ECE355: Lecture 17

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4 Pages

Contents

1 ch. 3.5: Properties of Continuous Time Fourier Series (CTFS)

Suppose we have the CTFS coefficients $\{a_k\}_{k=-\infty,\infty}$ of some signal, $x(t)$.

How can we find the CTFS coefficients of signals obtained by simple manipulations, e.g, $3x(t)+2$, $x(t-5)$, $Re\{x(t)\}$, $\frac{\mathrm{d}f}{\mathrm{d}x}$, ...?

There are many useful propertied of CTFS that you can use as shortcuts to find the CTFS coefficients for these transformed signals

First, for convenience, let's use a shorthand notation to indicate the relationship between a periodic signal. & its CTFS coefficients.

$$\begin{aligned} x(t) &\overset{\text{FS}}{\longleftrightarrow} a_k && (1)\\ x(t) &\ -\text{CT Periodic} && (2)\\ T &\ - Fundemental Period && (3)\\ \omega_0 = \frac{2\pi}{T} &\ - Fundemental Frequency && (4) \end{aligned}$$

1.1 Linearity:

If $y(t)$ has a fundamental period T (Same as $x(t)$):

Then:

$$Ax(t) + By(t) \overset{FS}{\longleftrightarrow} Aa_k + Bb_k \tag{1}$$

1.1.1 Proof:

$$\begin{aligned} \text{LHS} &= A(\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}) + B(\sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}) && (2)\\ &= \sum_{k=-\infty}^{\infty} (Aa_k + Bb_k) e^{jk\omega_0 t} && (3) \end{aligned}$$

1.2 Time Shifting:

$$x(t-t_0) \overset{FS}{\longleftrightarrow} a_k e^{-jk\omega_0 t_0} \tag{1}$$

Here there is no change to the magnitude of the FS coefficients.

1.2.1 Proof:

Finding FS coefficients for $x(t-t_0)$:

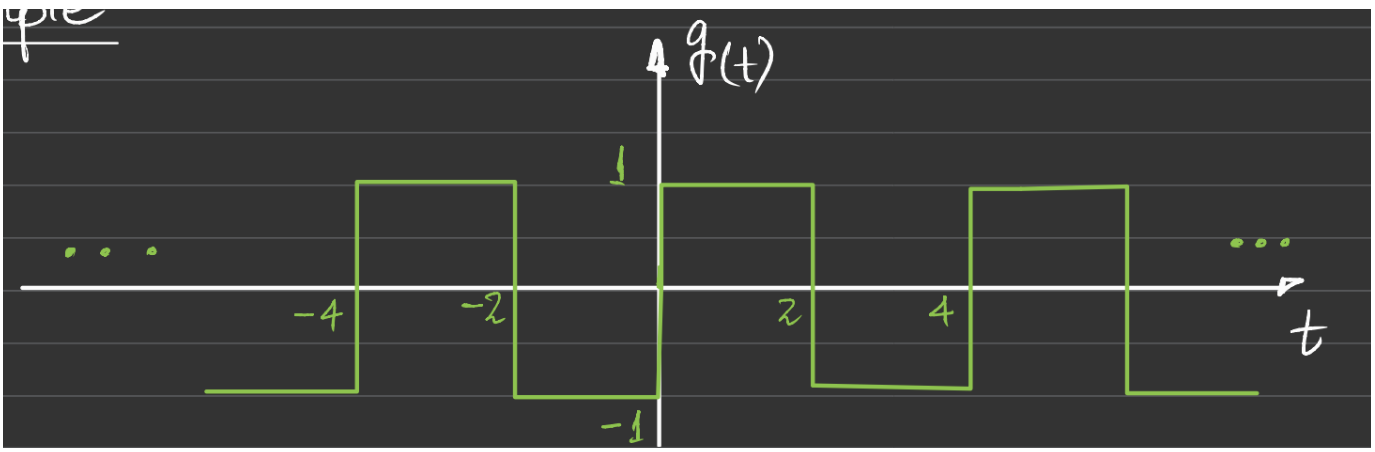
$$= \frac{1}{T}\int_T x(t-t_0)e^{-jk\omega_0 t}dt \tag{2}$$

$$\tau = t - t_0 \rightarrow = \frac{1}{T}\int_T x(\tau)e^{-jk\omega_0(\tau+t_0)}d\tau \tag{3}$$

