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A CONSTRAINT MONITOR ALGORITHM FOR THE CASSINI SPACECRAFT

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Abstract

The Cassini spacecraft is commanded to turn from one point in space to another by commanding attitude, rate, and acceleration profiles, which the Attitude Controller is required to execute faithfully. The Constraint Monitor algorithm performs appropriate checks to ensure the legality and the realizability of the instantaneous attitude, rate, and acceleration commands. When the command is found to be unrealizable because of hardware limitations or illegal because it may enter a constraint space, Constraint Monitor modifies it appropriately such that the legality of the command is maintained. In doing so it ensures that the rate and acceleration are not outside the capabilities of the attitude control hardware and that certain sensitive spacecraft boresights are protected from exposure to bright objects.

1. Introduction

1.1. Mission and Science Objectives

The Cassini spacecraft, scheduled for launch in October 1997, will arrive at Saturn in 2004. On its way to Saturn, it will fly by Venus, Earth, and Jupiter to pick up the needed gravity assists. The spacecraft will be carrying a probe intended for delivery into the Titan atmosphere. The probe entry into the Titan atmosphere will occur about four months after Saturn arrival. The spacecraft will conduct a tour of the Saturnian system for approximately four years. Several close flybys of Titan and Saturn's icy satellites are planned. The nominal mission will conclude in the year 2008.

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Cassini science objectives include investigation of Saturn's atmospheric composition, winds and temperature, configuration and dynamics of the magnetosphere, structure and composition of the rings, characterization of the icy satellites, and Titan's atmospheric constituent abundance. The radar mapper will perform surface imaging and altimetry during each Titan flyby.

Cassini was originally one of the two spacecraft of Mariner Mark II series intended for multi-mission purpose: the CRAF (Comet Rendezvous and Asteroid Flyby) and Cassini. The CRAF mission was to follow a comet and conduct scientific investigations for 120 days and later flyby an asteroid. NASA budget constraints necessitated the cancellation of CRAF and descoping of Cassini spacecraft. Both the high and low precision scan platforms and their structural booms were deleted, as was the turn table which carried a fields and particles experiment. The spacecraft basebody assumed the role of the observation platform and the entire spacecraft now had to turn in order to do earth pointing, star tracking, and remote science pointing. This makes the detection and avoidance of spacecraft attitude constraints ever more difficult since movement of one boresight requires turning the entire spacecraft.

1.2. Spacecraft Configuration and Modeling

Cassini is a flexible spacecraft containing four structural appendages and three propellant tanks. The fully deployed Cassini spacecraft with the Huygens Titan probe is shown in Figure 1. The four long booms are a magnetometer boom and three radio and plasma wave science antennas. The core structure of the spacecraft houses the propulsion module which consists of two bipropellant tanks. At the bottom of the propulsion module is the lower equipment module, which supports three radio-isotope thermoelectric generators, a set of reaction wheels, and two articulable (two axis) main engines for large trajectory correction maneuvers.

At launch, the spacecraft total mass is approximately 5530 kg and the inertia approximately 8780,9050,3770 kg-m^2 about the S/C x, y, z axes. Towards the end of the Saturn tour (probe released), the mass and inertia will be reduced to 2400 kg and 6940, 5720, 3600 kg-

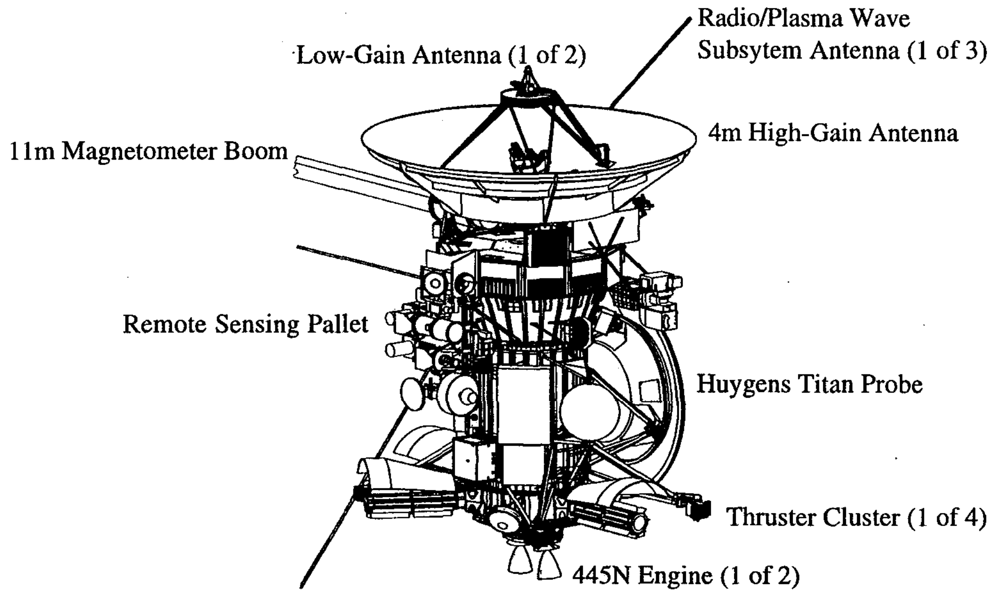

Figure 1. Cassini Spacecraft Configuration (Fully Deployed)

m^2. The magnetometer boom has a fundamental frequency at 0.65 Hz and damping between 0.2 and 1 %. The three radio and plasma wave antennas are much lower in mass and inertia, and have a frequency of 0.13 Hz or higher and a damping of 0.2 %.

At the beginning of the mission, the spacecraft propellant mass is heavier than the spacecraft dry mass. The bipropellant (Monomethyl Hydrazine (MMH) and Nitrogen Tetroxide (NTO)), consumed by the main engine, has a total launch mass of 3000 kg. The two bipropellant tanks are "stacked" along the spacecraft z-axis. Each of the two tanks contains an 8-panel propellant management device to alleviate propellant sloshing. The monopropellant (Hydrazine) launch mass is 132 kg. The monopropellant, consumed by the attitude control thrusters, is stored in a spherical tank off the S/C z-axis.

