Robot Poses, Workspace, Configuration space

Today’s topic

A robot (arm or mobile robot) is embedded in a physical environment: its actions and their effects strictly depend on where the robot is located in the environment

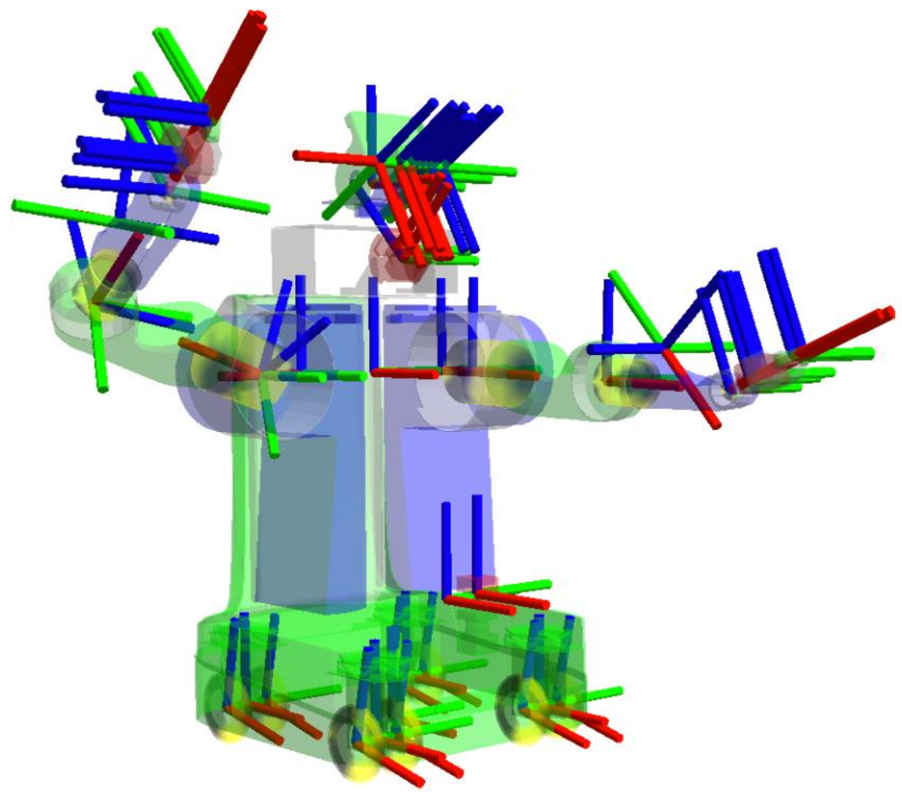
we care about ***position and orientation***

... with respect to **which reference**?

... of **which part** of the robot?

Robot pose / configuration

The pose / configuration of the robot is a complete specification of the location of **every** point of the robot



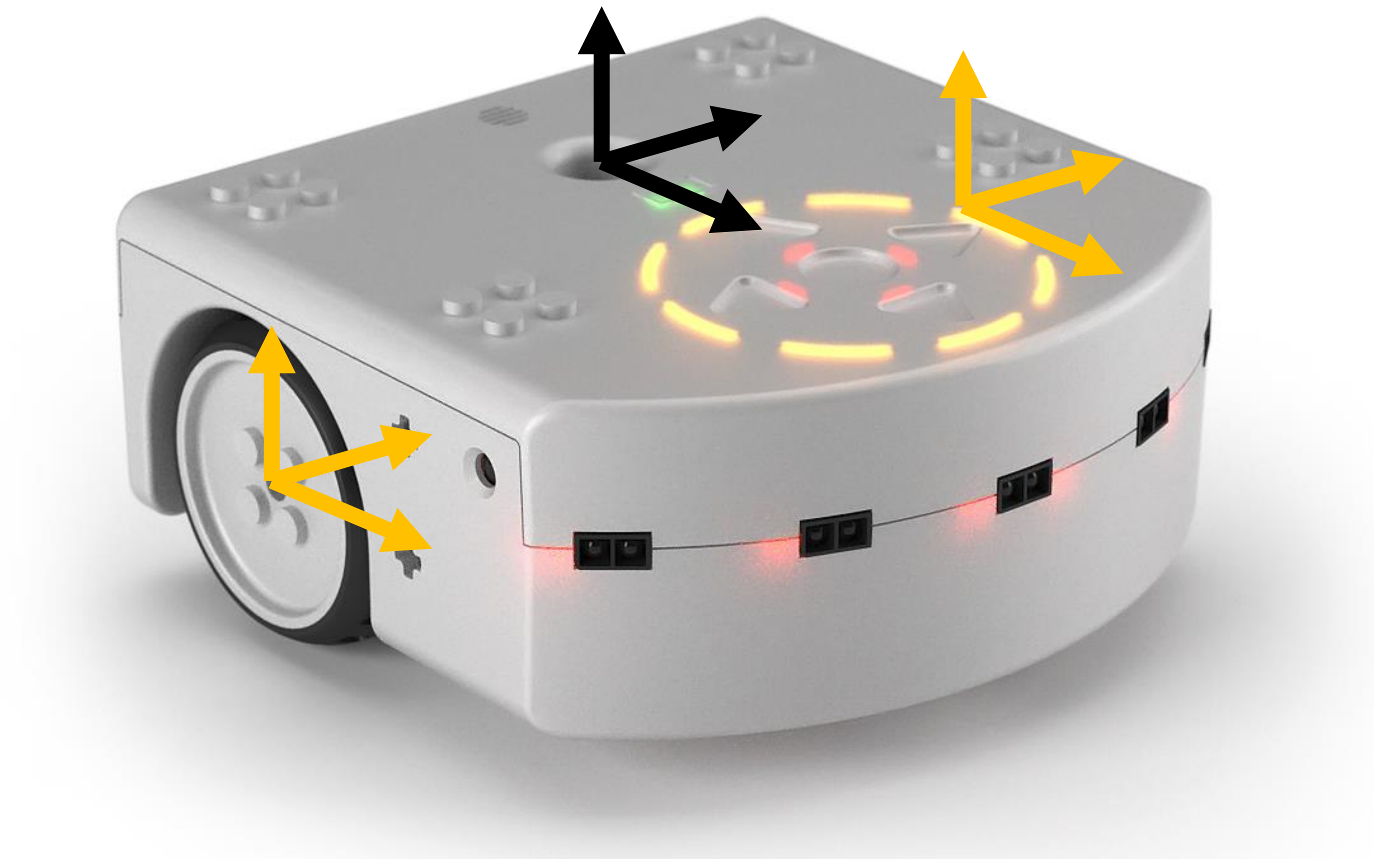
Rigid bodies

- An object is a rigid body if its constituent points maintain a constant relative position with respect to each other

  does not bend, deform, squeeze, articulate, ...

- This holds under any translation and rotation applied to the rigid body
- We can uniquely describe the pose of a rigid body by
  - choosing a reference point P on the rigid body (point abstraction)
  - describing the position of P
  - describing the orientation of the rigid body

Can we consider this a rigid body?



Yes...
If we ignore its wheels. Otherwise, it’s 3 rigid bodies.

Not rigid bodies

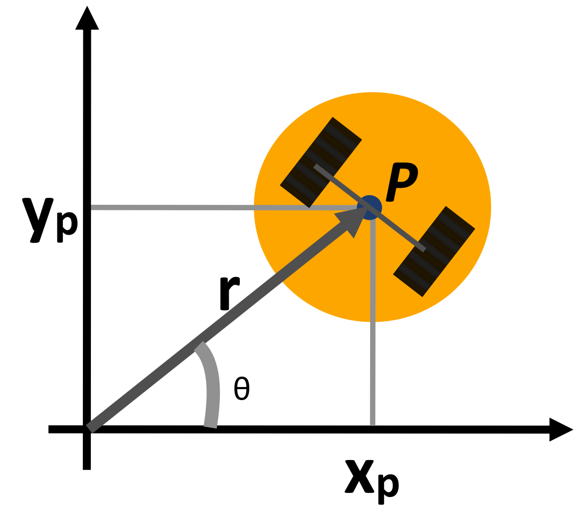
Thymio crane

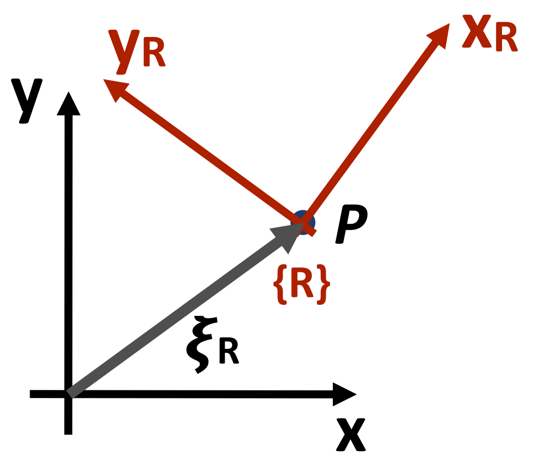
Thymio snake

https://youtu.be/ANM1hSSC_6o?t=40

The pose of a rigid body in 2D

- Define a fixed world reference coordinate frame {W}
- Center a (local) coordinate frame {R} in the robot's reference point P, oriented according to robot's natural orientation (usually, x points forwards)
- A point in space (= chosen reference point P) is described by a coordinate vector r representing the displacement of the point with respect to the reference coordinate frame {W}
- ξ_R is the **relative pose** of the frame/robot with respect to the reference coordinate frame



