线性系统的频域分析法

典型环节频率特性

Outline

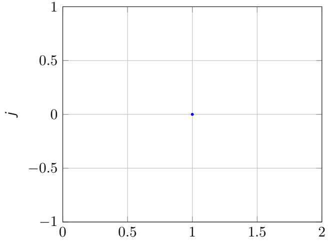
Topic

比例环节

$$
\begin{aligned}
G(s) &= K \\
G(j\omega) &= K \\
A(\omega) &= K \\
\phi(\omega) &= 0 \\
L(\omega) &= 20\lg K
\end{aligned}
$$

比例环节 (续) $G(j\omega) = K, K = 1$

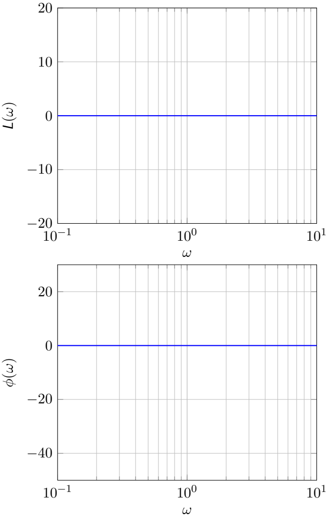
Nyquist 图



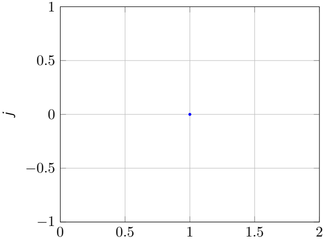
Bode 图

比例环节 (续) $G(j\omega) = K, K = 1$

Bode 图



Nyquist 图



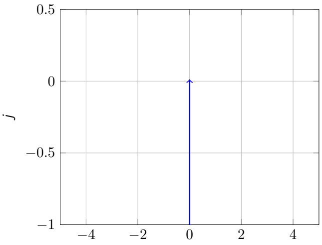
积分环节

$$
\begin{aligned}
G(s) &= \frac{1}{s} \\
G(j\omega) &= \frac{1}{j\omega} \\
A(\omega) &= \frac{1}{\omega} \\
\phi(\omega) &= -90^\circ \\
L(\omega) &= -20\lg\omega
\end{aligned}
$$

积分环节 (续) $G(j\omega) = \frac{1}{j\omega}$

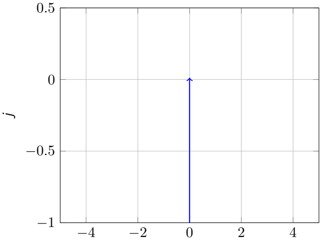
Bode 图

Nyquist 图

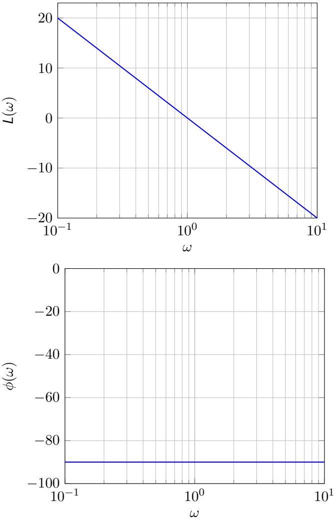


积分环节 (续)$G(j\omega)=\frac{1}{j\omega}$

Nyquist 图



Bode 图

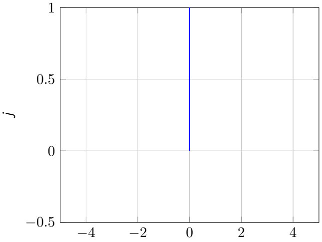


微分环节

$$\begin{aligned} G(s) &= s \\ G(j\omega) &= j\omega \\ A(\omega) &= \omega \\ \phi(\omega) &= 90^\circ \\ L(\omega) &= 20\lg\omega \end{aligned}$$

