Forward Kinematics and locomotion intro

Slides in part adapted from
Gianni di Caro, CMU

Part 1

Definitions

Definitions

- Locomotion

  Refers to the process of moving from one point to another, which requires the application of forces

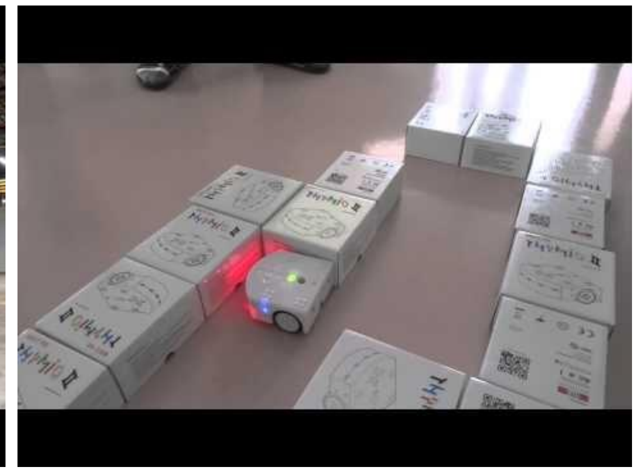
- Dynamics

  The study of motion by directly modeling the forces that cause it

- Kinematics

  The study of motion without taking into consideration the forces that cause it. It is based on geometric relations, positions, velocities, and accelerations

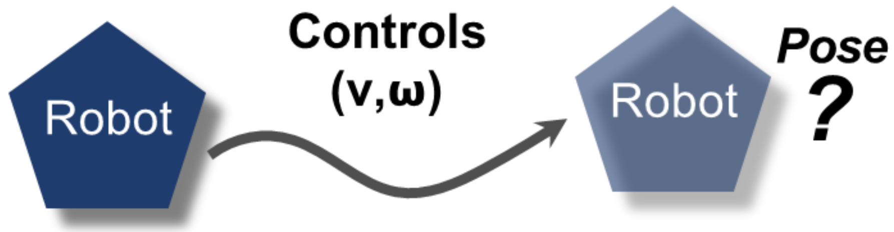
When is dynamics needed?



Definitions

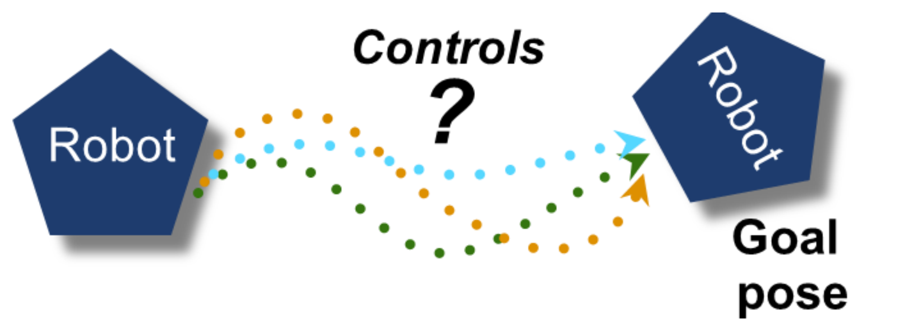
- Forward Kinematics

  Use of kinematic equations to determine / predict the final configuration/pose of a robot based on the specification of the values for the control variables (e.g., v,**ω**)



- Inverse Kinematics

  Given the desired final configuration (of the effectors/pose), make use of the kinematic equations to determine the values of the control variables that allow to achieve it.

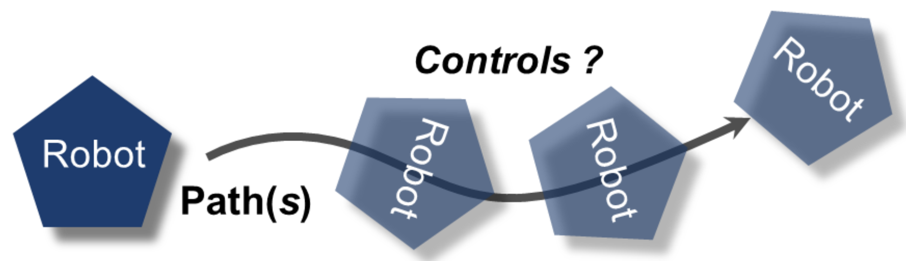


Definitions

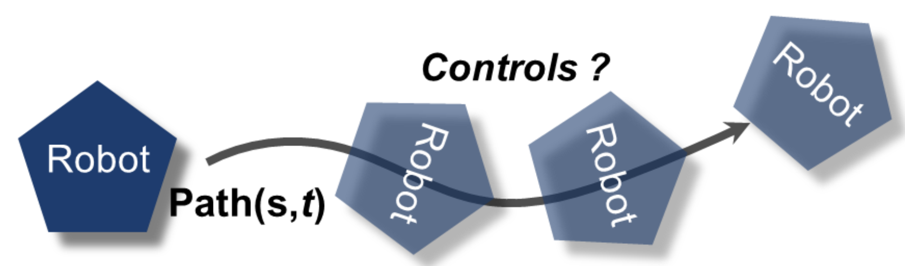
Motion Planning

The specification of the entire movement of the robot in terms of its control variables to achieve the desired configurations.

