riningan's blog

Prime Factorization In log(n) After Sieve

By riningan, 4 years ago,

We use Eratosthenes sieve for prime factorization, storing the primes in an array. But for that, we need to find the primes less than or equal to sqrt(n) which divide n. There are about n/log(n) primes less than or equal to n. So, the complexity is roughly sqrt(n)/log(sqrt(n))*log(n). But if n is asked to be factorized completely where n is within the Sieve range, then we can factorize n is log(n) complexity. And the trick is fairly small. Observe, that, we don't need to run a whole sqrt(n) loop for finding the prime divisors. Instead, we can even store them when n is in the range, say n<= 10^7. But the tricky part is not to store all the prime divisors of n. Let's see the following simulation. Take n = 60. We want to factorize n. We will store the **smallest prime factors only**. This does the trick. If n is composite, then it has such a prime factor, otherwise n is a prime and then the n itself is the smallest prime factor. It is obvious, for any even number n, sp(n)=2. Therefore, we only need to store these primes for odd n only. If we denote the smallest prime factor of n by sp(n), for odd 2 <= n <= 30, we get the following list.

sp(2n)=2, sp(3)=3, sp(5)=5, sp(7)=7, sp(9)=3, sp(11)=11, sp(13)=13, sp(15)=3, sp(17)=17, sp(19)=19, sp(21)=3, sp(23)=23, sp(25)=5, sp(27)=3, sp(29)=29.

Then the factorization is very simple. The optimization is needed only once, when the Sieve() function runs.

```
bool v[MAX];
int len, sp[MAX];

void Sieve(){
        for (int i = 2; i < MAX; i += 2)        sp[i] = 2;//even numbers
have smallest prime factor 2
        for (lli i = 3; i < MAX; i += 2){
                if (!v[i]){
                        sp[i] = i;
                        for (lli j = i; (j*i) < MAX; j += 2){
                                if (!v[j*i])    v[j*i] = true, sp[j*i] =
i;
                        }
                }
        }
}

int main(){
        Sieve();
        for (int i = 0; i < 50; i++)    cout << sp[i] << endl;

    return 0;
}
```

Now, notice the difference between the usual prime factorization and this one! The only problem is, you can't use this for n large enough in int range. Still, it seems nice to me and pleasured me when I first found this.

basic math sieve of eratosthenes

10	vepifanov	2992

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basic math, sieve of eratosthenes

+8 | riningan | 4 years ago | 36

Comments (36)

Write comment?

Helgui

4 years ago, # | +1

[:||||||:] David Gries, Jayadev Misra. A Linear Sieve Algorithm for Finding Prime Numbers, 1978. read this (in Russian)

→ Reply

riningan

4 years ago, # ^ | 0

hmm.

→ Reply

dalex

4 years ago, # | 0

So many minuses, why? It's very useful trick and I don't think that everyone knows it.

→ Reply

halfo

3 years ago, # | +5

Really nice trick! Thanks for sharing.

→ Reply

WARush143 → POI - Vacation

M.A.H.M.O.O.D → Do you know any good DP problems on codeforces ???

i.trofimow → a XOR problem

sgipc → Registration for SGIPC (No need to read)

singh1495 → hashing technique

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savinov

3 years ago, # | ← Rev. 2 0

It's better to precalculate not only smallest prime number, but also quotient $cp[i] = i / lp[i]$, to do not unnecessary and TOO SLOW operations of division, especially in case of big number of queries.

→ Reply

kien_coi_1997

3 years ago, # ^ | 0

I think that it is not important. Original source is easy to read and easy to understand. Also, you have to perform divide operations log(n) times only. It seems not too big.

→ Reply

mahfuzmohammad

3 years ago, # | 0

Dude, your tricks is really cool but I think there is some problem in your sample code. **Your Sieve() function doesn't store the smallest prime factors properly**. For 45, the smallest prime factor should be 3 where according to your sample code it stores 5!

→ Reply

riningan

3 years ago, # ^ | 0

that's because I forgot to check first if a number already has a smallest prime divisor. Now it is correct. Thanks for pointing the mistake out.

→ Reply

akhileshydv20

15 months ago, # | 0

how can we find factorization from sp[]..please explain?

