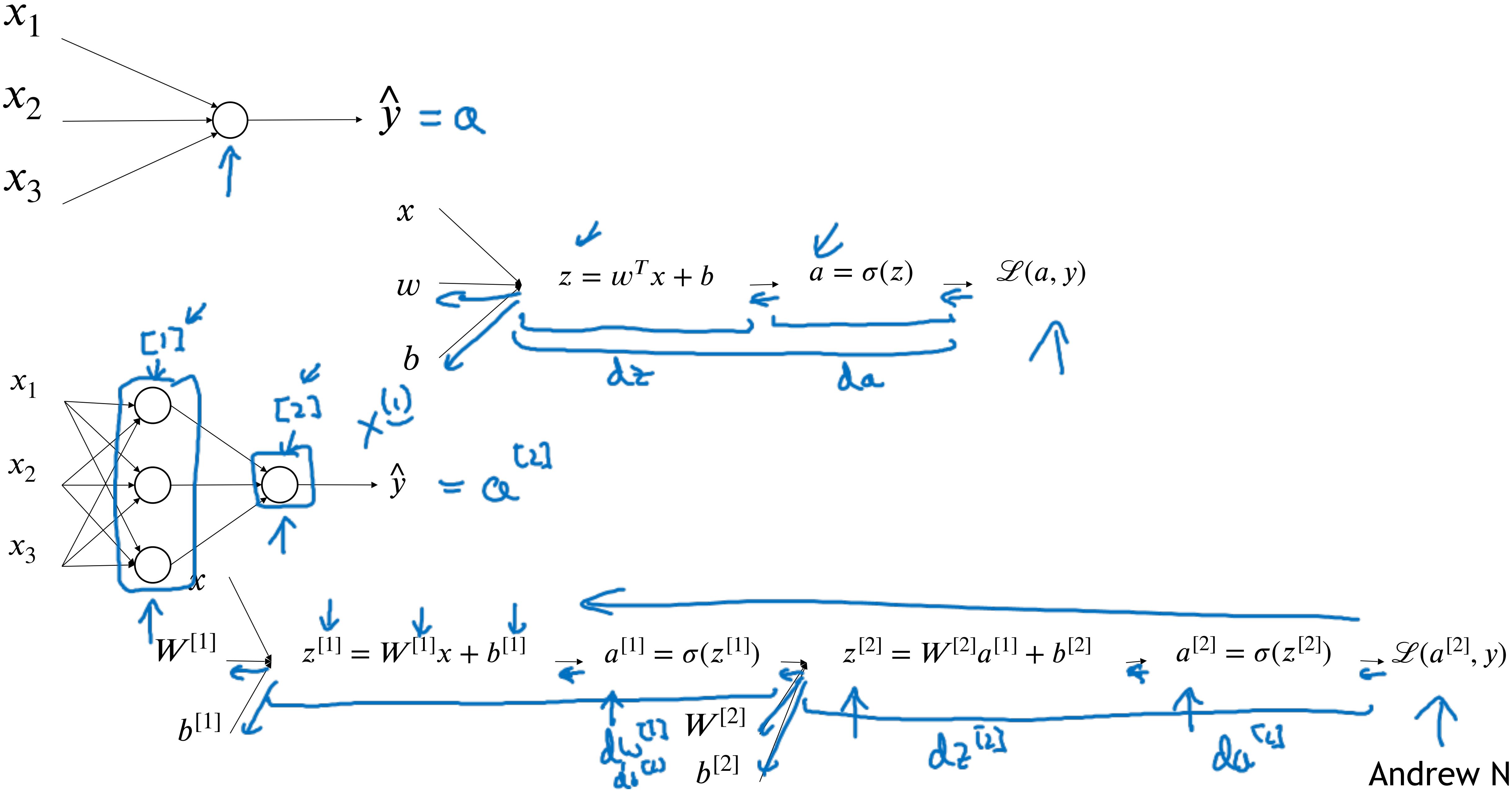
One hidden layer Neural Network

Neural Networks Overview

deeplearning.ai

What is a Neural Network?



