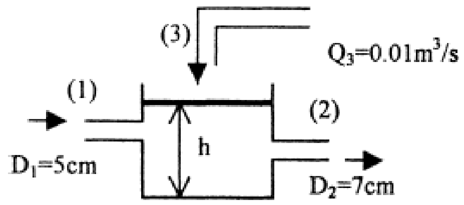
FM 题解

Chapter 3

P3. 14

P3.14 The open tank in the figure contains water at 20°C. For incompressible flow, (a) derive an analytic expression for *dh/dt* in terms of (Q1, Q2, Q3). (b) If *h* is constant, determine V2 for the given data if V1 = 3 m/s and Q3 = 0.01 m³/s.



Solution: For a control volume enclosing the tank,

$$\frac{d}{dt}\left(\int_{CV} \rho\, d\upsilon\right) + \rho(Q_2 - Q_1 - Q_3) = \rho\frac{\pi d^2}{4}\frac{dh}{dt} + \rho(Q_2 - Q_1 - Q_3),$$

$$\textit{solve} \quad \mathbf{\frac{dh}{dt} = \frac{Q_1 + Q_3 - Q_2}{(\pi d^2/4)}} \quad \textit{Ans.}\ \text{(a)}$$

If *h* is constant, then

$$Q_2 = Q_1 + Q_3 = 0.01 + \frac{\pi}{4}(0.05)^2(3.0) = 0.0159 = \frac{\pi}{4}(0.07)^2 V_2,$$

$$\textit{solve} \quad \mathbf{V_2 = 4.13\ m/s} \quad \textit{Ans.}\ \text{(b)}$$

P3. 16

P3.16 An incompressible fluid flows past an impermeable flat plate, as in Fig. P3.16, with a uniform inlet profile $u = U_o$ and a cubic polynomial exit profile

$$u \approx U_o\left(\frac{3\eta - \eta^3}{2}\right) \quad \text{where } \eta = \frac{y}{\delta}$$

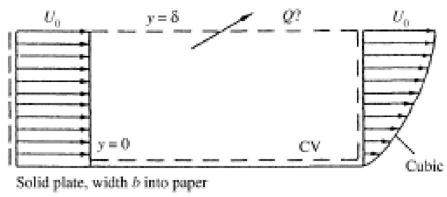


Fig. P3.16

Compute the volume flow *Q* across the top surface of the control volume.

Solution: For the given control volume and incompressible flow, we obtain

$$0 = Q_{top} + Q_{right} - Q_{left} = Q + \int_0^\delta U_o\left(\frac{3y}{2\delta} - \frac{y^3}{2\delta^3}\right) b\, dy - \int_0^\delta U_o b\, dy$$

$$= Q + \frac{5}{8}U_o b\delta - U_o b\delta, \quad \text{solve for } \mathbf{Q = \frac{3}{8}U_o b\delta} \quad \textit{Ans.}$$

P3. 45

P3.45 A 12-cm-diameter pipe, containing water flowing at 200 N/s, is capped by an orifice plate, as in Fig. P3.45. The exit jet is 25 mm in diameter. The pressure in the pipe at section 1 is 800 kPa-gage. Calculate the force F required to hold the orifice plate.

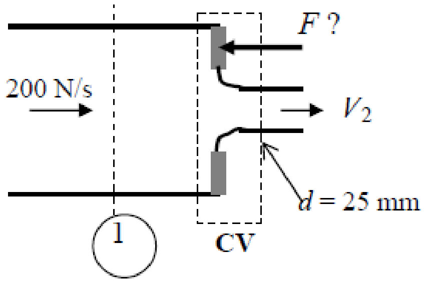


Fig. P3.45

Solution: For water take $\rho = 998$ kg/m^3. This is a straightforward x-momentum problem. First evaluate the mass flow and the two velocities:

$$\dot{m} = \frac{\dot{w}}{g} = \frac{200\,N/s}{9.81\,m/s^2} = 20.4\frac{kg}{s}\ ;\ V_1 = \frac{\dot{m}}{\rho A_1} = \frac{20.4\,kg/s}{(998\,kg/m^3)(\pi/4)(0.12m)^2} = 1.81\frac{m}{s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{20.4\,kg/s}{(998\,kg/m^3)(\pi/4)(0.025m)^2} = 41.6\frac{m}{s}$$

