Data Structures & Algorithms 1

Topic 6 – Sorting algorithms

Sorting

- To be useful, data should be sorted so that it is easy to search through
- Often we will get an array of data that is in a random order and we will need to sort it
- We need to develop some sorting algorithms to sort arrays - the more efficient the sorting algorithm, the better
- There are many different sorting algorithms and the correct one to choose depends on the following:
  - The amount of data that needs to be sorted
  - The amount of memory that is available
  - The amount of time that is available
  - The way the data is distributed

Sorting

- For starters we will consider three simple sorting algorithms
  - bubble sort
  - selection sort
  - insertion sort
- All of these algorithms have a complexity of $O(n^2)$
- This means that as the size of the array increases, the time taken to sort it will increase by that amount squared
  - An array that is twice as big will take four times longer to sort
  - An array that is three times bigger will take nine times longer to sort
- For that reason, these algorithms are only used to sort small amounts of data!

Swapping

- Swapping elements in an array is crucial to be able to sort it
- Lets swap slot 2 with slot 5

1	Backup slot 2 into temp	[5 4 8 9 1]	4
2	Copy slot 5 into slot 2	[5 1 8 9 1]	4
3	Copy temp into slot 5	[5 1 8 9 4]	4

Swapping

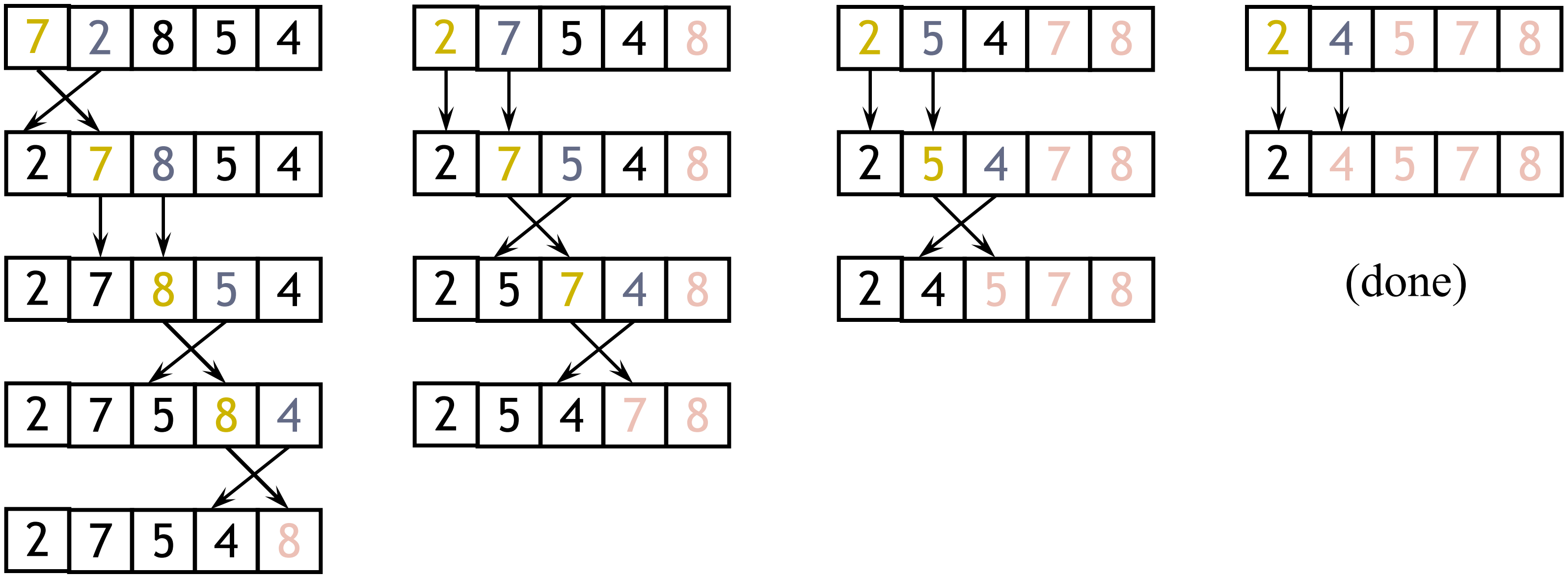
- In order to swap one variable with another in an array
  - Back-up slot A (the one that will be overwritten first) into a temporary variable
  - Overwrite slot A with the value of slot B
  - Use the temporary variable to overwrite the value of slot B with the original value of slot A

```
int temp = array[a];
array[a]=array[b];
array[b]=temp;
```

