线性系统的根轨迹法

特殊的根轨迹

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<.1>

1 一阶系统

$G(s)H(s) = \frac{K^*}{s}$

$$\begin{aligned} 1+\frac{K^*}{s} &= 0 && (1)\\ \frac{K^*}{s} &= -1 && (2)\\ \angle s &= (2k+1)\pi && (3)\\ s+K^* &= 0 && (4)\\ s &= -K^* && (5) \end{aligned}$$

<.2>

$G(s)H(s) = \frac{K^*}{s-c}$

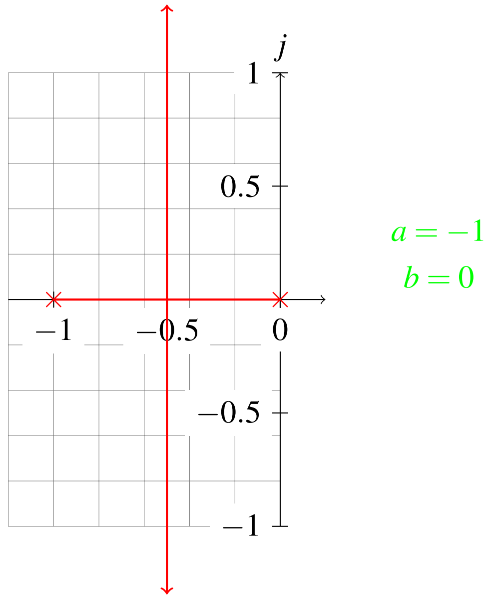
$$\begin{aligned} 1+\frac{K^*}{s-c} &= 0 && (6)\\ \frac{K^*}{s-c} &= -1 && (7)\\ \angle s-c &= (2k+1)\pi && (8)\\ s-c+K^* &= 0 && (9)\\ s &= c-K^* && (10) \end{aligned}$$

<.3>

2 二阶系统

2.1 无开环零点

$G(s)H(s)\frac{K^*}{(s+a)(s+b)}$



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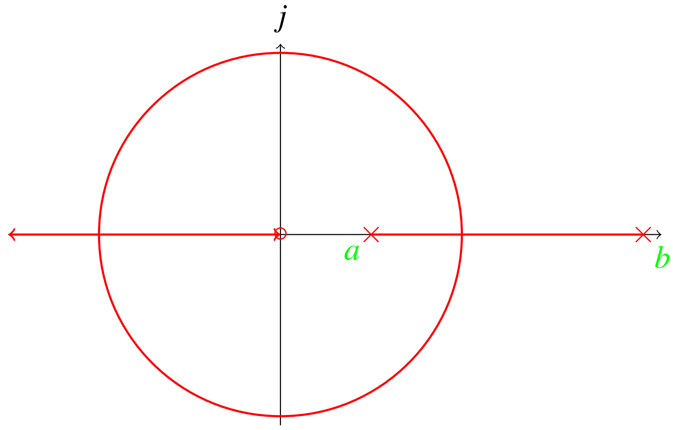
2.2 有开环零点

$G(s)H(s) = \frac{K^*(s+c)}{(s+a)(s+b)}$



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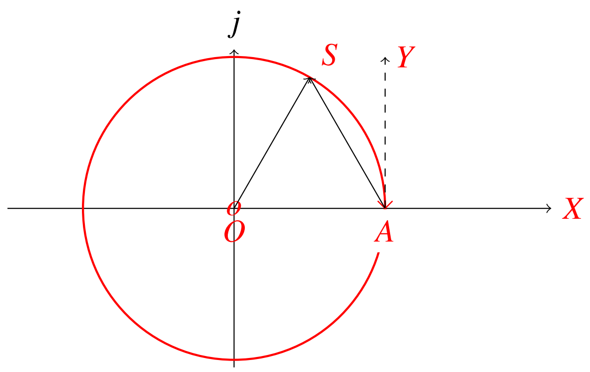
根轨迹为圆的证明 (重极点)



