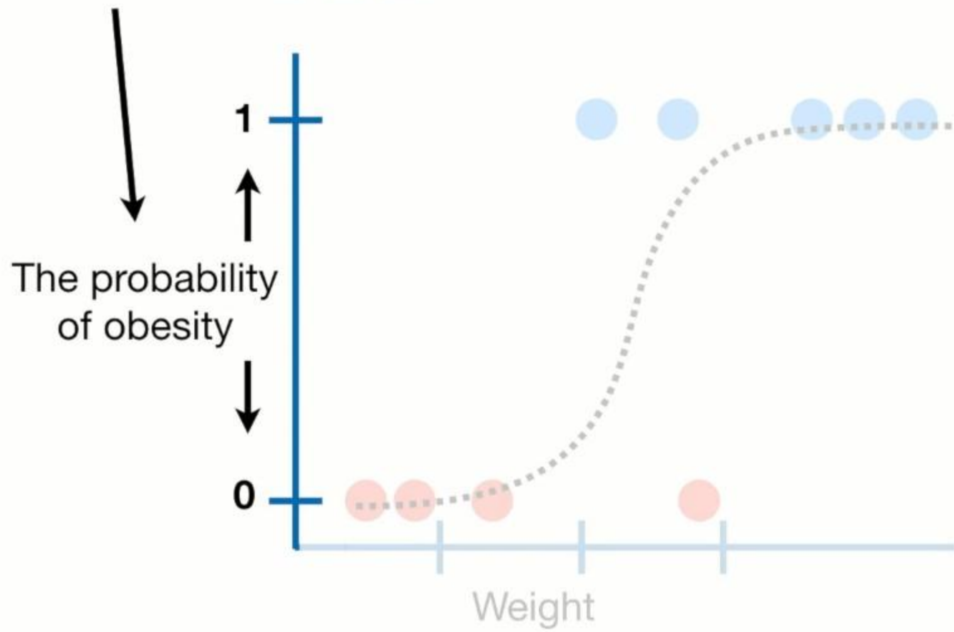
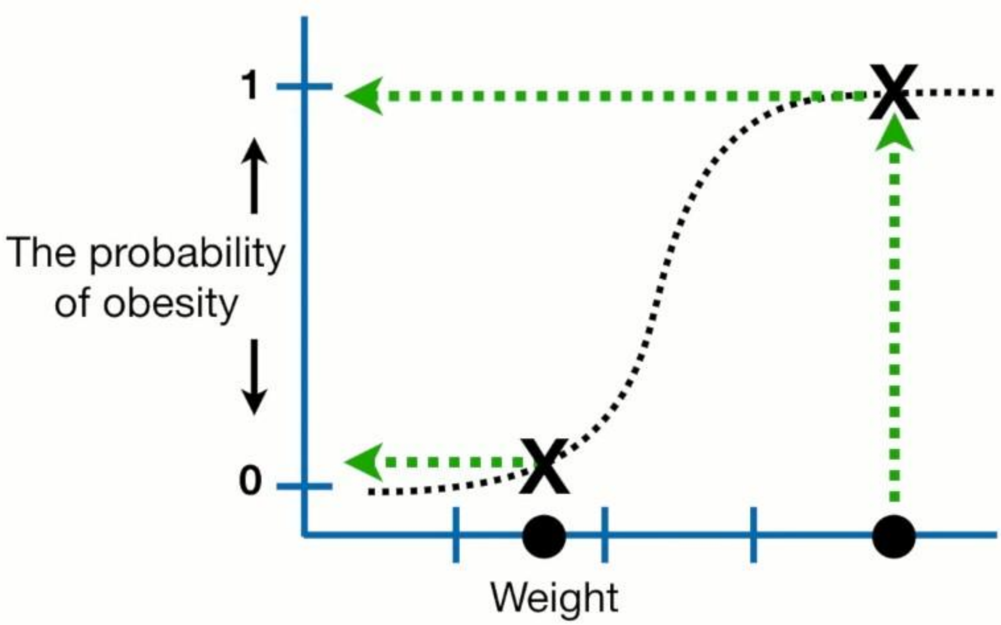
Materiale didattico per partecipante al corso "**TECNICO ESPERTO NELL'ANALISI E NELLA VISUALIZZAZIONE DEI DATI**" – Rif.P.A. 2021-15998/RER – approvata con DGR n. 1263 del 02/08/2021 di IFOA – Istituto Formazione Operatori Aziendali

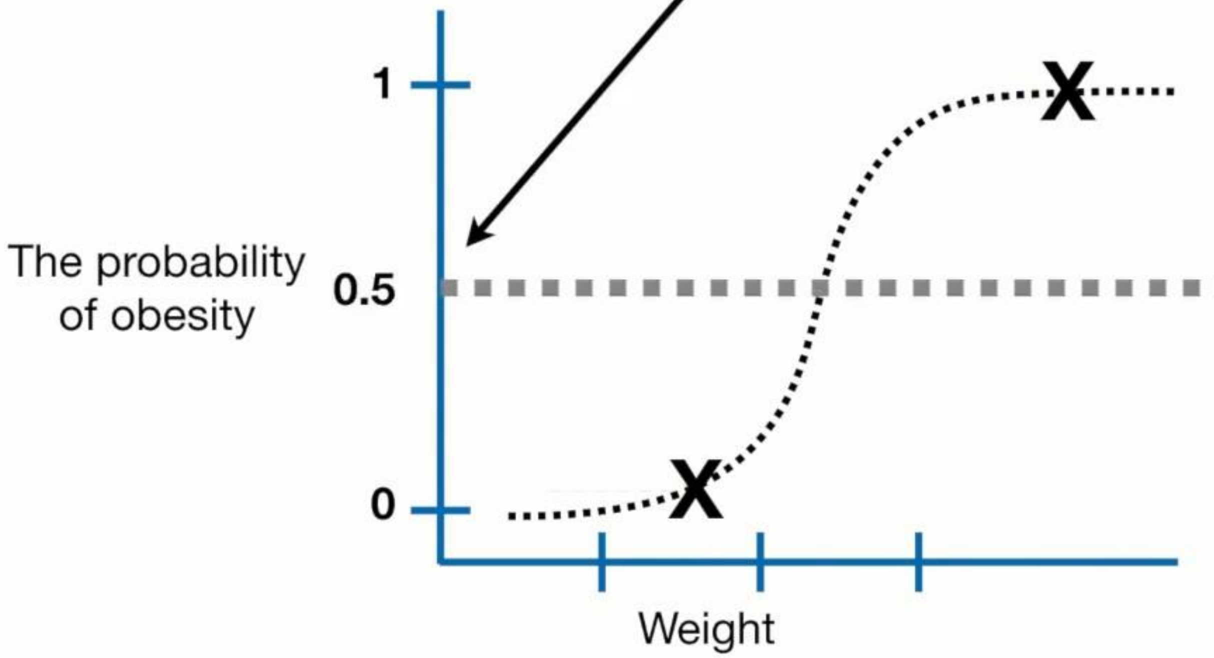
When we’re doing Logistic Regression, the y-axis is converted to the probability that a mouse **is obese**.