Bubble Sort

- The bubble sort algorithm works like this:
  - Start at the beginning of the array
  - Compare the first two numbers
  - If the one to the left is bigger, swap it with the one on the right
  - Move one position to the right and check the next two
  - Keep doing this until you reach the end of the array
  - The biggest number has now been ‘bubbled’ to the top
  - Go back to the beginning, do the same thing and bubble the next biggest number up
  - Stop short of the top part of the array where the ‘bubbles’ have arrived

Example of Bubble Sort



Implementing bubble sort

- How would you write this in Java?
- Because of all the swapping, it makes sense to write a swap() method

```
public void swap(int first, int second){
    int temp = array[first];
    array[first] = array[second];
    array[second] = temp;
}
```

Main bubble sort method

- Pseudo code might look something like this

```
outer 'bubbling' loop running from end of array backwards, bubbling
biggest element to the top each time it runs
{

    inner 'swapping' loop running from start of array up to last unsorted
    element, swapping two elements at a time
    {

                check if element[i] > element[i+1]
                if so, swap them
    }
}
```

Java Implementation

```
public void bubblesort(){
  int outer, inner;
  for(outer=nElems-1; outer>0;outer--){
      for(inner=0;inner<outer;inner++){
            if( array[inner] > array[inner+1]){
                  swap(inner,inner+1);
            }
      }
  }
```

Algorithm complexity

- The complexity of a sorting algorithm is the relationship between the size of the array to be sorted and the length of time it takes to sort
- This depends on the number of comparisons and swaps
- A comparison involves a memory read because you are checking the difference between two numbers
  - `if (a[inner] < a[min])`
- A swap involves a memory write because you are changing the arrangement of data in memory
  - `swap(inner,inner+1);`
- Swaps take far longer to execute than comparisons because memory writes take longer than memory reads

Complexity of bubble sort

- Assume there are 10 numbers to be sorted (i.e. n is 10)
- Because the outer loop decreases by one each time the number of comparisons performed by the inner loop will be
  - 9 + 8 +7 + 6 + 5 + 4 + 3 + 2 + 1 = 45
- In general this can be expressed as
  - (n -1) + (n -2) + (n – 3) + … + 1
  - or (n-1)*(n / 2) number of comparisons

Complexity of bubble sort

- On average, the algorithm will do a swap half of the time: $n*(n-1) / 4$ in total
- In the BEST CASE SCENARIO it won't have to do any swaps
- In the WORST CASE SCENARIO it will have to perform a swap after every comparison: $(n-1)*(n / 2)$ in total
- As the size of the array n increases, the number of comparisons and swaps increases by a factor of n^2
- Therefore bubble sort is $O(n^2)$ → very inefficient

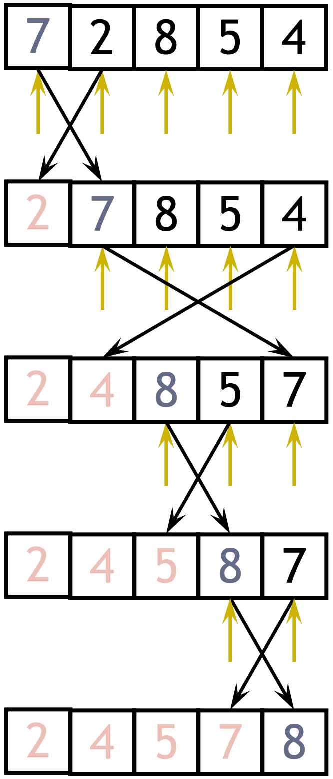
Selection Sort

- Selection sort improves on bubble sort by reducing number of swaps from $O(n^2)$ to $O(n)$
- Number of comparisons stays $O(n^2)$ but swaps are shorter than comparisons so this can be an important improvement
- Rather than continuously swapping to ‘bubble’ a number to the top, we find the smallest number and swap it into place

