Understanding

SENIOR ONE MATHEMATICS

BASED ON THE NEW LOWER SECONDARY CURRICULUM

KAZIBA STEPHEN

1^{ST} EDITION 2020

UNDERSTANDING

SENIOR ONE MATHEMATICS

BASED ON THE NEW LOWER SECONDARY CURRICULUM
by

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Key Topics covered in the book

- *NUMBER BASES*
- *WORKING WITH INTEGERS*
- *FRACTIONS, PERCENTAGES AND DECIMALS*
- *RECTANGULAR CARTESIAN COORDINATES IN 2 DIMENSIONS*
- *GEOMETRIC CONSTRUCTION SKILLS*
- *SEQUENCE AND PATTERNS*
- *BEARINGS*
- *GENERAL AND ANGLE PROPERTIES OF GEOMETRIC FIGURES*
- *DATA COLLECTION AND PRESENTATION .*
- *REFLECTION*
- *EQUATION OF LINES AND CURVES*
- *ALGEBRA 1*
- *BUSINESS ARITHMETIC*
- *TIME AND TIME TABLES*

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1st EDITION 2020

The only way to learn mathematics is to do mathematics.

Preface

Mathematics is increasingly useful in every day life.In modern society we rely heavily on mathematics to solve problems . The book covers 14 chapters of mathematics as per new upgraded curriculum prescribed by the National curriculum development center.All the changes made by the NCDC are implemented in this book.

This book is my attempt to enrich and enliven the teaching of mathematics ,and i have achieved this through including in new features that can stimulate the learning of the content.

At the beginning of each chapter ,the expected learning outcomes are given,introductory notes are given for each section explaining the concept in a very simple language.

I have included a Plenty of worked examples for each section and subsection and accompanied them with exercises at the end. Activities of integration for each chapter have been included and these activities can eitherbe done in pairs,groups or as a whole class. .

I would advice learners to go through all the problems in the exercises on their own ,before asking assistance from friends and teachers.I have not been able to include the answers for the exercises at this point,however in our next edition all the answers will be included
I feel confident that this book will be of immense value to both the students and the teachers

DEDICATION

To my beloved students ,Namutebi priscilla ,Twanza maria keziah,Muhanuzi daniel,Nabagereka alexadra,Nalubega sheillah,Akasha byona,Kirabo glen arthur,Nattabi pauline,Bisaso hannah grace, Mulumba mimmi ,Maisha alesi,Akabwai michael,Katumba victor and all the senior one students of Taibah international school ,the class of 2020 .

ACKNOWLEDGEMENTS

First of all I offer my sincere gratitude to the almighty God, who has brought me this far because with his grace anything is possible. I would firstly like to thank my friends more so,Mr Madoi geofrey , Mrs Kaggwa Sylvia, Mr Lukwago bashir ,Fr Kyazze Frank(Nyenga seminary),Mr sekirabi edward (Nyenga seminary),Mr Makumbi edward(Ndejje ss),,Mr Nshuguyika stephen(Mt st mary's Namagunga),and Mr ssempagala solomon(Namilyango high school) for your love , time and constant advice you shared with me during the course of writing. Thank you for the many helpful discussions , for your constant interest and time, encouragement ,suggestions and guidance . Really, you have both been amazing and an inspiration.Thank you so much am really indebted.

Iam thankful to and fortunate enough to get constant encouragement, support and guidance from all Teaching staffs of Taibah international school which helped me in successfully completing this work.

I must express my very profound gratitude to all the organisations ,authors and every one who has provided me with his/her resources to ensure that am able to come up with this book. This accomplishment would not have been possible without you. Thank you very much am really indebted .

Finally i would like to I welcome communications from students,parents and teachers especially when they concern errors or deficiencies that you find in this book.Feel free to let me know what still needs to be improved.

Contents

Chapter 1: NUMBER BASES

Learning objectives

By the end of this topic,the learners should be able to

- Identify numbers in any base using abacus
- Convert numbers from one base to another
- Manipulate numbers in different bases with respect to all four operations
- Identify place value in different bases

Introduction

[1]A number base is the number of digits or combination of digits that a system of counting uses to represent numbers. A base can be any whole number greater than 0. The most commonly used number system is the decimal system, commonly known as base 10. In everyday life, we count or estimate quantities using groups of ten items or units. This may be so because, naturally, we have ten fingers. For example, when we count ten, i.e. we write 10 meaning one group of 10 and no units. A quantity like twenty five, written as 25 means 2 groups of 10 and 5 units Suppose instead we had say 6 fingers

- How, in your opinion would we do our counting?
- If we had eight fingers, how would we count?

This is now what we are to cover under this topic.

NOTE

- The digits of a number in any base are less than the base itself
- The digits 10 and 11 are represented by **t** and **e** respectively in number bases
- For digits above 11 are represented by alphabetic letters of your choice
- The names of some number systems is as given below

NUMBER SYSTEM	NAME	BASE VALUE
Base three	trinary base	3
Base four	quaternary base	4
Base five	quinary base	5
Base six	seximal base	6
Base seven	septimal base	7
Base eight	octal base	8
Base nine	nonary base	9
Base ten	decimal base	10
Base twelve	duodecimal base	12
Base sixteen	hexadecimal base	16

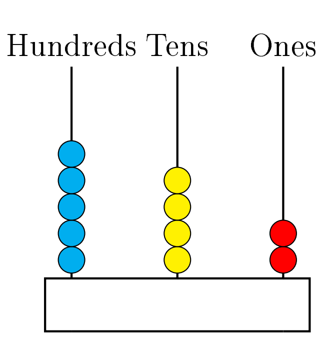
Activity:Getting familiar with number bases

Bases are used in day today life.Therefore copy and complete the table below by giving some real life situations were bases are used

Real life situation	Base	Reason for the base chosen
Days of the week	Base Seven	Seven days in a week
Football team	Base eleven	There are 11 players per team
Months of a year	Base twelve	Twelve months in a year

1.1 Identifying numbers of different bases on an abacus

1. Which possible base does each abacus below represent.



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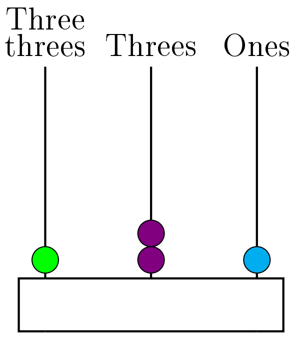

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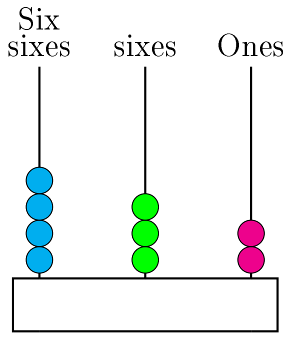

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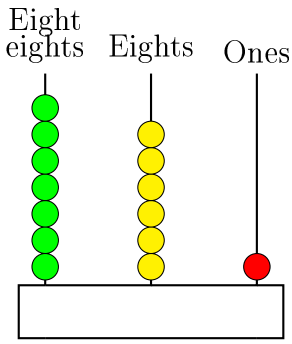

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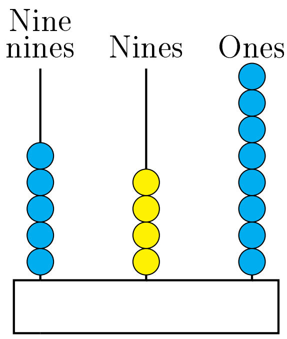

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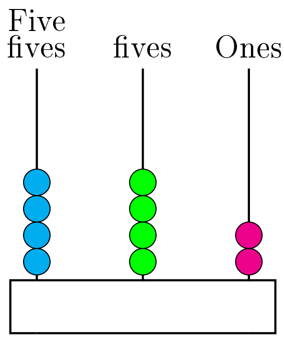
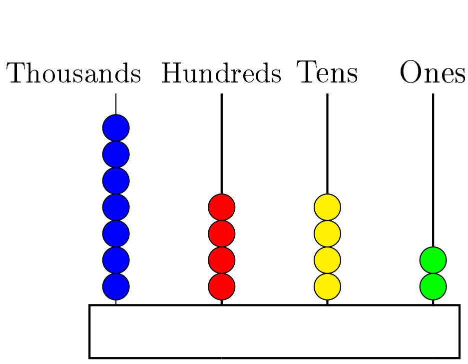


(f)

(b) Write down the numbers represented on the abaci above.

2. Write down the numbers represented on the abaci below.



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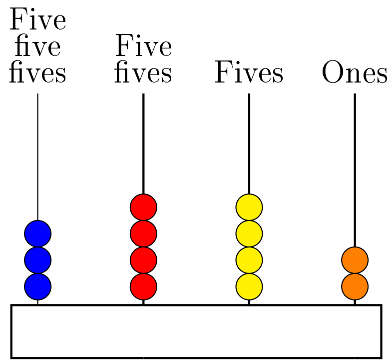

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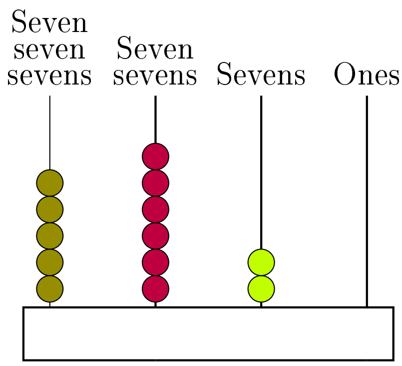

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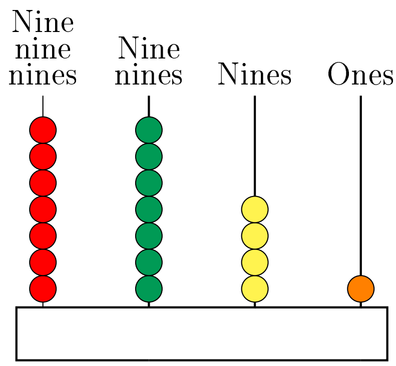


(d)

Activity:List the numerals for the following bases

Numerals are digits(or symbols) that are used for writing numbers in a given base.The digits are always less than the base itself. study the table below and fill in the gaps.

NUMBER SYSTEM	BASE VALUE	NUMERALS	EXAMPLE
Base 2	2	0,1	1111_{two}
Base 3	3		
Base 4	4		
Base 5		0,1,2,3,4	
Base 6			
Base 7			
Base 8			457_{eight}
Base 9			
Base 10			
Base 12	12	$0, 1, 2, 3, 4, 5, 6, 7, 8, 9, t, e$	$te5_{\text{twelve}}$
Base 16			

1.2 Place Values Using the Abacus

The representation of numbers on an abacus helps in identifying the place value of digits in any base.

Activity : Making abaci

1. Make abaci for the following number bases.

   (a) 220_{four} (d) 1110_{two} (g) 7562_{eight}

   (b) 6321_{ten} (e) 5552_{six} (h) 654_{nine}

   (c) 43_{five} (f) 6431_{seven} (i) 5974_{eleven}

2. Make an abacus for any number in base twelve

Activity : Reading and stating the value of digits in bases

1. State the place value and value of digit for each numeral in the following numbers:

   (a) 523

5 2 3	Place value	Value of digit
3	Ones	$3 \times 1 = 3$
2	Tens	$2 \times 10 = 20$
5	Hundreds	$5 \times 100 = 500$

(b) 5.12_{ten}

5 . 1 2	Place value	Value of digit
2	Hundredths	$2 \times \frac{1}{100} = 0.02$
1	Tenths	$1 \times \frac{1}{10} = 0.1$
5	ones	$5 \times 1 = 5$

1.1 Exercise Set

1. State the place value of each numeral in the following numbers:

   (a) 143_{five} (c) 5552_{six} (e) 4562_{eight} (g) 45.62_{eight}

   (b) 110_{two} (d) 431_{seven} (f) 4.1234_{ten} (h) 456.212_{ten}

2. State the value of each numeral in the following numbers:

(a) 1432_{five} (c) 3412_{six} (e) 6542_{eight} (g) 45.62_{eight}

(b) 111_{two} (d) 431_{seven} (f) 4.1234_{ten} (h) 456.212_{ten}

1.3 Converting Numbers

Numbers can be converted from one base to another, and when you do this, you get the same numbers written in different bases.

1.3.1 Converting from any base to base ten

EXAMPLES
Convert the following to base ten

1. 222_{four}

$$\begin{aligned}222_{\text{four}} &= (2\times 4^2)+(2\times 4^1)+(2\times 4^0)\\ &= (2\times 4\times 4)+(2\times 4)+(2\times 1)\\ &= 32+8+2\\ &= 42_{\text{ten}}\end{aligned}$$

2. $\text{ee}0_{\text{twelve}}$

$$\begin{aligned}\text{ee}0_{\text{twelve}} &= (\text{e}\times 12^2)+(\text{e}\times 12^1)+(0\times 12^0)\\ &= (11\times 12\times 12)+(11\times 12)+(0\times 1)\\ &= 1584+132+0\\ &= 1716_{\text{ten}}\end{aligned}$$

3. 1075_{eight}

$$\begin{aligned}1075_{\text{eight}} &= (1\times 8^3)+(0\times 8^2)+(7\times 8^1)+(5\times 8^0)\\ &= (1\times 8\times 8\times 8)+(0\times 8\times 8)+(7\times 8)+(5\times 1)\\ &= 512+0+56+5\\ &= 573_{\text{ten}}\end{aligned}$$

4. 45.4_{eight}

$$\begin{aligned}45.4_{\text{eight}} &= (4\times 8^1)+(5\times 8^0)+(4\times 8^{-1})\\ &= (4\times 8)+(5\times 1)+(4\times \frac{1}{8})\\ &= 32+5+0.5\\ &= 37.5_{\text{ten}}\end{aligned}$$

1.2 Exercise Set

1. Convert the following numbers to base ten

   (a) 1432_{five} (c) 3412_{six} (e) 6542_{eight} (g) 321_{four}

   (b) 111_{two} (d) 431_{seven} (f) 1202_{three} (h) 4518_{nine}

2. Convert 68.3_{nine} to base ten

3. Convert the following binary numbers to base 10:

   (a) 110 (d) 1101 (g) 1111111

   (b) 1111 (e) 10001 (h) 11001101

   (c) 1001 (f) 11011 (i) 111000111

4. A particular binary number has 3 digits.

   (a) What are the largest and smallest possible binary numbers?

   (b) Convert these numbers to base 10.

1.3.2 Converting from base ten to other bases

- We use BNR
- Divide the number repeatedly by the required bases
- The remainder in reverse order gives the required number

1. Convert 19_{ten} to base two

B	N	R
2	19	1
2	9	1
2	4	0
2	2	0
	1	

$$19_{\text{ten}} = 10011_{\text{two}}$$

2. Convert 85_{ten} to base eight

B	N	R
8	85	5
8	10	2
	1	

$$85_{\text{ten}} = 125_{\text{eight}}$$

3. Convert 762_{eight} to base seven

$$\begin{aligned} 762_{\text{eight}} &= (7 \times 8^2) + (6 \times 8^1) + (2 \times 8^0) \\ &= (7 \times 8 \times 8) + (6 \times 8) + (2 \times 1) \\ &= 448 + 48 + 2 \\ &= 498_{\text{ten}} \end{aligned}$$

B	N	R
7	498	1
7	71	1
7	10	3
	1	

$$762_{\text{eight}} = 1311_{\text{seven}}$$

4. Convert 32_{five} to base two

$$\begin{aligned} 32_{\text{five}} &= (3 \times 5^1) + (2 \times 5^0) \\ &= (3 \times 5) + (2 \times 1) \\ &= 15 + 2 \\ &= 17_{\text{ten}} \end{aligned}$$

B	N	R
2	17	1
2	8	0
2	4	0
2	2	0
	1	

$$32_{\text{five}} = 10001_{\text{two}}$$

5. Convert 5432_{six} to base twelve

$$\begin{aligned} 5432_{\text{six}} &= (5 \times 6^3) + (4 \times 6^2) + (3 \times 6^1) + (2 \times 6^0) \\ &= (5 \times 6 \times 6 \times 6) + (4 \times 6 \times 6) + (3 \times 6) + (2 \times 1) \\ &= 1080 + 144 + 18 + 2 \\ &= 1244_{\text{ten}} \end{aligned}$$

B	N	R
12	1244	8
12	103	7
	8	

$$5432_{\text{six}} = 878_{\text{twelve}}$$

1.3 Exercise Set

1. Convert the following numbers to the bases indicated:
   - (a) 19 base two
   - (b) 568 to base nine
   - (c) 1256 to base eleven
   - (d) 64_{10} to base three
   - (e) 27_{ten} to base eight
   - (f) 246_{ten} to base five
   - (g) 20_{twelve} to binary

2. Convert the following numbers to the bases indicated:
   - (a) 34_{five} base two
   - (b) 568_{nine} to base eleven
   - (c) 111_{two} to base four
   - (d) 234_{five} to base nine
   - (e) 64_7 to base three
   - (f) 276_{eight} to base twelve
   - (g) 341_{five} to base six
   - (h) $tt5_{\text{eleven}}$ to base twelve
   - (i) 5432_{six} to base twelve
   - (j) 554_{six} to base four

1.4 Operation on Numbers in Various Bases

In this section we are going to look at the four mathematical operations which include addition,subtraction,division and multiplication

Activity:James had two jackfruit trees in his compound. At one time one tree had 8 fruits ready and the other 7 fruits. He harvested them at the same time.

- If james puts the jack fruits in heaps of ten fruits. How many heaps of ten did he get and how many remained?
- If james puts the jack fruits in heaps of nine fruits. How many heaps of nine did he get and how many remained?
- If james puts the jack fruits in heaps of five fruits. How many heaps of five did he get and how many remained?

When you put the fruits in heaps of 10,9 and 5, you are adding in base 10,base 9 and base 5.

1.4.1 Addition of bases

- If the sum of the digits exceeds the base, divide that sum by the base then write down the remainder and carry the whole number.

