

# 线性系统时域分析法

## Outline

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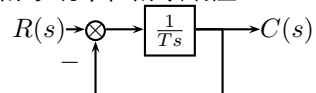
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## 1 动态性能分析

分析  $\sigma\%, t_s$  等指标,  $r(t) = 1, R(s) = \frac{1}{s}$

## 1.1 一阶系统动态性能

一阶系统单位阶跃响应



$$\begin{aligned}
 G(s) &= \frac{1}{Ts} \\
 \Phi(s) &= \frac{1}{Ts + 1} \\
 R(s) &= \frac{1}{s} \\
 C(s) &= \Phi(s)R(s) \\
 &= \frac{-T}{Ts + 1} + \frac{1}{s} \\
 c(t) &= 1 - e^{-t/T}
 \end{aligned}$$

一阶系统单位脉冲响应

$$\begin{aligned}
 R(s) &= 1 \\
 C(s) &= \Phi(s)R(s) \\
 &= \Phi(s) \\
 &= \frac{1}{Ts + 1} \\
 c(t) &= \frac{1}{T}e^{-t/T}
 \end{aligned}$$

一阶系统单位斜坡响应

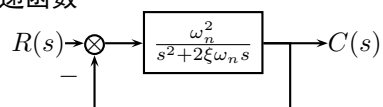
$$\begin{aligned}
 R(s) &= \frac{1}{s^2} \\
 C(s) &= \Phi(s)R(s) \\
 &= \frac{1}{(Ts + 1)s^2} \\
 &= \frac{1}{s^2} - \frac{T}{s} + \frac{T^2}{Ts + 1} \\
 c(t) &= (t - T) + Te^{-t/T}
 \end{aligned}$$

## 一阶系统单位加速度响应

$$\begin{aligned}
 R(s) &= \frac{1}{s^3} \\
 C(s) &= \Phi(s)R(s) \\
 &= \frac{1}{(Ts+1)s^3} \\
 &= \frac{1}{s^3} - \frac{T}{s^2} + \frac{T^2}{s} - \frac{T^3}{sT+1} \\
 c(t) &= \frac{1}{2}t^2 - Tt + T^2(1 - e^{-t/T})
 \end{aligned}$$

## 1.2 二阶系统时域分析

传递函数



- $\xi$ : 阻尼比
- $\omega_n$ : 自然频率, 无阻尼振荡频率

$$\begin{aligned}
 r(t) &= 1 \\
 R(s) &= \frac{1}{s} \\
 G(s) &= \frac{\omega_n^2}{s^2 + 2\xi\omega_n s} \\
 \Phi(s) &= \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \\
 p_{1,2} &= -\xi\omega_n \pm \omega_n \sqrt{\xi^2 - 1}
 \end{aligned}$$

$\xi \leq 0$

- $\xi < 0$  时有正实根, 不稳定
- $\xi = 0$  时有两个纯虚根, 无阻尼, 临界稳定, 等幅振荡, 频率为  $\omega_n$ ,

$$\begin{aligned}
 C(s) &= \frac{\omega_n^2}{s^2 + \omega_n^2} \cdot \frac{1}{s} \\
 &= \frac{-s}{s^2 + \omega_n^2} + \frac{1}{s} \\
 c(t) &= 1 - \cos \omega_n t
 \end{aligned}$$

$\xi > 1$

系统闭环极点为两个不同的实根. 过阻尼, 相当于两个一阶系统并联,  $\sigma\% = 0$

$$\begin{aligned}\Phi(s) &= \frac{\omega_n^2}{(s-p_1)(s-p_2)} \\ &= \frac{K_1}{s-p_1} + \frac{K_2}{s-p_2} \\ c(t) &= 1 - \frac{e^{p_1 t}}{1 - \frac{p_1}{p_2}} - \frac{e^{p_2 t}}{1 - \frac{p_2}{p_1}}\end{aligned}$$

$\xi = 1$

- 闭环极点有两个相同的负实根  $p_{1,2} = -\xi\omega_n = -\omega_n$

$$\begin{aligned}C(s) &= \frac{\omega_n^2}{(s + \omega_n^2)} \cdot \frac{1}{s} \\ c(t) &= 1 - e^{-\omega_n t}(1 + \omega_n t)\end{aligned}$$

- 且有:

$$\begin{aligned}\frac{dc(t)}{dt} &= \omega_n e^{-\omega_n t}(1 + \omega_n t) - \omega_n e^{-\omega_n t} = \omega_n^2 t e^{-\omega_n t} > 0 \\ c(0) &= 0 \\ c(\infty) &= 1 \\ \sigma\% &= 0 \\ t_s &= 4.75T \\ T &= \frac{1}{\omega_n}\end{aligned}$$

$0 < \xi < 1$

系统有一对实部小于零的共轭复根,  $p_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$

$$\begin{aligned}C(s) &= \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \cdot \frac{1}{s} \\ &= \frac{1}{s} - \frac{s + \xi\omega_n}{(s + \xi\omega_n)^2 + (1 - \xi^2)\omega_n^2} - \frac{\xi\omega_n}{(s + \xi\omega_n)^2 + (1 - \xi^2)\omega_n^2} \\ c(t) &= 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} (\sqrt{1-\xi^2} \sin \sqrt{1-\xi^2} \omega_n t \\ &\quad + \xi \cos \sqrt{1-\xi^2} \omega_n t) \\ &= 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin(\omega_d t + \beta) \\ \beta &= \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi}\end{aligned}$$

$$\omega_d = \sqrt{1 - \xi^2} \omega_n$$

### 1.3 二阶系统阶跃响应指标计算

二阶欠阻尼系统阶跃响应指标

$$c(t) = 1 - \frac{1}{\sqrt{1 - \xi^2}} e^{-\xi \omega_n t} \sin(\omega_d t + \beta)$$

- 欠阻尼.  $\omega_d$  称为有阻尼振荡频率. 最佳阻尼比  $\xi = 0.707$
- 指标:  $\sigma\%, t_s, t_p, t_r$  等

上升时间  $t_r$

- 100% 的  $t_r$ :  $c(t)$  首次达到  $c(\infty)$  的时间
- 90% 的  $t_r$ :  $c(t)$  首次达到 90% $c(\infty)$  的时间
- 70% 的  $t_r$ :  $c(t)$  首次达到 70% $c(\infty)$  的时间

$$\begin{aligned} c(t) &= c(\infty) \\ 1 - \frac{1}{\sqrt{1 - \xi^2}} e^{-\xi \omega_n t} \sin(\omega_d t + \beta) &= 1 \\ \sin(\omega_d t + \beta) &= 0 \\ \omega_d t + \beta &= k\pi \\ t_r &= \frac{\pi - \beta}{\omega_d} \end{aligned}$$