Selection Sort

- Given an array of length n
  - Search elements 0 through n-1 and select the smallest
    - Swap it with the element in location 0
  - Search elements 1 through n-1 and select the smallest
    - Swap it with the element in location 1
  - Search elements 2 through n-1 and select the smallest
    - Swap it with the element in location 2
  - Search elements 3 through n-1 and select the smallest
    - Swap it with the element in location 3
  - Continue in this fashion until there's nothing left to search

Selection Sort



Main selection sort method

- Pseudo code might look something like this

outer loop running through each place in the array looking for the correct element to swap into that place – starts at the beginning

```
{
    inner loop which always looks for the smallest remaining unsorted item
    {
            find minimum
            swap array[outer] with array[minimum]
    }
}
```

Code for Selection Sort

```
public static void selectionSort() {

        int min;

        for (int outer = 0; outer < array.length; outer++) {
// outer is the point where the unsorted numbers start
            min = outer;
// min's default value is the first slot to be checked
            for (int i = outer + 1; i < array.length; i++) {
// inner loop checks through the unsorted numbers
                if (array[i] < array[min]) {
                    min = i; //inner loop finds the minimum
                }
// min always refers to the min found so far
            }
            swap(outer, min);
// all items with slot numbers less than or equal to outer are
  sorted
        }
    }
```

Analysis of selection sort

- The outer loop executes n-1 times (i.e. we have to find the smallest number n-1 times)
- The inner loop executes about n/2 times on average (it executes n-1 times the first time and only once for the final time)
- Number of comparisons required is roughly (n-1)*(n/2)
- The algorithm is therefore $O(n^2)$

