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#### Al.Cash's blog

### Efficient and easy segment trees

By Al.Cash, 16 months ago, 314,

This is my first attempt at writing something useful, so your suggestions are welcome.

Most participants of programming contests are familiar with segment trees to some degree, especially having read this articles <a href="http://codeforces.com/blog/entry/15890">http://e-maxx.ru/algo/segment\_tree</a> (Russian only). If you're not — don't go there yet. I advise to read them after this article for the sake of examples, and to compare implementations and choose the one you like more (will be kinda obvious).

# Segment tree with single element modifications

Let's start with a brief explanation of segment trees. They are used when we have an array, perform some changes and queries on continuous segments. In the first example we'll consider 2 operations:

- 1. modify one element in the array;
- 2. find the sum of elements on some segment. .

## Perfect binary tree

I like to visualize a segment tree in the following way: image link

	1: [0, 16)																		
Г				2: [0	), 8)						3: [8	7: [12, 16)							
Γ	4: [0, 4) 5: [4, 8)								6: [8, 12) 7: [12, 16)										
Γ	8: 9:				1	0:	1	1:	12:		13:		14:		15:				
L	[0, 2)		[2, 4)		[4, 6)		[6, 8)		[8, 10)		[10, 12)		[12, 14)		[14, 16)				
Г	16:	17:	18:	19:	20:	21:	22:	23:	24:	25:	26:	27:	28:	29:	30:	31:			
L	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15			

Notation is  $node\_index$ :  $corresponding\ segment$  (left border included, right excluded). At the bottom row we have our array (0-indexed), the leaves of the tree. For now suppose it's length is a power of 2 (16 in the example), so we get perfect binary tree. When going up the tree we take pairs of nodes with indices (2\*i, 2\*i+1) and combine their values in their parent with index i. This way when we're asked to find a sum on interval [3,11), we need to sum up only values in the nodes 19, 5, 12 and 26 (marked with bold), not all 8 values inside the interval. Let's jump directly to implementation (in C++) to see how it works:

```
const int N = le5;  // limit for array size
int n;  // array size
int t[2 * N];

void build() {  // build the tree
  for (int i = n - 1; i > 0; --i) t[i] = t[i<<1] + t[i<<1|1];
}</pre>
```





	1									
#	User	Rating								
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2	Petr	3481								
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4	dotorya	3050								
5	Endagorion	3031								
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```
0
```

```
void modify(int p, int value) { // set value at position p
  for (t[p += n] = value; p > 1; p >>= 1) t[p>>1] = t[p] + t[p^1];
}
int query(int l, int r) { // sum on interval [l, r)
  int res = 0:
  for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
    if (l&1) res += t[l++];
    if (r\&1) res += t[--r];
  return res;
int main() {
 scanf("%d", &n);
  for (int i = 0; i < n; ++i) scanf("%d", t + n + i);</pre>
 build():
 modify(0, 1);
 printf("%d\n", query(3, 11));
  return 0:
}
```

That's it! Fully operational example. Forget about those cumbersome recursive functions with 5 arguments!

Now let's see why this works, and works very efficient.

- 1. As you could notice from the picture, leaves are stored in continuous nodes with indices starting with n, element with index i corresponds to a node with index i + n. So we can read initial values directly into the tree where they belong.
- 2. Before doing any queries we need to build the tree, which is quite straightforward and takes O(n) time. Since parent always has index less than its children, we just process all the internal nodes in decreasing order. In case you're confused by bit operations, the code in build() is equivalent to t[i] = t[2\*i] + t[2\*i+1].
- 3. Modifying an element is also quite straightforward and takes time proportional to the height of the tree, which is O(log(n)). We only need to update values in the parents of given node. So we just go up the tree knowing that parent of node p is p / 2 or p >> 1, which means the same.  $p^1$  turns 2\*i into 2\*i+1 and vice versa, so it represents the second child of p's parent.
- 4. Finding the sum also works in O(log(n)) time. To better understand it's logic you can go through example with interval [3, 11) and verify that result is composed exactly of values in nodes 19, 26, 12 and 5 (in that order).

General idea is the following. If l, the left interval border, is odd (which is equivalent to  $\lfloor \&1 \rfloor$ ) then l is the right child of its parent. Then our interval includes node l but doesn't include it's parent. So we add  $\lfloor t \lfloor 1 \rfloor$  and move to the right of l's parent by setting l = (l+1)/2. If l is even, it is the left child, and the interval includes its parent as well (unless the right border interferes), so we just move to it by setting l = l/2. Similar argumentation is applied to the right border. We stop once borders meet.

No recursion and no additional computations like finding the middle of the interval are involved, we just go through all the nodes we need, so this is very efficient.

## Arbitrary sized array

For now we talked only about an array with size equal to some power of 2, so the binary tree

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```
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```



was perfect. The next fact may be stunning, so prepare yourself.

#### The code above works for any size n.

Explanation is much more complex than before, so let's focus first on the advantages it gives us.

- 1. Segment tree uses exactly 2\*n memory, not 4\*n like some other implementations offer
- 2. Array elements are stored in continuous manner starting with index n.
- 3. All operations are very efficient and easy to write.

You can skip the next section and just test the code to check that it's correct. But for those interested in some kind of explanation, here's how the tree for n = 13 looks like: image link

1:															
			2: [3	, 11)			3:								
	4: [3, 7) 5: [7, 11)								6: 7: [1, 3)						
8	3:	9:		10:		11:		12:		13:	7	14:	15:		
[3, 5)		[5, 7)		[7, 9)		[9, 11)		[11, 13)		0		1	2		
16:	17:	18:	19:	20:	21:	22:	23:	24:	25:			T			
3	4	5	6	7	8	9	10	11	12						

It's not actually a single tree any more, but a set of perfect binary trees: with root 2 and height 4, root 7 and height 2, root 12 and height 2, root 13 and height 1. Nodes denoted by dashes aren't ever used in *query* operations, so it doesn't matter what's stored there. Leaves seem to appear on different heights, but that can be fixed by cutting the tree before the node 13 and moving its right part to the left. I believe the resulting structure can be shown to be isomorphic to a part of larger perfect binary tree with respect to operations we perform, and this is why we get correct results.

I won't bother with formal proof here, let's just go through the example with interval [0, 7). We have l=13, r=20,  $l\&1 \Rightarrow add t[13]$  and borders change to l=7, r=10. Again  $l\&1 \Rightarrow add t[7]$ , borders change to l=4, r=5, and suddenly nodes are at the same height. Now we have  $r\&1 \Rightarrow add t[4 = --r]$ , borders change to l=2, r=2, so we're finished.

# Modification on interval, single element access

Some people begin to struggle and invent something too complex when the operations are inverted, for example:

- 1. add a value to all elements in some interval;
- 2. compute an element at some position.

But all we need to do in this case is to switch the code in methods *modify* and *query* as follows:

```
void modify(int l, int r, int value) {
  for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
    if (l&1) t[l++] += value;
    if (r&1) t[--r] += value;
  }
}
int query(int p) {
  int res = 0;
  for (p += n; p > 0; p >>= 1) res += t[p];
  return res;
}
```



10

0

If at some point after modifications we need to inspect all the elements in the array, we can push all the modifications to the leaves using the following code. After that we can just traverse elements starting with index n. This way we reduce the complexity from O(nlog(n)) to O(n) similarly to using build instead of n modifications.

```
void push() {
  for (int i = 1; i < n; ++i) {
    t[i<<1] += t[i];
    t[i<<1|1] += t[i];
    t[i] = 0;
}</pre>
```

Note, however, that code above works only in case the order of modifications on a single element doesn't affect the result. Assignment, for example, doesn't satisfy this condition. Refer to section about lazy propagation for more information.

## Non-commutative combiner functions

For now we considered only the simplest combiner function — addition. It is commutative, which means the order of operands doesn't matter, we have a+b=b+a. The same applies to min and max, so we can just change all occurrences of + to one of those functions and be fine. But don't forget to initialize query result to infinity instead of 0.

However, there are cases when the combiner isn't commutative, for example, in the problem 380C - Sereja and Brackets, tutorial available here http://codeforces.com/blog/entry/10363. Fortunately, our implementation can easily support that. We define structure S and combine function for it. In method build we just change + to this function. In modify we need to ensure the correct ordering of children, knowing that left child has even index. When answering the query, we note that nodes corresponding to the left border are processed from left to right, while the right border moves from right to left. We can express it in the code in the following way:

```
void modify(int p, const S& value) {
   for (t[p += n] = value; p >>= 1; ) t[p] = combine(t[p<<1], t[p<<1|1]);
}

S query(int l, int r) {
   S resl, resr;
   for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
      if (l&1) resl = combine(resl, t[l++]);
      if (r&1) resr = combine(t[--r], resr);
   }
   return combine(resl, resr);
}
```

# Lazy propagation

Next we'll describe a technique to perform both range queries and range modifications, which is called lazy propagation. First, we need more variables:

```
int h = sizeof(int) * 8 - __builtin_clz(n);
int d[N];
```

h is a height of the tree, the highest significant bit in n. d[i] is a delayed operation to be propagated to the children of node i when necessary (this should become clearer from the examples). Array size if only N because we don't have to store this information for leaves — they don't have any children. This leads us to a total of 3\*n memory use.



Previously we could say that t[i] is a value corresponding to it's segment. Now it's not entirely true — first we need to apply all the delayed operations on the route from node i to the root of the tree (parents of node i). We assume that t[i] already includes d[i], so that route starts not with i but with its direct parent.

Let's get back to our first example with interval [3, 11), but now we want to modify all the elements inside this interval. In order to do that we modify t[i] and d[i] at the nodes 19, 5, 12 and 26. Later if we're asked for a value for example in node 22, we need to propagate modification from node 5 down the tree. Note that our modifications could affect t[i] values up the tree as well: node 19 affects nodes 9, 4, 2 and 1, node 5 affects 2 and 1. Next fact is critical for the complexity of our operations:

Modification on interval [l, r) affects t[i] values only in the parents of border leaves: l+n and r+n-1 (except the values that compose the interval itself — the ones accessed in *for* loop).

The proof is simple. When processing the left border, the node we modify in our loop is always the right child of its parent. Then all the previous modifications were made in the subtree of the left child of the same parent. Otherwise we would process the parent instead of both its children. This means current direct parent is also a parent of leaf then arguments apply to the right border.

OK, enough words for now, I think it's time to look at concrete examples.

### Increment modifications, queries for maximum

This is probably the simplest case. The code below is far from universal and not the most efficient, but it's a good place to start.

```
void apply(int p, int value) {
  t[p] += value;
  if (p < n) d[p] += value;
}
void build(int p) {
  while (p > 1) p >>= 1, t[p] = max(t[p<<1], t[p<<1|1]) + d[p];
void push(int p) {
  for (int s = h; s > 0; --s) {
    int i = p >> s;
    if (d[i] != 0) {
      apply(i<<1, d[i]);
      apply(i<<1|1, d[i]);
      d[i] = 0;
    }
  }
void inc(int l, int r, int value) {
  l += n, r += n;
  int 10 = 1, r0 = r;
  for (; l < r; l >>= 1, r >>= 1) {
    if (l&1) apply(l++, value);
    if (r&1) apply(--r, value);
  build(l0);
  build(r0 - 1);
```