Now apply the x-momentum relation for a control volume surrounding the plate:

$$\Sigma F_x = -F + p_{1,gage}A_1 = \dot{m}(V_2 - V_1)\ ,\quad \text{or :}$$

$$F = (800000Pa)\frac{\pi}{4}(0.12m)^2 - (20.4\frac{kg}{s})(41.6 - 1.81\frac{m}{s}) = 9048 - 812 = \mathbf{8240\,N}\quad Ans.$$

P3. 46

P3.46 When a jet strikes an inclined plate, it breaks into two jets of equal velocity V but unequal fluxes αQ at (2) and (1 – α)Q at (3), as shown. Find α, assuming that the tangential force on the plate is zero. Why doesn't the result depend upon the properties of the jet flow?

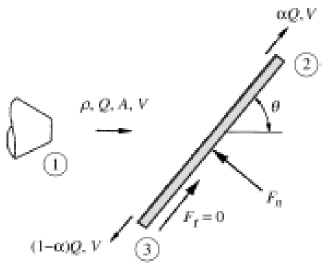


Fig. P3.46

Solution: Let the CV enclose all three jets and the surface of the plate. Analyze the force and momentum balance *tangential* to the plate:

$$\Sigma F_t = F_t = 0 = \dot{m}_2 V + \dot{m}_3(-V) - \dot{m}_1 V\cos\theta$$

$$= \alpha\dot{m}V - (1-\alpha)\dot{m}V - \dot{m}V\cos\theta = 0,\quad \text{solve for } \alpha = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{(1+\cos\theta)}\quad Ans.$$

The jet mass flow cancels out. Jet (3) has a fractional flow (1 – α) = (1/2)(1 – cosθ).

P3. 55

P3.55 In Fig. P3.55 the jet strikes a vane which moves to the right at constant velocity V_c on a frictionless cart. Compute (a) the force F_x required to restrain the cart and (b) the power P delivered to the cart. Also find the cart velocity for which (c) the force F_x is a maximum and (d) the power P is a maximum.

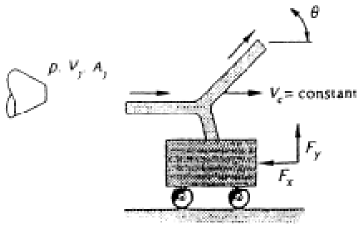


Fig. P3.55

Solution: Let the CV surround the vane and cart and move to the right at cart speed. The jet strikes the vane at *relative* speed Vj − Vc. The cart does not accelerate, so the horizontal force balance is

$$\Sigma F_x = -F_x = [\rho A_j(V_j - V_c)](V_j - V_c)\cos\theta - \rho A_j(V_j - V_c)^2$$

$$\text{or:}\quad \mathbf{F_x = \rho A_j(V_j - V_c)^2(1-\cos\theta)} \quad \textit{Ans. (a)}$$

The power delivered is $P = V_c F_x = \boldsymbol{\rho A_j V_c (V_j - V_c)^2(1-\cos\theta)}$ *Ans.* (b)

The maximum <u>force</u> occurs when the cart is fixed, or: $V_c = \mathbf{0}$ *Ans.* (c)

The maximum <u>power</u> occurs when $dP/dV_c = 0$, or: $V_c = \mathbf{V_j/3}$ *Ans.* (d)

P3. 121

P3.121 In Fig. P3.121 the flowing fluid is CO2 at 20°C. Neglect losses. If $p1$ = 170 kPa and the manometer fluid is Meriam red oil (SG = 0.827), estimate (a) $p2$ and (b) the gas flow rate in m^3/h.