峰值时间  $t_p$

$c(t)$  达到最大值的时间

$$\begin{aligned} \frac{dc(t)}{dt} &= 0 \\ -\xi \omega_n e^{-\xi \omega_n t} \sin(\omega_d t + \beta) + e^{-\xi \omega_n t} \omega_d \cos(\omega_d t + \beta) &= 0 \\ \omega_d \cos(\omega_d t + \beta) &= \xi \omega_n \sin(\omega_d t + \beta) \\ \tan(\omega_d t + \beta) &= \frac{\sqrt{1 - \xi^2}}{\xi} \\ \tan(\omega_d t + \beta) &= \tan \beta \end{aligned}$$

$$\begin{aligned}\omega_d t &= k\pi \\ t_p &= \frac{\pi}{\omega_d}\end{aligned}$$

超调量  $\sigma\%$

$$\begin{aligned}\sigma\% &= \frac{c_{max} - c(\infty)}{c(\infty)} \times 100\% = (c(t_p) - 1) \\ &= -\frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t_p} \sin(\omega_d t_p + \beta) \\ &= -\frac{1}{\sqrt{1-\xi^2}} e^{-\frac{\xi\omega_n \pi}{\omega_d}} \sin(\pi + \beta) \\ &= \frac{1}{\sqrt{1-\xi^2}} e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \sin(\beta) \\ &= e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \times 100\%\end{aligned}$$

分析:

- $\sigma\%$  只与  $\xi$  有关, 两者成反比关系
- 工程上一般取  $\xi \in [0.4, 0.8]$
- 最佳阻尼比  $\xi = 0.707, \sigma\% = 4.3\%$

调节时间  $t_s$

近似估算:

$$\begin{aligned}c(t) &= 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin(\omega_d t + \beta) \\ &\approx 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \\ e(t) &= c(\infty) - c(t) \\ &\approx \frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\end{aligned}$$

- $t_s$  与  $\omega_n, \xi$  有关: 通常取  $\xi\omega_n t_s = 3.5, t_s = \frac{3.5}{\xi\omega_n}$

## 二阶过阻尼系统阶跃响应指标

- $\sigma\% = 0$
- $\xi = 1$  时,

$$t_s = \frac{4.75}{\omega_n}$$

- $\xi > 1, |p_1| \ll |p_2|$  时, 系统降阶, 去掉极点  $p_2$ ,

$$t_s = \frac{3}{|p_1|}$$

## 1.4 二阶系统单位斜坡响应

### 欠阻尼单位斜坡响应

$$\begin{aligned} C(s) &= \frac{\omega_n^2}{s^2(s^2 + 2\xi\omega_n s + \omega_n^2)} \\ &= \frac{1}{s^2} - \frac{2\xi}{\omega_n s} + \frac{2\xi(s + \xi\omega_n) + \omega_n(2\xi^2 - 1)}{\omega_n(s^2 + 2\xi\omega_n s + \omega_n^2)} \\ c(t) &= t - \frac{2\xi}{\omega_n} + \frac{1}{\omega_n \sqrt{1 - \xi^2}} e^{-\xi\omega_n t} \sin(\omega_d t + 2\beta) \\ e(t) &= \frac{2\xi}{\omega_n} \left[ 1 - \frac{1}{2\xi \sqrt{1 - \xi^2}} e^{-\xi\omega_n t} \sin(\omega_d t + 2\beta) \right] \end{aligned}$$

### 临界阻尼单位斜坡响应

$$\begin{aligned} c(t) &= t - \frac{2}{\omega_n} + \frac{2}{\omega_n} \left( 1 + \frac{1}{2}\omega_n t \right) e^{-\omega_n t} \\ e(t) &= \frac{2}{\omega_n} \left[ 1 - \left( 1 + \frac{1}{2}\omega_n t \right) e^{-\omega_n t} \right] \end{aligned}$$

### 过阻尼单位斜坡响应

$$\begin{aligned} C(s) &= \frac{1}{s^2} - \frac{2\xi}{\omega_n s} + \frac{2\xi(s + \xi\omega_n) + \omega_n(2\xi^2 - 1)}{\omega_n(s - p_1)(s - p_2)} \\ p_1 &= -\omega_n \xi + \omega_n \sqrt{\xi^2 - 1} \\ p_2 &= -\omega_n \xi - \omega_n \sqrt{\xi^2 - 1} \end{aligned}$$

$$c(t) = t - \frac{2\xi}{\omega_n} + \frac{2\xi^2 - 1 + 2\xi\sqrt{\xi^2 - 1}}{2\omega_n\sqrt{\xi^2 - 1}}e^{p_1 t} - \frac{2\xi^2 - 1 - 2\xi\sqrt{\xi^2 - 1}}{2\omega_n\sqrt{\xi^2 - 1}}e^{p_2 t}$$

## 1.5 高阶系统时域分析（3 阶及以上系统）

### 三阶系统

- 根的几种情况
  - 3 个负实根  $p_1, p_2, p_3$
  - 1 个负实根, 一对共轭复根

$$-s_0, -\xi\omega_n \pm j\omega_n\sqrt{1 - \xi^2}, (0 < \xi < 1)$$

- 重点考虑有复根的情况.

### 三阶系统 ( $\Phi(s)$ ) 单位阶跃响应 ( $C(s)$ )

$$\begin{aligned}\Phi(s) &= \frac{s_0\omega_n^2}{(s + s_0)(s^2 + 2\xi\omega_n s + \omega_n^2)} \\ C(s) &= \frac{s_0\omega_n^2}{s(s + s_0)(s^2 + 2\xi\omega_n s + \omega_n^2)} \\ c(t) &= 1 - \frac{e^{-s_0 t}}{b\xi^2(b-2) + 1} - \frac{e^{-\xi\omega_n t}}{b\xi^2(b-2) + 1} \\ &\quad \left( b\xi^2(b-2) \cos \omega_d t + \frac{b\xi(\xi^2(b-2) + 1)}{\sqrt{1 - \xi^2}} \sin \omega_d t \right) \\ \omega_d &= \omega_n \sqrt{1 - \xi^2} \\ b &= \frac{s_0}{\xi\omega_n}\end{aligned}$$

### $b$ 对 $c(t)$ 的影响

- 复根比实根离虚轴近得多

$$b \gg 1$$

$$c(t) \approx 1 - e^{-\xi\omega_n t} \left( \cos \omega_d t + \frac{\xi}{\sqrt{1 - \xi^2}} \sin \omega_d t \right)$$

近似看作 2 阶欠阻尼系统.



- 实根比复根离虚轴近得多

$$\begin{aligned} b &\approx 0 \\ c(t) &\approx 1 - e^{-s_0 t} \end{aligned}$$

近似看作 1 阶系统

- 实根与复根与虚轴距离同

$$\begin{aligned} b &\approx 1 \\ c(t) &\approx 1 - \frac{e^{-\xi\omega_n t}}{1 - \xi^2} (1 + \xi \sin(\omega_d t - \beta)) \end{aligned}$$

### 主导极点法

- 目的: 分析高阶系统的性能
- 内容: 系统有多个极点, 其中某些极点决定了整个系统的性能, 对系统起主导作用, 称这些极点为主导极点.
- 确定方法: 主导极点离虚轴距离为  $a$ , 其它极点离虚轴距离  $\geq 5a$

## 2 稳定性分析

### 2.1 稳定性的概念

稳定性



定义: 系统处于平衡状态时, 若有干扰使系统偏离平衡状态, 当扰动消除后, 系统仍能回到原来的平衡状态, 则称该系统是稳定的, 反之称为不稳定.