```
0
```

```
int query(int l, int r) {
    l += n, r += n;
    push(l);
    push(r - 1);
    int res = -2e9;
    for (; l < r; l >>= 1, r >>= 1) {
        if (l&1) res = max(res, t[l++]);
        if (r&1) res = max(t[--r], res);
    }
    return res;
}
```

Let's analyze it one method at a time. The first three are just helper methods user doesn't really need to know about.

- 1. Now that we have 2 variables for every internal node, it's useful to write a method to *apply* changes to both of them.  $p \le n$  checks if p is not a leaf. Important property of our operations is that if we increase all the elements in some interval by one value, maximum will increase by the same value.
- 2. build is designed to update all the parents of a given node.
- 3. push propagates changes from all the parents of a given node down the tree starting from the root. This parents are exactly the prefixes of p in binary notation, that's why we use binary shifts to calculate them.

Now we're ready to look at main methods.

- 1. As explained above, we process increment request using our familiar loop and then updating everything else we need by calling *build*.
- 2. To answer the query, we use the same loop as earlier, but before that we need to push all the changes to the nodes we'll be using. Similarly to build, it's enough to push changes from the parents of border leaves.

It's easy to see that all operations above take O(log(n)) time.

Again, this is the simplest case because of two reasons:

- 1. order of modifications doesn't affect the result;
- 2. when updating a node, we don't need to know the length of interval it represents.

We'll show how to take that into account in the next example.

# Assignment modifications, sum queries

This example is inspired by problem Timus 2042

Again, we'll start with helper functions. Now we have more of them:

```
void calc(int p, int k) {
   if (d[p] == 0) t[p] = t[p<<1] + t[p<<1|1];
   else t[p] = d[p] * k;
}

void apply(int p, int value, int k) {
   t[p] = value * k;
   if (p < n) d[p] = value;
}</pre>
```



These are just simple O(1) functions to calculate value at node p and to apply a change to the node. But there are two thing to explain:

- 1. We suppose there's a value we never use for modification, in our case it's 0. In case there's no such value we would create additional boolean array and refer to it instead of checking d[p] = 0.
- 2. Now we have additional parameter k, which stands for the lenght of the interval corresponding to node p. We will use this name consistently in the code to preserve this meaning. Obviously, it's impossible to calculate the sum without this parameter. We can avoid passing this parameter if we precalculate this value for every node in a separate array or calculate it from the node index on the fly, but I'll show you a way to avoid using extra memory or calculations.

Next we need to update *build* and *push* methods. Note that we have two versions of them: one we introduces earlier that processes the whole tree in O(n), and one from the last example that processes just the parents of one leaf in O(log(n)). We can easily combine that functionality into one method and get even more.

```
void build(int l, int r) {
 int k = 2:
 for (l += n, r += n-1; l > 1; k <<= 1) {
    l >>= 1, r >>= 1;
    for (int i = r; i >= l; --i) calc(i, k);
 }
}
void push(int l, int r) {
 int s = h, k = 1 << (h-1);
 for (l += n, r += n-1; s > 0; --s, k >>= 1)
    for (int i = l >> s; i <= r >> s; ++i) if (d[i] != 0) {
      apply(i<<1, d[i], k);
      apply(i << 1|1, d[i], k);
      d[i] = 0;
    }
}
```

Both this methods work on any interval in O(log(n) + |r - l|) time. If we want to transform some interval in the tree, we can write code like this:

```
push(l, r);
... // do anything we want with elements in interval [l, r)
build(l, r);
```

Let's explain how they work. First, note that we change our interval to closed by doing r += n-1 in order to calculate parents properly. Since we process our tree level by level, is't easy to maintain current interval level, which is always a power of 2. *build* goes bottom to top, so we initialize k to 2 (not to 1, because we don't calculate anything for the leaves but start with their direct parents) and double it on each level. *push* goes top to bottom, so k's initial value depends here on the height of the tree and is divided by 2 on each level.

Main methods don't change much from the last example, but modify has 2 things to notice:

- Because the order of modifications is important, we need to make sure there are no old changes on the paths from the root to all the nodes we're going to update. This is done by calling *push* first as we did in *query*.
- 2. We need to maintain the value of k.

```
void modify(int l, int r, int value) {
  if (value == 0) return;
```

```
0
```

```
push(l, l + 1);
  push(r - 1, r);
  int 10 = 1, r0 = r, k = 1;
  for (l += n, r += n; l < r; l >>= 1, r >>= 1, k <<= 1) {
    if (l&1) apply(l++, value, k);
    if (r&1) apply(--r, value, k);
  build(l0, l0 + 1);
  build(r0 - 1, r0);
int query(int l, int r) {
  push(l, l + 1);
  push(r - 1, r);
  int res = 0;
  for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
    if (l&1) res += t[l++];
    if (r\&1) res += t[--r];
  return res;
}
```

One could notice that we do 3 passed in *modify* over almost the same nodes: 1 down the tree in *push*, then 2 up the tree. We can eliminate the last pass and calculate new values only where it's necessary, but the code gets more complicated:

```
void modify(int l, int r, int value) {
   if (value == 0) return;
   push(l, l + 1);
   push(r - 1, r);
   bool cl = false, cr = false;
   int k = 1;
   for (l += n, r += n; l < r; l >>= 1, r >>= 1, k <<= 1) {
      if (cl) calc(l - 1, k);
      if (cr) calc(r, k);
      if (l&1) apply(l++, value, k), cl = true;
      if (r&1) apply(--r, value, k), cr = true;
   }
   for (--l; r > 0; l >>= 1, r >>= 1, k <<= 1) {
      if (cl) calc(l, k);
      if (cr && (!cl || l != r)) calc(r, k);
   }
}</pre>
```

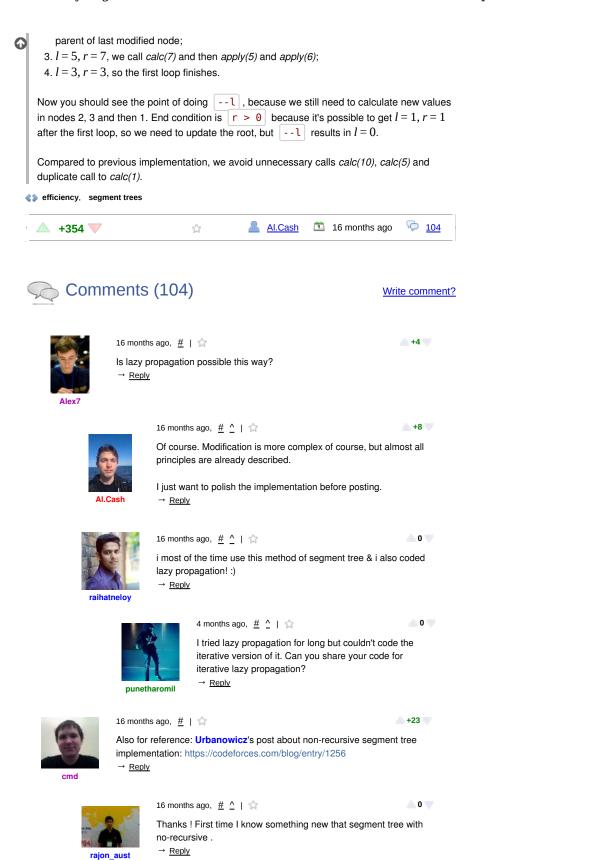
Boolean flags denote if we already performed any changes to the left and to the right. Let's look at an example: image link

ſ	1: [0, 16)															
				2: [0	), 8)			3: [8, 16)								
	4: [0, 4) 5: [4, 8)								6: [8, 12) 7: [12, 16)							
Γ	8: 9:				1	0:	1	1:	12:		13:		14:		15:	
L	[0, 2)		[2, 4)		[4, 6)		[6, 8)		[8, 10)		[10, 12)		[12, 14)		[14, 16)	
Γ	16:	17:	18:	19:	20:	21:	22:	23:	24:	25:	26:	27:	28:	29:	30:	31:
L	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

We call *modify* on interval [4, 13):