- Path following (only geometry)

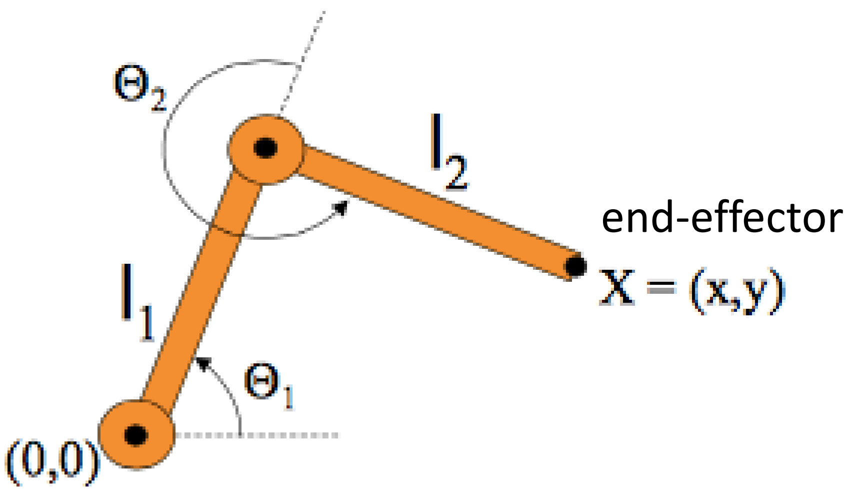


- Trajectory following (geometry+time)



Forward Kinematics: A known example

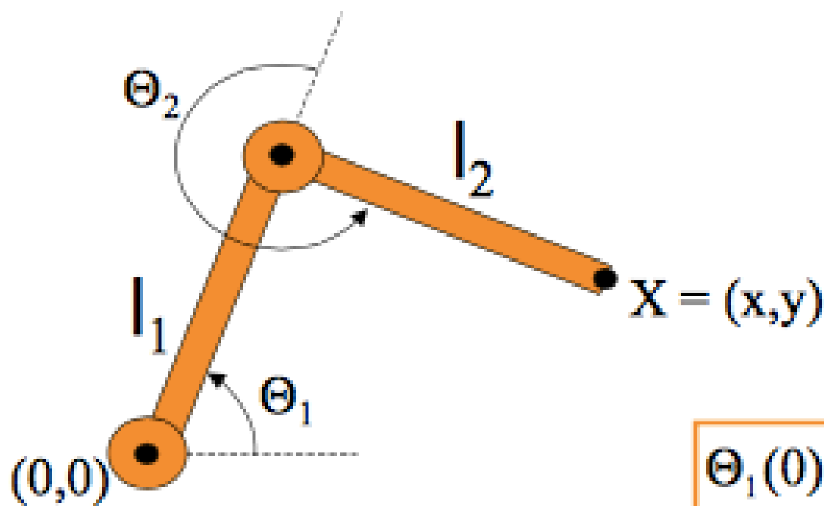
Given joint angles
determine the end-effector position



$$X = (l_1 \cos\Theta_1 + l_2 \cos(\Theta_1 + \Theta_2), l_1 \sin\Theta_1 + l_2 \sin(\Theta_1 + \Theta_2))$$

Forward Kinematics: A simple variation

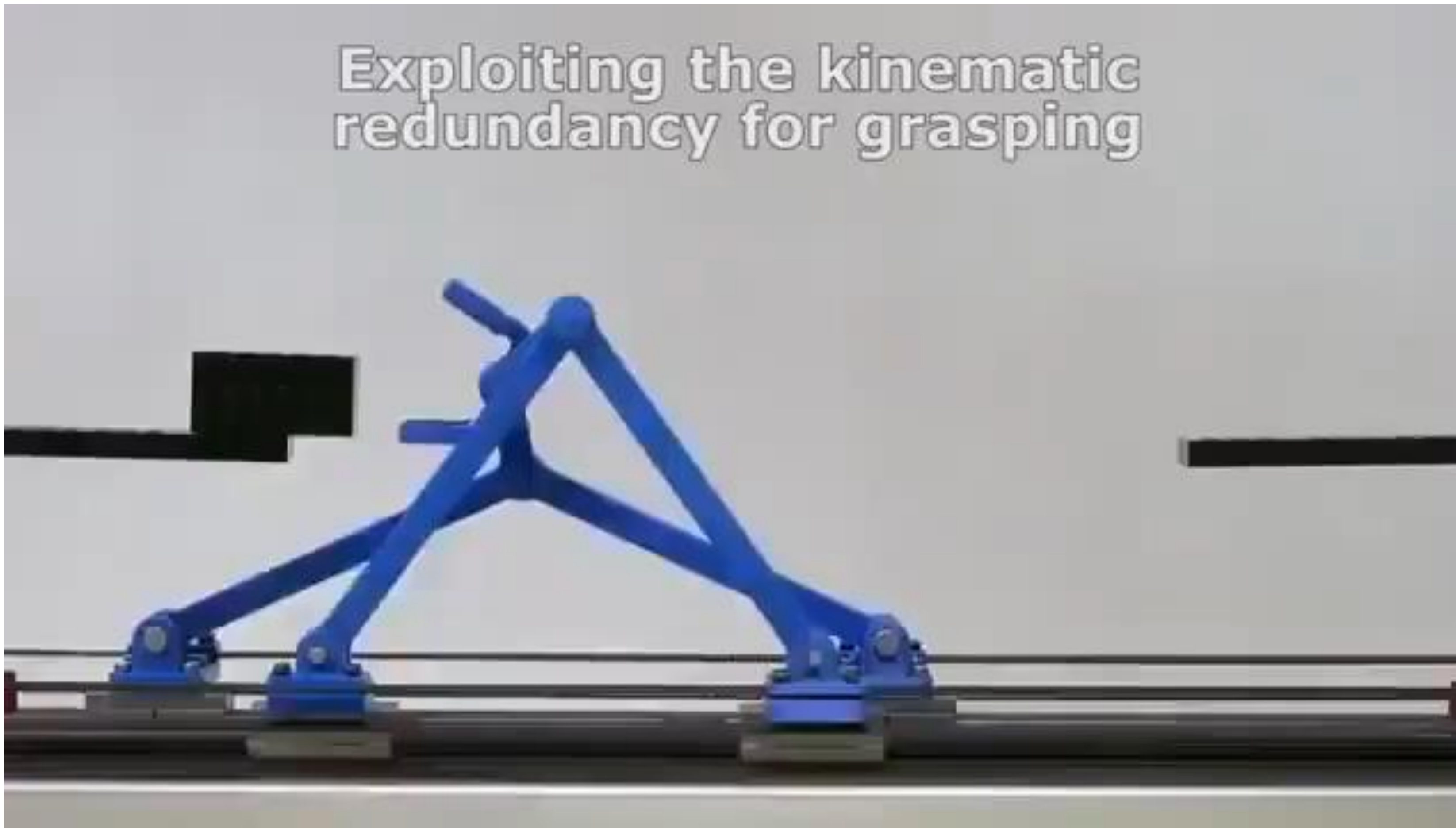
Given joint angles **and velocities**
determine the end-effector **trajectory in time**