EXAMPLES

1. Workout $234_{\text{five}} + 23_{\text{five}}$ leaving your answer in the base indicated

$$\begin{array}{r} 2\,3\,4_{\text{five}} \\ +\quad 2\,3_{\text{five}} \\ \hline 3\,1\,2_{\text{five}} \\ \hline \end{array}$$

2. Add 153_{seven} to 453_{seven}

$$\begin{array}{r} 1\,5\,3_{\text{seven}} \\ +\,4\,5\,3_{\text{seven}} \\ \hline 6\,3\,6_{\text{seven}} \\ \hline \end{array}$$

3. Add $98t_{\text{twelve}}$ to $98e_{\text{twelve}}$

$$\begin{array}{r} 9\,8\,t_{\text{twelve}} \\ +\,9\,8\,e_{\text{twelve}} \\ \hline 1\,7\,5\;\;9_{\text{twelve}} \\ \hline \end{array}$$

1.4 Exercise Set

1. Workout the following leaving your answer in the base indicated

   (a) $232_{\text{six}} + 451_{\text{six}}$

   (b) $66_{\text{seven}} + 56_{\text{seven}}$

   (c) $11101_{\text{two}} + 11010_{\text{two}}$

   (d) $577_{\text{eight}} + 165_{\text{eight}}$

   (e) $999_{\text{ten}} + 245_{\text{ten}}$

   (f) $684_{\text{twelve}} + 436_{\text{twelve}}$

   (g) $36.64_{\text{nine}} + 4.31_{\text{nine}}$

   (h) $66.45_{\text{eleven}} + 4.65_{\text{eleven}}$

2. Workout $233_{\text{four}} + 544_{\text{six}}$ giving your answer in base five

3. Calculate the binary numbers:

   (a) $111 + 101 + 100$

   (b) $11101 + 10011 + 110111$

4. Workout the following leaving your answer in the base indicated

   (a) $et4_{\text{twelve}} + tt3_{\text{twelve}}$

   (b) $234_{\text{five}} + 413_{\text{five}}$

1.4.2 Subtraction of bases

- In case of borrowing the new value is the sum of the base and the digit which was small

EXAMPLES

1. Workout $72_{\text{eight}} - 43_{\text{eight}}$ leaving your answer in the base indicated

$$\begin{array}{r} 7\ 2_{\text{eight}} \\ -\ 4\ 3_{\text{eight}} \\ \hline 2\ 7_{\text{ eight}} \\ \hline \end{array}$$

2. Workout $\text{t}45_{\text{twelve}} - 376_{\text{twelve}}$ leaving your answer in the base indicated

$$\begin{array}{r} \text{t}\ 4\ \ 5_{\text{twelve}} \\ -\ 3\ 7\ \ 6_{\text{twelve}} \\ \hline 6\ 8\ \text{e}_{\text{twelve}} \\ \hline \end{array}$$

3. Subtract $342_{\text{ eight}}$ from $537_{\text{ eight}}$

$$\begin{array}{r} 5\ 3\ 7_{\text{eight}} \\ -\ 3\ 4\ 2_{\text{eight}} \\ \hline 1\ 7\ 5_{\text{eight}} \\ \hline \end{array}$$

4. Subtract $432_{\text{ six}}$ from $514_{\text{ six}}$

$$\begin{array}{r} 5\ 1\ 4_{\text{six}} \\ -\ 4\ 3\ 2_{\text{six}} \\ \hline 4\ 2_{\text{six}} \\ \hline \end{array}$$

1.5 Exercise Set

1. Workout the following leaving your answer in the base indicated

   (a) $1022_{\text{ three}} - 210_{\text{ three}}$

   (b) $31_{\text{ eight}} - 17_{\text{ eight}}$

   (c) $11111_{\text{ two}} - 1010_{\text{ two}}$

   (d) $577_{\text{ eight}} - 165_{\text{ eight}}$

   (e) $999_{\text{ ten}} - 245_{\text{ ten}}$

   (f) $684_{\text{ twelve}} - 436_{\text{ twelve}}$

   (g) $36.64_{\text{ nine}} - 4.31_{\text{ nine}}$

   (h) $66.45_{\text{ eleven}} - 4.65_{\text{ eleven}}$

2. Subtract the following numbers in the given bases:

   (a) $354_{\text{ six}}$ from $553_{\text{ six}}$

   (b) $845_{\text{ twelve}}$ from $\text{t}43_{\text{ twelve}}$

3. Workout $221_{\text{three}} - 111_{\text{two}}$ giving your answer in base five

4. Workout $567_{\text{eight}} - 146_{\text{seven}}$ giving your answer in base six

5. Find the value of n ,$45_n = 29$

1.4.3 Multiplication of bases

- Find the product of any two numbers as we do in base ten
- Divide this product by the base number

- Write the remainder and carry the quotient to the next place value position

EXAMPLES

1. Workout $136_{\text{seven}} \times 4_{\text{seven}}$ leaving your answer in the base indicated

$$\begin{array}{rr} & 1\,3\,6_{\text{seven}} \\ \times & 4_{\text{seven}} \\ \hline & 6\,1\,3_{\text{seven}} \\ \hline \end{array}$$

2. Workout $32_{\text{five}} \times 14_{\text{five}}$ leaving your answer in the base indicated

$$\begin{array}{rr} & 3\,2_{\text{five}} \\ \times & 1\,4_{\text{five}} \\ \hline & 2\,3\,3 \\ + & 3\,2 \\ \hline & 1\,1\,0\,3_{\text{five}} \\ \hline \end{array}$$

3. Work out $et5_{\text{twelve}} \times 8t_{\text{twelve}}$

$$\begin{array}{rr} & e\,t\,5_{\text{twelve}} \\ \times & 8\,t_{\text{twelve}} \\ \hline & 9\,t\,8\,2 \\ + & 7\,t\,e\,4 \\ \hline & 8\,8\,t\,0\,2_{\text{twelve}} \\ \hline \end{array}$$

4. Work out $141_{\text{five}} \times 23_{\text{five}}$

$$\begin{array}{rr} & 1\,2\,3_{\text{five}} \\ \times & 2\,3_{\text{five}} \\ \hline & 1\,0\,2\,3 \\ + & 3\,3\,2\,0 \\ \hline & 4\,3\,4\,3_{\text{five}} \\ \hline \end{array}$$

1.6 Exercise Set

1. Fill in the missing numbers in this multiplication table in base twelve

×	0	1	2	3	4	5	6	7	8	9	t	e
0	0	0										
1		1_{twelve}										
2							10_{twelve}					
3												
4					14_{twelve}						34_{twelve}	
5												47_{twelve}
6				16_{twelve}								
7	0_{twelve}									53_{twelve}		
8									54_{twelve}			
9								53_{twelve}				
t												
e			$1t_{\text{twelve}}$			47_{twelve}						$t1_{\text{twelve}}$

2. Workout the following leaving your answer in the base indicated

(a) $1022_{\text{three}} \times 21_{\text{three}}$

(b) $315_{\text{eight}} \times 17_{\text{eight}}$

(c) $1111_{\text{two}} \times 10_{\text{two}}$

(d) $577_{\text{eight}} \times 165_{\text{eight}}$

(e) $999_{\text{ten}} \times 245_{\text{ten}}$

(f) $e84_{\text{twelve}} \times et_{\text{twelve}}$

(g) $3664_{\text{nine}} \times 31_{\text{nine}}$

(h) $8745_{\text{eleven}} \times t65_{\text{eleven}}$

3. Multiply:

(a) 1121_{three} by 212_{three}

(b) 312_{four} by 122_{four}

(c) 41_{five} by 12_{five}

(d) 1001_{two} by 11_{two}

4. Multiply:

(a) 11_{two} by 12_{three}

(b) 8787_{nine} by 435_{seven}

(c) 231_{four} by 235_{six}

(d) 1123_{four} by 234_{five}

(e) $tet4_{\text{twelve}}$ by $t67_{\text{eleven}}$

(f) 3421_{five} by 133_{four}

1.4.4 Division of bases

- Convert each number base to base ten
- Divide the two numbers in base ten
- Convert the result back to the required base

EXAMPLES

1. Divide 1331_{four} by 121_{four}
   First Convert 1331_{four} and 121_{four} to base ten and then finally express the answer in base

four.

Converting 1331_{four} to base ten

$$\begin{aligned}1331_{\text{four}} &= (1\times 4^3)+(3\times 4^2)+(3\times 4^1)+(1\times 4^0)\\ &= (1\times 4\times 4\times 4)+(3\times 4\times 4)+(3\times 4)+(1\times 1)\\ &= 64+48+12+1\\ &= 125_{\text{ten}}\end{aligned}$$

Converting 121_{four} to base ten

$$\begin{aligned}121_{\text{four}} &= (1\times 4^2)+(2\times 4^1)+(1\times 4^0)\\ &= (1\times 4\times 4)+(2\times 4)+(1\times 1)\\ &= 16+8+1\\ &= 25_{\text{ten}}\end{aligned}$$

Dividing the numbers in base ten

$$\begin{aligned}&= 125 \div 25\\ &= 5\end{aligned}$$

Converting to base four

B	N	R
4	5	1
	1	

$$1331_{\text{four}} \div 121_{\text{four}} = 11_{\text{four}}$$

2. Divide $\text{t}46_{\text{eleven}}$ by 26_{eleven}

First convert $\text{t}46_{\text{eleven}}$ and 26_{eleven} to base ten and then finally express the answer in base eleven .

Converting $\text{t}46_{\text{eleven}}$ to base ten

$$\begin{aligned}\text{t}46_{\text{eleven}} &= (t\times 11^2)+(4\times 11^1)+(6\times 11^0)\\ &= (10\times 11\times 11)+(4\times 11)+(6\times 1)\\ &= 1210+44+6\\ &= 1260_{\text{ten}}\end{aligned}$$

Converting 26_{eleven} to base ten

$$\begin{aligned}26_{\text{eleven}} &= (2\times 11^1)+(6\times 11^0)\\ &= (2\times 11)+(6\times 1)\\ &= 22+6\\ &= 28_{\text{ten}}\end{aligned}$$

Dividing the numbers in base ten

$$\begin{aligned}&= 1260 \div 28\\ &= 45\end{aligned}$$

Converting the number in base ten to base eleven

B	N	R
11	45	1
	4	

$$t46_{\text{eleven}} \div 26_{\text{eleven}} = 41_{\text{eleven}}$$

1.7 Exercise Set

1. Workout the following leaving your answer in the base indicated

   (a) $10000_{\text{two}} \div 1000_{\text{two}}$

   (b) $313_{\text{four}} \div 11_{\text{four}}$

   (c) $1111_{\text{two}} \div 101_{\text{two}}$

   (d) $1870_{\text{nine}} \div 35_{\text{nine}}$

   (e) $176_{\text{eight}} \div 11_{\text{eight}}$

   (f) $500_{\text{six}} \div 23_{\text{six}}$

2. Workout $201_{\text{four}} \div 1011_{\text{two}}$ giving your answer in base three.

Activity of intergration

On April 4, 2020 the Covid19 task force started the distribution of food in Kawempe Division(kampala district). Each member in the household was given a package containing 6 kgs of maize flour,and 3 kg of beans. There are 10 households in the community with 3, 5, 7, 4, 6, 5,8,12, 13,4 members respectively.

TASK

1. Determine the number of packages the task force distributed in kawempe division.
2. Determine the total weight of the maize flour that was distributed in the division.
3. In case there are some remaining packages,discuss what the task force should do with them.
4. The prices of, beans and maize flour was approximated to be at 4000UGX and 2500UGX perkilogram respectively.What is the total amount of money spent by the government on maize flour and beans in the 10 households.

Chapter 2: WORKING WITH INTEGERS

Learning objectives
By the end of this topic,the learners should be able to

- Identify ,read and write natural numbers as numerals and words in million,billion and trillion
- Differentiate between natural numbers and whole numbers /integers
- Identify directed numbers
- Use directed numbers (limited to integers) in real life situations
- Use the hierarchy of operations to carry out the four mathematical operations on integers
- Identify Even, Odd, Prime and Composite Numbers
- Find the prime factors of any number
- Relate common factors with HCF and multiples with LCM
- Work out and use of divisibility tests of some numbers

2.1 Natural numbers

Introduction
Natural numbers can be classified into various groups of numbers. In your primary education, you learnt numbers such as even, odd, prime and composite.

1. Natural Numbers
   These are numbers used in counting.e.g N=$\{1, 2, 3, 4 \cdots\}$
2. Whole Numbers
   These are counting numbers including zero.e.g W=$\{0, 1, 2, 3, 4 \cdots\}$

Activity:Writing and reading numbers

There are two boxes. In one box, number cards are written in figures and the others in words. In groups, a member picks one card from one of the boxes. After all the cards have been picked, one member displays his/her card; then the others check their cards, and the matching card is displayed.
EXAMPLES

1. Write $999, 444, 230, 999$ in words.

Billion			Million			Thousands			Ones		
H	T	O	H	T	O	H	T	O	H	T	O
9	9	9	4	4	4	2	3	0	9	9	9

Nine hundred ninety nine billion,four hundred forty four million Two hundred thirty thousand nine hundred ninety nine.

2. Write 940, 340, 400, 230, 886 in words.

Trillion			Billion			Million			Thousands			Ones		
H	T	O	H	T	O	H	T	O	H	T	O	H	T	O
9	4	0	3	4	0	4	0	0	2	3	0	8	8	6

Nine hundred fourty trillion three hundred forty billion,four hundred million Two hundred thirty thousand eight hundred eighty six.

3. Write: Nine hundred ninety nine trillion seven hundred eighty eight billion,five hundred ninety nine million nine hundred ninety nine thousand eight hundred eighty six in figures.

	999	000	000	000	000
		788	000	000	000
			599	000	000
				999	000
+					886
	999	788	599	999	886

2.1 Exercise Set

1. Write the following in words:

(a) 6,800
(b) 9,888,008
(c) 722,820,060
(d) 76000
(e) 8,888,888
(f) 9,770,500
(g) 8,999,909,700
(h) 6,745,842,003
(i) 3,730,284,654,040

2. Write the following in figures

(a) Seven hundred three million seven thousand and six

(b) Four billion seventy-nine million five thousand six

(c) One trillion three hundred forty billion seven hundred seventy-five million two hundred sixty thousand

(d) Nine hundred ninety- nine trillion seven hundred eighty eight billion five hundred ninety nine million nine hundred ninety nine thousand eight hunded eighty six

(e) Seventy seven million two hundred sixty seven thousand nine hundred eighty

2.1.1 Differentiating between natural numbers and whole numbers/integers

Activity:Relating natural numbers and integers

Two learners Hannah and Ritah went to the school canteen to buy some snacks for their breakfast. Ritah bought 3 pancakes at UGX.200 each and 1 ban at UGX. 300. Hannah checked her bag and found out that her money was stolen. She borrowed some money from Ritah. She bought four 4 pancakes and 2 bans.
Questions

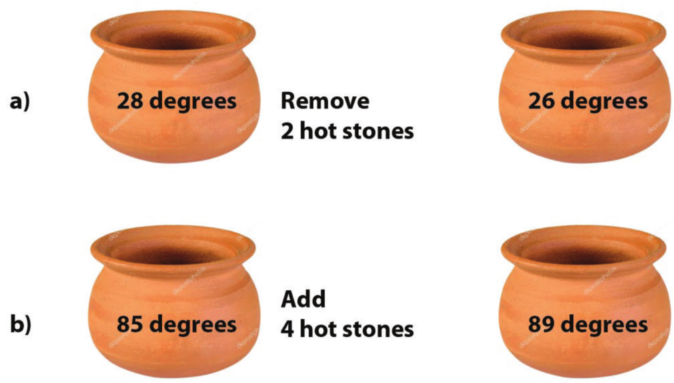
1. Which of the two learners had more money?
2. How much money did Hannah borrow from Ritah?
3. Ritah said that Hannah had negative UGX. 1400. Was she correct?
4. Give reasons for your answer.

2.2 Use Directed Numbers (Limited to Integers) in Real-life Situations

Numbers which have a direction and a size are called directed numbers. Once a direction is chosen as positive (+), the opposite direction is taken as negative (-)

Activity: Integers in real-life situations

Read the story below and answer the questions.
Once upon a time, there lived an old woman. She had hot and cold stones and a big pot of water. If she put one hot stone in the water, the temperature of the water would rise by 1 degree. If she took the hot stone out of the water again, the temperature would go down by 1 degree.i.e if the temperature of the water was 28 degrees and the old woman removes 2 hot stones ,the temperature would drop to 26 degrees ,and if the temperature of the water was at 85 degrees and the old woman adds 4 hot stones,the temperature would rise to 89 degrees.



Questions

1. If the temperature of the water is 24 degrees and the old woman adds 5 hot stones, what is the new temperature of the water?
2. Now imagine that the temperature of the water is at 29 degrees. The old woman takes a spoon and takes out 3 of the hot stones from the pot.What is the temperature of the water when the old woman removes 3 hot stones? Explain your answer.
3. The old woman also had cold stones. If she adds 1 cold stone to the water, the temperature goes down by 1 degree. The temperature of the water was 26 degrees. Then the old woman added 4 cold stones.What is the temperature of the water after the old woman added 4 cold stones
4. Give a reason for your answer.
5. Imagine that the temperature of the water was 22 degrees and the old woman removes 3 cold stones. What happens to the temperature of the water?
6. What is the new temperature of the water? Explain your answer.

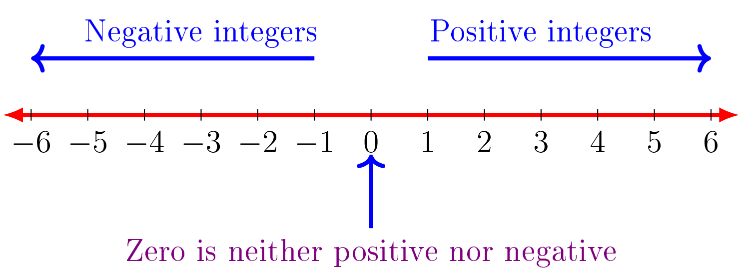
2.3 Use the Hierarchy of Operations to Carry out the Four Mathematical Operations on Integers

Activity: Operation on integers

1. Sarah moved 5 steps to the right from a fixed point. Then she moved 9 steps to the left.
   (a) How far is Sarah from the fixed point?
   (b) Peter gave his answer as 4 steps to the left of the fixed point and John as –4 (negative 4). Who is correct? Give reasons for your answer.
2. A group of learners of Geography went for a tour to Kabale. They found out that the temperature at one time was 13^0C. At around mid-night the temperature was 10^0C. By how many degrees had the temperature dropped?

2.4 Number line

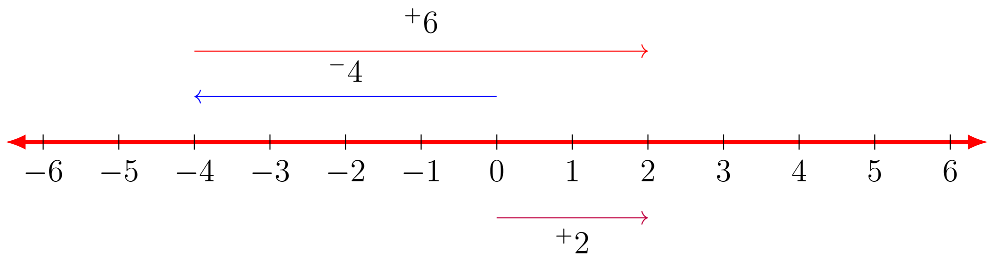
Integers can be illustrated on a number line (number scale) as shown below



Positive integers are to the right of a zero and negative integers are to the left of zero.Positive integers are shifts to the right while negative integers are shifts to the left

2.4.1 Addition of numbers on a number line

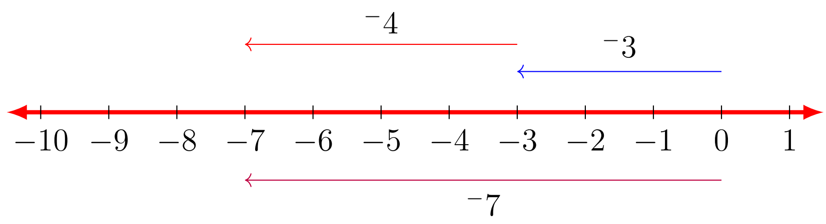
Work out $^{-}4 +^{+} 6$ using a numberline



$^{-}4 +^{+} 6 =^{+} 2$

2.4.2 Subtraction of numbers on a number line

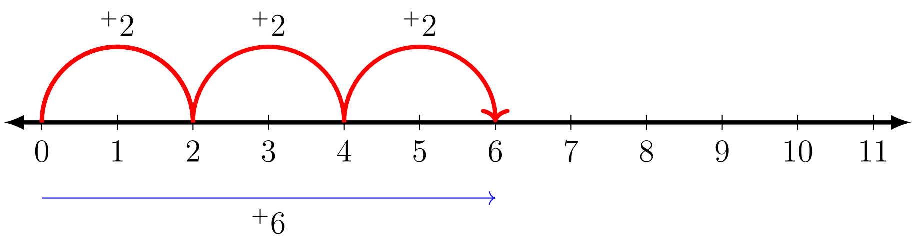
Work out $^{-}3 - 4$ using a number line



$^{-}3 +^{-} 4 =^{-} 7$

2.4.3 Multiplication of numbers on a number line

Multiplication is interpreted as repeated addition of positive or negative numbers.
Work out $^{+}2 \times^{+} 3$
SOLUTION
$^{+}2 \times^{+} 3 =^{+} 2 +^{+} 2 +^{+} 2$



$^{+}2 \times^{+} 3 = 6$

2.4.4 Division of numbers on a number line

Division is interpreted as repeated subtraction of positive or negative numbers

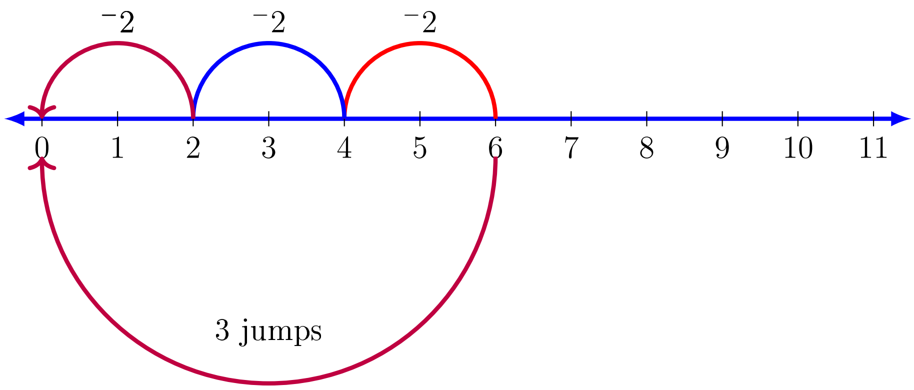
STEPS

- Draw an empty number line.
- Start from the right hand side of the number line.i.e From the Dividend
- Subtract by groups i.e subtract the divisor from the dividend up to when you reach zero.
- Count the jumps made from the dividend

(i) Workout $^{+}6 \div ^{+}2$ using a numberline

$$\begin{aligned} ^{+}6 &= 6 - 2 - 2 - 2 \\ &= 0 \\ ^{+}6 \div ^{+}2 &= 3 \end{aligned}$$

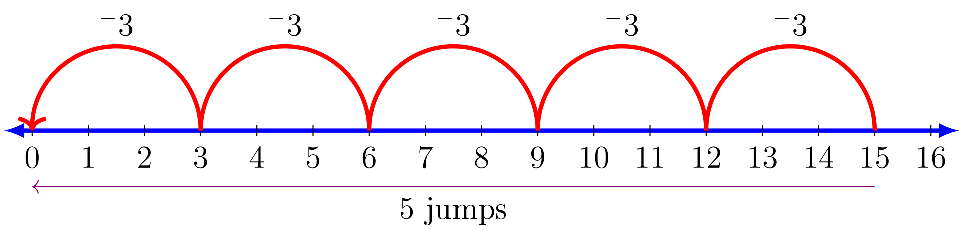
3 is the number of times you can subtract 2 from 6 before you get to zero i.e 3 represents the jumps made (skip 2 digits from 6)



