DISTRIBUTED CONTROL OF A MULTI-AGENTS MAGNETIC LEVITATION SYSTEM

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1 Introduction

Our aim is to test the behavior of a multi-agents cyber-physical system implementing both a distributed observer and then a local observer. In order to check the response of the system in different contexts, we used various topologies and we analysed their impact on the global performances.

2 Theory of SBVF Protocol

The system that we are going to study is a CPS made up of 7 magnetic levitation systems. It has to be modeled as a multi-agents system with the following features:

- One leader node S_0
- 6 maglev system as follower nodes S_i with i = 1,2,...,N

Each agent is characterized by the state-space equations obtained by linearizing the physical equation around a suitable equilibrium point:

$$\dot{x_i} = Ax_i + Bu_i \tag{1}$$

$$y_i = Cx_i \tag{2}$$

where A = $\begin{bmatrix} 0 & 1 \\ 880.87 & 0 \end{bmatrix}$,B = $\begin{bmatrix} 0 \\ -9.9453 \end{bmatrix}$,C = $\begin{bmatrix} 708.27 & 0 \end{bmatrix}$ and D = $\begin{bmatrix} 0 \end{bmatrix}$
Each topology is described by:

- Adjacency matrix associated to the diagraph $\mathscr{G}$, $\mathscr{A} = [a_{ij}] \in R^n$, where a_{ij} is the weight for the edge (j,i) ;
- Augmented graph $\overline{G} = (\overline{V}, \overline{E})$ where $\overline{V} = \{v_0, v_1, ..., v_N\}$;
- Laplacian matrix of $\mathscr{G}$, L = $[I_{ij}] = D - \mathscr{A}$, D = diag$(d_1, d_2, ..., d_N)$, d_i in-degree of node i;
- Pinning matrix G = diag$(g_1, g_2, ..., g_N)$.

Our goal is to design a distributed control protocol such that our system performs the cooperative tracking problem: the follower nodes are expected to track the reference given by the leader node. In both systems there is a leader node for which we have defined a dynamic regulator (controller and observer) in order to estimate its state that will be useful to build the input for the follower nodes. All the follower nodes, on their turn, have to be stabilized using the same identical state feedback protocol.
Each node computes its neighborhood tracking error in this way:

$$\epsilon_i = \sum_{j=1}^{N} a_{ij}(x_j - x_i) + g_i(x_0 - x_i) \tag{3}$$

The control law applied to each node is then

$$u_i = cK\epsilon_i \tag{4}$$

where $c > 0$ is the coupling gain.
The gain matrix K is computed as

$$K = R^{-1}B'P \tag{5}$$

where P is the unique positive definite solution of the algebraic Riccati equation:

$$A'P + PA + Q - PB^{-1}B'P = 0 \tag{6}$$

and Q and R are parameters to tune in order to give more weight to state energy or command activity, respectively.

A necessary and sufficient condition to reach convergence is that the matrix

$$Ac = I_N \otimes A - c(L + G) \otimes BK \tag{7}$$

is Hurwitz: all the eigenvalues must have strictly negative real part.

Moreover we know that state variables are not directly accessible for measurement. That's why we need to build observers, in a distributed or local way, to obtain a proper estimate of x, that will be compared to its true value. In any of the two cases the leader node is provided with is local observer.

- DISTRIBUTED OBSERVER
  The state estimation is computed in a distributed way, in the sense that each observer only requires its local output estimation error and the output estimation errors of the neighborhood agents. The so called neighborhood output estimation error is computed as follows:

$$\xi_i = \sum_{j=1}^{N} a_{ij}(\tilde{y}_j - \tilde{y}_i) + g_i(\tilde{y}_0 - \tilde{y}_i) \tag{8}$$

  where $\tilde{y}_0$ is the estimated output of the leader, a_{ij} is the value in position (i,j) of the adjacency matrix associated to the graph, and g_i is the pinning gain, which is the weight of the link between the leader node and the follower node i. The estimated state of the node i is then described by the following equation

$$\dot{\hat{x}}_i = Ax_i + Bu_i - cF\xi_i, \tag{9}$$

  where u is as in equation 4 The observer gain F is designed as follows:

