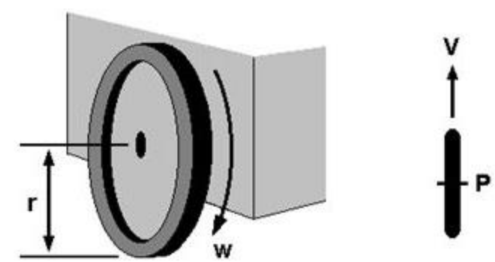
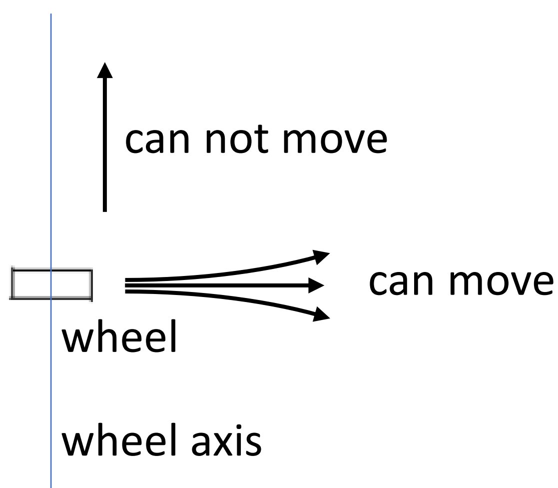
Kinematics of wheeled robots

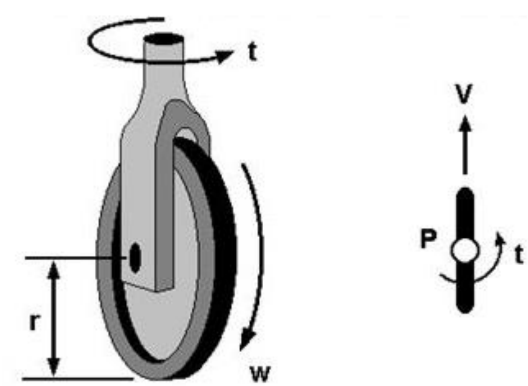
Slides in part adapted from
Gianni di Caro, CMU

Recap: assumptions for wheels

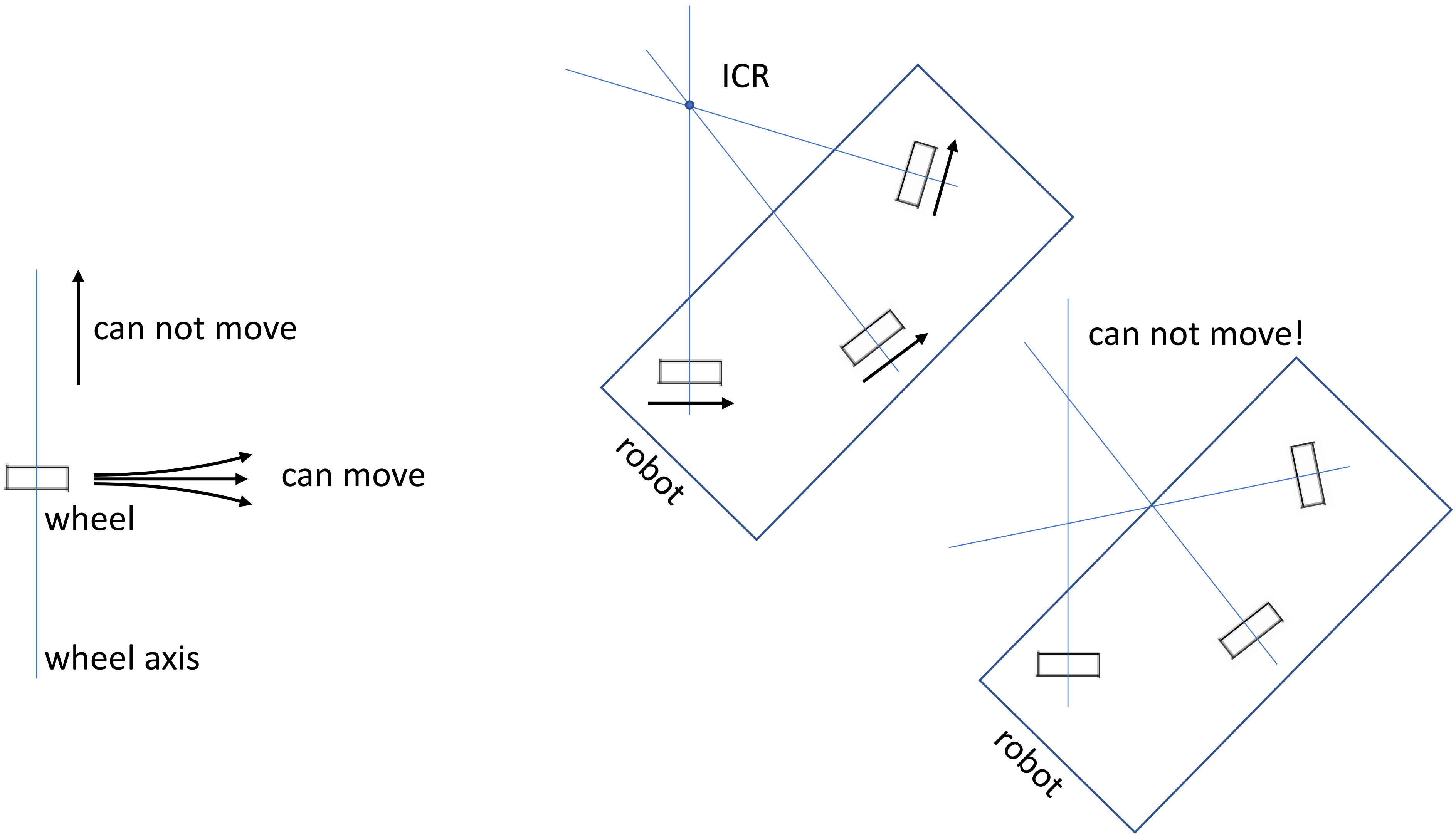
- Wheels does not slip
- The wheel can not move in directions orthogonal to the wheel plane
- Wheel plane orthogonal to ground → wheel axis parallel to ground
- For steered wheels: steering axis passes through the center of the wheel and is orthogonal to ground