$$\Theta_1(0) = 60^\circ \qquad \Theta_2(0) = 250^\circ$$

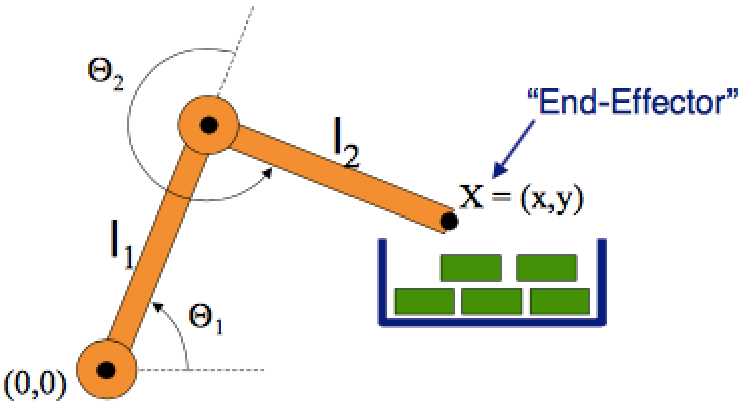
$$\frac{d\Theta_1}{dt} = 1.2 \qquad \frac{d\Theta_2}{dt} = -0.1$$

A robot arm with complicated kinematics

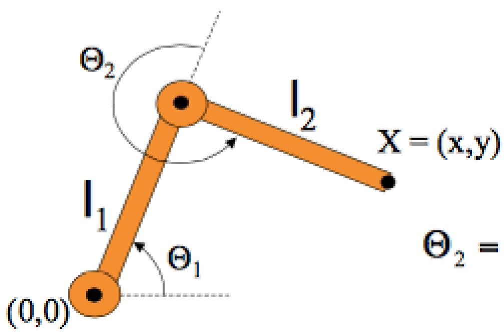


https://www.youtube.com/watch?v=_vp1ELEtDN4

Inverse Kinematics for arms



Given the desired position X of the end-effector, determine the values for the joint variables



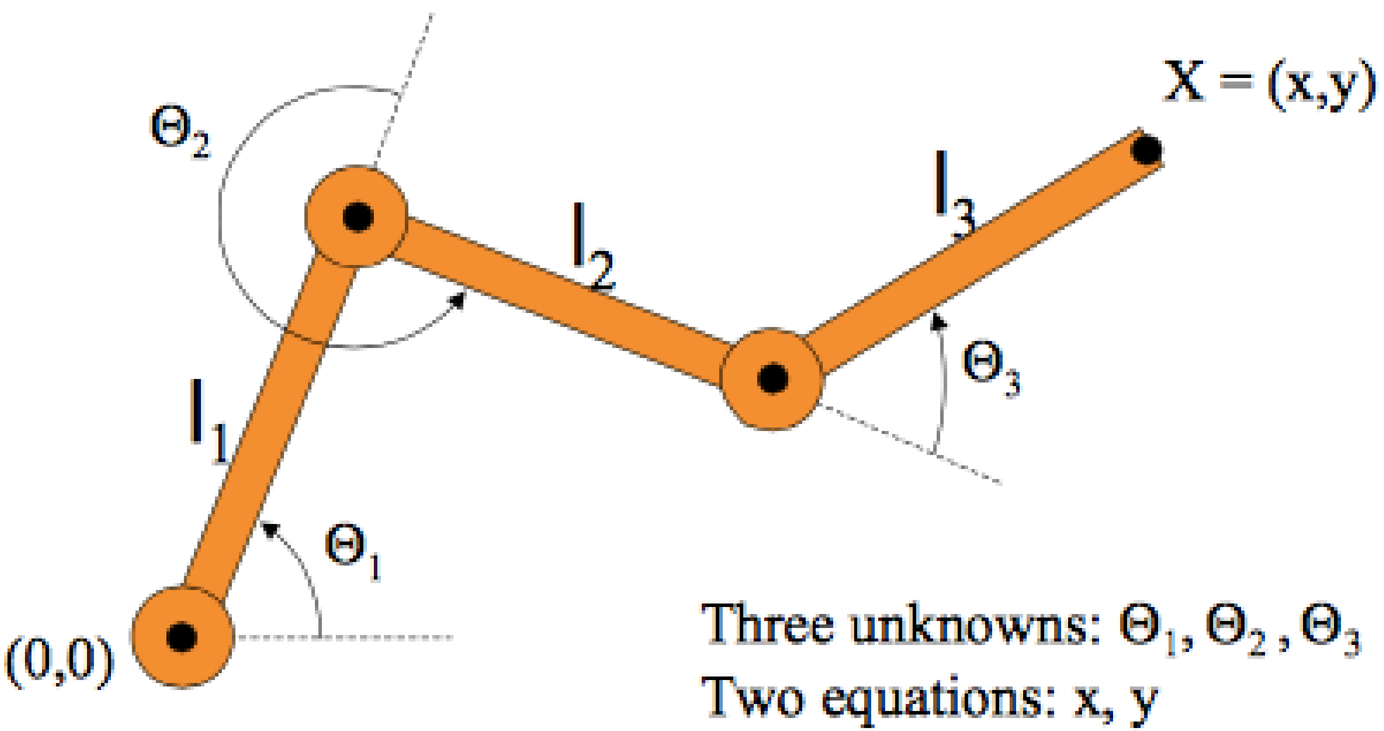
$$\Theta_2 = \cos^{-1}\left(\frac{x^2 + x^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

$$\Theta_1 = \frac{-(l_2 \sin(\Theta_2)x + (l_1 + l_2 \cos(\Theta_2))y}{(l_2 \sin(\Theta_2))y + (l_1 + l_2 \cos(\Theta_2))x}$$

Solve the kinematic equations wrt the final pose

Inverse Kinematics for arms

In the general case, IK problems admit multiple solutions!