(ii) Work out 15÷3

$$\begin{aligned} 15 &= 15 - 3 - 3 - 3 - 3 - 3 \\ &= 0 \\ 15 \div 3 &= 5 \end{aligned}$$

5 is the number of times you can subtract 3 from 15 before you get to zero i.e 5 represents the jumps made (skip 3 digits from 15)

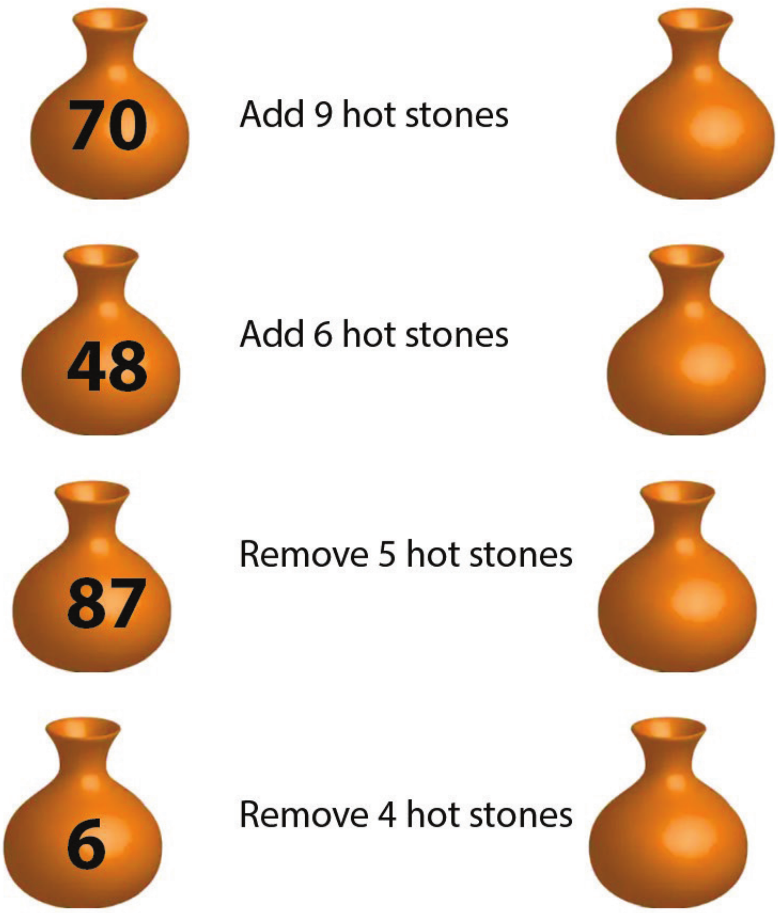


Summary for the rules of multiplication and division of integers

RULE	RESULT	EXAMPLE
A positive$\times$ A positive	A positive	$^{+}2 \times ^{+} 4 = ^{+} 8$
A positive$\times$ A negative	A negative	$^{+}2 \times ^{-} 3 = ^{-} 6$
A negative $\times$ A negative	A positive	$^{-}2 \times ^{-} 2 = ^{+} 4$
A positive$\div$A positive	A positive	$^{+}4 \div ^{+} 2 = ^{+} 2$
A positive $\div$ A negative	A negative	$^{+}4 \div ^{-} 2 = ^{-} 2$
A negative $\div$ A positive	A negative	$^{-}4 \div ^{+} 2 = ^{-} 2$
A negative $\div$A negative	A positive	$^{-}4 \div ^{-} 2 = ^{+} 2$

2.2 Exercise Set

1. Work out the following in degrees



2. Workout the following numbers using a number line

(a) $^{+}3 + ^{+} 5$

(b) $^{+}4 - 6$

(c) $^{-}7 - ^{-} 3$

(d) $^{+}4 - ^{+} 8$

(e) $^{+}5 - ^{-} 3$

(f) $4 \times ^{+} 2$

(g) $^{+}3 \times ^{-} 3$

(h) $^{+}4 \times ^{-} 2$

(i) $^{+}23 \times ^{+} 6$

(j) $^{+}3 - ^{-} 4$

(k) $^{+}9 \div 3$

(l) $24 \div 4$

3. Work out the following:

(a) $^{+}32 +^{-} 5$
(b) $^{+}84 - 6$
(c) $^{-}17 -^{-} 13$
(d) $^{+}104 -^{+} 5$
(e) $^{+}51 -^{-} 32$
(f) $42 \times^{+} 2$
(g) $^{+}13 \times^{-} 3$
(h) $^{+}74 \times^{-} 2$
(i) $^{+}123 \times^{+} 6$
(j) $^{+}73 -^{-} 4$
(k) $^{+}99 \div 3$
(l) $124 \div 4$

4. Work out
Hint:BODMAS MUST BE APPLIED

(a) $^{+}3 \times^{-} 4 \times^{-} 6$
(b) $^{+}4 \times^{-} 2 \times^{+} 5$
(c) $^{+}7 \times^{-} 8 \times^{+} 4$
(d) $^{-}20 \times^{-} 6 \div^{+} 2$
(e) $^{-}25 \div 5 \times^{-} 8$
(f) $^{-}34 \times^{+} 2 \div^{+} 2$
(g) 24 of $13^{\smile}(18 \div 6 + 3) \div (9 \times 3 - 25)$
(h) $89 - (99 - 84 \div 2 + 2)$
(i) $6 \div (2 + (2 \times 6 - 2))$
(j) 4 of $(4 + 3) - 2(1 + 9) \div 4$

5. In a certain mathematics test a correct answer scores 5 marks and an incorrect answer, the child gets a penalty of two marks deducted. Joy guessed all the answers. She got 12 correct and 8 wrong. Work out her total marks.

2.5 Identify Even, Odd, Prime and Composite Numbers

Introduction
Natural numbers can be classified into various groups of numbers. In your primary education, you learnt numbers such as even, odd, prime and composite.

Activity: Identifying even, odd, prime and composite numbers

1. Natural Numbers
These are numbers used in counting.e.g N=$\{1, 2, 3, 4 \cdots\}$
2. Whole Numbers
These are counting numbers including zero.e.g W=$\{0, 1, 2, 3, 4 \cdots\}$
3. Square Numbers
These are numbers got after multiplying a natural number by itself.e.g S=$\{1, 4, 9, 16 \cdots\}$
4. Cube Numbers
These are numbers got after multiplying a natural number three times.e.g C=$\{1, 8, 27, 64 \cdots\}$
5. Even Numbers
This is a number that is exactly divisible by two .e.g E=$\{2, 4, 6, 8 \cdots\}$
6. Prime Numbers
This is a number with only two factors one and itself .e.g E=$\{2, 3, 5, 7 \cdots\}$

7. Composite Numbers
   This is a number with more than two factors .e.g $\{4, 6, 8 \cdots\}$

EXAMPLES

1. Identifying prime and composite numbers

Prime and Composite Numbers

Prime | Composite | Neither prime nor composite

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

2. Are there numbers that belong to more than one group?
3. Wha is the 6^{th} prime number?
4. Wha is the 19^{th} composite number?

2.3 Exercise Set

1. (a) The table below shows the natural numbers from 1 to 100.Color the numbers

Even numbers   Odd numbers

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

(b) What is the 7^{th} odd number?

(c) What is the 20^{th} even number?

(d) Are there numbers that belong to more than one group?

2. Write down a number which is both an even number and a prime number

3. List the first 10 composite numbers

2.6 Finding the Prime Factors and multiples of any Number

- Factors
  Factors of numbers are all numbers that divide exactly into it,leaving no remainder..e.g 6 is divisible by $\{1, 2, 3, 6\}$ Therefore the factors of 6 are F_6=$\{1, 2, 3, 6\}$
- Prime Factor
  This is a factor which is a prime number.
- Prime Factorisation
  Expresses a number as a product of only its prime factors .
- Multiple of a Number
  Is that number multiplied by another integer .i.e When two numbers are multiplied together, the product is called multipl.e.g Multiples of 5 include $\{5, 10, 15, 20 \cdots\}$.

EXAMPLES

1. List all the factors of the following numbers

   (a) 12
   $F_{12} = \{1, 2, 3, 4, 6, 12\}$

   (b) 32
   $F_{32} = \{1, 2, 4, 8, 16, 32\}$

   (c) 60
   $F_{60} = \{1, 2, 3, 4, 5, 10, 12, 15, 30, 60\}$

2. List the multiples of the following numbers

   (a) 2
   $M_2 = \{2, 4, 6, 8, 10, \cdots\}$

   (b) 3
   $M_3 = \{3, 6, 9, 12, \cdots\}$

   (c) 12
   $M_{12} = \{12, 24, 36, 48, 60, \cdots\}$

3. Express each of the following numbers as a product of its prime factors

   (a) 36

Prime Factor	Number
2	36
2	18
3	9
3	3
	1

$$36 = 2 \times 2 \times 3 \times 3$$
$$36 = 2^2 \times 3^2$$

(b) 108

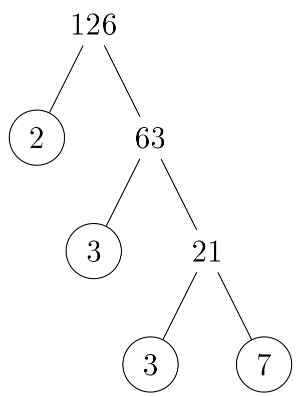
Prime Factor	Number
2	108
2	54
3	27
3	9
3	3
	1

$$108 = 2 \times 2 \times 3 \times 3 \times 3 \tag{1}$$
$$108 = 2^2 \times 3^3 \tag{2}$$

NB

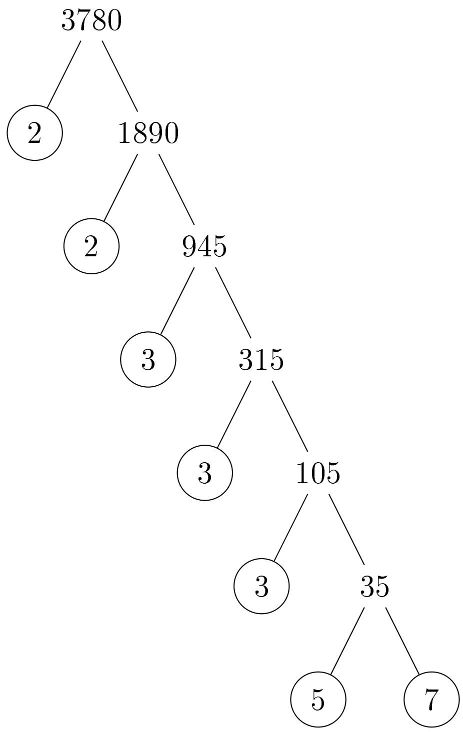
- Equation (1) expresses our answer as a product of prime factors
- Equation (2) expresses our answer in power notation
- The above method is known as prime factorisation

4. 126



$$126 = 2 \times 3 \times 3 \times 7$$
$$3780 = 2^1 \times 3^2 \times 7^1$$

5. 3780
   We can as well use the prime factor tree to find the factors of a number



$$3780 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 7$$
$$3780 = 2^2 \times 3^3 \times 5^1 \times 7^1$$

NOTE
To express our answer in power notation we apply the law of indices as below

LAW	Example
$a^m \times a^n = a^{(m+n)}$	$2^3 \times 2^4 = 2^7$
$a^m \div a^n = a^{(m-n)}$	$2^5 \div 2^2 = 2^3$
$(a^m)^n = a^{m \times n}$	$(2^3)^3 = 2^9$
$a^1 = a$	$2^1 = 2$
$a^0 = 1$	$2^0 = 1$
$(\frac{a}{b})^m = \frac{a^m}{b^m}$	$(\frac{2}{3})^2 = \frac{2^2}{3^2}$
$a^{-m} = \frac{1}{a^m}$	$2^{-4} = \frac{1}{2^4}$

2.7 Work Out and Use Divisibility Tests of Some Numbers

- If a number is divisible by 2, its last digit is even(2,4,6,8) or 0 .
- If a number is divisible by 3, the sum of its digits will be a multiple of 3.

- If a number is divisible by 4, the last two digits will be a multiple of 4.
- If a number is divisible by 5, it will end in 0 or 5.
- If a number is divisible by 6, its last digit is even and the sum of its digits is divisible by 3
- If a number is divisible by 8, its lastthree digits form a number divisible by 8 .
- If a number is divisible by 9, the sum of its digits will be a multiple of 9.
- If a number is divisible by 10, its last digit is 0

2.4 Exercise Set

1. List all the common divisors/ factors of the following:

   (a) 16 (b) 60 (c) 112 (d) 225 (e) 90 (f) 100 (g) 18 (h) 48

2. List down multiples of the following numbers that are less than 50

   (a) 5 (b) 20 (c) 9 (d) 10 (e) 13 (f) 7 (g) 24 (h) 11

3. Find the prime factors of the following numbers. Give your answer in power form(Power notation).

   (a) 28 (b) 54 (c) 204 (d) 156 (e) 225 (f) 1020 (g) 132 (h) 90 (i) 1232 (j) 993 (k) 2145 (l) 780

4. (a) List all the factors of each of the following numbers: 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

   (b) Which of these numbers are prime?

5. Explain why 99 is not a prime number.
6. Which of the following are prime numbers: 33, 35, 37, 39 ?
7. Find the prime factors of 72.
8. (a) Find the prime factors of 40.

   (b) Find the prime factors of 70.

   (c) Which prime factors do 40 and 70 have in common?

9. Find the prime factors that 48 and 54 have in common.
10. A number has prime factors 2, 5 and 7. Which is the smallest number that has these prime factors?

11. The first 5 prime numbers are 2, 3, 5, 7 and 11. Which is the smallest number that has these prime factors?
12. Write down the first two prime numbers which are greater than 100.
13. Which is the first prime number that is greater than 200?
14. Use a factor tree to find the prime factors of:

    (a) 102 (c) 30 (e) 200
    (b) 60 (d) 80 (f) 72

15. A number is expressed as the product of its prime factors as $5^2 \times 7^2$.What is the number?
16. A number is expressed as the product of its prime factors as $2^3 \times 3^5$.What is the number?
17. The prime factors of a number are 2, 7 and 11. Which are the three smallest numbers with these prime factors?
18. Given the following numbers: 12, 132, 1212, 3243, 1112, 81, 18, 27, 279, 2580, 5750. Find out which of them are divisible by:

    (a) 2 (d) 5 (g) 8
    (b) 3 (e) 6 (h) 9
    (c) 4 (f) 7 (i) 10

2.8 Relate Common Factors with HCF and Multiples with LCM

In this section we deal with finding LCM and HCF.We use the knowledge of multiples and factors.

2.8.1 Highest Common Factor(HCF)

- Highest Common Factor(H.C.F)
  Is the highest number that divides exacly in two or more numbers .H.C.F is also called Greatest Common Divisor(G.C.D) or The highest common factor (HCF) of two numbers is the largest number that is a factor of both.

STEPS
To find the HCF of two or more numbers:

- Express each of the numbers as a product of prime factors,
- Pick out the least power of each common factor. The product of these gives the HCF or GCF

EXAMPLES

1. Find the HCF of 12 and 15.

SOLUTION

$F_{12} = \{1, 2, 3, 4, 6, 12\}$
$F_{15} = \{1, 3, 5, 15\}$

The common factors are $\{1, 3\}$. The highest of these is 3. Therefore,the HCF of 12 and 15 is 3.

2. Find the HCF of 20 and 30

SOLUTION

$F_{20} = \{1, 2, 4, 5, 10, 20\}$
$F_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$

The HCF of 20 and 30 is 10.

3. Find the HCF of 210 and 360

SOLUTION

Prime Factor	Number
2	210
3	105
5	35
7	7
	1

Prime Factor	Number
2	360
2	180
2	90
3	45
3	15
5	5
	1

$$210 = 2^1 \times 3^1 \times 5^1 \times 7^1$$

$$360 = 2^3 \times 3^2 \times 5^1$$

The common factors are 2,3 and 5.So we pick out those with the lowest(smaller) power .i.e On 2^3 and 2^1,we choose the one with the smaller power ,which is 2^1

$$\begin{aligned} HCF &= 2^1 \times 3^1 \times 5^1 \\ &= 2 \times 3 \times 5 \\ HCF &= 30 \end{aligned}$$

2.5 Exercise Set

1. (a) Write the factors of 8 and 12
   (b) Identify the common factors of 8 and 12
   (c) What is the highest common factor

2. Find the HCF of the following:

    (a) 96, 57
    (b) 49 ,84
    (c) 72,144,288
    (d) 42 ,63 ,105
    (e) 28,42,98
    (f) 132, 156,204,228
    (g) 54, 48
    (h) 42 ,63 ,105
    (i) 90,126,270

3. Find the HCF of:

    (a) 6 and 9
    (b) 14 and 18
    (c) 30 and 24
    (d) 15 and 10
    (e) 90 and 120
    (f) 96 and 72
    (g) 56 and 60
    (h) 77 and 50
    (i) 300 and 550
    (j) 320 and 128
    (k) 46 and 62
    (l) 124 and 72

4. (a) Use a factor tree to find the prime factorisation of 42.

    (b) Use a factor tree to find the prime factorisation of 90.

    (c) Find the HCF of 42 and 90.

5. Stephen has two pieces of cloth. One piece is 36 inches wide and the other piece is 24 inches wide. He wants to cut both pieces into strips of equal width that are as wide as possible. How wide should he cut the strips?

6. Determine the smallest sum of money out of which a number of men, women and children may receive UGX. 750, Ush.900 and Ush.700 each.

2.8.2 Lowest Common Multiple(LCM)

Lowest common multiple(L.C.M)

The lowest common multiple (LCM) of two numbers is the smallest number that is a multiple of both.

EXAMPLES

1. What is the LCM of 5 and 7

    $M_5 = \{5, 10, 15, 20, 25, 30, 35, 40 \cdots\}$

    $M_7 = \{7, 14, 21, 28, 35, 42 \cdots\}$

    The LCM of 5 and 7 is 35.

2. Find the LCM of 16, 12 and 24.

	16	12	24
2	8	6	12
2	4	3	6
2	2	3	3
2	1	3	3
3	1	1	1

$$LCM = 2 \times 2 \times 2 \times 2 \times 2 \times 3$$
$$LCM = 48$$

3. Find the LCM of 210 and 360

SOLUTION

	210	360
2	105	180
2	105	90
2	105	45
3	35	15
5	35	5
7	7	1
	1	1

$$LCM = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$$
$$LCM = 2520$$

2.6 Exercise Set

1. (a) List the first 10 multiples of 8.
   (b) List the first 10 multiples of 6.
   (c) What is the LCM of 6 and 8 ?
2. What is the LCM of:
   (a) 5 and 3
   (b) 9 and 6
   (c) 8 and 10
   (d) 12 and 9
   (e) 15 and 20
   (f) 6 and 11 ?
3. (a) Use a factor tree to find the prime factorisation of 66.
   (b) Use a factor tree to find the prime factorisation of 40.
   (c) Find the LCM of 40 and 66.
4. Find the LCM of:

(a) 28 and 30
(b) 16 and 24
(c) 20 and 25
(d) 60 and 50
(e) 12 and 18
(f) 21 and 35
(g) 14, 21
(h) 18, 24, 96
(i) 60, 72, 84, 112

5. Two lighthouses can be seen from the top of a hill. The first flashes once every 8 seconds, and the other flashes once every 15 seconds. If they flash simultaneously, how long is it until they flash again at the same time?
6. At Namboole stadium race track, Victor completes a lap in 40 seconds; Ethan completes a lap in 30 seconds, and Joel completes a lap in 50 seconds. If all three start a lap at the same time, how long is it before
   (a) Victor overtakes joel,
   (b) Ethan overtakes victor?
7. Martin exercises every 12 days and Daniel every 8 days. Martin and Daniel both exercised today. How many days will it be until they exercise together again?
8. At Taibah international school, two bells are rung to change lessons at intervals of 60 minutes and 120 minutes respectively.After how many minutes will the bells be rung together again?
9. Daniel, Ethan and Michael start to jog around a circular stadium. They complete their rounds in 36 seconds, 48 seconds and 42 seconds respectively. After how many seconds will they be together at the starting point?

Activity of intergration

Stephen is planning a graduation party and wants to give his guests some snacks on arrival for the party.He buys 72 cup cakes,144 apples and 288 chocolate bars

- Support: Each plate must have exactly the same number of chocolate bars,apples,and cup cakes .There must not be any left overs.
- Knowledge: Knowledge of factors,highest common factor and numbers
- Tasks:
  1. What is the greatest number of guests stephen must invite for the graduation party
  2. Write down the number of guests in words

Chapter 3: FRACTIONS, PERCENTAGES AND DECIMALS.