稳定的充要条件

- 系统的传递函数

$$\Phi(s) = \frac{C(s)}{R(s)} = \frac{k_g \prod_{i=1}^m (s - z_i)}{\sum_{j=1}^n (s - p_j)}$$

- 设系统输入为单位脉冲信号:  $r(t) = \delta(t), R(s) = 1$

$$C(s) = \Phi(s)R(s) = \frac{k_g \prod_{i=1}^m (s - z_i)}{\sum_{j=1}^n (s - p_j)}$$

$$= k_g \sum_{j=1}^n \frac{k_j}{s - p_j}$$

$$c(t) = k_g \sum_{j=1}^n k_j e^{p_j t}$$

- 当  $\Re(p_j) < 0$  时, 有  $\lim_{t \rightarrow \infty} c(t) = 0$
- 稳定性充要条件: 系统的闭环极点均具有负实部

## 2.2 古尔维茨判据及劳斯判据

特征多项式

$$G(s) = \frac{M(s)}{N(s)}$$

$$M(s) = b_0 s^m + \cdots + b_n$$

$$N(s) = a_0 s^n + \cdots + a_n$$

其中:

- $N(s)$  称为特征多项式
- $N(s) = 0$  称为特征方程 (只与  $a_0, \cdots, a_n$  有关)

古尔维茨判据 (代数判据):

- 稳定的必要条件: 特征多项式中各项系数大于零.
- 稳定的充要条件: 特征多项式中各项系数所构成的主行列式及顺序主子式全部大于零.
- 主行列式:

$$\Delta_n = \begin{vmatrix} a_1 & a_3 & a_5 & \cdots & 0 \\ a_0 & a_2 & a_4 & \cdots & 0 \\ 0 & a_1 & a_3 & \cdots & 0 \\ \vdots & a_0 & a_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & & 0 \\ 0 & 0 & \cdots & a_{n-2} & a_n \end{vmatrix}$$

- 顺序主子式有  $n-1$  个:  $\Delta_1, \cdots, \Delta_{n-1}$
- 李纳德-威帕特判据: 若所有奇次古尔维茨行列式为正, 则偶次古尔维茨行列式必为正, 反之亦然

劳斯 -古尔维茨判据, 简称劳斯判据

- 构造劳斯表判断系统是否稳定

$$\begin{array}{ccccccc}
 s^n & & a_0 & & a_2 & & a_4 & \cdots \\
 s^{n-1} & & a_1 & & a_3 & & a_5 & \cdots \\
 s^{n-2} & c_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} & & c_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} & & \cdots & & \\
 s^{n-3} & d_1 = \frac{c_1 a_3 - a_1 c_2}{c_1} & & d_2 = \frac{c_1 a_5 - a_1 c_3}{c_1} & & \cdots & & \\
 \vdots & & \vdots & & & & \cdots & \\
 s^0 & & a_n & & & & & 
 \end{array}$$

- 劳斯判据:
  - 系统稳定的充要条件: 劳斯表中第一列元素均大于零
  - 若第一列元素有小于零的, 则系统不稳定, 且不稳定根的个数等于第一列元素变号的次数.

劳斯表与多项式除法

$$\begin{array}{ccccccc}
 s^n & & a_0 & & a_2 & & a_4 & \cdots \\
 s^{n-1} & & a_1 & & a_3 & & a_5 & \cdots \\
 s^{n-2} & c_1 = a_2 - \frac{a_0 a_3}{a_1} & & c_2 = a_4 - \frac{a_0 a_5}{a_1} & & \cdots & & \\
 s^{n-3} & d_1 = a_3 - \frac{a_1 c_2}{c_1} & & d_2 = a_5 - \frac{a_1 c_3}{c_1} & & \cdots & & \\
 \vdots & & \vdots & & & & \cdots & \\
 s^0 & & a_n & & & & & 
 \end{array}$$

即:

$$\begin{aligned}
 a_0 s^n + a_2 s^{n-2} + \cdots &= s \frac{a_0}{a_1} (a_1 s^{n-1} + a_3 s^{n-3} + \cdots) + c_1 s^{n-2} + c_2 s^{n-4} + \cdots \\
 c_1 s^{n-2} + c_2 s^{n-4} + \cdots &= s \frac{c_1}{d_1} (d_1 s^{n-3} + d_2 s^{n-5} + \cdots) + \cdots
 \end{aligned}$$

## 2.3 劳斯判据示例

**Routh 判据示例 1:**  $3s^4 + 10s^3 + 5s^2 + s + 2 = 0$

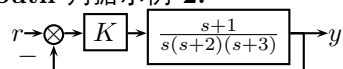
解:

- 劳斯表:

$$\begin{array}{cccc}
 s^4 & 3 & 5 & 2 \\
 s^3 & 10 & 1 & \\
 s^2 & 4.7 & 2 & \\
 s^1 & -\frac{15.3}{4.7} & & \\
 s^0 & 2 & & 
 \end{array}$$

- 结论 系统不稳定, 有 2 个不稳定根.

**Routh 判据示例 2:**



求使系统稳定的  $K$  的范围

解:

- 传递函数

$$G(s) = \frac{K(s+1)}{s(s+2)(s+3)}$$

$$\Phi(s) = \frac{K(s+1)}{s^3 + 5s^2 + (6+K)s + K}$$

- 劳斯表:

$s^3$	1	$6+K$
$s^2$	5	$K$
$s^1$	$\frac{30+4K}{5}$	0
$s^0$	$K$	

稳定条件:

$$\begin{aligned} 30 + 4K &> 0 \\ K &> 0 \end{aligned}$$

得:  $K > 0$

## 2.4 劳斯判据应用中的特殊情况

**Routh 表特殊情况:** 第一列元素有零元, 系统不稳定:

- 先用  $\epsilon$  代替, 再令  $\epsilon \rightarrow 0$
- 系统升阶:  $(s+a)D(s), a > 0$
- 例: 系统特征方程:  $D(s) = s^4 + 2s^3 + s^2 + 2s + 1 = 0$  判断系统的稳定性, 求出不稳定根的个数.
- Roth 表:

$s^4$	1	1
$s^3$	2	2
$s^2$	$0(\epsilon)$	1
$s^1$	$\frac{2\epsilon-2}{\epsilon}$	0
$s^0$	1	

- $(s+1)D(s) = s^5 + 3s^4 + 3s^3 + 3s^2 + 3s + 1$

$$\begin{array}{r} s^5 \\ s^4 \\ s^3 \\ s^2 \\ s^1 \\ s^0 \end{array} \begin{array}{ccc} 1 & 3 & 3 \\ 3 & 3 & 1 \\ 2 & \frac{8}{3} & \\ -1 & 1 & \\ \frac{14}{3} & 0 & \\ 1 & & \end{array}$$