```
1. l = 20, r = 29, we call apply(28);
```

<sup>2.</sup> l = 10, r = 14, we call calc(14) — first node to the right of current interval is exactly the







True, my implementation is basically a refinement of the one in that post and in [user:Alias,2015-05-26]'s comment to that post. But for some reason it's still not well known and not really searchable, and I wasn't aware of that post. Also I hope to provide more knowledge, especially after I add the section about lazy

propagation. → Reply





+12



Just wondering, how did you write codes like this?

16 months ago, # ^ | \*\*

for 
$$(t[p += n] = value; p > 1; p >>= 1) t[p>>1] = t[p] + t[p^1];$$

Did you write it in the first attempt or you wrote something else and compressed that?

→ Reply

16 months ago, # ^ | 🏫



I don't remember, but of course it wasn't the first version, the code was revisited several times.



I think at first it looked like the code as I posted for non-commutative combiners, which is also used in build method.

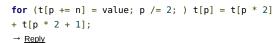
^1 trick is used in max flow implementation where edge is stored adjacent to it's reverse, so we can use ^1 to get from one to another. I'm not sure, but probably it came from there.

→ Reply



+19 16 months ago, # ^ | 🏠

I believe smth like this is more understandable and is equivalently short:





16 months ago, # ^ | 😭

A+10

Well, it's a matter of style. I prefer not to write modifications inside a loop condition.

→ Reply



16 months ago, # \_^ | 🏠

A 0

I'm just saying that this way it's more similar to traditional segment tree implementation and thus easier to follow the logic. But you are right, it's just a matter of style :)

→ Reply



16 months ago, # | 🏫 it so useful for non-red participants -16

• ← Rev. 2 16 months ago, # | 😭 Hello, can you explain this line ? ~~~~  $t[p>>1] = t[p] + t[p^1]$  ~~~~~ Specifically, why are you xor-ing it? → Reply I\_love\_Pro 16 months ago, # ^ | 🏠 A +6 p>>1 is a parent of p in the tree. Another son of p>>1 is p^1, since x^1 gives you x with reversed parity(last bit). <u></u> 0 🖤 16 months ago, # ^ | 🏫 So basically, p^1 equals p/2 + 1? → Reply I love Pro 16 months ago, # ^ | 🏠 A +18 W No, p^1 equals to p+1 if p is even, and p-1 if p is odd. → Reply 16 months ago, # ^ | 🏠 ♣ +1 ▼ xor will give the other child of the parent. For example if p is left child, p^1 will give the right one. Quite an interesting way of doing this in fact. → Reply **0** 16 months ago, # ^ | 🏠 Thank you! → Reply I\_love\_Pro <u>0</u> 16 months ago, # | 🏠 Hello, sorry to ask another silly question. Al.Cash says: "I, the left interval border, is odd (which is equivalent to I&1)" I am not familiar with bit operations too much, and I am having difficulty how x AND 1 gives odd/even value. Thanks in advance :) → Reply 16 months ago, # ^ | 🏠 A +3 Every odd value ends with 1(3 = 11, 5 = 101) and every even value ends with 0(2 = 10, 4 = 100). (xxx1 and 0001) --> 1 // odd edgarciarod (xxx0 and 0001) --> 0 // even → Reply 16 months ago, # | 🏫 -13 Hi, I am new in competitive programming, I was reading this article and trying to use it, but i dont understand how the function query works, for example if I have these numbers 1 47 45 23 348 and I would like the sum alanor1



from 47 to 23, whats numbers should I put in the arguments???? please help, thanks



well your array starts from position 0. Position of 47 is 1 and of 23 is 3. The Query will give you an answer for [I, r) so you should pass as arguments (1, 4). Hope it helps!

→ Reply



9 months ago, # ^ | 🏫 thanks so much → Reply



16 months ago, # | 😭 ← Rev. 2 Section about lazy propagation has been added.

→ Reply



16 months ago, # | 🏠

A+11 T

+25

0

I did some speed comparison between recursive and non-recursive lazy segment trees. With array size of 1<<18 and 1e7 randomly chosen operations between modifications and queries. Array size of 1<<18 is of course easier for my recursive code which uses arrays of the size 2<sup>n</sup> but on practise it doesn't affect much to the speed of the code.



My recursive ( http://paste.dy.fi/BUZ ): 6 s

Non-recursive from the blog: ( http://paste.dy.fi/kBY ): 3 s

That's quite big difference in my opinion. Unfortunately the non-recursive one seems to be a bit more painful to code but maybe it's just that I'm used to code it recursively.

→ Reply

16 months ago, # | 🏠





I think your modification on all element in an interval is not incrementing every element in an interval. You should check it.

Ex: n=8 0 1 2 3 4 5 6 7 modif(0,8,5); It should increment all places by 5. According to your algorithm array t is- 33 6 22 1 5 9 13 0 1 2 3 4 5 6 7 but clearly all element isn't incremented by 5.

→ Reply







Probably you're talking about the last example, but the operation there is assignment, not increment. And array looks absolutely correct except it should start with 40 not 33 (I believe 33 is a typo - let me know if it's not the case).

That's the point of 'lazy propagation' — not to modify anything until we really need it. You shouldn't access tree elements directly unless you called push(0, 8).

→ Reply





zeura

16 months ago, # ^ | 🏠 I was talking about this modify function: void modify(int I, int r, int value) { for  $(1 += n, r += n; 1 < r; 1 >>= 1, r >>= 1) {$ if (l&1) t[l++] += value; if (r&1) t[--r] += value; } → Reply



16 months ago, # ^ | 🏠

I see. Again, if you want to get an element call query, don't access the tree directly. And you don't need to call build in this example. → Reply



16 months ago, # ^ | 🏠 Thanks, I think I got it. :D → Reply





16 months ago, # | 🏠



0

Is it possible to extend this form to Multiple dimensions!

→ Reply

#### siddharths067



16 months ago, # ^ | 😭



Sure, just change one loop everywhere into two nested loops for different coordinates.

→ Reply



16 months ago, # | 😭



How to perform find-kth query? For n that is not the power of 2.

→ Reply



satirmo

16 months ago, # | 🏠



Hi. I have a question. When should it be considered necessary to use lazy propagation?

→ Reply



16 months ago, # ^ | 🏫



When you have both range modifications and range queries, or just range modifications for which order is important (for example, assignment).

→ Reply



16 months ago, # | 🏠

the child nodes..

← Rev. 2

HAPPINESS IS

..... EETTING & CREEN Is it possible to generalize the structure for the Lazy propagation  $\mbox{\sc Loop}$  , as in both the cases the propagation format of the loop changes. In recursion,

no matter what the context the function has the same structure for lazy

propagation namely, checking the Boolean flag and changing the value of

It would be better if you generalize it ....

siddharths067

For Example in Function Build:

•