Foward Kinematics for a differential drive mobile robot

Given:

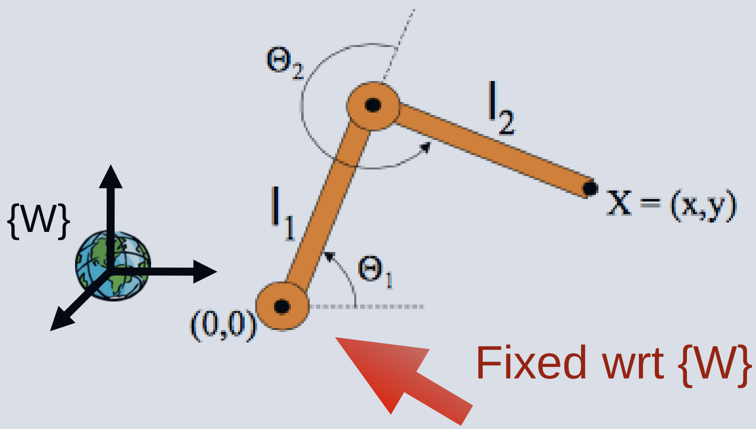
- the geometric parameters: number and type of wheels, wheel(s) radius, length of axes, …
- the initial conditions: pose and velocity
- and assigned the spinning speeds of each wheel: $\dot{\varphi}_1, \dot{\varphi}_2$

a forward kinematic model aims to predict the robot's generalized velocity (rate of pose change) in the global reference frame:

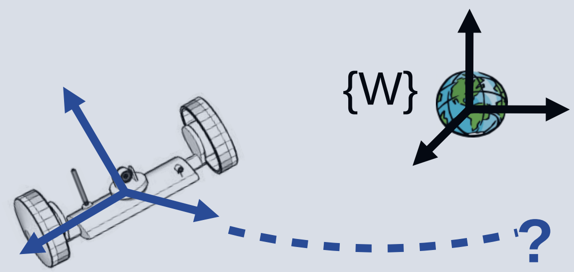
$$\dot{\xi}_W = \begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}^T = f(l, r, \theta, \dot{\varphi}_1, \dot{\varphi}_2)$$

FK for arms vs mobile robots

- ***Arm*: Constrained workspace** ➔ Measures of all intermediate joints + Kinematic equations

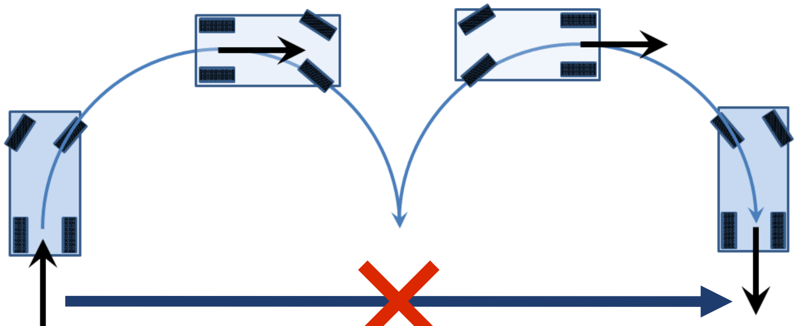


- ***Mobile*: It can span the entire environment**, no direct/obvious way to measure its position instantaneously/exactly ➔ Integrate motion over time + include uncertainties and errors (e.g., due to wheels slippage)

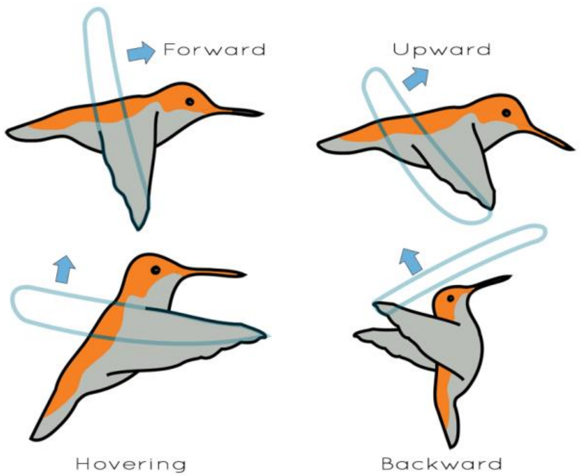


It is a much harder task!

Limits to the robot motion



Two-moves car
parking:
no side-way motion



No easy side-way motion in 3D

My car can't
do this

Formalizing constraints

- Geometric constraints, aka **Holonomic**

  Impose restrictions on the achievable configurations of the robot.
  Based on a functional relation among configuration variables

- Kinematic constraints, aka Non-holonomic

  Impose restrictions on the achievable **velocities** of the robot.

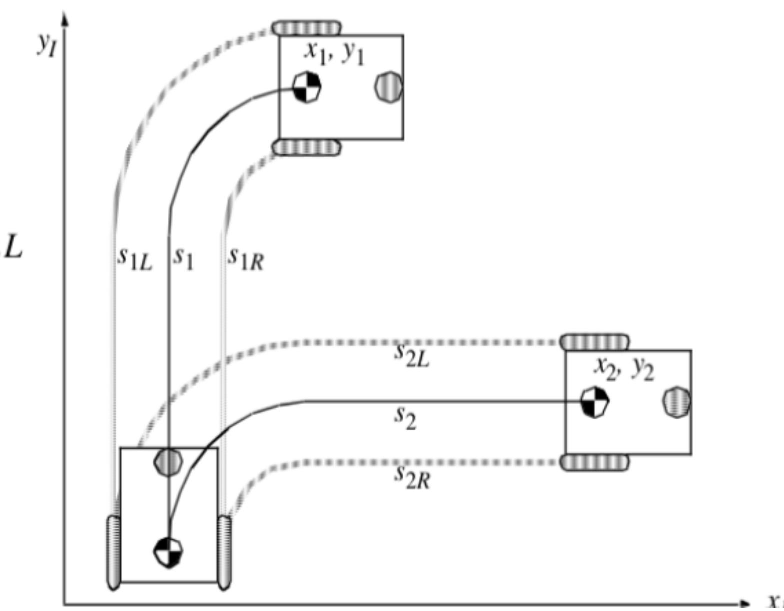
  Based on a functional relation among configuration variables **and their derivatives**