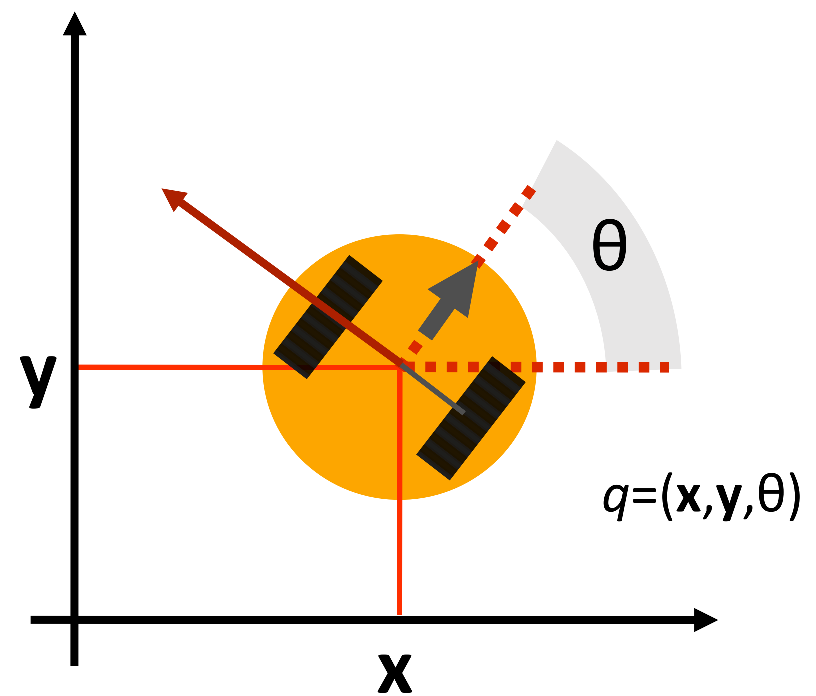


The pose of a rigid body in 2D

The pose of a single-body mobile robot in a two-dimensional coordinate systems is fully defined by:

- position (x,y)
- orientation angle θ, with respect to the world coordinate axes

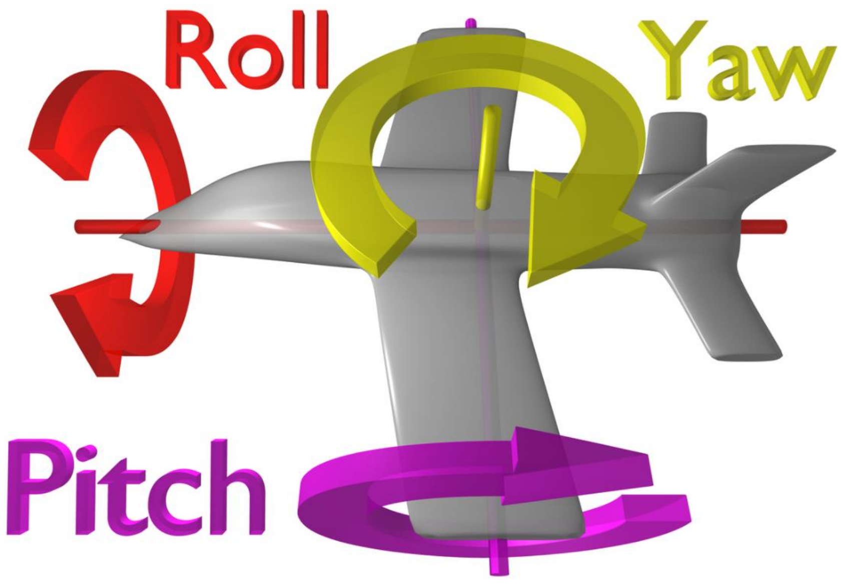
The triple q=(x,y,θ) is the robot pose



The pose of a rigid body in 3D

In 3D the pose / configuration of a single-body object is a a 6-dimensional vector:

$$\vec{q} = (\mathbf{x}, \mathbf{y}, \mathbf{z}, \alpha, \beta, \gamma)$$



Definitions

Generalized coordinates: The n parameters q = (q1, q2, …, qn), that uniquely describe the configuration of the system

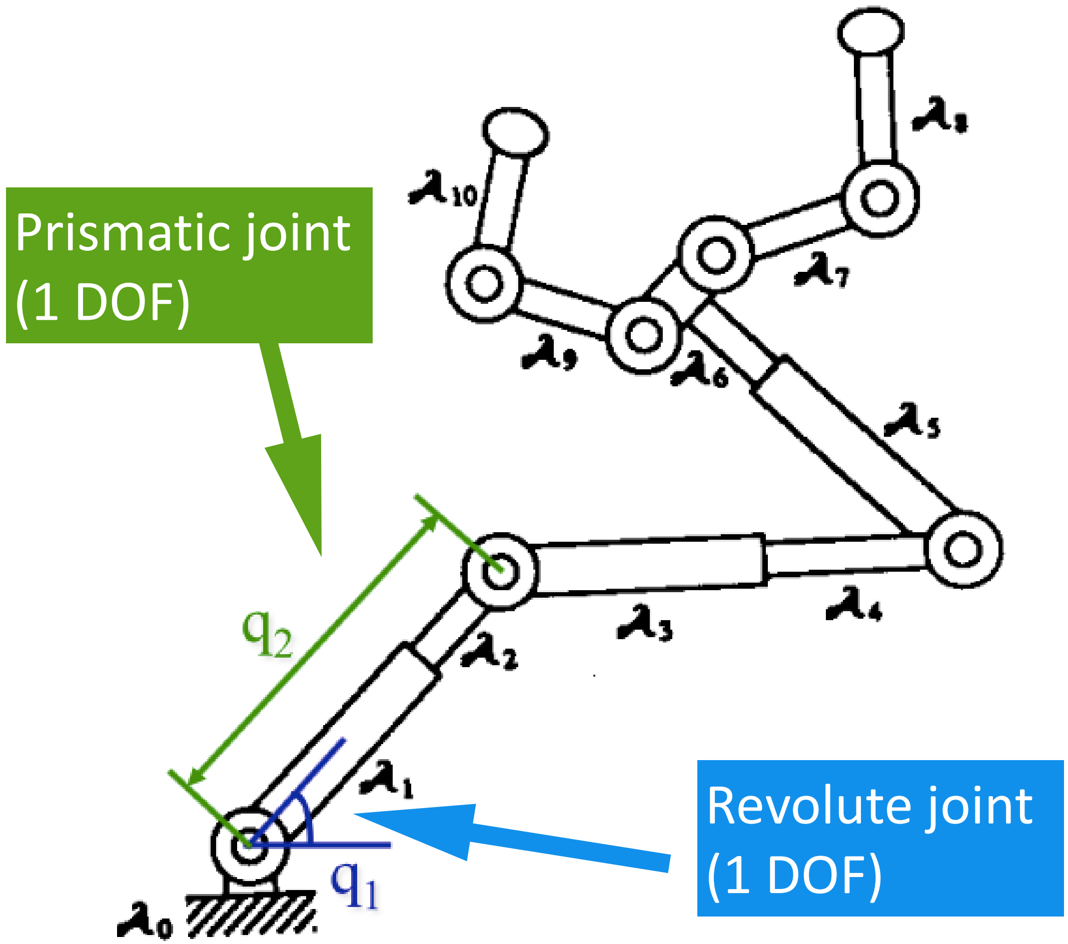
Generalized velocities: The time derivatives of generalized coordinates.

State of the system: Described by (generalized coordinates, generalized velocities)

Degrees of freedom (DoF): A system whose configuration is described by n independent generalized coordinates has n degrees of freedom.