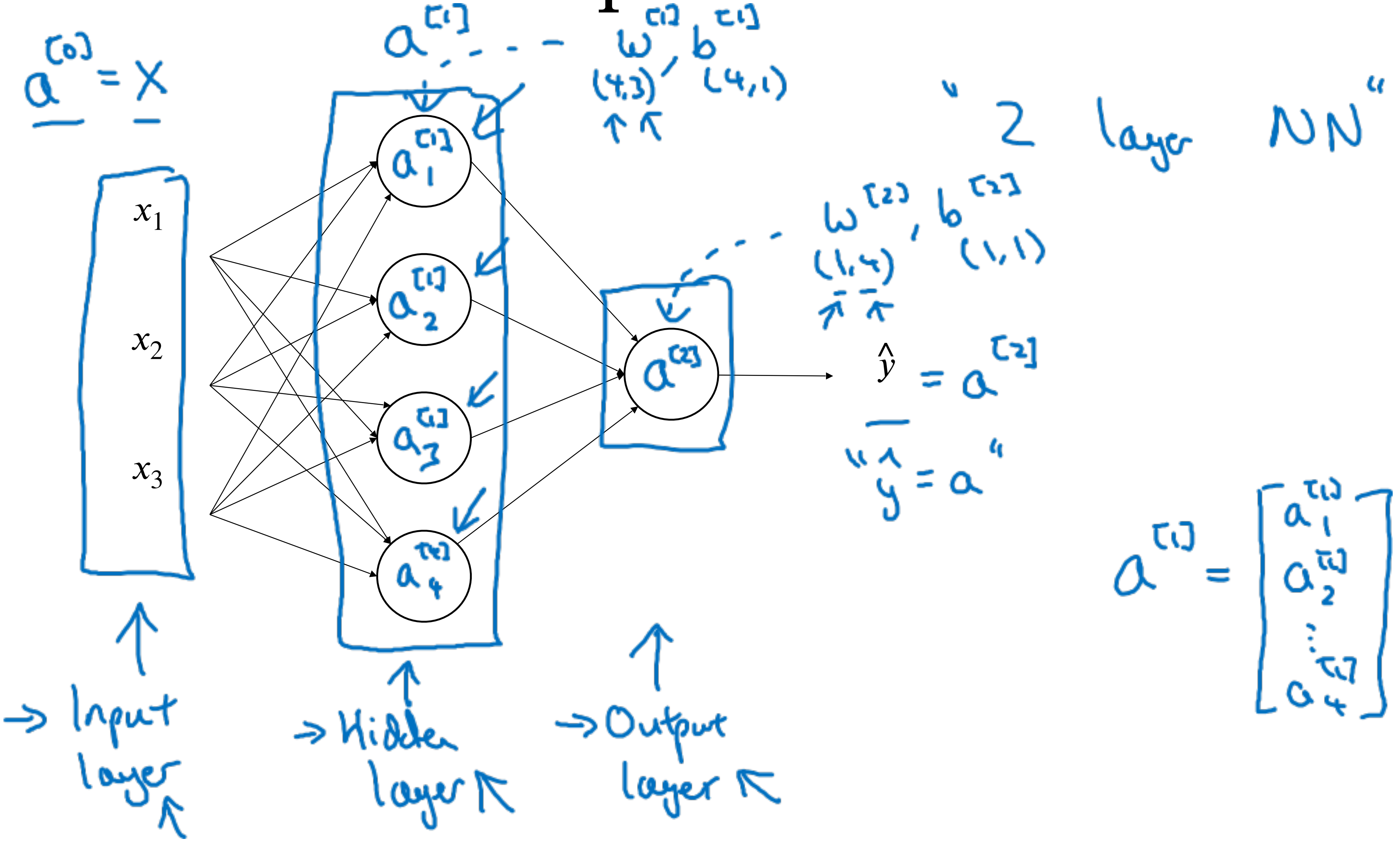
Andrew Ng

deeplearning.ai

One hidden layer Neural Network

Neural Network Representation

Neural Network Representation



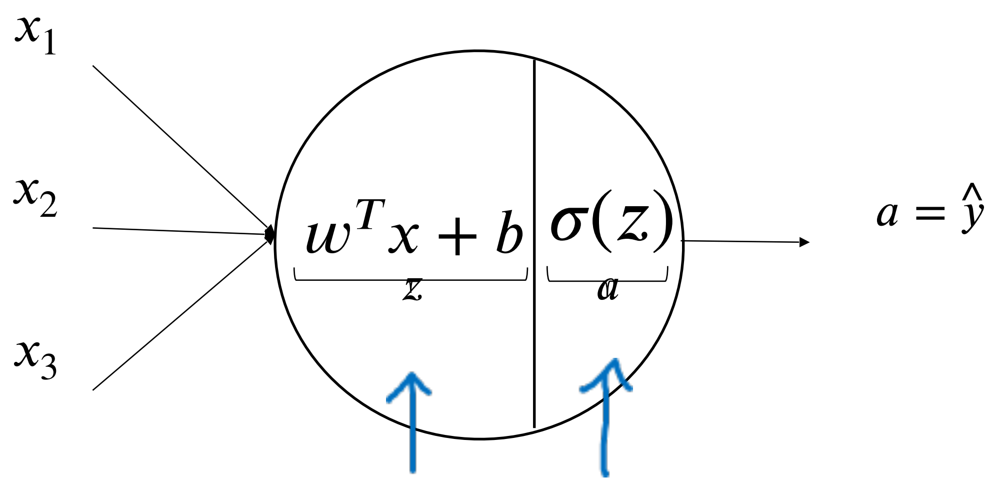
Andrew Ng

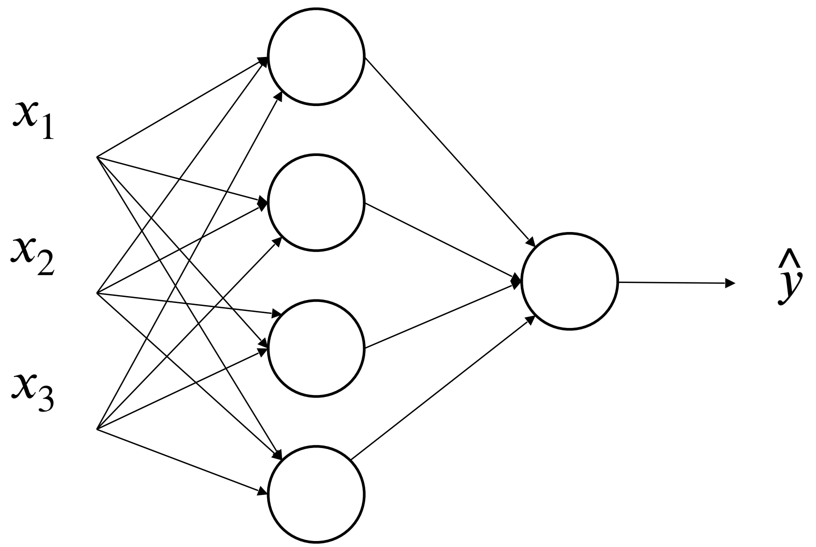
One hidden layer Neural Network

Computing a Neural Network's Output

deeplearning.ai

Neural Network Representation

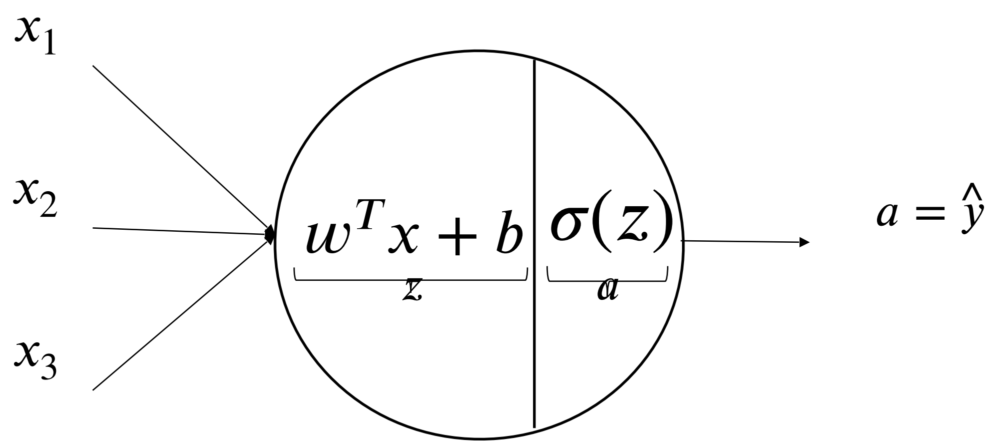




$$z = w^T x + b$$

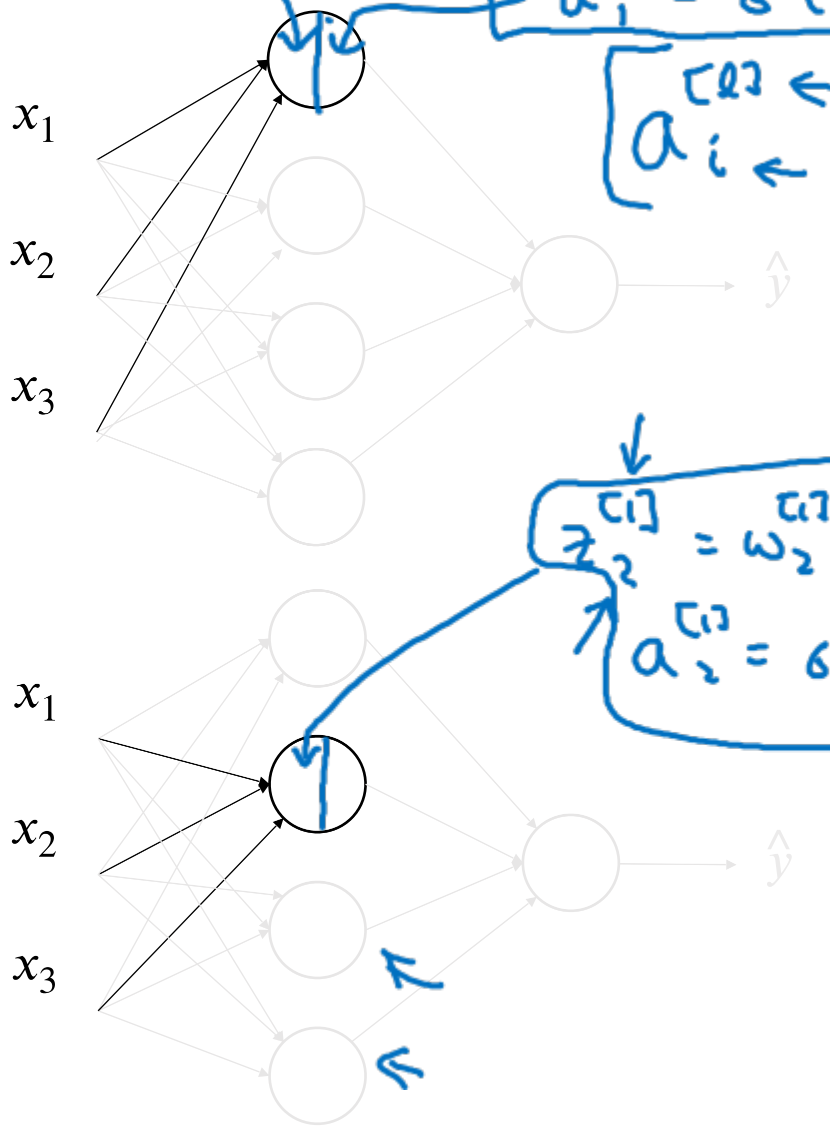
$$a = \sigma(z)$$

Neural Network Representation



$$z = w^T x + b$$

$$a = \sigma(z)$$



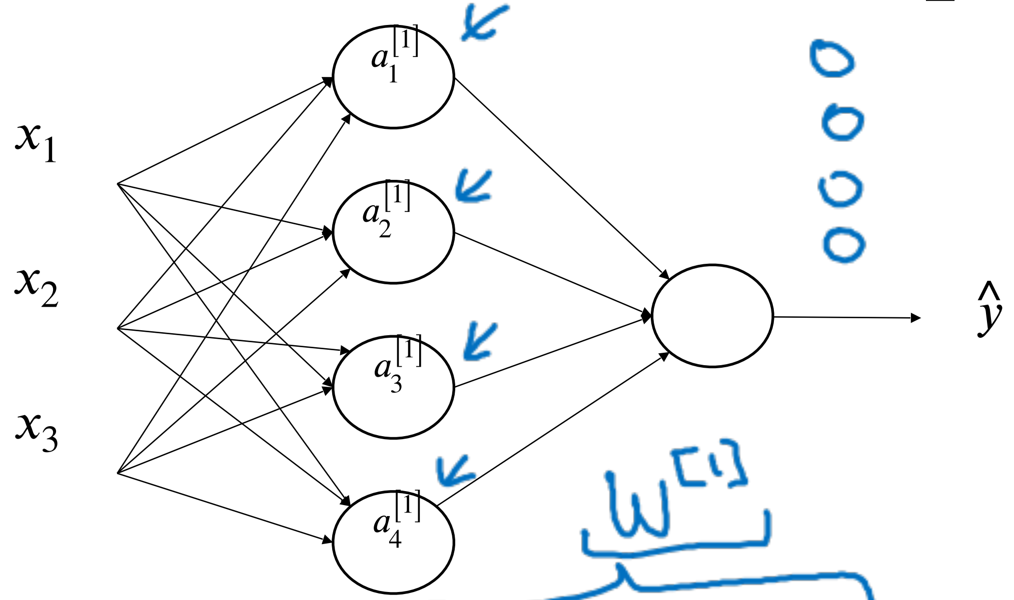
$$\boxed{\begin{aligned} z_1^{[1]} &= w_1^{[1]T} x + b_1^{[1]} \\ a_1^{[1]} &= \sigma(z_1^{[1]}) \end{aligned}}$$