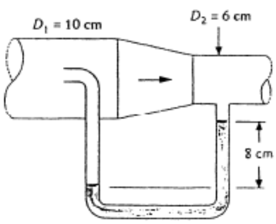


Fig. P3.121

Solution: Estimate the CO2 density as $\rho = p/RT = (170000)/[189(293)] \approx 3.07\ kg/m^3$. The manometer reading gives the downstream pressure:

$$p_1 - p_2 = (\rho_{oil} - \rho_{CO_2})gh = [0.827(998) - 3.07](9.81)(0.08) \approx 645\ Pa$$

$$\text{Hence}\quad p_2 = 170{,}000 - 645 \approx \mathbf{169400\ Pa} \quad \textit{Ans. (a)}$$

Now use Bernoulli to find V2, assuming p1 ≈ stagnation pressure (V1 = 0):

$$p_1 + \frac{1}{2}\rho(0)^2 \approx p_2 + \frac{1}{2}\rho V_2^2,$$

$$\text{or:}\quad V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho}} = \sqrt{\frac{2(645)}{3.07}} \approx 20.5\ \frac{m}{s}$$

$$\text{Then}\quad Q = V_2 A_2 = (20.5)(\pi/4)(0.06)^2 = 0.058\ m^3/s \approx \mathbf{209\ \frac{m^3}{hr}} \quad \textit{Ans. (b)}$$

P3. 131

P3.131 In Fig. P3.131 both fluids are at 20°C. If V_1 = 1.7 ft/s and losses are neglected, what should the manometer reading h ft be?

Solution: By continuity, establish V_2:

$$V_2 = V_1(D_1/D_2)^2 = 1.7(3/1)^2 = 15.3\ \frac{\text{ft}}{\text{s}}$$

Now apply Bernoulli between 1 and 2 to establish the pressure at section 2:

$$p_1 + \frac{\rho}{2}V_1^2 + \rho g z_1 = p_2 + \frac{\rho}{2}V_2^2 + \rho g z_2,$$

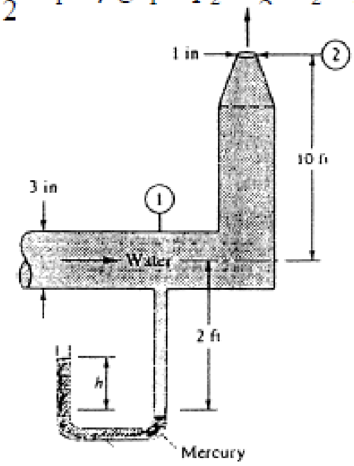


Fig. P3.131

or: $p_1 + (1.94/2)(1.7)^2 + 0 \approx 0 + (1.94/2)(15.3)^2 + (62.4)(10), \quad p_1 = 848\ \text{psf}$

This is gage pressure. Now the manometer *reads* gage pressure, so

$p_1 + (62.4)(2\ ft) - 846\,h = 848 + 124.8 - 846\,h = p_a = 0$

solve for $h = \dfrac{973}{846} = 1.15\ ft$ *Ans.*

P3. 154

P3.154 Water at 20°C flows at 30 gal/min through the 0.75-in-diameter double pipe bend of Fig. P3.154. The pressures are $p1$ = 30 lbf/in^2 and $p2$ = 24 lbf/in^2. Compute the torque T at point B necessary to keep the pipe from rotating.

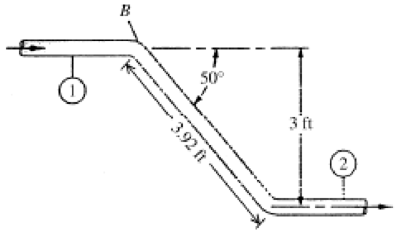


Fig. P3.154

Solution: This is similar to Example 3.13, of the text. The volume flow Q = 30 gal/min = 0.0668 ft^3/s, and ρ = 1.94 slug/ft^3. Thus the mass flow ρQ = 0.130 slug/s. The velocity in the pipe is

$$V_1 = V_2 = Q/A = \frac{0.0668}{(\pi/4)(0.75/12)^2} = 21.8\ \frac{ft}{s}$$

If we take torques about point B, then the distance "h1," from p. 143, = 0, and h2 = 3 ft. The final torque at point B, from "Ans. (a)" on p. 143 of the text, is

$$T_B = h_2(p_2A_2 + \dot{m}V_2) = (3\text{ ft})[(24\text{ psi})\frac{\pi}{4}(0.75\text{ in})^2 + (0.130)(21.8)] \approx \mathbf{40\ ft \cdot lbf} \quad \textit{Ans.}$$