Learning objectives

By the end of this topic,the learner should be able to

- Describe different types of fractions.
- Convert improper fractions to mixed numbers and vice versa.
- Work out problems from real-life situations.
- Add, subtract, divide and multiply decimals.
- Convert fractions to decimals and vice versa.
- Identify and classify decimals as terminating, non-terminating and recurring decimals.
- Convert recurring decimals into fractions.
- Convert fractions and decimals into percentages and vice versa.
- Finding the Percentage Increase and Decrease
- Work out real-life problems involving percentages.

Introduction

In this topic, you will use knowledge of place values to manipulate fractions, decimals and percentages. You will convert fractions to decimals, decimals to percentages and vice versa.

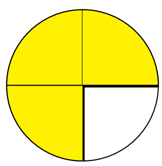
1. A fraction is a number in the form $\frac{a}{b}$ where a and b are whole numbers and b is not zero.
2. In a fraction the top number is called the numerator(a) and the bottom number is called the denominator(b)
3. A fraction is in simplest form (lowest terms) when the top and bottom cannot be any smaller

3.1 Types of fraction

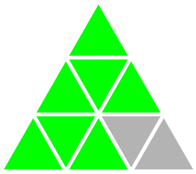
- *Proper fraction*

  In a proper fraction the numerator is less than the denominator. Thus$\frac{3}{4}$ and $\frac{7}{9}$ are both proper fractions.

  1. Shade $\frac{3}{4}$

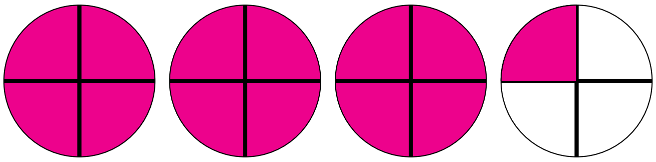


2. Shade $\frac{7}{9}$

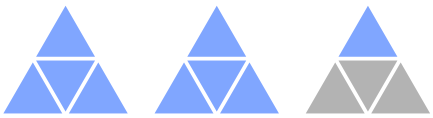


- *Improper fraction*
  In an improper fraction the numerator is greater than the denominator. Thus $\frac{13}{4}$,$\frac{5}{3}$ and $\frac{9}{5}$are improper fractions.

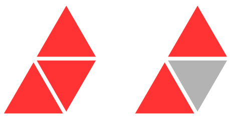
1. Shade $\frac{13}{4}$



2. Shade $\frac{9}{5}$



3. shade $\frac{5}{3}$



- *Equivalent fractions*
  Equivalent fractions have the same value. In an equivalent fraction both the numerator and denominator are multiplied or divided by the same number.Thus $\frac{5}{8}$ and $\frac{10}{16}$ are equivalent fractions

NOTE

In a mixed number a whole number is followed by a proper fraction. Thus $1\frac{3}{4}$ and $3\frac{5}{8}$ are both mixed numbers. A mixed number can be converted into an improper fraction and vice versa

3.1 Exercise Set

1. Roberta shades $\frac{3}{7}$ of a shape. What fraction of the shape is left unshaded?
2. A cake is divided into 12 equal parts. Hannah eats $\frac{3}{12}$ of the cake and Priscilla eats another $\frac{1}{12}$. What fraction of the cake is left?

3. A car park contains 20 spaces. There are 17 cars parked in the car park.

   (a) What fraction of the car park is full?

   (b) What fraction of the car park is empty?

4. Benjamin eats $\frac{3}{10}$ of the sweets in a packet. Shakur eats another $\frac{4}{10}$ of the sweets.

   (a) What fraction of the sweets has been eaten?

   (b) What fraction of the sweets is left?

5. Draw diagrams to show these improper fractions:

   (a) $\frac{7}{2}$

   (b) $\frac{8}{3}$

6. Draw a square with its four lines of symmetry.

   (a) Shade $\frac{3}{8}$ of the shape.

   (b) Shade another $\frac{2}{8}$ of the shape.

   (c) What is the total fraction now shaded?

   (d) How much is left unshaded?

7. Shade the following fractions

   (a) $\frac{3}{10}$ (c) $\frac{1}{2}$ (e) $\frac{14}{10}$ (g) $\frac{9}{8}$

   (b) $\frac{10}{3}$ (d) $\frac{4}{3}$ (f) $\frac{5}{6}$ (h) $\frac{8}{9}$

3.2 Converting Improper Fractions to Mixed Numbers and Vice Versa

Summary

$$\frac{(D \times W) + N}{D}$$
$$Where:$$
$$D = denominator$$
$$N = numerator$$
$$W = wholenumber$$

EXAMPLES

1. Convert $3\frac{2}{5}$ into an improper fraction
   SOLUTION

$$\begin{aligned}
&\frac{(D \times W) + N}{D} \\
&Where: \\
D &= 5 \\
N &= 2 \\
W &= 3 \\
\frac{(D \times W) + N}{D} &= \frac{(5 \times 3) + 2}{5} \\
&= \frac{15 + 2}{5} \\
&= \frac{17}{5}
\end{aligned}$$

2. Express $\frac{11}{4}$ as a mixed number.
   We are required to express our answer in the form $W\frac{R}{D}$

$$\begin{aligned}
\frac{11}{4} &= 2remainder3 \\
&= 2\frac{3}{4}
\end{aligned}$$

3. Reduce $\frac{5}{10}$ to its simplest form

$$\begin{aligned}
\frac{5}{10} &= \frac{5 \div 5}{10 \div 5} \\
&= \frac{1}{2}
\end{aligned}$$

4. Find the equivalent fractions for $\frac{1}{3}$
   We can find the equivalent fractions by multiplying the numerator and denominator by the same number

$$\begin{aligned}
\frac{1}{3} &= \frac{1 \times 2}{3 \times 2} = \frac{2}{6} \\
\frac{1}{3} &= \frac{1 \times 4}{3 \times 4} = \frac{4}{12} \\
\frac{1}{3} &= \frac{1 \times 5}{3 \times 5} = \frac{5}{15} \\
Therefore \frac{1}{3} &= \frac{2}{6} = \frac{4}{12} = \frac{5}{15}
\end{aligned}$$

3.2 Exercise Set

1. Convert these mixed numbers to improper fractions.

   (a) $1\frac{3}{5}$ (b) $7\frac{1}{3}$ (c) $3\frac{4}{5}$ (d) $6\frac{4}{9}$ (e) $10\frac{3}{7}$ (f) $9\frac{2}{3}$ (g) $5\frac{3}{5}$ (h) $7\frac{8}{12}$

2. Write these fractions in order of increasing size. $6\frac{1}{2}, \frac{18}{5}, 3\frac{1}{4}, 5\frac{1}{3}, \frac{17}{3}$

3. Arrange the fractions $\frac{5}{6}, \frac{4}{9}, \frac{7}{8}$ and $\frac{1}{2}$ in descending order of magnitude

4. A young child is 44 months old. Find the age of the baby in years as a mixed number in the simplest form.

5. In an office there are $3\frac{1}{2}$ reams of paper. There are 500 sheets of paper in each full ream. How many sheets of paper are there in the office?

6. Express the following improper fractions as a mixed number.

   (a) $\frac{7}{2}$ (b) $\frac{22}{3}$ (c) $\frac{54}{7}$ (d) $\frac{27}{13}$ (e) $\frac{10}{3}$ (f) $\frac{4}{3}$ (g) $\frac{14}{10}$ (h) $\frac{9}{8}$

7. Change these mixed numbers to vulgar fractions

   (a) $6\frac{3}{5}$

   (b) $3\frac{2}{17}$

8. Express the following improper fractions as a mixed number.

   (a) $\frac{38}{9}$ (b) $\frac{231}{15}$ (c) $\frac{54}{7}$ (d) $\frac{29}{13}$

3.3 Operations on Fractions

- For fractions with plus (+) and minus (−) signs only, find the LCM and workout
- For fractions with combined operations, the BODMAS rule must be observed.

Activity:Work out problems from real-life situations

EXAMPLES

1. Find $\frac{1}{10}$ of UGX. 10000

SOLUTION

$$= \frac{1}{10} \text{of } 10000$$
$$= \frac{1}{10} \times 10000$$
$$= \frac{1}{1\cancel{0}} \times 1000\cancel{0}$$
$$= \text{UGX}1000$$

2. Find $\frac{4}{8}$ of UGX. 16,000

SOLUTION

$$= \frac{4}{8} \text{of } 16,000$$
$$= \frac{4}{8} \times 16,000$$
$$= \frac{4}{\cancel{8}} \times \cancel{16000}^{2000}$$
$$= 4 \times 2000$$
$$= \text{UGX}8000$$

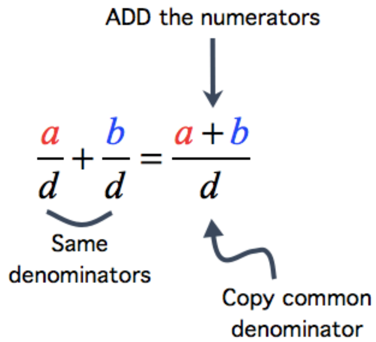
3.3 Exercise Set

1. Find:
   (a) $\frac{1}{2}$ of UGX. 16,000
   (b) $\frac{1}{3}$ of 15
   (c) $\frac{6}{7}$ of 49
   (d) $\frac{4}{8}$ of 800
   (e) $\frac{3}{4}$ of UGX. 2,500,000
2. In a test, there are 40 marks. Mimmi gets $\frac{3}{4}$ of the marks. How many marks does she get?
3. At Taibah international school school there are 850 pupils. If $\frac{3}{50}$ of the pupils are left-handed, how many left-handed pupils are there in the school?
4. There are 600 pupils in a school. How many school lunches must be prepared if:
   (a) $\frac{3}{4}$ of the pupils have school lunches
   (b) $\frac{2}{3}$ of the pupils have school lunches
5. A school has 800 pupils. The Headteacher decides to send a questionnaire

   to $\frac{2}{5}$ of the pupils. How many pupils receive a questionnaire?

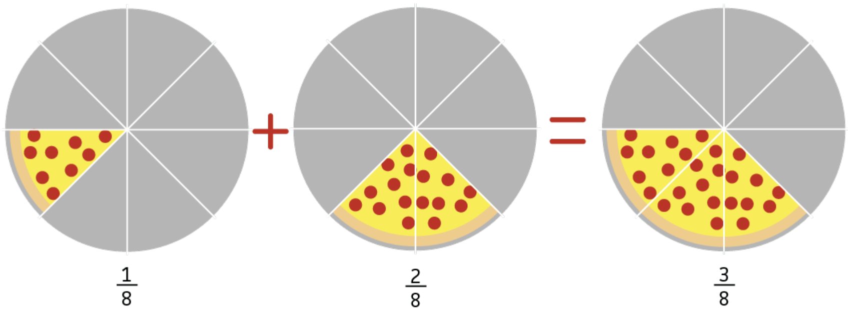
3.3.1 Addition of Fractions with the Same Denominators

To add fractions with like or the same denominator, simply add the numerators then copy the common denominator. Always reduce your final answer to its lowest term.



EXAMPLES

1. Work out $\frac{1}{8} + \frac{2}{8}$



2. Work out $\frac{2}{9} + \frac{1}{9}$

$$\begin{aligned}\frac{2}{9} + \frac{1}{9} &= \frac{2+1}{9} \\ &= \frac{3}{9} \qquad \text{Reduce the fraction to its lowest term} \\ &= \frac{\cancel{3}^{1}}{\cancel{9}^{3}} \\ &= \frac{1}{3}\end{aligned}$$

3.4 Exercise Set

1. Work out

(a) $\frac{4}{8} + \frac{2}{8}$

(b) $2\frac{1}{3} + 3\frac{2}{3}$

(c) $\frac{5}{12} + \frac{3}{12} + \frac{1}{12}$

(d) $\frac{5}{6} + \frac{2}{6} + \frac{7}{6}$

(e) $1\frac{1}{3} + 3\frac{2}{3} + 5\frac{1}{3}$

(f) $\frac{4}{28} + \frac{3}{28}$

2. If Praise sells $\frac{5}{8}$ of her clothes to Maria keziah,and $\frac{2}{8}$ of it to Alexadra.What fraction of her clothes has she sold.

3. Tendo ate $\frac{1}{3}$ of a fish for lunch and another $\frac{1}{3}$ of the fish for supper.What fraction of the fish did Tendo eat altogether

3.3.2 Addition of Fractions with different Denominators

Given two unlike fractions where the denominators are NOT the same,the fractions can be solved using two methods.

- LCM method
- Cross Multiplication method

The following steps are followed when using the LCM method
Steps for Adding Fractions with Unlike Denominators

- Identify the least common denominator by finding the least common multiple for the denominators.
- Write equivalent fractions (making sure that each equivalent fraction contains the least common denominator (LCM))
- Add the equivalent fractions that you wrote in step 2. (The denominators should now be the same.)
- Reduce the fraction to its lowest term

EXAMPLES

1. Add $\frac{3}{4} + \frac{1}{3}$
   STEP 1:Finding the LCM of 4 and 3.
   LCM=12

   Divide the denominator by the LCM and then multiply it with the numerator

$$\begin{aligned}\frac{3}{4} + \frac{1}{3} &= \frac{(12 \div 4) \times 3 + (12 \div 3) \times 1}{12} \\ &= \frac{3 \times 3 + 4 \times 1}{12} \\ &= \frac{9 + 4}{12} \\ &= \frac{13}{12}\end{aligned}$$

2. Add $\frac{3}{5} + \frac{2}{9}$
   Cross multiplying method

$$\frac{3}{5} \quad \frac{2}{9}$$

$$
\begin{aligned}
&= \frac{(3 \times 9) + (5 \times 2)}{5 \times 9} \\
&= \frac{27 + 10}{45} \\
&= \frac{37}{45}
\end{aligned}
$$

3. Add $\frac{1}{8} + \frac{1}{3}$

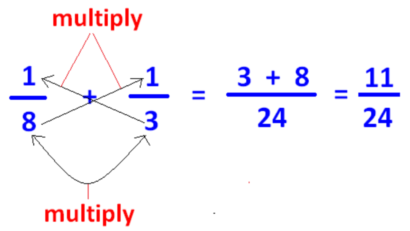
   LCM method

   STEP 1:Finding the LCM of 8 and 3.

   LCM=24

   Divide the denominator by the LCM and then multiply it with the numerator

$$
\begin{aligned}
\frac{1}{8} + \frac{1}{3} &= \frac{(24 \div 8) \times 1 + (24 \div 3) \times 1}{24} \\
&= \frac{3 \times 1 + 8 \times 1}{24} \\
&= \frac{3 + 8}{24} \\
&= \frac{11}{24}
\end{aligned}
$$



4. Add $\frac{1}{2} + \frac{1}{3}$

   STEP 1:Finding the LCM of 2 and 3.

   LCM=6

   Divide the denominator by the LCM and then multiply it with the numerator

$$
\begin{aligned}
\frac{1}{2} + \frac{1}{3} &= \frac{(6 \div 2) \times 1 + (6 \div 3) \times 1}{6} \\
&= \frac{3 \times 1 + 2 \times 1}{6} \\
&= \frac{3 + 2}{6} \\
&= \frac{5}{6}
\end{aligned}
$$

3.5 Exercise Set

1. Calculate

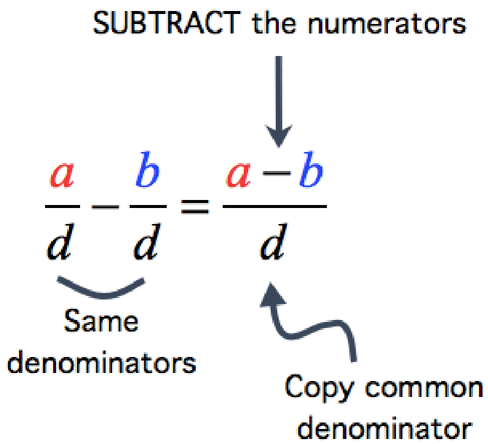
(a) $\frac{5}{7}+\frac{2}{3}$
(b) $\frac{3}{5}+\frac{2}{9}$
(c) $\frac{7}{8}+\frac{4}{6}$
(d) $\frac{1}{2}+\frac{2}{3}$
(e) $\frac{3}{7}+\frac{1}{5}$
(f) $\frac{4}{9}+\frac{2}{3}$
(g) $\frac{1}{4}+\frac{5}{8}$
(h) $\frac{3}{4}+\frac{4}{5}$
(i) $\frac{7}{8}+\frac{3}{10}$

2. If christine sells $\frac{5}{8}$ of her clothes to Maria keziah,and $\frac{1}{4}$ of it to Alexadra.What fraction of her clothes has she sold.

3. Tendo ate $\frac{1}{3}$ of a fish for lunch and another $\frac{1}{6}$ of the fish for supper.What fraction of the fish did Tendo eat altogether

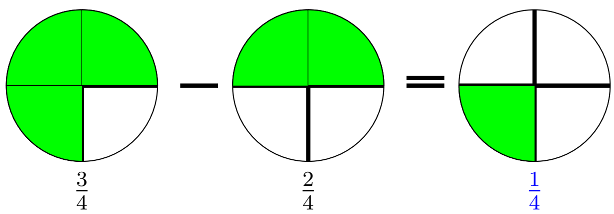
3.3.3 Subtraction of Fractions with Same Denominators

To subtract fractions with like or the same denominator, simply subtract the numerators then copy the common denominator. Always reduce your final answer to its lowest term.



EXAMPLES

1. Work out $\frac{3}{4}-\frac{2}{4}$



2. Work out $\frac{2}{9}-\frac{1}{9}$

$$\frac{2}{9}-\frac{1}{9}=\frac{2-1}{9}$$
$$=\frac{1}{9}$$

3.6 Exercise Set

1. Calculate

(a) $\frac{5}{7} - \frac{2}{7}$

(b) $4\frac{5}{7} - 2\frac{2}{7}$

(c) $\frac{4}{8} - \frac{2}{8}$

(d) $5\frac{1}{3} - 3\frac{2}{3}$

(e) $\frac{5}{12} - \frac{3}{12}$

(f) $\frac{5}{6} - \frac{2}{6}$

(g) $\frac{4}{28} - \frac{3}{28}$

2. Kristi had $\frac{9}{10}$ of a cake ,she ate $\frac{7}{10}$ of it.What fraction remained

3.3.4 Subtraction of Fractions with different Denominators

Given two unlike fractions where the denominators are NOT the same,we follow the same steps as in addition.

EXAMPLES

1. Work out $\frac{6}{11} - \frac{3}{22}$
STEP 1:Finding the LCM of 11 and 22.
LCM=22

Divide the denominator by the LCM and then multiply it with the numerator

$$\begin{aligned}\frac{6}{11} - \frac{3}{22} &= \frac{(22 \div 11) \times 6 - (22 \div 22) \times 3}{22} \\ &= \frac{2 \times 6 - 1 \times 3}{22} \\ &= \frac{12 - 3}{22} \\ &= \frac{9}{22}\end{aligned}$$

Cross multiplication method

$$\frac{6}{11} - \frac{3}{22}$$

$$\begin{aligned}\frac{6}{11} - \frac{3}{22} &= \frac{(22 \times 6) - (11 \times 3)}{22 \times 11} \\ &= \frac{132 - 33}{242} \\ &= \frac{99}{242} \\ &= \frac{\cancel{99}^{9}}{\cancel{242}^{22}} \\ &= \frac{9}{22}\end{aligned}$$

2. Workout $\frac{5}{6} - \frac{1}{3}$
STEP 1:Finding the LCM of 6 and 3.
LCM=6

Divide the denominator by the LCM and then multiply it with the numerator

$$\begin{aligned}\frac{5}{6}-\frac{1}{3}&=\frac{(6\div 6)\times 5-(6\div 3)\times 1}{6}\\&=\frac{1\times 5-2\times 1}{6}\\&=\frac{5-2}{6}\\&=\frac{3}{6}\\&=\frac{\cancelto{1}{3}}{\cancelto{2}{6}}\\&=\frac{1}{2}\end{aligned}$$

Cross multiplication method

$$\begin{aligned}\frac{5}{6}-\frac{1}{3}&=\frac{(3\times 5)-(6\times 1)}{3\times 6}\\&=\frac{15-6}{18}\\&=\frac{9}{18}\\&=\frac{\cancelto{1}{9}}{\cancelto{2}{18}}\\&=\frac{1}{2}\end{aligned}$$

3.7 Exercise Set

1. Work out

(a) $\frac{7}{11}-\frac{4}{22}$ (c) $\frac{3}{4}-\frac{1}{2}$ (e) $\frac{3}{4}-\frac{1}{3}$

(b) $4\frac{5}{6}-\frac{1}{3}$ (d) $\frac{5}{6}-\frac{3}{4}$ (f) $\frac{4}{5}-\frac{3}{6}$

3.3.5 Addition of Mixed Fractions

1. Work out $1+\frac{3}{5}$

$$1+\frac{3}{5}=1\frac{3}{5}$$

2. Work out $5+\frac{5}{9}$

$$5+\frac{5}{9}=5\frac{5}{9}$$

3. Work out $3 + \frac{7}{8}$

$$3 + \frac{7}{8} = 3\frac{7}{8}$$

4. Work out $1\frac{1}{2} + 2\frac{3}{5}$

Convert the mixed fractions into an improper fraction

$$= \frac{(2 \times 1) + 1}{2} + \frac{(2 \times 5) + 3}{5}$$
$$= \frac{3}{2} + \frac{13}{5}$$

Finding the LCM of 5 and 2

LCM=10

$$= \frac{(10 \div 2) \times 3 + (10 \div 5) \times 13}{10}$$
$$= \frac{5 \times 3 + 2 \times 13}{10}$$
$$= \frac{15 + 26}{10}$$
$$= \frac{41}{10}$$

3.8 Exercise Set

1. Calculate

(a) $2\frac{1}{2} - 1\frac{1}{2}$

(b) $4\frac{3}{4} - 3\frac{1}{4}$

(c) $2\frac{3}{8} - 1\frac{2}{8}$

(d) $1\frac{5}{7} + 4\frac{2}{7}$

(e) $6\frac{3}{4} + 4\frac{1}{3}$

2. Daniel bought $6\frac{1}{4}$kg of chicken on monday and $7\frac{3}{4}$kg on tuesday.How many kilograms did he buy altogether.