If generalized coordinates are not independent and there are m functional relations among the generalized coordinates, then the number of DoF becomes n-m

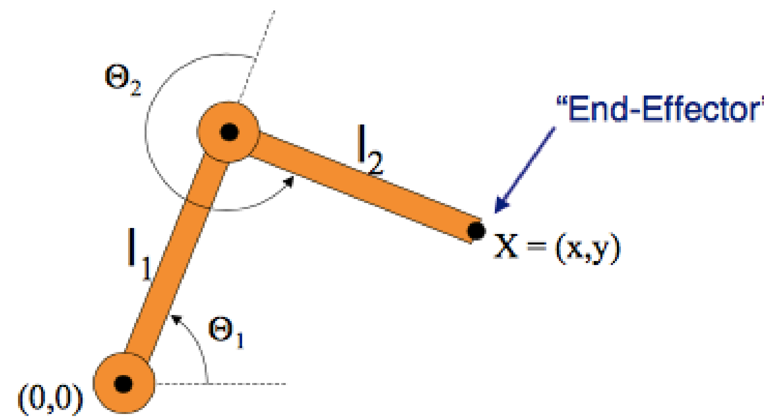
The pose of a robot arm



The configuration of a *robot arm* is defined by its joints and their characteristics

$$\vec{q} = (q_1, q_2, \ldots, q_{10})$$

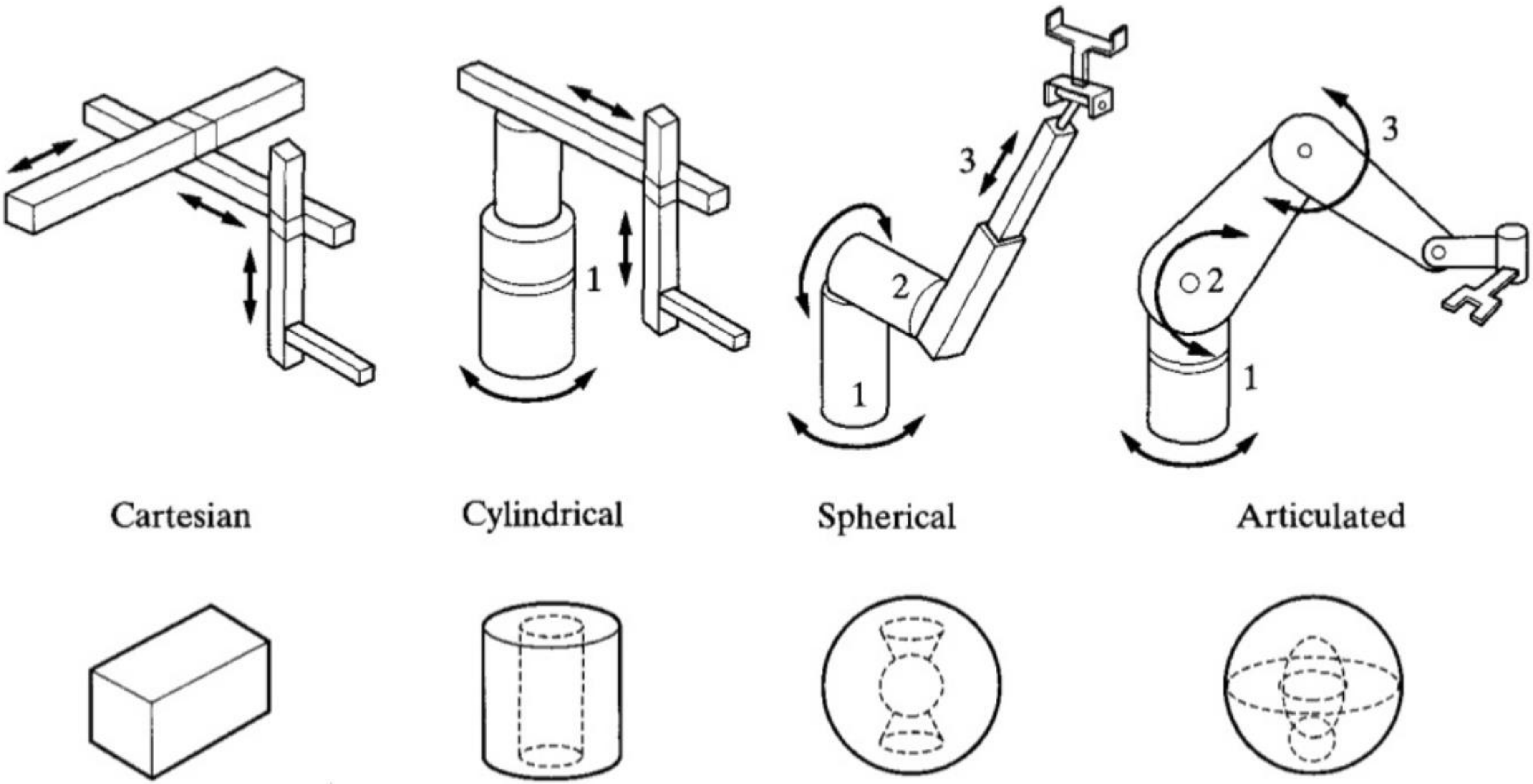
Multiple translational and rotational DOFs, each q is either an angle or a distance



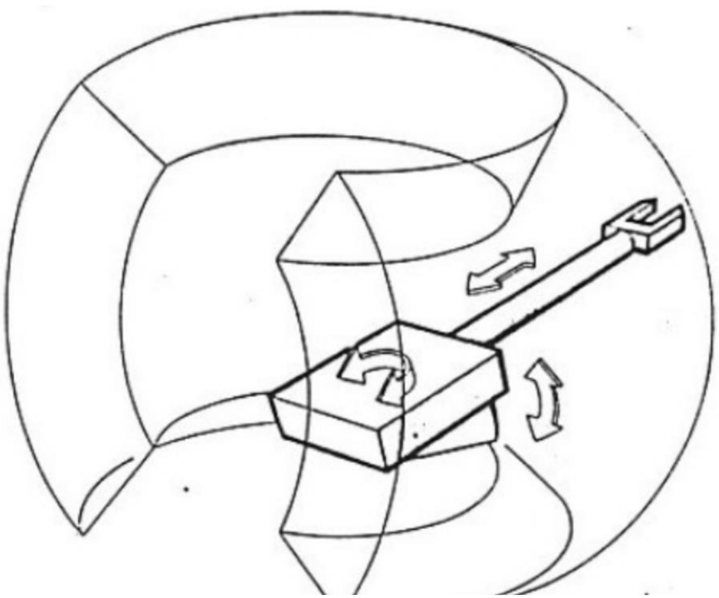
Pose of the manipulator: Configuration of all joints (*n* DoF for *n* joints)
Pose of the end-effector: Can be obtained by the pose of manipulator (reverse doesn't hold! multiple manipulator poses can generate the same endeffector-pose)

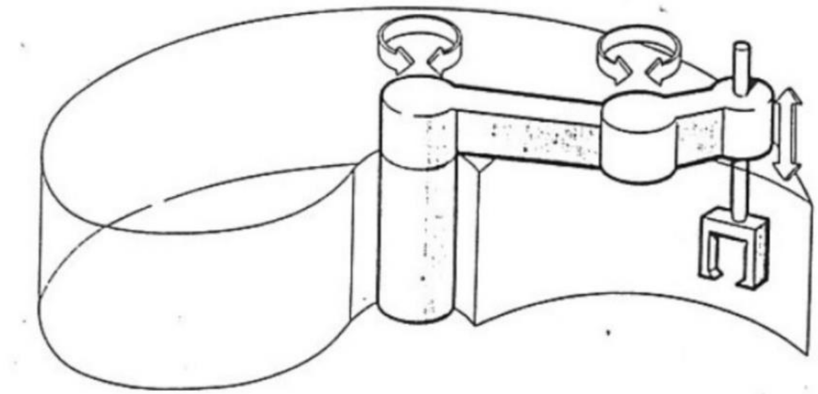
Workspace of a Robot Arm

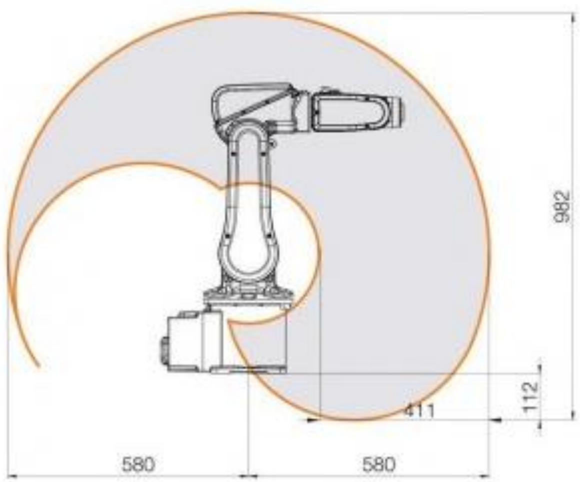
Workspace: A robot's workspace (or workspace envelope) is the set of all points the end effector of the robot can reach in the physical embedding volume.