特殊情况: 全行为零时:

- 全零行的数值由上一行求导代替
- 辅助方程: 全零行的上一行可构成辅助方程, 辅助方程的解全部为系统特征根
- 例:  $D(s) = s^4 + 5s^3 + 5s^2 - 5s - 6$  求系统不稳定根的个数
- Routh 表

$$\begin{array}{r} s^4 \\ s^3 \\ s^2 \\ s^1 \\ s^0 \end{array} \begin{array}{ccc} 1 & 5 & -6 \\ 5 & -5 & \\ 6 & -6 & \\ 0(12) & 0 & \\ -6 & & \end{array}$$

- 辅助方程
  - 对  $6s^2 - 6$  求导得  $12s$
  - 辅助方程  $6s^2 - 6 = 0$ , 得  $s_{1,2} = \pm 1$  系统不稳定根为1

## 2.5 劳斯判据求解系统参数

例:  $D(s) = s^4 + 2s^3 + ks^2 + s + 2$  求  $K$  值稳定范围  
解:

- Routh 表

$$\begin{array}{r} s^4 \\ s^3 \\ s^2 \\ s^1 \\ s^0 \end{array} \begin{array}{ccc} 1 & K & 2 \\ 2 & 1 & \\ \frac{2K-1}{2} & 2 & \\ 1 - \frac{8}{2K-1} & 0 & \\ 2 & & \end{array}$$

$$\begin{array}{l} 2K-1 > 0 \\ 1 - \frac{8}{2K-1} > 0 \end{array}$$

得:  $K > \frac{9}{2}$

- 当  $K = \frac{9}{2}$  时: Routh 表:

$$\begin{array}{ccc} s^4 & 1 & 4.5 & 2 \\ s^3 & 2 & 1 & \\ s^2 & 4 & 2 & \\ s^1 & 0(8) & 0 & \\ s^0 & 2 & & \end{array}$$

其中辅助方程为  $4s^2 + 2 = 0$ , 可解得  $s = \pm \frac{\sqrt{2}}{2}j$

例:  $D(s) = Ts^3 + s^2 + K = 0$

- Routh 表

$$\begin{array}{ccc} s^3 & T & 0 \\ s^2 & 1 & K \\ s^1 & -TK & \\ s^0 & K & \end{array}$$

- 无解

$$\begin{array}{ccc} T & > & 0 \\ K & > & 0 \\ TK & < & 0 \end{array}$$

## 2.6 相对稳定性

例:  $D(s) = s^3 + 5.5s^2 + 8.5s + 3$

- 判断系统是否具有相对稳定性:  $\sigma = 1$

解:

- Routh 表

$$\begin{array}{ccc} s^3 & 1 & 8.5 \\ s^2 & 5.5 & 3 \\ s^1 & 8.5 - \frac{3}{5.5} & 0 \\ s^0 & 3 & \end{array}$$

- 将  $s = z - \sigma$  代入  $D(s)$  中得  $D(z) = z^3 + 2.5z^2 + 0.5z - 1$ , 不稳定.

– Routh 表:

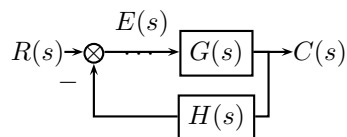
$$\begin{array}{ccc} s^3 & 1 & 0.5 \\ s^2 & 2.5 & -1 \\ s^1 & 0.5 + \frac{1}{2.5} & 0 \\ s^0 & -1 & \end{array}$$

– 结论 有一个不稳定极点.

### 3 稳态误差

#### 3.1 误差传递函数

系统误差



- ignore

系统误差有两种:

- 输入端定义:  $E_2(s) = E(s)$
- 输出端定义:  $E_1(s) = C_{expect} - C_{real}$
- 不加特别说明, 系统误差指的是输入端定义.

- $E(s)$  与  $E_1(s)$

$$\begin{aligned} C_{expect} &= \frac{R(s)}{H(s)} \\ E_1(s) &= \frac{R(s)}{H(s)} - C(s) \\ &= \frac{R(s) - C(s)H(s)}{H(s)} \\ &= \frac{E(s)}{H(s)} \end{aligned}$$

误差传递函数:

$$\begin{aligned} \Phi_e(s) &= \frac{E(s)}{R(s)} \\ &= \frac{1}{1 + G(s)H(s)} \\ &= \frac{R(s) - H(s)C(s)}{R(s)} \\ &= 1 - H(s)\Phi(s) \end{aligned}$$

- 系统误差:  $E(s) = \Phi_e(s)R(s)$

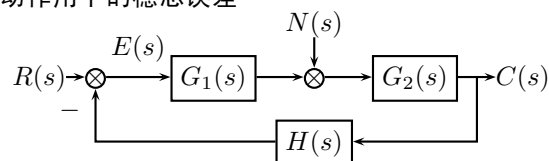
稳态误差:

$$e_{ss} = \lim_{t \rightarrow \infty} e(t)$$

$$\begin{aligned}
&= \lim_{s \rightarrow 0} sE(s) \\
&= \lim_{s \rightarrow 0} s\Phi_e(s)R(s)
\end{aligned}$$

- 稳态误差与输入信号有关
- 求稳态误差前要判断系统稳定性

扰动作用下的稳态误差



- 输入端定义:

$$\begin{aligned}
E(s) &= E_R(s) + E_N(s) \\
E_R(s) &= \Phi_e(s)R(s) \\
E_N(s) &= \Phi_{en}(s)N(s)
\end{aligned}$$

- 输出端定义: 令  $R(s) = 0$ , 计算  $N(s)$  单独引起的  $e_{ss}$ , 此时  $C_{expect}(s) = 0$

$$\begin{aligned}
E(s) &= 0 - C(s) \\
&= -\Phi_N(s)N(s) \\
\Phi_N(s) &= \frac{G_2}{1 + G_1G_2} \\
e_{ss} &= \lim_{s \rightarrow 0} s(-\Phi_N(s)N(s))
\end{aligned}$$

### 3.2 系统类型与静态误差系数

阶跃输入:

$$\begin{aligned}
r(t) &= A \\
R(s) &= \frac{A}{s} \\
e_{ss} &= \lim_{s \rightarrow 0} s \cdot \frac{1}{1 + G_{open}(s)} \cdot \frac{A}{s} \\
&= \lim_{s \rightarrow 0} \frac{A}{1 + G_{open}(s)}
\end{aligned}$$



速度输入

$$\begin{aligned}r(t) &= vt \\R(s) &= \frac{v}{s^2} \\e_{ss} &= \lim_{s \rightarrow 0} s \cdot \frac{1}{1 + G_{open}(s)} \cdot \frac{v}{s^2} \\&= \lim_{s \rightarrow 0} \frac{A}{s + sG_{open}(s)} \\&= \lim_{s \rightarrow 0} \frac{A}{sG_{open}(s)}\end{aligned}$$