3.3.6 Subtraction of Mixed Fractions

1. Subtract$6\frac{5}{7} - 2\frac{1}{7}$

Changing the mixed fractions into an improper fraction

$$= \frac{47}{7} - \frac{15}{7}$$
$$= \frac{47 - 15}{7}$$
$$= \frac{32}{7}$$
$$= 4\frac{4}{7}$$

2. Subtract$4\frac{3}{5} - 2\frac{1}{5}$

Changing the mixed fractions into an improper fraction

$$\begin{aligned} &= \frac{23}{5} - \frac{11}{5} \\ &= \frac{23-11}{5} \\ &= \frac{12}{5} \\ &= 2\frac{2}{5} \end{aligned}$$

3.9 Exercise Set

1. Work out

(a) $2\frac{2}{3} - 1\frac{1}{9}$

(b) $5\frac{3}{7} - 2\frac{1}{2}$

(c) $4\frac{1}{4} - 2\frac{2}{3}$

(d) $4\frac{1}{3} - 3\frac{1}{2}$

(e) $2\frac{3}{8} - 2\frac{1}{4}$

(f) $5\frac{4}{5} - 2\frac{3}{6}$

(g) $3\frac{1}{5} - 1\frac{7}{8}$

(h) $6\frac{3}{4} - 4\frac{1}{3}$

3.3.7 Multiplication of Fractions

When multiplying fractions,the numerator and the denominator are multiplied separately.

1. Work out $\frac{3}{5} \times \frac{2}{7}$

$$\begin{aligned} \frac{3}{5} \times \frac{2}{7} &= \frac{3 \times 2}{5 \times 7} \\ &= \frac{6}{35} \end{aligned}$$

2. Work out $\frac{6}{9} of \frac{3}{7}$

$$\begin{aligned} \frac{6}{9} of \frac{3}{7} &= \frac{6}{9} \times \frac{3}{7} \\ &= \frac{6 \times 3}{9 \times 7} \\ &= \frac{18}{63} \\ &= \frac{\cancelto{2}{18}}{\cancelto{7}{63}} \\ &= \frac{2}{7} \end{aligned}$$

3.10 Exercise Set

1. Work out $\frac{2}{3} \times \frac{5}{7}$
2. Work out $\frac{3}{4} \times \frac{8}{9}$
3. Evaluate the following fractions.Answers should be as simplest as possible.
   (a) $\frac{2}{3} \times \frac{5}{7} \times \frac{21}{32}$
   (b) $\frac{2}{3}$ of $\frac{4}{5}$
   (c) $\frac{1}{4}$ of $\frac{4}{5}$

3.3.8 Multiplying Mixed Fractions

Convert the mixed fraction into an improper fraction ,and then apply the multiplication rules

1. Work out$4\frac{3}{5} \times 2\frac{1}{5}$

   Changing the mixed fractions into an improper fraction
$$= \frac{23}{5} \times \frac{11}{5}$$
$$= \frac{23 \times 11}{5 \times 5}$$
$$= \frac{253}{25}$$
$$= 10\frac{3}{25}$$

2. Work out$2\frac{3}{5} \times 2\frac{3}{7}$

   Changing the mixed fractions into an improper fraction
$$= \frac{13}{5} \times \frac{17}{7}$$
$$= \frac{13 \times 17}{5 \times 7}$$
$$= \frac{221}{35}$$
$$= 6\frac{11}{35}$$

3.11 Exercise Set

1. Work out $1\frac{7}{8} \times 3\frac{2}{7}$
2. Work out $8\frac{7}{5} \times 4\frac{2}{3}$
3. Calculate
   (a) $2\frac{5}{14} \times \frac{7}{66} \times 4$

(b) $7\frac{1}{8} \times \frac{5}{16}$

(c) $5\frac{1}{4}$ of $10\frac{4}{5}$

3.3.9 Division of Fractions

Flip And Multiply

1. Work out $\frac{6}{9} \div \frac{7}{3}$

   Flip the second fraction and then multiply

$$\begin{aligned}\frac{6}{9} \div \frac{7}{3} &= \frac{6}{9} \times \frac{3}{7} \\ &= \frac{6 \times 3}{9 \times 7} \\ &= \frac{18}{63} \\ &= \frac{\cancelto{2}{18}}{\cancelto{7}{63}} \\ &= \frac{2}{7}\end{aligned}$$

2. Work out $\frac{7}{8} \div \frac{4}{3}$

   Flip the second fraction and then multiply

$$\begin{aligned}\frac{7}{8} \div \frac{4}{3} &= \frac{7}{8} \times \frac{3}{4} \\ &= \frac{7 \times 3}{8 \times 4} \\ &= \frac{21}{32}\end{aligned}$$

3.12 Exercise Set

1. Work out

   (a) $\frac{3}{8} \div 2\frac{1}{4}$

   (b) $\frac{4}{7} \div 2\frac{1}{3}$

   (c) $21 \div \frac{7}{3}$

   (d) $3\frac{2}{3} \div \frac{2}{5}$

   (e) $1 \div 6\frac{12}{30}$

   (f) $\frac{2}{5} \div \frac{1}{2}$

   (g) $21 \div \frac{7}{9}$

   (h) $1\frac{2}{7} \div 6$

3.4 Add, Subtract, Divide and Multiply Decimals

3.4.1 Fractions and decimals

A decimal number is a number with a decimal point. Thus 1.56 is a decimal number

Activity: Convert Fractions to Decimals
To convert a fraction to decimal, divide the numerator by the denominator.
EXAMPLES

1. Using a calculator, convert the following fractions into decimal numbers:

   (a) $\frac{1}{2}$

   $$\begin{aligned}\frac{1}{2} &= 1 \div 2 \\ &= 0.5\end{aligned}$$

   (b) $\frac{1}{4}$

   $$\begin{aligned}\frac{1}{4} &= 1 \div 4 \\ &= 0.25\end{aligned}$$

   (c) $\frac{3}{4}$

   $$\begin{aligned}\frac{3}{4} &= 3 \div 4 \\ &= 0.75\end{aligned}$$

3.13 Exercise Set

1. Write these fractions as decimals:

   (a) $\frac{8}{10}$ (b) $\frac{71}{100}$ (c) $\frac{3}{1000}$ (d) $\frac{408}{10000}$

2. Write these fractions as decimals:

   (a) $\frac{3}{500}$ (b) $\frac{7}{20}$ (c) $\frac{9}{50}$ (d) $\frac{61}{200}$

3. Write these improper fractions as decimals:

   (a) $\frac{12}{10}$ (b) $\frac{212}{100}$ (c) $\frac{2008}{100}$ (d) $\frac{418}{10}$

4. Write these improper fractions as decimals:

   (a) $\frac{7}{2}$ (b) $\frac{21}{20}$ (c) $\frac{16}{5}$ (d) $\frac{32}{25}$

5. Using a calculator, convert the following fractions into decimal numbers:

   (a) $\frac{5}{8}$
   (b) $\frac{7}{4}$
   (c) $\frac{47}{20}$
   (d) $\frac{3}{8}$
   (e) $\frac{17}{50}$

Activity: Convert Decimals to Fractions

A decimal number can be exact or inexact An exact decimal or terminating decimal is a decimal that ends. This decimal is converted into a fraction as follows:

- Divide the decimal by one to get $\frac{\text{Decimal}}{1}$
- Multiply both top and bottom by 10 for every number after the decimal point.e.g 0.2 ,it has one number after the decimal point so we shall multiply by 10 .for 1.25,it has two numbers after the decimal point so we multiply by 100

EXAMPLES

1. Convert the following decimals to fractions

   (a) 0.5

$$= \frac{\text{Decimal}}{1}$$
$$0.5 = \frac{0.5}{1}$$

   we have one number after the decimal point so we multiply the denominator and numerator by 10

$$= \frac{0.5 \times 10}{1 \times 10}$$
$$= \frac{5}{10}$$
$$= \frac{\cancel{5}^{1}}{\cancel{10}^{2}}$$
$$= \frac{1}{2}$$

   (b) 1.05

$$= \frac{\text{Decimal}}{1}$$
$$1.05 = \frac{1.05}{1}$$

we have two numbers after the decimal point so we multiply the denominator and numerator by 100

$$= \frac{1.05 \times 100}{1 \times 100}$$
$$= \frac{105}{100}$$
$$= \frac{\cancel{105}^{21}}{\cancel{100}^{20}}$$
$$= \frac{21}{20}$$

(c) 0.625

$$= \frac{\text{Decimal}}{1}$$
$$0.625 = \frac{0.625}{1}$$

we have three numbers after the decimal point so we multiply the denominator and numerator by 1000

$$= \frac{0.625 \times 1000}{1 \times 1000}$$
$$= \frac{625}{1000}$$
$$= \frac{\cancel{625}^{5}}{\cancel{1000}^{8}}$$
$$= \frac{5}{8}$$

3.14 Exercise Set

1. Write each of these decimals as a fraction, giving them in their simplest form:

   (a) 0.1 (c) 0.3 (e) 0.017 (g) 0.87
   (b) 0.25 (d) 0.07 (f) 0.71 (h) 0.201

2. Write each of these decimals as a fraction and simplify where possible:

   (a) 0.4 (c) 0.012 (e) 0.328 (g) 0.014
   (b) 0.08 (d) 0.108 (f) 0.108 (h) 0.162

3. Write these numbers as improper fractions in their simplest form:

   (a) 1.2 (b) 3.02 (c) 4.008 (d) 3.62 (e) 5.015

4. Convert these decimals to fractions in their simplest form:

   (a) 0.00102 (b) 0.00000006 (c) 0.00002246 (d) 0.000006

5. Convert the following decimal numbers into fractions in their lowest terms:

   (a) 0.125
   (b) 0.08
   (c) 0.75
   (d) 0.375
   (e) 0.625
   (f) 1.75
   (g) 2.35
   (h) 0.3
   (i) 0.37
   (j) 0.0225

3.4.2 Addition and subtraction of decimals

To add or subtract two decimal numbers, line up the decimal points and then workout.

1. Without using a calculator, evaluate:

   (a) $3.21 + 4.5$

$$\begin{array}{r} 3.21 \\ +4.50 \\ \hline 7.71 \\ \hline \end{array}$$

   (b) $0.32 + 12.965 + 1.1$

$$\begin{array}{r} 0.32 \\ +\,12.965 \\ 1.1 \\ \hline 14.385 \\ \hline \end{array}$$

2. Without using a calculator, evaluate:

   (a) $8.97 - 2.82$

$$\begin{array}{r} 8.97 \\ -2.82 \\ \hline 6.15 \\ \hline \end{array}$$

   (b) $76.3 - 34.1$

$$\begin{array}{r} 76.3 \\ -34.1 \\ \hline 42.2 \\ \hline \end{array}$$

3.15 Exercise Set

1. Calculate, giving your answers as decimals and as fractions:

   (a) 0.7+0.6
   (b) 0.8-0.3
   (c) 1.7+0.21
   (d) 0.06+0.3
   (e) 8.06-0.2
   (f) 0.71+0.62

2. Without using a calculator, evaluate: $13.79 - 12.547$

3. Without using a calculator, evaluate: $136 - 14.54$

4. Without using a calculator, evaluate: $308.6 + 20.475 + 1.36$

3.4.3 Multiplication and Division of decimals

To multiply or divide two decimal numbers, express the decimal numbers in fractions and then workout.

EXAMPLES

1. Without using a calculator, evaluate: 0.5×0.08

   Convert the decimals into fractions

$$\begin{aligned} 0.5 \times 0.08 &= \frac{0.5 \times 10}{1 \times 10} \times \frac{0.08 \times 100}{1 \times 100} \\ &= \frac{5}{10} \times \frac{8}{100} \end{aligned}$$

   Multiply the numerators and denominators separately

$$\begin{aligned} &= \frac{5 \times 8}{10 \times 100} \\ &= \frac{40}{1000} \end{aligned}$$

   Reduce the fraction in its lowest term

$$\begin{aligned} &= \frac{\cancel{40}^{1}}{\cancel{1000}^{25}} \\ &= \frac{1}{25} \end{aligned}$$

2. Without using a calculator, evaluate:0.25×0.004

   Convert the decimals into fractions

$$\begin{aligned} 0.25 \times 0.004 &= \frac{0.25 \times 100}{1 \times 100} \times \frac{0.004 \times 1000}{1 \times 1000} \\ &= \frac{25}{100} \times \frac{4}{1000} \end{aligned}$$

   Multiply the numerators and denominators separately

$$\begin{aligned} &= \frac{25 \times 4}{100 \times 1000} \\ &= \frac{100}{100000} \end{aligned}$$

Reduce the fraction in its lowest term

$$= \frac{\cancel{100}^{1}}{\cancel{100000}^{1000}}$$
$$= \frac{1}{1000}$$

3. Without using a calculator, evaluate: $\frac{0.032}{0.16}$

Convert the decimals into fractions

$$\frac{0.032}{0.16} = \frac{32}{1000} \div \frac{16}{100}$$

Flip the second fraction

$$= \frac{32}{1000} \times \frac{100}{16}$$
$$= \frac{32 \times 100}{1000 \times 16}$$
$$= \frac{3200}{16000}$$
$$= \frac{1}{5}$$

3.16 Exercise Set

1. Without using a calculator, evaluate giving your answer as a fraction

   (a) 0.35×0.05 (b) 0.45×0.10 (c) 0.00044×10.00 (d) 0.5×0.45

2. Without using a calculator, evaluate:

   (a) $\frac{0.45}{0.9}$ (b) $\frac{0.0035}{0.015}$ (c) $\frac{0.100}{0.12}$ (d) $\frac{0.5400}{0.03}$

3.5 Identify and Classify Decimals as Terminating, Non-terminating and Recurring Decimals

- *A terminating decimal(An exact decimal)* is a decimal number that contains a finite number of digits after the decimal point.Fractions like $\frac{3}{5}, \frac{1}{2}, \frac{3}{8}$ can be converted into decimals and they end or terminate: $\frac{3}{5} = 0.6, \frac{1}{2} = 0.5, \frac{3}{8} = 0.375$
- *Non-terminating decimal*: is a decimal number that never repeats. Example : $0.076923\cdots$, $0.05882352\cdots$,$1.4223213345\cdots$

- *Recurring Decimal(Repeating decimals)*:is a decimal number that contains an infinite number of digits.Fractions like$\frac{2}{3}$,$\frac{2}{15}$,$\frac{1}{11}$ do not end or terminate when converted into decimals: $\frac{2}{3} = 0.66666\cdots$, $\frac{2}{15} = 0.13333\cdots$, $\frac{1}{11} = 0.09090\cdots$

3.17 Exercise Set

1. Using a calculator Write the following fractions as recurring decimals:

(a) $\frac{36}{99}$ (c) $\frac{1}{6}$ (e) $\frac{5}{9}$

(b) $\frac{2}{11}$ (d) $\frac{45}{99}$ (f) $\frac{256}{999}$

3.5.1 Converting Recurring Decimals into Fractions

- A recurring decimal is a decimal with endless repeating digits after the decimal point.
- A recurring decimal $0.363636\cdots$ is the same as $0.\overline{36}$ or $0.\dot{3}\dot{6}$
- A recurring decimal is converted into a fraction as follows:

  1 Let x = recurring decimal.

  2 Let n= the number of recurring digits

  3 Multiply the recurring decimal by 10^n E.g when only one number is repeating i.e n=1,$10^1 = 10$ so we multiply through out by 10,when n=2,$10^2 = 100$,so we multiply through out by 100

  4 Eliminate the recurring part by subtracting (3)-(1)

  5 Solve for x,expressing your answer as a fraction in its lowest form

EXAMPLES

1. Convert $0.5555\cdots$ into a fraction.

SOLUTION

Let the fraction be x

$$x = 0.5555\cdots \tag{3.1}$$

n=1,since we have only one repeating digit i.e 5 so $10^n, 10^1 = 10$

Multiply through Equation (3.1) by 10

$$x \times 10 = 0.5555\cdots \times 10 \tag{3.2}$$

$$10x = 5.555\cdots \tag{3.3}$$

Subtracting Equation (3.3)-Equation(3.1)

$$\tag{3.4}$$

$$-\begin{cases} 10x = 5.555\cdots \\ \quad x = 0.555\cdots \end{cases}$$

$$9x = 5$$

Divide through out by 9

$$\frac{9x}{9} = \frac{5}{9}$$
$$\frac{\cancel{9}x}{\cancel{9}} = \frac{5}{9}$$
$$x = \frac{5}{9}$$

2. Express $0.363636\cdots$ as a fraction in its simplest form

SOLUTION

Let the fraction be x

$$x = 0.363636\cdots - - - - - - - - - - - - - - -(1)$$

n=2,since we have only two repeating digit i.e 3 and 6 so $10^n, 10^2 = 100$

Multiply through Equation (1) by 100

$$x \times 100 = 0.363636\cdots \times 100$$
$$100x = 36.363636\cdots - - - - - - - - - - - - - - (3)$$

Subtracting Equation (3)-Equation(1)

$$-\begin{cases} 100x = 36.363636\cdots \\ \quad x = 0.363636\cdots \end{cases}$$

$$99x = 36$$

Divide through out by 99

$$\frac{99x}{99} = \frac{36}{99}$$
$$\frac{\cancel{99}x}{\cancel{99}} = \frac{\cancelto{4}{36}}{\cancelto{11}{99}}$$
$$x = \frac{4}{11}$$

3. Express $0.\overline{891}$ as a fraction in its simplest form

SOLUTION

Let the fraction be x

$$x = 0.\overline{891} -(1)$$

n=3,since we have only three repeating digits i.e 8,9 and 1 so $10^n, 10^3 = 1000$

Multiply through Equation (1) by 1000

$$x \times 1000 = 0.891891\cdots \times 1000$$

$$1000x = 891.891891\cdots ------------------------- (3)$$

Subtracting Equation (3)-Equation(1)

$$-\begin{cases} 1000x = 891.891891\cdots \\ x = 0.891891\cdots \end{cases}$$

$$999x = 891$$

Divide through out by 999

$$\frac{999x}{999} = \frac{891}{999}$$

$$\frac{\cancel{999}x}{\cancel{999}} = \frac{\cancel{891}^{33}}{\cancel{999}^{37}}$$

$$x = \frac{33}{37}$$

4. Express $1.\dot{2}\dot{7}$ as a fraction in its simplest form

SOLUTION

Let the fraction be x

$$x = 1.\dot{2}\dot{7} --------------------------- (1)$$

n=2,since we have only two repeating digits i.e 2 and 7 so $10^n, 10^2 = 100$

Multiply through Equation (1) by 100

$$x \times 100 = 1.2727\cdots \times 100$$

$$100x = 127.2727\cdots ------------------------- (3)$$

Subtracting Equation (3)-Equation(1)

$$-\begin{cases} 100x = 127.2727\cdots \\ x = 1.2727\cdots \end{cases}$$

$$99x = 126$$

Divide through out by 99

$$\frac{99x}{99} = \frac{126}{99}$$

$$\frac{\cancel{99}x}{\cancel{99}} = \frac{\cancel{126}^{14}}{\cancel{99}^{11}}$$

$$x = \frac{14}{11}$$

5. Express $0.1\dot{6}$ as a fraction in its simplest form

SOLUTION

NOTE:For this question on the Right Hand Side(RHS) of the decimal point we have only one digit that is recurring,so for us to solve it we ought to remain with the recurring part on the RHS of the decimal point

Let the fraction be x

$$x = 0.1\dot{6} ------------------------------ (1)$$

Both sides of the equation are multiplied by 10 (Since we have only one number after the decimal point that is not recurring) so that the repeating part of the number is immediately next to the decimal.

$$x \times 10 = 0.1666\cdots \times 10$$

$$10x = 1.666\cdots ------------------------------ (2)$$

n=1,since we have only one repeating digit i.e 6 so $10^n, 10^1 = 10$

Multiply through Equation (2) by 10

$$10x \times 10 = 1.666\cdots \times 10$$

$$100x = 16.666\cdots ------------------------------ (3)$$

Subtracting Equation (3)-Equation(2)

$$-\begin{cases} 100x = 16.666\cdots \\ 10x = 1.666\cdots \end{cases}$$

$$90x = 15$$

Divide through out by 90

$$\frac{90x}{90} = \frac{15}{90}$$

$$\frac{\cancel{90}x}{\cancel{90}} = \frac{\cancel{15}^{1}}{\cancel{90}^{6}}$$

$$x = \frac{1}{6}$$

6. Express $2.014545\cdots$ as a fraction in its simplest form

SOLUTION

NOTE:For this question on the Right Hand Side(RHS) of the decimal point we have only one digit that is recurring,so for us to solve it we ought to remain with the recurring part on the RHS of the decimal point

Let the fraction be x

$$x = 2.014545 ------------------------------ (1)$$

Both sides of the equation are multiplied by 100 (Since we have two numbers after the decimal point that are not recurring) so that the repeating part of the number is immediately next to the decimal.

$$x \times 100 = 2.014545\cdots \times 100$$
$$100x = 201.4545\cdots - (2)$$

n=2,since we have only two repeating digits i.e 4 and 5 so $10^n, 10^2 = 100$

Multiply through Equation (2) by 100

$$100x \times 100 = 201.4545\cdots \times 100$$
$$10000x = 20145.4545\cdots - (3)$$

Subtracting Equation (3)-Equation(2)

$$-\begin{cases} 10000x = 20145.4545\cdots \\ 100x = 201.4545\cdots \end{cases}$$

$$9900x = 19944$$

Divide through out by 9900

$$\frac{9900x}{9900} = \frac{19944}{9900}$$
$$\frac{\cancel{9900}x}{\cancel{9900}} = \frac{\cancel{19944}^{554}}{\cancel{9900}^{275}}$$
$$x = \frac{554}{275}$$

3.18 Exercise Set

1. Convert the following recurring decimals into fractions

   (a) $0.777\cdots$ (c) $0.1333\cdots$ (e) $3.4373737\cdots$

   (b) $0.4444\cdots$ (d) $1.2565656\cdots$ (f) $0.0131313\cdots$

2. Convert the following numbers into recurring decimals

   (a) $\frac{7}{9}$ (b) $\frac{1}{3}$ (c) $\frac{2}{6}$ (d) $\frac{15}{99}$

3. Express the following recurring decimals as a fraction in their simplest form

   (a) $1.633\cdots$ (c) $2.13535\cdots$ (e) $2.\overline{43}$ (g) $0.\overline{63}$

   (b) $0.7444\cdots$ (d) $0.\dot{3}\dot{8}$ (f) $0.\overline{45}$ (h) $0.3\dot{7}$

4. Express $0.3181818\cdots$ as a fraction in its simplest form

3.6 Percentages

The word 'percentage' means 'per hundred'. In this section we concentrate in converting between decimals, fractions and percentages.