2. Attitude Control Functions

The primary functions of the Cassini Attitude and Articulation Control Subsystem (AACS) are attitude determination, attitude commanding, attitude control, spacecraft burn maneuvers, constraint violations detection and avoidance, fault protection, command processing, telemetry generation and data handling. The attitude control functional block diagram is shown in Figure 2.

Once the spacecraft attitude with respect to the J2000 celestial coordinate system has been initialized using the sun and the stars, the spacecraft attitude estimator maintains attitude knowledge using star identification and, when necessary, gyro outputs. The AACS software generates the required turn commands upon receiving the uplinked ground commands. The Attitude Controller follows the commanded attitude profile. The attitude profile can command a task as simple as pointing at Earth for downlink or more complicated tasks like surface mapping or probe tracking by the high gain antenna.

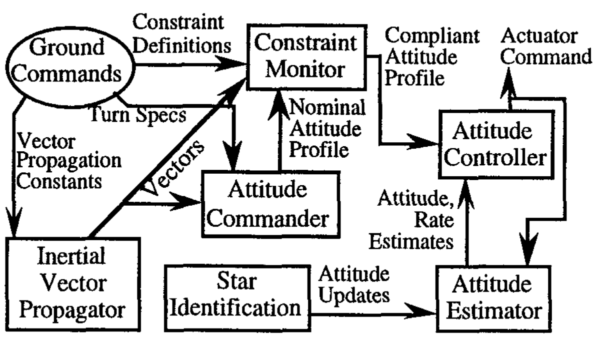

Figure 2. Attitude Control Functional Block Diagram

Large angle slews and coarse pointing are performed using the reaction control system (RCS) thrusters, and precision pointing and slewing for remote science observations by reaction wheels (RWA).

Constraint violations of turn commands are checked and an avoidance path is generated, if necessary, by onboard software prior to execution by the Attitude Controller. Profile commands generated by the Attitude Commander are continuously monitored by Constraint Monitor. By design, the attitude commanding and constraint monitoring tasks are not coupled. The actions of constraint monitor downstream of the nominal attitude profile generation do not effect the nominal attitude profile generation. Ideally, the commander-generated profile should never violate a constraint region if the ground command sequence is generated and checked properly. However, if the ground-commanded profile is in error, or must be abandoned due to a fault, and the attitude profile were to bring an instrument boresight or other sensitive aperture into a constraint region (e.g. an exclusion zone around Sun), the software would generate an alternative path to avoid the excluded region. It is the alternate path, traversed at an acceptable rate and acceleration, which is passed along to the attitude controller. The spacecraft is commanded to follow the alternate path until it can acceptably merge with the attitude commander-generated profile.

This paper considers the design of constraint detection and avoidance aspects of the software. The intent is to give the reader a clear understanding of the two steps involved in the process -- the detection of constraint violations and, once a violation has been detected, the steps involved in the avoidance process. The avoidance step is where an acceptable alternate path is generated.

In the following, underlined capital letters denote matrices, underlined lowercase letters denote vectors (in our usage, an n-vector is an n by 1 matrix), a ()′ implies transposition. The subscript b is used to denote a vector in S/C coordinates (as in $\underline{x}_b$). When no subscript is present, the vector is assumed to be in inertial coordinates. The symbol '×' implies vector cross product. A quaternion multiplication is implied whenever two quaternions appear together. In our usage the first three elements of a quaternion represent the vector component.

3. Constraint Monitor

The Constraint Monitor (CMT) function, which exists between the Attitude Commander (ACM) and the Attitude Controller (ACL) functions, implements part of the AACS fault protection functions. It determines what commanded motions are acceptable, when action should be taken to interrupt the ACM-commanded pointing profile so that an unacceptable motion is not realized, and determines how to re-join with the ACM-commanded motion, should the nominal path become acceptable again. Nominally, the Constraint Monitor should not alter the ACM-commanded motion. It is during a fault response that CMT is more likely to intervene and appropriately alter the ACM-commanded path. The Constraint Monitor function on-board Cassini

- ensures that the attitude commands received by the attitude controllers do not violate a celestial constraint.
- ensures that attitude commands passed to the controllers are smooth in attitude and rate, within tolerances (rate limited) and realizable (acceleration limited).

The two functions performed by the algorithm may be partitioned further as the Detection function and the Avoidance function. The Detection function provides the capability to detect constraint violations for ACM-commanded motions. The algorithm is capable of detecting imminent violations so that action may be taken well in advance. The Avoidance function generates the alternate path which satisfies all constraints when ACM-commanded path has been found to be in violation. The computational needs of the Avoidance algorithm are significantly greater than that of the Detection algorithm. In order to make CMT run in real-time, only the Detection algorithm is exercised every control cycle (125 millisec). The avoidance algorithm on the other hand executes in the background at a slower pace (once every 2 seconds). The end product of the Avoidance algorithm is an acceleration command which is to be applied until the next Avoidance computation 2 seconds later. The attitude and rate commands are needed every 125 millisec, however. The required commands are kinematic extrapolations, carried out every 125 millisec, which use the last commanded attitude and rate, and the latest avoidance acceleration computation.

The nature of all constraints enforced by CMT may be classified as celestial or dynamic. Flight rules restricting the pointing of spacecraft hardware at celestial objects (which can also include direction of incoming particles, e.g. at ring-plane crossing) are translated into celestial constraints enforced by the CMT. Enforcement of these constraints protect certain spacecraft fixed directions from exposure to celestial objects. There are two types of celestial constraints - non-timed and timed. A non-timed constraint may be expressed as follows.

- The angular separation between body vector B and celestial vector C shall never be less than Θ

A timed constraint, which protects certain spacecraft (S/C) axes from *extended* exposure to celestial objects, may be expressed as :

- The angular separation between body vector B and celestial vector C shall never be less than Θ for a time period greater than T. Time is accumulated at the rate of a 1 sec/sec inside the constraint to a maximum of T, and when outside the time is credited back (i.e. decremented) at a rate of R sec/sec to a minimum of zero.