加速度输入

$$\begin{aligned}r(t) &= \frac{1}{2}at^2 \\R(s) &= \frac{a}{s^2} \\e_{ss} &= \lim_{s \rightarrow 0} s \cdot \frac{1}{1 + G_{open}(s)} \cdot \frac{a}{s^2} \\&= \lim_{s \rightarrow 0} \frac{A}{s^2 + s^2G_{open}(s)} \\&= \lim_{s \rightarrow 0} \frac{A}{s^2G_{open}(s)}\end{aligned}$$

系统类型

- 由开环传递函数定义

$$\begin{aligned}G_{open} &= G(s)H(s) \\&= \frac{K \prod_{i=1}^m (\tau_i s + 1)}{s^\nu \prod_{j=1}^{n-\nu} (T_j s + 1)}\end{aligned}$$

- 其中  $K$  为开环增益.
- 定义:
  - $\nu = 0$  称为 0 型系统
  - $\nu = 1$  称为 I 型系统
  - $\nu = 2$  称为 II 型系统

## 静态误差系数

- 静态位置误差系数

$$\begin{aligned} r(t) &= A \\ e_{ss} &= \frac{A}{1 + K_p}, \quad K_p = \lim_{s \rightarrow 0} G_{open}(s) \end{aligned}$$

- 静态速度误差系数

$$\begin{aligned} r(t) &= vt \\ e_{ss} &= \frac{v}{K_v}, \quad K_v = \lim_{s \rightarrow 0} sG_{open}(s) \end{aligned}$$

- 静态加速度误差系数

$$\begin{aligned} r(t) &= \frac{1}{2}at^2 \\ e_{ss} &= \frac{a}{K_a}, \quad K_a = \lim_{s \rightarrow 0} s^2G_{open}(s) \end{aligned}$$

## 零型系统 ( $\nu = 0$ )

- $r(t) = A$  时:

$$\begin{aligned} K_p &= \lim_{s \rightarrow 0} G_o(s) = \lim_{s \rightarrow 0} \frac{K \prod_{i=0}^m (\tau_i s + 1)}{\prod_{j=1}^n (\tau_j s + 1)} = K \\ e_{ss1} &= \frac{A}{1 + K_p} \end{aligned}$$

称为有差系统.

- $r(t) = vt$  时:

$$\begin{aligned} K_v &= \lim_{s \rightarrow 0} sG_o(s) = 0 \\ e_{ss2} &= \infty \end{aligned}$$

- $r(t) = \frac{1}{2}at^2$  时:

$$\begin{aligned} K_a &= \lim_{s \rightarrow 0} s^2G_o(s) = 0 \\ e_{ss3} &= \infty \end{aligned}$$

## I 型系统 ( $\nu = 1$ )

- $r(t) = A$  时:

$$K_p = \lim_{s \rightarrow 0} G_o(s) = \lim_{s \rightarrow 0} \frac{K \prod_{i=0}^m (\tau_i s + 1)}{s \prod_{j=1}^{n-1} (\tau_j s + 1)} = \infty$$

$$e_{ss1} = \frac{1}{1 + K_p} = 0$$

无差系统.

- $r(t) = vt$  时:

$$K_v = \lim_{s \rightarrow 0} s G_o(s) = K$$

$$e_{ss2} = \frac{v}{K_v} = \frac{v}{K}$$

- $r(t) = \frac{1}{2}at^2$  时:

$$K_a = \lim_{s \rightarrow 0} s^2 G_o(s) = 0$$

$$e_{ss3} = \infty$$

## II 型系统 ( $\nu = 2$ )

$$K_p = \infty$$

$$e_{ss1} = 0$$

$$K_v = \infty$$

$$e_{ss2} = 0$$

$$K_a = K$$

$$e_{ss3} = \frac{a}{K}$$

小结:

- 零型:

$$e_{ss1} = \frac{A}{1 + K}, e_{ss2} = e_{ss3} = \infty$$

- I 型:

$$e_{ss1} = 0, e_{ss2} = \frac{v}{K}, e_{ss3} = \infty$$

- II 型:

$$e_{ss1} = e_{ss2} = 0, e_{ss3} = \frac{a}{K}$$

例:

若  $G(s)H(s) = \frac{10K_h}{s+1}$ ,  $K_h \in \{0.1, 1\}$ , 求单位阶跃下的  $e_{ss}$ .

解:

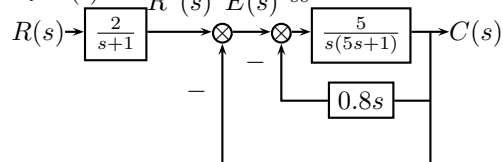
- 解法 1 零型系统,  $r(t) = 1, e_{ss} = \frac{1}{1+K_p}$

$$\begin{aligned} K_p &= \lim_{s \rightarrow 0} G(s)H(s) \\ &= 10K_h \\ &= \begin{cases} 1 & K_h = 0.1 \\ 10 & K_h = 1 \end{cases} \\ e_{ss} &= \begin{cases} 0.5 & K_h = 0.1 \\ \frac{1}{11} & K_h = 1 \end{cases} \end{aligned}$$

- 解法 2:

$$\begin{aligned} e_{ss} &= \lim_{s \rightarrow 0} s\Phi_e(s)R(s) \\ &= \lim_{s \rightarrow 0} s \frac{1}{1 + G(s)H(s)} R(s) \\ &= \lim_{s \rightarrow 0} s \frac{s+1}{s+1+10K_h} \frac{1}{s} \\ &= \frac{1}{1+10K_h} \\ &= \begin{cases} 0.5 & K_h = 0.1 \\ \frac{1}{11} & K_h = 1 \end{cases} \end{aligned}$$

例: 求  $r(t) = 2 + 3t$  时的  $e_{ss}$



解:

$$\begin{aligned} G(s) &= \frac{C(s)}{E(s)} \\ &= \frac{\frac{5}{s(5s+1)}}{1 + \frac{4s}{s(5s+1)}} \\ &= \frac{5}{5s^2 + 5s} \end{aligned}$$

$$= \frac{1}{s(s+1)}$$

例：计算稳态误差  
判断稳定性：

$$\begin{aligned}\Phi(s) &= \frac{C(s)}{R^*(s)} = \frac{1}{s(s+1)+1} \\ \Phi_e(s) &= \frac{s(s+1)}{s(s+1)+1}\end{aligned}$$

系统稳定.

$$\begin{aligned}R(s) &= \frac{2s+3}{s^2} \\ e_{ss} &= \lim_{s \rightarrow 0} s \Phi_e(s) R^*(s) \\ &= \lim_{s \rightarrow 0} s \cdot \frac{s(s+1)}{s(s+1)+1} \cdot \frac{2}{s+1} \cdot \frac{2s+3}{s^2} \\ &= 6\end{aligned}$$

### 3.3 动态误差系数

动态误差系数

动态误差系数可描述系统稳态误差随时间变化的规律，静态误差可看作动态误差的一个特例.

$$\begin{aligned}E(s) &= \Phi_e(s) R(s) \\ \Phi_e(s) &= \frac{E(s)}{R(s)} \\ &= \frac{1}{1+G(s)H(s)} \\ &= \frac{M(s)}{N(s)}\end{aligned}$$