→ Reply

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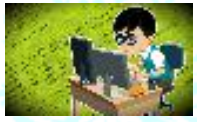
Detailed →

15 months ago, # ^ | 0

```
vector <int> factorize(int k) {
        vector <int> ans;
        while(k>1) {
                ans.push_back(sp[k]);
                k/=sp[k];
        }
        return ans;
}
```

→ Reply

-Secta-



15 months ago, # ^ | 0

Gotcha...thanks :-)

→ Reply

akhileshydv20

14 months ago, # | 0

How large can MAX be?

→ Reply

i_love_emilia_clarke

14 months ago, # ^ | 0

10^7

→ Reply

Dushyant

14 months ago, # ^ | 0

Hi Dushyant, If the limit is 10^7 then why this code is not working. I have commented out the rest part which is not concerned....

i_love_emilia_clarke

→ Reply

-Secta-

14 months ago, # ^ | 0

Signed integer overflow — http://ideone.com/FXLHXO :)

→ Reply

quantic

7 months ago, # ^ | 0

what should i do for nos of 10^9 range?

→ Reply

Fekete

7 months ago, # ^ | 0

It can be Pollard's "Ro" algorithm or smth like that.

→ Reply

quantic

7 months ago, # ^ | 0

I got Pollard's "Ro" algorithm.really nice one.thank u @fekete

→ Reply

Fekete

3 months ago, # ^ | 0

Is it a sarcasm?

→ Reply

14 months ago, # | 0

luismo

Any problems to solve with this technique ???

→ Reply

SomeRandomGuy

14 months ago, # ^ | 0

http://codeforces.com/contest/546/problem/D

→ Reply

Dushyant

14 months ago, # ^ | ← Rev. 3 0

Medium Factorization

One more

Simple Sum

→ Reply

Taube

14 months ago, # ^ | 0

http://codeforces.com/problemset/problem/222/C

→ Reply

mshibli786

14 months ago, # | 0

hey smallest prime factor for 567 is 3 but you program is outputing 7...plz correct it

→ Reply

14 months ago, # ^ | 0

Sorry but you are mistaken.It is giving 3 as the output.

→ Reply

Mocking_Jay

14 months ago, # ^ | ▲ 0 ▼

actually i am converting it in java code may be due to i am getting this...if u can convert this in java then it would be very helpful for me and for othes..plz do it soon

→ Reply

mshibli786

14 months ago, # ^ | ← Rev. 2 ▲ -6 ▼

Whats Wrong With this logic every time exception was occuring or it is Same as ABove logic but not Working for java

```
static void Sieve() {
    for (int i = 2; i < MAX; i += 2)
       sp[i] = 2;// even numbers have smallest
prime factor 2
    for (int i = 3; i < MAX; i += 2) {
       if (!v[i]) {
         sp[i] = i;
         for (int j = i; (j * i) < MAX; j+=2) {
          if (!v[j * i])
              v[j * i] = true;
          sp[j * i] = i;
         }
       }
    }
}
```

→ Reply

mshibli786

-Secta-

14 months ago, # ^ | 0

```
if (!v[j*i]) v[j*i] = true, sp[j*i] = i;
```

→ Reply

Mocking_Jay

14 months ago, # ^ | ← Rev. 3 0

He has pointed out the mistake.

→ Reply

additya1998

14 months ago, # | 0

This is really nice! Thanks for sharing.

→ Reply

guddu1996

12 months ago, # | ← Rev. 2 0

I don't know why I m getting segmentation fault for the spf() function...
http://codepad.org/cKUBvEJ2

→ Reply

ayu15

4 months ago, # | 0

I am not able to understand that why is it log(n) ???

→ Reply

4 months ago, # ^ | +6

Consider the prime factorization n = p1 * p2 * ... * pk, where p1, p2, ... pk are the prime factors. n has at most k = log(n) prime

Flatfoot

factors.

To understand this think of how you can maximize the number of prime factors. You'll get the most number of prime factors for p1 = p2 = ... = pk = 2. So we have n = 2^k. Solving for k yields k = log(n).

→ Reply

ayu15

4 months ago, # ^ | +3

Amazing... thanks :)

→ Reply

prak_blah

3 weeks ago, # ^ | -10

what is the overall complexity of the Sieve() function mentioned above

→ Reply

saand_nitd

3 months ago, # | +3

I think this can be done without extra space :)

→ Reply