- Percentage is a fraction whose denominator is 100.
- The Symbol for percentage is written as %.

3.6.1 Convert Fractions and Decimals into Percentages and Vice Versa

- To change a percentage into a fraction or decimal divide by 100. Thus

Percentage	Fraction	Fraction in lowest term	Decimal
60%	$\frac{60}{100}$	$\frac{3}{5}$	0.6
75%	$\frac{75}{100}$	$\frac{3}{4}$	0.75
15%	$\frac{15}{100}$	$\frac{3}{20}$	0.15

- To change a fraction into a percentage multiply by 100. Thus

Fraction	Conversion	Percentage
$\frac{3}{5}$	$\frac{3}{5} \times 100$	60%
$\frac{3}{4}$	$\frac{3}{4} \times 100$	75%
$\frac{1}{2}$	$\frac{1}{2} \times 100$	50%

- To change a decimal into a percentage multiply by 100. Thus

Decimal	Conversion	Percentage
0.5	0.5×100	50%
0.84	0.84×100	84%
0.125	0.125×100	125%

3.19 Exercise Set

1. Express each percentage as a fraction in its simplest form

   (a) 16% (b) 30% (c) 24% (d) 15.5%

2. Express each percentage as a decimal

   (a) 67% (b) 25% (c) 84.5% (d) 50%

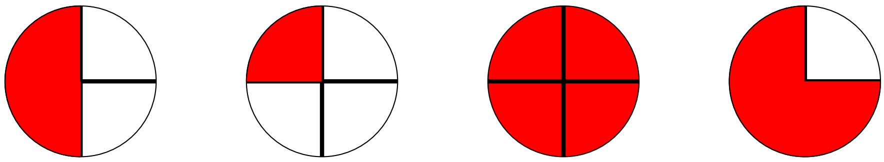
3. Express each fraction as a percentage

   (a) $\frac{9}{10}$ (b) $\frac{49}{50}$ (c) $\frac{11}{20}$ (d) $\frac{14}{25}$

4. Express each decimal as a percentage

   (a) $o.25$ (b) 0.125 (c) 0.486 (d) 0.34

5. Find 10% of 40,000
6. Find 25% of 120
7. Express 44 as a percentage of 80
8. If 76% of a rectangle is shaded, what percentage is not shaded?
9. For each diagram, state the percentage that is shaded:



(a) (b) (c) (d)

10. In a class, 90 students are boys and 25% are girls. Find the number of girls in the class
11. A football team is based on a squad of 20 players. In one season 8 players are shown a red or yellow card.
    (a) What percentage of the squad is shown a red or yellow card?
    (b) What percentage of the squad is not shown a red or yellow card?
12. In a class of 25 pupils there are 8 individuals who play in the school hockey team. What percentage of the class play in the hockey team?
13. Ben and Adam spend their Saturdays cleaning cars. They agree that Adam will have 60% of the money they earn and that Ben will have the rest.
    (a) What percentage of the money will Ben have?
    (b) How much do they each have if they earn UGX200,000?
    (c) How much do they each have if they earn UGX 350,000?
14. Copy and complete this table:
15. There are 200 children in a school hall, eating lunch. Of these children, 124 have chosen chicken as part of their lunch.
    (a) What fraction of the children have chosen chicken?
    (b) What percentage of the children have chosen chicken?
    (c) What percentage of the children have not chosen chicken?

Fraction	Decimal	Percentage
	0.05	
		25%
$\frac{1}{4}$		
		85%
$\frac{4}{5}$		
	0.95	

16. Priscilla scores 32 out of 80 in a test. Express his score as a percentage.

17. An athlete has completed 250 m of a 400 m race. What percentage of the distance has the athlete run?

18. In a car park, 40% of the cars are red and $\frac{7}{20}$ of the cars are blue.

    (a) What percentage are blue?

    (b) What fraction are red?

19. A double decker bus has 72 seats; there are 18 empty seats on the bus.

    (a) What percentage of the seats are empty?

    (b) What percentage of the seats are being used?

20. Andy buys a bag of 12 apples at a supermarket; there are 4 bruised apples in the bag.

    (a) What percentage of the apples are bruised?

    (b) What percentage of the apples are not bruised?

21. Copy and complete the table.The column headings will help you.You are required to fill in the fraction and percentage columns as done in the first three rows

Tens	Ones	Tenth($\frac{1}{10}$)	Hundredth ($\frac{1}{100}$)	Thousandth($\frac{1}{1000}$)	Fraction	Percentage
		5			$\frac{1}{2}$	50
1	2	4			$12\frac{2}{5}$	1240
		2	5		$\frac{1}{4}$	25
		1	5	2	—	—
		5			—	—
					—	80
					$\frac{17}{20}$	—
					—	64
		0	0	4	—	—
					$\frac{3}{10}$	–
4	0	3			—	—
	0	6	4		—	—

3.7 Finding the Percentage Increase and Decrease

The percentage of a quantity can always be calculated in terms of percentage increase or percentage decrease.Thus this is referred to as a percentage change

- Percentage change $= \frac{\text{Change in value}}{\text{Original value}} \times 100$
- Change in value =|New value-Old value|
- Percentage increase $= \frac{\text{increase in value}}{\text{Original value}} \times 100$
- An increase of 20% means the new value is 120% of the old value
- Percentage decrease $= \frac{\text{decrease in value}}{\text{Original value}} \times 100$
- A decrease of 20% means the new value is 80% of the old value

EXAMPLES

1. Stephen had 60 goats. Now he has 63 goats. What is the percentage increase?

$$\begin{aligned}
\text{Increase in value} &= \text{New value} - \text{Old value} \\
&= 63 - 60 \\
&= 3 \\
\text{Percentage increase} &= \frac{\text{increase in value}}{\text{Original value}} \times 100 \\
&= \frac{3}{60} \times 100 \\
&= 5\%
\end{aligned}$$

2. The price of bread increased from Shs 3800 to Shs 4000. Find the percentage increase in the price of the item

$$\begin{aligned}
\text{Increase in value} &= \text{New value} - \text{Old value} \\
&= 4000 - 3800 \\
&= 200 \\
\text{Percentage increase} &= \frac{\text{increase in value}}{\text{Original value}} \times 100 \\
&= \frac{200}{3800} \times 100 \\
&= 5.263\%
\end{aligned}$$

3. The price of an item reduced from Shs 8,000 to Shs 6,000. Find the percentage decrease in

the price of the item

$$\begin{aligned}
\text{Decrease in value} &= \text{Old value} - \text{New value} \\
&= 8000 - 6000 \\
&= 2000 \\
\text{Percentage decrease} &= \frac{\text{Decrease in value}}{\text{Original value}} \times 100 \\
&= \frac{2000}{8000} \times 100 \\
&= 25\%
\end{aligned}$$

4. An item costing Shs 3,000 is reduced by 20%. Find its new cost price
   A decrease(reduction) of 20% means the new value is 80% of the old value

$$\begin{aligned}
\text{New cost price} &= \frac{80}{100} \times \text{Old value} \\
&= \frac{80}{100} \times 3000 \\
&= \text{Shs}2400
\end{aligned}$$

3.20 Exercise Set

1. The price of a drink increases from 4000 to 4500. What is the percentage increase?
2. The number of pupils in a school increases from 820 to 861. Calculate the percentage increase.
3. Although the lion is thought of as an African animal, there is a small population in India and elsewhere in Asia. The number of lions in India decreased from 6000 to 3900 over a 10-year period. Calculate the percentage decrease in this period.
4. The table below shows the marks obtained by some students of s.1 at Taibah international school in two mathematics tests. For each one, calculate the percentage differene(change) and make a conclusion whether it is an increase or a decrease.

Student	Test 1	Test 2	Change in value	Percentage change	Conclusion
Mimmi	92	97	–	–	–
Cooper	100	92	–	–	–
Praise	92	83	–	–	–
Tendo	100	72	–	–	–
Hannah	100	67	–	–	–

5. The population of a school increased from 1,200 to 1,500 students. Find the percentage increase in the population of the school
6. The number of books in a school library is increased from 2220 to 2354. What is the percentage increase in the number of books?

7. The price of an item reduced from Shs 4,000 to Shs 3,400. Find the percentage decrease in the price of the item
8. In a closing-down sale, a shop offers 50% cut of the original prices. What fraction is taken off the prices?
9. In a survey one in five people said they preferred a particular brand of Coca Cola. What is this figure as a percentage?
10. Peter pays tax at the rate of 25% of his income. What fraction of Peter's income is this?
11. When Carol was buying a house, she had to make a deposit of $\frac{1}{10}$ of the value of the house. What percentage was this?
12. I bought a coat in the January sales with $\frac{1}{5}$ price cut of the selling price. What percentage was taken off the price of the coat?
13. Akasha bought some fabric that was 1.75 metres long. How could this be written as a fraction?
14. An item costing Shs 8,000 is increased by 15%. Find its new cost price

3.8 Work out real-life problems involving percentages.

Real life problems on percentage will help us to solve different types of problems related to the real-life situations.

3.21 Exercise Set

1. Max scored 6 marks more than what he did in the previous examination in which he scored 30. Maria scored 30 marks more than she did in the previous examination in which she scored 60. Who showed less improvement?
2. In a closing-down sale, a shop offers 50% cut of the original prices. What fraction is taken off the prices?
3. In a survey one in five people said they preferred a particular brand of Coca Cola. What is this figure as a percentage?
4. Peter pays tax at the rate of 25% of his income. What fraction of Peter's income is this?
5. When Carol was buying a house, she had to make a deposit of $\frac{1}{10}$ of the value of the house. What percentage was this?
6. I bought a coat in the January sales with $\frac{1}{3}$ price cut of the selling price. What percentage was taken off the price of the coat?
7. Adikinyi bought some fabric that was 1.75 metres long. How could this be written as a fraction?
8. In a class of 50 students, 40% are girls. Find the number of girls and number of boys in the class?

9. In final exam of senior one there are 50 students 10% students failed. How many students passed to senior two?
10. Victor gets 92% marks in examinations. If these are 460 marks, find the maximum marks.
11. There are 50 students in a class. If 14% are absent on a particular day, find the number of students present in the class.
12. In a basket of apples, 12% of them are rotten and 66 are in good condition. Find the total number of apples in the basket.
13. In an examination, 300 students appeared. Out of these students; 28% got first division, 54% got second division and the remaining just passed. Assuming that no student failed; find the number of students who just passed.
14. In an election, candidate A got 70% of the total valid votes. 20% of the total votes were declared invalid. If the total number of votes is 600000, find the number of valid votes polled in favour of the candidate.

ACTIVITY OF INTERGRATION

- Taibah international school has two sections, that is, Lower UNEB (S.1-S.4) and Upper UNEB (S.5-S.6). The Director of studies of the school needs to draw a timetable for the online lessons for both sections. The sections should start and end their morning lessons at the same time before break time, start and end their break time at the same time. The after break lessons should start at the same time. The lunchtime for both sections should start at the same time and end at the same time.The after Lunch lessons should start at the same time and end at the same time.Math must have 3 hours in a week in each class

- Support: The time to start lessons for the two sections is 8.30am and lessons end at 4:30pm. The duration of the lesson for the Lower UNEB section is 1 hour and that of the Upper UNEB is 2 hours.Assume the following subjects to be offered

Math	English	History	Art	Geography	CRE	Music
Biology	Chemistry	Physics	Entrepreneurship	Home mgt	Psychology	P.E

- Resources: Knowledge of fractions, percentages, natural numbers, factors, multiples, lowest common multiples, and the subjects taught in all classes and of time.

- Tasks:
  - Help the Director of studies by drawing the timetable for the week (monday to friday) for the two sections.
  - How many lessons does each section have up to lunchtime?
  - What is the total number of hours in a week for the lower section

Chapter 4: RECTANGULAR CARTESIAN COORDINATES IN 2 DIMENSIONS

Learning objectives
By the end of this topic,the learners should be able to

- Draw and label the cartesian plane
- Identify the $x-$ axis and $y-$ axis
- Read and plot points on the cartesian plane/coordinate grid
- Complete shapes on a coordinate grid
- Choose and uses appropriate scale for a bi-variate data set

Introduction
This topic is key in building the concept of location. The knowledge achieved from this topic can be used in locating places. In order to locate places you need a starting point (reference point).

- A pair of values written in the form (x, y) is called coordinates
- A point with given coordinates can be plotted on the $x - y$ plane
- The $x - y$ plane is the same as the coordinate plane or the rectangular Cartesian plane
- On the $x - y$ plane, the horizontal axis is called the $x-$axis and the vertical axis is called the $y-$ axis.
- The $x-$axis meets the $y-$axis at a point called the origin. The coordinates of the origin are (0, 0)
- On the $x-$axis, values to the right of the origin are positive and those to the left are negative
- On the $y-$axis, values above the origin are positive and those below are negative

4.1 Identifying the $x-$ axis and $y-$axis

Activity : Plotting Points
STEPS:

- Find the value of x on the $x-$ axis.i.e Start from the origin (0,0) and move the required steps along the $x-$ axis
- Locate the value of y on the $y-$ axis.i.e Start from the origin (0,0) and move the required steps along the $y-$ axis

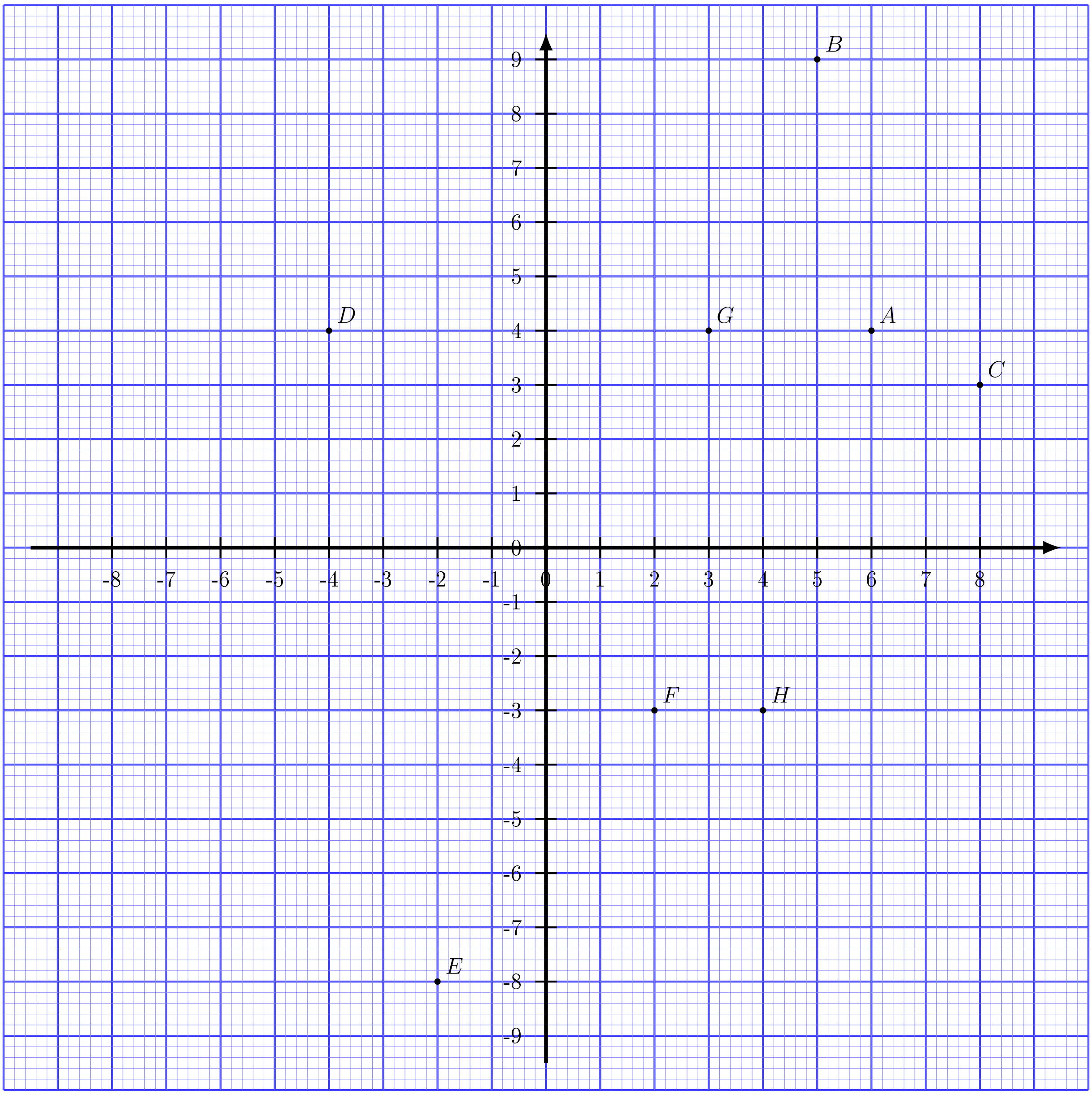
- The intersection of the x and y values is the point

EXAMPLE

Plot the following points on a graph paper A(6,4), B(5,9), C(8,3), D(-4,4), E(-2,-8), F(2,-3), G(3,4),and H(4,-3)

SOLUTION

A(6,5). Start from the origin and first move 6 units to the right (because its positive) ,then 4 units upwards .The intersection is point A



4.1 Exercise Set

1. (a) Plot the following points on a graph paper A(-4,2), B(-3,5), C(1,5), D(2,2), E(-5,-5), F(-3,-2),G(-1,-5), H(2,-2), I(8,2) , J(8,-4) and K(2,-4)