Name	Celestial Object, C	Body Object, B	Half Cone Angle Θ (deg)	Max Allowable Time, T (sec)	Decay Rate R, (sec/sec)	Type	Drop/ Keep
const 1	SUN	B1	30	0	0	AVOID	KEEP
const 4	SUN	B2	30	0	0	AVOID	DROP
const 2	SUN	B4	85	300	0.01	DETECT	KEEP
const 3	SUN	B3	70	75	1.0	OFF	DROP
const 5	INERT 1	B5	30	45	3.0	DETECT	DROP

Table 1. Constraint Table Example

Celestial constraints define a conical region around the celestial vector. The body vector is required to stay out of the cone for non-timed constraints and for timed constraints it is required that the body vector not stay inside the cone for longer than the specified duration. CMT also enforces dynamic constraints to ensure that the acceleration and rates commanded by the ACM are not excessive. These constraints are specified as rate and acceleration ellipsoids in the spacecraft fixed coordinates. The sizes of these ellipsoids can be controlled via ground commands.

The celestial constraints enforced by CMT are defined by ground commands and stored in a table on board (Table 1). The table is of a fixed size (10 entries) and stores all relevant attributes of celestial constraints, which are, the constraint name (unique), the name of the body object, the name of the inertial object, the spatial extent (the half-cone angle of the constraint cone), the temporal extent (timed constraints only - the constraint is treated as a hard constraint when this attribute is zero), and the decay-rate (used in enforcing only the timed constraints). The constraint table need not be full. In addition, the ground commands also specify whether a constraint is to be avoided (avoidance action taken when the ACM-commanded path is about to violate the constraint) or only detected (no avoidance action taken but a violation is reported). A constraint can also be turned OFF (neither DETECT nor AVOID) The celestial and the body vector identifiers (C and B, respectively, in the examples above) and measures Θ, T, and R are commandable parameters. The last attribute of a constraint is the Drop/Keep flag which can have two possible values: DROP or KEEP. This allows CMT to ignore the constraint marked DROP in certain situations (Section 3.2.4). Any attribute of an existing constraint can be modified. Constraints can be removed from this table only by ground commands.

Each constraint is identified by a unique name. Notice that a constraint's inertial and body vectors are also referred to by *name*. The appropriate vector values are obtained from the Inertial Vector Propagator (IVP). Two separate tables in IVP maintain the appropriate values for the inertial and body vectors. The body vector table is not propagated with time but the inertial vector table is. In other words the positions and velocities of the objects stored therein are in general time-dependent. The vector name and epoch are supplied by CMT and the appropriate vector value is returned by IVP.

Once a constraint violation has been detected, the Avoidance function allows the ACM-commanded attitude, rate and acceleration to be modified such that the resulting attitude, rate and acceleration are not in violation of any celestial / dynamic constraints. The Avoidance machinery does not engage unless CMT is currently in Avoidance and / or an AVOIDable celestial constraint violation or a dynamic constraint violation has been detected. When "DETECT only" celestial constraints are violated, the ACM-computed attitude is not altered but such a violation is reported in telemetry. The CMT output is fed directly to the appropriate attitude controller as the attitude commands that the controller must follow.

3.1. Detection Function

This function provides the capability to anticipate AVOIDable celestial constraint and dynamic constraint violations. Two types of constraints are covered: AVOIDable celestial constraints and dynamic constraints. The celestial constraint checks are based on geometry and extrapolated attitude motion (this is an extrapolation based on the current ACM-commanded attitude, rate, and acceleration). The dynamic constraint checks are more "immediate" in nature.

3.1.1. Celestial Constraints

There is clearly the need for early detection of AVOIDable celestial constraint violations. We would like to start taking evasive action early enough so that the constraint cone is not entered for non-timed constraints and not entered for more than the allowed time for timed-constraints. This requires extrapolation of the current path to a point in time which is 2 background cycles (4 seconds) later. This allows sufficient time for the Avoidance algorithm to complete, should a violation be declared imminent. Constraint regions are always avoided by using maximum allowable acceleration in a direction which moves the offending body vector *radially* away from the celestial vector. Imminent violation of a non-timed constraint can therefore be detected early when the predicted (4 seconds later) celestial-body separation angle is less than the sum of the constraint angle and the maximum deceleration *stopping* distance. This stopping distance is the predicted angular motion in the direction of the constraint under the assumption that a radial acceleration is applied away from it until the rate towards the constraint is nulled. A closed-form solution for this distance is not possible. Instead a first order approximation is used. It works very well, especially for the relatively weak acceleration and rate capabilities of Cassini. The cross product of the celestial and the body vector (unit vectors $\underline{c}$ and $\underline{b}$, respectively) provides the direction (henceforth referred to as the "escape" direction) used in the evaluation of the stopping distance. Let the $\underline{\lambda}$ denote a unit vector in this direction.

$$\underline{\lambda} \quad = Unit\,(\,\underline{c} \times \underline{b}\,),$$

where *Unit* is the normalization operator and predicted future values of $\underline{c}$ and $\underline{b}$ are used in this evaluation. The Avoidance function applies maximum available acceleration in the $\underline{\lambda}$ direction to move the body vector away from the constraint cone when a celestial constraint violation has occurred or is imminent (it is not necessarily time-optimal; it would be, if all axes had the same acceleration capability). Computation of the stopping distance requires the relative angular rate between the celestial and the body vectors (let the two rates be $\underline{v}$ and $\underline{\omega}$, respectively). The scalar rate which we are trying to annul by rotating about the escape direction, should the body vector get too-close to the constraint edge, is the projection of the differential rate onto the escape direction.

$$\omega \quad = (\,\underline{v} - \underline{\omega}\,)'\,\underline{\lambda}.$$

If the body vector is moving away from the celestial vector ($\omega < 0$) then there can not be an imminent violation. Consequently the stopping distance is set to zero, else it is computed as the angular distance moved such that the application of maximum deceleration about $\underline{\lambda}$ annuls the projected rate ω. The prediction requires the use of maximum available acceleration along the escape axis. The use of largest available acceleration might be optimistic since, while trying to escape, the escape direction may change as the offending body vector is pushed away from the celestial constraint it is trying to avoid. To avoid this optimism (and therefore future violations) and err on the conservative side, the smallest available acceleration (α) is used <u>only</u> in predicting violations. The exact rate and acceleration constraints are used in constraint avoidance. The stopping distance δ is therefore evaluated as

$$\delta \quad = 0.5\,\omega^2/\alpha, \qquad \text{if } \omega > 0.$$

A violation is declared imminent if the predicted future c-b separation

$$\theta \quad = Cos^{-1}(\,\underline{c}'\,\underline{b}) \;<\; \delta + \Theta.$$