```
for (l += n, r += n-1; s > 0; --s, k >>= 1)
for (int i = l >> s; i <= r >> s; ++i) if (d[i] != 0)
```

From Sum Queries

for (; l < r; l >>= 1, r >>= 1)

To RMQ

There is a difference in initial values of I and r in each loop.

Note: My Concern here is generalizing a loop for propagating the Segment Tree Lazily, I don't Care about the additional factor K for Sum queries or any additional parameter for a particular purpose.

← Rev. 2

0

<u>0</u>

A 0

**△** 0 **▼** 

<u>0</u> 0

→ Reply



15 months ago, # | 🏠

Understood. + for you!

15 months ago, # | 🏠

→ Reply



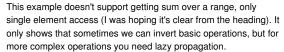
praveen14078

Can anyone please explain how the code in section "Modification on interval, single element access" actually work with a small example if possiple explaining how to we get sum over a range after the update(which also I am not able to understand!) ?

→ Reply



15 months ago, # ^ | 🏠



→ Reply



praveen14078

15 months ago, # 🛕 | 🏫

can you give an example of single element access after some update, i am not able to grasp how query() would return a[p] (as u mentioned in some earlier comment to call query to access an element or simply what does the query() do ?

→ Reply



15 months ago, # ^ | 👚

query() doesn't simply access the element — it calculates it as a sum of all the updates applied to that element.

→ Reply



15 months ago, # | 😭

← Rev. 2 0

Can someone explain logic(bit operations) here (t[p += n] = value; p > 1; p

The above expression is from the below function void modify(int p, int value)  $\{ // \text{ set value at position p for } (t[p += n] = value; p > 1; p >>= 1) t[p >> 1] = t[p]$ + t[p^1]; }

→ Reply

<u>0</u>

A 0 🐨

A 0

<u></u> 0 🛡

```
Efficient and easy segment trees - Codeforces
            •
                                            15 months ago, # _^ | 😭
                                            It is equivalent to this simplified code:
                                            void modify(int p, long new_val)
                                                      {
                                                               p += n:
                                                               tree[p] = new_val;
                                                               for (; p > 1; p/= 2)
                                                                {
                                                                         tree[p/2] = tree[p] +
                                            tree[p^1];
                                                               }
                                                      }
                                            For the explanation of using xor(^) you can check the previous
                                            comment that I made.
                                             → Reply
                                                     15 months ago, # _^ | 🏠
                                                     Can u explain query function a bit more clearly Thanks
                                                     for ur previous answer
                                                      → Reply
                                    nishanthyydana
                                   15 months ago, # | 🏠
                                   Thanks for this post, it is very useful for beginners like me. But there is a
                                   small problem; the picture of the segment tree at the very beginning of the
                                   post is not showing. It would be nice if you fixed that :)
                                   → Reply
                    I love Pro
                                                                                        ← Rev. 2
                                   15 months ago, # | 😭
                                   an interval [l,r]to a constant value v using lazy propagation.
                                   Thanks.
                                   → Reply
```

Please can somebody explain how to change all the elements of an array in