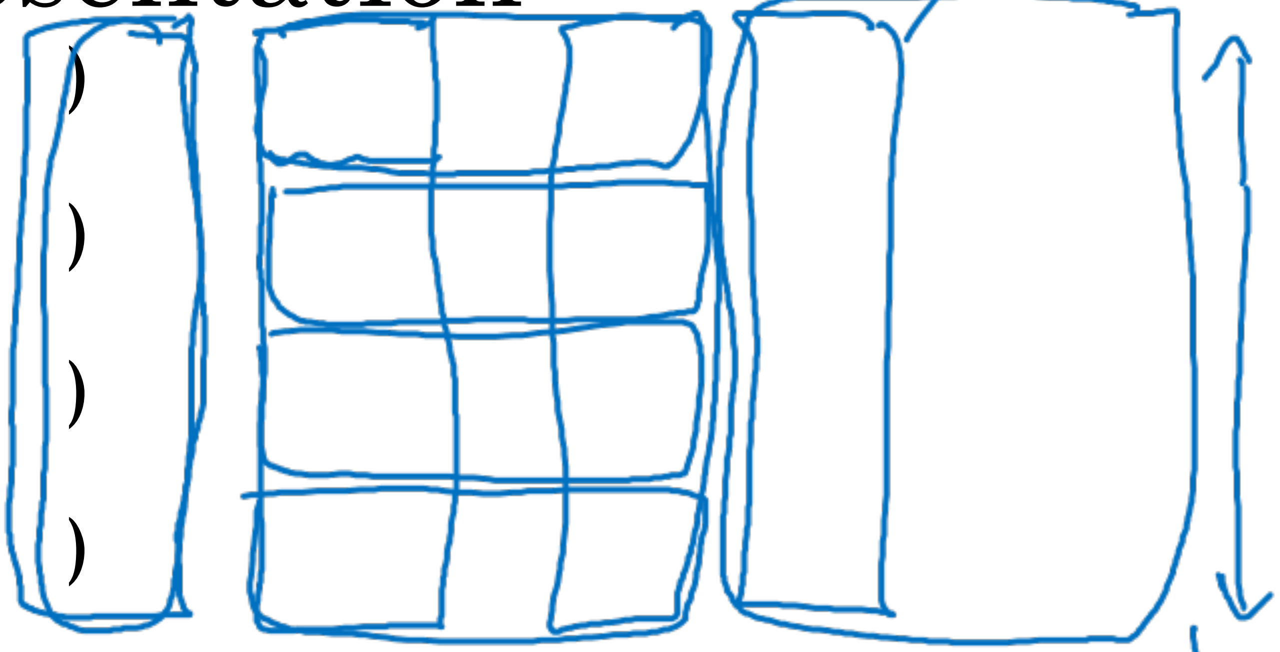
$a_i^{[l]}$ ← layer, i ← node in layer.

$$\boxed{\begin{aligned} z_2^{[1]} &= w_2^{[1]T} x + b_2^{[1]} \\ a_2^{[1]} &= \sigma(z_2^{[1]}) \end{aligned}}$$

Neural Network Representation

$(w_1^{[1]})^T x$ $\quad a^{[1]}$

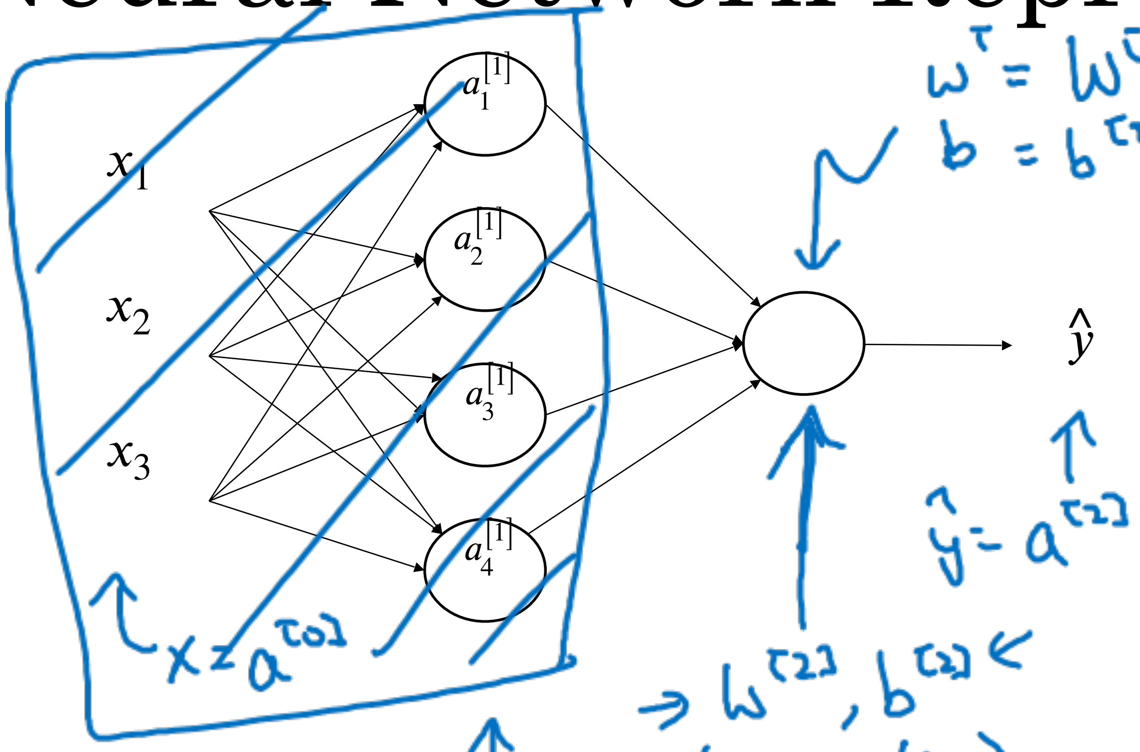




$$z^{[1]} = \overbrace{\begin{bmatrix} — w_1^{[1]T} — \\ — w_2^{[1]T} — \\ — w_3^{[1]T} — \\ — w_4^{[1]T} — \end{bmatrix}}^{W^{[1]}}_{(4,3)} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \\ b_4^{[1]} \end{bmatrix}}_{b^{[1]} \; (4,1)} = \begin{bmatrix} w_1^{[1]T} x + b_1^{[1]} \\ w_2^{[1]T} x + b_2^{[1]} \\ w_3^{[1]T} x + b_3^{[1]} \\ w_4^{[1]T} x + b_4^{[1]} \end{bmatrix} = \begin{bmatrix} z_1^{[1]} \\ z_2^{[1]} \\ z_3^{[1]} \\ z_4^{[1]} \end{bmatrix}$$

$$a^{[1]} = \begin{bmatrix} a_1^{[1]} \\ \vdots \\ a_4^{[1]} \end{bmatrix} = \sigma(z^{[1]})$$