To detect imminent violations of timed constraints, the additional time that a body vector can spend inside the timed constraint is also required. Note that it is legal to enter the constraint space here but not for more than the specified time. The time accumulation t is updated as follows. If the angle θ is smaller than the constraint angle Θ, implying that the body vector is predicted to be inside the constraint cone, the time must be accumulated. If the body vector is outside the constraint cone, the accumulated time must be reduced at the rate R specified in the constraint definition. In other words

$$t \quad = t + \Delta, \qquad \text{if } \Theta > \theta,$$
$$\quad = t - R\,\Delta, \qquad \text{if } \Theta < \theta,$$

where Δ is the time elapsed since the last evaluation. This accumulation is bounded by 0 from below and Δ from above. At each cycle, the following question is asked. If an acceleration in the escape direction is applied, would the constraint be entered as the S/C decelerates to a zero rate in the escape direction. If not then there can not be a violation. If the constraint is predicted to be entered but can be exited in the time remaining (T - t) using the allowable rate and acceleration then there can not be a violation imminent, otherwise an imminent violation is declared and Avoidance machinery is invoked.

3.1.2. Dynamic Constraints

The dynamic constraints checks are relatively simple in nature. The rate and acceleration constraints are specified as S/C fixed ellipsoids. The ACM-commanded rate and acceleration commands must never lie outside their respective ellipsoids. A dynamic constraint violation is declared when the commands are outside, e.g. let $\{\alpha_x, \alpha_y, \alpha_z\}'$ be the acceleration command and (A_x, A_y, A_z) be the semi-major axes of the acceleration constraint ellipsoid, then the acceleration command is outside the acceleration ellipsoid (a violation) if

$$\{\alpha_x/A_x\}^2 + \{\alpha_y/A_y\}^2 + \{\alpha_z/A_z\}^2 > 1.$$

To detect imminent rate violations, the rate command at the next evaluation time is also estimated (assuming constant current acceleration) and verified against the rate ellipsoid. A dynamic violation is declared whenever the acceleration vector is outside the acceleration ellipsoid, or the current rate command or the predicted rate command is outside the rate constraint ellipsoid. There are two types of attitude controllers on-board, one which uses reaction wheels and the other which uses thrusters. The maximum rate and acceleration capabilities are different for each. Although the rate and acceleration constraints imposed by the algorithm are control-law specific, the fundamental behavior of the algorithm does not change when we transition from thrusters to reaction-wheels.

3.2. Avoidance Function

The Avoidance function is invoked when a violation is declared imminent by the Detection function. CMT stops calling this function only when ACM commands are in compliance of all constraints and CMT-propagated motion is close to the ACM-commanded motion. The purpose of this function is to modify the ACM-commanded attitude, rate, and acceleration commands such that the eventual attitude, rate, and acceleration commands (i.e. the commands relayed to the Attitude Controller) are in conformance with all constraints. The end product of the Avoidance evaluations is an acceptable acceleration command which will prevent the attitude from violating celestial constraints and the rate and acceleration commands from exceeding the hardware limits. This function executes at a slower pace (once every 16 Detection cycles). On initial entry into avoidance, the acceleration command is set to zero until the function returns with the correct acceleration command 2 seconds later. This delay is accounted for in the avoidance acceleration evaluation.

The corrective action taken is designed to be such that the constraint in question is not violated eventually. The corrective action consists of three steps, which may be repeated several times until the CMT-commanded motion merges with the ACM-commanded motion. The end goal is to always try to move towards a point in space which is not violating any constraints and is close to the ACM commanded path. When the ACM command is not in violation, the goal is of course the ACM command itself. The first action taken when a celestial constraint violation is declared imminent is the so-called Escape where an acceleration is commanded such that it moves the offending body vector radially away from the inertial vector. While in this mode, maximum available acceleration along the escape axis is used until the constraints are no longer in danger of imminent violations. When several constraints are to be "escaped" simultaneously, the appropriate escape directions are combined linearly to obtain the resultant escape direction. This action is maintained until the constraints in question are no longer in imminent violation. If at this point it is possible to move along a great circle towards the goal attitude and not violate any constraint in the immediate future (next 2 seconds), a Clear mode is declared where the motion is a great circle arc heading towards the goal. When this is not possible, the constraint or a collection of constraints have to be circum-navigated until it is possible to transition to the Clear mode. This intermediate mode is called the Circulate mode. While in this mode the CMT attitude is such that all body vectors remain outside their respective celestial constraints. The timed-constraints are allowed to be penetrated (under certain conditions) but non-timed constraints are generally followed at a fixed distance. This behavior is referred to as "...following the flow-field". While in Circulate mode, the attitude is continually checked for imminent violations (transition back to Escape) or possibility of movement towards the goal attitude (transition to Clear). The rate and acceleration constraints are never violated. The goal attitude that the CMT attitude is always striving for is such that it is in compliance of all AVOIDable celestial constraints. When there are no constraint violations, the attitude and rate goals coincide with the ACM commands. When one or more celestial constraint violations are present, the goal attitude is such that all celestial constraints (which are to be AVOIDed) are satisfied. The rate goal then is zero rate. This has been assumed since the attitude goal can exhibit rapid movement which may be discontinuous at times. The discontinuities are an artifact of trying to compute the attitude goal in a finite number of steps. It makes no sense therefore to compute a rate goal consistent with the variation in the attitude goal. Only the violated constraints are considered in determining the attitude goal. The attitude goal is not "close" to the ACM command necessarily but it does satisfy all violated constraints (only in rare cases have we found the algorithm to yield a non-compliant attitude: rapidly moving celestial constraints are one example). There may also be situations where no solution exists for a goal attitude. The algorithm is not interrupted in such an event since the rest of the CMT machinery would not allow the CMT attitude to acquire a non-compliant attitude.