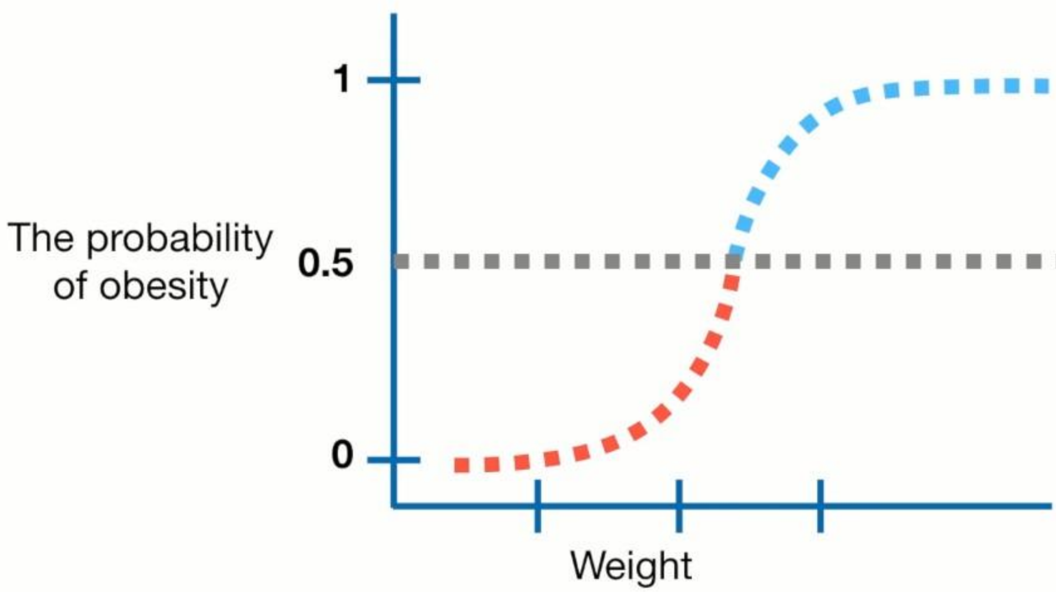
So this Logistic Regression tells us the ***probability*** that a mouse is **obese** based on its weight.

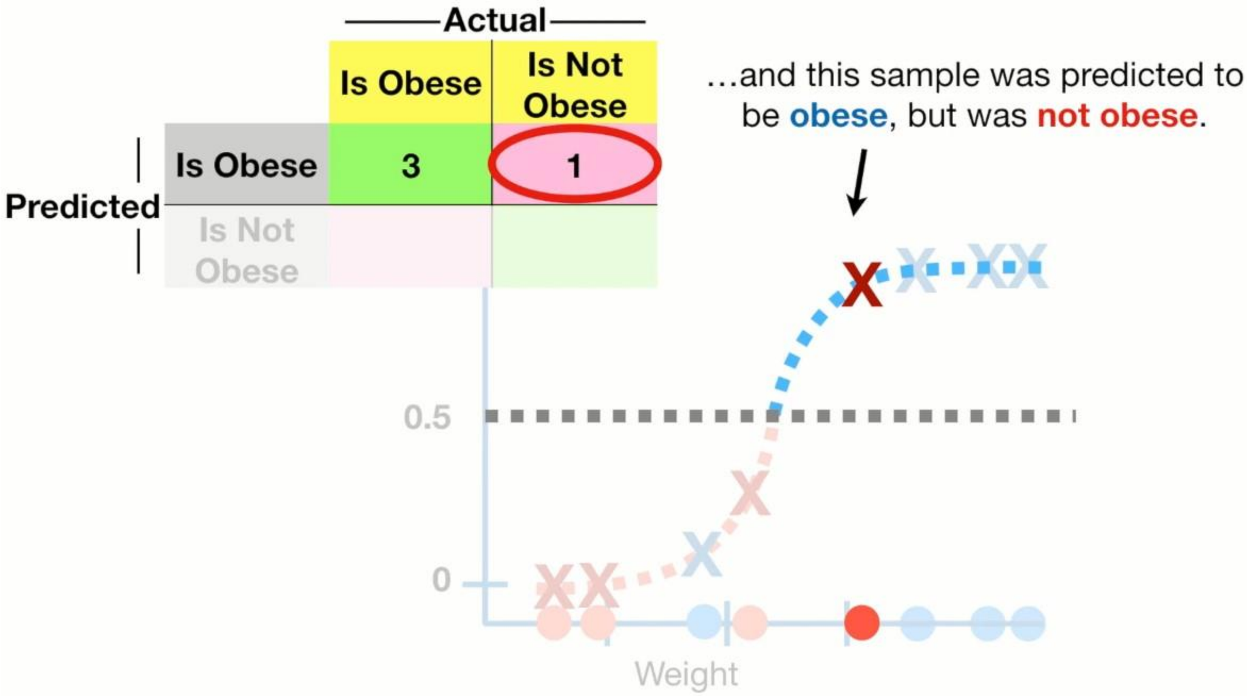


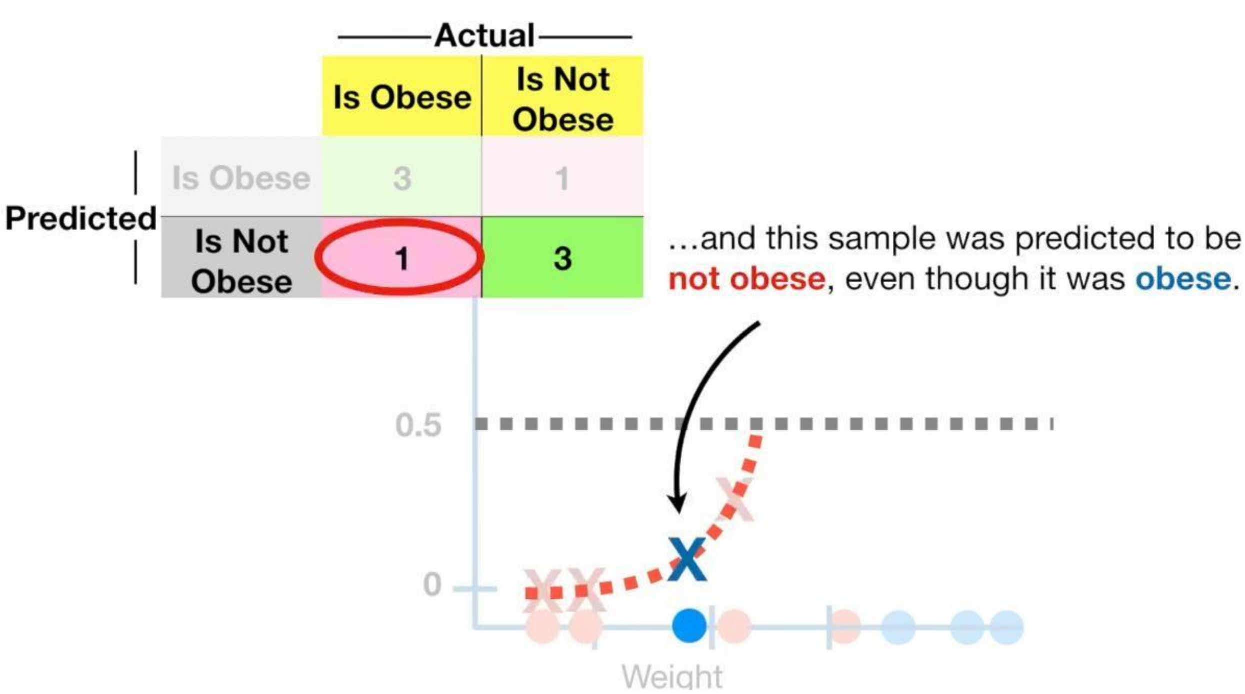
One way to classify mice is to set a threshold at **0.5**...