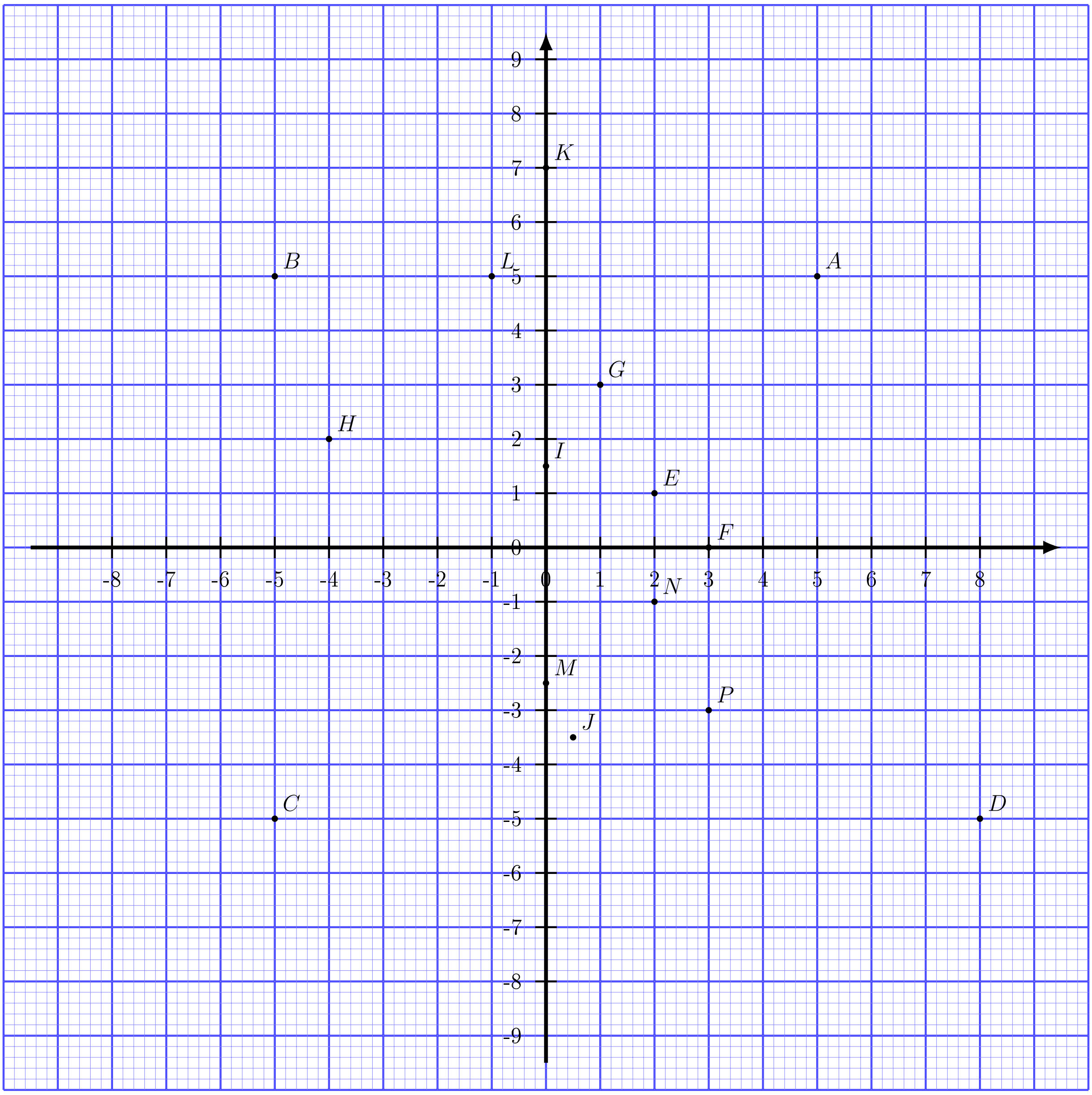
   (b) Join points ABCDA,EFGE,HIJKH

   (c) Name the figures formed in each case

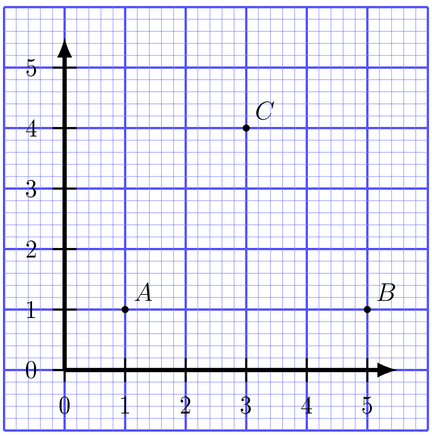
2. (a) Write down the points plotted on the graph paper below

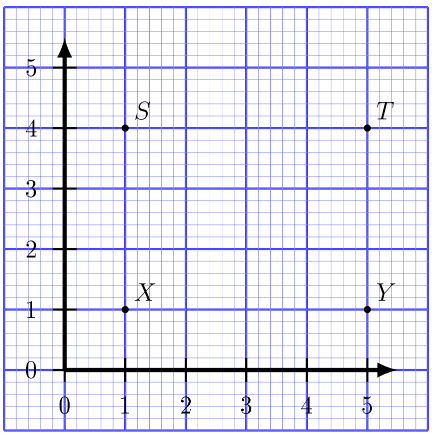
   (b) Join points ABCDA

   (c) Name the figure formed in each case



3(a) Join points ABC and XYTS ,and name the figures formed in each case





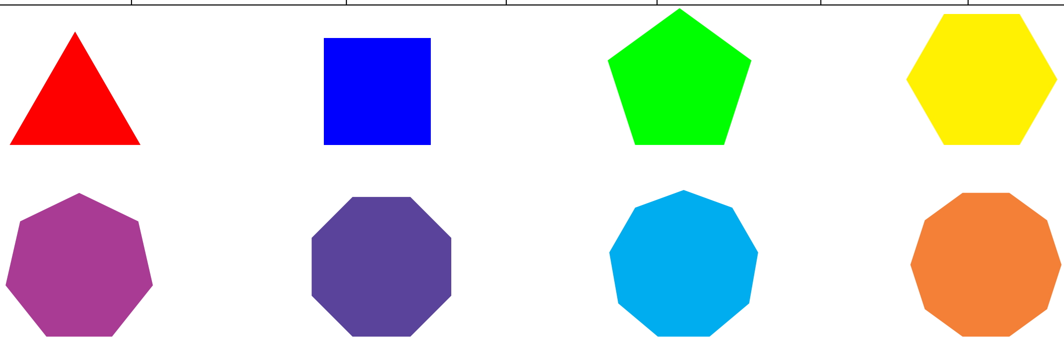
(b) Write down the plotted points

(a) A
(b) B
(c) C
(d) X
(e) Y
(f) T
(g) S

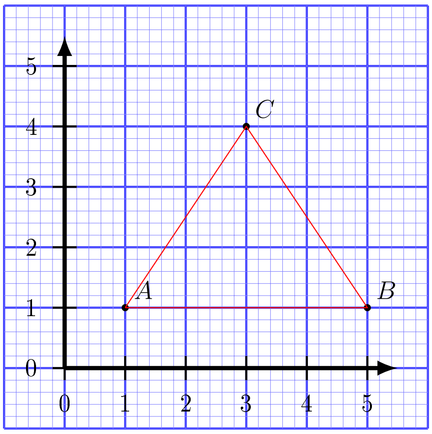
4.2 Plotting Polygons (shapes)

A regular polygon is a polygon which is equiangular (all angles are of the same size) and equilateral (all sides have the same length).

Sides	3	4	5	6	7	8	9	10
Name	Triangle	Quadrilateral	Pentagon	Hexagon	Heptagon	Octagon	Nonagon	Decagon



EXAMPLE:
Join the points A(1,1) ,B(5,1) and C(3,4) to form a triangle

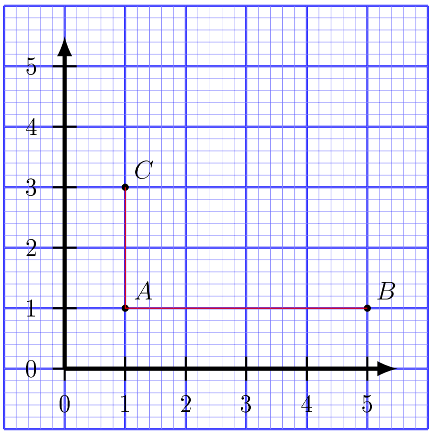


4.2 Exercise Set

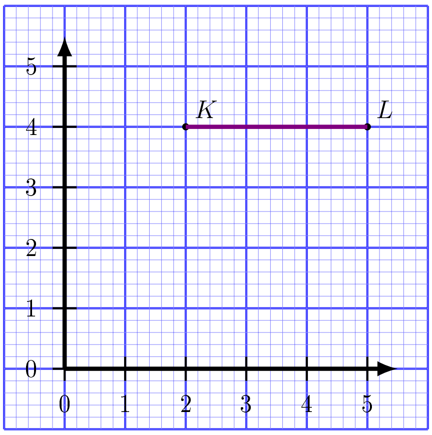
1. In each case the coordinates of 3 corners of a square are given. Find the coordinates of the other corner.

(a) (2, -2), (2, 3) and (-3, 3)
(b) (2, 3), (3, 4) and (1, 4)
(c) (2, 2), (4, 4) and (4, 0)
(d) (-6, 2), (-5, -5) and (1, 3)
(e) (-5, -2), (-2, -1), and (-1, -4)

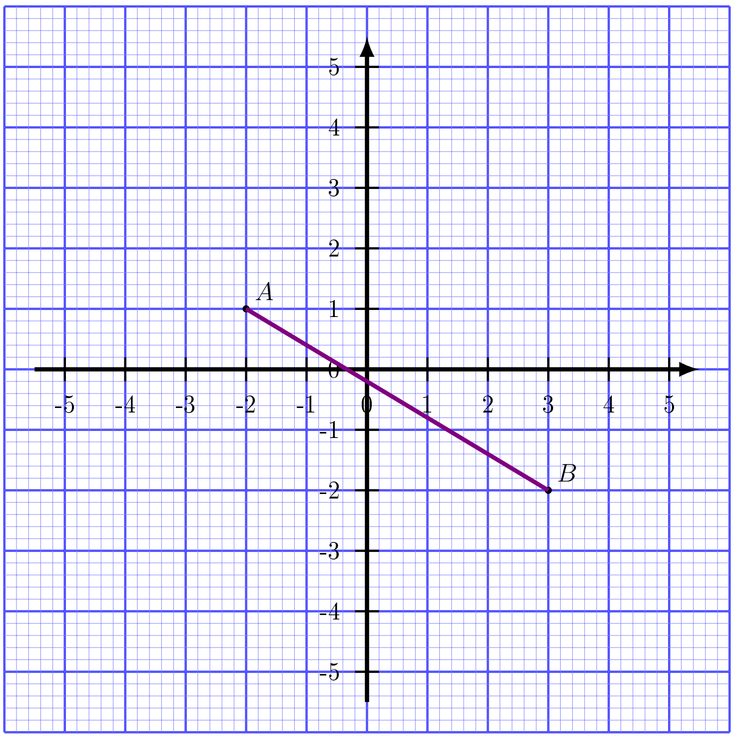
2. What is the possible coordinate of the corner of the rectangle ABCD?



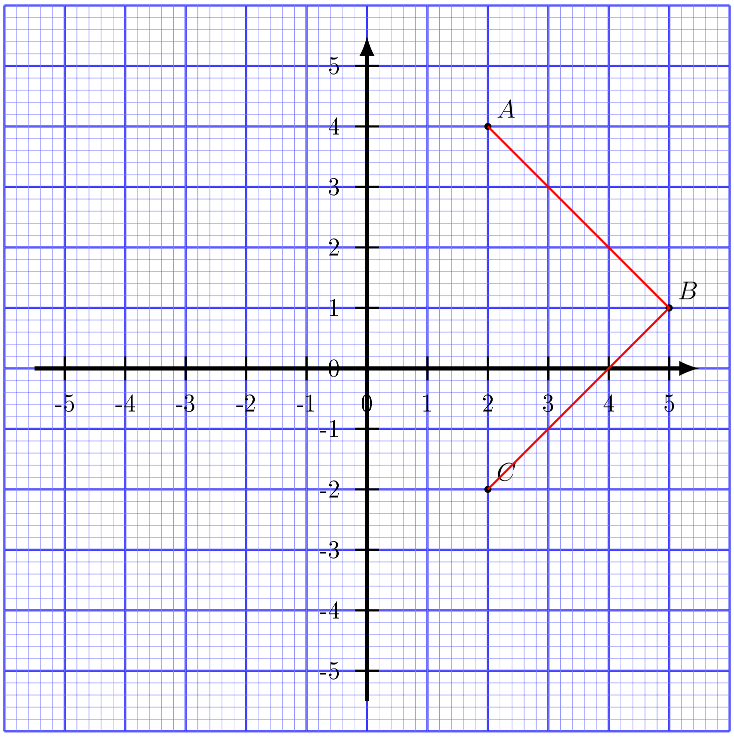
3. What are the possible coordinates of the corners of the square KLMN?



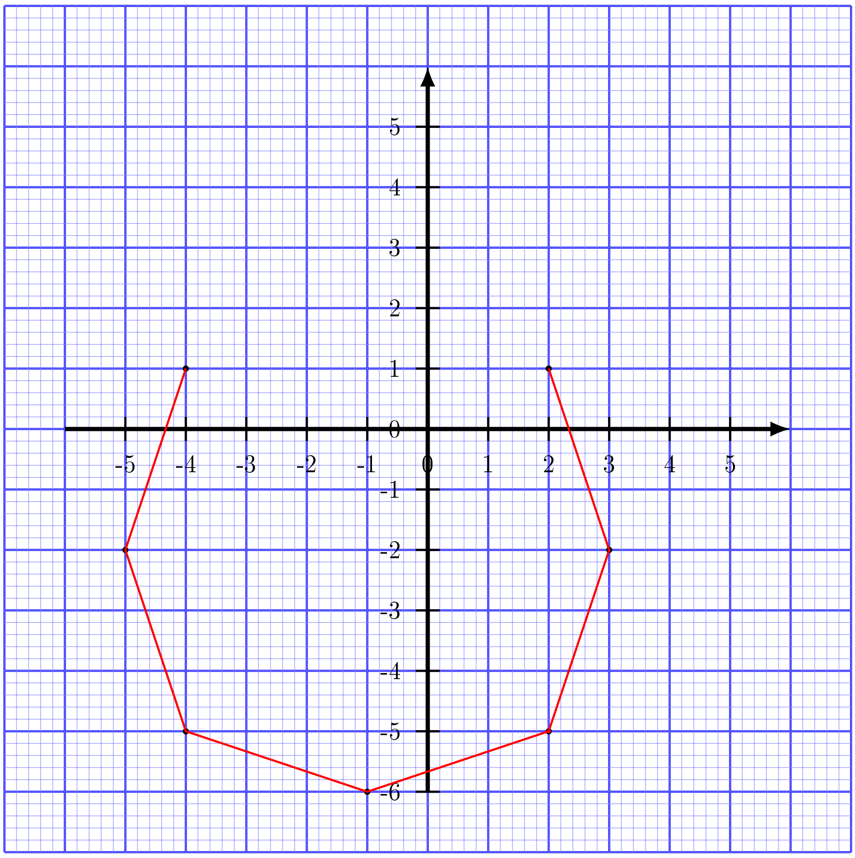
4. What are the possible coordinates of the corners of the square ABCD?



5. Write down the coordinates of the fourth corner of the square.?



6. The sides of an octagon are all the same length. The diagram below shows part of the octagon.Complete the octagon and record the coordinates of the missing corner.



7. The coordinates of 3 corners of a rectangle are given below. Find the coordinates of the other corner of each rectangle.

(a) (-4, 2), (-4, 1) and (6, 1)

(b) (0, 2), (-2, 0) and (4, -6)

(c) (-4, 5), (-2, -1) and (1, 0)

(d) (-5, 1), (-2, 5) and (6, -1)

8. (a) The coordinates of 2 corners of a square are (-4, 4) and (1, -1). Explain why it is possible to draw three different squares using these two points.

   (b) Draw the three different squares.

   (c) If the coordinates of the corners had been (-5, 1) and (1, 3) would it still be possible to draw 3 squares? Draw the possible squares.

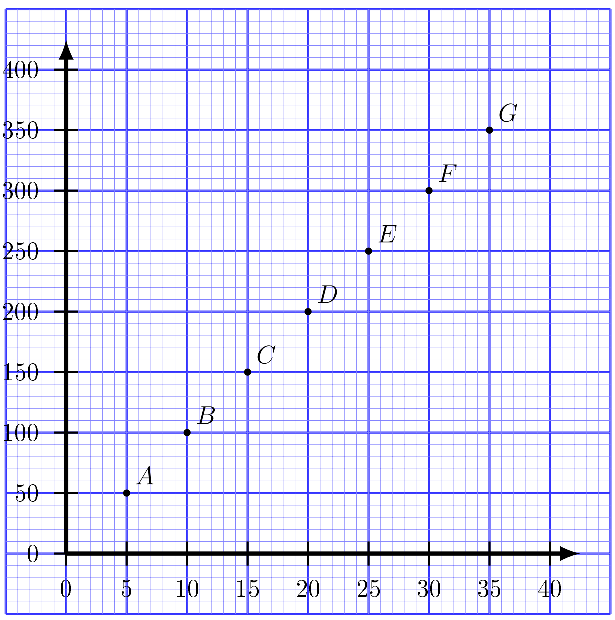
9. Half of an Irregular octagon with one line of symmetry can be drawn by joining the points with coordinates: (0,-2), (-2, 0), (-2, 2), (0, 4) . Join the coordinates. You have drawn one half of the Irregular octagon. Complete the Irregular octagon. Write down the coordinates.

10. On the same axes, plot the points P(-3, 2), Q(-5, 0), R(-4, -3) S(-2, -3), T(-1,0) Join the points and name the formed figure PQRSTP.

11. On the same axes, plot the points P(3, 4), Q(5, 4), R(6, 2) and S(2, 2),Join the points and name the formed figure PQRS.

4.3 Use of Appropriate Scale for Given Data

At times we encounter large values for x and y ,and for such cases we are required to use a convenient scale such that all our values can be able to fit on the graph paper.
EXAMPLE:
Plot the following points on the axes: A(5, 50), B(10,100), C(15,150), D(20,200),E(25,250) ,F(30, 300),G(35,350).
you realise that on the horizontal axis(x- values) there are 5 units for each space and On the vertical axis (y- values)there are 50 units for each space
Horizontal scale : 1cm:5
Vertical axis :1cm :50



4.3 Exercise Set

1. For each part, draw a pair of axes with suitable scales and plot the points:
   (a) A(1, 15), B(4, 35), C(8, 45)
   (b) M(15, 100), N(35, 500), P(40, 700)
2. Plot the points X(2, 60), Y(4, 50), Z(0, 70), T(7, 60)
3. On the same axes, plot the following points A(4, 10), B(-2, -40), C(3, 0), D(0, 30), E(-3, 15) and F(0, -20).Use a scale of 1cm to represent 1 unit on the x- axis and 1cm to represent 5 units on the y- axis
4. A quadrilateral has vertices A(-10, 0), B(-10, 25), C(15, 25) and D(25, -10). Plot the points of the quadrilateral and identify it. Use a scale of 2cm to represent 10 units on both axes
5. A quadrilateral has vertices A(1, 20), B(-3, 30), C(-2, -10) and D(2, -20). Plot the points of the quadrilateral and identify it.
6. Plot the sixteen points below on the graph paperAnd join them to form a pointed star $(4, 0), (-4, 0), (0, 4), (0, -4), (1, 2), (1, -2), (3, 3), (3, -3)(2, 1), (2, -1), (-1, 2), (-1, -2)$ $(-3, 3), (-3, -3), (-2, 1), (-2, -1)$

Situation of Integration

A Senior One learner has reported in her class and has settled at her desk.

- Support: The classroom is arranged in rows and columns. It is a big class with each learner having his/ her own desk.
- Resources: Knowledge of horizontal and vertical lines i.e. rows and columns, coordinates
- Knowledge: counting numbers
- Task: The mathematics teacher has asked her to explain how she can access her seat, starting from the entrance of the class. Discuss whether there are other ways of reaching her seat.

Chapter 5: GEOMETRIC CONSTRUCTION SKILLS

Learning objectives

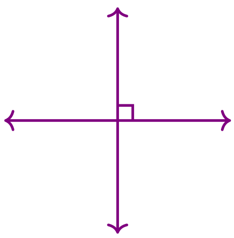
By the end of this topic,the learners should be able to

- Draw perpendicular and parallel lines
- Construct perpendiculars,angle bisectors,mediators and parallel lines
- Use a pair of compasses and a ruler to construct special angles
- Describe a locus
- Relate parallel lines,perpendicular bisector ,angle bisector,straight lines and a circle as loci
- Draw polygons
- Measure lengths and angles
- Construct geometrical figures such as triangle,square ,rectangle,rhombus ,parallelogram

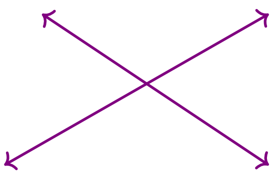
In this topic you will learn how to construct lines, angles and geometric figures. Skills developed from this topic can be applied in day-to-day life.

