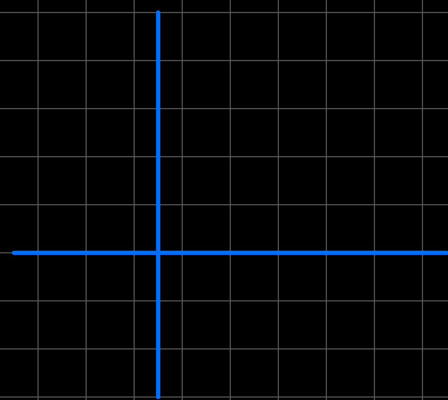
Problem 1:



Part 5:

$$\mathcal{L}(w,b,a) = \frac{1}{2}\|w\|^2 - \sum_{n=1}^{N} \alpha_n\left(t^n(w^T x^{(n)} + b) - 1\right)$$

$$= \frac{1}{2} w^T w - \sum \alpha_n w^T x^n t^n + \alpha_n b t^n - \alpha_n$$

$$= \frac{1}{2} w^T w + \sum \alpha_n - \sum \alpha_n w^T x^{(n)} t^{(n)} - \underbrace{\sum \alpha_n b t^{(n)}}_{\text{we know from part 4, this is equal to } G = 0}$$

$$= \frac{1}{2} w^T w - \sum \alpha_n w^T x^{(n)} t^{(n)} + \sum \alpha_n$$

$$= w^T\left(\frac{1}{2} w - \sum \alpha_n x^{(n)} t^{(n)}\right) + \sum \alpha_n$$

↳ Part 4: $w = A = \sum \alpha_n t^{(n)} x^{(n)}$

$$= w^T\left(\frac{1}{2}\sum \alpha_n t^{(n)} x^{(n)} - \sum \alpha_n t^{(n)} x^{(n)}\right) + \sum \alpha_n$$

$$= w^T\left(-\frac{1}{2}\sum \alpha_n t^{(n)} x^{(n)}\right) + \sum \alpha_n$$

↳ Part 4: $w = \sum \alpha_n t^{(n)} x^{(n)} \Rightarrow w^T = \sum \alpha_n t^{(n)} x^{T(n)}$

$$= \sum \alpha_n - \frac{1}{2}\sum_{m=1}^{M}\sum_{n=1}^{N} \alpha_n \alpha_m t^{(n)} t^{(m)} x^{(n)} x^{T(m)}$$

$\therefore$ we have $A_n = \alpha_n$, $B_n = \alpha_n$, $C_m = \alpha_m$, $D_n = t^{(n)}$, $E_m = t^{(m)}$, $F = x^{T(m)}$, $G = x^{(n)}$

Part 6:

Since $x^{(2)} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ is not a support vector, we can set $\alpha_2 = 0$

Part 7:

$$\sum \alpha_n t^{(n)} = 0$$

$$\Rightarrow \alpha_1(1) + \cancel{\alpha_2(-1)}^{0} + \alpha_3(-1) + \alpha_4(-1) = 0$$

$$\Rightarrow \alpha_1 = \alpha_3 + \alpha_4$$

Part 8:

$$\max_{\alpha}\min_{w,b} \mathcal{L}(w,b,\alpha) =$$

$$= \max_{\alpha} \sum^{N} \alpha_n - \frac{1}{2}\sum^{M}\sum^{N} \alpha_n \alpha_m t^{(n)} t^{(m)} x^{T(m)} x^{(n)}$$