$$F = PC'R^{-1} \tag{10}$$

  where P is the unique positive definite solution of the algebraic Riccati Equation

$$AP + PA' + Q - PC'R^{-1}CP = 0 \tag{11}$$

- LOCAL OBSERVER
  Each node does the computation locally on his own, not considering his neighbors. The observer is simply designed by means of appropriate eigenvalue placement, tuned by simulation. The ones chosen for this study are $\lambda_L = [-2 - 4]$. The estimated state of the node i is then described by the following equation

$$\dot{\hat{x}} = A\hat{x} + Bu(t) - L(\hat{y}(t) - y(t)) \tag{12}$$

To check the resilience of the system, we tested the cooperative algorithm implementing different topologies.

3 Analysis of different network structures

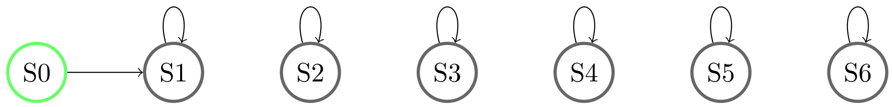
We considered three different kinds of steady state behaviors of the leader node, obtained by choosing appropriate eigenvalues for the state feedback gain:

- **CONSTANT STEADY STATE**
  Amplitude $R_0 = 2$
  λ = [0 -20]
  $x0_{leader}$ = [R_0 0]'
  The two states will be set respectively to the initial conditions of the leader node, which is, in this case, x1 = R0, x2 = 0;
- **RAMP STEADY STATE**
  Slope $R_0 = 1$; λ = [0 0]
  $x0_{leader}$ = [0 R_0]'
  The first state is going to converge to a ramp with slope R0 since the second state, which is its derivative, is converging to 1;
- **SINUSOIDAL STEADY STATE**
  Frequency $\omega_0 = 1$;
  Amplitude $R_0 = 1$;
  λ = [ω_0i -ω_0i]
  $x0_{leader}$ = [R_0 0]';
  Since the eigenvalues are complex conjugates with $Re(\lambda) = 0$ and $Im(\lambda) = \omega_0$, the first state of the leader node is expected to converge to a sinusoid with amplitude R_0 and frequency ω_0

We studied the behaviour of the distributed algorithm considering four different network structures.
We ran some simulations to stress the system with randomly generated errors on the output measurements of each one of the nodes. In order to derive some results, we considered as convergence time the time required by the system to drive the global disagreement error below a 5% threshold (with respect to the ideal behaviour). Furthermore, for each one of the reference behaviours, we analyzed the difference between the global system steady-state dynamics and the ideal steady-state dynamics we wanted to obtain. If the sufficient conditions for global disagreement error convergence are satisfied, the follower nodes behave inevitably like the leader node. However, since the leader node is subject to errors, analyzing the steady-state dynamics with respect to the ideal one is a useful additional metrics. In particular we calculated the mean absolute difference between follower nodes states dynamics and ideal steady-state behaviour, starting from convergence time until end of simulation time.
The results have been averaged over 5 runs. All the systems were tested using these parameters:

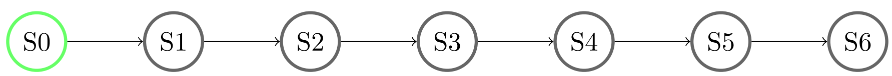
- Initial conditions followers:
$$X0_{follower} = \begin{bmatrix} 2.0936 & -2.2397 & 1.5510 & -3.8100 & 4.5974 & 0.8527 \\ 2.5469 & 1.7970 & -3.3739 & -0.0164 & -1.5961 & -2.7619 \end{bmatrix}$$
- Coupling gain = 2*cmin where
$$cmin = \frac{1}{2 * minRe(\lambda_i)} \tag{13}$$
  is imposed by the algorithm and λ_i are the eigenvalues of the matrix L+G;
- Q = q*I, where I is the identity matrix and q = 1 ;
- R = $\frac{q}{5} = \frac{1}{5}$;

Topology 1: Only self loops
The leader node (S0, green in the figure below) is connected to the first node and all the follower nodes are just connected to themselves.