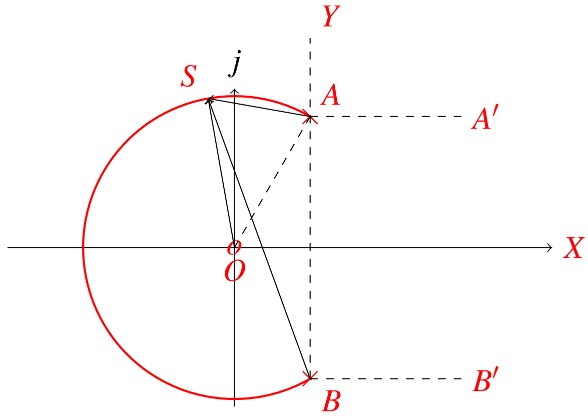
$$\angle(s-a)+\angle(s-b)-\angle(s-O) = 2\angle SAX - \angle SOX$$

$$\angle SAX = \angle SAY + \frac{\pi}{2}$$

$$2\angle SAY = \angle SOX$$

<.6>

根轨迹为圆的证明 (共轭极点)



$$\begin{aligned} \angle SAY &= \angle SBY + \angle BSA \\ \angle BSA &= \angle AOX \\ 2\angle SBA &= \angle SOA \end{aligned}$$

<.7>

3 开环重极点

3.1 原点为极点

$G(s)H(s) = \frac{K^*}{s^n}$

$$1 + K^* \frac{1}{s^n} = 0 \tag{11}$$
$$K^* \frac{1}{s^n} = -1 \tag{12}$$
$$\angle s^n = (2k+1)\pi \tag{13}$$
$$n\angle s = (2k+1)\pi \tag{14}$$
$$\angle s = \frac{(2k+1)\pi}{n} \tag{15}$$
$$s^n = -K^* \tag{16}$$
$$s = \sqrt[n]{K^*} e^{j\frac{(2k+1)\pi}{n}} \tag{17}$$

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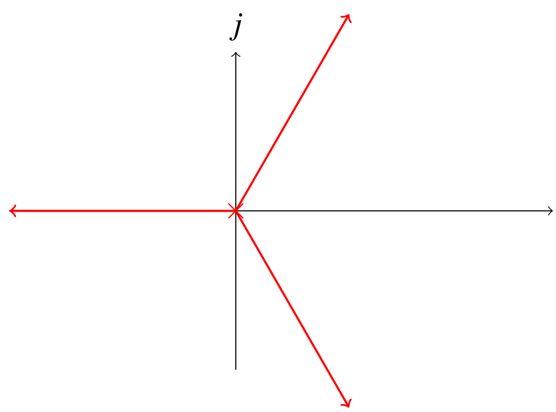
示例 $G(s)H(s) = \frac{K^*}{s^3}$



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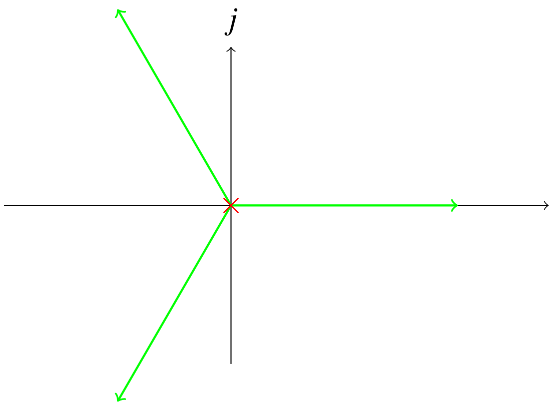
示例 $G(s)H(s)=\frac{K^*}{s^3}, K^*\in(-\infty,0)$



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3.2 极点平移

$G(s)=\frac{K^*}{(s-c)^n}$

$$
\begin{aligned}
1+K^*\frac{1}{(s-c)^n} &= 0 && (18)\\
K^*\frac{1}{(s-c)^n} &= -1 && (19)\\
\angle(s-c)^n &= (2k+1)\pi && (20)\\
n\angle(s-c) &= (2k+1)\pi && (21)\\
\angle(s-c) &= \frac{(2k+1)\pi}{n} && (22)\\
(s-c)^n &= -K^* && (23)\\
s &= c+\sqrt[n]{K^*}e^{j\frac{(2k+1)\pi}{n}} && (24)
\end{aligned}
$$