Recap: Instantaneous Center of Rotation (ICR), aka ICC

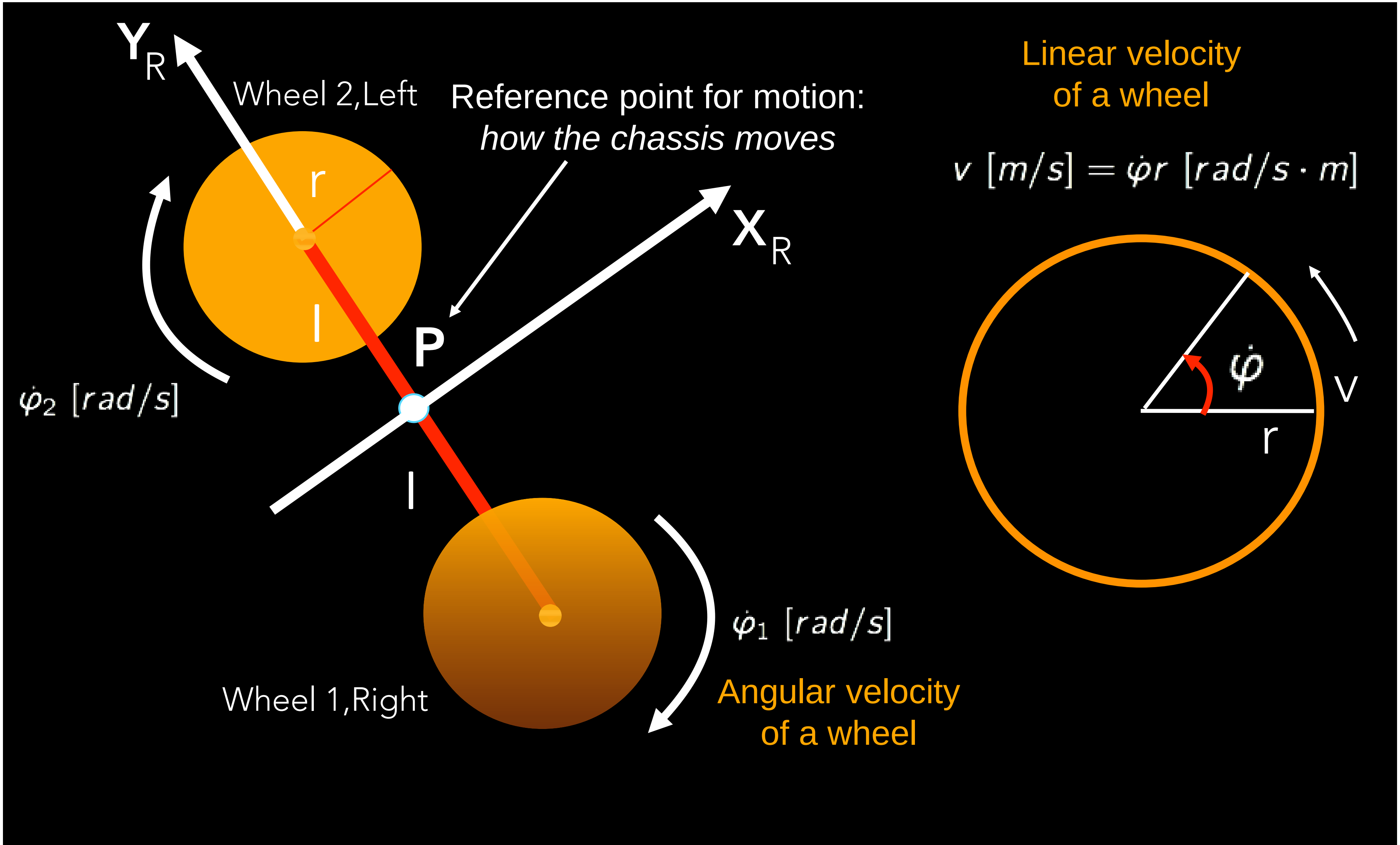


Differential drive Kinematics

Differential Drive Robots

two standard wheels mounted on a single axis are independently powered and controlled, providing both drive and steering functions through the motion difference between the wheels

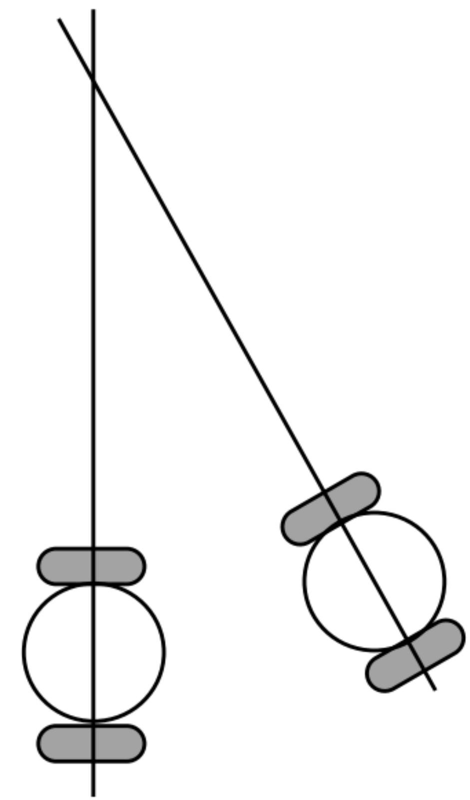
Model



Y_R
Wheel 2,Left
Reference point for motion:
how the chassis moves
Linear velocity
of a wheel
$v\ [m/s] = \dot{\varphi} r\ [rad/s \cdot m]$
r
X_R
l
P
$\dot{\varphi}$
v
$\dot{\varphi}_2\ [rad/s]$
r
l
$\dot{\varphi}_1\ [rad/s]$
Wheel 1,Right
Angular velocity
of a wheel

Instantaneous Center of Rotation

- ICR (aka ICC, Instantaneous Center of Curvature) defines the zero motion line drawn through the horizontal axis perpendicular to the plane of each wheel
- At any time t, the robot reference point P moves along a circumference of radius R(t) with center on the ICR(t). Similarly, other points on the rigid chassis follow circular trajectories
- The ICR changes over time as a function of the individual wheel velocities, and, in particular, of their relative difference