P3. 183

P3.183 The pump in Fig. P3.183 creates a 20°C water jet oriented to travel a maximum horizontal distance. System friction head losses are 6.5 m. The jet may be approximated by the trajectory of frictionless particles. What power must be delivered by the pump?

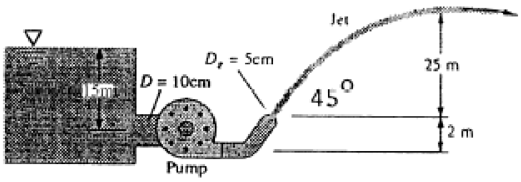


Fig. P3.183

Solution: For maximum travel, the jet must exit at θ = 45°, and the exit velocity must be

$$V_2 \sin\theta = \sqrt{2g\Delta z_{max}} \quad \text{or:} \quad V_2 = \frac{[2(9.81)(25)]^{1/2}}{\sin 45°} \approx 31.32\ \frac{m}{s}$$

The steady flow energy equation for the piping system may then be evaluated:

$$p_1/\gamma + V_1^2/2g + z_1 = p_2/\gamma + V_2^2/2g + z_2 + h_f - h_p,$$

or: $0 + 0 + 15 = 0 + (31.32)^2/[2(9.81)] + 2 + 6.5 - h_p$, solve for $h_p \approx 43.5$ m

$$\text{Then}\quad P_{pump} = \gamma Q h_p = (9790)\left[\frac{\pi}{4}(0.05)^2(31.32)\right](43.5) \approx \mathbf{26200\ W} \quad \textit{Ans.}$$

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Fundamentals of Enginerring Exam (FEE)Problems

FE3.1 (a) 2.6 m/s

FE3.2 (d) 22 kPa

FE3.3 (b) 36 N

FE3.4 (c) 2.8 m/s

FE3.5 (b) 5.7 m/s

FE3.6 (c) 32 m

FE3.7 (a) 187 gal/min

FE3.8 (c) 154 gal/min

FE3.10 (c) 263 gal/min

Chapter 4

P4.1

P4.1 An idealized velocity field is given by the formula

$$\mathbf{V} = 4tx\mathbf{i} - 2t^2y\mathbf{j} + 4xz\mathbf{k}$$

Is this flow field steady or unsteady? Is it two- or three-dimensional? At the point $(x, y, z) = (-1, +1, 0)$, compute (a) the acceleration vector and (b) any unit vector normal to the acceleration.

Solution: (a) The flow is unsteady because time t appears explicitly in the components. (b) The flow is three-dimensional because all three velocity components are nonzero. (c) Evaluate, by laborious differentiation, the acceleration vector at (x, y, z) = (−1, +1, 0).

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = 4x + 4tx(4t) - 2t^2y(0) + 4xz(0) = 4x + 16t^2x$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -4ty + 4tx(0) - 2t^2y(-2t^2) + 4xz(0) = -4ty + 4t^4y$$

$$\frac{dw}{dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = 0 + 4tx(4z) - 2t^2y(0) + 4xz(4x) = 16txz + 16x^2z$$

$$\text{or: } \frac{d\mathbf{V}}{dt} = (4x + 16t^2x)\mathbf{i} + (-4ty + 4t^4y)\mathbf{j} + (16txz + 16x^2z)\mathbf{k}$$

at (x, y, z) = (−1, +1, 0), we obtain $\dfrac{d\mathbf{V}}{dt} = -4(1 + 4t^2)\mathbf{i} - 4t(1 - t^3)\mathbf{j} + 0\mathbf{k}$ *Ans.* (c)