Neural Network Representation learning



$w^T = W^{[2]}$

$b = b^{[2]}$

$\hat{y} = a^{[2]}$

$x = a^{[0]}$

$\underset{(1,4)}{W^{[2]}}, \underset{(1,1)}{b^{[2]}}$

$z = w^T x + b$

$\hat{y} = a = \sigma(z)$

Given input x:

$$\underset{(4,1)}{z^{[1]}} = \underset{(4,3)}{W^{[1]}} \underset{(3,1)}{\overset{a^{[0]}}{x}} + \underset{(4,1)}{b^{[1]}}$$

$$\underset{(4,1)}{a^{[1]}} = \sigma(\underset{(4,1)}{z^{[1]}})$$

$$\underset{(1,1)}{z^{[2]}} = \underset{(1,4)}{W^{[2]}} \underset{(4,1)}{a^{[1]}} + \underset{(1,1)}{b^{[2]}}$$

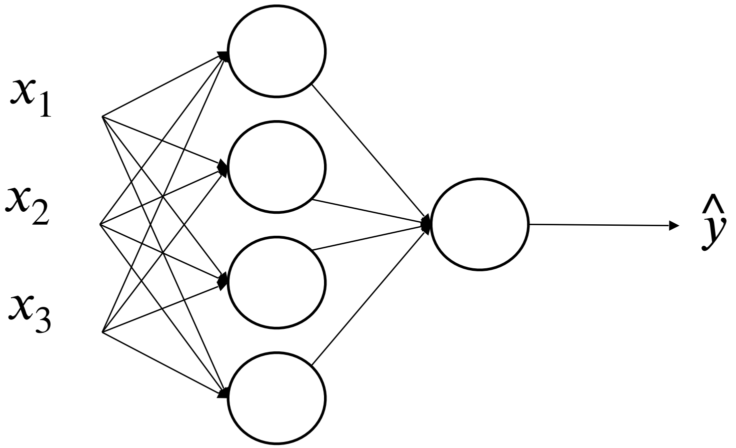
$$\underset{(1,1)}{a^{[2]}} = \sigma(\underset{(1,1)}{z^{[2]}})$$

One hidden layer Neural Network

deeplearning.ai

Vectorizing across multiple examples

Vectorizing across multiple examples



$$z^{[1]} = W^{[1]}x + b^{[1]}$$
$$a^{[1]} = \sigma(z^{[1]})$$
$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$
$$a^{[2]} = \sigma(z^{[2]})$$

$x \longrightarrow a^{[2]} = \hat{y}$

$x^{(1)} \longrightarrow a^{[2](1)} = \hat{y}^{(1)}$

$x^{(2)} \longrightarrow a^{[2](2)} \quad \hat{y}^{(2)}$

$\vdots$

$x^{(m)} \longrightarrow a^{[2](m)} \quad \hat{y}^{(m)}$

$a^{[2](i)}$ — example i, layer 2

for $i = 1$ to m,

$z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}$

$a^{[1](i)} = \sigma(z^{[1](i)})$

$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$

$a^{[2](i)} = \sigma(z^{[2](i)})$

Andrew Ng

Vectorizing across multiple examples

```
for i = 1 to m:
```

$$\boxed{z^{[1](i)}} = W^{[1]}x^{(i)} + b^{[1]}$$

$$\boxed{a^{[1](i)}} = \sigma(z^{[1](i)})$$

$$\boxed{z^{[2](i)}} = W^{[2]}a^{[1](i)} + b^{[2]}$$

$$\boxed{a^{[2](i)}} = \sigma(z^{[2](i)})$$

$$\boxed{\begin{aligned} Z^{[1]} &= W^{[1]}X + b^{[1]} \\ A^{[1]} &= \sigma(Z^{[1]}) \\ Z^{[2]} &= W^{[2]}A^{[1]} + b^{[2]} \\ A^{[2]} &= \sigma(Z^{[2]}) \end{aligned}}$$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix} \quad (n_x, m)$$

training examples

hidden units.

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

hidden units

Andrew Ng

deeplearning.ai

One hidden layer Neural Network

Explanation for vectorized implementation

Justification for vectorized implementation

$z^{[1](1)} = W^{[1]} x^{(1)} + \cancel{b^{[1]}}$, $\quad z^{[1](2)} = W^{[1]} x^{(2)} + \cancel{b^{[1]}}$, $\quad z^{[1](3)} = W^{[1]} x^{(3)} + \cancel{b^{[1]}}$

$0 \qquad 0 \qquad 0$

$$W^{[1]} = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} \qquad W^{[1]} x^{(1)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad W^{[1]} x^{(2)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad W^{[1]} x^{(3)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

$$W^{[1]} \underbrace{\begin{bmatrix} | & | & | & \\ x^{(1)} & x^{(2)} & x^{(3)} & \cdots \\ | & | & | & \end{bmatrix}}_{X} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} = \begin{bmatrix} | & | & | & \\ z^{[1](1)} & z^{[1](2)} & z^{[1](3)} & \cdots \\ | & | & | & \end{bmatrix} = Z^{[1]}$$

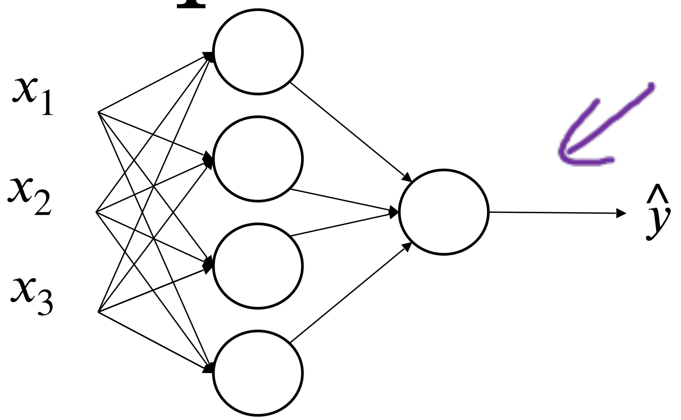
$+b^{[1]} \qquad +b^{[1]} \qquad +b^{[1]}$

$Z^{[1]} = W^{[1]} X + b^{[1]}$

$W^{[1]} x^{(i)} = z^{[1](i)}$

Andrew Ng

Recap of vectorizing across multiple examples



for i = 1 to m

$\boxed{z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}}$

$a^{[1](i)} = \sigma(z^{[1](i)})$

$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$

$a^{[2](i)} = \sigma(z^{[2](i)})$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$$\begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \dots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$A^{[0]}$ $\quad X = a^{[0]}$ $\quad x^{(i)} = a^{[0](i)}$

