Proj 1
Problem

4

Rewriting the result from problem 3:

$$-\left(\frac{v_{i+1} - 2v_i + v_{i-1}}{h^2}\right) = f_i$$

$$-v_{i+1} + 2v_i - v_{i-1} = h^2 f_i \qquad (*)$$

We know the boundary points (v_0, v_a) (a = steps = highest index n.)

This gives us $a-1$ unknowns

We can then use $(*)$ to produce a set of $a-1$ equations where $i = 1, 2, \dots, a-1$:

$$\begin{aligned}
(*)\ (i=1)&: +2v_1 - v_2 &&= h^2 f_1 + v_0\\
(i=2)&: -v_1 + 2v_2 - v_3 &&= h^2 f_2\\
(i=3)&: -v_2 + 2v_3 - v_4 &&= h^2 f_3\\
&\vdots\\
(i=a-1)&: -v_{a-2} + 2v_{a-1} &&= h^2 f_{a-1} + v_a
\end{aligned}$$

As the structure of this set illustrates, this can be written as: $(h^2 f_i = g_i)$

$$\underbrace{\begin{bmatrix} 2 & -1 & 0 & \cdots & 0 \\ -1 & 2 & -1 & \cdots & 0 \\ 0 & -1 & 2 & \ddots & 0 \\ \vdots & & \ddots & \ddots & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_{a-1} \end{bmatrix}}_{\vec{v}} = \underbrace{\begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ \vdots \\ g_{a-1} \end{bmatrix}}_{\vec{g}}$$

$$\Rightarrow A\vec{v} = \vec{g}$$

matrix A has a diagonal with the values 2 and a sub- and superdiagonal with values -1