Several cycles of Escape - Circulate - Clear (not necessarily in this order) may be required until the goal attitude is reached. Escape and Circulate are entered only when celestial constraint violations are imminent. When the CMT attitude is *far* from the goal it is trying to attain, largest possible rate is commanded until it is in the vicinity of it where a rate proportional to the distance from the goal is commanded.

3.2.1. Flow-Fields

It was noted earlier that while in the Circulate mode, the constraints have to be circum-navigated until a transition to either the Escape or Clear mode occurs. Computation of the direction in which to move around constraints is not straightforward and requires visualization of constraint cones in the Rodriguez parameter space (if $\underline{q}$ is the inertial-to-body attitude quaternion, then $\varepsilon(i) = q(i)/q(4)$, $i = 1, 2, 3$ are the Rodriguez parameters) - referred henceforth as the Q-space. The Rodriguez parameters form the basis of the Q-space. The constraint cones have the appearance of hyperboloids of one-sheet in the Q-space. The Q-space is obviously not finite but infinity here simply implies a π rotation. Attitude is a point in this space and the evolution of $\underline{\varepsilon}$, the "attitude", is required to stay outside a collection of hyperboloids.

In general the attitude $\underline{\varepsilon}$ is in compliance (i.e. outside the constraint) when the following quadratic form is negative, i.e.

$$\underline{\varepsilon}'\,\underline{U}\,\underline{\varepsilon} + 2\,\underline{v}'\,\underline{\varepsilon} + w < 0 \quad \Leftrightarrow \quad \Theta < \theta,$$

where the matrix $\underline{U}$, vector $\underline{v}$, and scalar w are dependent on celestial and body vectors and the constraint angle. Let $\underline{c}$ be the constraint celestial vector in inertial coordinates, $\underline{b}_b$ be the constraint body vector *in S/C coordinates* and Θ be the constraint angle. Then it can be shown that

$$\underline{U} = \underline{c}\,\underline{b}_b' + \underline{b}_b\,\underline{c}' - \{\underline{c}'\,\underline{b}_b + Cos(\Theta)\}\,\underline{I},$$

$$\underline{v} = \underline{b}_b \times \underline{c},$$

$$w = \underline{c}'\,\underline{b}_b - Cos(\Theta),$$

where $\underline{I}$ is a 3 x 3 identity matrix. Attitude has to "flow" a certain way around a collection of such surfaces as $\underline{\varepsilon}$ makes its way towards the goal attitude. The direction ($\underline{c}$+$\underline{b}_b$) forms the symmetry axis of the constraint hyperboloid. It makes sense to pick a direction such that the flow around the constraints moves the attitude in positive sense about the ($\underline{c}$+$\underline{b}_b$) axis. This argument sets the flow-field direction in a relative sense. The correct orientation (i.e. one way or the other) is determined when the first imminent celestial constraint violation is declared.

3.2.2. The Goal Attitude

When CMT is in avoidance the attitude does not wander aimlessly, it is actually trying to move towards an attitude goal as it negotiates the constraints between the current attitude and the attitude goal. When the ACM-commanded attitude is in compliance with all AVOIDable celestial constraints, the goal attitude and rate are the ACM-commanded values. The goal attitude and rate are not close, necessarily, to the ACM attitude (it is, in single constraint situations). Rather, the algorithm provides an attitude which is in compliance with all AVOIDable celestial constraints. The solution process is iterative but the algorithm is exited after a fixed number of iterations, regardless of whether a goal attitude has been found or not. As a consequence of the fixed number of iterations, the alternate attitude can exhibit discontinuities.

The algorithm starts by accepting the ACM attitude and rate as the starting point for the goal-attitude iterations. At each instance of an AVOIDable celestial constraint violation (by the candidate goal attitude that is), the candidate goal attitude is updated such that the offending body vector (at the modified goal) lies slightly outside the edge of the constraint cone. The movement of the offending body vector is accomplished by rotating the body vector about the escape axis $\underline{\lambda}$. Two solutions are possible: either move the short distance about the positive escape direction or the long distance about the negative escape axis. The path taken (short / long) is such that the movement of one body vector will not push a previously adjusted body vector back inside it's constraint. The iterations to compute a goal attitude are repeated until there are no violations during an iteration or the iteration count exceeds the number of constraints under consideration (whichever comes first). The goal rate is zero whenever the goal attitude is not the ACM-commanded attitude.

3.2.3. Desired Direction of Motion

Once a goal attitude is available, the direction in which CMT should try to head towards is needed. The direction is a function of the current state, the goal state (attitude and rate), and the location of the constraints which are to be avoided. The desired direction of motion is prescribed in terms of a rate vector which CMT should try to attain by the end of next Avoidance cycle except in Escape mode, where the desired direction is actually a direction in which to accelerate in.

When in the Escape mode, the desired direction of travel is the weighted sum of all applicable escape directions and a maximum possible acceleration is commanded in this direction until either the constraints are no longer in imminent violation or the rate limit is reached.

When the constraints have been exited (i.e. an Escape is no longer needed), a transition to either the Circulate or the Clear mode takes place. These modes were described earlier. A Clear mode is trasitioned to if it is possible to move towards the goal attitude through a single-axis rotation and not be in imminent violation of constraints which may lie between the current attitude and the goal.

When far from the goal, maximum allowable rate is prescribed. In the vicinity of the goal, the prescribed rate ($\underline{\omega}^p{}_b$) becomes a linear function of the separation from the goal.

$$\underline{\omega}^p{}_b = \underline{\omega}^g{}_b + K\, \underline{q}^g\, \underline{q}^*, \quad (1)$$

where $\underline{\omega}^g{}_b$ is the goal rate in S/C coordinates $\underline{q}^g$ is the goal attitude quaternion and $\underline{q}$ the current CMT-commanded attitude quaternion. The ()* is the conjugation operation. Note that only the vector part of the quaternion product $\underline{q}^g\, \underline{q}^*$ is used in (1). The gain K is such that stability of motion is guaranteed as CMT commanded attitude closes in on the goal attitude (more on this later) and the approach transients are acceptable.