$Z^{[1]} = W^{[1]}X + b^{[1]} \leftarrow W^{[1]}A^{[0]} + b^{[1]}$

$A^{[1]} = \sigma(Z^{[1]})$

$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$

$A^{[2]} = \sigma(Z^{[2]})$

Andrew Ng

One hidden layer Neural Network

Activation functions

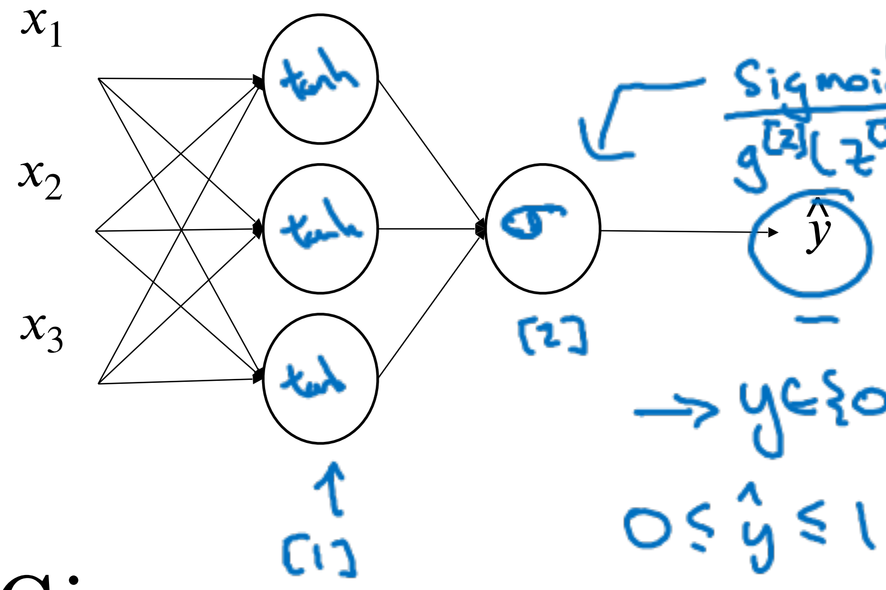
deeplearning.ai

Activation functions

$g^{[1]}(z^{[1]}) = \tanh(z^{[1]})$

sigmoid

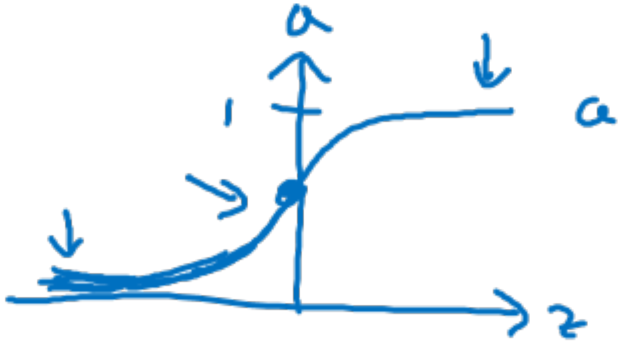
$g^{[2]}(z^{[2]}) = \sigma(z^{[2]})$



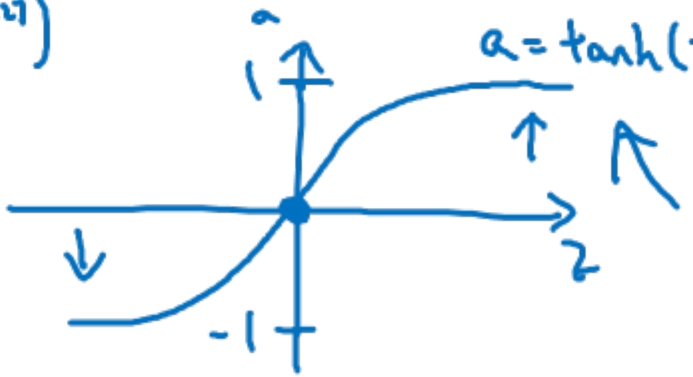
$\rightarrow y \in \{0, 1\}$

$0 \le \hat{y} \le 1$

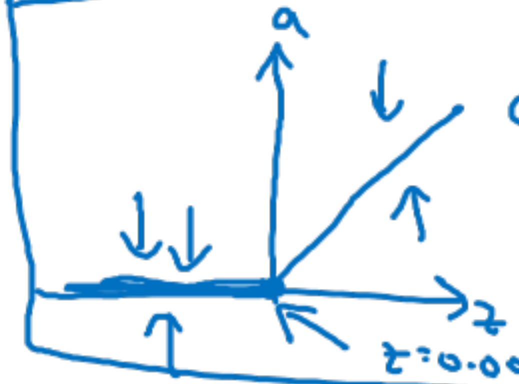
$-1 \qquad 1$



$a = \frac{1}{1+e^{-z}}$



$a = \tanh(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$



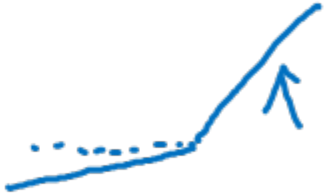
$z > 0$

$a = \max(0, z)$

ReLU

Rectified Linear Unit

$z = 0.0000000000$



Leaky ReLU

Given x:

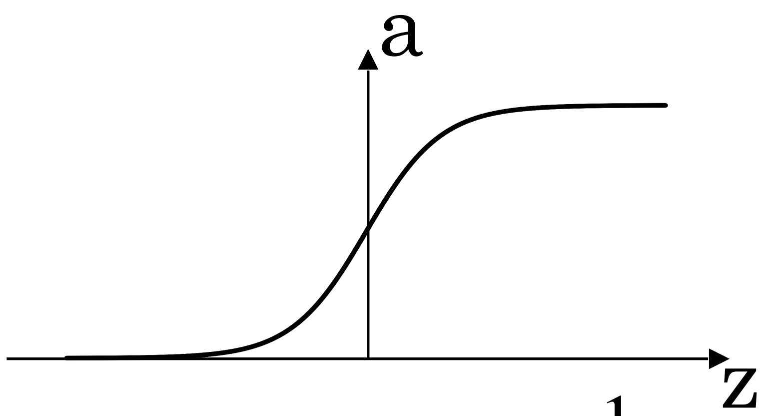
$$z^{[1]} = W^{[1]}x + b^{[1]}$$

$$\rightarrow \quad a^{[1]} = \cancel{\sigma(z^{[1]})}\ g^{[1]}(z^{[1]})$$

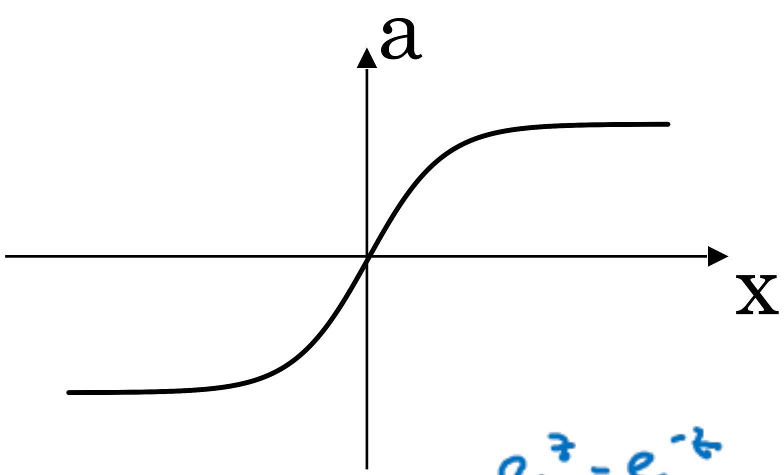
$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$\rightarrow \quad a^{[2]} = \cancel{\sigma(z^{[2]})}\ g^{[2]}(z^{[2]})$$