Note (teaser)

In a wheeled robot, the measure of the traveled distance of each wheel is not sufficient to calculate the final position of the robot: one has also to know **how this movement was executed as a function of time**.

$$s_1 = s_2, s_{1R} = s_{2R}, s_{1L} = s_{2L}$$

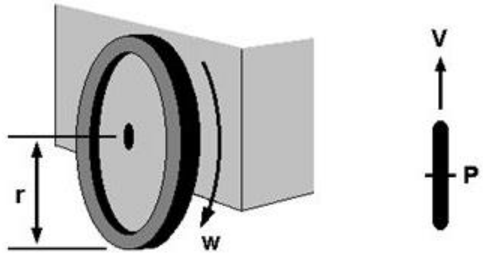
$$x_1 \neq x_2, y_1 \neq y_2$$



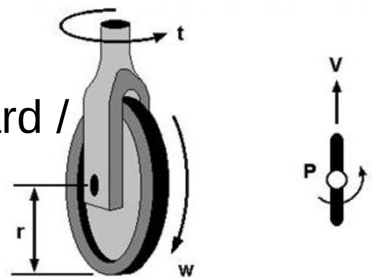
Part 2

Locomotion and wheels

Types of wheels

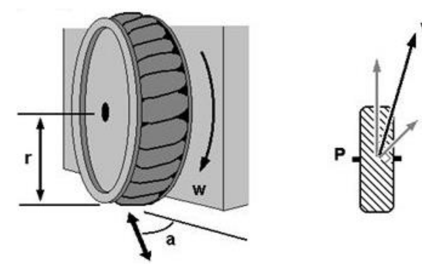


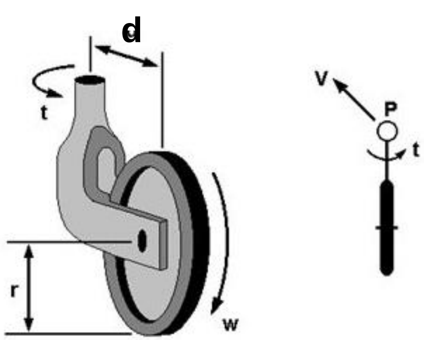
Fixed standard



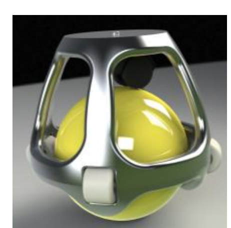
Steered standard /
Orientable

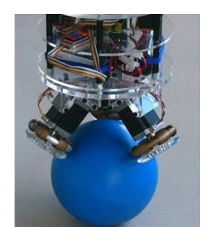
Mecanum/Swedish





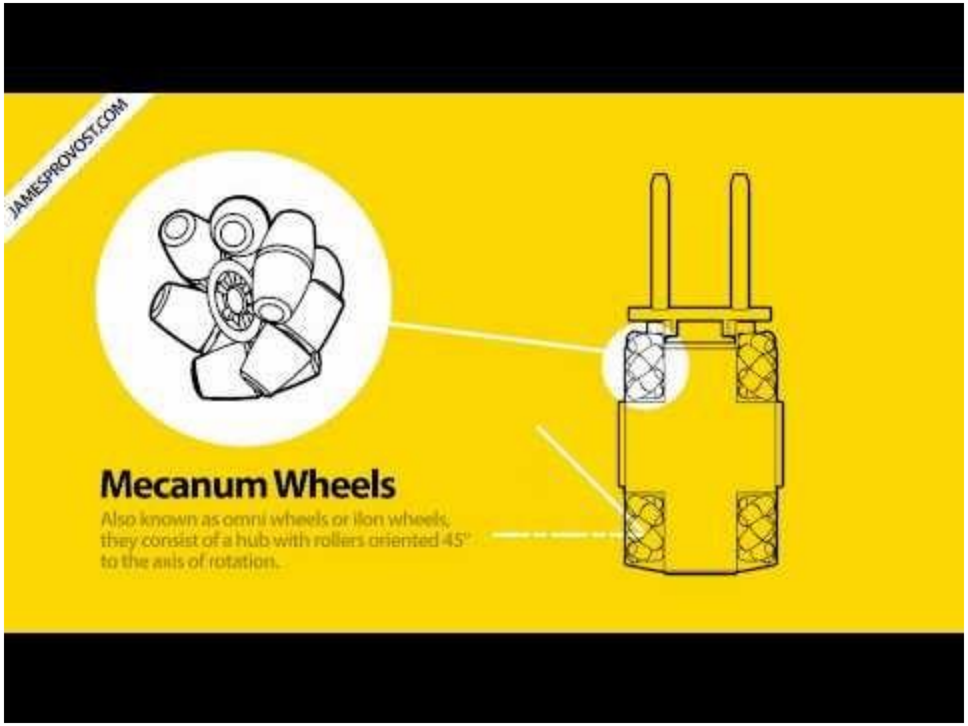
Castor /
Off-centered orientable





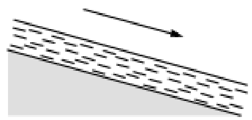
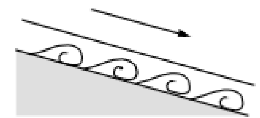
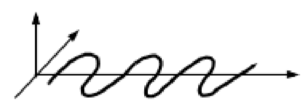
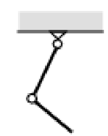
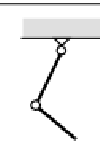
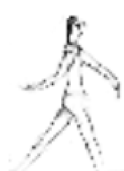
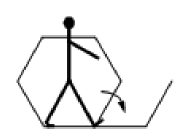
Spherical

Mecanum wheels



https://www.youtube.com/watch?v=o-j9TReI1aQ

Locomotion is more than wheels!