(d) At (−1, +1, 0) there are <u>many</u> unit vectors normal to dV/dt. One obvious one is **k**. *Ans.*

P4. 2

P4.2 Flow through the converging nozzle in Fig. P4.2 can be approximated by the one-dimensional velocity distribution

$$u \approx V_o\left(1+\frac{2x}{L}\right) \quad \upsilon \approx 0 \quad w \approx 0$$

(a) Find a general expression for the fluid acceleration in the nozzle. (b) For the specific case V_O = 10 ft/s and L = 6 in, compute the acceleration, in g's, at the entrance and at the exit.

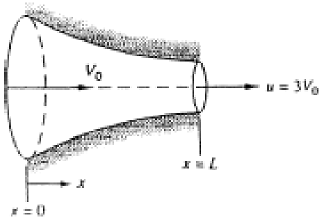

Fig. P4.2

Solution: Here we have only the single 'one-dimensional' convective acceleration:

$$\frac{du}{dt}=u\frac{\partial u}{\partial x}=\left[V_o\left(1+\frac{2x}{L}\right)\right]\frac{2V_o}{L}=\frac{\mathbf{2V_o^2}}{\mathbf{L}}\left(\mathbf{1}+\frac{\mathbf{2x}}{\mathbf{L}}\right) \quad \textit{Ans. (a)}$$

$$For\ L=6''\ and\ V_o=10\ \frac{ft}{s},\quad \frac{du}{dt}=\frac{2(10)^2}{6/12}\left(1+\frac{2x}{6/12}\right)=400(1+4x),\ \ with\ x\ in\ feet$$

At x = 0, du/dt = **400** ft/s^2 (12 g's); at x = L = 0.5 ft, du/dt = **1200** ft/s^2 (37 g's). *Ans.* (b)

P4. 3

P4.3 A two-dimensional velocity field is given by

$$\mathbf{V}=(x^2-y^2+x)\mathbf{i}-(2xy+y)\mathbf{j}$$

in arbitrary units. At (x, y) = (1, 2), compute (a) the accelerations ax and ay, (b) the velocity component in the direction θ = 40°, (c) the direction of maximum velocity, and (d) the direction of maximum acceleration.

Solution: (a) Do each component of acceleration:

$$\frac{du}{dt}=u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=(x^2-y^2+x)(2x+1)+(-2xy-y)(-2y)=a_x$$

$$\frac{dv}{dt}=u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=(x^2-y^2+x)(-2y)+(-2xy-y)(-2x-1)=a_y$$

At (x, y) = (1, 2), we obtain **ax** = 18**i** and **ay** = 26**j** *Ans.* (a)

(b) At (x, y) = (1, 2), **V** = –2**i** – 6**j**. A unit vector along a 40° line would be **n** = cos40°**i** + sin40°**j**. Then the velocity component along a 40° line is

$$V_{40°}=\mathbf{V}\cdot\mathbf{n}_{40°}=(-2\mathbf{i}-6\mathbf{j})\cdot(\cos 40°\mathbf{i}+\sin 40°\mathbf{j})\approx \mathbf{5.39\ units} \quad \textit{Ans. (b)}$$

(c) The maximum velocity is [(-2)2 + (-6)2]1/2 = 5.32 units, at an angle in the third quadrant,

θ = 180° + arctan(-6/-2) = 180° + 71.6° = .251.6°. Ans. (c)

(d) The maximum acceleration is amax . [182 262]1/2 . 31.6 units at .55.3. Ans. (c, d)

P4. 9

P4.9 An idealized incompressible flow has the proposed three-dimensional velocity distribution

$$\mathbf{V}=4xy^2\mathbf{i}+f(y)\mathbf{j}-zy^2\mathbf{k}$$

Find the appropriate form of the function $f(y)$ which satisfies the continuity relation.