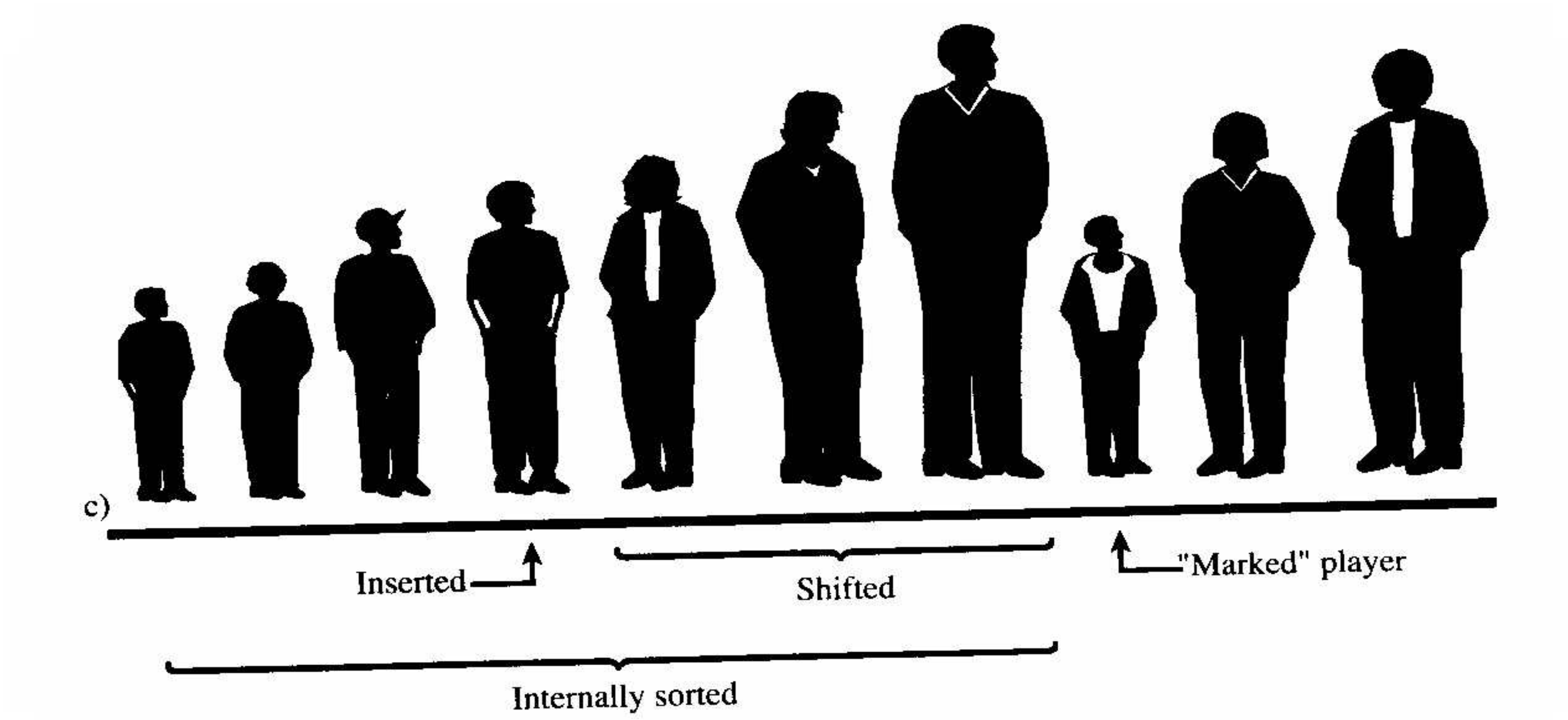
Complexity of Selection Sort

- Selection sort performs the same number of comparisons as bubble sort : $n*(n-1)/2$
- However, we have minimized the number of swaps required – we only swap something into its correct location once – a total of $n-1$ swaps
- The algorithm is still $O(n^2)$ but it is a faster $O(n^2)$ than bubble sort since a swap takes far longer than a comparison and there are less swaps involved