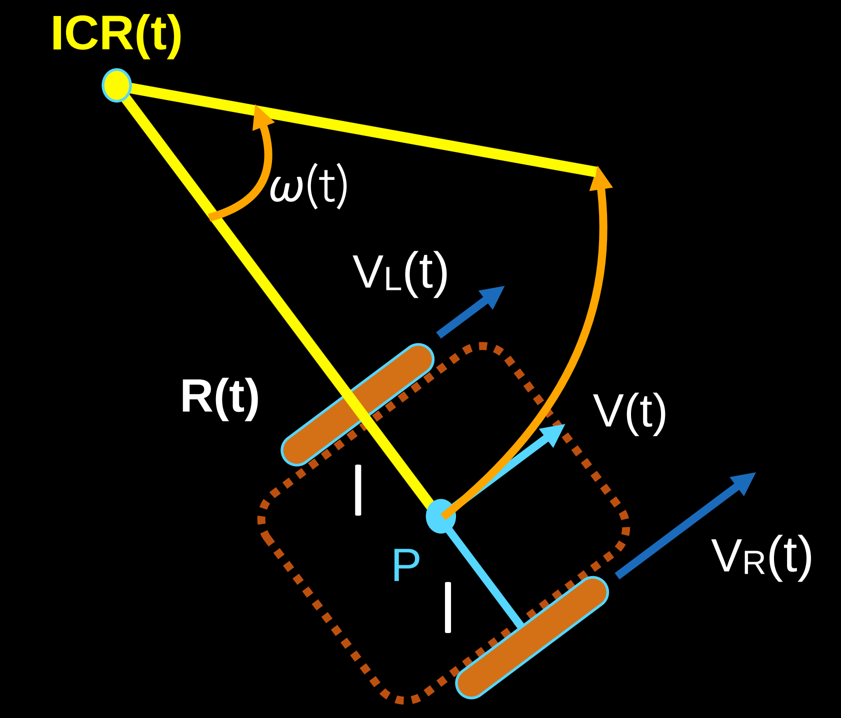
Model

- **$V_L = V_R$** ➔ $R = \infty$
  there is effectively no rotation, $\omega = 0$
  Forward linear motion in a straight line

- **$V_L = -V_R$** ➔ $R = 0$
  ICR coincides with P
  $\omega = -V/l$
  Rotation about the midpoint of the wheel axis (in place rotation)

- **$V_L = 0$** ➔ $R = l$ (in the center of left wheel)
  $\omega = V_R/2l$
  Counterclockwise rotation about the left wheel

- **$V_R = 0$** ➔ $R = -l$ (in the center of right wheel)
  $\omega = -V_L/2l$
  Clockwise rotation about the right wheel



$$\omega(t)(R(t) + \ell) = V_R(t)$$
$$\omega(t)(R(t) - \ell) = V_L(t)$$

At any specific time instant *t:*

$$R(t) = \ell \frac{V_R(t)+V_L(t)}{V_R(t)-V_L(t)}$$

$$\omega(t) = \frac{V_R(t)-V_L(T)}{2\ell}$$

Composition of angular velocities

- If only the right wheel spins: $\omega_1 = \frac{r\dot{\varphi}_1}{2l}$

- If only the left wheel spins: $\omega_2 = -\frac{r\dot{\varphi}_2}{2l}$

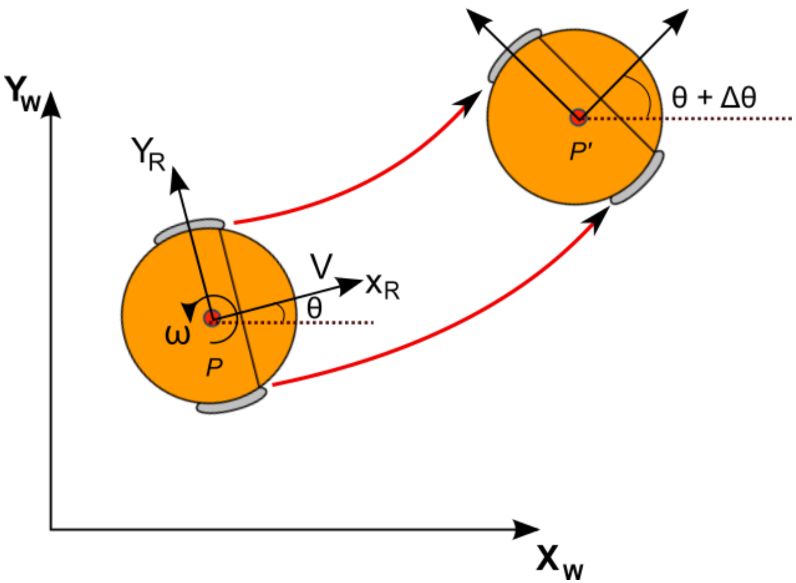
- The contributions of each wheel to the angular velocity in P can be computed independently and added up (signed)