在  $s=0$  处展开，得：

$$\phi_e(s) = \Phi_e(0) + \dot{\Phi}_e(0)s + \cdots + \frac{\Phi_e^{(n)}(0)s^n}{n!} + \cdots$$

$$\begin{aligned}
E(s) &= \Phi_e(0)R(s) + \dot{\Phi}_e(0)sR(s) + \cdots + \frac{\Phi_e^{(n)}(0)s^n R(s)}{n!} + \cdots \\
e_{ss}(t) &= \Phi_e(0)r(t) + \dot{\Phi}_e(0)\dot{r}(t) + \cdots + \frac{\Phi_e^{(n)}(0)r^{(n)}(t)}{n!} + \cdots \\
&= \sum_{i=1}^{\infty} C_i r^{(i)}(t), \quad C_i = \frac{\Phi_e^{(i)}(0)}{i!}
\end{aligned}$$

- 其中  $C_i$  称为动态误差系数.
  - $C_0$  动态位置误差系数
  - $C_1$  动态速度误差系数
  - $C_2$  动态加速度误差系数

动态误差系数示例:

- 零型系统  $r(t) = 1$  则

$$e_{ss}(t) = C_0, C_0 = \frac{1}{1 + K_p}$$

- I 型系统  $r(t) = t$  则

$$e_{ss}(t) = C_0 t + C_1, C_0 = 0, C_1 = \frac{1}{K_v}$$

- II 型系统  $r(t) = t^2$  则

$$e_{ss}(t) = C_0 \frac{1}{2} a t^2 + C_1 a t + C_2 a, C_0 = C_1 = 0, C_2 = \frac{1}{K_a}$$

讨论:  $C_i$  的计算

$$\begin{aligned}
\Phi_e(s) &= \frac{M(s)}{N(s)} \\
&= C_0 + C_1 s + C_2 s^2 + \cdots
\end{aligned}$$

例:  $G(s)H(s) = \frac{1}{s(s+1)}$

综合除法:

divident		divisor		quotient		remainder
$s^2 + s$	$\div$	$s^2 + s + 1$	$\rightarrow$	$s$		$s^2 + s - s(1 + s + s^2)$
$-s^3$	$\div$	$s^2 + s + 1$	$\rightarrow$	$-s^3$		$-s^3 - (-s^3)(1 + s + s^2)$
$s^4 + s^5$	$\div$	$s^2 + s + 1$	$\rightarrow$	$s^4$		$\cdots$
$\cdots$	$\div$	$s^2 + s + 1$	$\rightarrow$	$\cdots$		$\cdots$

得:

$$\Phi_e(s) = s - s^3 + s^4 + \dots$$

例:  $G(s)H(s) = \frac{1}{s(s+1)}$  另一种写法:

$$\begin{aligned} \frac{s^2 + s}{s^2 + s + 1} &= s + \frac{-s^2 + s - s(1 + s + s^2)}{s^2 + s + 1} \\ \frac{-s^3}{s^2 + s + 1} &= -s^3 + \frac{-s^3 - (-s^3)(1 + s + s^2)}{s^2 + s + 1} \\ \frac{s^4 + s^5}{s^2 + s + 1} &= s^4 + \dots \\ \dots &= \dots \\ \Phi_e(s) &= s - s^3 + s^4 + \dots \end{aligned}$$

例:  $G(s)H(s) = \frac{1}{s(s+1)}$  长除法

$$\begin{array}{r} \begin{array}{ccccccc} & s & & -s^3 & +s^4 & & \dots \\ 1+s+s^2 & \overline{) \begin{array}{ccccccc} s & +s^2 & & & & & \\ s & +s^2 & +s^3 & & & & \\ \hline & & -s^3 & & & & \\ & & -s^3 & -s^4 & -s^5 & & \\ & & & s^4 & +s^5 & & \\ & & & s^4 & +s^5 & +s^6 & \\ & & & & & -s^6 & \\ & & & & & & \dots \end{array} \end{array} \end{array}$$

例:

单位负反馈系统开环传递函数:  $G_o(s) = \frac{100}{s(0.1s+1)}$  求输入信号为  $\sin(5t)$  时的稳态误差.

解: 系统稳定,

$$\begin{aligned} r(t) &= \sin(\omega t), \omega = 5 \\ E(s) &= \Phi_e(s)R(s) \\ e_{ss} &= \sum_{i=0}^{\infty} C_i r^{(i)} \\ \Phi_e(s) &= \frac{1}{1 + G_o(s)} \\ &= \frac{0.1s^2 + s}{0.1s^2 + s + 100} \end{aligned}$$

### 解法 1

$$\begin{aligned}
 & \bullet \frac{0.1s^2+s}{0.1s^2+s+100} = 0.01s + \frac{0.1s^2+s-0.01s(0.1s^2+s+100)}{0.1s^2+s+100} \\
 & \bullet \frac{-10^{-3}s^3+0.09s^2}{0.1s^2+s+100} = 9 \times 10^{-4}s^2 + \frac{-10^{-3}s^3+0.09s^2-9 \times 10^{-4}s^2(0.1s^2+s+100)}{0.1s^2+s+100} \\
 & \bullet \frac{-9 \times 10^{-5}s^4-1.9 \times 10^{-3}s^3}{0.1s^2+s+100} = -1.9 \times 10^{-5}s^3 + \dots
 \end{aligned}$$

所以

$$\begin{aligned}
 & \bullet \Phi_e(s) = 0 + 0.01s + 9 \times 10^{-4}s^2 - 1.9 \times 10^{-5}s^3 + \dots \\
 & \bullet e_{ss}(t) = (C_0 - C_2\omega^2 + C_4\omega^4 + \dots) \sin(\omega t) + (C_1 - C_3\omega^3 + C_5\omega^5 + \dots) \cos(\omega t) \\
 & \bullet e_{ss}(t) = -0.055 \cos(5t - 249^\circ)
 \end{aligned}$$

### 解法 2:

$$\begin{aligned}
 E(s) &= \Phi_e(s)R(s) \\
 &= \frac{s^2 + 10S}{s^2 + 10S + 1000} \cdot \frac{5}{s^2 + 25} \\
 &= \frac{-0.0498s - 0.1115}{s^2 + 25} + \frac{as + b}{s^2 + 10s + 1000} \\
 e_{ss}(t) &= -0.055 \cos(5t - 249^\circ) + \Delta
 \end{aligned}$$

其中:  $\lim_{t \rightarrow \infty} \Delta = 0$

## 3.4 减小稳态误差的措施

减小  $e_{ss}$  的措施

- 增大开环增益
- 提高系统类型
- 串级控制抑制扰动
- 复合控制

增大开环增益

$$\begin{aligned}
 G(s) &= \frac{K \prod_{i=1}^m (\tau_i s + 1)}{s^\nu \prod_{j=1}^{n-\nu} (T_j s + 1)} \\
 e_{ss} &= \begin{cases} \frac{1}{1+K} & \nu = 0, R(s) = \frac{1}{s} \\ \frac{1}{K} & \nu = 1, R(s) = \frac{1}{s^2} \\ \frac{1}{K} & \nu = 2, R(s) = \frac{1}{s^3} \end{cases}
 \end{aligned}$$

