Proj 1
Problem
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Derive a discretized version of the Poisson eq.

$$-\frac{d^2u}{dx^2} = f(x) \quad (*)$$

Using Taylor expansion $(x \pm h)$ and solve it for $u''(x)$

Taylor expansion:

$$u(x+h) = \sum_{n=0}^{\infty} \frac{1}{n!} u^{(n)}(x)\, h^n \quad (**)$$

$$\overset{(**)}{\Rightarrow} u(x+h) = u(x) + u'(x)h + \frac{1}{2}u''(x)h^2 + \frac{1}{6}u'''(x)h^3 + O(h^4)$$

$$\overset{(**)}{\Rightarrow} u(x-h) = u(x) - u'(x)h + \frac{1}{2}u''(x)h^2 - \frac{1}{6}u'''(x)h^3 + O(h^4)$$

$(-1)^2 = +1$

Addition:

Giving a three-point evaluation instead of two-point. This kind of evaluation gives higher accuracy, but at the cost of more operations / evaluations.

$$u(x+h) + u(x-h) = 2u(x) + u''(x)h^2 + O(h^4)$$

$$\Rightarrow u''(x) = \frac{u(x+h) - 2u(x) + u(x-h) + O(h^4)}{h^2}$$

$$= \frac{u(x+h) - 2u(x) + u(x-h)}{h^2} + O(h^2)$$

Disc.

$$u_i'' = \frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} + O(h^2)$$

Approx.

$$v_i'' = \frac{v_{i+1} - 2v_i + v_{i-1}}{h^2} \xrightarrow{(*)} -\left(\frac{v_{i+1} - 2v_i + v_{i-1}}{h^2}\right) = f_i$$

The approximation is to not take the truncation error into consideration