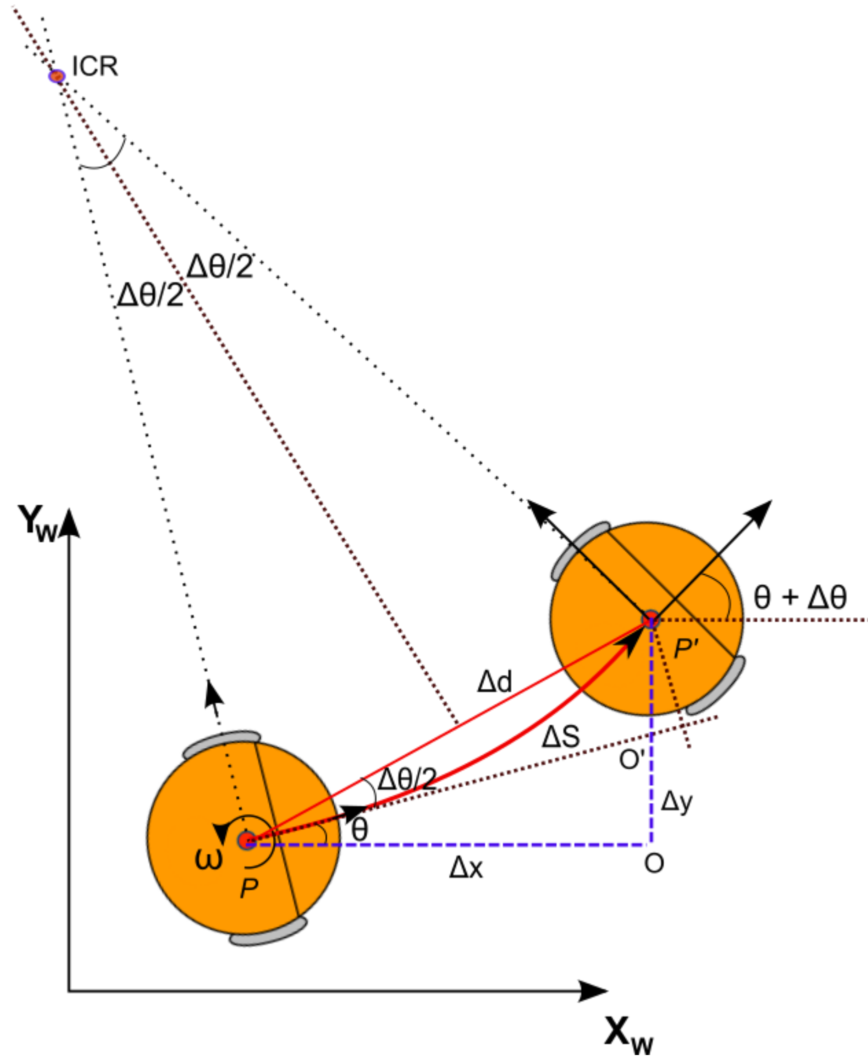
$$\omega_P = \frac{r\dot{\varphi}_1 - r\dot{\varphi}_2}{2\ell}$$

Composition of linear velocities

- If only the left (or right) wheel spins, the linear velocity of P is half its tangential velocity (because P is in the middle of the robot.
- The contributions of each wheel to the linear velocity in P can be computed independently and added up (signed)

$$v_P = \frac{r\dot{\varphi}_1 + r\dot{\varphi}_2}{2}$$





At a time t, an instantaneous motion of duration $\boldsymbol{\delta}$t results in an infinitesimal change in orientation equal to $\Delta\theta$, and in an infinitesimal displacement ΔS.

What is the robot pose ${}^{W}\boldsymbol{\xi}_{R}$ at time $(t + \boldsymbol{\delta}t)$?

The ICR will not change, and the new pose is the result of a rotation $\Delta\theta = \omega\boldsymbol{\delta}t$ of the robot about the ICR (ω is constant during the infinitesimal interval).