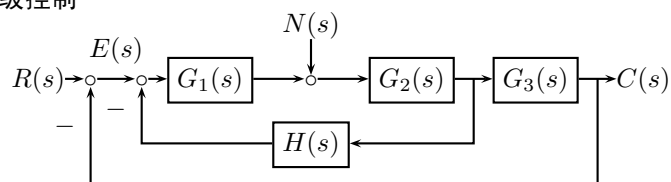


提高系统类型

$$G(s) = \frac{1}{s} \frac{K \prod_{i=1}^m (\tau_i s + 1)}{s^\nu \prod_{j=1}^{n-\nu} (T_j s + 1)}$$

$$e_{ss} = \begin{cases} \frac{1}{K} & \nu = 0, R(s) = \frac{1}{s^2} \\ \frac{1}{K} & \nu = 1, R(s) = \frac{1}{s^3} \end{cases}$$

串级控制



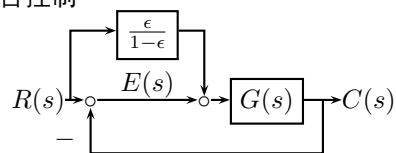
$$C(s) = (G_1(s)E'(s) + N(s))G_2(s)G_3(s)$$

$$E'(s) = E(s) - \frac{H(s)}{G_3(s)}C(s)$$

$$C(s) = \frac{(G_1(s)E(s) + N(s))G_2(s)G_3(s)}{1 + G_1(s)G_2(s)H(s)}$$

$$C(s) \approx \frac{G_3(s)E(s)}{H(s)} \quad (G_1(s) \gg 1)$$

复合控制



$$\epsilon = \frac{r(\infty) - c(\infty)}{r(\infty)}$$

$$c(\infty) = (1 - \epsilon)r(\infty)$$

$$r(t) = \frac{r'(t)}{1 - \epsilon}$$

$$c(\infty) = r'(\infty)$$

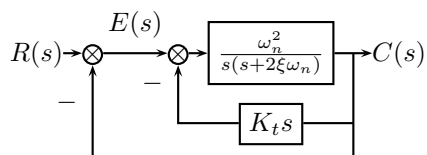
$$e'_{ss} = r'(\infty) - c(\infty)$$

$$= 0$$

## 4 示例

### 4.1 速度反馈

速度反馈分析

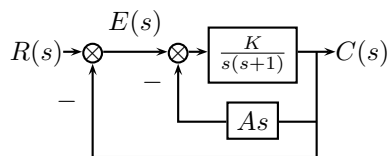


- 结构图
- 分析:

$$\Phi(s) = \frac{\omega_n^2}{s^2 + 2\xi_t\omega_n s + \omega_n^2}$$

$$\xi_t = \xi + \frac{1}{2}K_t\omega_n$$

速度反馈示例



- 结构图

$As$  为系统的速度反馈, 求解:

- $A = \{0.8, 0\}$  时系统稳态误差及动态品质指标 ( $K = 1$ , 求  $e_{ss}$  时  $R(s) = \frac{1}{s^2}$ )
- 若要求系统的  $\sigma\% = 20\%$ ,  $t_s \leq 1s$ , 确定  $A, K$  的值

- 解:

$$G(s) = \frac{K}{s(s+1) + KAs}$$

$$= \frac{K}{s^2 + (KA+1)s}$$

$$\Phi(s) = \frac{K}{s^2 + (KA+1)s + K}$$

系统稳定, 为 I 型系统.

(续) 计算稳态误差:

$R(s) = \frac{1}{s^2}$  时,

$$\begin{aligned} e_{ss} &= \frac{1}{K_v} \\ K_v &= \lim_{s \rightarrow 0} sG(s) \\ &= \lim_{s \rightarrow 0} \frac{K}{s + KA + 1} \\ &= \frac{K}{KA + 1} \end{aligned}$$

- $K = 1, A = 0.8$  时,  $e_{ss} = \frac{KA+1}{K} = 1.8$
- $K = 1, A = 0$  时,  $e_{ss} = \frac{KA+1}{K} = 1$

(续): 计算动态品质:

$$\begin{aligned} \Phi(s) &= \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \\ &= \frac{K}{s^2 + (KA + 1)s + K} \\ \sigma\% &= e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \\ t_s &= \frac{3.5}{\xi\omega_n} \end{aligned}$$

- $K = 1, A = 0$  时,  $\omega_n = 1, \xi = 0.5, \sigma\% = 16.3\%, t_s = 7$
- $K = 1, A = 0.8$  时,  $\omega_n = 1, \xi = 0.9, \sigma\% = 0.15\%, t_s = 3.89$

(续) 确定  $A, K$  值

$$\begin{aligned} e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} &= 20\% \\ \frac{3.5}{\xi\omega_n} &= 1 \end{aligned}$$

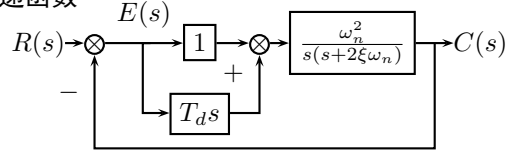
得:  $\xi = 0.456, \omega_n = 7.68$

$$\begin{aligned} 2\xi\omega_n &= AK + 1 \\ K &= \omega_n^2 \end{aligned}$$

得:  $A = 0.102, K = 58.9$

## 4.2 比例 -微分控制

传递函数



比较原系统与 PD 控制系统的稳态性能与动态性能  
解:

$$G(s) = \frac{\omega_n^2(1 + T_d s)}{s^2 + 2\xi\omega_n s}$$

$$\Phi(s) = \frac{\omega_n^2(1 + T_d s)}{s^2 + (2\xi + T_d\omega_n)\omega_n s + \omega_n^2}$$

稳态误差

系统稳定, 为 I 型系统.

$$e_{ss} = \frac{1}{K_v}$$

$$K_v = \lim_{s \rightarrow 0} sG(s)$$

$$= \lim_{s \rightarrow 0} \frac{\omega_n^2(1 + T_d s)}{s + 2\xi\omega_n}$$

$$= \frac{\omega_n}{2\xi}$$

$e_{ss}$  与  $T_d$  无关.

动态性能分析

考虑如下三个系统:

$$\Phi_1(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\Phi_2(s) = \frac{\omega_n^2(1 + T_d s)}{s^2 + (2\xi + T_d\omega_n)\omega_n s + \omega_n^2}$$

$$\Phi_3(s) = \frac{\omega_n^2}{s^2 + (2\xi + T_d\omega_n)\omega_n s + \omega_n^2}$$

- 与系统 1 相比, 系统 3 的阻尼比较大, 两者的自然频率相同.