```
<u></u> 0 🕎
15 months ago, # ^ | 🏫
```

The last example does exactly that, assignment on an interval. → Reply



15 months ago, # | 😭

how to scale values in a given range say [L,R] with constant C by segment tree or BIT . Thanks in advance :D

→ Reply



I dont know for BIT but for segtree you can store additional factor for all nodes (initialized by 1). For each scale query, just scale this factor properties as always.

→ Reply







When processing the left border, the node we modify is always the right child of its parent. Then all the previous modifications were made in the subtree of the left child of the same parent. Otherwise we would process the parent instead of both its children. This means current direct parent is also a parent of leaf l+n → Reply



abinash\_nitr

I am new to segment tree and algorithms in general. I see a difference in implementation of query function at http://codeforces.com/blog/entry/1256. In my observation query function in this post does not give correct result. Did anyone else notice that?

→ Reply

13 months ago, # | 🏠

14 months ago, # | 🏠

I have a little question. In this problem, 533A — Berland Miners, I used this. I used this before, so I thought it works perfectly.

My submission with  $N = 10^6 + 5$  got WA. When I changed it to  $N = 2^{20}$ , it



 $N = 10^6 + 5 --> \text{http://codeforces.com/contest/533/submission/12482628}$ 

 $N = 2^{20}$  --> http://codeforces.com/contest/533/submission/12483080

 $2^{20}$  is bigger than  $10^6 \pm 5$  but problem isn't this. I tried it with  $N = 2 * 10^6 + 5$  but I got WA again.

 $N = 2 * 10^6 + 5 --> \text{http://codeforces.com/contest/533/submission}$ /12483105

Do you have any idea about it?

→ Reply

13 months ago, # ^ | 🏠 A +5 🐨

As said in this post:

It's not actually a single tree any more, but a set of perfect binary



Urbanowicz

Not sure, where exactly that fails for you, but I guess it's here:

int mn = min(op[x + x], cl[x + x + 1]);

Seems like you intend to take left child of op and right child of cl , but when you have a set of trees it doesn't always work. → Reply



13 months ago, # 🛕 | 🏫

I didn't understand the problem on here. Is it different from maximum of range or another segment operation? → Reply





13 months ago, #  $^{\circ}$  |  $^{\circ}$  | But it works with  $N=3\cdot 10^6+5...$  12484538  $^{\rightarrow}$  Reply

A +5

A +5

+10

Urbanowicz

13 months ago,  $\# \land | \Leftrightarrow$ I think the problem is in this line: if(op[1] == 0)



Node 1 is a true root only if N is a power of 2, otherwise it's not trivial to define what's stored there (just look at the picture).

Everything is guaranteed to work only if you use queries and don't access tree element directly (except for leaves).

→ Reply



Great and efficient implementation. Could you please say more about "Modification on interval [I,r) affects t[i] values only in the parents of border leaves: I+n and r+n-1.". Because, the inc method modifies more than the parents of the border leaves.



If I well understood, you want to explain the fact that in query method (max implementation) we need only to push down ONLY to the left border and right border nodes. Since when computing the query we will be using only the nodes along the route. Moreover, this explains the fact that when we finish increment method, we need ONLY to propagate up to to root of the tree starting from the left and right border leaves, so that the tree and the lazy array will be consistent and representing correct values.

→ Reply



12 months ago, # ^ | 🏠

You're right. I wanted to say what values are affected except the ones we modify directly in the loop (the ones that compose the interval). Will think how to formulate it better.

← Rev. 2

← Rev. 2 **0** 

→ Reply

11 months ago, # | 🏠



// In case someone needs: query with [l, r] inclusive
interval:
int query(int l, int r) { // sum on interval [l, r]
 int res = 0;
 for (l += n, r += n; l <= r; l >>= 1, r >>= 1) {
 if (l&l) res += t[l++];
 if (!(r&l)) res += t[r--];
 }
 return res;
}

→ Reply



If you prefer closed interval, just change r += n to r += n+1, otherwise it's easy to make mistakes.

11 months ago, # ^ | 😭

Al.Cash



```
0
                  4 months ago, # ^ | 🏫
                  Is any problem to use query function mention by
                  rvelloso? does it fails in any test case?
                  → Reply
   sajalhsn13
                          4 months ago, # ^ | 🏠
                          int query(int I, int r) { int res = 0; for (I += n, r +=
                          n; l <= r; l >>= 1, r >>= 1) { if (l&1) res += t[l++]; }
                          if (!(r&1)) res += t[r--]; } return res; }
                          This implementation works fine. I solve a
            sajalhsn13
                          problem with this implimentation.
                           → Reply
10 months ago, # | 😭
You said that range modifications and point updates are simple, but the
following test case doesn't give a correct example (if I understood the
problem correctly): - 5 nodes - 1 2 4 8 16 are t[n, n + 1, n + 2, n + 3, n + 4] -
one modification: add 2 to [0, 5) - query for 0
returns 59, while it should be 3.
Code (same as all the functions given above, but for unambiguity:
#include <cstdio>
const int N = 1e5; // limit for array size
int n; // array size
int t[2 * N];
void build() { // build the tree
  for (int i = n - 1; i > 0; --i) t[i] = t[i << 1] +
t[i<<1|1];
void modify(int l, int r, int value) {
  for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
    if (l&1) t[l++] += value;
    if (r&1) t[--r] += value;
  }
}
int query(int p) {
  int res = 0;
  for (p += n; p > 0; p >>= 1) res += t[p];
  return res;
}
int main() {
  scanf("%d", &n);
  for (int i = 0; i < n; ++i) scanf("%d", t + n + i);</pre>
  build();
  modify(0, n, 2);
  printf("%d\n", query(0));
  return 0;
}
Input: 5 1 2 4 8 16
```



→ Reply



10 months ago, #  $^{\wedge}$  |  $^{\wedge}$  | Just remove build — it doesn't belong to this example.

Al.Cash

10 months ago, # | 😭

Al.Cash, I am finding the code inside the header of for-loop too confusing, Can you please tell me the easy and the common version of theirs?

Like this:



for (l += n, r += n; l < r; l >>= 1, r >>= 1)

What does it mean? Its tough for me to understand. Would be great if you could clarify this.

Thanks Alot. PS. Nice work on the tutorial, CF needs more articles like this one :)

→ Reply

10 months ago, # ^ | 🏠

+8

Step-by-step, in the simpler case (when  $n=2^k$  and we have full binary tree). Consider n=16:

- First operation of for is executed exactly once before any iteration (as per definition). That is: add n (16 in our case) to both and r, i.e. convert "array coordinates" to numbers of vertices in the tree.
- 3. Then we iterate while current segment is not-empty, that is l < r.
- 4. After each iteration we move 'up' the tree. We know that parent of vertex x is  $\left[\frac{x}{2}\right]$ , so we should divide both  $\left[\frac{x}{2}\right]$  and  $\left[\frac{x}{2}\right]$  means "bitwise shift by one bit", which works exactly like division by two with rounding down for non-negative integers.
- → Reply

10 months ago, # | 🏠

← Rev. 3

Hi, I am trying to solve SPOJ GSS1 according to this tutorial on Segement Trees. But I am not able to write a query method. Following is my code so far. I understood the logic but not able to write query method. Please help.

import java.util.Scanner;



public class GSS1\_CanYouAnswerTheseQueriesI {