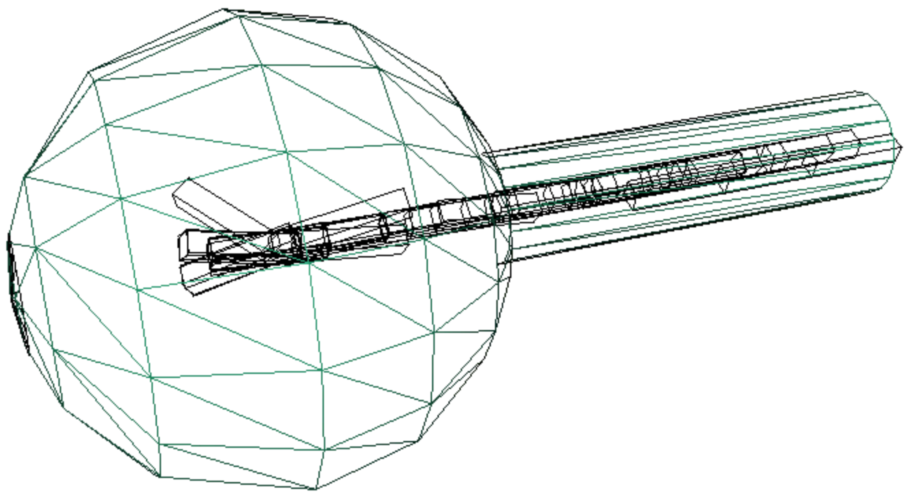


Examples of workspaces



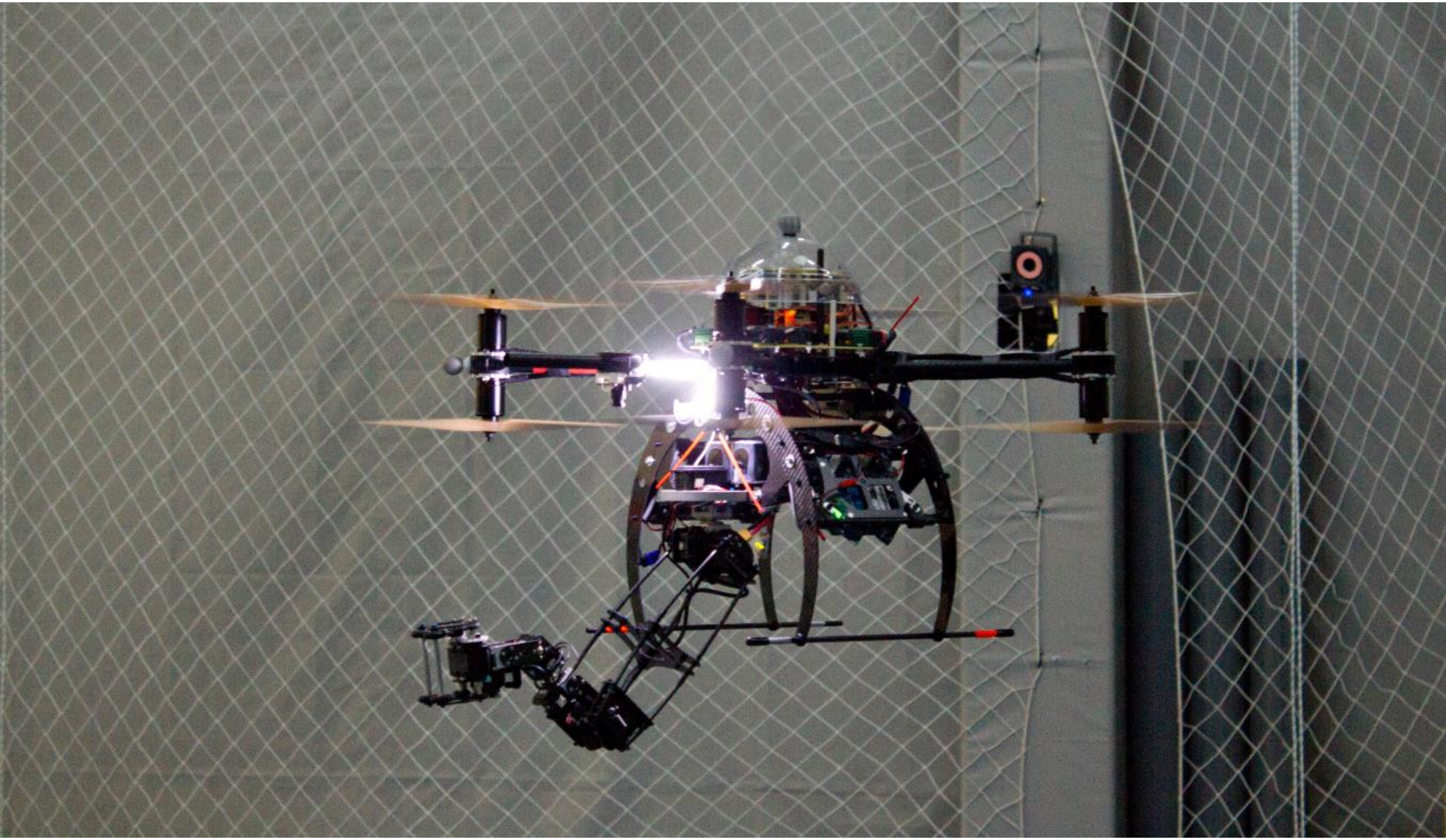






https://www.slideshare.net/mkazree/dek3223-chapter-2-robotic

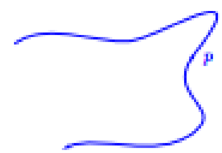
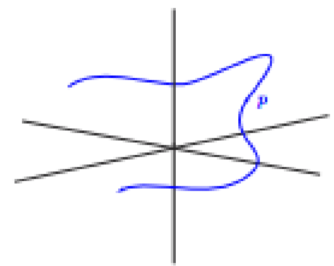
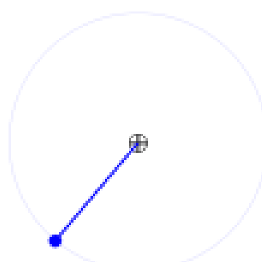
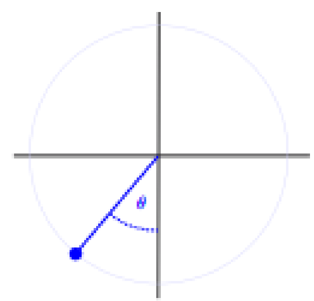
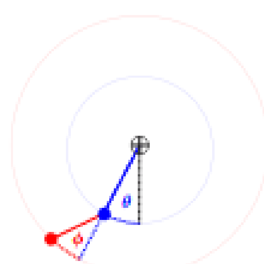
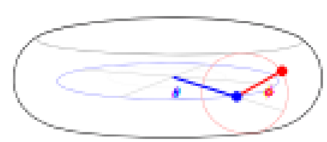
A robot with a pretty large workspace



https://phys.org/news/2014-09-robots-humans.html

Configuration Space

Configuration space (C-space): the set of all possible robot configurations.
If the robot has n DOFs, then the configuration space is n-dimensional
... but it may be a non-Cartesian manifold

	Physical System	Configuration space	How many DOF?
Classical particle			3
Single pendulum			1
Double pendulum			2

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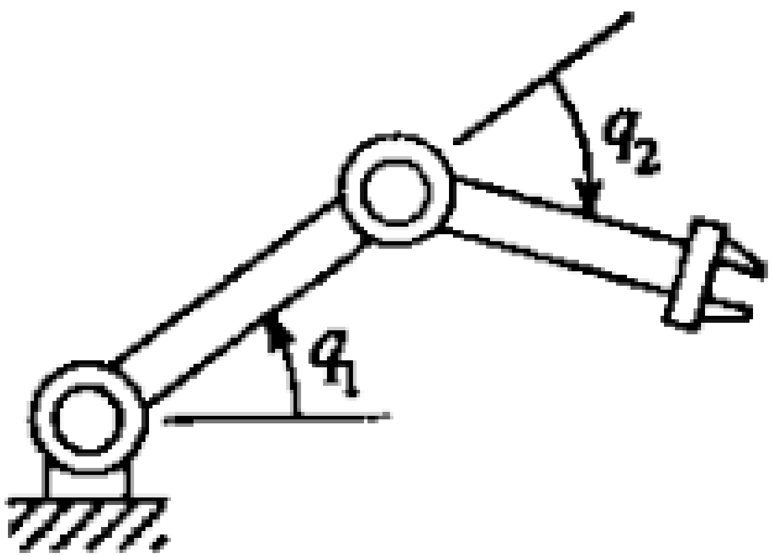
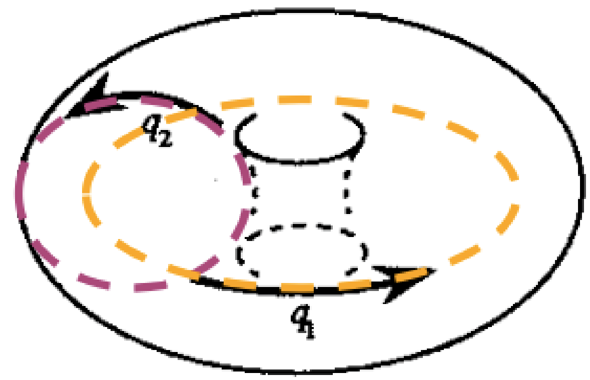


