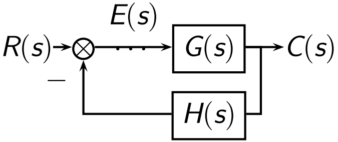
线性系统时域分析法

系统的稳态误差计算

Outline

Topic

系统误差



- 输入端定义:$E_2(s) = E(s)$
- 输出端定义:$E_1(s) = C_{expect} - C_{real}$
- 不加特别说明, 系统误差指的是输入端定义.

$E(s)$ 与 $E_1(s)$

$$
\begin{aligned}
C_{expect} &= \frac{R(s)}{H(s)} \\
E_1(s) &= \frac{R(s)}{H(s)} - C(s) \\
&= \frac{R(s) - C(s)H(s)}{H(s)} \\
&= \frac{E(s)}{H(s)}
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$$

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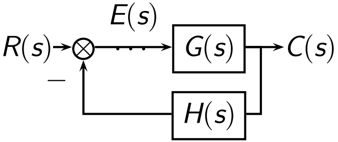

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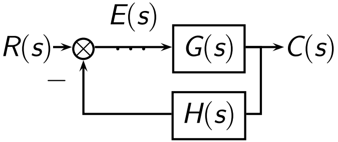

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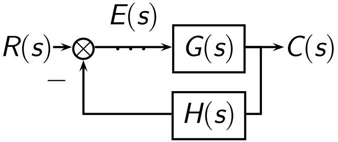

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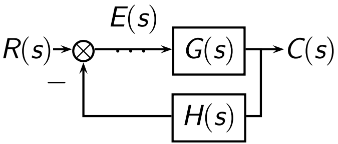

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误差传递函数:

$$\begin{aligned}\Phi_e(s) &= \frac{E(s)}{R(s)} \\ &= \frac{1}{1+G(s)H(s)} \\ &= \frac{R(s)-H(s)C(s)}{R(s)} \\ &= 1-H(s)\Phi(s)\end{aligned}$$

- 系统误差:$E(s)=\Phi_e(s)R(s)$

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- 系统误差:$E(s)=\Phi_e(s)R(s)$

稳态误差:

$$
\begin{aligned}
e_{ss} &= \lim_{t\to\infty} e(t) \\
&= \lim_{s\to 0} sE(s) \\
&= \lim_{s\to 0} s\Phi_e(s)R(s)
\end{aligned}
$$

- 稳态误差与输入信号有关
- 求稳态误差前要判断系统稳定性

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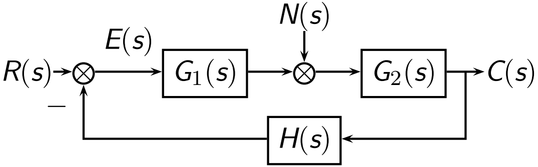
扰动作用下的稳态误差



输入端定义:

$$
\begin{aligned}
E(s) &= E_R(s) + E_N(s) \\
E_R(s) &= \Phi_e(s)R(s) \\
E_N(s) &= \Phi_{en}(s)N(s)
\end{aligned}
$$

输出端定义:

令 $R(s)=0$，计算 $N(s)$ 单独引起的 e_{ss}，此时 $C_{expect}(s)=0$

$$
\begin{aligned}
E(s) &= 0 - C(s) \\
&= -\Phi_N(s)N(s) \\
\Phi_N(s) &= \frac{G_2}{1+G_1G_2} \\
e_{ss} &= \lim_{s\to 0} s(-\Phi_N(s)N(s))
\end{aligned}
$$

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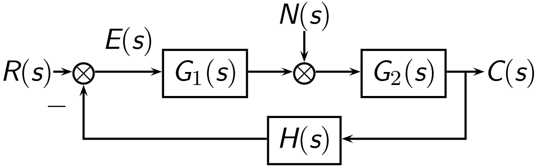