```
this.right = right;
                        this.segsum = segsum;
                        this.bestsum = bestsum;
                public SegNode(){
                        this.left = Integer.MIN_VALUE;
                        this.right = Integer.MIN_VALUE;
                        this.segsum = Integer.MIN_VALUE;
                        this.bestsum = Integer.MIN_VALUE;
                private int left, right, segsum, bestsum;
        };
        //array size
        static int n;
        static SegNode[] nodes;
        public static void main(String[] args) {
                  Scanner sc = new Scanner(System.in);
                  n = sc.nextInt();
                  //Height of segment tree
              int x = (int) (Math.ceil(Math.log(n) /
Math.log(2)));
              //Maximum size of segment tree
              int max_size = 2 * (int) Math.pow(2, x) - 1;
              nodes = new SegNode[max_size];
                  for (int i = 0; i < n; ++i){
                          int in = sc.nextInt();
                          SegNode node = new
SegNode(in,in,in,in);
                          //nodes[n + i - 1] = node;
                          nodes[n + i] = node;
                  }
                  build();
                  int M = sc.nextInt();
                  for(int i=0;i<M;i++){</pre>
                          int l = sc.nextInt();
                          int r = sc.nextInt();
                          System.out.println(query(l,r));
                  }
        }
        public static SegNode merge(SegNode cl,SegNode cr)
                SegNode newNode = new SegNode();
                if(cl!=null && cr!=null){
                        newNode.segsum =
cl.segsum+cr.segsum;
                        newNode.left =
Math.max(cl.segsum+cr.left,cl.left);
                        newNode.right =
Math.max(cr.segsum+cl.right,cr.right);
                        newNode.bestsum =
max3(cl.bestsum,cr.bestsum,cl.right+cr.left);
                        return newNode;
```

10

```
•
```

```
if(cl==null){
                        return cr;
                }else if(cr==null){
                        return cl;
                }
                return newNode;
        }
        public static int max3(int a,int b,int c)
                return Math.max(Math.max(a,b),c);
        public static void build() { // build the tree
                for (int i = n - 1; i >= 0; --i){
                        nodes[i] = new SegNode();
                        nodes[i] =
merge(nodes[i<<1],nodes[i<<1|1]);</pre>
                        //nodes[i] =
merge(nodes[2*i+1], nodes[2*i+2]);
                }
        }
        public static int query(int l, int r) { // sum on
interval (l, r)
                SegNode leftinMergeNode = null;
                SegNode rightinMerge = null;
                for (l += n-1, r += n-1; l <= r; l >>= 1,
r >>= 1) {
                        //if l is the right child
                        if ((l&1)>0) {
                                 leftinMergeNode =
merge(leftinMergeNode,nodes[l++]);
                        //if r is the left child
                        if ((r&1)==0) {
                                 rightinMerge =
merge(rightinMerge,nodes[r--]);
        }
                SegNode res =
merge(leftinMergeNode,rightinMerge);
                return res.bestsum;
→ Reply
                                                       <u></u> 0 🖤
10 months ago, # | 🏠
What a nice! I'm fucking wet.
→ Reply
```

10 1

LaFuckinGioconda



9 months ago, # | 🈭

5 elements. (Read as 'count of 0 is 5') 0->5 1->0 2->0 3->0 4->0. Therefore for query <1, the answer should be 4 cuz 4 slots have value <1 (1,2,3 and 4).



according to this implementation when array is not of size 2<sup>n</sup>, the tree is coming out to be wrong. n=5. so we start filling at n=5.

We have 4 slots with value [0,1) and 1 slot with value [5-6). The array would be like: (started filling at 5 cuz n=5, representing [0,1)) 0 1 2 3 4 5 6 7 8 9 10 0 0 0 0 0 4 0 0 0 0 1

<u>0</u>

now when making segment for array index 10, the value at index 5 gets overwritten. How to handle this?

→ Reply

9 months ago, # | 🌣

To this line

Modification on interval [l, r) affects t[i] values only in the parents of border leaves: l+n and r+n-1 (except the values that compose the interval itself – the ones accessed in for loop).

I have a question: If the **apply** operation does not satisfy the associative law, is it still work?

For example: modify(3, 11, value1)

1 If I use push(3, 11), operations on node 5 is:

```
1.1 apply(2, d[2]); ( in the push loop);
```

1.2 d[5] += d[2] (in the apply);

1.3 d[2] is passed to 5's son, d[5] = 0 (in the |apply(5, d[5])|;

1.4 d[5] = value (in the | modify | loop);



value is passed to 5's son 10 and 11, so:

1.5 d[10] = (d[10] + d[2]) + value; (same to 11)

2 If I use push(3, 4), push(10, 11), operations on node 5 is:

2.1 apply(2, d[2]); (in the | push | loop);

2.2 d[5] += d[2]; (in the apply);

2.3 d[5] += value; (in the modify loop);

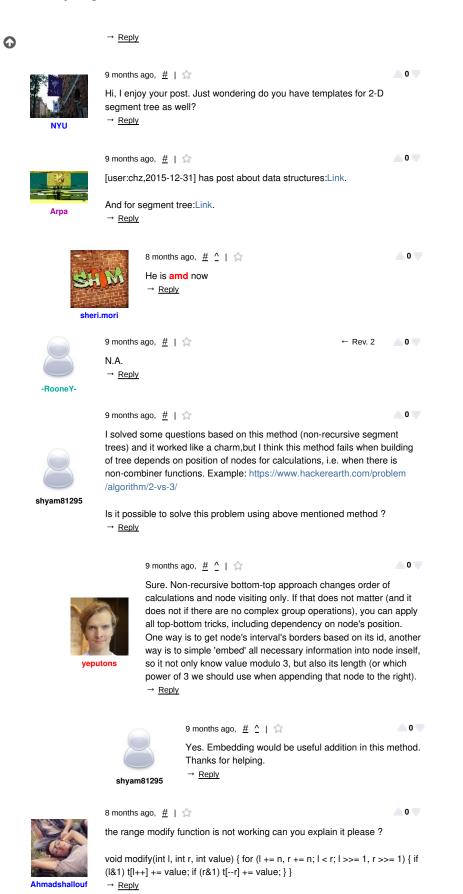
•••

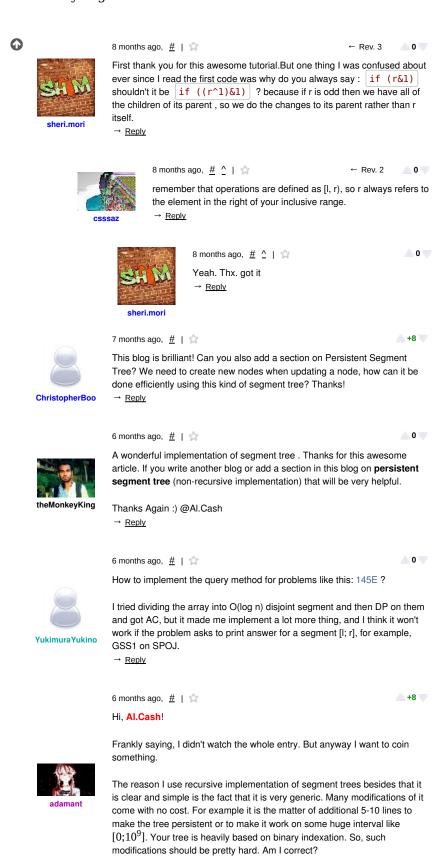
d[5] is passed to 5's son 10 and 11, so:

2.4 d[10] = d[10] + (d[2] + value);

1.5 is equal to 2.4, because (a+b)+c = a+(b+c), but if I replace the + to other special operation which does not satisfy the associative law, what should I do in + Lazy propagation + Lazy pro

10 \_\_\_\_\_





→ Reply





True, is't impossible to modify this approach to support persistency or arbitrary intervals. However it handles all other cases better and they are the vast majority. Especially it's noticeable in the simplest (and the most common) case.

The choice is up to you, of course :)

6 months ago, # ^ | 🏫

→ Reply



6 months ago, # ^ | 😭

BTW you can just use unordered\_map instead of array in order to make it work on some huge interval like  $[0;10^9]$ . AFAIK it's well known trick used with Fenwick Tree.

→ Reply





Noooo. Very, very, very bad idea. The constant is too large even for Fenwick. Many problems where [0;10<sup>9</sup>] expect participant to compress the data instead of using dynamic structure, so it is usually hard to get accepted even with fair dynamic tree. Using unordered\_map instead of it is simply waste of time, in my opinion.

6 months ago, # ^ | 🏠

→ Reply



6 months ago, # 🐴 | 🏫

Yes, you are right, data compression of course better. But I guess that perfomance of BIT + unordered\_map is not so much worse than performance of dynamic tree. From another hand it's extremely easy way to modify this data structure and it's also possible for some problems.

→ Reply



6 months ago, # | 🏠

**40** 

A +5

A+5 W

can we find sum of elements(of different sets respectively) of a power set using segment tree?Explain.

→ Reply



6 months ago, # | 🏠

← Rev. 2

I'm not that much into C++, but shouldn't this statement



kocko

```
scanf("%d", t + n + i);
```

be actually:

scanf("%d", t[n + i]); // since t is an array

?

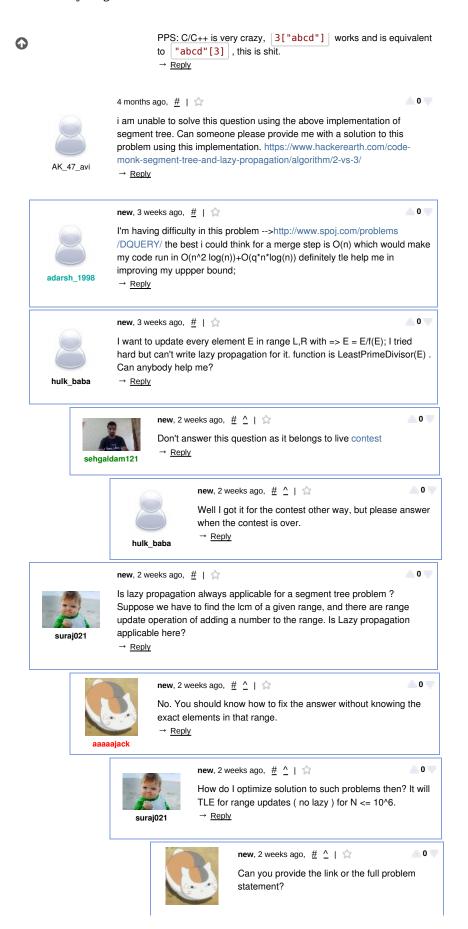
→ Reply

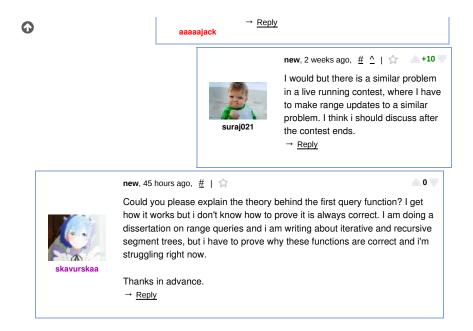


Those are equivalent. In the first case C treats t as a pointer.

PS: in the second you would need to write &t[n+i], that's why the first one is easier.

25 of 27





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The only programming contests Web 2.0 platform
Server time: Sep/24/2016 20:32:20<sup>UTC+5.5</sup> (p1).
Desktop version, switch to mobile version.