Stochastic Methods SP 2024

Assignment 3

1: Consider probability density functions f and g, and let h be an arbitrary function such that the condition $g(x) = 0$ implies $f(x)h(x) = 0$.

a) Derive the importance sampling identity for continuous random variables:

$$E_f[h(X)] = E_g\left[\frac{h(X)f(X)}{g(X)}\right],$$

where $E_f[\cdot]$ and $E_g[\cdot]$ denote the expected value with respect to the probability density functions f and g, respectively.

2: A store that stocks a certain commodity uses the following (s, S) ordering policy: if its supply at the beginning of a time period is x, then it orders

$$\begin{cases} 0 & \text{if } x \geq s, \\ S - x & \text{if } x < s. \end{cases}$$

The order is immediately filled. The daily demands are independent and equal j with probability α_j. All demands that cannot be immediately met are lost. Let X_n denote the inventory level at the end of the n-th time period. Argue that $\{X_n, n \geq 1\}$ is a Markov chain and compute its transition probabilities.

3: A particle moves along the following graph so that at each step it is equally likely to move to any of its neighbors.

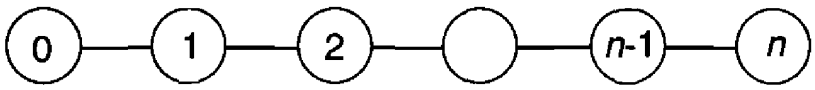

Figure 1: Particle moving along a line graph.

Starting at 0, show that the expected number of steps it takes to reach n is n^2.

(Hint: Let T_i denote the number of steps it takes to go from vertex $i-1$ to vertex i, $i = 1, \ldots, n$. Determine $E[T_i]$ recursively, first for $i = 1$, then $i = 2$, and so on.)

4: Suppose that whether or not it rains today depends on previous weather conditions through the last three days. Show how this system may be analyzed by using a Markov chain. How many states are needed?

5. A certain town never has two sunny days in a row. Each day is classified as being either sunny, cloudy (but dry), or rainy. If it is sunny one day, then it is equally likely to be either cloudy or rainy the next day. If it is rainy or cloudy one day, then there is one chance in two that it will be the same the next day, and if it changes then it is equally likely to be either of the other two possibilities. In the long run, what proportion of days are sunny? What proportion are cloudy?