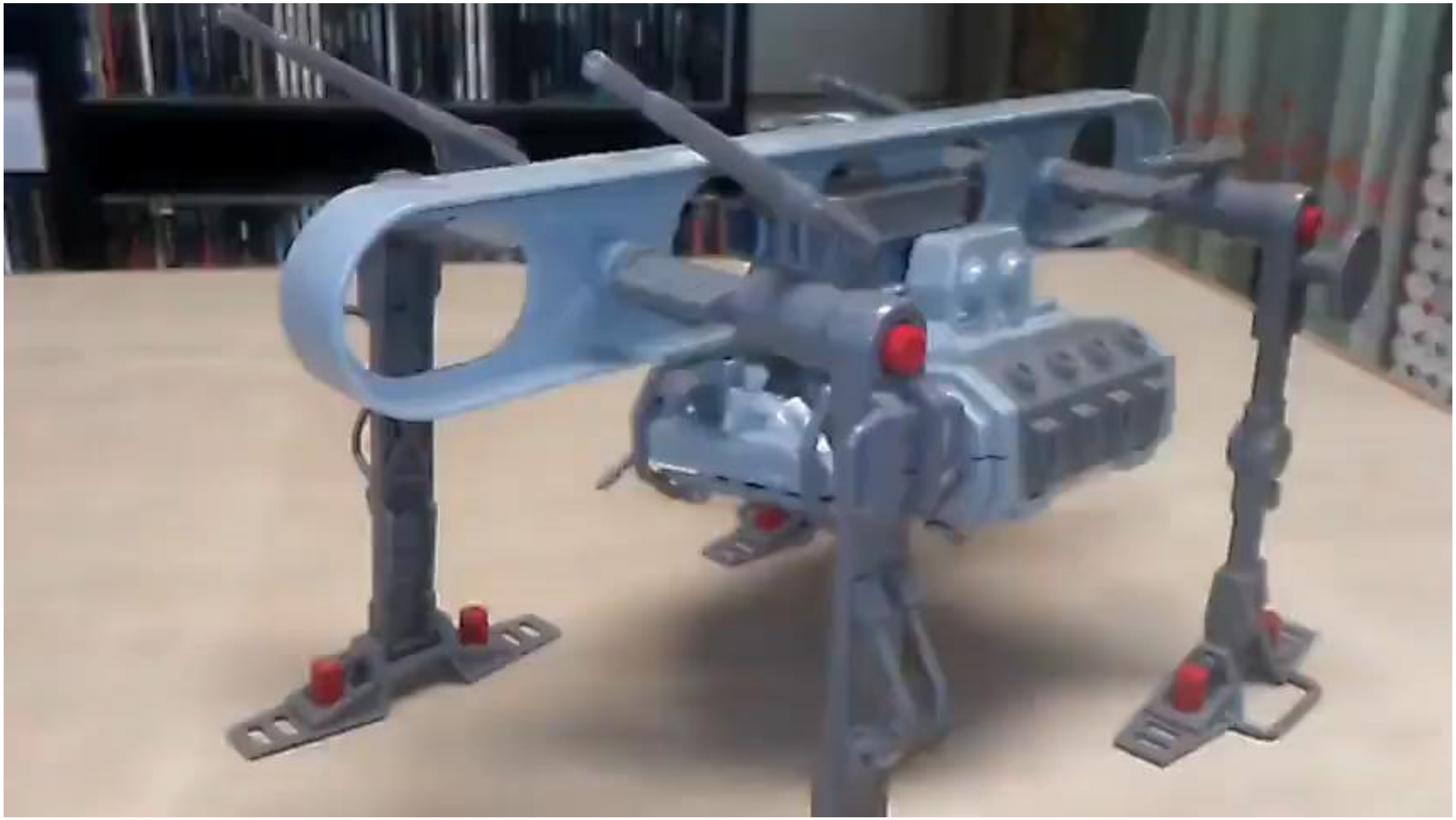
Type of motion	Resistance to motion	Basic kinematics of motion
Flow in a Channel 	Hydrodynamic forces	Eddies 
Crawl 	Friction forces	 Longitudinal vibration
Sliding 	Friction forces	 Transverse vibration
Running 	Loss of kinetic energy	Oscillatory movement of a multi-link pendulum 
Jumping 	Loss of kinetic energy	Oscillatory movement of a multi-link pendulum 
Walking 	Gravitational forces	Rolling of a polygon (see figure 2.2) 

Locomotion is more than wheels!

Karl Sims - Evolved Virtual Creatures, Evolution Simulation, 1994
https://www.youtube.com/watch?v=JBgG_VSP7f8&ab_channel=MediaArtTube

This video shows results from a research project involving simulated Darwinian evolutions of virtual block creatures. A population of several hundred creatures is created within a supercomputer, and each creature is tested for their ability to perform a given task, such the ability to swim in a simulated water environment. Those that are most successful survive, and their virtual genes containing coded instructions for their growth, are copied, combined, and mutated to make offspring for a new population. The new creatures are again tested, and some may be improvements on their parents. As this cycle of variation and selection continues, creatures with more and more successful behaviors can emerge.

Locomotion is more than wheels!



The Revolt Bear Mobil is a 1987 1/60 Bandai model kit with a very peculiar locomotion system

… but many robots DO use wheels

Because it’s easy!

- Three wheels are sufficient to guarantee stability
- With less than three wheels stability may be an issue
- With more than three wheels an appropriate suspension is required on nonflat ground
- Most arrangements are **non holonomic!**

A dynamically-stable 2-wheeled robot

Quiz: does a statically-stable 2-wheeled robot exist?