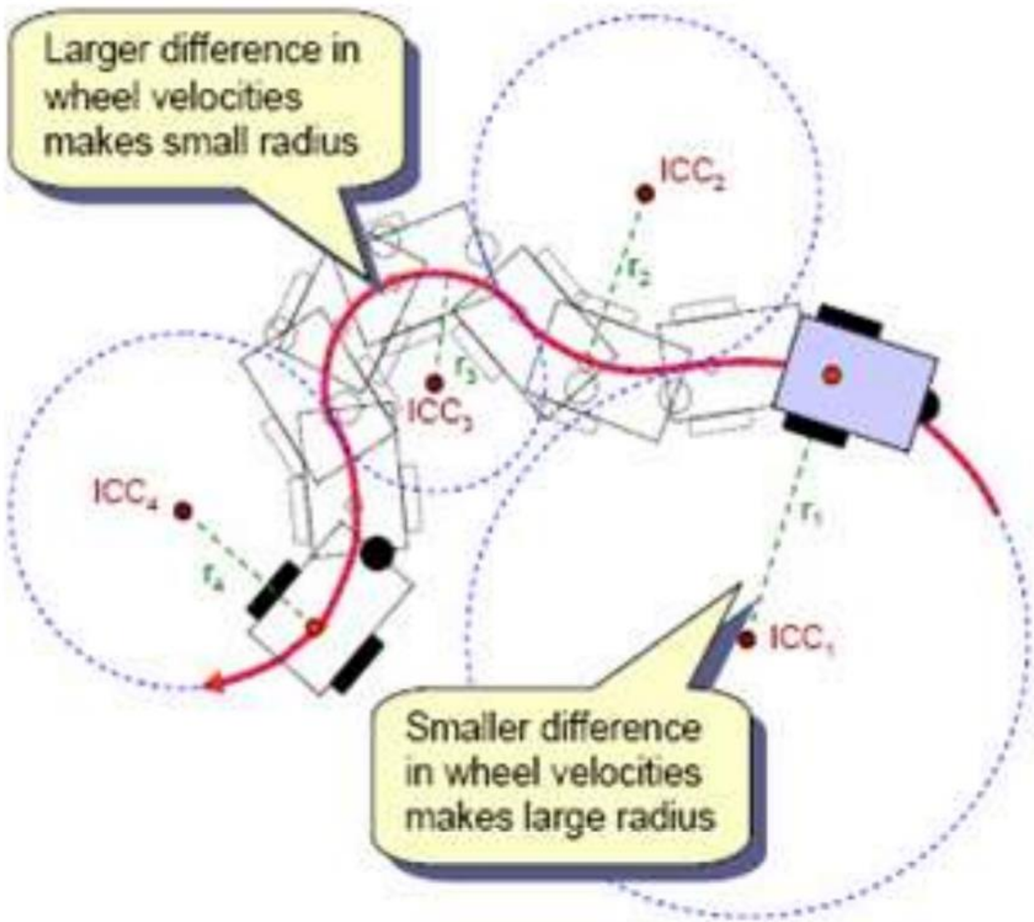
Steps to compute pose transform

1. Move the robot to the ICR

   I.e. translate by x,y = (0,R) with respect to the robot’s own reference frame.
   Note: we know R from the kinematic equations!