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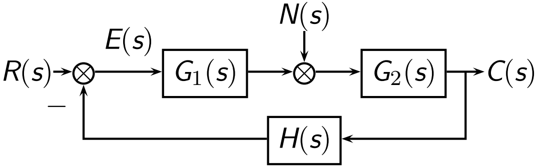

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Topic

阶跃输入:

$$
\begin{aligned}
r(t) &= A \\
R(s) &= \frac{A}{s} \\
e_{ss} &= \lim_{s\to 0} s \cdot \frac{1}{1+G_{open}(s)} \cdot \frac{A}{s} \\
&= \lim_{s\to 0} \frac{A}{1+G_{open}(s)}
\end{aligned}
$$

速度输入

$$
\begin{aligned}
r(t) &= vt \\
R(s) &= \frac{v}{s^2} \\
e_{ss} &= \lim_{s\to 0} s \cdot \frac{1}{1+G_{open}(s)} \cdot \frac{v}{s^2} \\
&= \lim_{s\to 0} \frac{A}{s+sG_{open}(s)} \\
&= \lim_{s\to 0} \frac{A}{sG_{open}(s)}
\end{aligned}
$$

加速度输入

$$
\begin{aligned}
r(t) &= \frac{1}{2}at^2 \\
R(s) &= \frac{a}{s^2} \\
e_{ss} &= \lim_{s\to 0} s\cdot\frac{1}{1+G_{open}(s)}\cdot\frac{a}{s^3} \\
&= \lim_{s\to 0}\frac{A}{s^2+s^2G_{open}(s)} \\
&= \lim_{s\to 0}\frac{A}{s^2G_{open}(s)}
\end{aligned}
$$

系统类型

- 由开环传递函数定义

$$
\begin{aligned}
G_{open} &= G(s)H(s) \\
&= \frac{K\prod_{i=1}^{m}(\tau_i s+1)}{s^{\nu}\prod_{j=1}^{n-\nu}(T_j s+1)}
\end{aligned}
$$

- 其中 K 为开环增益.
- 定义:
  - $\nu=0$ 称为 0 型系统
  - $\nu=1$ 称为 I 型系统
  - $\nu=2$ 称为 II 型系统

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静态误差系数

- 静态位置误差系数

$$
\begin{aligned}
r(t) &= A \\
e_{ss} &= \frac{A}{1+K_p}, \qquad K_p = \lim_{s\to 0} G_{open}(s)
\end{aligned}
$$

- 静态速度误差系数

$$
\begin{aligned}
r(t) &= vt \\
e_{ss} &= \frac{v}{K_v}, \qquad K_v = \lim_{s\to 0} sG_{open}(s)
\end{aligned}
$$

- 静态加速度误差系数

$$
\begin{aligned}
r(t) &= \frac{1}{2}at^2 \\
e_{ss} &= \frac{a}{K_a}, \qquad K_a = \lim_{s\to 0} s^2 G_{open}(s)
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零型系统 ($\nu = 0$)

- $r(t) = A$ 时:

$$
\begin{aligned}
K_p &= \lim_{s\to 0} G_o(s) = \lim_{s\to 0} \frac{K\prod_{i=0}^{m}(\tau_i s + 1)}{\prod_{j=1}^{n}(\tau_j s + 1)} = K \\
e_{ss1} &= \frac{A}{1+K_p}
\end{aligned}
$$

- $r(t) = vt$ 时:

$$
\begin{aligned}
K_v &= \lim_{s\to 0} sG_o(s) = 0 \\
e_{ss2} &= \infty
\end{aligned}
$$

- $r(t) = \frac{1}{2}at^2$ 时:

$$
\begin{aligned}
K_a &= \lim_{s\to 0} s^2 G_o(s) = 0 \\
e_{ss3} &= \infty
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$$

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I 型系统 ($\nu = 1$)

- $r(t) = A$ 时:

$$
\begin{aligned}
K_p &= \lim_{s\to 0} G_o(s) = \lim_{s\to 0} \frac{K\prod_{i=0}^{m}(\tau_i s+1)}{s\prod_{j=1}^{n-1}(\tau_j s+1)} = \infty \\
e_{ss1} &= \frac{1}{1+K_p} = 0
\end{aligned}
$$