To evaluate the effectiveness of this Logistic Regression, with the classification threshold set to **0.5**, we can test it with mice that we know are **obese** or **not obese**.

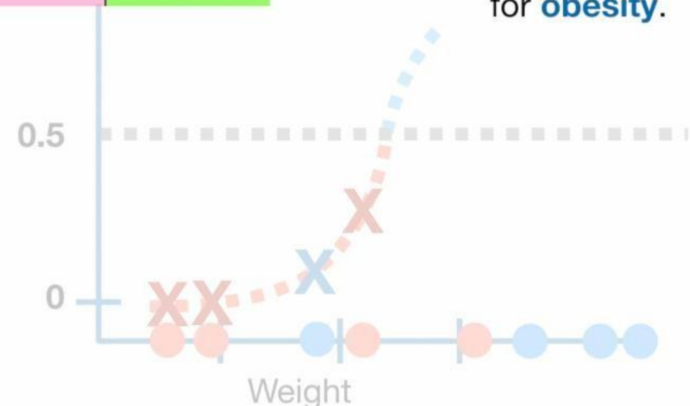



Actual
Is Obese
Is Not Obese
Predicted
Is Obese
3
1
Is Not Obese
…and this sample was predicted to be obese, but was not obese.
0.5
0
Weight


Actual
Is Obese
Is Not Obese
Predicted
Is Obese
3
1
Is Not Obese
1
3
…and this sample was predicted to be not obese, even though it was obese.
0.5
0
Weight

		Actual: Is Obese	Actual: Is Not Obese
Predicted	Is Obese	3	1
	Is Not Obese	1	3

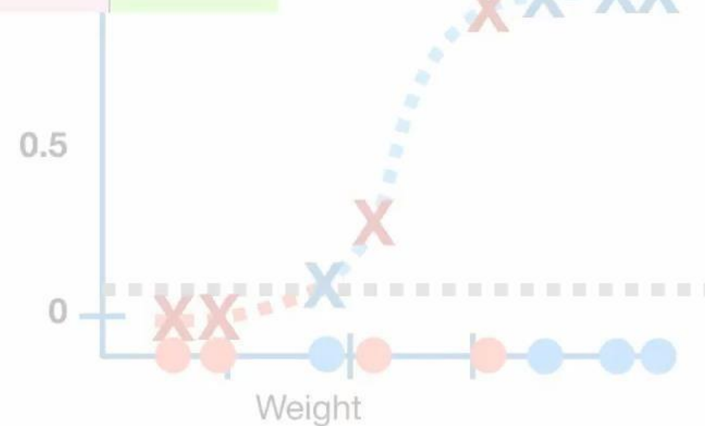
Once the **Confusion Matrix** is filled in, we can calculate **Sensitivity** and **Specificity** to evaluate this Logistic Regression when **0.5** is the threshold for **obesity**.



Now let's talk about what happens when we use a different threshold for deciding if a sample is obese or not.

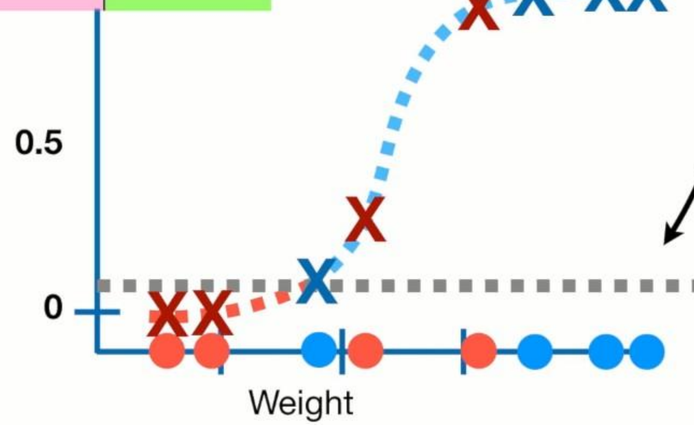
		Actual: Is Obese	Actual: Is Not Obese
Predicted	Is Obese	4	2
	Is Not Obese		

...but it would also increase the number of **False-Positives**.



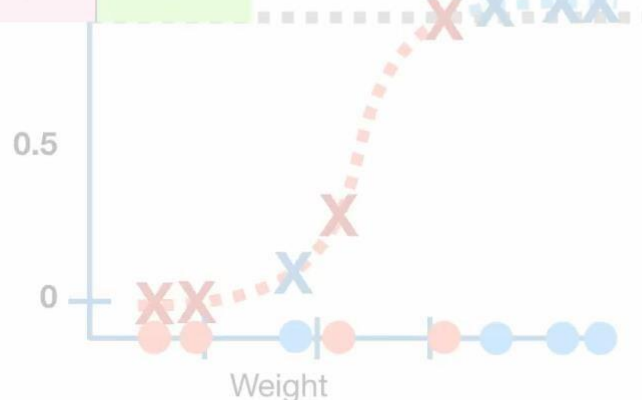
…and that means lowering the threshold, even if that results in more **False Positives**.

