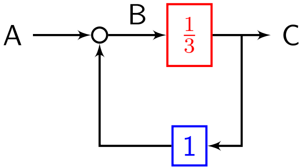
补充例子

Outline

$$R(s) \longrightarrow * + [Fo] \longrightarrow * + [F]\frac{1}{s} \xrightarrow{\qquad\qquad} C(s)$$

Topic

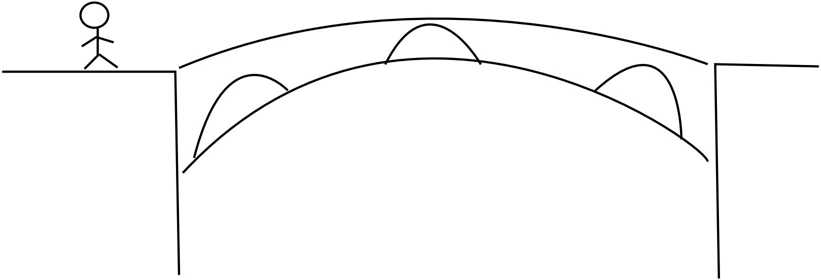
反馈与方程（静态）



$$
\begin{aligned}
C &= \frac{B}{3} \quad (1)\\
A + C &= B \quad (2)\\
A &= 10 \quad (3)\\
C &= ? \quad (4)
\end{aligned}
$$

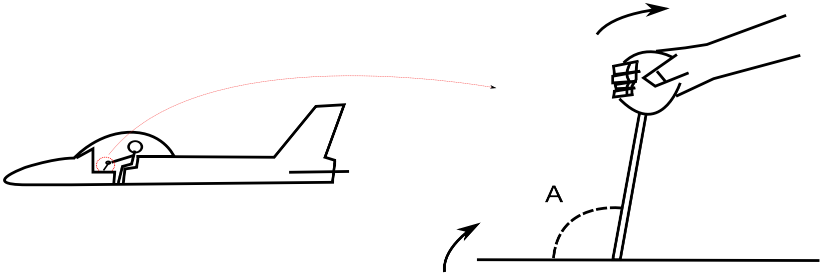
笑话：过桥

路人甲要过桥，发现桥很长，问桥边路人乙桥长多少，乙说：50 米。甲走上桥后不一会儿，乙追了上来，说，桥长 100 米，你要是过了 50 米就转弯，就掉下去了。



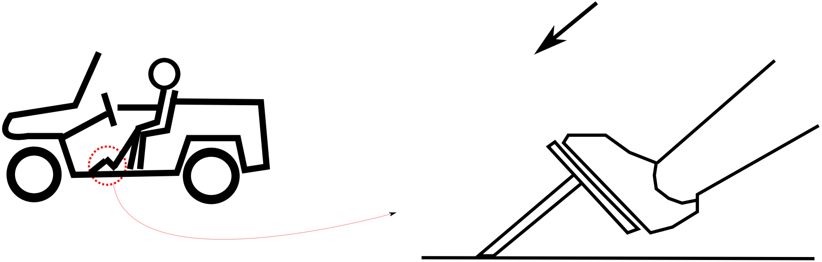
操纵杆

飞行员拉动操纵杆，飞机机头上扬，结果减弱了拉动操纵杆的效果，即：拉杆 ->A 增大 -> 机头上扬 ->A 减小



油门、刹车

司机踩刹车，汽车减速，司机因为惯性会有前冲的趋势，易导致踩刹车的力度变大。
司机踩油门，汽车加速，司机因为惯性会有后仰的趋势，易导致踩油门的力度变小。



Topic

稳定性与平衡点

$$
\begin{aligned}
\dot{x}(t) - x(t) &= r(t) \\
r &= 1 \\
x(0) &= -1 \\
x(t) &= -1
\end{aligned}
$$

- 通解：$x_1(t) = a_0 e^t$
- 特解：$x_2(t) = -1$
- $x_1(0) + x_2(0) = x(0)$ 得 $a_0 = 0$

正反馈与离散系统稳定性

$$
\begin{aligned}
x(n+1) - kx(n) &= r(n) \\
r(n) &= 0 \\
x(n) &= x(0)k^n
\end{aligned}
$$

正反馈与延迟系统稳定性

$$
\begin{aligned}
x(t+a) - kx(t) &= r(t) \\
r(t) &= 0 \\
x(na) &= x(0)k^n
\end{aligned}
$$

Topic

卷积与脉冲响应

- 卷积

$$\begin{aligned} x(t) * y(t) &= \int_{-\infty}^{\infty} x(\tau) y(t-\tau) d\tau \\ x(t) * \delta(t) &= x(t) \end{aligned}$$