When a transition from Escape to Clear mode is not possible it is only because a constraint violation may be *immediately* imminent had CMT tried to move towards the goal. A transition to Circulate mode is then declared until it is possible to move towards the goal (Clear mode). A desired rate vector has to be computed again. The desired rate magnitude is the largest possible when far from the goal, else the magnitude is the norm of $\underline{\omega}^p{}_b$ (1). The direction of the desired rate vector is a weighted sum of the so-called "circulation vectors". The weights bear an inverse square relationship to the distance from the edge of the constraints. Each constraint has a unique circulation vector which is defined such that it yields a minimum twist as the body vector follows the constraint boundary. Let $\underline{c}_b$ and $\underline{b}$ be the celestial and body vectors in the S/C coordinates now. Consider a perturbed attitude which is reached by following a sequence of two rotations: a rotation ψ about $\underline{c}_b$, followed by another rotation $-\psi$ about $\underline{b}$. It can be shown that the sequence yields a minimum twist offset between the starting and the perturbed attitude when ψ is small. The attitude perturbation (quaternion) can be shown to be:

$$\underline{q}^p = [\, (\underline{c}_b\text{-}\underline{b})\, s\, c + s^2(\underline{b}\times\underline{c}_b),\ c^2+s^2(\underline{c}_b{}'\underline{b})],$$

where $s = Sin(\psi/2)$, $c = Cos(\psi/2)$. Clearly, as ψ becomes small, the perturbation approaches:

$$\underline{q}^p = [\, (\underline{c}_b\text{-}\underline{b})\, \psi/2,\ 1],$$

which implies a rotation about the direction $\underline{c}_b$-$\underline{b}$ is S/C coordinates. This direction is the required circulation vector.

3.2.4. Avoidance Acceleration

Once the required rate vector has been computed in the previous step, all that is left to do is to compute the required acceleration command. Evaluations are different depending on the avoidance mode (Escape/Circulate/Clear) CMT happens to be in. An Escape mode implies that a constraint has been entered or is about to be entered and the commanded acceleration must move the violating body vector radially away from the celestial vector. The acceleration direction in this case is the sum of all applicable escape directions ($\underline{\lambda}_s$). Largest possible acceleration is applied in this direction until the mode can be transitioned to Circulate or Clear or maximum rate is reached.

For non-Escape situations, the desired rate vector (1) is used to compute the acceleration command:

$$\underline{\alpha}^p{}_b = (\, \underline{\omega}^p{}_b - \underline{\omega}_b\,) / \Delta, \quad (2)$$

where Δ is the avoidance cycle (2 seconds in this case), $\underline{\omega}^p{}_b$ is the prescribed rate (1) and $\underline{\omega}_b$ is the current CMT-commanded rate in S/C coordinates. It can be shown that, in the proximity of the goal attitude, the approach law (1), when used in (2) to evaluate the desired acceleration, results in a stable behavior when

$$0 < K\Delta < 4, \quad \text{(Cassini: } K = 0.05,\ \Delta = 2)$$

It is entirely possible that the acceleration suggested by (2) violates the CMT acceleration constraints, i.e. lies outside the acceleration ellipsoid. The direction of $\underline{\omega}^p{}_b$ must be preserved if possible. A geometric solution is possible sometimes[2] which allows the length of the $\underline{\omega}^p{}_b$ to be shortened such that $\underline{\alpha}^p{}_b$ lies exactly on the acceleration ellipsoid. When this is not possible the length of $\underline{\alpha}^p{}_b$ is shortened such that the acceleration command lies on the acceleration ellipsoid[2].

3.2.5. Avoidance End

Avoidance is declared over only when the ACM-commanded path is not in imminent violation and the CMT-commanded path (in attitude and rate) is *close* to the ACM-commanded path. The maximum rate and acceleration limits enforced by CMT are used to quantify *closeness* in this case.

Since sequences are checked rigorously on the ground before uplink, likelihood of a constraint violation during normal operations is very slight. Although chances are slight, we expect a dynamic constraint violation to be more likely to occur than a celestial constraint violation. It is not hard to construct impossible situations for CMT to handle. Pointing constraints can be constructed such that the CMT-commanded attitude never attains the CMT-computed attitude goal. Protection against such occurrences is provided by another attribute of the constraints (the DROP/KEEP column in Table 1). When a situation such as the one described here arises (i.e. away from the goal attitude for too-long) all non-Sun based constraints (i.e. constraints for which the inertial vector is not the S/C to Sun direction) and all Sun-based constraints whose DROP/KEEP attribute is DROP are marked

DETECT only (i.e. the type is changed to DETECT) by CMT. No ground intervention is needed in this step. The remaining constraints which have to be negotiated are therefore all Sun-based and it can be shown that CMT-commanded attitude should always be able to merge with the CMT-computed goal in this case.

4. Examples

The actions of the algorithm are demonstrated below via some examples. First consider a simple case where only one constraint is to be enforced. The constraint region and the path taken to avoid it are shown in Figure 3. The constraint on the left is a hard constraint and the one on the right a timed one which is allowed to be entered but for not more than 40 seconds. The constraint is defined as a cone of 30 degrees about the inertial Z-axis. The body vector to be kept out of it is the spacecraft x-axis. First consider a case where ACM commands a path which moves the spacecraft x-axis, initially outside, through the constraint to a final point outside the constraint. The two avoidance examples for this ACM command are shown in the top two sub-plots in Figure 3. The intersections of the spacecraft x-axis

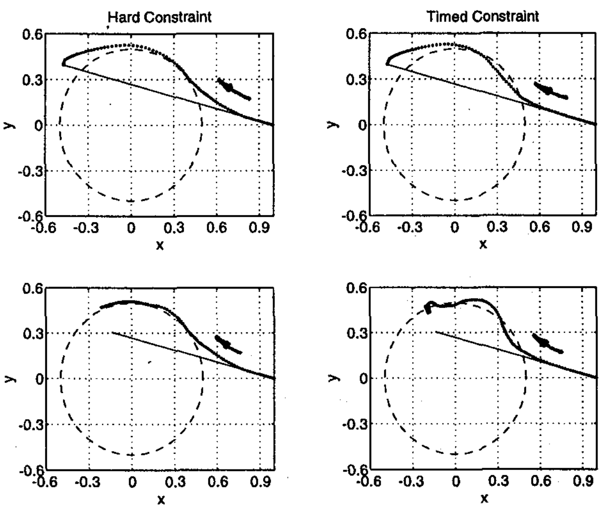

Figure 3. Single Constraint Avoidance

with the unit celestial sphere projected onto the inertial XY plane are shown here. The constraint edge is a dashed circle centered at (0,0) in this plane. Inside of this circle is the forbidden area. The ACM-commanded x-axis path goes from right to left and is shown as the thin solid line and the CMT action appears as a dot plotted every 2 seconds. The hard constraint is never entered and CMT rejoins with ACM after an Escape-Circulate-Clear sequence. The time constraint (top right) is indeed entered but for only 36 seconds (it is exited with 4 seconds to spare). It also rejoins with the ACM point later on.