- 因为:

$$\begin{aligned}\Phi_2(s) &= (1 + T_d s) \Phi_3(s) \\ c_2(t) &= c_3(t) + T_d \frac{dc_3(t)}{dt}\end{aligned}$$

由于存在微分作用, 因此对高频噪声有放大作用.

动态性能计算: 阶跃响应

$$\begin{aligned}\Phi &= \frac{\omega_n^2}{z} \left( \frac{s+z}{s^2 + 2\xi_d \omega_n s + \omega_n^2} \right) \quad (\xi_d = \xi + \frac{\omega_n}{2z}, z = \frac{1}{T_d}) \\ C(s) &= \frac{\omega_n^2}{s(s^2 + 2\xi_d \omega_n s + \omega_n^2)} + \frac{1}{z} \frac{s\omega_n^2}{s(s^2 + 2\xi_d \omega_n s + \omega_n^2)} \\ h(t) &= 1 + r e^{-\xi_d \omega_n t} \sin(\omega_n \sqrt{1 - \xi_d^2} t + \psi) \\ r &= \frac{\sqrt{z^2 - 2\xi_d \omega_n z + \omega_n^2}}{z \sqrt{1 - \xi_d^2}} \\ \psi &= -\pi + \arctan \frac{\omega_n \sqrt{1 - \xi_d^2}}{z - \xi_d \omega_n} + \arctan \frac{\sqrt{1 - \xi_d^2}}{\xi_d}\end{aligned}$$

动态性能计算: 峰值时间

$$\begin{aligned}\frac{dh(t)}{dt} &= 0 \\ \tan(\omega_n \sqrt{1 - \xi_d^2} t_p + \psi) &= \frac{\sqrt{1 - \xi_d^2}}{\xi_d} \\ t_p &= \frac{\beta_d - \psi}{\omega_n \sqrt{1 - \xi_d^2}} \quad \left( \beta_d = \arctan \frac{\sqrt{1 - \xi_d^2}}{\xi_d} \right)\end{aligned}$$

动态性能计算: 超调量

$$\begin{aligned}h(t_p) &= 1 + r e^{-\xi_d \omega_n t_p} \sin(\omega_n \sqrt{1 - \xi_d^2} t_p + \psi) \\ &= 1 + r e^{-\xi_d \omega_n t_p} \sin(\beta_d) \\ \sigma\% &= r \sqrt{1 - \xi_d^2} e^{-\xi_d \omega_n t_p} \times 100\%\end{aligned}$$

动态性能计算：调节时间

$$\begin{aligned}\Delta &= |h(t) - h(\infty)| \\ &= |re^{-\xi_d \omega_n t} \sin(\omega_n \sqrt{1 - \xi_d^2} t + \psi)| \\ &\leq re^{-\xi_d \omega_n t} \\ t_s &= \frac{3 + \ln r}{\xi_d \omega_n} \quad (\Delta = 0.05)\end{aligned}$$

比例 -微分控制示例

设单位反馈系统开环传递函数：  $G(s) = \frac{K(T_d s + 1)}{s(1.67s + 1)}$  若要求单位斜坡函数输入时  $e_{ss} \leq 0.2, \xi_d = 0.5$  求  $K, T_d$  的值，并估算系统在阶跃函数作用下的动态性能。

解：

- 求  $K$  :

$$\begin{aligned}e_{ss} &= \frac{1}{K} \leq 0.2 \\ K &= 5\end{aligned}$$

- $T_d = 0$

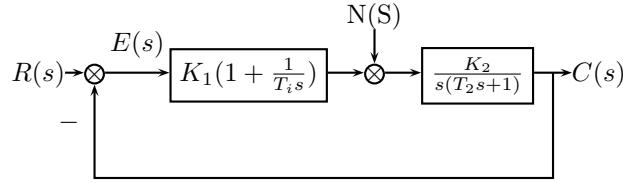
$$\begin{aligned}s^2 + 0.6s + 3 &= 0 \\ \xi &= 0.173 \\ \omega_n &= 1.732 \\ t_p &= 1.84s \\ \sigma\% &= 57.6\% \\ t_s &= 11.7s\end{aligned}$$

- $T_d \neq 0$

$$\begin{aligned}\xi_d &= 0.5 \\ T_d &= \frac{2(\xi_d - \xi)}{\omega_n} \\ t_p &= 1.63s \\ \sigma\% &= 22\% \\ t_s &= 3.49s\end{aligned}$$

### 4.3 比例 -积分控制

比例 -积分控制分析



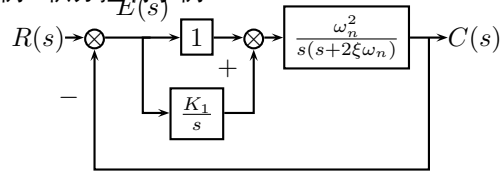
$$E_n(s) = -\frac{K_2 T_i s}{T_i T_2 s^3 + T_i s^2 + K_1 K_2 T_i s + K_1 K_2} N(s)$$

系统稳定时:

$$e_{ssn} = \lim_{s \rightarrow 0} s E_n(s) = 0 \quad (N(s) = \frac{n_0}{s})$$

$$e_{ssn} = \lim_{s \rightarrow 0} s E_n(s) = -\frac{n_1 T_i}{K_1} \quad (N(s) = \frac{n_1}{s^2})$$

比例 -积分控制示例



已知参数  $\xi = 0.2, \omega_n = 86.6$  求:

- 使闭环系统稳定的  $K_1$  范围
- 使闭环系统极点实部全部小于 -1 的  $K_1$  范围

解:

$$\Phi = \frac{\omega_n(s + K_1)}{s^3 + 2\xi\omega_n s^2 + \omega_n^2 s + K_1\omega_n^2}$$

$$D(s) = s^3 + 2\xi\omega_n s^2 + \omega_n^2 s + K_1\omega_n^2 = 0$$

$$D(s) = s^3 + 34.6s^2 + 7500s + 7500K_1 = 0$$

比例 -积分控制示例 (稳定性)

Routh 表:

$$\begin{array}{r|rr} s^3 & 1 & 7500 \\ s^2 & 34.6 & 7500K_1 \\ s^1 & \frac{34.6 \times 7500 - 7500K_1}{34.6} & 0 \\ s^0 & 7500K_1 & \end{array}$$

得:

$$0 < K_1 < 34.6$$

比例 -积分控制示例 (相对稳定性)

设:  $s = s_1 - 1$

$$\begin{aligned} (s_1 - 1)^3 + 34.6(s_1 - 1)^2 + 7500(s_1 - 1) + 7500K_1 &= 0 \\ s_1^3 + 31.6s_1^2 + 7433.8s_1 + (7500K_1 - 7466.4) &= 0 \end{aligned}$$

Routh 表:

$$\begin{array}{r|rr} s^3 & 1 & 7433.8 \\ s^2 & 31.6 & 7500K_1 - 7466.4 \\ s^1 & \frac{31.6 \times 7433.8 - (7500K_1 - 7466.4)}{31.6} & 0 \\ s^0 & 7500K_1 - 7466.4 & \end{array}$$

得:

$$1 < K_1 < 32.3$$