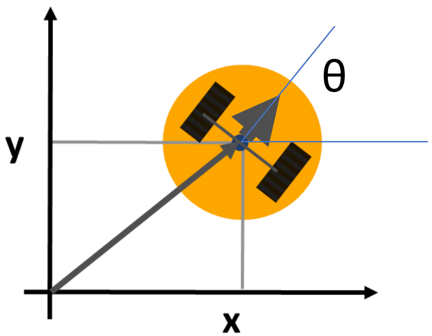
group of 2d rotations

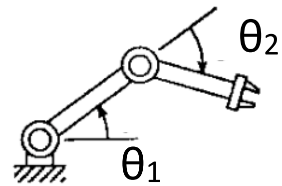
$C = S^1 \times S^1$

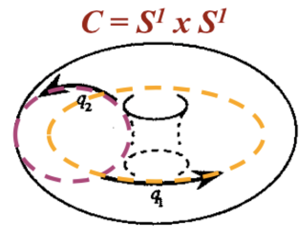


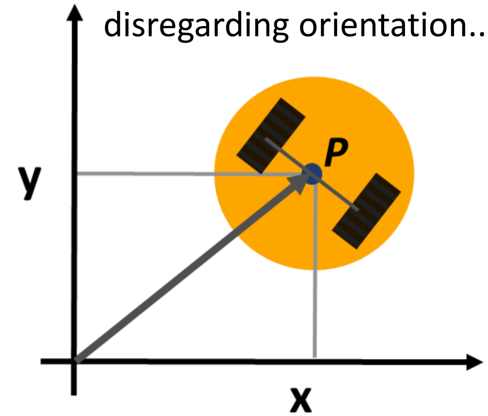
configuration space

Configuration space examples









$\boldsymbol{q} = (x, y, \theta)$

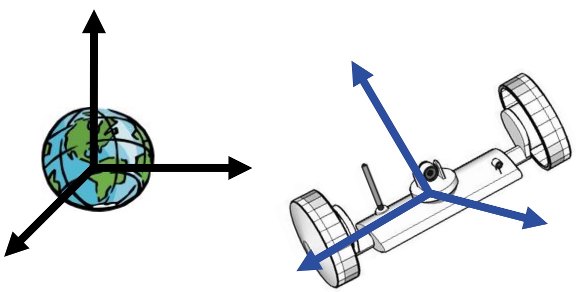
$C = \mathbb{R}^2 \times S$

$\boldsymbol{q} = (\theta_1, \theta_2)$

$C = S \times S$

$\boldsymbol{q} = (x, y)$

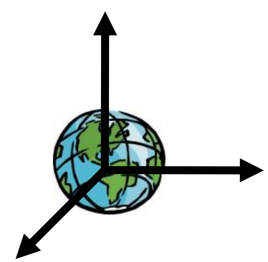
$C = \mathbb{R} \times \mathbb{R}$



$\boldsymbol{q} = (x, y, z, \alpha, \beta, \gamma)$

$C = \mathbb{R}^3 \times SO(3)$

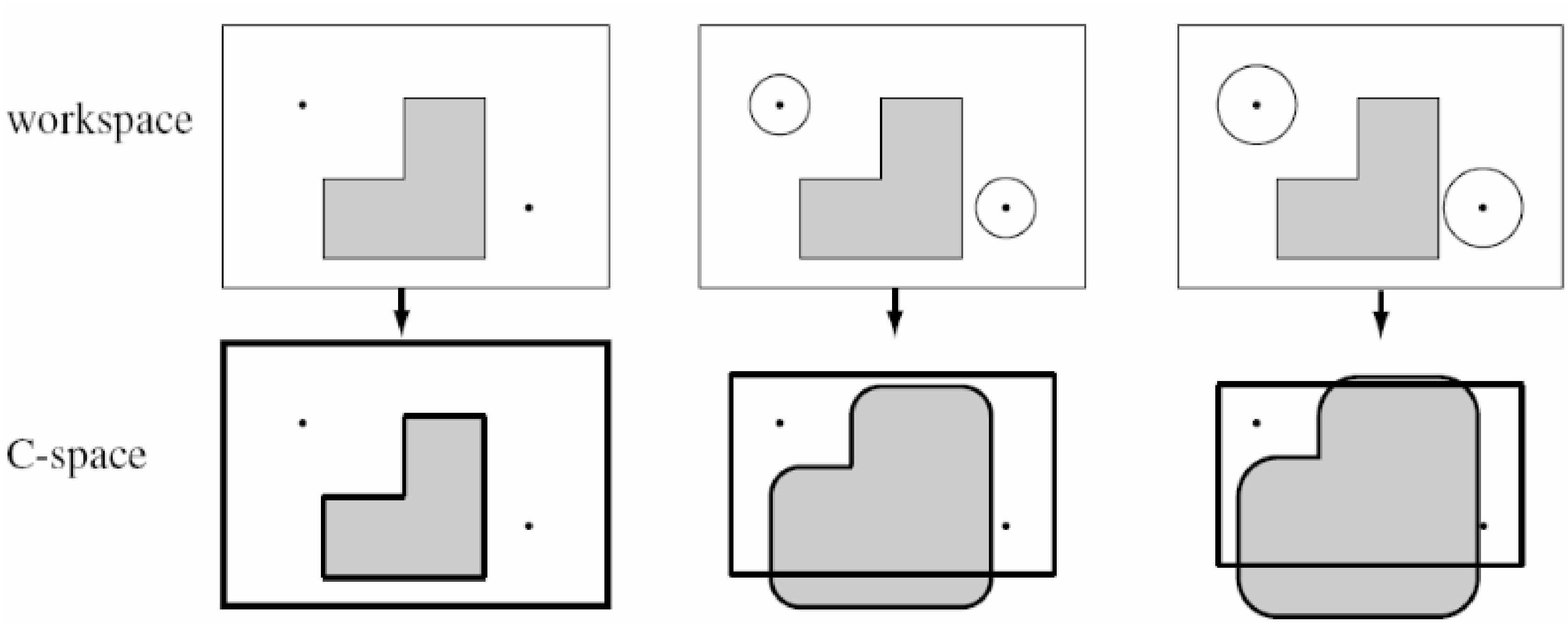
group of 3d rotations



$C = \mathbb{R}^3 \times SO(3) \times \mathbb{R}^{34}$

https://en.wikipedia.org/wiki/Rotation_group_SO(3)

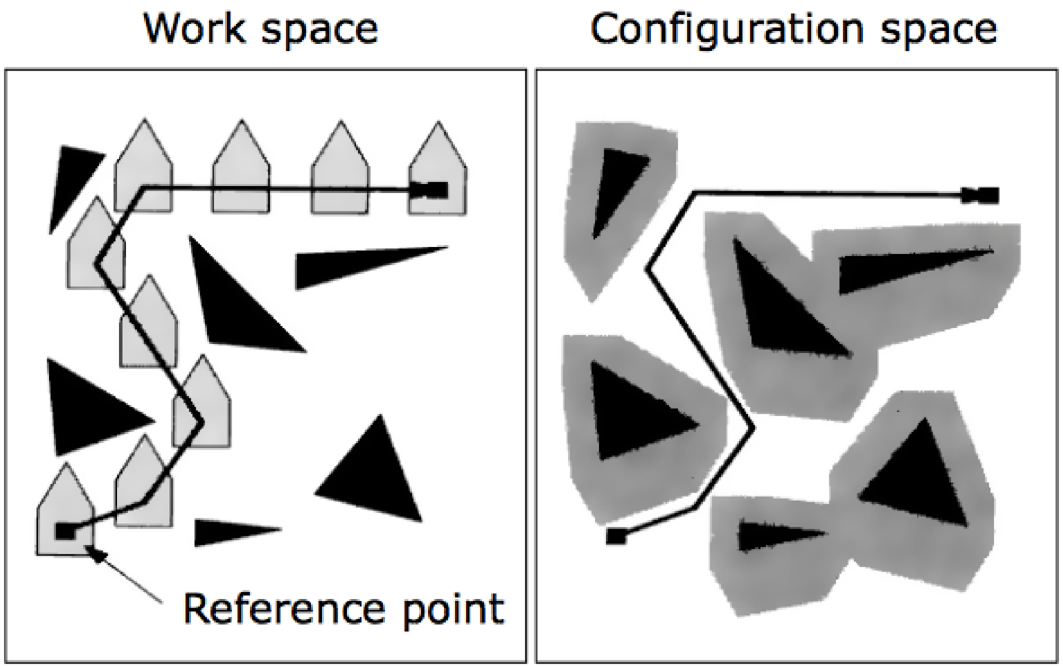
Free C-space with obstacles for a circular 2D robot



One *reference point* on the robot is selected, then the C-space is obtained wrt the reference point by sliding the robot along the edge of the obstacle regions "blowing them up" by the robot radius.
In image processing, the operation would be described as a *binary morphological dilation* with a robot-shaped structuring element.