- 脉冲响应设线性时不变 (Linear Time Invariant,LTI) 系统脉冲响应为 $h(t)$：

$$\begin{aligned} h(t) &= LTI[\delta(t)] \\ h(t-\tau) &= LTI[\delta(t-\tau)] \end{aligned}$$

卷积与系统响应

设输入信号为 $x(t)$ 时，输出为 $y(t)$:

$$
\begin{aligned}
y(t) &= LTI[x(t)] \\
&= LTI\left[\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau\right] \\
&= \int_{-\infty}^{\infty} LTI[x(\tau)\delta(t-\tau)]d\tau \\
&= \int_{-\infty}^{\infty} x(\tau)LTI[\delta(t-\tau)]d\tau \\
&= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
&= x(t) * h(t)
\end{aligned}
$$

Topic

阶跃输入

$$
\begin{aligned}
m\dot{v} &= f \\
m\dot{v} &= r - v \\
m\dot{v} &= 1 - v \\
m\frac{d}{dt}(v-1) &= 1 - v \\
m\frac{d}{dt}(1-v) &= -(1 - v) \\
m\dot{E} &= -E \\
E &= e^{-\frac{t}{m}}
\end{aligned}
$$

斜坡输入

$$
\begin{aligned}
m\dot{v} &= f \\
m\dot{v} &= r - v \\
m\dot{v} &= t - v \\
m\frac{d}{dt}v - t + m &= t - v \\
m\frac{d}{dt}t - v &= -(t - v) + m \\
m\dot{E} &= -E + m \\
E &= (1 - m)e^{-\frac{t}{m}} + m
\end{aligned}
$$

Topic

Fourier 级数 (三角形式)

$$f_T(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos(n\omega t) + b_n\sin(n\omega t))$$

其中：

$$\omega = \frac{2\pi}{T}$$

$$a_0 = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f_T(t)dt$$

$$a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f_T(t)\cos(n\omega t)dt$$

$$b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f_T(t)\sin(n\omega t)dt$$

Fourier 级数 (复指数形式)

$$
\begin{aligned}
f_T(t) &= \sum_{n=-\infty}^{+\infty} c_n e^{j\omega_n t} \\
f_T(t) &= \frac{1}{T}\sum_{n=-\infty}^{+\infty}\left[\int_{-\frac{T}{2}}^{\frac{T}{2}} f_T(\tau)e^{-j\omega_n\tau}d\tau\right]e^{j\omega_n t}
\end{aligned}
$$

Fourier 积分

$$
\begin{aligned}
\lim_{T\to+\infty} f_T(t) &= f(t)\\
f(t) &= \lim_{T\to+\infty}\frac{1}{T}\sum_{n=-\infty}^{+\infty}\left[\int_{-\frac{T}{2}}^{\frac{T}{2}} f_T(\tau)e^{-j\omega_n\tau}d\tau\right]e^{j\omega_n t}\\
\Delta\omega &= \frac{2\pi}{T}\\
f(t) &= \lim_{\Delta\omega\to 0}\frac{1}{2\pi}\sum_{n=-\infty}^{+\infty}\left[\int_{-\frac{T}{2}}^{\frac{T}{2}} f_T(\tau)e^{-j\omega_n\tau}d\tau\right]e^{j\omega_n t}\Delta\omega\\
f(t) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{\infty} f(\tau)e^{-j\omega_n\tau}d\tau\right]e^{j\omega_n t}d\omega
\end{aligned}
$$

Fourier 变换定义

$$
\begin{aligned}
f(t) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{\infty} f(\tau)e^{-j\omega_n\tau}d\tau\right]e^{j\omega_n t}d\omega \\
F(j\omega) &= \int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt \\
f(t) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(j\omega)e^{j\omega t}d\omega \\
F(j\omega) &= \mathcal{F}[f(t)] \\
f(t) &= \mathcal{F}^{-1}[F(j\omega)]
\end{aligned}
$$

常用函数的 Fourier 变换

- 单位脉冲函数 $f(t) = \delta(t) \to F(j\omega) = 1$
- 阶跃函数 $f(t) = A, (t \geq 0) \to F(j\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$
- 指数函数 $f(t) = e^{at}, (t \geq 0) \to F(j\omega) = \frac{1}{j\omega - a}$
- 正弦函数 $f(t) = \sin(\omega_0 t) \to F(j\omega) = \pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$