		Actual: Infected	Actual: Not Infected
Predicted	Infected	4	2
	Not Infected	0	2

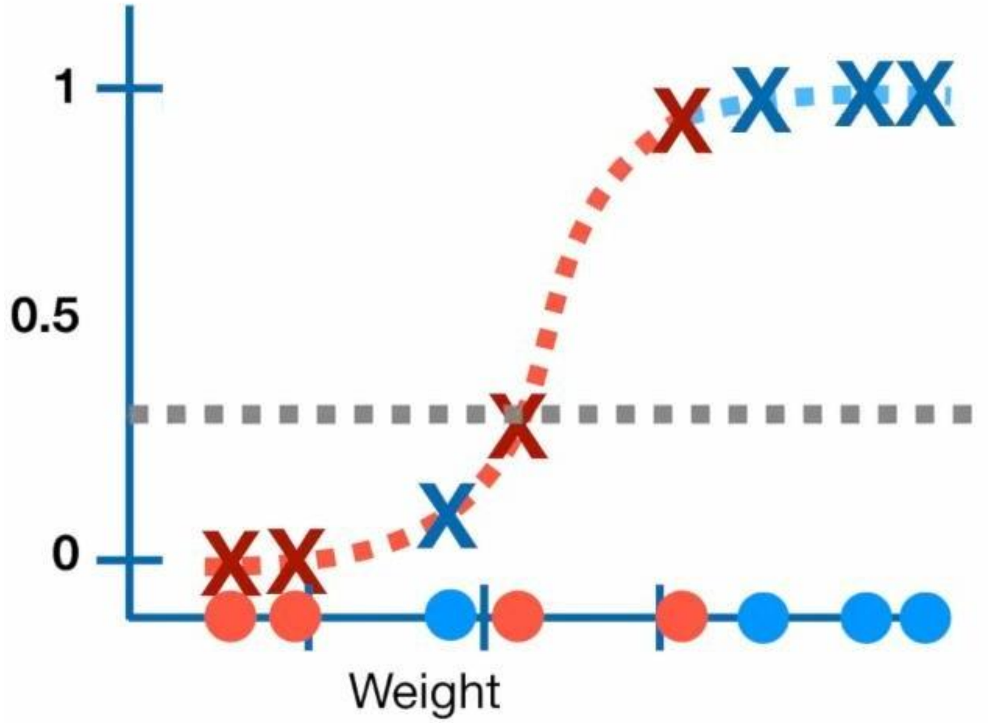


In this case, we would correctly classify the same number of **obese** samples as when the threshold was set to **0.5**…

		Actual: Is Obese	Actual: Is Not Obese
Predicted	Is Obese	3	0
	Is Not Obese	1	4

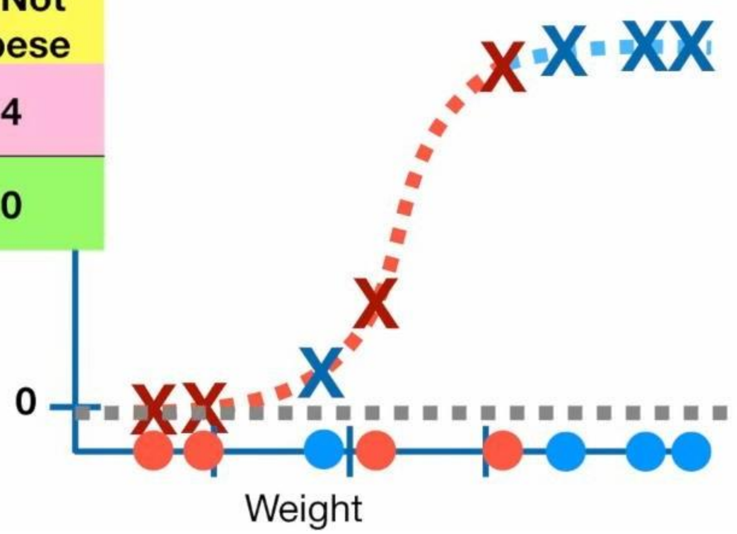


...but the threshold could be set to anything between 0 and 1.



But even if we made one confusion matrix for each threshold that mattered, it would result in a confusingly large number of confusion matrices.

	Is Obese	Is Not Obese
Is Obese	4	4
Is Not Obese	0	0



But even if we made one confusion matrix for each threshold that mattered, it would result in a confusingly large number of confusion matrices.

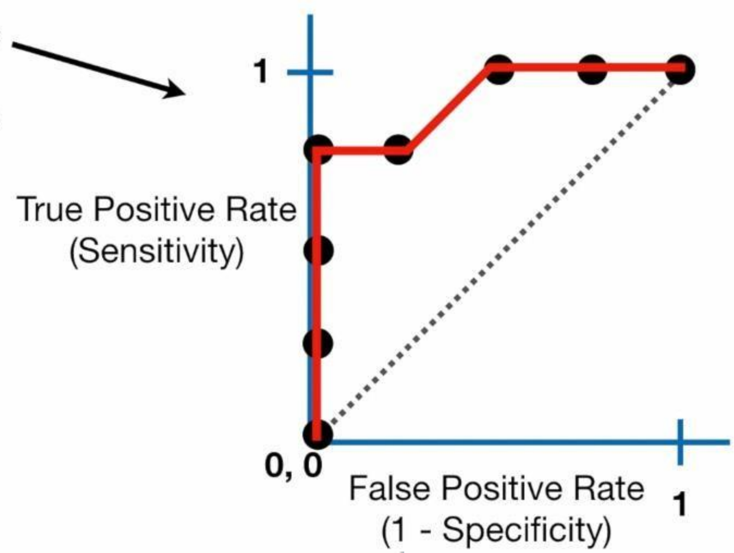
	Is Obese	Is Not Obese
Is Obese	4	[illegible]
Is Not Obese	0	[illegible]

	Is Obese	Is Not Obese
Is Obese	4	2
Is Not Obese	0	2

	Is Obese	Is Not Obese
Is Obese	4	[illegible]
Is Not Obese	0	[illegible]

	Is Obese	Is Not Obese
Is Obese	3	2
Is Not Obese	1	2

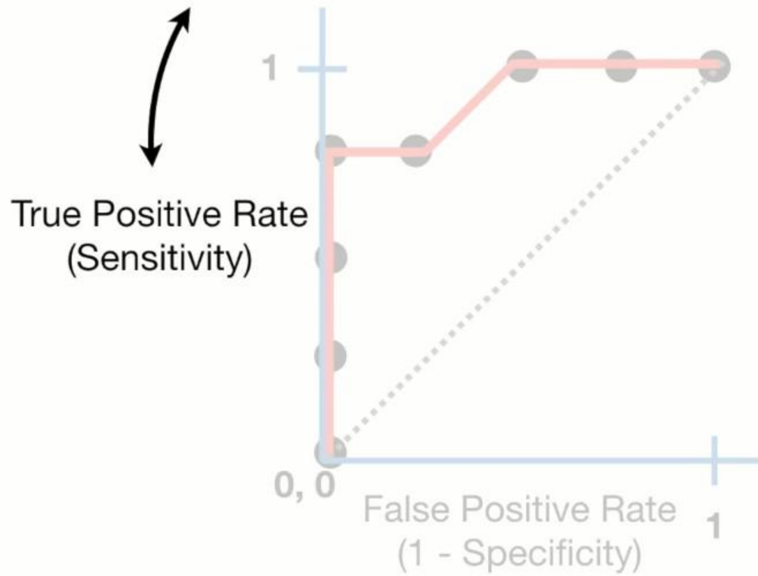
So instead of being overwhelmed with confusion matrices, **Receiver Operator Characteristic** (**ROC**) graphs provide a simple way to summarize all of the information.