C-space and point reduction

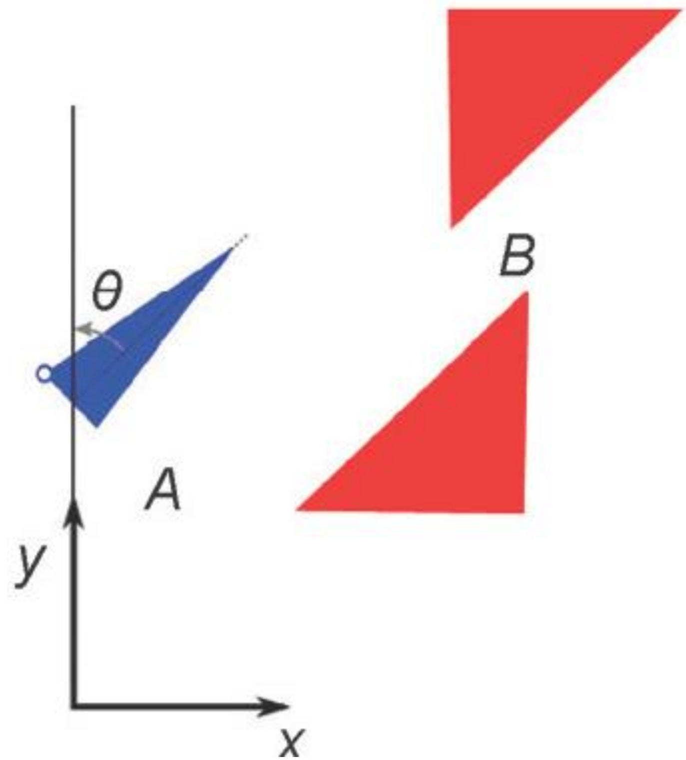
Special case: The robot is a *polygonal* one and can only *translate*



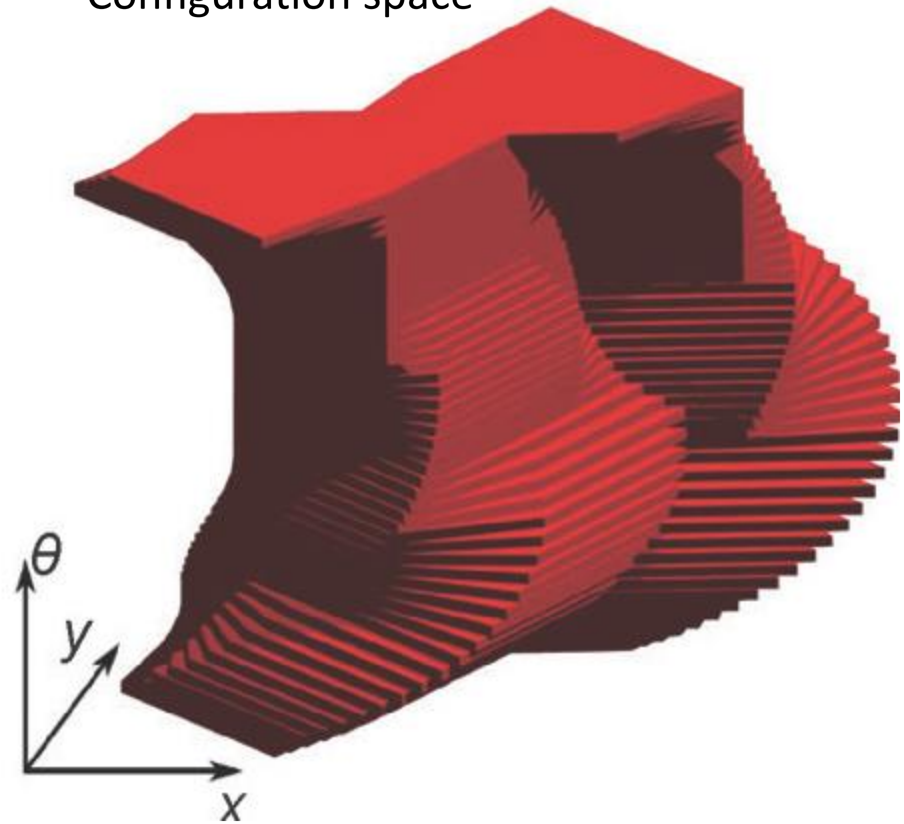
For *motion planning and navigation* a mobile robot of any shape can be “reduced” to a **point**

