HomeWork 01

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Problem 1.1

Norm Requirements

1. Positivity: $\|x\|_p \geq 0$ and $\|x\|_p = 0 \iff x = 0$
2. Homogeneity: $\|\alpha x\|_p = |\alpha| \|x\|_p$
3. Sub-additivity: $\|x + y\|_p \leq \|x\|_p + \|y\|_p$

Proof: l_1

1. Positivity: $\|x\|_1 = \sum_{i=1}^{n} |x_i| \geq 0$ and $\|x\|_1 = 0 \iff x_i = 0 \ , \forall i \in \{1, ..., n\}$
2. Homogeneity: $\|\alpha x\|_1 = \sum_{i=1}^{n} |\alpha x_i| = |\alpha| \sum_{i=1}^{n} |x_i| = |\alpha| \|x\|_1$
3. Sub-additivity:

$$\|x + y\|_1 = \sum_{i=1}^{n} |x_i + y_i| \leq \sum_{i=1}^{n} |x_i| + |y_i| = \|x\|_1 + \|y\|_1$$