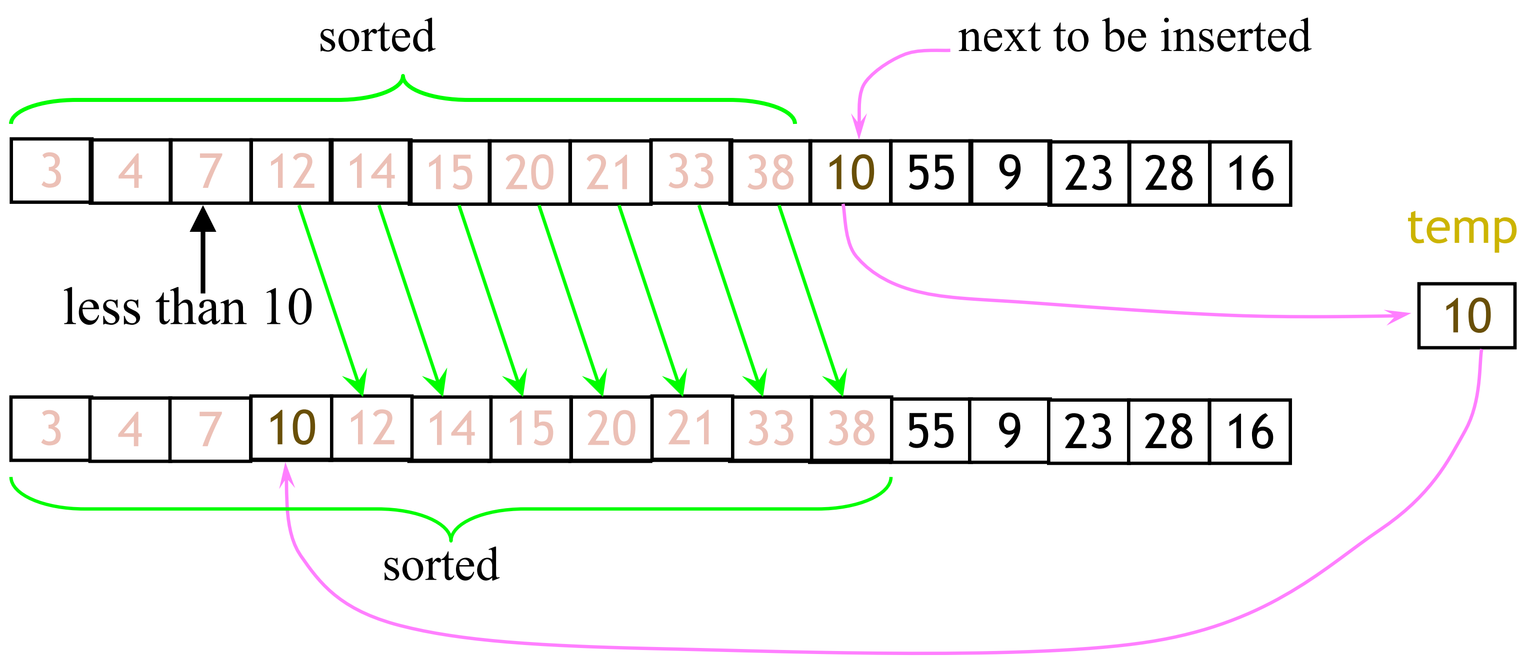
Insertion Sort

- Still $O(n^2)$ but can be faster than bubble and selection sort
- Usually used as the final stage of more sophisticated sorts, like quicksort
- Doesn’t use a swap – instead shifts elements up to make space
- The idea is that the elements on the left of a marker are already sorted
- You make space to slot in the new unsorted element by moving all the other elements up one
- Exactly like insertion into an ordered array

Insertion Sort



Insertion Sort



Code for Insertion Sort

```
public static void insertionSort() {

    for (int outer = 1; outer < array.length; outer++) {
           // outer is the next element to be sorted

        int temp = array[outer];  //back it up
        int inner = outer; // inner used to track shifts
        while (inner > 0 && array[inner - 1] >= temp) {

             array[inner] = array[inner - 1];// swap
             inner--;

        }   //shift them all right until one is smaller
        array[inner] = temp;
    }
}
```

Analysis of insertion sort

- We run once through the outer loop, inserting each of n elements
- On average, there are $n/2$ elements already sorted
  - The inner loop checks and moves half of these
  - This gives a second factor of $n/4$
- Hence, the time required for an insertion sort of an array of n elements is proportional to $n^2/4$
- Discarding constants, we find that insertion sort is $O(n^2)$
- For already sorted data, runs in $O(n)$ time
- For data arranged in inverse order, runs no faster than bubble sort

Remembering

- Bubble Sort
  - Bubbles the biggest to the end by swapping every time
- Selection Sort
  - Selects the min and swaps that element to the beginning
- Insertion Sort
  - Finds where the element should go in the sorted part and moves all the elements up one to make space
- All three algorithms are considered $O(n^2)$

Comparison

- Bubble sort is useful for small amounts of data because it is easy to code
  - Comparisons: Always $O(n^2)$
  - Swaps: Depends on how sorted the list is → varies from 0 to $O(n^2)$
- Selection sort can be used when amount of data is small and memory writing is far more time consuming than memory reading
  - Comparisons: Always $O(n^2)$
  - Swaps: Always $n-1$
- Insertion sort performs well when the list is already partially sorted
  - Comparisons: Depends on how sorted the list is → varies from $O(n)$ to $O(n^2)$
  - Swaps: Depends on how sorted the list is → varies from $O(n)$ to $O(n^2)$