Homework 02

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Problem 2.1

Consider the list of n $n - vectors$

$$a_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, a_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \ldots, a_n = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{1}$$

(The vector a_i has its first i entries equal to one, and the remaining entries zero.) Describe what happens when you run the Gram–Schmidt algorithm on this list of vectors, *i.e.*, say what $q_1, \ldots, q_n$ are. Is $a_1, \ldots, a_n$ a basis?

Solution:

If we apply the Gram-Schmidt algorithm on the given list of vectors, we will through each iteration remove the linearly dependent elements of each a_i vector until we arrive at the following result:

$$q_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, q_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \ldots, q_n = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \tag{2}$$

Since the set of $a_1, \ldots, a_n$ is entirely linearly independent it is a basis for $\mathbb{R}^n$.

Problem 2.2

Problem 2.3

Problem 2.4

Problem 2.5