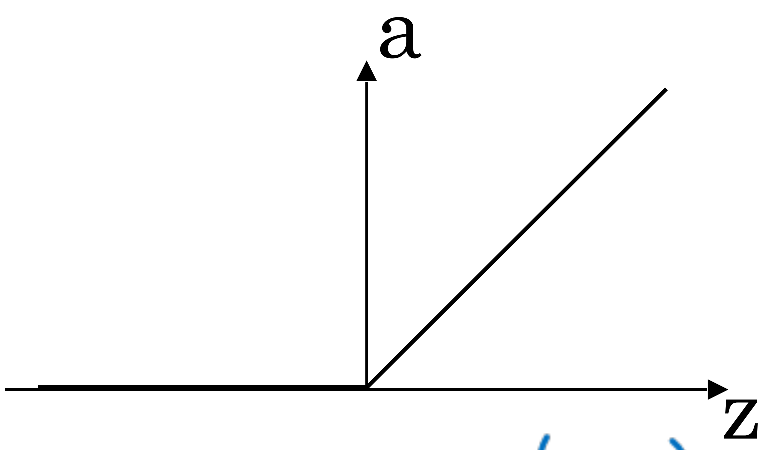
Pros and cons of activation functions



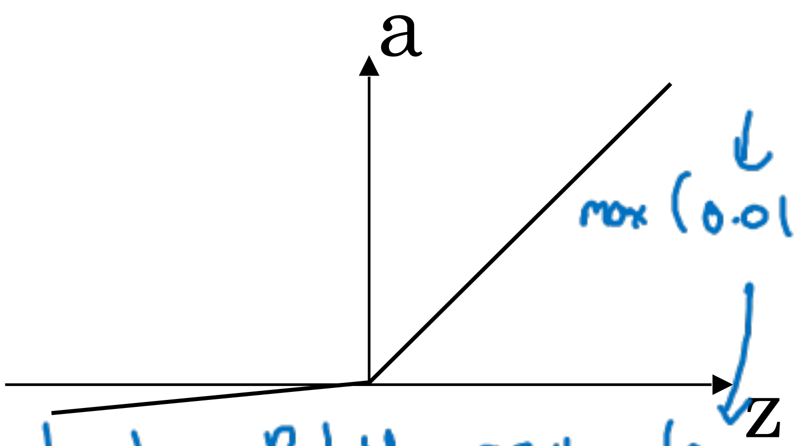
sigmoid: $a = \dfrac{1}{1+e^{-z}}$



tanh: $a = \dfrac{e^z - e^{-z}}{e^z + e^{-z}}$



ReLU $a = \max(0, z)$



$\max(0.01z, z)$

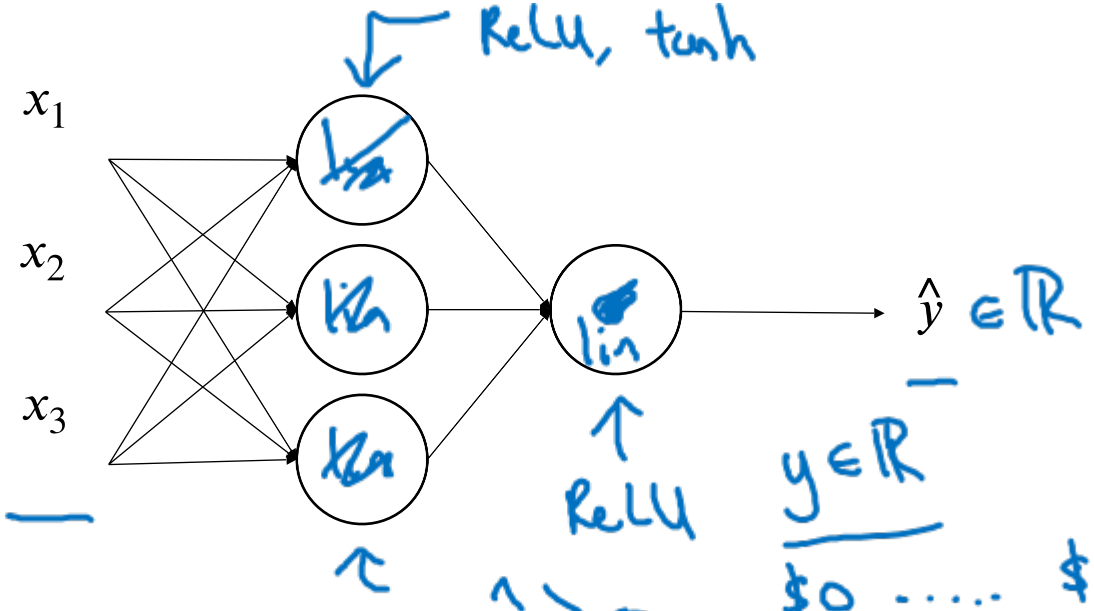
Leaky ReLU $a = \max(0.01z, z)$

deeplearning.ai

One hidden layer Neural Network

Why do you need non-linear activation functions?

Activation function



$y \in \mathbb{R}$

\$0 ..... \$1,000,000

$\hat{y} \geq 0$.

Given x:

$$\rightarrow z^{[1]} = W^{[1]}x + b^{[1]}$$

$$\rightarrow a^{[1]} = \cancel{g^{[1]}(z^{[1]})}\ z^{[1]}$$

$$\rightarrow z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$\rightarrow a^{[2]} = \cancel{g^{[2]}(z^{[2]})}\ z^{[2]}$$

$g(z) = z$

"linear activation function"

$$\underbrace{a^{[1]} = z^{[1]} = W^{[1]}x + b^{[1]}}$$

$$a^{[2]} = z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = W^{[2]}(\underbrace{W^{[1]}x + b^{[1]}}_{a^{[1]}}) + b^{[2]}$$

$$= \underbrace{(W^{[2]}W^{[1]})}_{W'}x + \underbrace{(W^{[2]}b^{[1]} + b^{[2]})}_{b'}$$

$$= W'x + b'$$

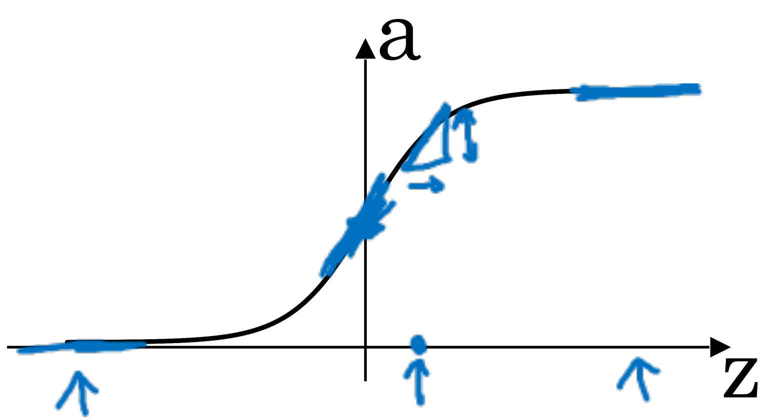
$g(z) = z$

deeplearning.ai

One hidden layer Neural Network

Derivatives of activation functions

Sigmoid activation function



$$g(z) = \frac{1}{1+e^{-z}}$$

$$a = g(z) = \frac{1}{1+e^{-z}}$$

$$\boxed{g'(z)} = \boxed{\frac{d}{dz} g(z)} = \text{slope of } g(x) \text{ at } z$$

$$= \frac{1}{1+e^{-z}}\left(1 - \frac{1}{1+e^{-z}}\right)$$

$$= \underbrace{g(z)\,(1-g(z))}$$

$$= \boxed{a\,(1-a)}$$

$$g'(z) = a(1-a)$$

$z = 10 \quad g(z) \approx 1$

$$\frac{d}{dz} g(z) \approx 1\,(1-1) \approx 0$$

$z = -10 \quad g(z) \approx 0$

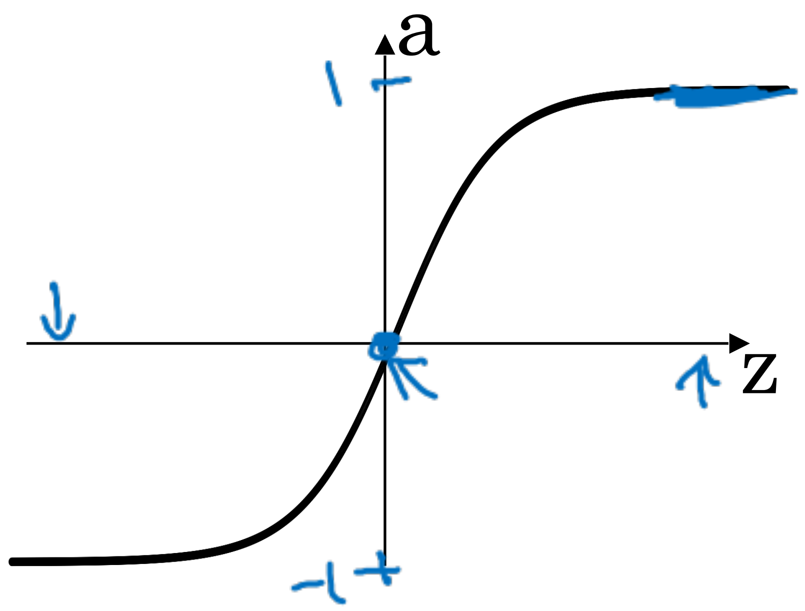
$$\frac{d}{dz} g(z) \approx 0 \cdot (1-0) \approx 0$$

