Stochastic Methods SP 2024

Assignment 1, Due: 15th March

1. A gambler wins each game with probability p. In each of the following cases, determine the expected total number of wins.

   a) The gambler will play n games; if he wins X of these games, then he will play an additional X games before stopping.

   b) The gambler will play until he wins; if it takes him Y games to get this win, then he will play an additional Y games.

2. Let X be a Poisson random variable with parameter λ. Show that $P(X = i)$ increases monotonically and then decreases monotonically as i increases, reaching its maximum when i is the largest integer not exceeding λ.

    - *Hint:* Consider $\frac{P(X=i)}{P(X=i-1)}$.

3. . Let the probability density of X be given by

$$f(x) = \begin{cases} c(4x - 2x^2), & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) What is the value of c?

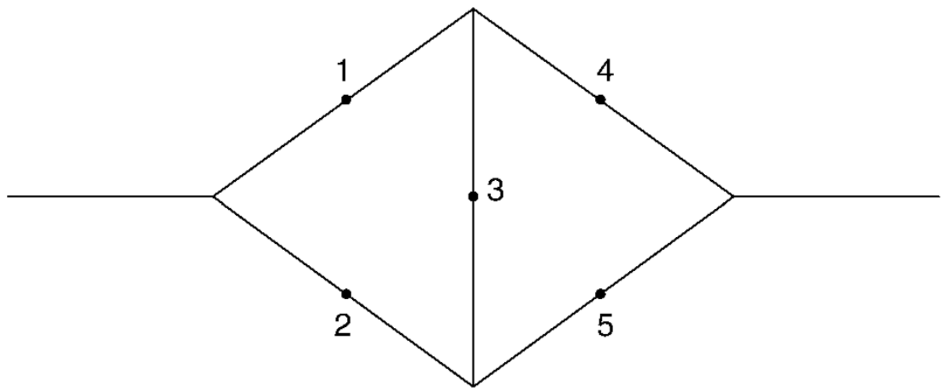
(a) $P\left(\frac{1}{2} \leq X < \frac{3}{2}\right) =?$

4. Let $X_i, i \geq 0$ be independent and identically distributed random variables with probability mass function

$$p(j) = P(X_i = j), \quad j = 1, \ldots, m, \quad \sum_{j=1}^{m} P(j) = 1$$

Find $E[N]$, where $N = \min\{n > 0 : X_n = X_0\}$.

5. There are five components. The components act independently, with component i working with probability p_i, $i = 1, 2, 3, 4, 5$. These components form a system as shown in the Figure.



The system is said to work if a signal originating at the left end of the diagram can reach the right end, where it can pass through a component only if that component is working. (For instance, if components 1 and 4 both work, then the system also works.) What is the probability that the system works?