Skip $n, m = 2$ since $\alpha_2 = 0$

$$= \max_{\alpha} (\alpha_1 + \alpha_3 + \alpha_4)$$

$* \; x^{T(3)} \cdot x^{(4)} = x^{T(4)} \cdot x^{(3)} = 0$

$x^{(m)} \cdot x^{T(n)} = 1$

$$-\frac{1}{2}\Big[\alpha_1 \alpha_1 \cancelto{1}{t^{(1)} t^{(1)}} \cancelto{2}{x^{T(1)} x^{(1)}} + \alpha_1 \alpha_3 \cancelto{-1}{t^{(1)} t^{(3)}} \cancelto{1}{x^{T(1)} x^{(3)}}$$

$$\alpha_1 \alpha_4 \cancelto{-1}{t^{(1)} t^{(4)}} \cancelto{1}{x^{T(1)} x^{(4)}} + \alpha_3 \alpha_1 \cancelto{-1}{t^{(3)} t^{(1)}} \cancelto{1}{x^{T(3)} x^{(1)}}$$

$$\alpha_3 \alpha_3 \cancelto{+1}{t^{(3)} t^{(3)}} \cancelto{1}{x^{T(3)} x^{(3)}} + \cancel{\alpha_3 \alpha_4 t^{(3)} t^{(4)} x^{T(3)} x^{(4)}}$$

$$\alpha_4 \alpha_1 \cancelto{-1}{t^{(4)} t^{(1)}} \cancelto{1}{x^{T(4)} x^{(1)}} + \cancel{\alpha_4 \alpha_3 t^{(4)} t^{(3)} x^{T(4)} x^{(3)}}$$

$$+ \alpha_4 \alpha_4 \cancelto{+1}{t^{(4)} t^{(4)}} \cancel{x^{T(4)} x^{(4)}} \Big]$$

$$= \max_{\alpha} (\alpha_1 + \alpha_3 + \alpha_4) - \frac{1}{2}\left[2\alpha_1^2 - \alpha_1\alpha_3 - \alpha_1\alpha_4 - \alpha_3\alpha_1 + \alpha_3^2 - \alpha_4\alpha_1 + \alpha_4^2\right]$$

$$= \max_{\alpha} (\alpha_1 + \alpha_3 + \alpha_4) - \alpha_1^2 + \alpha_1\alpha_3 + \alpha_1\alpha_4 - \frac{1}{2}\left(\alpha_3^2 + \alpha_4^2\right)$$

$* \; \alpha_1 = \alpha_3 + \alpha_4$

$$= 2\alpha_3 + 2\alpha_4 - (\cancel{\alpha_3 + \alpha_4})^2 + \alpha_3^2 + \cancel{\alpha_3\alpha_4} + \alpha_4^2 + \cancel{\alpha_3\alpha_4} - \frac{1}{2}\left(\alpha_3^2 + \alpha_4^2\right)$$

$$= \max_{\alpha} -\frac{3}{2}\cdot\alpha_3^2 - \frac{3}{2}\alpha_4^2 + 2\alpha_3 + 2\alpha_4 \Rightarrow \frac{\partial \mathcal{L}}{\partial \alpha} = 0 \Rightarrow \begin{bmatrix} -\alpha_3 + 2 \\ -\alpha_4 + 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \alpha_3 = \alpha_4 = 2 \Rightarrow \alpha_1 = \alpha_3 + \alpha_4 = 4$$

$$\therefore \alpha = \begin{bmatrix} 4 \\ 0 \\ 2 \\ 2 \end{bmatrix}$$

Part 9:

$$w = \sum \alpha_n t^{(n)} x^{(n)}$$

$$w = 4\begin{bmatrix}1\\1\end{bmatrix}(1) + \cancel{0\begin{bmatrix}-1\\-1\end{bmatrix}(-1)} + 2\begin{bmatrix}1\\0\end{bmatrix}(-1) + 2\begin{bmatrix}0\\1\end{bmatrix}(-1)$$

$$= \begin{bmatrix}4-2\\4-2\end{bmatrix} = \begin{bmatrix}2\\2\end{bmatrix}$$

Part 10: Since $(w^T x^{(n)} + b) = t^{(n)}$

Then $b = t^{(n)} - w^T x^{(n)}$

$$= 1 - [2\ 2]\begin{bmatrix} 1 \\ 1 \end{bmatrix} = -3$$

$$= -1 - [2\ 2]\begin{bmatrix} 1 \\ 0 \end{bmatrix} = -3$$

$$= -1 - [2\ 2]\begin{bmatrix} 0 \\ 1 \end{bmatrix} = -3$$

$\therefore b = -3$