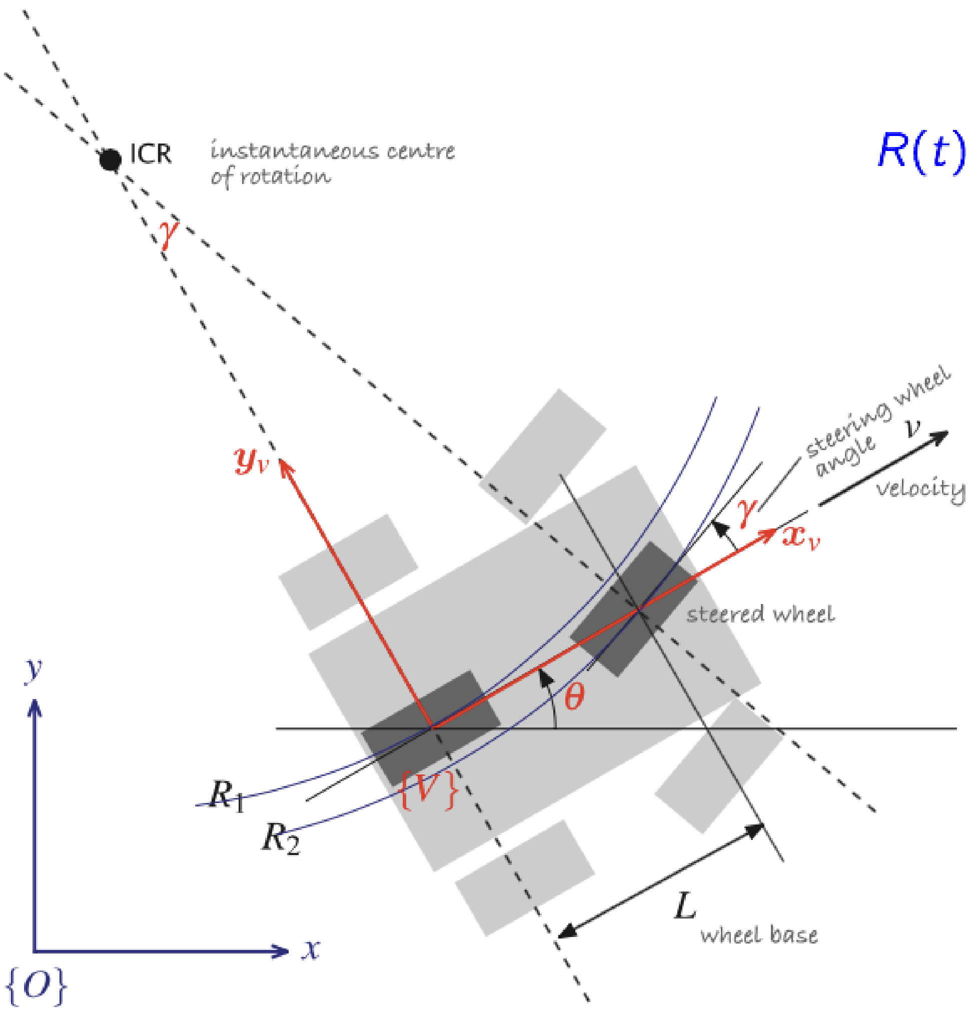
2. Rotate the robot in place by $\Delta\theta$
3. Translate the robot back by x,y = (0,-R) with respect to the robot’s **new** (rotated) reference frame.