性质

- 线性: $f(t) = af_1(t) + bf_2(t) \rightarrow F(j\omega) = aF_1(j\omega) + bF_2(j\omega)$, 其中 a, b 为常数
- 时移: $g(t) = f(t \pm a) \rightarrow G(s) = F(j\omega)e^{\pm j\omega a}$
- 频移：$\mathcal{F}[e^{\pm\omega_0 t}f(t)] = F(j(\omega \mp \omega_0))$
- 时域微分: $g(t) = f(t)' \rightarrow G(j\omega) = j\omega F(j\omega)$
- 频域微分：$\mathcal{F}[tf(t)] = j\frac{dF(j\omega)}{d\omega}$
- 时域积分: $g(t) = \int_{-\infty}^{t} f(\tau)d\tau \rightarrow G(j\omega) = \frac{F(j\omega)}{j\omega} + \pi F(0)\delta(\omega)$
- 卷积：$\mathcal{F}[f_1(t) * f_2(t)] = \mathcal{F}[f_1(t)]\mathcal{F}[f_2(t)]$

Topic

基本信号 $e^{j\omega t}$ 通过线性系统

$$
\begin{aligned}
f(t) &= e^{j\omega t}, \qquad -\infty < t < \infty \\
H(j\omega) &= \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt \\
&= |H(j\omega)| e^{j\phi(\omega)} \\
y_f(t) &= e^{j\omega t} * h(t) \\
&= \int_{-\infty}^{\infty} h(\tau) e^{j\omega(t-\tau)} d\tau \\
&= e^{j\omega t} \int_{-\infty}^{\infty} h(\tau) e^{-j\omega t} d\tau \\
&= H(j\omega) e^{j\omega t} \\
&= |H(j\omega)| e^{j(\omega t + \phi(\omega))}
\end{aligned}
$$

正弦（余弦）信号通过线性系统

$$
\begin{aligned}
f(t) &= A\cos\omega t, \qquad -\infty < t < \infty \\
&= \frac{A}{2}(e^{j\omega t} + e^{-j\omega t}) \\
y_f(t) &= \frac{A}{2}(H(j\omega)e^{j\omega t} + H(-j\omega)e^{-j\omega t}) \\
&= \frac{A}{2}|H(j\omega)|(e^{j\omega t+\phi(\omega)} + e^{-j\omega t-\phi(\omega)}) \\
&= A|H(j\omega)|\cos(\omega t + \phi(\omega))
\end{aligned}
$$

非正弦周期信号通过线性系统

$$
\begin{aligned}
f(t) &= \sum_{n=-\infty}^{\infty} F_n e^{jn\omega t} \\
F_n &= \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)e^{-jn\omega t}dt \\
&= |F_n|e^{j\theta(n\omega)} \\
y_f(t) &= \sum_{n=-\infty}^{\infty} F_n H(jn\omega)e^{jn\omega t} \\
&= \sum_{n=-\infty}^{\infty} |F_n||H(jn\omega)|e^{jn\omega t+\phi(n\omega)+\theta(n\omega)} \\
&= F_0 + \sum_{n=-\infty}^{\infty} 2|F_n||H(jn\omega)|\cos(jn\omega t+\phi(n\omega)+\theta(n\omega))
\end{aligned}
$$

系统对非周期信号的响应

$$
\begin{aligned}
y(t) &= f(t) * h(t) \\
Y(j\omega) &= F(j\omega) H(j\omega) \\
y(t) &= \mathcal{F}^{-1}[Y(j\omega)] \\
H(j\omega) &= \frac{Y(j\omega)}{F(j\omega)}
\end{aligned}
$$

利用 Fourier 变换计算零状态响应

某线性时不变系统的脉冲响应 $h(t) = (e^{-2t} - e^{-3t})U(t)$，求输入信号 $f(t) = e^{-t}U(t)$ 时系统的零状态响应。其中 $U(t)$ 为单位阶跃函数。

解：

$$
\begin{aligned}
F(j\omega) &= \mathcal{F}[f(t)] = \frac{1}{j\omega + 1} \\
H(j\omega) &= \mathcal{F}[h(t)] = \frac{1}{j\omega + 2} - \frac{1}{j\omega + 3} = \frac{1}{(j\omega + 2)(j\omega + 3)} \\
Y(j\omega) &= F(j\omega)H(j\omega) = \frac{1}{(j\omega + 1)(j\omega + 2)(j\omega + 3)} \\
&= \frac{1/2}{j\omega + 1} + \frac{-1}{j\omega + 2} + \frac{1/2}{j\omega + 3} \\
y(t) &= (\frac{1}{2}e^{-t} - e^{-2t} + \frac{1}{2}e^{-3t})U(t)
\end{aligned}
$$