C-space and point reduction

Work space

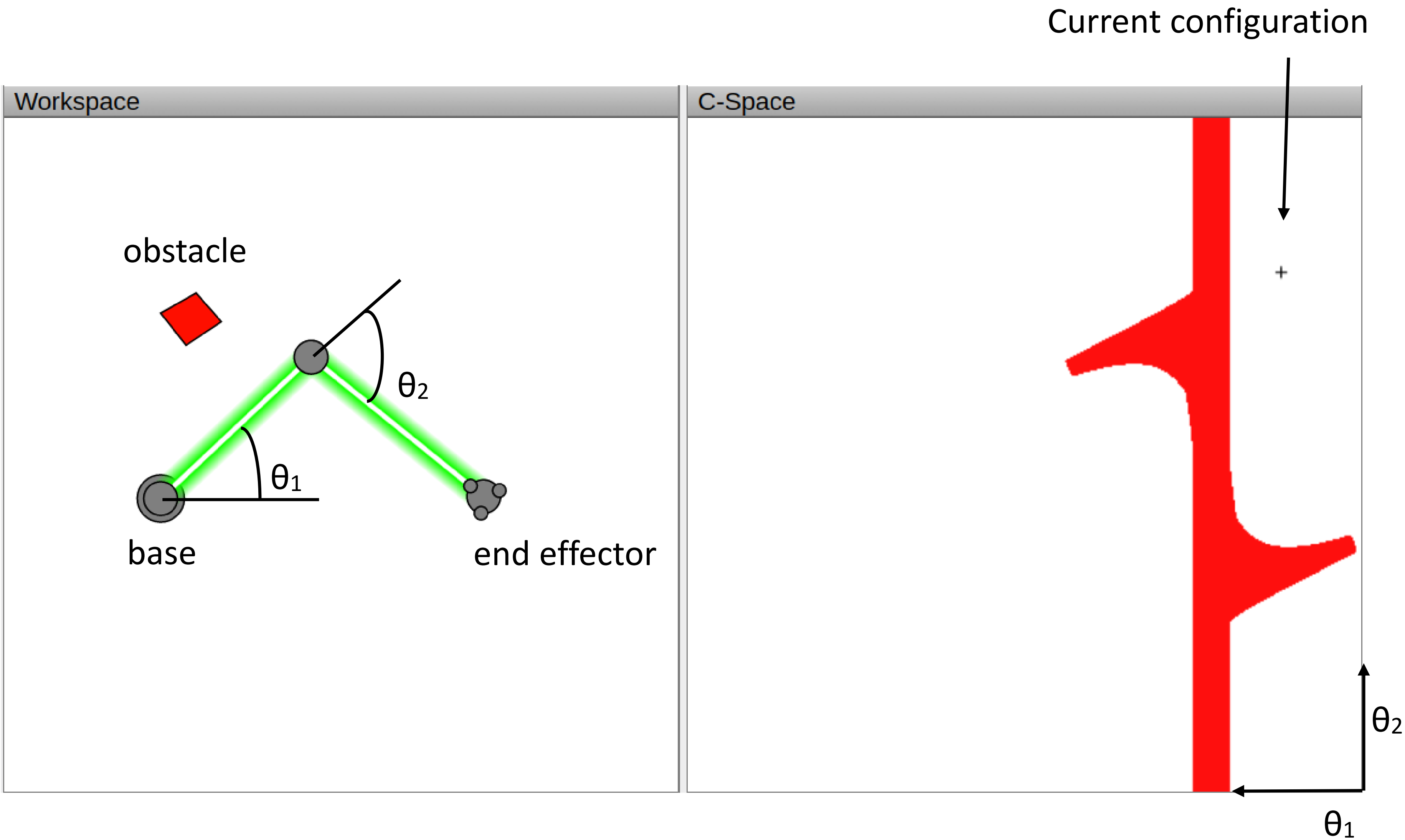


Configuration space



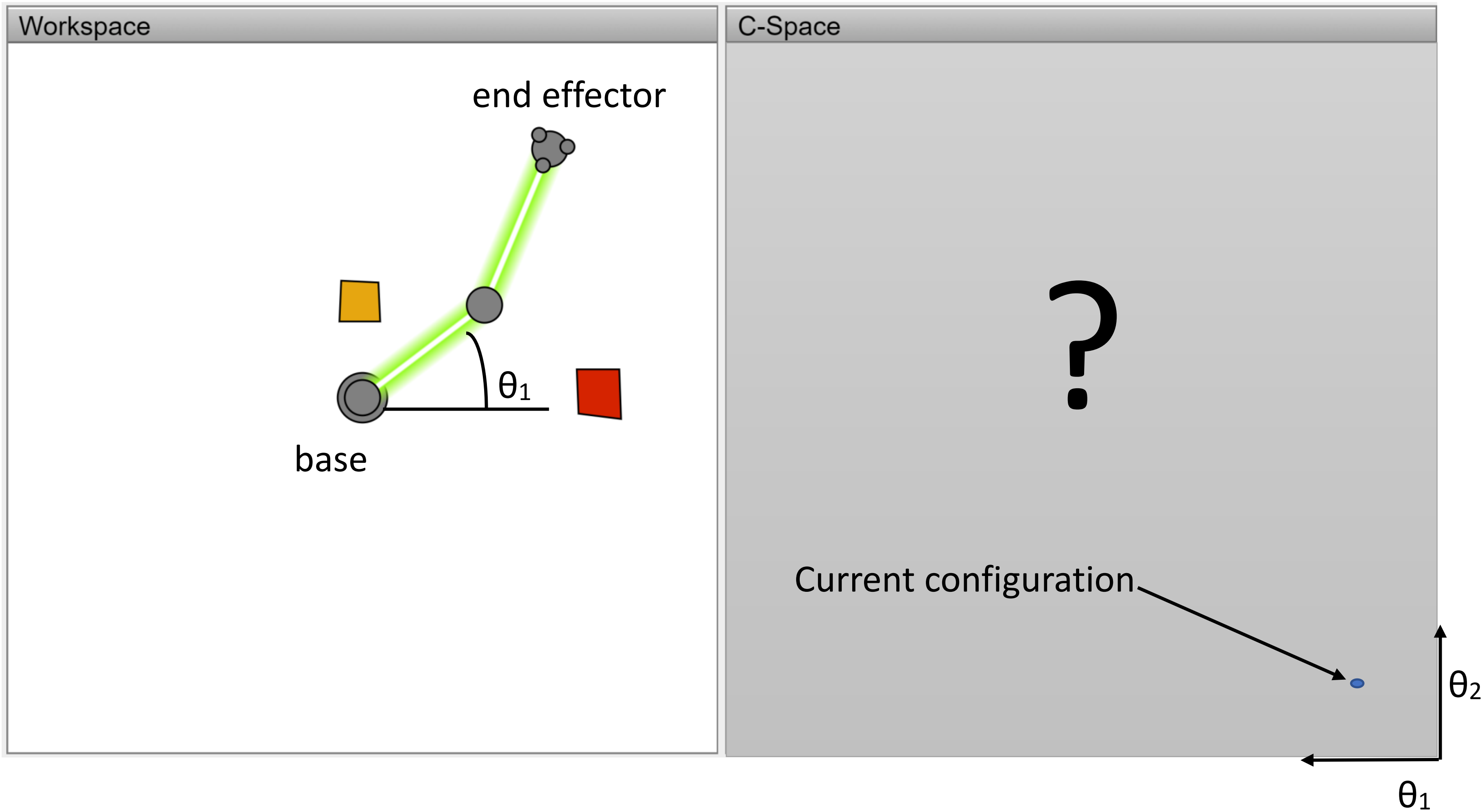
The robot configuration is represented by a point in the configuration space. When the point is in the obstacle, it means that the robot is hitting the obstacles

Let’s play with C-spaces for arms



https://www.cs.unc.edu/~jeffi/c-space/robot.xhtml

Let's play with C-spaces for arms



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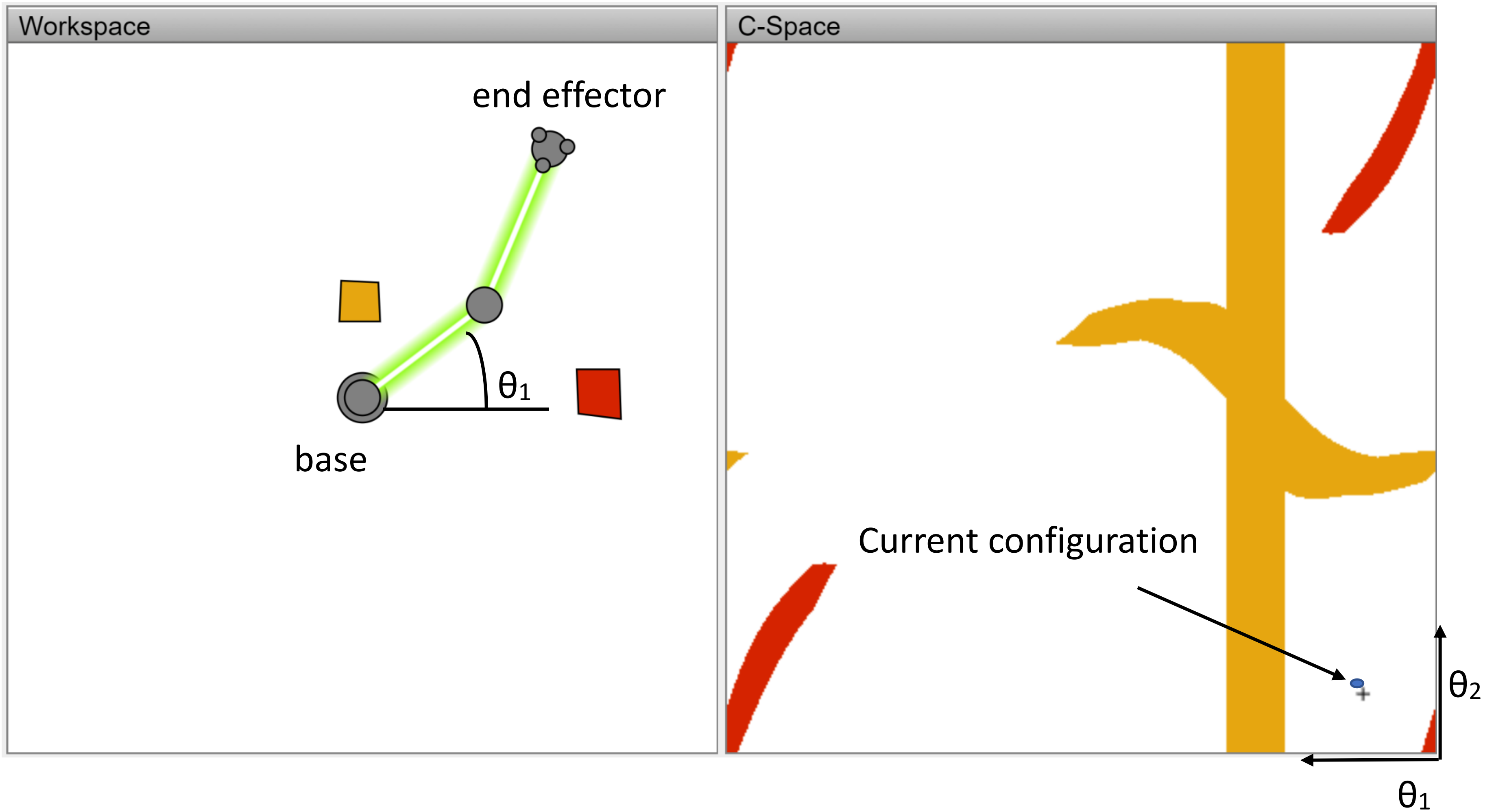