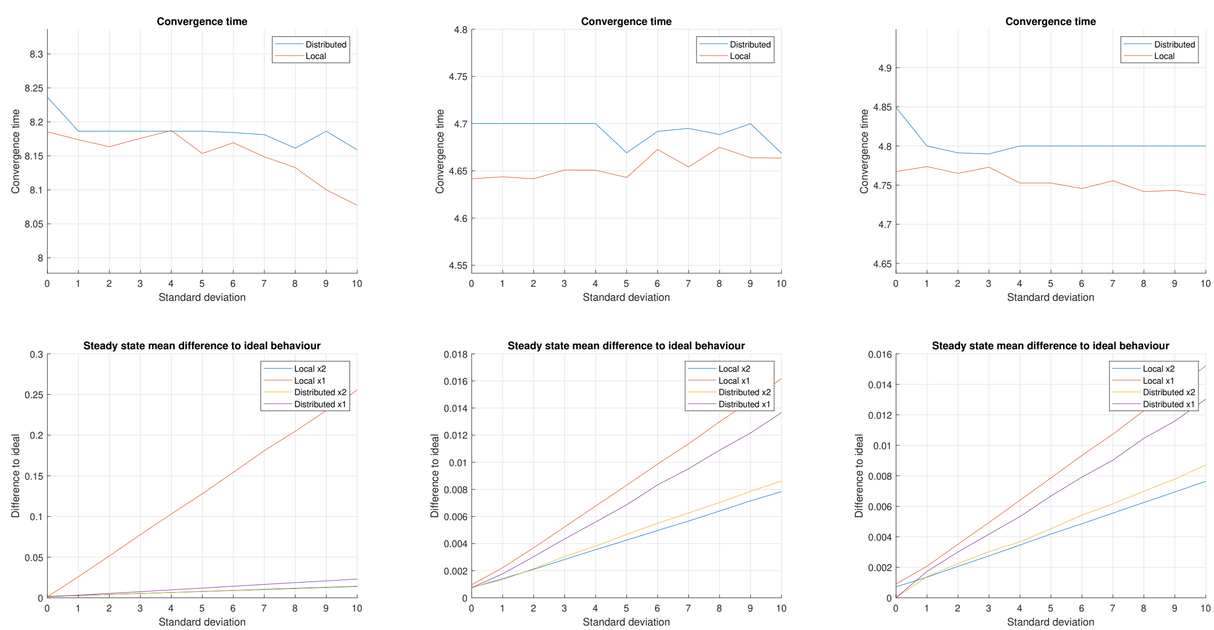


The information that the leader node exchanges with the first node is not propagated to the network, therefore they are only able to asymptotically converge to 0 thanks to their own state feedback loop. No other reference behaviour, which are constant different from 0, ramp or sinusoid, can be obtained.
It is important to note that the matrix A_c of the global controlled system is in fact non Hurwitz: the sufficient condition of the cooperative tracking problem is not satisfied.
No further discussion is provided as this topology is not feasible.

Topology 2: Line
All nodes are connected in a line and the leader node (S0, green in the figure below) is connected to the first follower node



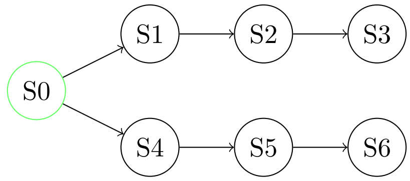
This network topology is able to ensure convergence to all the reference steady-state behaviours proposed. Performance is good and computational load is evenly distributed over all the nodes, since all agents, except for the leader and the last of the chain, are just connected to the previous and the following ones.
In general, the multi-agents system with distributed observer takes little more time to converge, as seen in the graphics below. As error's standard deviation increases the convergence time does not change significantly, but the behaviour with respect to the ideal reference is indeed different. Considering all the three possible reference behaviours, the multi-agent system with distributed observer is able to better recover from the action of the errors, obtaining a steady-state dynamics generally closer to the ideal one. In particular the 6th node, the last in the line, combines data already processed by all the other nodes, and it is therefore the one able to better neglect the effect of the errors.



