线性系统的频域分析法

开环频率特性

1. 开环系统 Nyquist 图
   - 开环系统 Nyquist 图 $G_o(s) = \frac{K\prod_{j=1}^{m}(\tau_j s+1)}{s^{\nu}\prod_{i=1}^{n-\nu}(T_i s+1)}$
   - 开环系统 Nyquist 图 (续) $G_o(s) = \frac{K\prod_{j=1}^{m}(\tau_j s+1)}{s^{\nu}\prod_{i=1}^{n-\nu}(T_i s+1)}$
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   - 开环系统 Nyquist 图, 例 1
     - $G_o(s) = \frac{10}{s^{\nu}(0.1s+1)}$ BMCOL:B_BLOCK
     - $G_o(s) = \frac{10}{(0.1s+1)^n}$ BMCOL:B_BLOCK
   - 开环系统 Nyquist 图, 例 2:$G(s) = \frac{10}{s(s+1)(2s+1)(4s+1)}$
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- Nyquist 图 BMCOL:B_BLOCK
- 局部放大: BMCOL:B_BLOCK

2. 开环系统 Bode 图
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- 当 $\nu = 0$ 时, 为零型系统:

$$
\begin{aligned}
A(\omega)|_{\omega=0} &= K \\
\phi(\omega)|_{\omega=0} &= 0 \\
\lim_{\omega\to\infty} A(\omega) &= 0 \\
\lim_{\omega\to\infty} \phi(\omega) &= -(n-m)\times\frac{\pi}{2}
\end{aligned}
$$

- 当 $\nu = 1$ 时, 为 I 型系统:

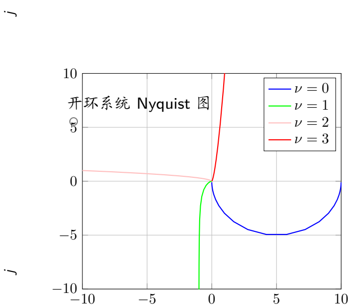
$$
\begin{aligned}
\lim_{\omega\to 0} A(\omega) &= \infty \\
\lim_{\omega\to 0} \phi(\omega) &= -\frac{\pi}{2} \\
\lim_{\omega\to\infty} A(\omega) &= 0 \\
\lim_{\omega\to\infty} \phi(\omega) &= -(n-m)\times\frac{\pi}{2}
\end{aligned}
$$

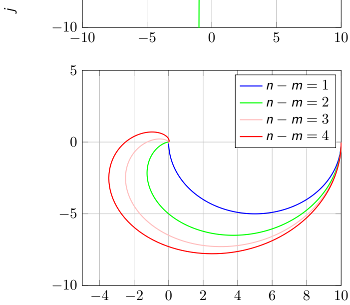
- 当 $\nu = 2$ 时, 为 II 型系统:

$$\begin{aligned}
\lim_{\omega\to 0} A(\omega) &= \infty \\
\lim_{\omega\to 0} \phi(\omega) &= -\pi \\
\lim_{\omega\to \infty} A(\omega) &= 0 \\
\lim_{\omega\to \infty} \phi(\omega) &= -(n-m)\times\frac{\pi}{2}
\end{aligned}$$

- 当 $\nu = 3$ 时, 为 III 型系统:

$$\begin{aligned}
\lim_{\omega\to 0} A(\omega) &= \infty \\
\lim_{\omega\to 0} \phi(\omega) &= -\frac{3}{2}\pi
\end{aligned}$$

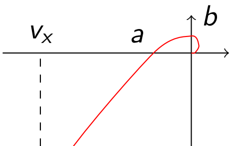

j
开环系统 Nyquist 图
$\nu = 0$
$\nu = 1$
$\nu = 2$
$\nu = 3$
10
5
0
−5
−10
−10
−5
0
5
10


j
$n - m = 1$
$n - m = 2$
$n - m = 3$
$n - m = 4$
5
0
−5
−10
−4
−2
0
2
4
6
8
10

- 由于 $\nu=1$

$$
\begin{aligned}
\lim_{\omega\to 0} A(\omega) &= \infty \\
\lim_{\omega\to 0} \phi(\omega) &= -\frac{\pi}{2} \\
\lim_{\omega\to \infty} A(\omega) &= 0 \\
\lim_{\omega\to \infty} \phi(\omega) &= -2\pi
\end{aligned}
$$