$z = 0 \quad g(z) = \frac{1}{2}$

$$\frac{d}{dz} g(z) = \frac{1}{2}\left(1-\frac{1}{2}\right) = \frac{1}{4}$$

Andrew Ng

Tanh activation function



$g(z) = \tanh(z)$

$$= \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$$

$$g'(z) = \frac{d}{dz} g(z) = \text{slope of } g(z) \text{ at } z$$

$$= 1 - (\tanh(z))^2 \leftarrow$$

$a = g(z)$, $\quad g'(z) = 1 - a^2$

$z = 10 \quad \tanh(z) \approx 1$

$g'(z) \approx 0$

$z = -10 \quad \tanh(z) \approx -1$

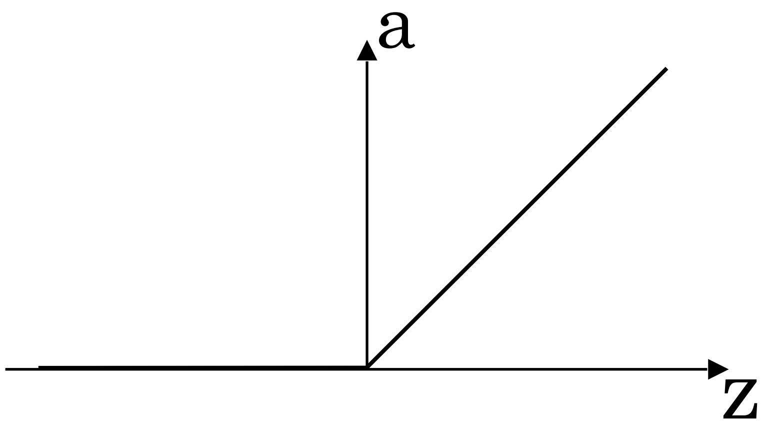
$g'(z) \approx 0$

$z = 0 \quad \tanh(z) = 0$

$g'(z) = 1$

Andrew Ng

ReLU and Leaky ReLU

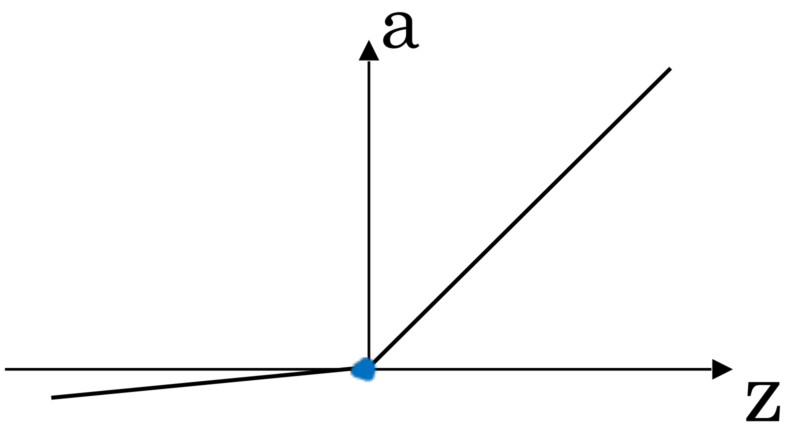


ReLU

$$g(z) = \max(0, z)$$

$$g'(z) = \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \\ \cancel{\text{undef}} & \cancel{\text{if } z = 0} \end{cases}$$

$$z = 0.00000\ldots0$$



Leaky ReLU

$$g(z) = \max(0.01z, z)$$

$$g'(z) = \begin{cases} 0.01 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$$

Andrew Ng

deeplearning.ai

One hidden layer Neural Network

Gradient descent for neural networks

Gradient descent for neural networks

Parameters: $\underset{(n^{[1]}, n^{[0]})}{W^{[1]}}, \underset{(n^{[1]}, 1)}{b^{[1]}}, \underset{(n^{[2]}, n^{[1]})}{W^{[2]}}, \underset{(n^{[2]}, 1)}{b^{[2]}}$ $\qquad n_x = n^{[0]}, n^{[1]}, \underline{n^{[2]} = 1}$

Cost function: $J(W^{[1]}, b^{[1]}, \underline{W^{[2]}}, \underline{b^{[2]}}) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\hat{y}, y)$ $\quad \hat{y} \leftarrow a^{[2]}$

Gradient descent:

→ Repeat {

→ Compute predicts $(\hat{y}^{(i)}, i = 1 \dots, m)$

$\underline{dW^{[1]}} = \frac{\partial J}{\partial W^{[1]}}, \quad \underline{db^{[1]}} = \frac{\partial J}{\partial b^{[1]}}, \dots$

$W^{[1]} := W^{[1]} - \alpha \, dW^{[1]}$

$b^{[1]} := b^{[1]} - \alpha \, db^{[1]}$

$W^{[2]} := \dots \qquad b^{[2]} := \dots$

}

Andrew Ng

Formulas for computing derivatives

Forward propagation:

$Z^{[1]} = W^{[1]}X + b^{[1]}$

$A^{[1]} = g^{[1]}(Z^{[1]})$

$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$

$A^{[2]} = g^{[2]}(Z^{[2]}) = \sigma(Z^{[2]})$

Back propagation:

$dZ^{[2]} = A^{[2]} - Y$ $\qquad Y = [y^{(1)}\ y^{(2)} \dots, y^{(m)}]$

$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]T}$ $\qquad (n^{[2]},)$

$db^{[2]} = \frac{1}{m}$ np.sum($dZ^{[2]}$, axis=1, keepdims=True) $\qquad (n^{[2]}, 1)$

$dZ^{[1]} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]}, m)} * \underbrace{g^{[1]\prime}(Z^{[1]})}$ $(n^{[1]}, m)$

element-wise product

$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$

$db^{[1]} = \frac{1}{m}$ np.sum($dZ^{[1]}$, axis=1, keepdims=True)

$(n^{[1]}, 1)$ $\qquad (n^{[1]},)$ $\qquad$ reshape

Andrew Ng

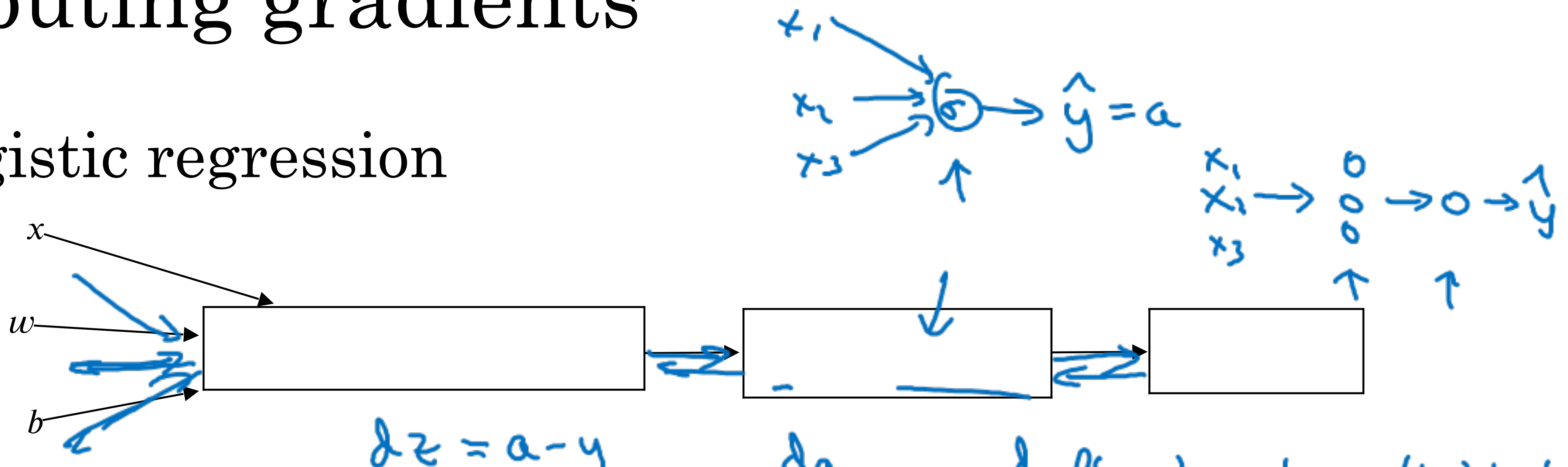
deeplearning.ai

One hidden layer Neural Network

Backpropagation intuition (Optional)

Computing gradients

Logistic regression



$$dz = a - y$$

$$dw = dz \cdot x$$

$$db = dz$$

$$\boxed{dz = da \cdot g'(z)}$$

$$g(z) = \sigma(z)$$

$$\boxed{\frac{\partial \mathcal{L}}{\partial z} = \frac{\partial \mathcal{L}}{\partial a} \cdot \boxed{\frac{da}{dz}}}$$

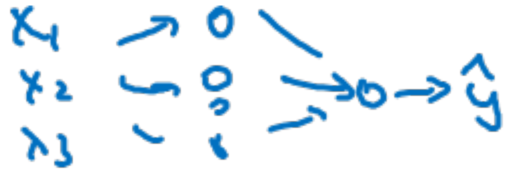
"dz" = "da"

$$\frac{d}{dz} g(z) = g'(z)$$

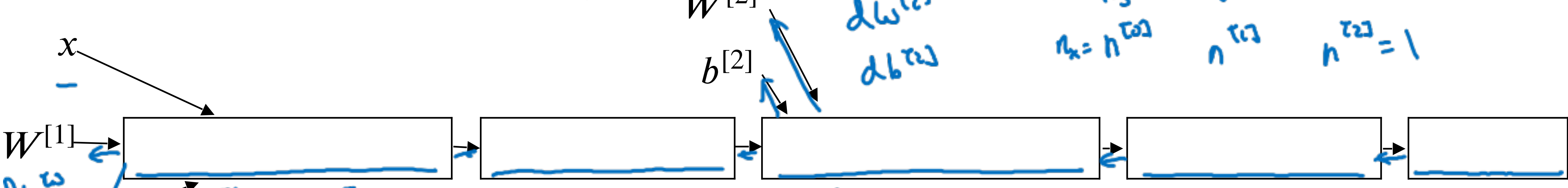
$$da = \frac{d}{da} \mathcal{L}(a,y) = -y\log a - (1-y)\log(1-a)$$

$$= -\frac{y}{a} + \frac{1-y}{1-a}$$

Neural network gradients



$n_x = n^{[0]}$ $\quad n^{[1]}$ $\quad n^{[2]} = 1$



$dz^{[1]} = W^{[2]T} dz^{[2]}$ $\quad da^{[1]} = \ldots$ $\quad dz^{[2]} = a^{[2]} - y$ $\quad da^{[2]}$