Solution: Simply substitute the given velocity components into the incompressible continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=\frac{\partial}{\partial x}(4xy^2)+\frac{\partial f}{\partial y}+\frac{\partial}{\partial z}(-zy^2)=4y^2+\frac{df}{dy}-y^2=0$$

$$or:\ \frac{df}{dy}=-3y^2.\quad \text{Integrate: } f(y)=\int(-3y^2)dy=\mathbf{-y^3+constant}\quad Ans.$$

P4. 27

P4.27 A frictionless, incompressible steady-flow field is given by

$$\mathbf{V}=2xy\mathbf{i}-y^2\mathbf{j}$$

in arbitrary units. Let the density be ρ_0 = constant and neglect gravity. Find an expression for the pressure gradient in the x direction.

Solution: For this (gravity-free) velocity, the momentum equation is

$$\rho\left(u\frac{\partial \mathbf{V}}{\partial x}+v\frac{\partial \mathbf{V}}{\partial y}\right)=-\nabla p,\quad \text{or:}\quad \rho_o[(2xy)(2y\mathbf{i})+(-y^2)(2x\mathbf{i}-2y\mathbf{j})]=-\nabla p$$

$$\text{Solve for}\quad \nabla p=-\rho_o(2xy^2\mathbf{i}+2y^3\mathbf{j}),\quad \text{or:}\quad \boldsymbol{\frac{\partial p}{\partial x}=-\rho_o 2xy^2}\quad Ans.$$

P4. 28

P4.28 Consider the incompressible flow field of Prob. P4.6, with velocity components $u = 2y$, $v = 8x$, $w = 0$. Neglect gravity and assume constant viscosity. (*a*) Determine whether this flow satisfies the Navier-Stokes equations. (*b*) If so, find the pressure distribution $p(x, y)$ if the pressure at the origin is p_o.

Solution: In Prob. P4.6 we found the accelerations, so we can proceed to Navier-Stokes:

$$\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=\rho[0+(8x)(2)]=-\frac{\partial p}{\partial x}+\rho g_x+\mu\nabla^2 u=-\frac{\partial p}{\partial x}+0+0;\ \frac{\partial p}{\partial x}=-16\rho x$$

$$\rho(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=\rho[(2y)(8)+0]=-\frac{\partial p}{\partial y}+\rho g_y+\mu\nabla^2 v=-\frac{\partial p}{\partial y}+0+0;\ \frac{\partial p}{\partial y}=-16\rho y$$

Noting that

$\partial^2 p/(\partial x\partial y)=0$ in both cases, we conclude **Yes, satisfies Navier - Stokes.** *Ans.(a)*

(*b*) The pressure gradients are simple, so we may easily integrate:

$$dp=\frac{\partial p}{\partial x}dx+\frac{\partial p}{\partial y}dy,\ \text{or: } p=\int-16\rho x\,dx+\int-16\rho y\,dy=-8\rho(x^2+y^2)+const$$

$$\text{If } p(0,0)=p_o,\ then\quad p=p_o-8\rho(x^2+y^2)\qquad Ans.(b)$$

This is an exact solution, but it is *not* Bernoulli's equation. The flow is *rotational*.

P4.61

P4.61 For the incompressible plane flow of Prob. P4.6, with velocity components $u = 2y$, $v = 8x$, $w = 0$, determine (*a*) if a velocity potential exists. (*b*) If so, determine the form of the velocity potential, and (*c*) plot a few representative potential lines.

Solution: (*a*) A velocity potential exists if the vorticity is zero. Here, for plane flow in (*x*, *y*) coordinates, we need only evaluate rotation around the *z* axis:

$$\varsigma_z = 2\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 8 - 2 = +6 \neq 0 \qquad \text{Rotational, } \phi \text{ does not exist.} \quad \mathit{Ans.}(a)$$

(*b, c*) There is **no velocity potential** – no plot, no formula. The flow has constant vorticity.

Fundamentals of Enginerring Exam (FEE) Problems

FE4.1 (e) - 3

FE4.2 (c) 0

FE4.3 (c) 0

FE4.4 (d) - B

FE4.5 (b) A^2x