- $r(t) = vt$ 时:

$$
\begin{aligned}
K_v &= \lim_{s\to 0} sG_o(s) = K \\
e_{ss2} &= \frac{v}{K_v} = \frac{v}{K}
\end{aligned}
$$

- $r(t) = \frac{1}{2}at^2$ 时:

$$
\begin{aligned}
K_a &= \lim_{s\to 0} s^2 G_o(s) = 0 \\
e_{ss3} &= \infty
\end{aligned}
$$

I 型系统 ($\nu = 1$)

- $r(t) = A$ 时:

$$K_p = \lim_{s\to 0} G_o(s) = \lim_{s\to 0} \frac{K\prod_{i=0}^{m}(\tau_i s+1)}{s\prod_{j=1}^{n-1}(\tau_j s+1)} = \infty$$

$$e_{ss1} = \frac{1}{1+K_p} = 0$$

- $r(t) = vt$ 时:

$$K_v = \lim_{s\to 0} sG_o(s) = K$$

$$e_{ss2} = \frac{v}{K_v} = \frac{v}{K}$$

- $r(t) = \frac{1}{2}at^2$ 时:

$$K_a = \lim_{s\to 0} s^2 G_o(s) = 0$$

$$e_{ss3} = \infty$$

I 型系统 $(\nu = 1)$

- $r(t) = A$ 时:

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$$
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K_a &= \lim_{s\to 0} s^2 G_o(s) = 0 \\
e_{ss3} &= \infty
\end{aligned}
$$

II 型系统 ($\nu = 2$)

$$
\begin{aligned}
K_p &= \infty \\
e_{ss1} &= 0 \\
K_v &= \infty \\
e_{ss2} &= 0 \\
K_a &= K \\
e_{ss3} &= \frac{a}{K}
\end{aligned}
$$

小结:

- 零型:

$$e_{ss1} = \frac{A}{1+K}, e_{ss2} = e_{ss3} = \infty$$

- I 型:

$$e_{ss1} = 0, e_{ss2} = \frac{v}{K}, e_{ss3} = \infty$$

- II 型:

$$e_{ss1} = e_{ss2} = 0, e_{ss3} = \frac{a}{K}$$

小结:

- 零型:

$$e_{ss1} = \frac{A}{1+K}, e_{ss2} = e_{ss3} = \infty$$

- I 型:

$$e_{ss1} = 0, e_{ss2} = \frac{v}{K}, e_{ss3} = \infty$$

- II 型:

$$e_{ss1} = e_{ss2} = 0, e_{ss3} = \frac{a}{K}$$

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- 零型:

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- II 型:

$$e_{ss1} = e_{ss2} = 0, e_{ss3} = \frac{a}{K}$$

小结:

- 零型:

$$e_{ss1} = \frac{A}{1+K}, e_{ss2} = e_{ss3} = \infty$$

- I 型:

$$e_{ss1} = 0, e_{ss2} = \frac{v}{K}, e_{ss3} = \infty$$

- II 型:

$$e_{ss1} = e_{ss2} = 0, e_{ss3} = \frac{a}{K}$$

例:

若 $G(s)H(s)=\frac{10K_h}{s+1}, K_h\in\{0.1,1\}$，求单位阶跃下的 e_{ss} .

解法1

零型系统,
$r(t)=1, e_{ss}=\frac{1}{1+K_p}$

$$
\begin{aligned}
K_p &= \lim_{s\to 0} G(s)H(s)\\
&= 10K_h\\
&= \begin{cases} 1 & K_h=0.1\\ 10 & K_h=1 \end{cases}\\
e_{ss} &= \begin{cases} 0.5 & K_h=0.1\\ \frac{1}{11} & K_h=1 \end{cases}
\end{aligned}
$$

解法 2:

$$
\begin{aligned}
e_{ss} &= \lim_{s\to 0} s\Phi_e(s)R(s)\\
&= \lim_{s\to 0} s\frac{1}{1+G(s)H(s)}R(s)\\
&= \lim_{s\to 0} s\frac{s+1}{s+1+10K_h}\frac{1}{s}\\
&= \frac{1}{1+10K_h}\\
&= \begin{cases} 0.5 & K_h=0.1\\ \frac{1}{11} & K_h=1 \end{cases}
\end{aligned}
$$

例:

若 $G(s)H(s)=\frac{10K_h}{s+1}, K_h\in\{0.1,1\}$, 求单位阶跃下的 e_{ss} .