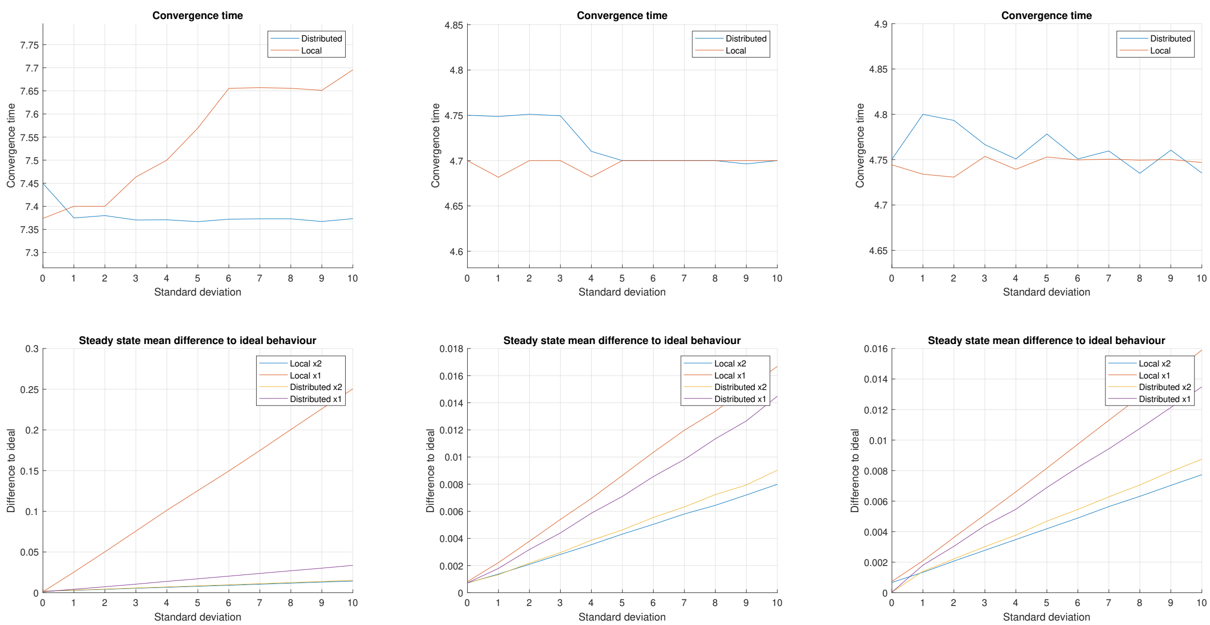
Constant Ramp Sinusoid

Topology 3: Tree

The leader node (S0, green in the figure below) is connected to two subsets of three follower nodes arranged in a line-like configuration



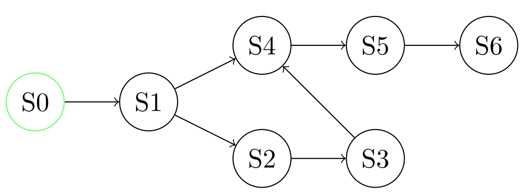
The multi-agents system is able to achieve convergence to all the three reference behaviours considered. Overall the convergence time is better than in the previous topology, in particular when looking at the constant steady-state behaviour (first couple of plots from the left). Every other consideration is similar to the previous case. The system with distributed observer in generally able to better recover the ideal dynamics with respect to the system with local observers