微分环节 (续) $G(j\omega) = j\omega$

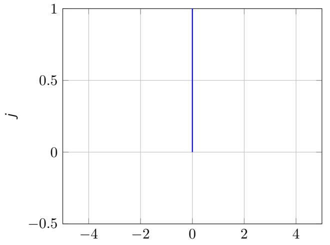
Nyquist 图



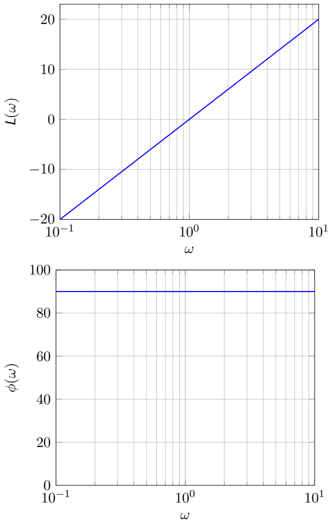
Bode 图

微分环节（续）$G(j\omega) = j\omega$

Nyquist 图



Bode 图



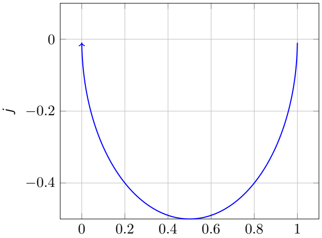
Topic

惯性环节

$$
\begin{aligned}
G(s) &= \frac{1}{Ts+1} \\
G(j\omega) &= \frac{1}{j\omega T+1} \\
A(\omega) &= \sqrt{\frac{1}{1+\omega^2 T^2}} \\
\phi(\omega) &= -\arctan \omega T \\
L(\omega) &= -20\lg\sqrt{1+\omega^2 T^2} \\
L_a(\omega) &= \begin{cases} 0 & \omega < \frac{1}{T} \\ -20\lg \omega T & \omega > \frac{1}{T} \end{cases}
\end{aligned}
$$

惯性环节（续）$G(j\omega) = \frac{1}{j\omega T+1}, T = 1$

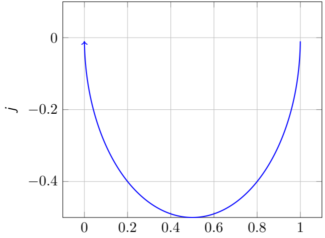
Nyquist 图



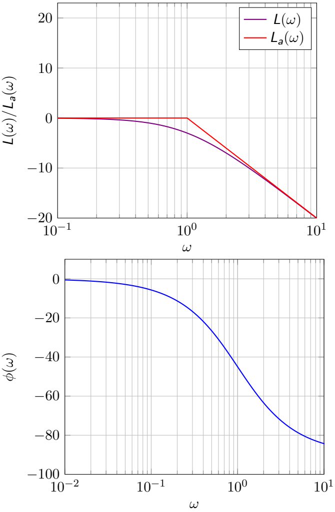
Bode 图

惯性环节（续）$G(j\omega)=\frac{1}{j\omega T+1}, T=1$

Nyquist 图



Bode 图

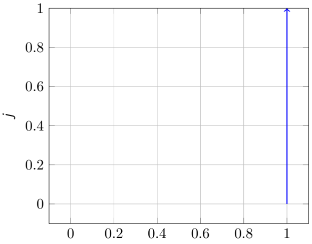


一阶微分环节

$$
\begin{aligned}
G(s) &= Ts+1 \\
G(j\omega) &= j\omega T+1 \\
A(\omega) &= \sqrt{1+\omega^2T^2} \\
\phi(\omega) &= \arctan\omega T \\
L(\omega) &= 20\lg\sqrt{1+\omega^2T^2} \\
L_a(\omega) &= \begin{cases} 0 & \omega < \frac{1}{T} \\ 20\lg\omega T & \omega > \frac{1}{T} \end{cases}
\end{aligned}
$$

一阶微分环节（续）$G(j\omega) = j\omega T + 1, T = 1$

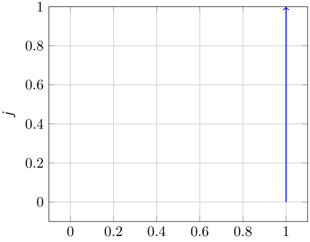
Nyquist 图



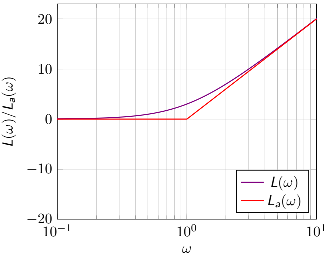
Bode 图

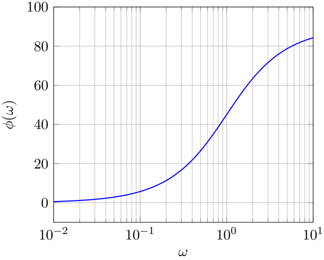
一阶微分环节（续）$G(j\omega) = j\omega T + 1, T = 1$