$* \, g^{[1]\prime}(z^{[1]})$

element wise product

$dW^{[2]} = dz^{[2]} a^{[1]T}$ $\quad \rightarrow$ "$dw = dz \cdot x$"

$db^{[2]} = dz^{[2]}$

$w^{[1]}: [\quad]$

$W^{[2]}$ $\quad (n^{[2]}, n^{[1]})$

$\rightarrow z^{[2]}, dz^{[2]}$ $\quad (n^{[2]}, 1)$ $\quad - (1,1)$

$\rightarrow z^{[1]}, dz^{[1]}$ $\quad (n^{[1]}, 1)$

$dz^{[1]} = \boxed{W^{[2]T} dz^{[2]}} * \boxed{g^{[1]\prime}(z^{[1]})}$

$(n^{[1]}, 1) = (n^{[1]}, n^{[2]}) \; (n^{[2]}, 1) * (n^{[1]}, 1)$

foo $\quad W^{[1]}$

dfoo $\quad dW^{[1]}$

$dW^{[1]} = dz^{[1]} \cdot x^T$ $\quad a^{[0]T}$

$db^{[1]} = dz^{[1]}$

Andrew Ng

Summary of gradient descent

$$dz^{[2]} = a^{[2]} - y$$

$$dW^{[2]} = dz^{[2]}a^{[1]^T}$$

$$db^{[2]} = dz^{[2]}$$

$$dz^{[1]} = W^{[2]T}dz^{[2]} * g^{[1]\prime}(z^{[1]})$$

$$dW^{[1]} = dz^{[1]}x^T$$

$$db^{[1]} = dz^{[1]}$$

Vectorized Implementation:

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[1]} = g^{[1]}(z^{[1]})$$

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]})$$

Andrew Ng

Summary of gradient descent

$$dz^{[2]} = a^{[2]} - y$$

$$dW^{[2]} = dz^{[2]}a^{[1]^T}$$

$$db^{[2]} = dz^{[2]}$$

$$dz^{[1]} = W^{[2]T}dz^{[2]} * g^{[1]\prime}(z^{[1]})$$

$(n^{[1]}, 1)$

$$dW^{[1]} = dz^{[1]}x^T$$

$$db^{[1]} = dz^{[1]}$$

$$dZ^{[2]} = A^{[2]} - Y$$

$$J(\ldots) = \frac{1}{m}\sum_{i=1}^{n} \mathcal{L}(\hat{y}, y)$$

$$dW^{[2]} = \frac{1}{m}dZ^{[2]}A^{[1]^T}$$

$$db^{[2]} = \frac{1}{m}np.sum(dZ^{[2]}, axis = 1, keepdims = True)$$

elementwise product

$$\underbrace{dZ^{[1]}}_{(n^{[1]},m)} = \underbrace{W^{[2]T}dZ^{[2]}}_{(n^{[1]},m)} * \underbrace{g^{[1]\prime}(Z^{[1]})}_{(n^{[1]},m)}$$

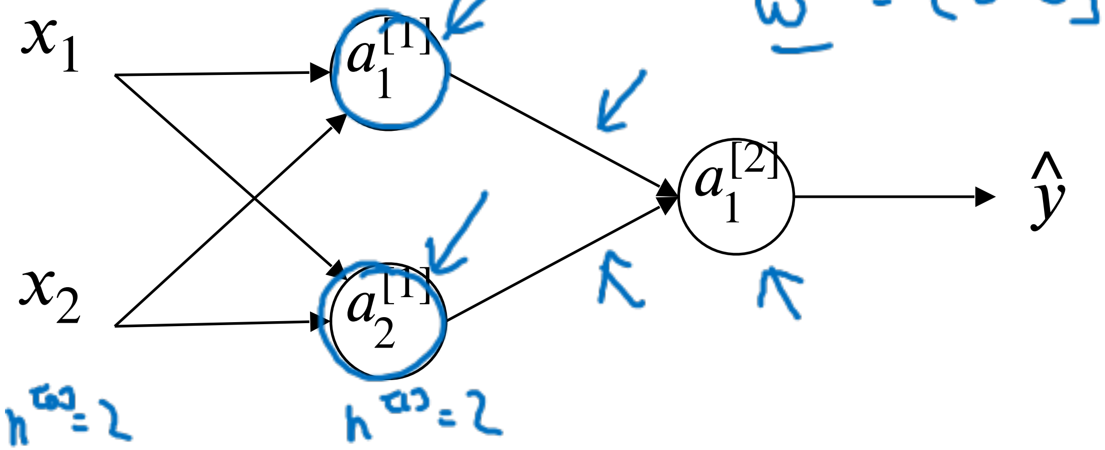
$$dW^{[1]} = \frac{1}{m}dZ^{[1]}X^T$$

deeplearning.ai

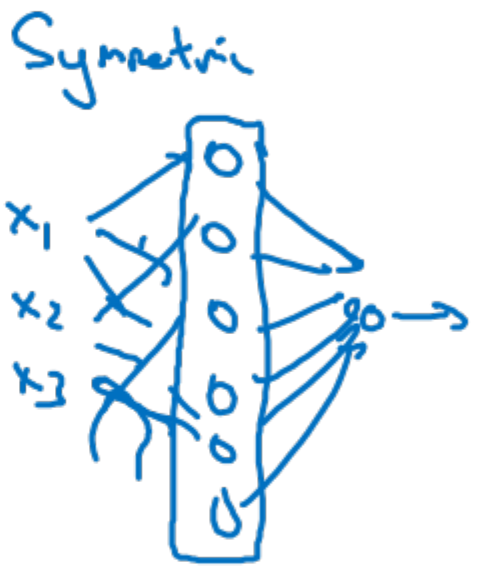
One hidden layer Neural Network

Random Initialization

What happens if you initialize weights to zero?



$W^{[2]} = \begin{bmatrix} 0 & 0 \end{bmatrix}$



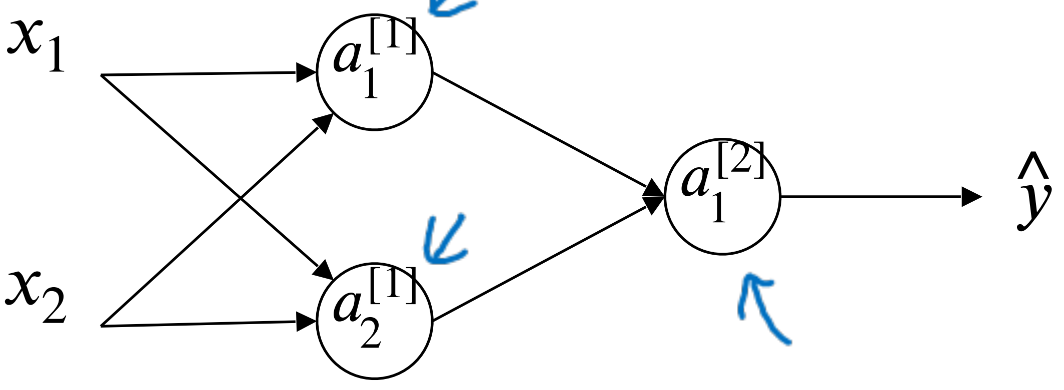
$W^{[1]} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $b^{[1]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$a_1^{[1]} = a_2^{[1]}$ $dz_1^{[1]} = dz_2^{[1]}$

$dW = \begin{bmatrix} u & v \\ u & v \end{bmatrix}$ $W^{[1]} = W^{[1]} - \alpha dW$

$W^{[1]} = \begin{bmatrix} \cdots \\ \cdots \end{bmatrix}$

Random initialization



$\rightarrow W^{[1]} =$ np.random.randn((2,2)) $* \, 0.01$

100?

$b^{[1]} =$ np.zeros((2,1))

$W^{[2]} =$ np.random.randn((1,2)) $* \, 0.01$

$b^{[2]} = 0$

$\rightarrow z^{[1]} = W^{[1]} x + b^{[1]}$

$a^{[1]} = g^{[1]}(z^{[1]})$