ECE421: Assignment 03

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Problem 1

Part 1:

Points:

$$x^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, x^{(2)} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, x^{(3)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, x^{(4)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{1}$$

$$t^{(1)} = 1, t^{(2)} = -1, t^{(3)} = -1, t^{(4)} = -1 \tag{2}$$

Graph: The equation for the decision boundary is: $y = -x + 1.5$. And the equation

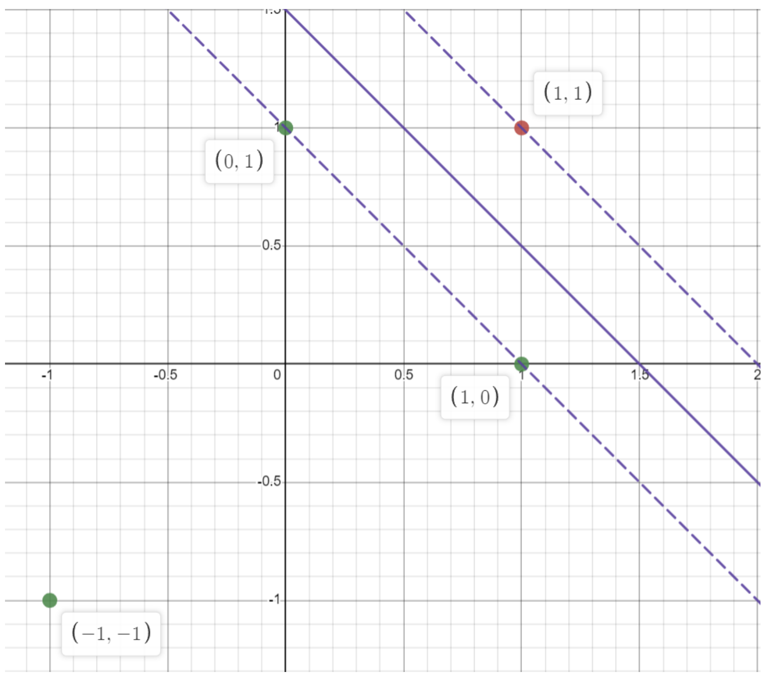

Figure 1: Plot of the points

for the margins are: $y = -x + 1.5 \pm 0.5$.

Part 2:

From the above figure 1, we can see that the support vectors are $x^{(1)}$, $x^{(3)}$, and $x^{(4)}$.

Part 3:

We have $\text{argmin}_{\omega,b}\frac{1}{2}\left\|\omega\right\|^2$ while the constraint of $t^{(n)}(\omega^T x^{(n)}+b) \geq 1 \ \forall n$ holds over the support vectors. We can rewrite this as the following Lagrangian:

$$\mathcal{L}(\omega, b, \alpha) = \frac{1}{2}\left\|\omega\right\|^2 - \sum_{n=1}^{N} \alpha_n \left(t^{(n)}(\omega^T x^{(n)} + b) - 1\right) \tag{3}$$

Where each α_n is the Lagrange multiplier for the nth data point. We can then take the partial derivatives of $\mathcal{L}$ with respect to ω and b and set them to zero to find the optimal ω and b.

Part 4:

To solve the inner minimization problem, we can first take the partial derivative of $\mathcal{L}$ with respect to ω and set it to zero:

$$\frac{\partial \mathcal{L}}{\partial \omega} = \frac{\partial}{\partial \omega}(\frac{1}{2}\left\|\omega\right\|^2 - \sum_{n=1}^{N} \alpha_n \left(t^{(n)}(\omega^T x^{(n)} + b) - 1 \right)) = 0 \tag{4}$$

$$= \omega - \sum_{n=1}^{N} \alpha_n(\omega^T x^{(n)} t^{(n)} + bt^{(n)} - 1) = 0 \tag{5}$$

$$= \omega - \sum_{n=1}^{N} \alpha_n t^{(n)} x^{(n)} = 0 \tag{6}$$

$$\therefore \omega = \sum_{n=1}^{N} \alpha_n t^{(n)} x^{(n)} \tag{7}$$

Additionally, we can take the partial derivative of $\mathcal{L}$ with respect to b and set it to zero:

$$\frac{\partial \mathcal{L}}{\partial b} = \frac{\partial}{\partial b}(\frac{1}{2}\left\|\omega\right\|^2 - \sum_{n=1}^{N} \alpha_n \left(t^{(n)}(\omega^T x^{(n)} + b) - 1 \right)) = 0 \tag{8}$$

$$= -\sum_{n=1}^{N} \alpha_n t^{(n)} = 0 \tag{9}$$

$$\therefore \; 0 = \sum_{n=1}^{N} \alpha_n t^{(n)} \tag{10}$$

From these results we can reorganize the relational equations as:

$$A = \sum_{n=1}^{N} B_n C^{(n)} D^{(n)}, G = \sum_{n=1}^{N} E_n F^{(n)} \tag{11}$$

Such that:

$$A = \omega, B_n = \alpha_n, C^{(n)} = t^{(n)}, D^{(n)} = x^{(n)}, E_n = \alpha_n, F^{(n)} = t^{(n)}, G = 0 \tag{12}$$