ECE358 PS4

**Q1.**

**a)**

G and H share the same set of minimum spanning trees. First it must be proved that any minimum spanning tree for G is also a minimum spanning tree for H and then it must be proved that any minimum spanning tree for H is also a minimum spanning tree for G.

Let be the summation of all weights of a tree in the graph G, and be the summation of all weights of a tree in the graph H.

Let be a minimum spanning tree for G

**Proof:**

Recall, by properties of a MST, an MST for G or H will have V-1 edges. Because each edge in H has a weight that is 1 greater than the weight in G,

**ATAC:** Assume there is a minimum spanning tree A for the graph H that is not a minimum spanning tree for G. By definition, and .

The terms cancel:

But by definition:

Contradiction! If A is not a minimum spanning tree of G, it also cannot be a minimum spanning tree for H.

**ATAC**: Assume there is a minimum spanning tree B for the graph G that is not a minimum spanning tree for H. By definition, and .

The terms cancel:

But by definition:

Contradicition! If B is not a minimum spanning tree of H, it also cannot be a minimum spanning tree for G.

This means that there cannot be any graph that is a minimum spanning tree for H but not G or a minimum spanning tree for G but not H.

Therefore, G and H share the same set of minimum spanning trees.

**b)**

G and H do not always share the same shortest path between all pairs of vertices.

Consider the following graph G

A diagram of a diagram

Description automatically generated

The shortest path from A → F is: A → B → C → D → F, having a total weight of 4.

This same graph in H becomes:

A diagram of a diagram

Description automatically generated

Now the path A → B → C → D → F has weight 8, while the path A → E → F has weight 7. This means that the shortest path A → F is A → E → F. This counter example proves that G and H do not always share the same shortest path between all pairs of vertices.