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解法 2:

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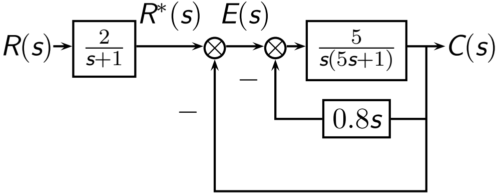
例: 求 $r(t) = 2 + 3t$ 时的 e_{ss}



解:

$$
\begin{aligned}
G(s) &= \frac{C(s)}{E(s)} \\
&= \frac{\frac{5}{s(5s+1)}}{1+\frac{4s}{s(5s+1)}} \\
&= \frac{5}{5s^2+5s} \\
&= \frac{1}{s(s+1)}
\end{aligned}
$$

例: 计算稳态误差

$$
\begin{aligned}
\Phi(s) &= \frac{C(s)}{R^*(s)} = \frac{1}{s(s+1)+1} \\
\Phi_e(s) &= \frac{s(s+1)}{s(s+1)+1}
\end{aligned}
$$

系统稳定.

$$
\begin{aligned}
R(s) &= \frac{2s+3}{s^2} \\
e_{ss} &= \lim_{s\to 0} s\Phi_e(s)R^*(s) \\
&= \lim_{s\to 0} s\cdot\frac{s(s+1)}{s(s+1)+1}\cdot\frac{2}{s+1}\cdot\frac{2s+3}{s^2} \\
&= 6
\end{aligned}
$$

Topic

动态误差系数

动态误差系数可描述系统稳态误差随时间变化的规律，静态误差可看作动态误差的一个特例.

$$
\begin{aligned}
E(s) &= \Phi_e(s)R(s) \\
\Phi_e(s) &= \frac{E(s)}{R(s)} \\
&= \frac{1}{1+G(s)H(s)} \\
&= \frac{M(s)}{N(s)}
\end{aligned}
$$

在 $s=0$ 处展开, 得:

$$
\begin{aligned}
\phi_e(s) &= \Phi_e(0)+\dot{\Phi}_e(0)s+\cdots+\frac{\Phi_e^{(n)}(0)s^n}{n!}+\cdots \\
E(s) &= \Phi_e(0)R(s)+\dot{\Phi}_e(0)sR(s)+\cdots+\frac{\Phi_e^{(n)}(0)s^nR(s)}{n!}+\cdots \\
e_{ss}(t) &= \Phi_e(0)r(t)+\dot{\Phi}_e(0)\dot{r}(t)+\cdots+\frac{\Phi_e^{(n)}(0)r^{(n)}(t)}{n!}+\cdots \\
&= \sum_{i=1}^{\infty} C_i r^{(i)}(t), \qquad C_i=\frac{\Phi_e^{(i)}(0)}{i!}
\end{aligned}
$$

- 其中 C_i 称为动态误差系数.
  - C_0 动态位置误差系数
  - C_1 动态速度误差系数
  - C_2 动态加速度误差系数

动态误差系数示例:

- 零型系统 $r(t)=1$ 则

$$e_{ss}(t)=C_0, C_0=\frac{1}{1+K_p}$$

- I 型系统 $r(t)=t$ 则

$$e_{ss}(t)=C_0t+C_1, C_0=0, C_1=\frac{1}{K_v}$$

- II 型系统 $r(t)=t$ 则

$$e_{ss}(t)=C_0\frac{1}{2}at^2+C_1at+C_2a, C_0=C_1=0, C_2=\frac{1}{K_a}$$

动态误差系数示例:

- 零型系统 $r(t) = 1$ 则

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$$e_{ss}(t) = C_0 t + C_1, C_0 = 0, C_1 = \frac{1}{K_v}$$

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$$e_{ss}(t) = C_0 \frac{1}{2} a t^2 + C_1 a t + C_2 a, C_0 = C_1 = 0, C_2 = \frac{1}{K_a}$$

动态误差系数示例:

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动态误差系数示例:

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- II 型系统 $r(t) = t$ 则

$$e_{ss}(t) = C_0 \frac{1}{2} a t^2 + C_1 a t + C_2 a, C_0 = C_1 = 0, C_2 = \frac{1}{K_a}$$

讨论: C_i 的计算

$$\begin{aligned}\Phi_e(s) &= \frac{M(s)}{N(s)} \\ &= C_0 + C_1 s + C_2 s^2 + \cdots\end{aligned}$$

例: $G(s)H(s) = \frac{1}{s(s+1)}$

综合除法:

dividend		divisor		quotient	remainder
s^2+s	$\div$	s^2+s+1	$\rightarrow$	s	$s^2+s-s(1+s+s^2)$
$-s^3$	$\div$	s^2+s+1	$\rightarrow$	$-s^3$	$-s^3-(-s^3)(1+s+s^2)$
s^4+s^5	$\div$	s^2+s+1	$\rightarrow$	s^4	$\cdots$
$\cdots$	$\div$	s^2+s+1	$\rightarrow$	$\cdots$	$\cdots$

得:

$$\Phi_e(s) = s - s^3 + s^4 + \cdots$$

例: $G(s)H(s)=\frac{1}{s(s+1)}$ 另一种写法:

$$\begin{aligned}
\frac{s^2+s}{s^2+s+1} &= s+\frac{-s^2+s-s(1+s+s^2)}{s^2+s+1}\\
\frac{-s^3}{s^2+s+1} &= -s^3+\frac{-s^3-(-s^3)(1+s+s^2)}{s^2+s+1}\\
\frac{s^4+s^5}{s^2+s+1} &= s^4+\cdots\\
\cdots &= \cdots\\
\Phi_e(s) &= s-s^3+s^4+\cdots
\end{aligned}$$

例：$G(s)H(s)=\frac{1}{s(s+1)}$ 长除法

$$
\begin{array}{r|rrrrrr}
 & s & & -s^3 & +s^4 & & \cdots \\
\cline{2-7}
1+s+s^2 & s & +s^2 & & & & \\
 & s & +s^2 & +s^3 & & & \\
\cline{2-4}
 & & & -s^3 & & & \\
 & & & -s^3 & -s^4 & -s^5 & \\
\cline{4-6}
 & & & & s^4 & +s^5 & \\
 & & & & s^4 & +s^5 & +s^6 \\
\cline{5-7}
 & & & & & & -s^6 \\
 & & & & & & \cdots
\end{array}
$$

例:

单位负反馈系统开环传递函数: $G_o(s) = \frac{100}{s(0.s1+1)}$ 求输入信号为 $\sin(5t)$ 时的稳态误差.
解: 系统稳定,

$$\begin{aligned}
r(t) &= sin(\omega t), \omega = 5 \\
E(s) &= \Phi_e(s) R(s) \\
e_{ss} &= \sum_{i=0}^{\infty} C_i r^{(i)} \\
\Phi_e(s) &= \frac{1}{1 + G_o(s)} \\
&= \frac{0.1s^2 + s}{0.1s^2 + s + 100}
\end{aligned}$$

解法 1

- $\frac{0.1s^2+s}{0.1s^2+s+100} = 0.01s + \frac{0.1s^2+s-0.01s(0.1s^2+s+100)}{0.1s^2+s+100}$
- $\frac{-10^{-3}s^3+0.09s^2}{0.1s^2+s+100} =$
  $9\times10^{-4}s^2 + \frac{-10^{-3}s^3+0.09s^2-9\times10^{-4}s^2(0.1s^2+s+100)}{0.1s^2+s+100}$
- $\frac{-9\times10^{-5}s^4-1.9\times10^{-3}s^3}{0.1s^2+s+100} = -1.9\times10^{-5}s^3+\cdots$