Complex trajectories



Bicycle Kinematics, Car-like robots, Ackermann Steering

The bicycle model



$$R(t) = R_1(t) = \frac{L}{\tan(\gamma(t))}, \quad \omega(t) = \frac{v(t)}{R(t)}$$

The front wheel must follow a longer path, and therefore must rotate faster than the rear wheel.

$$R_2(t) > R_1(t)$$

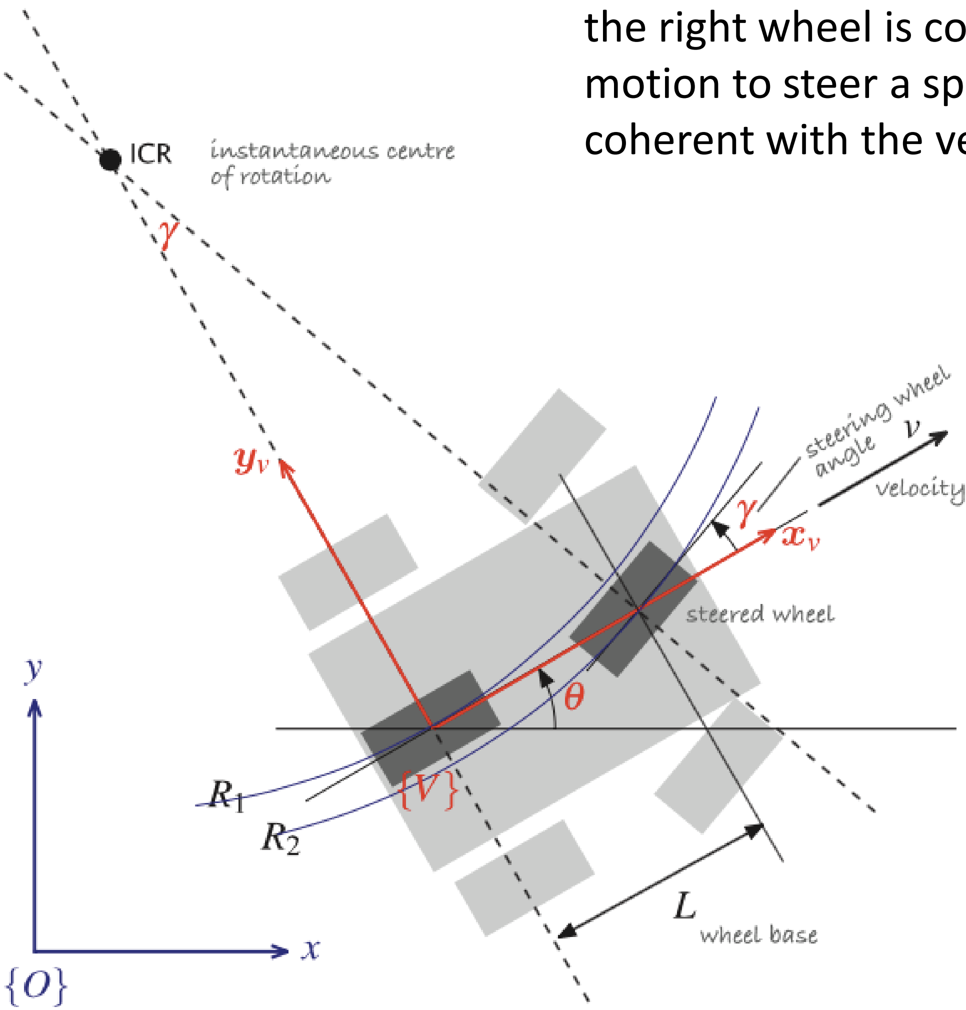
The limited range of $\boldsymbol{\gamma}$ limits maneuverability: parking problem, complex inverse kinematics

