Homework 04

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Part A:

The continuous-time equations of motion for a unit mass subject to a piecewise constant force $f(t) = p_n$ over intervals of length 1 second are given by:

$$m \cdot x''(t) = f(t)$$

Substituting $m = 1$ (unit mass), integrating once to find velocity, and then integrating again to find position, we get:

$$x'(t) = \int f(t)\, dt = p_n$$

$$x(t) = \int x'(t)\, dt = \int p_n\, dt = p_n \cdot t + C$$

Applying the initial conditions $x(0) = 0$ and $x'(0) = 0$, we find $C = 0$, so:

$$x(t) = \sum_{k=1}^{n} p_k \cdot t$$

Now, discretizing this system over intervals of 1 second and evaluating at $t = n$, we get the discrete-time equations:

$$x(n) = x(n-1) + \sum_{k=n-1}^{n} p_k$$

$$x'(n) = p_n$$

These are consistent with the provided state-space equations:

$$\begin{bmatrix} x(n) \\ x'(n) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x(n-1) \\ x'(n-1) \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} p_n$$

Part B:

The optimization problem is to minimize the objective function

$$\sum_{n=1}^{10} (p_n)^2$$

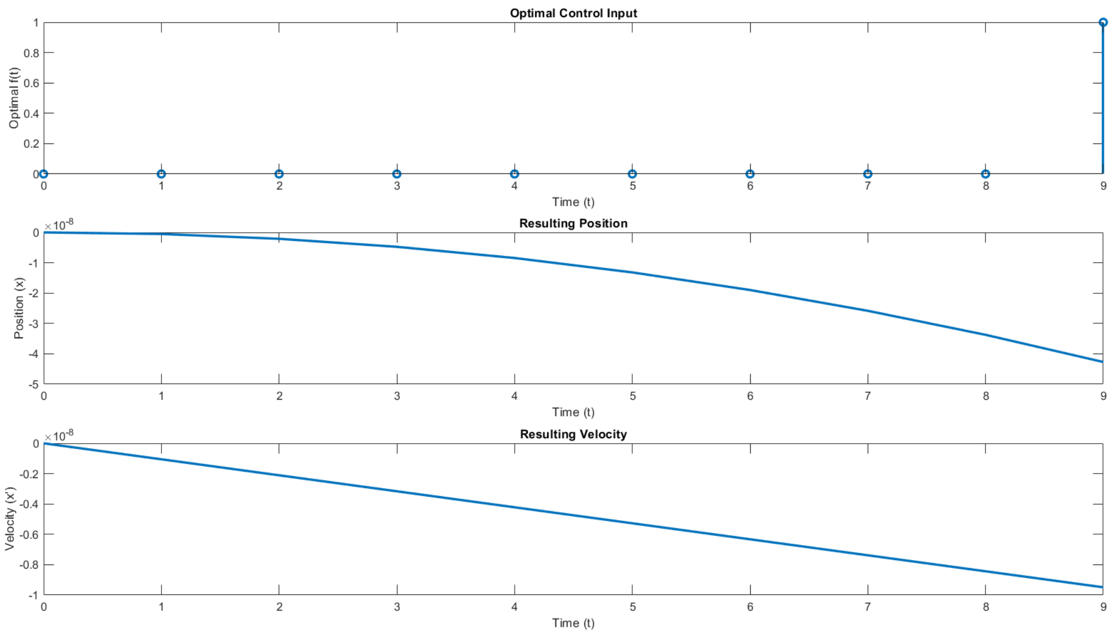
subject to the constraints $x(10) = 1$ and $x'(10) = 0$. We can formulate this problem using the method of Lagrange multipliers. The Lagrangian is given by

$$L(p, \lambda_1, \lambda_2) = \sum_{n=1}^{10} (p_n)^2 + \lambda_1 \cdot (x(10) - 1) + \lambda_2 \cdot x'(10)$$

where λ_1 and λ_2 are the Lagrange multipliers. The necessary conditions for optimality are obtained by taking partial derivatives and setting them equal to zero:

$$\frac{\partial L}{\partial p_n} = 2p_n + \lambda_1 = 0, \quad \text{for } n = 1, 2, \ldots, 10$$
$$\frac{\partial L}{\partial \lambda_1} = x(10) - 1 = 0$$
$$\frac{\partial L}{\partial \lambda_2} = x'(10) = 0$$

Solving this system of equations will yield the optimal values for p_n. Once we have these optimal values, we can use them to simulate the system and plot the optimal control input $f(t)$, resulting position $x(t)$, and velocity $x'(t)$.


Optimal Control Input
Optimal f(t)
Time (t)
Resulting Position
Position (x)
Time (t)
Resulting Velocity
Velocity (x')
Time (t)