所以

- $\Phi_e(s) = 0+0.01s+9\times10^{-4}s^2-1.9\times10^{-5}s^3+\cdots$
- $e_{ss}(t) = (C_0 - C_2\omega^2 + C_4\omega^4 + \cdots)\sin(\omega t) + (C_1 - C_3\omega^3 + C_5\omega^5 + \cdots)\cos(\omega t)$
- $e_{ss}(t) = -0.055\cos(5t - 249^\circ)$

解法 2:

$$
\begin{aligned}
E(s) &= \Phi_e(s)R(s) \\
&= \frac{s^2+10S}{s^2+10S+1000}\cdot\frac{5}{s^2+25} \\
&= \frac{-0.0498s-0.1115}{s^2+25}+\frac{as+b}{s^2+10s+1000} \\
e_{ss}(t) &= -0.055\cos(5t-249^\circ)+\Delta
\end{aligned}
$$

其中: $\lim_{t\to\infty}\Delta = 0$

Topic

减小 e_{ss} 的措施

- 增大开环增益
- 提高系统类型
- 串级控制抑制扰动
- 复合控制

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增大开环增益

$$G(s) = \frac{K\prod_{i=1}^{m}(\tau_i s + 1)}{s^{\nu}\prod_{j=1}^{n-\nu}(T_j s + 1)}$$

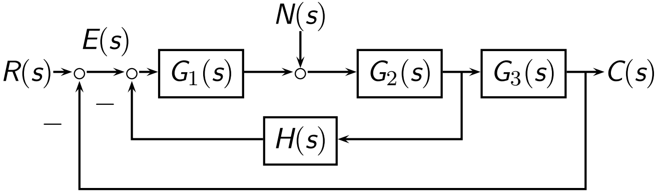
$$e_{ss} = \begin{cases} \frac{1}{1+K} & \nu = 0, R(s) = \frac{1}{s} \\ \frac{1}{K} & \nu = 1, R(s) = \frac{1}{s^2} \\ \frac{1}{K} & \nu = 2, R(s) = \frac{1}{s^3} \end{cases}$$

提高系统类型

$$G(s) = \frac{1}{s}\frac{K\prod_{i=1}^{m}(\tau_i s+1)}{s^{\nu}\prod_{j=1}^{n-\nu}(T_j s+1)}$$

$$e_{ss} = \begin{cases} \frac{1}{K} & \nu = 0, R(s) = \frac{1}{s^2} \\ \frac{1}{K} & \nu = 1, R(s) = \frac{1}{s^3} \end{cases}$$

串级控制



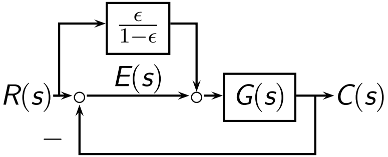
$$C(s) = \left(G_1(s)E'(s) + N(s)\right)G_2(s)G_3(s)$$

$$E'(s) = E(s) - \frac{H(s)}{G_3(s)}C(s)$$

$$C(s) = \frac{\left(G_1(s)E(s) + N(s)\right)G_2(s)G_3(s)}{1 + G_1(s)G_2(s)H(s)}$$

$$C(s) \approx \frac{G_3(s)E(s)}{H(s)} \qquad (G_1(s) >> 1)$$

复合控制



$$
\begin{aligned}
\epsilon &= \frac{r(\infty) - c(\infty)}{r(\infty)} \\
c(\infty) &= (1-\epsilon) r(\infty) \\
r(t) &= \frac{r'(t)}{1-\epsilon} \\
c(\infty) &= r'(\infty) \\
e'_{ss} &= r'(\infty) - c(\infty) \\
&= 0
\end{aligned}
$$