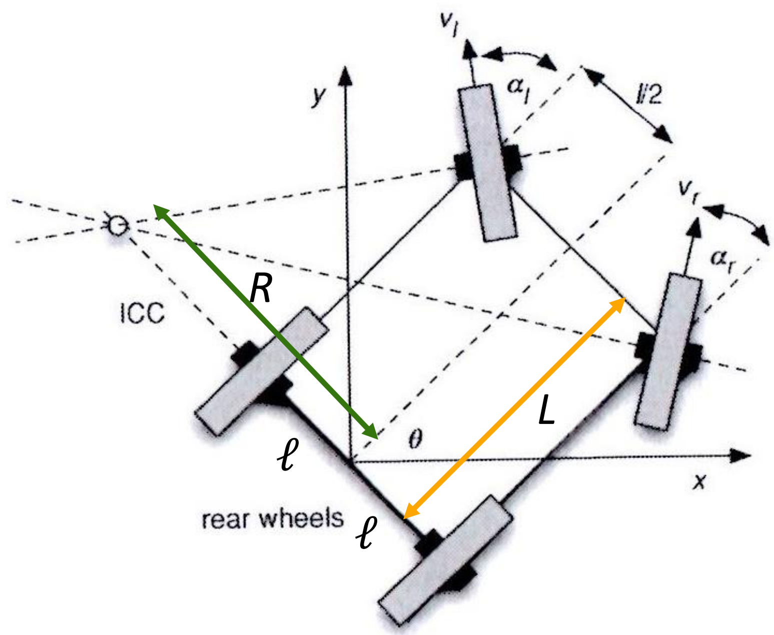
Car-like robots: Ackermann steering

Once set the steering for the left wheel, the right wheel is constrained by rolling motion to steer a specific angle which is coherent with the vehicle's ICR

$$R(t) - \ell = \frac{L}{\tan(\alpha_l(t))}$$

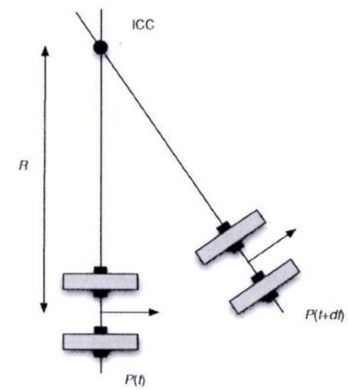
$$R(t) + \ell = \frac{L}{\tan(\alpha_r(t))}$$



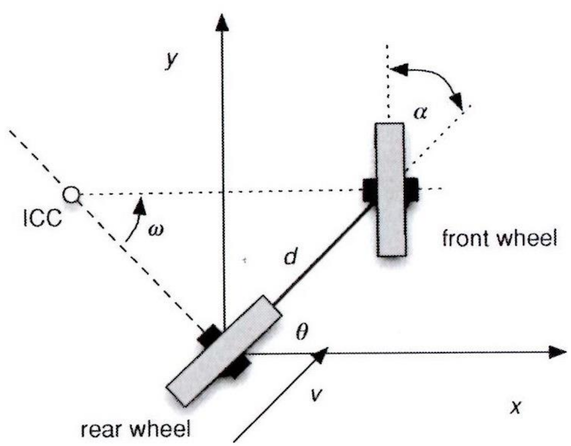


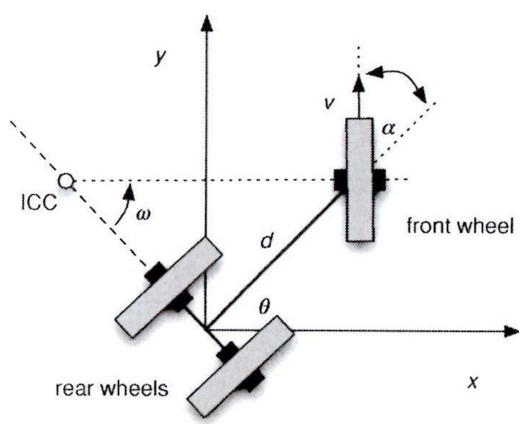
Summary of important kinematic models

Differential drive
(left and right wheel speeds actuated separately)

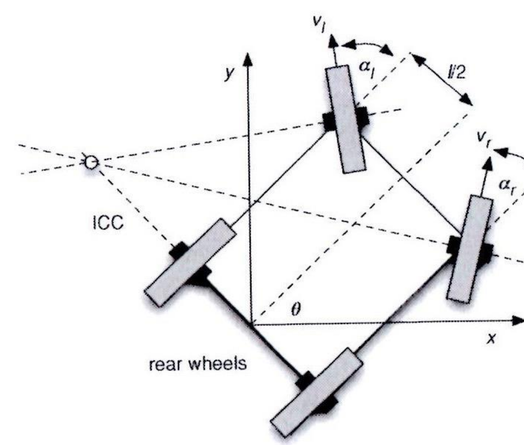


Bicycle and derivatives (one wheel speed actuated, one wheel steered)





(a) Tricycle schematic



(a) Ackerman steering

Implementation

See Jupyter notebook