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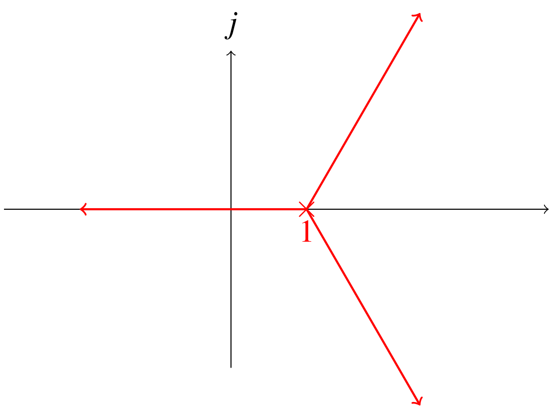
示例 $G(s)H(s)=\frac{K^*}{(s-1)^3}$



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4 $G(s+a)H(s+a)$

$G(s+a)H(s+a)$ 的根轨迹

$$
\begin{aligned}
s' &= s+a \\
1+K^*G(s')H(s') &= 0 \\
s' &= f(K^*) \\
s &= f(K^*)-a
\end{aligned}
$$

<.13>

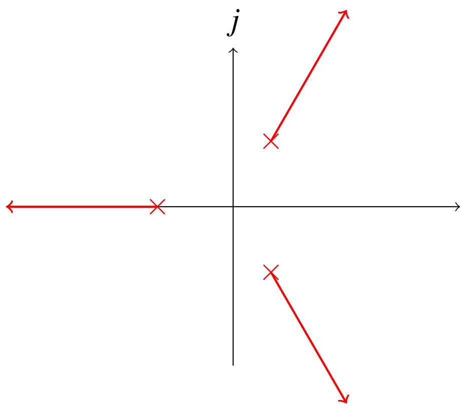
5 $1+(K^*+a)G(s)H(s)$

$1+(K^*+a)G(s)H(s)$ 的根轨迹

$$
\begin{aligned}
K' &= K^*+a \\
s &= f(K') \\
s &= f(K^*+a)
\end{aligned}
$$

<.14>

$\frac{K^*}{s^3+1}$ 的根轨迹



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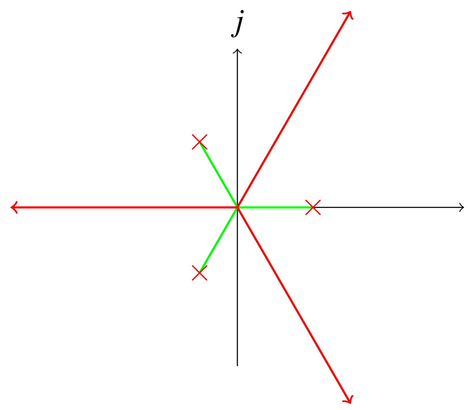
当 $K^* \in (1,\infty)$ 时，$\frac{K^*}{s^3}$ 的根轨迹

$$
\begin{aligned}
\frac{K^*}{s^3+1} &= -1 \\
K^* &= -s^3-1 \\
K^*+1 &= -s^3 \\
\frac{K^*+1}{s^3} &= -1 \qquad (K^* \in (0,+\infty)) \\
\frac{K'}{s^3} &= -1 \qquad (K' \in (1,+\infty)) \\
s &= \sqrt[3]{K'}e^{j\frac{(2k+1)\pi}{3}} \qquad (K' \in (1,+\infty))
\end{aligned}
$$

<.16>

$\frac{K^*}{s^3-1}$ 的根轨迹



<.17>

当 $K^* \in (-1, \infty)$ 时 $\frac{K^*}{s^3}$ 的根轨迹

$$
\begin{aligned}
\frac{K^*}{s^3-1} &= -1 \\
K^* &= -s^3+1 \\
K^*-1 &= -s^3 \\
\frac{K^*-1}{s^3} &= -1 \qquad (K^* \in (0, +\infty)) \\
\frac{K'}{s^3} &= -1 \qquad (K' \in (-1, +\infty)) \\
s &= \begin{cases} \sqrt[3]{-K'} e^{j\frac{2k\pi}{3}} & (K' \in (-1, 0)) \\ \sqrt[3]{K'} e^{j\frac{(2k+1)\pi}{3}} & (K' \in [0, +\infty)) \end{cases}
\end{aligned}
$$

<.18>