T = 1/2$:

$$x(t) = \cos(4\pi t),$$
$$y(t) = \sin(4\pi t),$$
$$z(t) = x(t)y(t).$$

a) Determine the Fourier series coefficients of $x(t)$.

b) Determine the Fourier series coefficients of $y(t)$.

c) Use the results of parts (a) and (b), along with the multiplication property of the continuous-time Fourier series, to determine the Fourier series coefficients of $z(t) = x(t)y(t)$.

d) Determine the Fourier series coefficients of $z(t)$ through direct expansion of $z(t)$ in trigonometric form, and compare your result with that of part (c).

Problem 3

(Problem 3.26 - Textbook)

Let $x(t)$ be a periodic signal whose Fourier series coefficients are

$$a_k = \begin{cases} 2, & k = 0 \\ j(\frac{1}{2})^{|k|}, & \text{otherwise} \end{cases}$$

Use Fourier series properties to answer the following questions:

a) Is x(t) real?

b) Is x(t) even?

c) Is $dx(t)/dt$ even?

Problem 4

(Problem 3.44 - Textbook)

Suppose we are given the following information about a signal $x(t)$:

1) $x(t)$ is a real signal.

2) $x(t)$ is periodic with period $T = 6$ and has Fourier coefficients a_k.

3) $a_k = 0$ for $k = 0$ and $k > 2$.

4) $x(t) = -x(t-3)$.

5) $\frac{1}{6}\int_{-3}^{3} |x(t)|^2\, dt = \frac{1}{2}$.

6) a_1 is a positive real number.

Show that $x(t) = A\cos(Bt + C)$, and determine the values of the constants $A, B,$ and$C$.

Problem 5

(Problem 4.1 - Textbook)

Use the Fourier transform analysis equation to calculate the Fourier transforms of:

a) $e^{-2(t-1)}u(t-1)$

b) $e^{-2|t-1|}$

Sketch and label the magnitude of each Fourier transform.

Problem 6

(Problem 4.3 - Textbook)

Determine the Fourier transform of each of the following periodic signals:

a) $\sin(2\pi t + \frac{\pi}{4})$

b) $1 + \cos(6\pi t + \frac{\pi}{8})$

Problem 7

(Problem 4.4 - Textbook)

Use the Fourier transform synthesis equation to determine the inverse Fourier transforms of:

a) $X_1(j\omega) = 2\pi\delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)$

b) $X_2(j\omega) = \begin{cases} 2, & 0 \le \omega \le 2 \\ -2, & -2 \le \omega < 0 \\ 0, & |\omega| > 2 \end{cases}$

Problem 8

(Problem 4.5 - Textbook)

Use the Fourier transform synthesis equation to determine the inverse Fourier transform of $X(j\omega) = |X(j\omega)|e^{j\sphericalangle X(j\omega)}$, where

$$|X(j\omega)| = 2\{u(\omega + 3) - u(\omega - 3)\},$$
$$\sphericalangle X(j\omega) = -\frac{3}{2}\omega + \pi.$$

Use your answer to determine the values of t for which $x(t) = 0$.

Textbook

Alan V. Oppenheim, Alan S. Willsky, and S. Hamid Nawab, Signals & Systems, 2nd Ed., Prentice-Hall, 1996 (ISBN 0-13-814757-4)