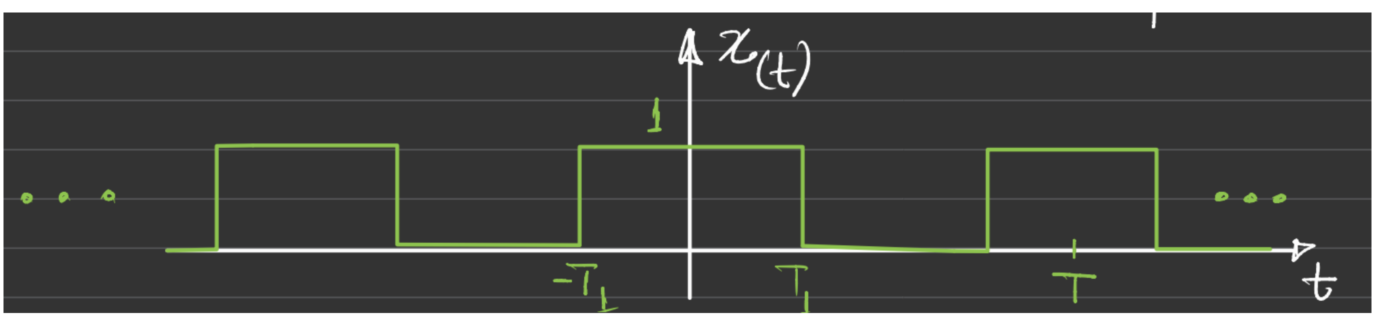
$$= e^{-jk\omega_0 t_0}\underbrace{\frac{1}{T}\int_T x(\tau)e^{-jk\omega_0 \tau}d\tau}_{=a_k} \tag{4}$$

$$= a_k e^{-jk\omega_0 t_0} \tag{5}$$

1.2.2 Example:



- To evaluate its FS coefficients. recall the examsple of lecture 15.



$$FS \rightarrow a_k = \begin{cases} \frac{\sin(\frac{k\pi}{2})}{k\pi} & , \ k \neq 0 \\ \frac{x-a}{b-a} & , \ k = 0 \end{cases} \tag{6}$$

- Relating $g(t)$ to $x(t)$: Let $T = 4$ and $\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$. &

$$g(t) = \underbrace{2x(t-1)}_{\mathbf{I}} \underbrace{-1}_{\mathbf{II}} \leftrightarrow C_k \tag{7}$$

$$\mathbf{I}.\ 2x(t-1) \overset{FS}{\longleftrightarrow} 2a_k e^{-jk\frac{\pi}{2}} \tag{8}$$

$$\mathbf{II}.\ -1x \overset{FS}{\longleftrightarrow} \left\{ \begin{array}{ll} -1 & , \ k \neq 0 \\ 0 & , \ k = 0 \end{array} \right| \text{using FS analysis equation} \tag{9}$$

$$\therefore\ \mathbf{I}\ \&\ \mathbf{II} \rightarrow \left\{ \begin{array}{ll} c_0 = 2a_0 - 1 = 1 - 1 = 0 & ;\ k = 0 \\ c_k = 2e^{-jk\frac{\pi}{2}} \frac{\sin(k\frac{k\pi}{2})}{k\pi} & ;\ k \neq 0 \end{array} \right. \tag{10}$$

1.3 Time Scaling:

For $\alpha > 0$, $x(\alpha t) \overset{FS}{\longleftrightarrow} a_k$, with fundamental frequency, $\alpha\omega_0$

1.3.1 <u>Proof:</u>

$$x(\alpha t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0(\alpha t)} \tag{1}$$

$$= \sum_{k=-\infty}^{\infty} a_k e^{jk \underbrace{(\alpha\omega_0)}_{\text{Fundemental frequency is now } \alpha\omega_0} t} \tag{2}$$

1.4 Time Reversal: