P425/2
APPLIED MATHEMATICS
Paper 2
Nov./ Dec. 2020
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and any* **five** *questions from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take acceleration due to gravity g, to be 9.8 ms $^{-2}$.

Turn Over

SECTION A: (40 MARKS)

Answer **all** *the questions in this section.*

1. The position vector of a particle at any time t seconds is given by

$$r(t) = (t^2 + 4t)\,\boldsymbol{i} + (3t - t^3)\,\boldsymbol{j} \text{ metres.}$$

Calculate the speed of the particle when $t = 3$ seconds. *(05 marks)*

2. The table below shows the values of x and the corresponding values of a function $f(x)$.

x	0.3	0.6	0.9	1.2
$f(x)$	3.00	3.22	3.69	4.06

Use linear interpolation to find;

(a) $f(x)$ when $x = 0.4$ *(03 marks)*

(b) x when $f(x) = 3.82$ *(02 marks)*

3. The table below shows the price indices of beans, maize, rice and meat with the corresponding weights.

Item	**Price index 2008 (2007 = 100%)**	**Weight**
Beans	105	4
Maize	x	7
Rice	104	2
Meat	113	5

Calculate the;

(a) value of x given that the price indices of maize in 2007 and 2008 using 2006 as the base year are 112 and 130 respectively. *(02 marks)*

(b) weighted price index for 2008 using 2007 as the base year. *(03 marks)*

4. A particle moves with simple harmonic motion (SHM) about a mean position O with periodic time of $\frac{2\pi}{3}$ seconds. When the particle is 0.8 m from one extreme end, its speed is 3.6 ms $^{-1}$. Determine the amplitude of the motion. *(05 marks)*

5. The numbers $X = 1.2$, $Y = 1.33$ and $Z = 2.245$ have been rounded off to the given decimal places. Find the maximum possible value of

$$\frac{Y}{Z - X}$$

correct to **three** decimal places. *(05 marks)*

6. Two events are such that $P(A) = 0.7$, $P(B) = 0.2$ and $P(A/B) = 0.1$. Find:

(a) $P(A \cup B)$. *(03 marks)*

(b) $P(A \cap B')$. *(02 marks)*

7. A particle of weight 20 N is placed on a rough plane inclined at an angle of 40° to the horizontal. The coefficient of friction between the plane and the particle is ¼. When a horizontal force P is applied on the particle, it rests in equilibrium. Calculate the value of P. *(05 marks)*

8. A mobile phone dealer imports Nokia and Motorola phones. In a given consignment, 55% were Nokia and 45% were Motorola phones. The probability that a Nokia phone is defective is 4%. The probability that a Motorola phone is defective is 6%. A phone is picked at random from the consignment. Determine the probability that it is;

(a) defective. *(03 marks)*

(b) a Motorola given that it is defective. *(02 marks)*

SECTION B: (60 MARKS)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

9. The table below shows marks obtained by 100 students in a Mathematics test.

Marks	Number of students
20 – < 40	5
40 – < 50	15
50 – < 55	10
55 – < 60	15
60 – < 70	25
70 – < 90	25
90 – < 100	5

(a) Calculate the mean mark. *(05 marks)*

(b) Draw a cumulative frequency curve (Ogive) and use it to find the;

(i) median mark.

(ii) range of the middle 40% of the marks.

(07 marks)

10. Two bodies A and B of masses 3 kg and 2 kg respectively are 7 m apart on a smooth horizontal surface. A is moving directly towards B with a speed of 2 ms $^{-1}$ and an acceleration of 0.3 ms^{-2}. B is moving in the same direction as A with a speed of 5 ms^{-1} and a retardation of 0.2 ms^{-2}. If the bodies collide and coalesce, calculate the;

(a) time taken before collision occurs. *(08 marks)*

(b) common velocity immediately after the collision. *(04 marks)*

11. (a) Use the trapezium rule with 6-ordinates to estimate

$$\int_{0.1}^{0.5} \frac{1}{2x+1}\,dx$$

correct to **three** significant figures. *(06 marks)*

(b) Evaluate $\displaystyle\int_{0.1}^{0.5} \frac{1}{2x+1}\,dx$

correct to **three** significant figures. *(02 marks)*

(c) (i) Determine the percentage error in the estimation in (a) above, correct to **two** decimal places.

(ii) Suggest how the percentage error may be reduced. *(04 marks)*

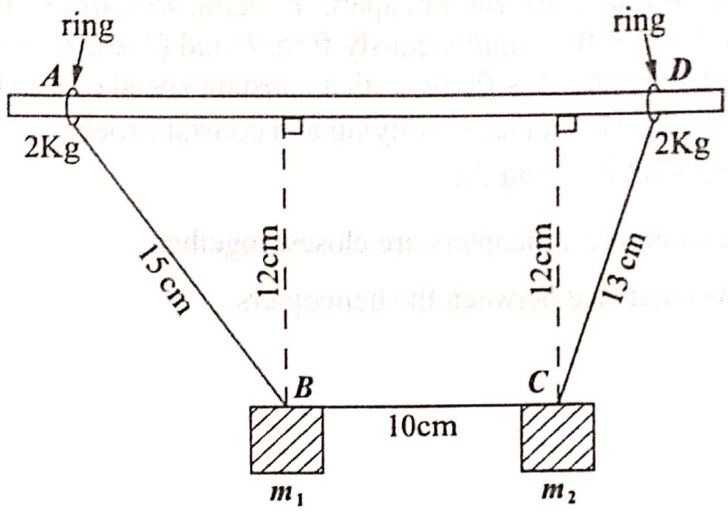
12. A continuous random variable X has a cumulative distribution function given by

$$F(x) = \begin{cases} 0; & x \leq 0 \\ \frac{k}{2}x^2; & 0 \leq x \leq 2 \\ k(6x - x^2 - 6); & 2 \leq x \leq 3 \\ 1; & x \geq 3 \end{cases}$$

(a) Determine the value of k.
Hence sketch the graph of $F(x)$. *(08 marks)*

(b) Find the probability density function (pdf) of X. *(04 marks)*

13. The diagram below shows three strings $AB = 15$ cm, $BC = 10$ cm and $CD = 13$ cm. A and D are fixed to small rings each of mass 2 kg which can slide on a rough horizontal rail AD. Masses m_1 and m_2 are attached at B and C respectively. The system rests in equilibrium with BC at a distance of 12 cm below AD.



(a) Show that $9m_1 = 5m_2$. *(07 marks)*

(b) If the coefficient of friction between each ring and the rail is ¼ and the ring at A is on the point of slipping, determine the value of m_1.

(05 marks)

14. (a) (i) Draw on the same axes the graphs of $y = x\sin x$ and $y = e^x - 2$ for $0.5 \le x \le 1.5$.

(ii) Use your graphs to find an approximate root of the equation $2 - e^x + x\sin x = 0$. *(06 marks)*

(b) Using Newton Raphson iterative formula and your approximate root in a (ii) above as the initial value, calculate the root of the given equation correct to **three** decimal places. *(06 marks)*

15. A certain football team has three matches to play. The probabilities of winning the first, second and third matches are $\frac{3}{5}, \frac{2}{5}$ and $\frac{1}{5}$ respectively.

(a) Find the probability that the team wins;

(i) exactly two matches.

(ii) all matches.

(iii) no match.

(07 marks)

Turn Over

0.5, 0.6 0.7 0.8

(b) If a random variable X is defined as "the number of matches won",

(i) construct a probability distribution table for X.

(ii) calculate the expectation of X, $E(X)$. *(05 marks)*

16. Two airstrips P and Q are 100 km apart, P being west of Q. Two helicopters A and B fly simultaneously from P and Q respectively, at 11.00 a.m. Helicopter A is flying with a constant speed of 400 kmh^{-1} in a direction N50°E. Helicopter B is flying at a constant speed of 500 kmh^{-1} in the direction N70°W. Find the;

(a) time when the helicopters are closest together. *(08 marks)*

(b) closest distance between the helicopters. *(04 marks)*

END