Some outdoor mobile robots

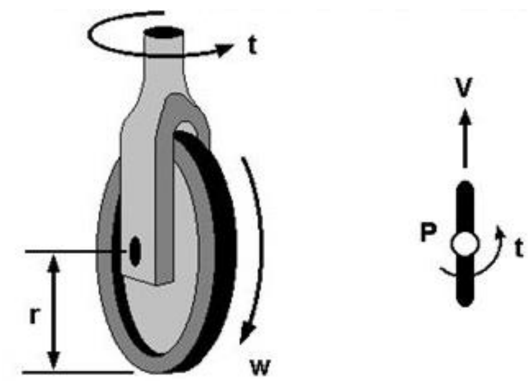
- Two wheels (differential drive)
  - Simple model
  - Suffers terrain irregularities
  - Cannot translate laterally

- Tracks
  - Suited for outdoor terrains
  - Not accurate movements (with rotations)
  - Complex model
  - Cannot translate laterally

- Omnidirectional (synchro drive)
  - Can exploit all degrees of freedom (3DoF)
  - Complex model
  - Complex structure

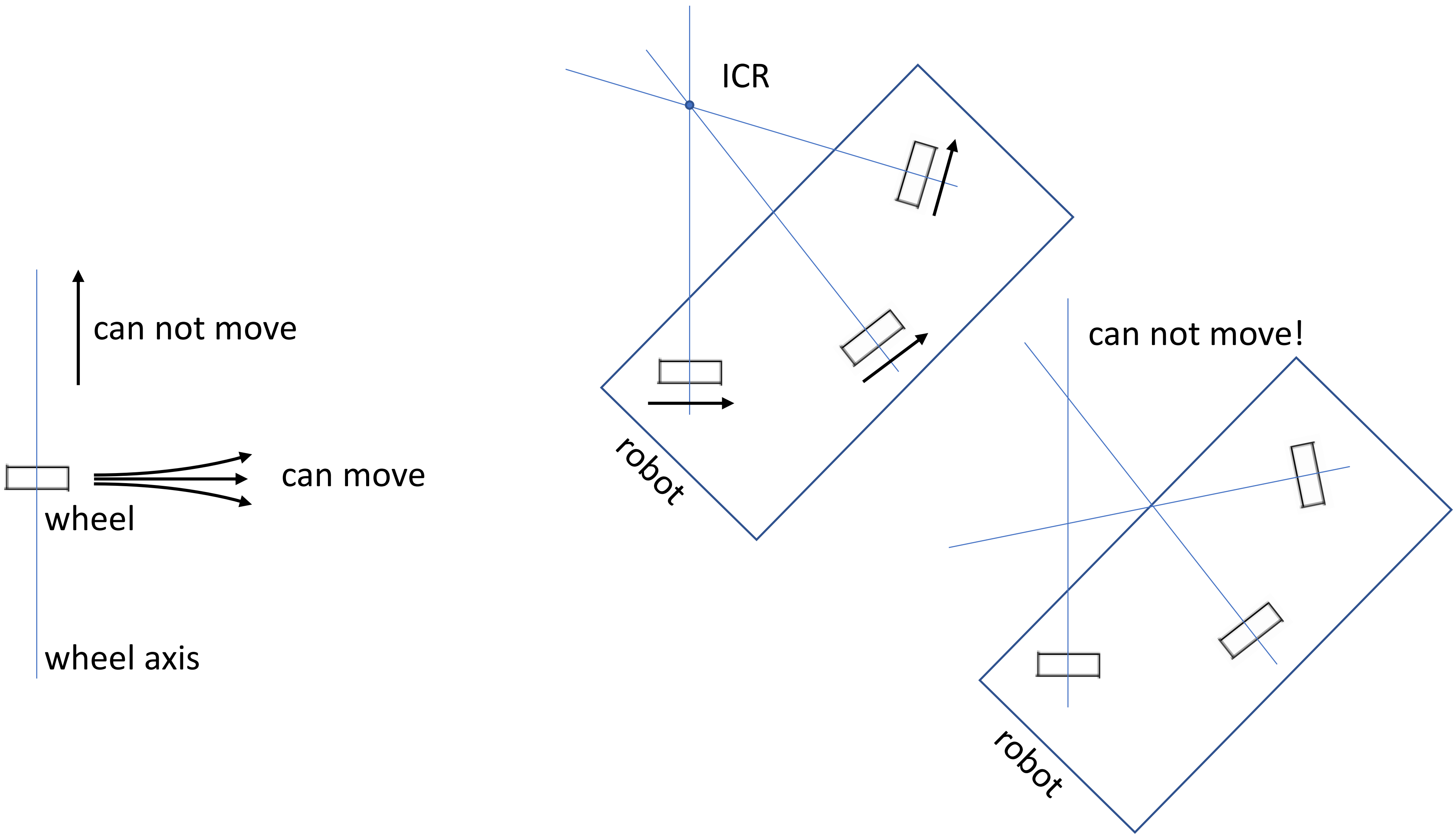
Basic assumptions for wheels

- Wheel does not slip
- The wheel can not move in directions orthogonal to the wheel plane
- Wheel plane orthogonal to ground → wheel axis parallel to ground
- For steered wheels: steering axis passes through the center of the wheel and is orthogonal to ground