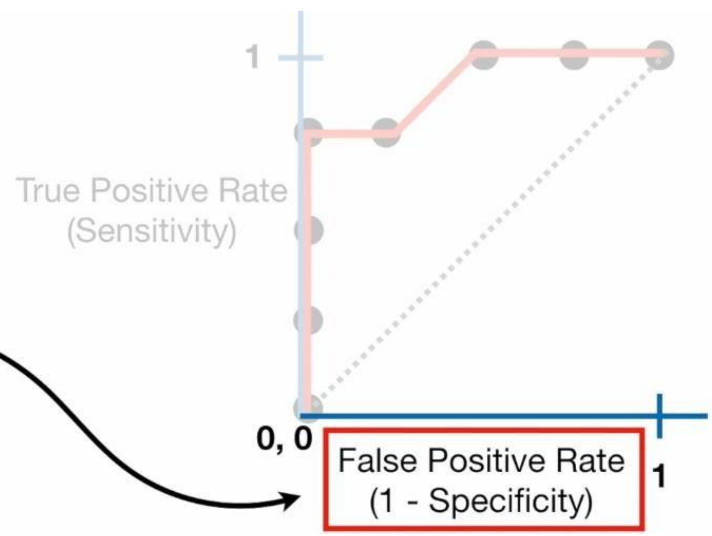
The y-axis shows the **True Positive Rate**, which is the same thing as **Sensitivity**.

True Positive Rate (Sensitivity)

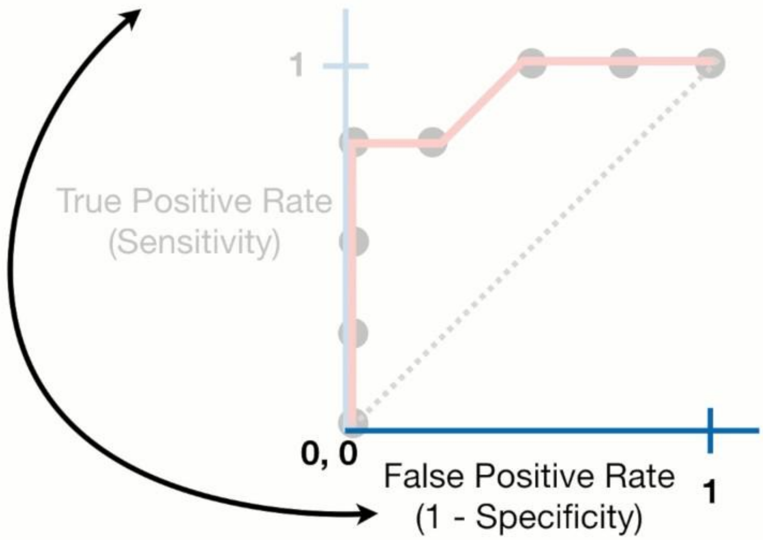
$$\text{True Positive Rate} = \text{Sensitivity} = \frac{\text{True Positives}}{\text{True Positives} + \text{False Negatives}}$$



The x-axis shows the **False Positive Rate**, which is the same thing as **1 - Specificity**.



$$\text{False Positive Rate} = (1 - \text{Specificity}) = \frac{\text{False Positives}}{\text{False Positives} + \text{True Negatives}}$$

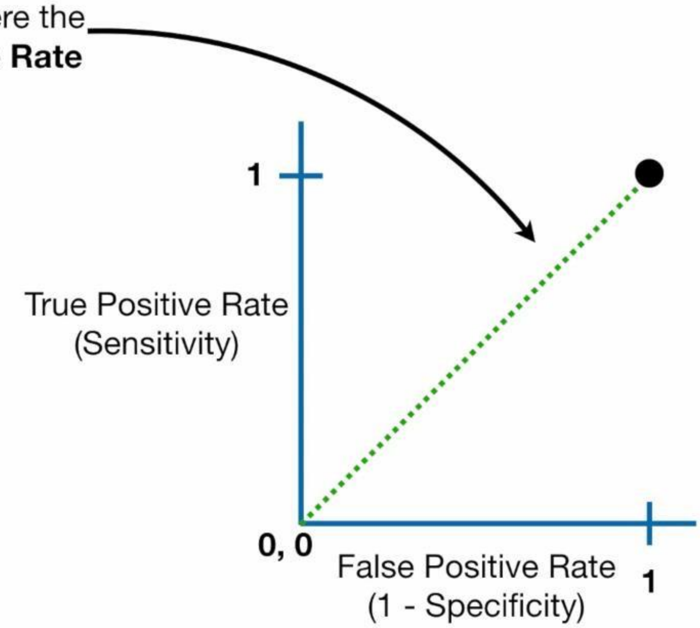


$$\text{False Positive Rate} = (1 - \text{Specificity}) = \frac{\text{False Positives}}{\text{False Positives} + \text{True Negatives}}$$

		Actual: Is Obese	Actual: Is Not Obese
Predicted	Is Obese	True Positives	**False Positives**
	Is Not Obese	False Negatives	**True Negatives**

The **False Positive Rate** tells you the proportion of **not obese** samples that were incorrectly classified and are **False Positives**.

This **green diagonal line** shows where the
True Positive Rate = **False Positive Rate**

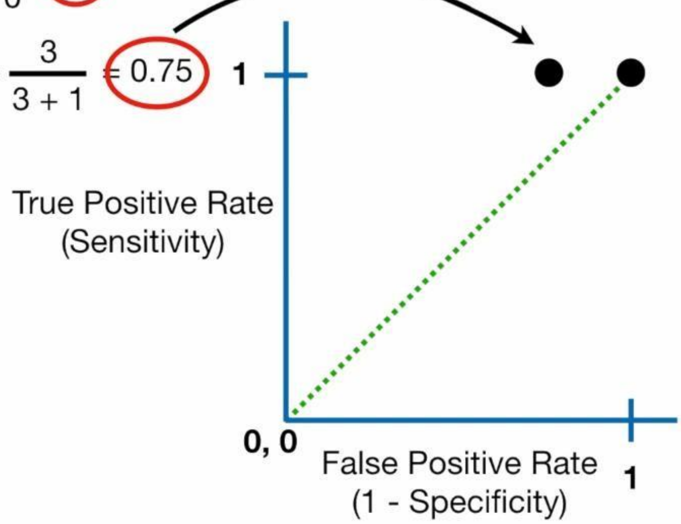


Any point on this **line** means that the **proportion** of ***correctly*** classified **obese** samples is the same as the **proportion** of ***incorrectly*** classified samples that are **not obese**.