Constant Ramp Sinusoid

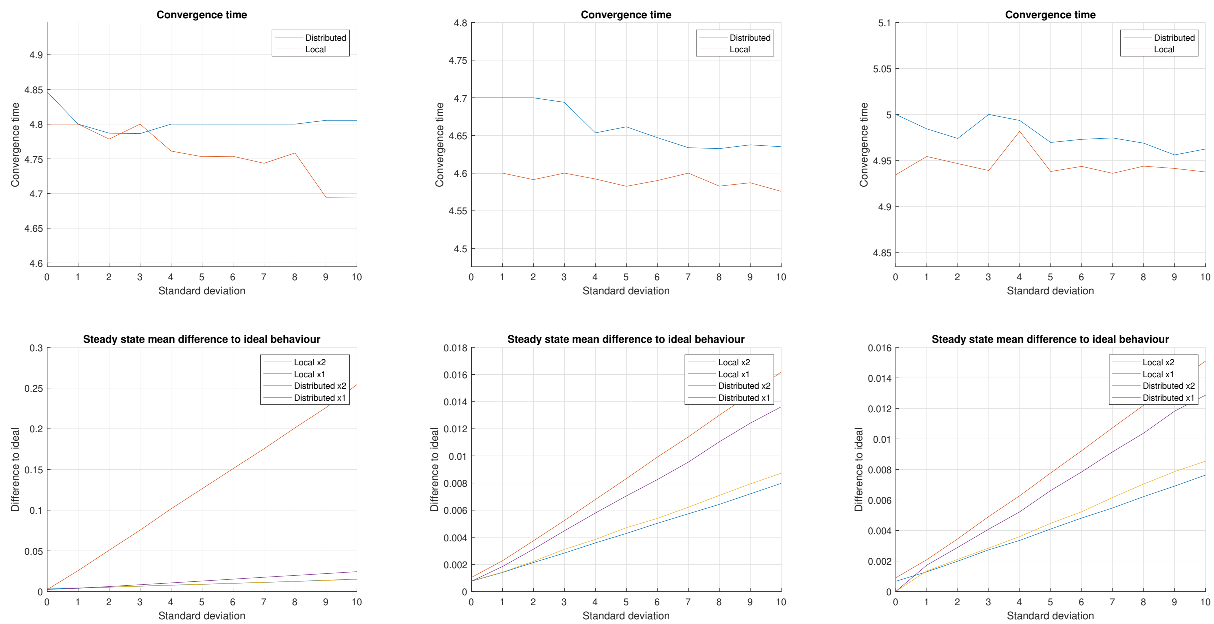
Topology 4: Complex tree

The leader node (S0, green in the figure below) is only connected to the first node. On its turn it is connected to two subsets of two and three follower nodes, with a cross connection between the two branches.



With the multi-agent system displaced in this topology we obtain the best results among all the network structures studied. The system converges pretty fast to all the three reference dynamics. In particular,

considering constant steady-state behaviour, this topology enables a 35% faster convergence with respect to the other networks considered, both for distributed and local observer implementations. This is also the only case in which the system with distributed observer has overall almost the same convergence time as the one with local observer, if not even less in some cases (see graphics below).
Since this network structure is particularly performing, we provide further discussion on it in the following sections.



Constant Ramp Sinusoid

4 Effect of R/Q ratio and coupling gain on performance

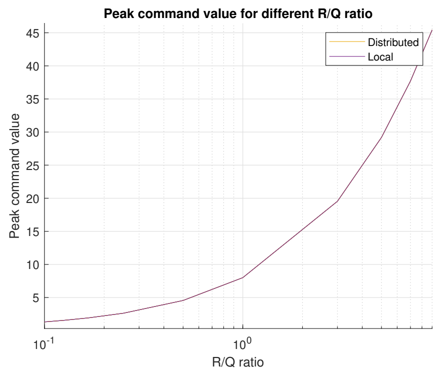
In this section we are going to study the effect of different values of R,Q weights and coupling gain c on the performance of the distributed algorithm. All the results have been derived using the topology 4 as network structure.
The weights Q and R are taken into account in the calculation of the optimal solution for the linear quadratic control problem

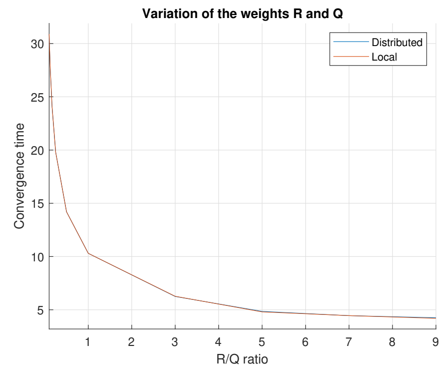
$$u(t) = arg \quad \min_{\mathrm{u(t)}}[\frac{1}{2}\int_0^\infty x^T(t)Qx(t) + u^T(t)Ru(t)\,dt] \tag{14}$$