概略 Nyquist 图:



- 起始点实部 v_x:

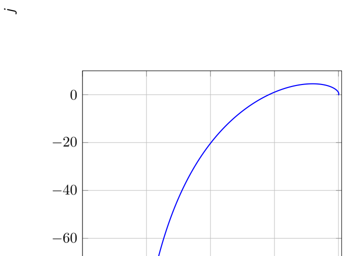
$$\begin{aligned} G(j\omega) &= \frac{10}{j\omega(j\omega+1)(2j\omega+1)(4j\omega+ \\ &= \frac{10\omega(8\omega^2-7)+10(14\omega^2-}{\omega(1+\omega^2)(1+4\omega^2)(1+16} \\ \lim_{\omega\to 0}\Re[G(j\omega)] &= -70 \end{aligned}$$

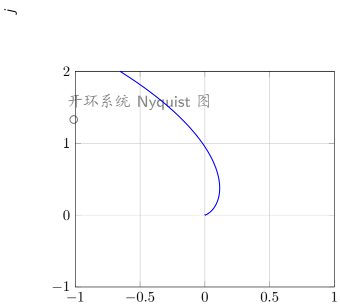
- 与实轴交点 a:

$$\begin{aligned} \Im[G(j\omega)] &= 0 \\ \frac{10(14\omega^2-1)}{(1+\omega^2)(1+4\omega^2)(1+16\omega^2)} &= 0 \\ \omega &= \sqrt{\frac{1}{ } } \end{aligned}$$

- 与虚轴交点 b:

$$
\begin{aligned}
\Re[G(j\omega)] &= 0 \\
\frac{10\omega(8\omega^2-7)}{(1+\omega^2)(1+4\omega^2)(1+16\omega^2)} &= 0 \\
8\omega^2-7 &= 0 \\
\omega &= \sqrt{\frac{7}{8}} \\
G(j\sqrt{\frac{7}{8}}) &\approx 0.95j
\end{aligned}
$$





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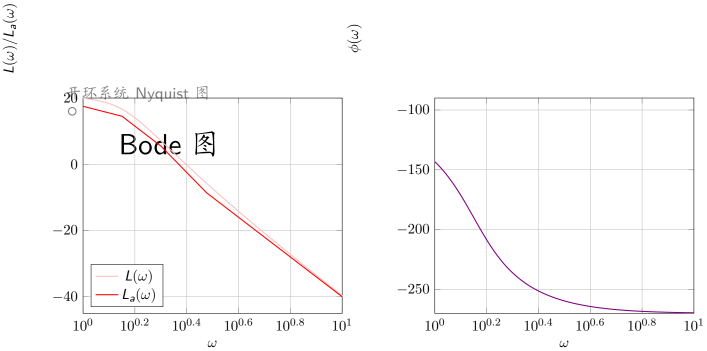
$$
\begin{aligned}
G_o(s) &= G_1(s)G_2(s)G_3(s)\cdots Gn(s)\\
A(\omega) &= A_1(\omega)A_2(\omega)A_3(\omega)\cdots A_n(\omega)\\
L(\omega) &= 20\lg A_1(\omega)+\cdots+20\lg A_n(\omega)\\
\phi(\omega) &= \phi_1(\omega)+\cdots+\phi_n(\omega)
\end{aligned}
$$

- 结论:
  - 系统的低频段由系统的类型和开环增益 K 决定, 代
    能. 由初始斜率可得系统类型.
  - 系统的高频段反映系统的抗噪能力, 下降速度要快.

绘制 Bode 图:

1. 改写为标准形式: $G_o(s) = \frac{7.5(\frac{s}{3}+1)}{s(0.5s+1)(0.5s^2+0.5s+1)}$
2. 写出转折频率: $\omega = \sqrt{2}, 2, 3$



开环系统 Nyquist 图
Bode 图
$L(\omega)/L_a(\omega)$
20
0
−20
−40
$L(\omega)$
$L_a(\omega)$
10^0
$10^{0.2}$
$10^{0.4}$
$10^{0.6}$
$10^{0.8}$
10^1
ω
$\phi(\omega)$
−100
−150
−200
−250
10^0
$10^{0.2}$
$10^{0.4}$
$10^{0.6}$
$10^{0.8}$
10^1
ω