Nyquist 图



Bode 图





Topic

二阶振荡环节

$$
\begin{aligned}
G(s) &= \frac{\omega_n^2}{\omega_n^2 + 2\xi\omega_n s + s^2} = \frac{1}{(Ts)^2 + 2\xi Ts + 1} \\
G(j\omega) &= \frac{1}{1 + 2j\xi\omega T - \omega^2 T^2} \\
A(\omega) &= \sqrt{\frac{1}{(1-\omega^2 T^2)^2 + (2\xi\omega T)^2}} \\
\phi(\omega) &= \begin{cases} -\arctan\frac{2\xi\omega T}{1-\omega^2 T^2} & \omega T < 1 \\ -90^\circ & \omega T = 1 \\ -180 - \arctan\frac{2\xi\omega T}{1-\omega^2 T^2} & \omega T > 1 \end{cases} \\
L(\omega) &= -20\lg\sqrt{(1-\omega^2 T^2)^2 + (2\xi\omega T)^2} \\
L_a(\omega) &= \begin{cases} 0 & \omega T < 1 \\ -40\lg\omega T & \omega T > 1 \end{cases}
\end{aligned}
$$

二阶振荡环节 (续) $G(j\omega) = \frac{1}{1+2j\xi\omega T-\omega^2 T^2}$

- Nyquist 曲线与虚轴交点:

$$\begin{aligned} \Re[G(j\omega)] &= 0 \\ 1-\omega^2 T^2 &= 0 \\ \omega T &= 1 \\ G(j\frac{1}{T}) &= -\frac{1}{2\xi}j \end{aligned}$$

二阶振荡环节 (续) $G(j\omega) = \frac{1}{1+2j\xi\omega T-\omega^2T^2}$

- 谐振频率与谐振峰值

$$A(\omega) = \sqrt{\frac{1}{(1-\omega^2T^2)^2+(2\xi\omega T)^2}}$$

$$\frac{dA(\omega)}{d\omega} = -\frac{-2(1-\omega^2T^2)\omega T^2+4\xi^2\omega T^2}{\sqrt{(1-\omega^2T^2)^2+(2\xi\omega T)^2}}$$

- 令 $\frac{dA(\omega)}{d\omega} = 0$，得
  - 谐振频率:$\omega_r = \omega_n\sqrt{1-2\xi^2}$，其中 $0 < \xi \le \frac{\sqrt{2}}{2}$
  - 谐振峰值:$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$

二阶振荡环节 (续) $G(j\omega) = \frac{1}{1+2j\xi\omega T-\omega^2 T^2}$

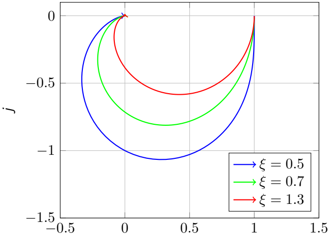
- 谐振频率与谐振峰值

$$
\begin{aligned}
A(\omega) &= \sqrt{\frac{1}{(1-\omega^2 T^2)^2 + (2\xi\omega T)^2}} \\
\frac{dA(\omega)}{d\omega} &= -\frac{-2(1-\omega^2 T^2)\omega T^2 + 4\xi^2\omega T^2}{\sqrt{(1-\omega^2 T^2)^2 + (2\xi\omega T)^2}}
\end{aligned}
$$

- 令 $\frac{dA(\omega)}{d\omega} = 0$，得
  - 谐振频率:$\omega_r = \omega_n\sqrt{1-2\xi^2}$，其中 $0 < \xi \leq \frac{\sqrt{2}}{2}$
  - 谐振峰值:$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$

二阶振荡环节 (续): $G(j\omega) = \frac{1}{1+2j\xi\omega T-\omega^2 T^2}$, $T = 1$

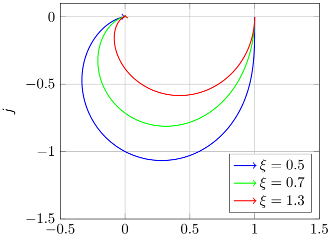
Nyquist 图



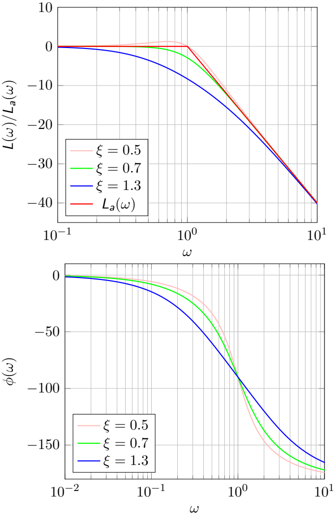
Bode 图

二阶振荡环节 (续): $G(j\omega) = \frac{1}{1+2j\xi\omega T-\omega^2 T^2}$, $T = 1$

Nyquist 图



Bode 图

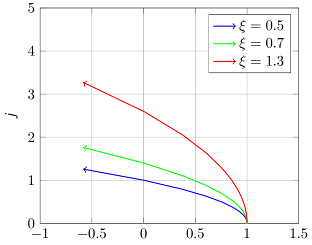


二阶微分环节

$$
\begin{aligned}
G(s) &= (Ts)^2 + 2\xi Ts + 1 \\
G(j\omega) &= 1 + 2j\xi\omega T - \omega^2 T^2
\end{aligned}
$$