Next consider the same example as above except that this time ACM commands a motion to leave the x axis at rest inside the constraint. In this case (the bottom plots), CMT seeks the goal attitude, just outside the constraint but close to the ACM point, and comes to rest there. Again, the hard constraint is never entered but the timed constraint is entered but exited in time (0.875 seconds to spare). Note that repeated attempts are made by CMT to approach the ACM commanded location inside the timed-constraint. This is of course perfectly legal as long as the spacecraft x-axis exits the timed-constraint in time.

Consider a more complicated example next, where three constraints have to be avoided. The constraint table is as shown in Table 2.

Name	C	B	Θ	T	R	Type	Dp/Kp
A	C1	B1	30°	0	0	AVOID	KEEP
B	C2	B1	45°	0	0	AVOID	KEEP
C	C3	B2	15°	50s	1.2	AVOID	KEEP

Table 2. Multiple Constraints Example - I

C1	0.0000	0.0000	1.0000
C2	0.4332	-0.2512	0.8661
C3	0.0000	-0.9662	0.2588
B1	-0.2418	0.6645	0.7071
B2	0.1228	0.6964	0.7071

Table 3. Vector Components for Constraints in Table 2

The constraint cone projections and projections of various axes on the unit J2000 celestial sphere are shown in Figure 4. Constraint regions are regions

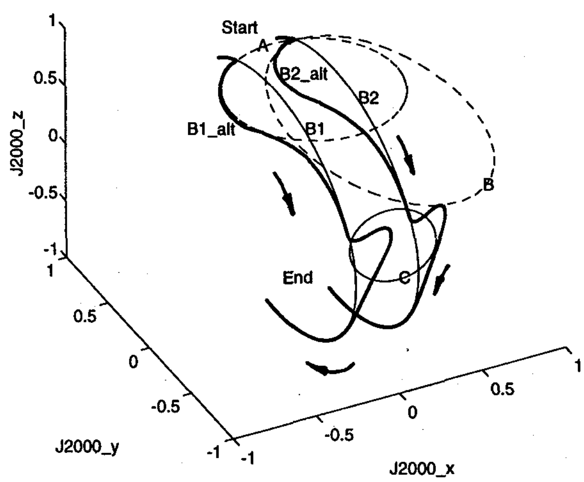

Figure 4. Multiple Constraint Avoidance (Table 2)

inside the dashed circles labeled A, B, C representing the three constraints A, B, C, respectively. The ACM-commanded paths, shown as thin arcs, begin at the top. The B1 path cuts across all three constraints but it is required to stay out of constraints A and B. B2 path also cuts across all three constraints but it is required to not remain in C for more than 50 seconds. The avoidance paths taken by vectors B1 and B2, shown as the thick line, are labeled B1_alt and B2_alt. The algorithm does its job by keeping B1 within 0.3 mrad and 18.3 mrad of violating constraints A and B respectively. Body vector B2 remained inside constraint C for 47 seconds, it exited with 3 seconds to spare. The time histories of the angular separation between the nominal ACM-commanded attitude (which is in violation some of the time) and the alternate path suggested by the CMT-commanded attitude (i.e. the separation between the thick and thin paths in Figure 4) and the CMT-mode are shown below in Figure 5. In this case CMT-commanded path merges with the ACM-commanded path at the 750 sec mark.

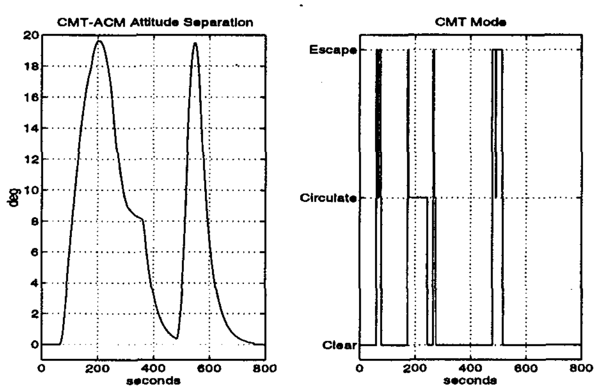

Figure 5. Attitude Separation and CMT Modes

As the final example consider a 2-constraint case where a complicated slew profile is commanded by ACM. The constraints in this case are as listed in Tables 4-6. Two variants are considered. In the first (Table 4), both constraints A and B are non-timed, and in the second (Table 6), Constraint B is allowed to be entered but not for more than 110 seconds. The two constraints share the same inertial vector.

Name	C	B	Θ	T	R	Type	Dp/Kp
A	C1	B1	90°	0	0	AVOID	KEEP
B	C1	B2	110°	0	0	AVOID	KEEP

Table 4. Multiple Constraints Example - II

C1	0.0000	0.0000	1.0000
B1	0.1710	0.0030	0.9848
B2	-0.3830	-0.3214	0.8660

Table 5. Vector Components for Tables 4, 6

The motion of the inertial vector C1 in S/C x-z plane is depicted (solid line) in Figure 6. The dashed lines in this figure denote the constraint boundaries (the edges closest to the inertial vector are marked A and B, respectively, for the two constraints A and B). The ACM-commanded path resembles a mosaic during which the inertial vector position is at the lower right corner (outside both constraints) at the start, it then zig-zags in and out of the constraint space, eventually returning back to the starting position.