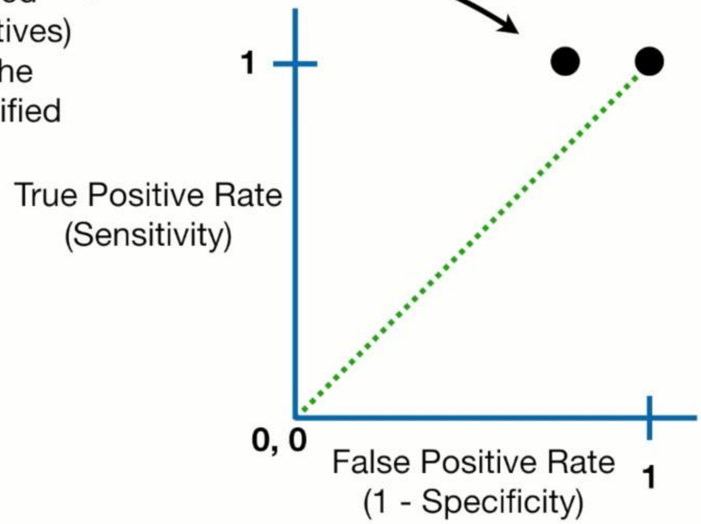
True Positive Rate = Sensitivity = $\frac{4}{4 + 0} = 1$

False Positive Rate = 1 - Specificity = $\frac{3}{3 + 1} = 0.75$

…and plot a point at **0.75, 1**.

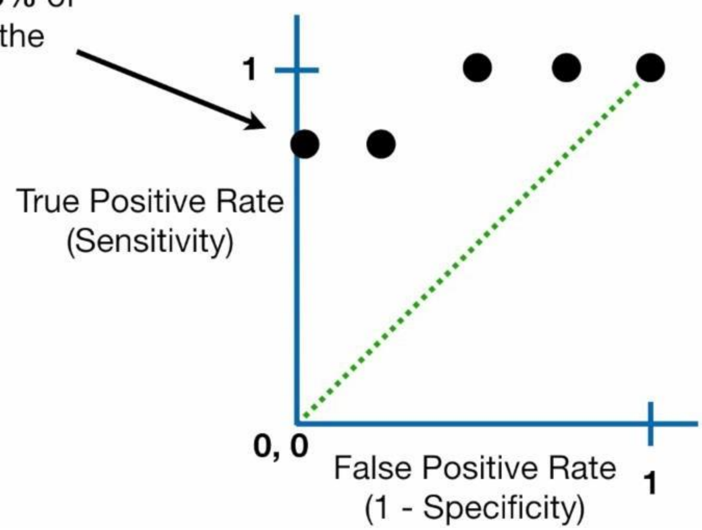


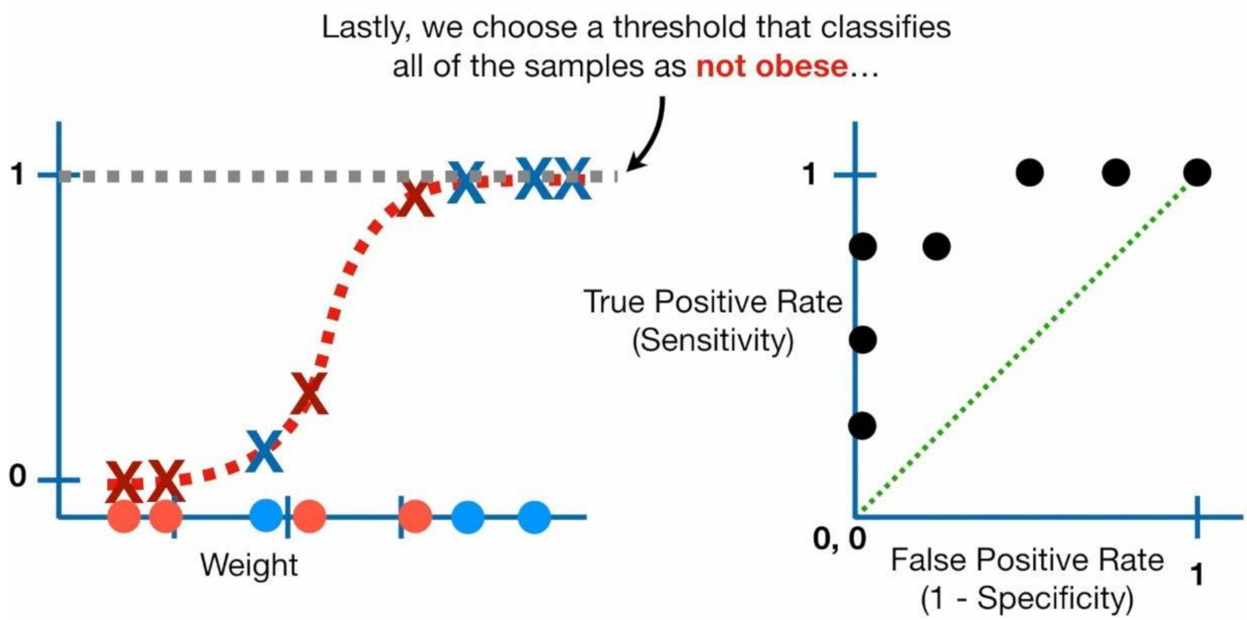
Since the new point (**0.75, 1**) is to the left of the **dotted green line**, we know that the proportion of correctly classified samples that were **obese** (true positives) *is greater* than the proportion of the samples that were incorrectly classified as **obese** (false positives).



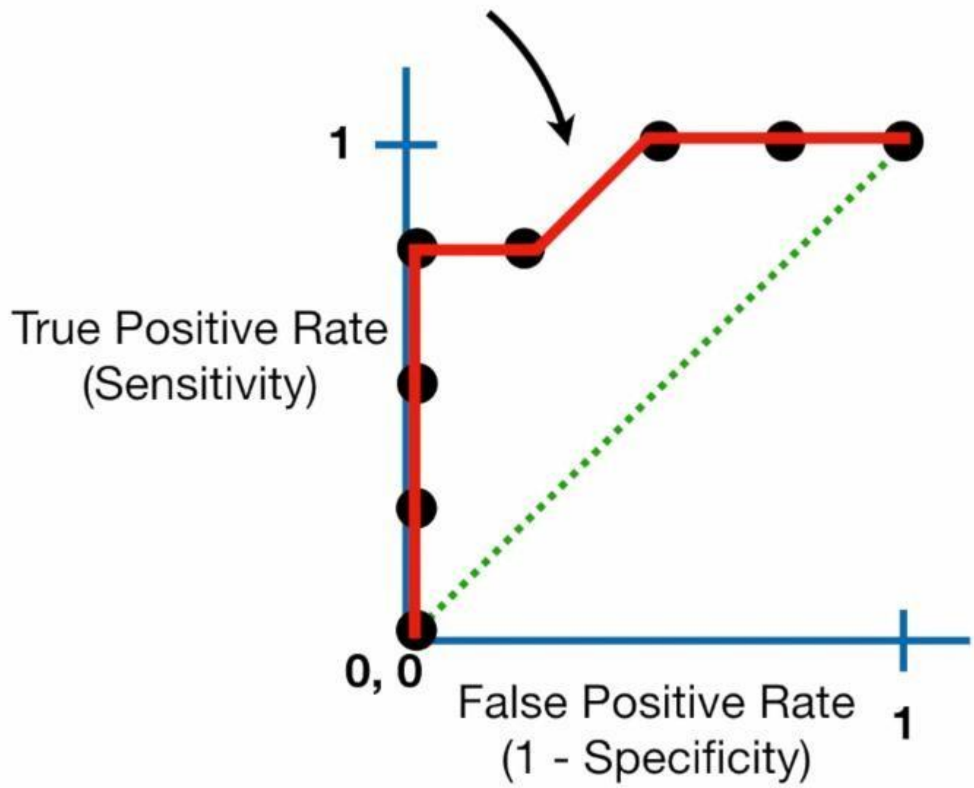
In other words, the new threshold for deciding if a sample is **obese** or **not** is better than the first one.