二阶微分环节 (续) $G(j\omega) = 1 + 2j\xi\omega T - \omega^2 T^2, T = 1$

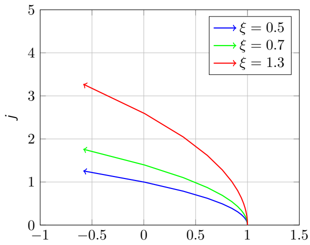
Nyquist 图



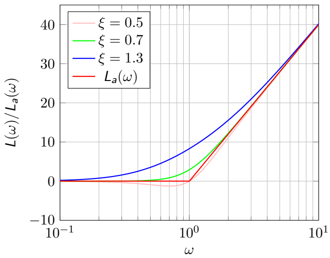
Bode 图

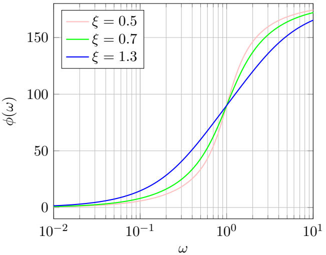
二阶微分环节 (续) $G(j\omega) = 1 + 2j\xi\omega T - \omega^2 T^2, T = 1$

Nyquist 图



Bode 图





Topic

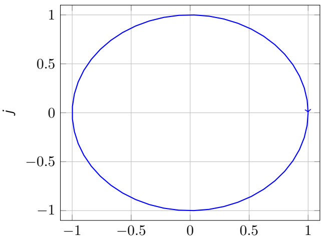
1. 比例, 积分微分环节
2. 惯性, 一阶微分环节
3. 二阶环节
4. 非最小相位环节

延迟环节

$$
\begin{aligned}
G(s) &= e^{-\tau s} \\
G(j\omega) &= e^{-j\omega\tau} \\
A(\omega) &= 1 \\
\phi(\omega) &= -\omega\tau
\end{aligned}
$$

延迟环节 (续) $G(j\omega) = e^{-j\omega\tau}, \tau = 1$

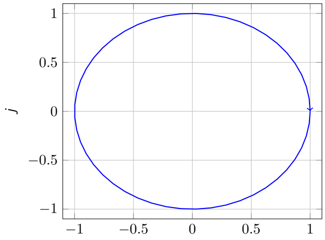
Nyquist 图



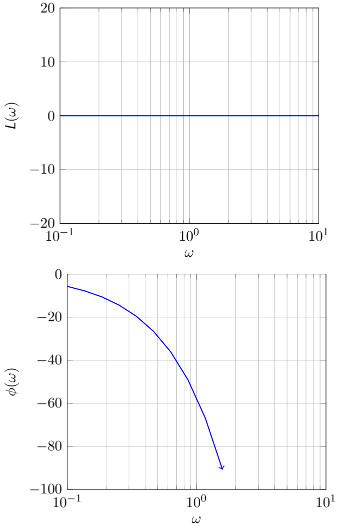
Bode 图

延迟环节（续）$G(j\omega)=e^{-j\omega\tau},\tau=1$

Nyquist 图



Bode 图



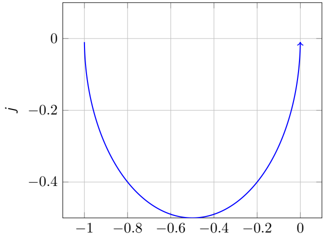
非最小相位惯性环节

最小相位系统：在右半平面无零极点

$$
\begin{aligned}
G(s) &= \frac{1}{Ts-1} \\
G(j\omega) &= \frac{1}{j\omega T-1} \\
A(\omega) &= \sqrt{\frac{1}{1+\omega^2 T^2}} \\
\phi(\omega) &= -180^\circ + \arctan \omega T \\
L(\omega) &= -20 \lg \sqrt{1+\omega^2 T^2} \\
L_a(\omega) &= \begin{cases} 0 & \omega < \frac{1}{T} \\ -20 \lg \omega T & \omega > \frac{1}{T} \end{cases}
\end{aligned}
$$

非最小相位惯性环节 (续) $G(j\omega)=\frac{1}{j\omega T-1}, T=1$

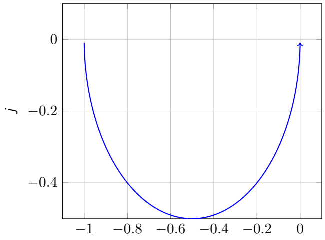
Nyquist 图



Bode 图

非最小相位惯性环节 (续) $G(j\omega) = \frac{1}{j\omega T - 1}, T = 1$

Nyquist 图



Bode 图