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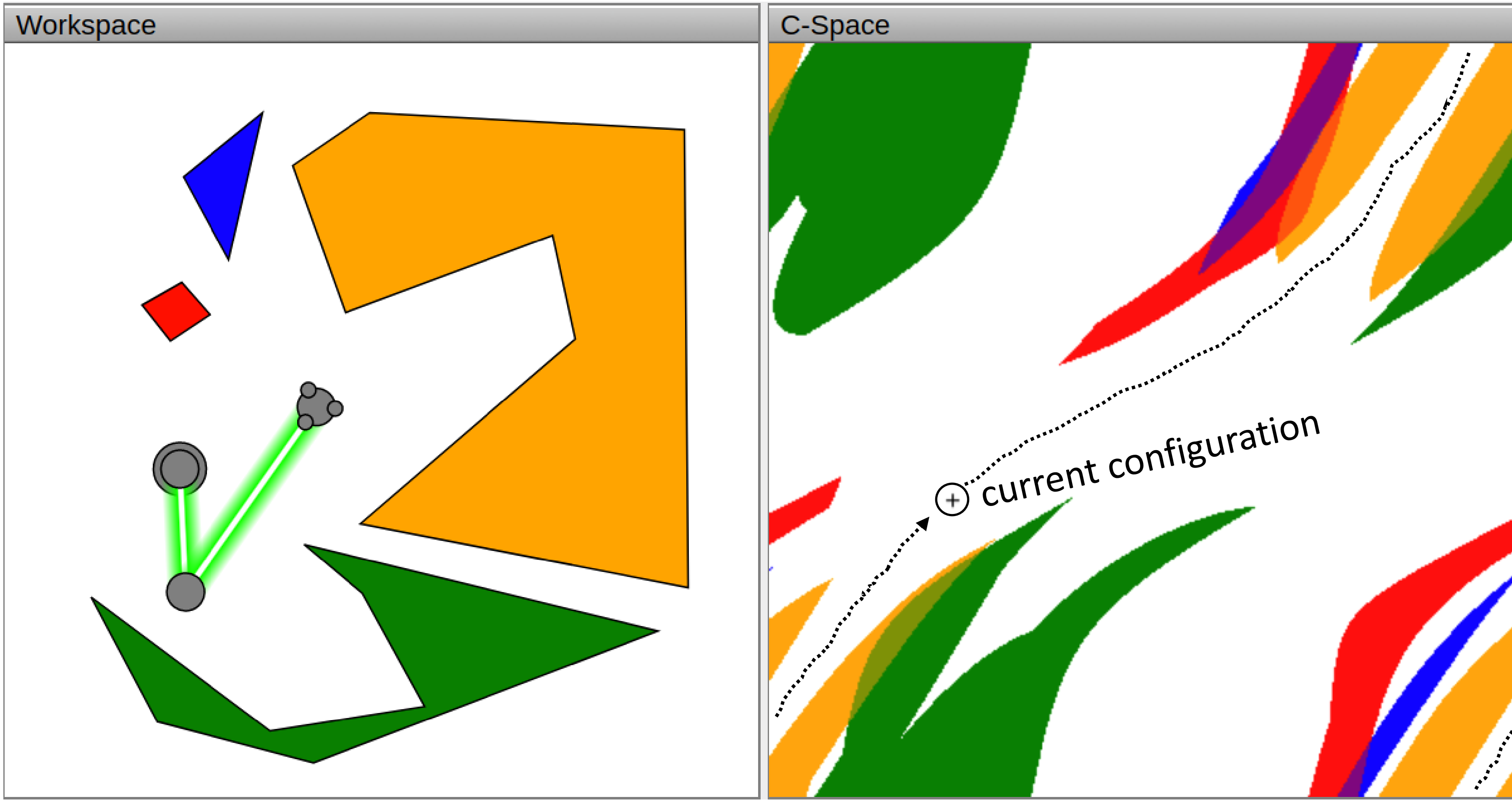
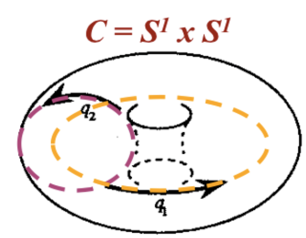


Can you find out what motion the dotted line corresponds to?

(remember the toroidal topology of this C-space!)



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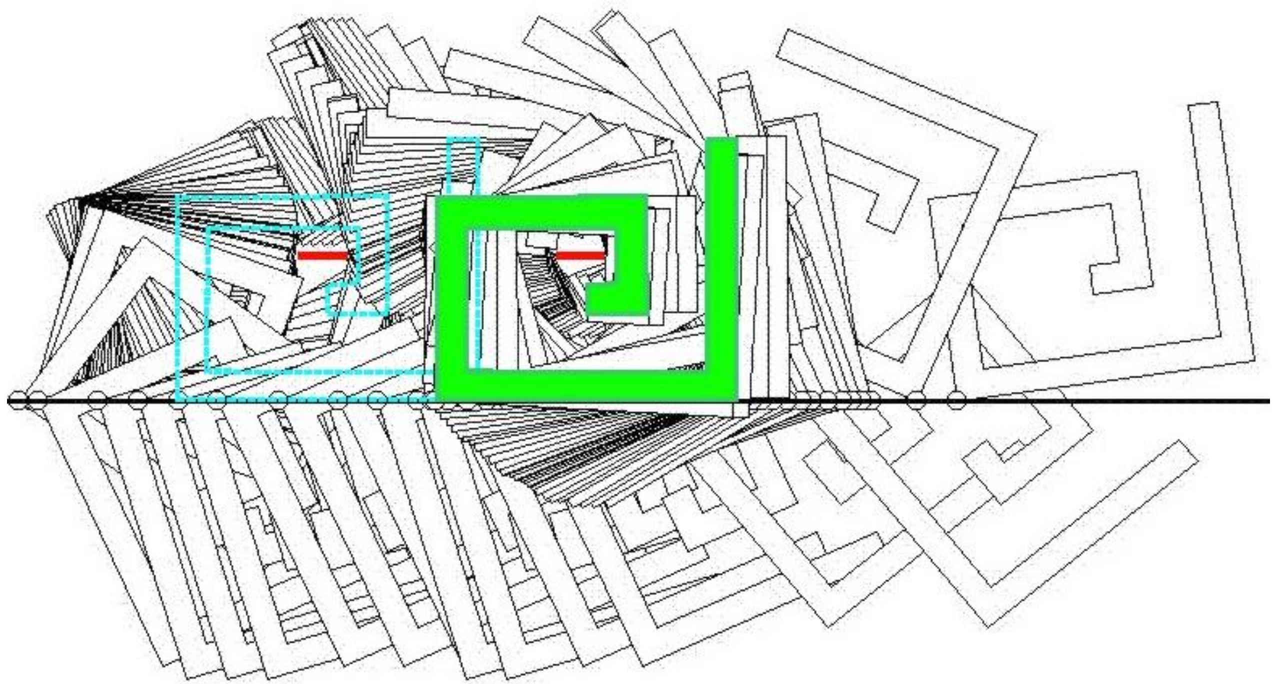
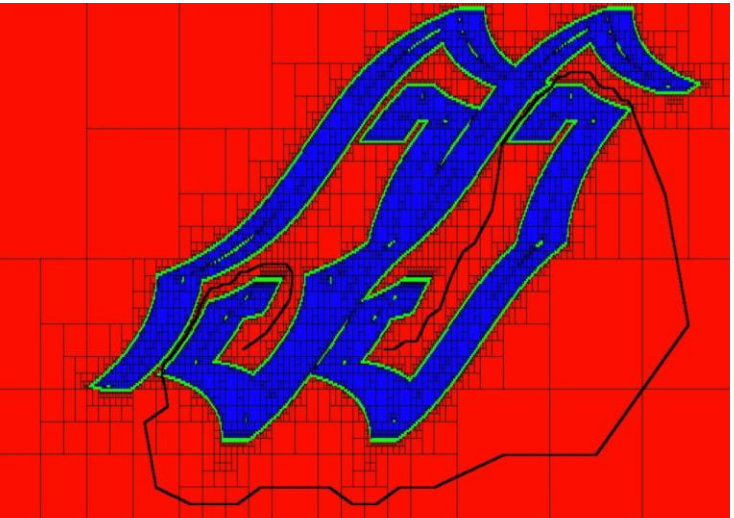
A tight path in a 2D C-space

The object is constrained with its lower-left corner on the line:
2 DoF → 2D C-space.

(what is its topology?)





C-space, and path (black) from initial position to outline.

https://en.wikipedia.org/wiki/Motion_planning

A C-space in 6D

https://www.youtube.com/watch?v=_hGo7Pg5HTs

Fix the pose of one of the two parts. The pose of the other part is defined by a point in a 6D C-space.

$$C = \mathbb{R}^3 \times SO(3)$$

Solving the puzzle “just” consists in finding a path in this C-space moving from the initial configuration (tangled) to the final configuration (untangled)