The threshold represented by the new point (**0, 0.75**) correctly classified **75%** of the **obese** samples and **100%** of the samples that were **not obese**.



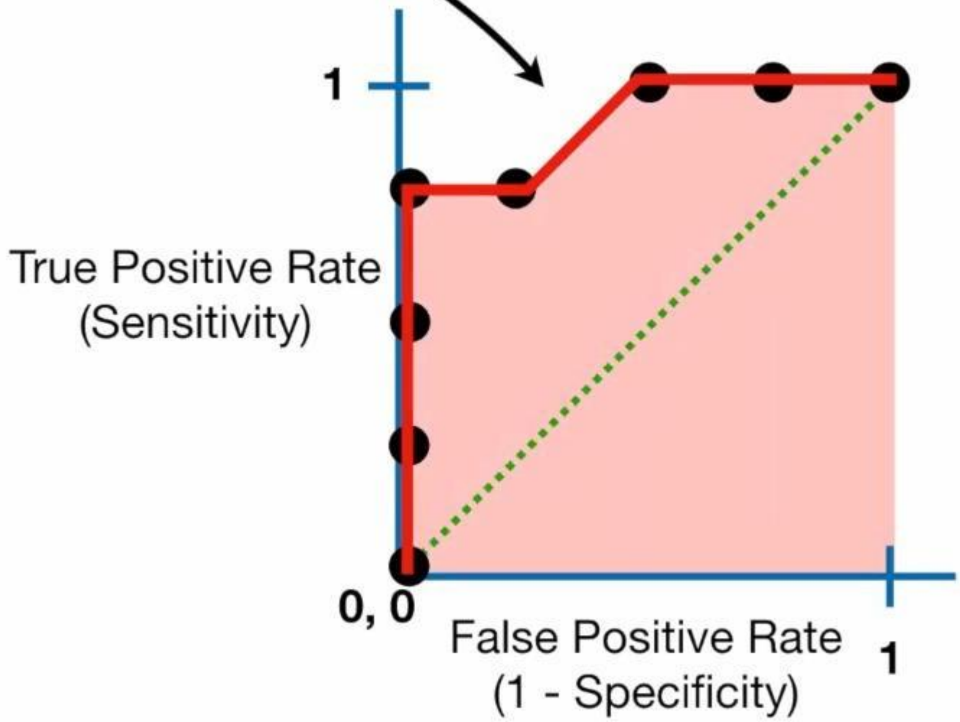


If we want, we can connect the dots…

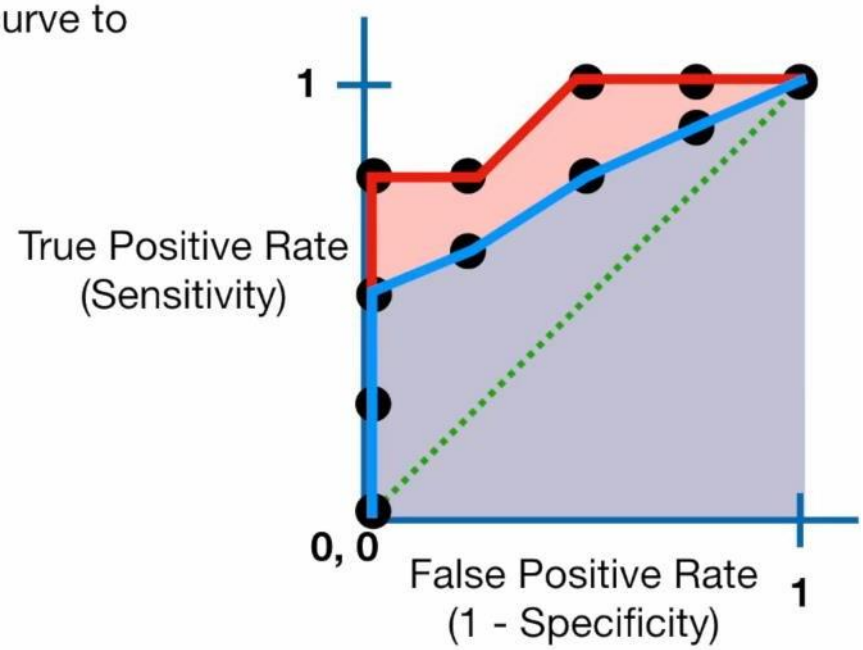


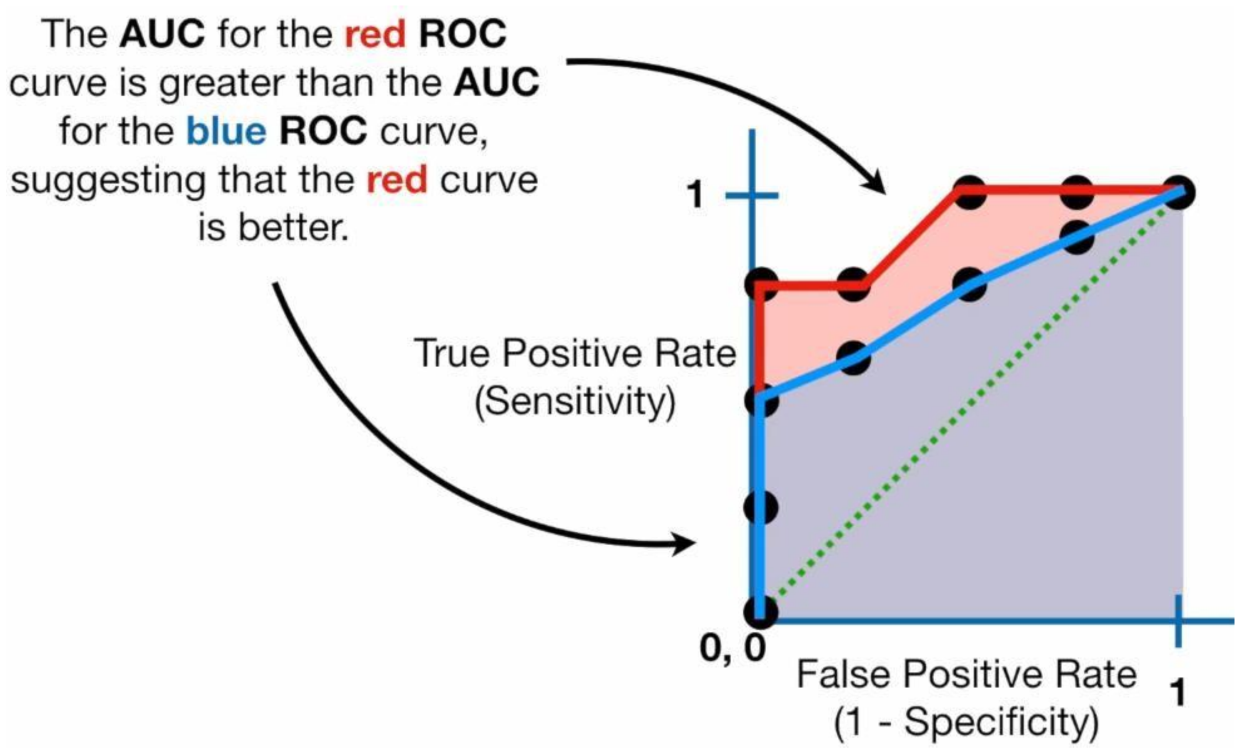
The **ROC** graph summarizes all of the confusion matrices that each threshold produced.

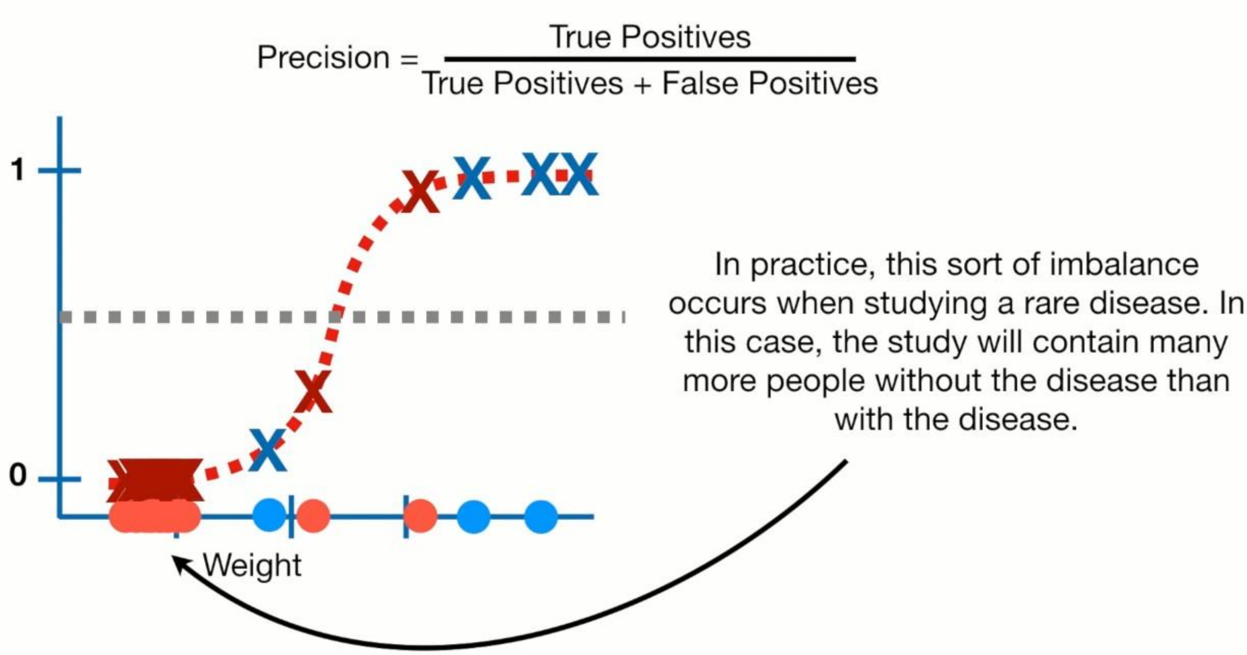
The **AUC** (Area Under the Curve) is **0.9**



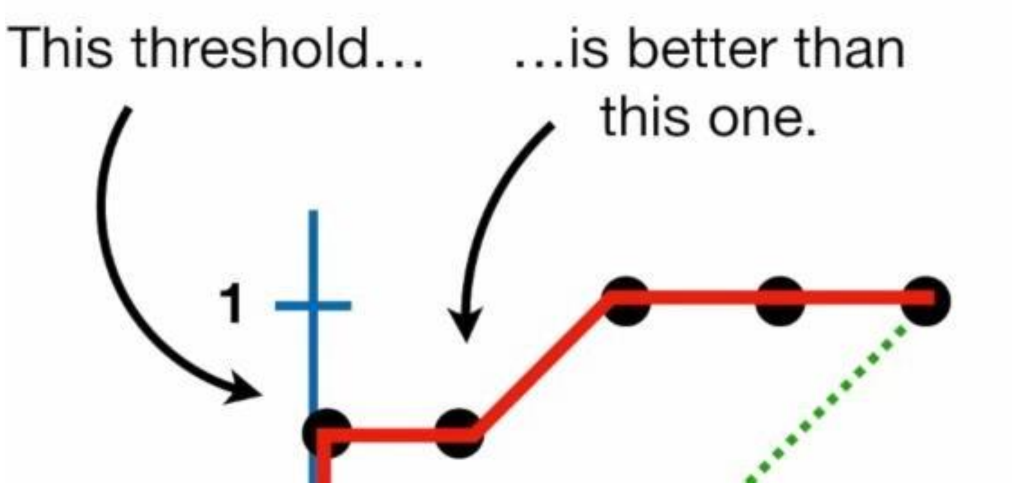
The **AUC** makes it easy to compare one **ROC** curve to another.




The AUC for the red ROC curve is greater than the AUC for the blue ROC curve, suggesting that the red curve is better.
1
True Positive Rate
(Sensitivity)
0, 0
False Positive Rate
(1 - Specificity)
1


Precision = True Positives / True Positives + False Positives
1
0
Weight
In practice, this sort of imbalance occurs when studying a rare disease. In this case, the study will contain many more people without the disease than with the disease.

ROC curves make it easy to identify the best threshold for making a decision…



…and the **AUC** can help you decide which categorization method is better.