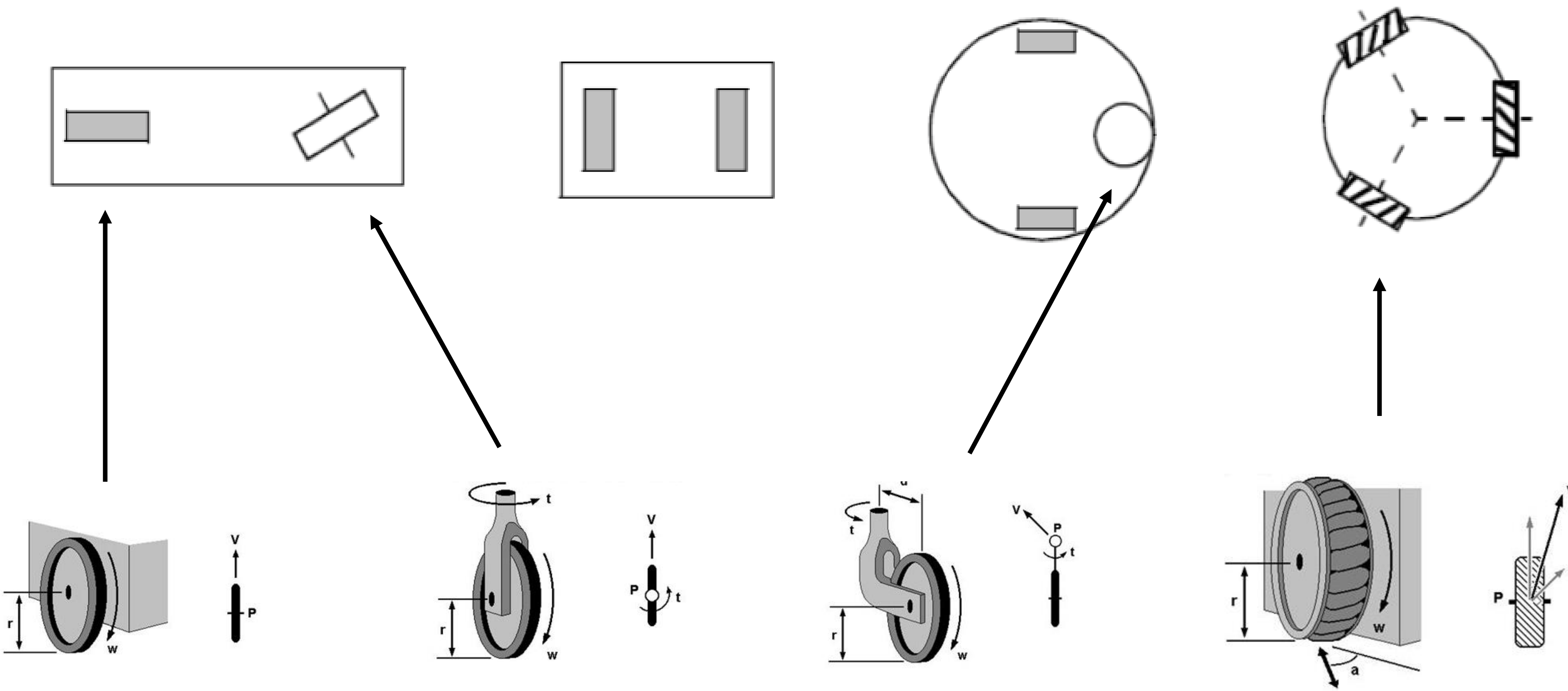


When the wheel moves in directions orthogonal to the wheel plane...

Instantaneous Center of Rotation (ICR), aka ICC



Arrangements of wheels: legend



Fixed standard wheel

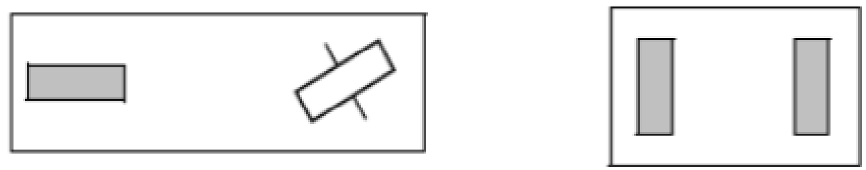
Steered standard wheel

Castor wheel

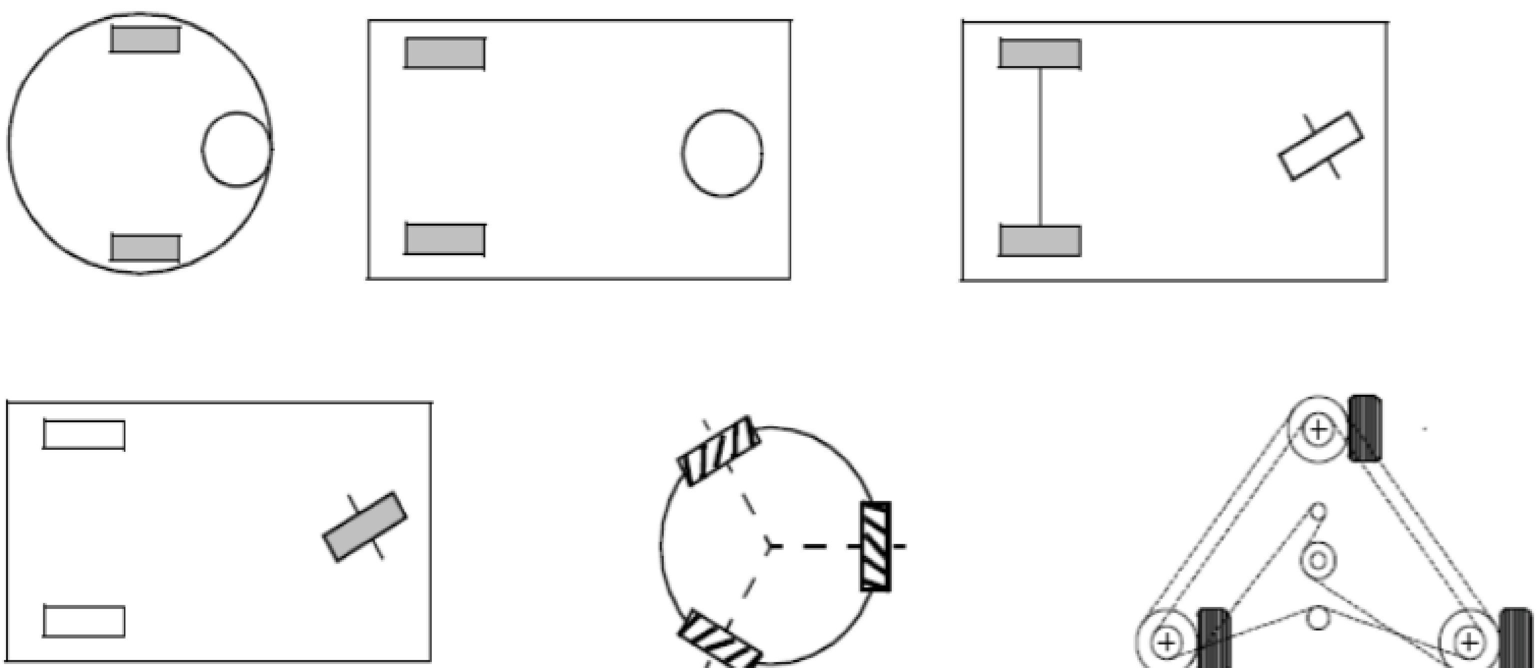
Mecanum wheel

Arragements of wheels

Two wheels



Three wheels



Arrangements of wheels

Four wheels