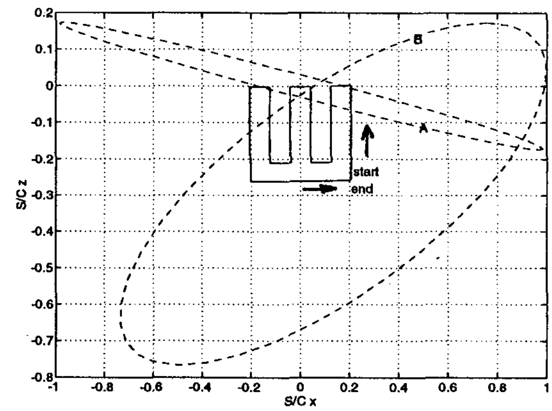

Figure 6. Constraints and the Commanded Path

The avoidance motion and the related time histories for this case are shown in the subplots on the left in Figure 7. The top figure depicts the constraint inertial vector path commanded by CMT in S/C x-z plane (dark line) superimposed on Figure 6. An 'x' and 'o' have been placed at 50 second intervals, respectively, along the ACM- and CMT-commanded paths. The minimum distance from the edges of the constraints in this case was 1 and 2.6 mrad, respectively. The time history of the angular separation between the ACM-commanded path and the CMT-commanded path is shown in the middle and the progression of CMT-mode appears at the bottom. ACM-commanded profile is rejoined soon after the 800 sec mark. As another variation of the example above, consider the following constraint table. It is exactly as in Table 4, except that Constraint B is now a

Name	C	B	Θ	T	R	Type	Dp/Kp
A	C1	B1	90°	0	0	AVOID	KEEP
B	C1	B2	110°	110s	1	AVOID	KEEP

Table 6. Multiple Constraints Example - III

timed-constraint allowed to be ventured into for not more than 110 seconds. The related motions and time histories appear as the subplots on the right in Figure 7. Note that Constraint B is entered this time but it is exited with 1.25 seconds to spare. ACM-commanded path remains inside Constraint B for about 120 seconds.

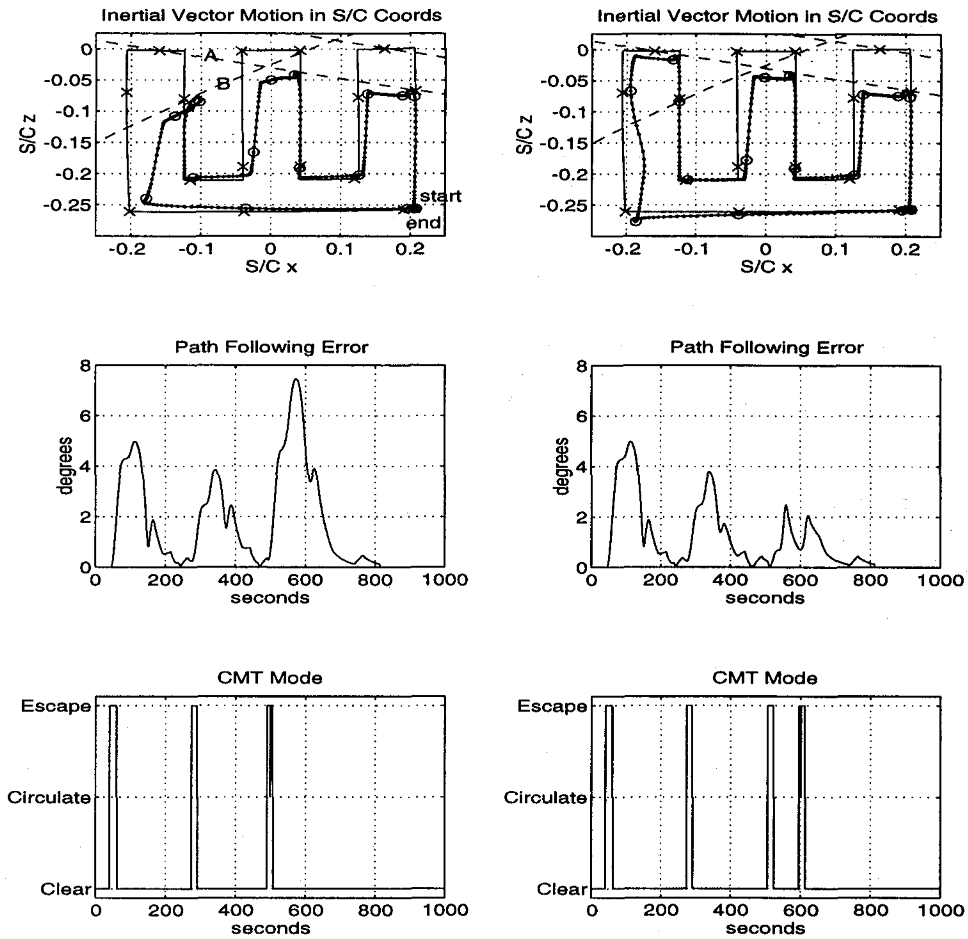

Figure 7. Multiple-Constraint Avoidance Example II

5. Summary

This paper presented the design of an autonomous avoidance algorithm. The algorithm is slated to fly on-board the Cassini spacecraft in October 1997. The algorithm does more than simply avoid pointing constraints. It prevents excessive rate and attitude commands from being sent to the attitude controllers. To our knowledge, autonomous constraint avoidance capability of the kind described here has never been designed or flown on an inter-planetary spacecraft. The algorithm is unique in the sense that it reacts to current S/C attitude relative to the constraints. No action is taken unless it is almost absolutely necessary to do so. The algorithm has been rigorously tested on many testbeds used to validate the Cassini Attitude Control Flight Software and it's performance has been very satisfying indeed.

6. Acronyms

S/C	Spacecraft
AACS	Attitude and Articulation Control Subsystem
ACM	Attitude Commander
ACL	Attitude Controller
CMT	Constraint Monitor
IVP	Inertial Vector Propagator

7. Acknowledgments

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8. References

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[2] Singh, G. and Macala, G.A., "Cassini Constraint Monitor Algorithm", IOM 3456-95-071, 21 December 1995