whose solution is

$$u(t) = -Kx(t) \tag{15}$$

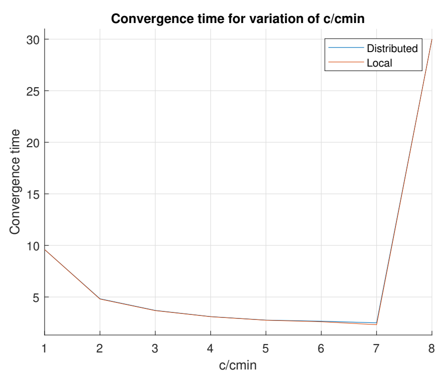
where K is the solution of the Algebraic Riccati Equation 6. As it can be seen in the equation 9, the value of Q and R weights is only relevant one with respect to the other, which is why it is their ratio R/Q that matters.
For values of R/Q ratio ranging from 1/10 to 10, we analyzed the effect on peak command value (left plot, logarithmic scale) and the effect on the convergence time.

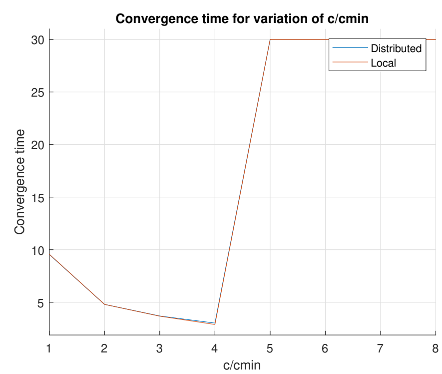


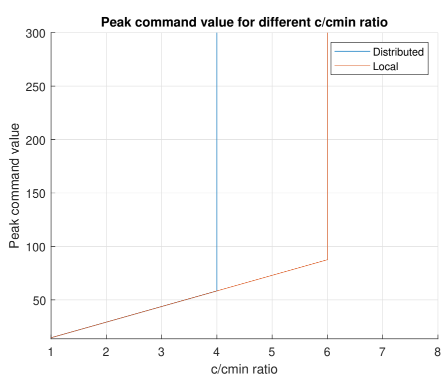


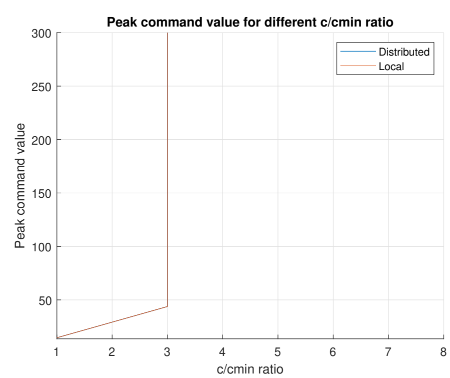
As it can be seen from the above plots, for higher values of R/Q ratio we get an exponentially lower convergence time, but at the same time a linear increase (exponential in logarithmic scale) in peak command value. Considerations on R/Q values to implement in a real multi-agents system of this kind have to be derived according to limitations imposed by real world physical devices.

On another hand we addressed an analysis on the variation of the coupling gain parameter and its effects on the system. The SBVF theory imposes a constraint on the minimum value of the coupling gain, as stated in equation 6. We plotted the values of convergence time and peak command value for different values of c/cmin, in case of an ideal system (left couple of plots) and in case of a system subject to errors on output measurements (right couple of plots).









No error

With error

As we can see in the above plots, we experimentally found that there is a certain critical threshold of the

coupling gain value after which the system starts to behave in an unstable way, even if all the sufficient conditions for stability are satisfied. Oscillations in the control input get amplified exponentially and lead to instability of the system. It is important to note that, in factm for this computation we had to strengthen the eigenvalues of the local observers, in order to better resist against these oscillations.
Of course, as easily visible in the graphics above, this critical threshold is even lower in case of a system that is affected by errors. For an ideal system the threshold sets at around c/cmin = 7, even if command activity begins to highly increase even earlier, whereas for a system subject to errors the threshold sets at c/cmin = 4.