5.1 Parallel ,Perpendicular and Intersecting lines

Parallel lines are lines in a plane that are always the same distance apart. Parallel lines never intersect



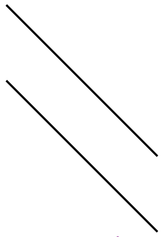
Perpendicular lines are lines that intersect at a right (90^0) angle.

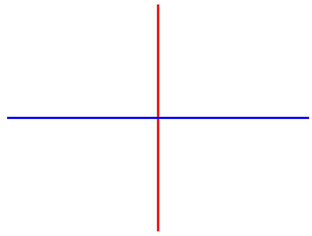


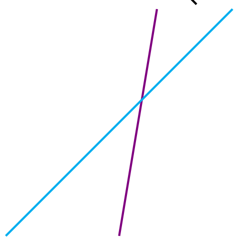
Intersecting lines are two lines that share exactly one point. This shared point is called the point of intersection.

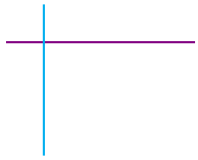
5.1 Exercise Set

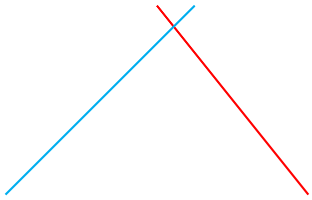
Identify the lines below

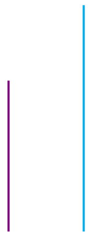
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

5.2 Construction of Perpendicular Lines

Activity : Construction of perpendicular line from an external point to a given line

Given line segment AB and point P outside the line, construct a perpendicular line from point P to line AB.
STEPS

1. Taking the centre as P and any radius,Place the campus at point P and draw two arcs to cut line AB .
2. Taking A as the centre and any radius, draw an arc below or above the line opposite point P .
3. Without changing the radius and taking B as the centre, draw an arc to intersect the previous arc at point Q.
4. Join the intersection of the arcs from point P to Q

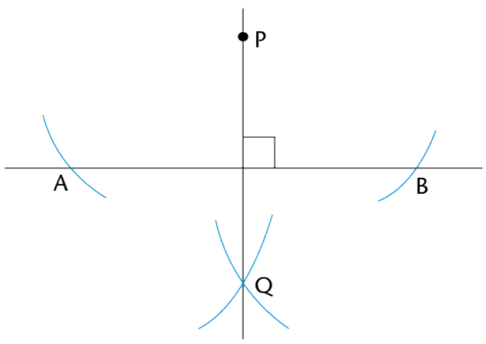


Figure 5.1: Construction of perpendicular line from an external point to a given line

Activity : Construction of a Perpendicular line to a given point on a given line segment

Given line AB and point P on AB.construct a perpendicular line from point P on AB
STEPS

1. Taking P as the centre and any radius, draw two arcs on either side of P name the arcs A and B .
2. Taking A as the centre and any radius draw an arc either above or below the line.
3. Without changing the radius and taking B as the centre draw an arc to meet the previous arc at point Q
4. Join the intersection of the arcs to from P to Q.

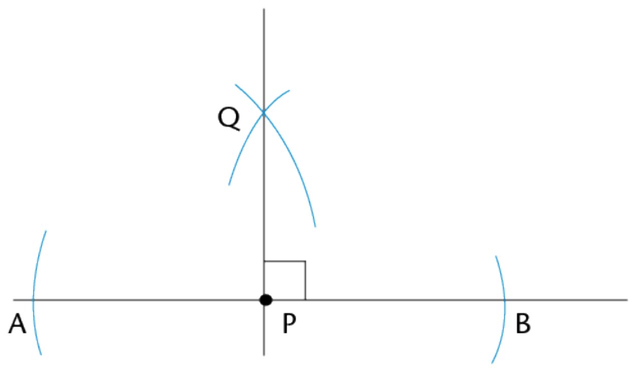


Figure 5.2: Construction of a Perpendicular line to a given point on a given line segment

Activity : Construction of a Perpendicular Bisector

Given line segment AB ,draw a perpendicular bisector
STEPS

1. Draw a line segment with end points A and B .
2. Place the point of compass at A ,stretch out the compass until more than half of the length of AB.
3. Draw an arc on either side of the line segment
4. Keeping the radius of the compass constant,place the point of the compass at point B and draw an arc on either sides of the line segment.
5. Join the intersection of the arc at point C and D.

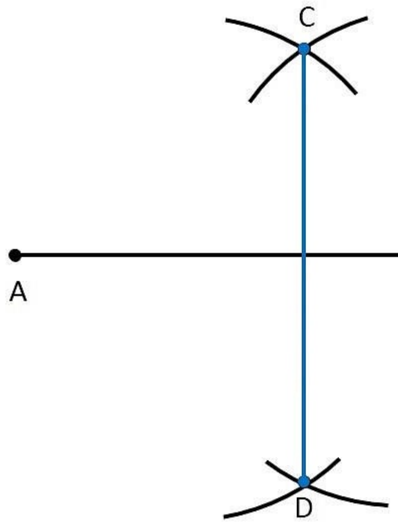
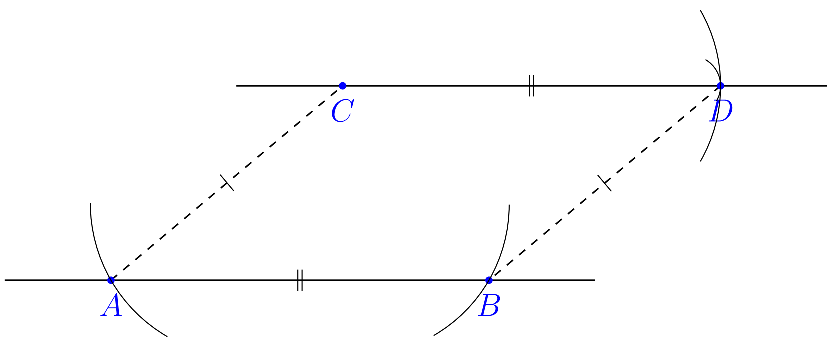


Figure 5.3: Construction of a Perpendicular bisector

5.3 Construction of parallel lines

STEPS

1. Draw a line segment with end points AB and a point C outside the line .
2. Draw an arc at point A taking AB as radius and C as the centre.
3. Taking A as the centre and AB as the radius ,draw an arc at point B
4. Taking AC as the radius and B as the centre ,draw an arc above B.
5. Taking AB as the radius and and C as the centre ,draw an arc to meet the previous arc at D.
6. Join the intersection of the arcs at point D to C.
7. The line segment AB is parallel to CD



5.4 Construction of special angles

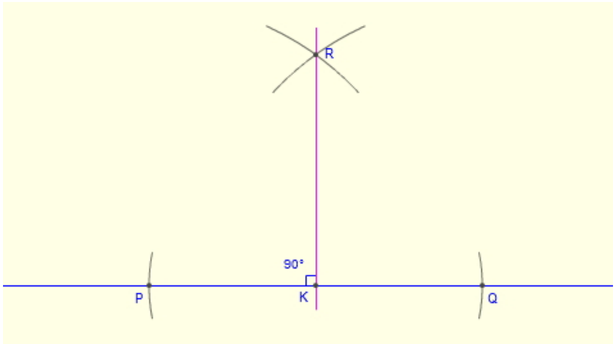
1. In construction, of angles, we use a ruler, pencil and pair of compasses only.

2. The angle bisector method can be used to create other angles. Thus, an angle of 30^0 is obtained by bisecting an angle of 60^0.
3. The supplementary angle construction method can be used to get obtuse angles. Thus, an angle of 120^0 is obtained by constructing an angle of 60^0.

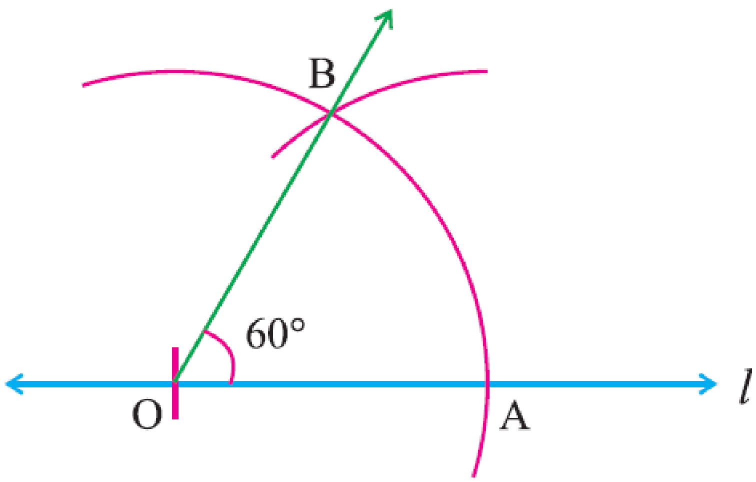
Activity : Construction of special angles

Using a pencil, ruler and pair of compasses only, construct the following angles:

1. 90^0



2. 60^0



5.2 Exercise Set

1. Using a pencil, ruler and pair of compasses only, construct the following angles:

(a) 30^0 (d) 45^0 (g) 120^0 (j) 150^0

(b) 15^0 (e) 75^0 (h) 135^0

(c) 7.5^0 (f) $22\frac{1}{2}^0$ (i) 165^0

2. Using a potractor measure the angles constructed above

5.5 Describing a Locus

Locus of a point is the path which it describes as it moves

Activity : Discovering what Locus is
EXAMPLE 1

- *What is the path traced out by the tip of the seconds-hand of a clock in the course of each minute?*

- The Second hands of a clock moves around the clock and creates a circular path. The tip of each hand is always the same distance (equidistant) from the centre of the clock. The locus the second hand of a clock create is a circle

Figure 5.4: Path traced by second hand of a Clock

EXAMPLE 2

- *Describe what happens if a cow is tied to a rope of length 4 metres and around the place where the cow is, there are gardens at a distance of 4 metres.*

- The cow rotates around creating a circular path.Therefore the locus at a distance of 4 metres from the centre (the stake), is a circle with a center and a radius, of 4 metres.

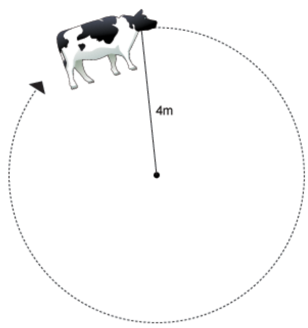


Figure 5.5: Path traced by a cow

Activity : Sketching and Describing Loci
Sketch and describe what happens about the following:

1. A mark on the floor as the door opens and closes.
2. The centre of a bicycle wheel as the bicycle travels along a straight line.
3. A man is walking and keeping the same distance from two trees P and Q.
4. A student is walking in the class keeping the same distance from two opposite walls.

5.6 Relating Lines and Angles to Loci

locus is a set of points which satisfies a certain condition.
Activity : Demonstration of some simple Loci

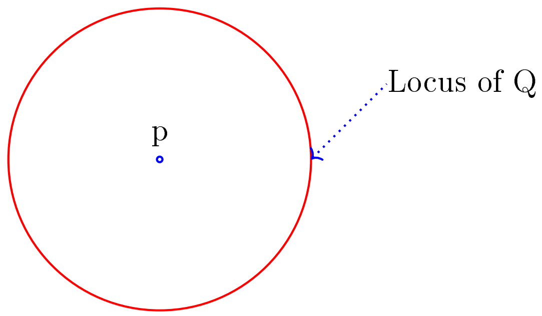
1. Demonstrate how one can walk the same distance from a given point.
2. How one can walk the same distance from two fixed points.

3. How one can walk the same distance from a line.
4. How one can walk the same distance from two intersecting lines

5.7 Construction of Loci

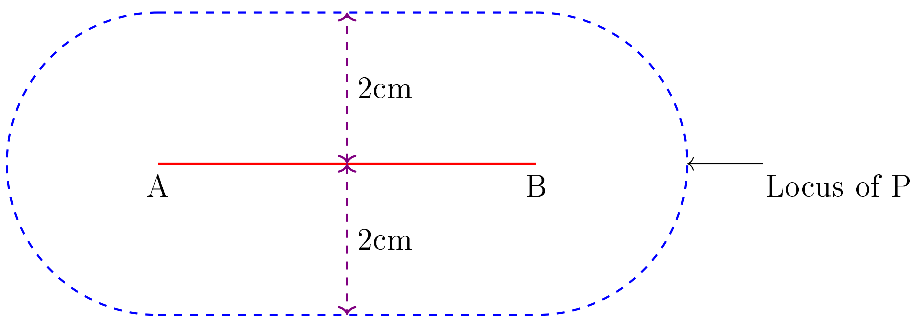
Activity : Construction of a locus at a point equidistant from a fixed point.
The locus of points that are at a constant distance from a fixed point is a circle with radius equals to constant distance.

EXAMPLE
Construct the locus of a point Q at a constant distance of 2 cm from a fixed point P.



Activity : Construction of a locus of a point equidistant from a given line
The locus of points that are at a constant distance from a straight line is a pair of parallel lines at a constant distance from the given straight line.

EXAMPLE
Construct the locus of a point P that moves a constant distance of 2 cm from a straight line AB



NOTE

- The locus of points at a fixed distance, d, from the point, P is a circle with the given point P as its center and d as its radius.i.e Given a fixed point, the locus of points is a circle.
- The locus of the points at a fixed distance, d, from a line with end points AB, is a pair of parallel lines at a distance,d(apart) from AB and on either side of AB.i.e Given a straight line, the locus of points is two parallel lines.

- The locus of points equidistant from two given points, A and B, is the perpendicular bisector of the line segment that joins the two points..i.e Given two points, the locus of points is a straight line midway between the two points.
- The locus of points equidistant from two intersecting lines, L_1 and L_2, is a pair of bisectors that bisect the angles formed by line L_1 and L_2.i.e Given two intersecting lines, the locus of points is a pair of lines that cut the intersecting lines in half.

5.3 Exercise Set

1. Construct the locus of a point Q that moves a constant distance of 3 cm from a straight line XY
2. A dog is on a lead tethered to a post in the corner of a garden. The lead is 5 cm long. Describe the locus of the dog with a sketch.
3. Construct the locus of a point equidistant from two intersecting lines.

5.8 Construction of Geometric Figure

Construction of figures is an application of the locus,since during inscribing and circumscribing we use the knowelge of angle bisector.
Activity : Construction of geometrical figures
Steps for circumscribing a circle on a triangle. .

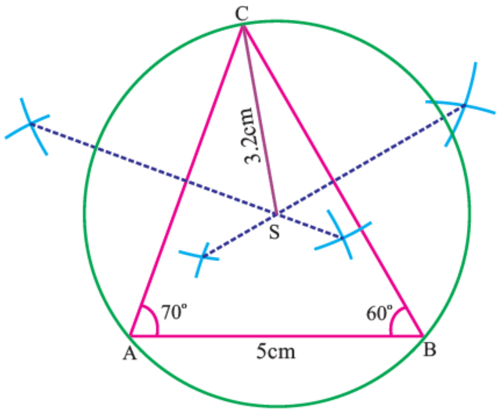
- Construct the perpendicular bisector of one side of triangle.
- Construct the perpendicular bisector of another side.
- Where they cross is the center of the Circumscribed circle.
- Place the compass on the center point, adjust its length to reach any vertex of the triangle, and draw your Circumscribed circle

EXAMPLE
Using a pair of compasses ,ruler and pencil only,construct a triangle ABC in which AB=5cm, $\angle$ BAC=$70^0$and $\angle$ ABC =50^0

1. Measure and record th lengths BC and AC
2. Construct a perpendicular bisector of the line segments BC and AC
3. Using the meeting point of the perpendicular bisectors as your center,draw a circle to pass through the vertices of the triangle
4. Measure and record the radius of the circle
5. Calculate the area of the circle

SOLUTION

1.



2. BC= 6.1cm and AC=5.6cm
3. Radius=3.2cm
4.

$$\begin{aligned} A &= \pi r^2 \\ &= \frac{22}{7} \times 3.2^2 \\ &= \frac{22}{7} \times 10.24 \\ &= 32.183 cm^2 \end{aligned}$$

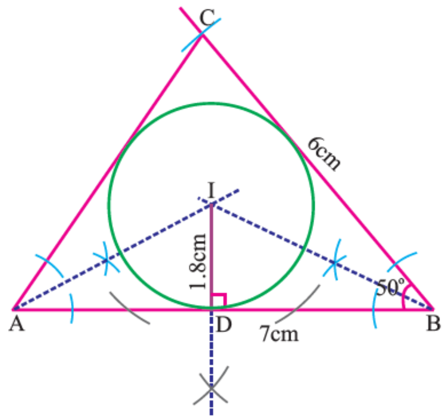
Steps for inscribing a circle in a triangle.

- Bisect two angles of a triangle
- The angle bisector will intersect at the incenter(centre point)
- Construct a perpendicular from the centre point to one side of the triangle..
- Place the compass at the centre point and adjust its length up to where the perpendicular crosses the triangle, and then draw the inscribed circle.

EXAMPLE
Using a pair of compasses ,ruler and pencil only,construct a triangle ABC in which AB=7cm, $\angle$ ABC =50^0 and BC=6cm .

1. Bisect $\angle$ ABC and $\angle$ BAC
2. Construct a perpendicular from the centre point to one side of the triangle
3. Measure and record the radius of the circle
4. Calculate the area of the circle

SOLUTION



1.

2. Radius=1.8cm

3.

$$
\begin{aligned}
A &= \pi r^2 \\
&= \frac{22}{7} \times 1.8^2 \\
&= \frac{22}{7} \times 3.24 \\
&= 10.183cm^2
\end{aligned}
$$

5.4 Exercise Set

1. Using a ruler,pencil and compass ,construct a triangle ABC where AB = 7cm, AC=5cm ,$\angle$BAC = 60^0 . Find the point with in the triangle where the distance from that point to all the vertices of the triangle is equal. Taking that point as the centre and the distance from the centre to the vertices as the radius draw a circle.

   HINT: We need to construct a circle inscribed in triangle and this can be done by making angle bisector of two sides, the point where it intersect will be incentre. (The angle bisector is the locus where points are equidistant from two sides)

2. Construct a perpendicular bisector of any line segment. Measure the distance from the perpendicular line to any of the points on either side of the perpendicular bisector. What have you found out? Construct an equilateral triangle with length 6cm. Construct a circumcircle of the triangle. What type of locus is applied here?

3. Construct a triangle ABC in which AB = 8.5cm, BC = 6cm and $\angle$ABC =30^0 Construct a circle through the vertices of the triangle. Work out the area of the circle.

   HINT: Area of a circle: A=πr^2 ,where r=radius of the circle,$\pi = \frac{22}{7}$

4. Using a ruler and a pair of compasses only, construct a triangle ABC in which BC = 7.2cm, AC = 8.4cm and $\angle$ ABC = 75^0.

(a) Measure length AB and angle ACB

(b) Draw a perpendicular from A onto BC to meet it at D. Measure length AD

(c) Draw a circle circumscribing triangle ABC. Measure the radius of the circle

(d) Calculate the area of the circle.

5. Construct triangle PQR with PQ = QR= 7cm angle Q = 45^0. Construct a circumcircle of the triangle.

6. Construct a parallelogram ABCD in which AB=5cm, BC=4cm and angle B is 120^0.

7. Construct an equilateral triangle ABC of sides 7cm.Bisect AB and BC and let the bisectors intersect at X. With X as the centre and radius XA, draw a circle.

8. Using a ruler and a pair of compasses only, draw a triangle PQR such that PQ = QR = 8.5 cm and $\angle$PQR=120^0 . Draw the incircle of triangle PQR and measure its radius. Calculate the area of the incircle.

9. Using a pair of compasses ,ruler and pencil only,construct a triangle ABC in which AB=10cm, $\angle$ABC=60^0 and $\angle$CAB=45^0 .

   (a) Measure and state lengths AC and BC

   (b) Circumscribe triangle ABC

   (c) Measure and state the radius of the circle

   (d) Calculate the area of the circle

   (e) Calculate the perimeter(circumference) of the circle
   HINT: Perimeter of a circle: P=$2\pi r$,where r=radius of the circle,$\pi = \frac{22}{7}$

Situation of Integration

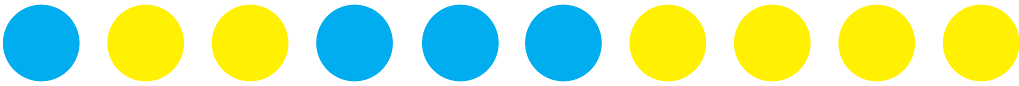
In a village, there is an old man who wants to construct a rectangular small house of wattle and mud.

- Support: A string, sticks, panga, tape measure and human resource.

- Resources: Knowledge of horizontal and vertical lines i.e. rows and columns, knowledge of construction of geometric figures

- Task: The community asks you to accurately construct the foundation plan for this old man's house.Explain how you have accurately constructed the foundation plan. Discuss whether there are other ways of constructing an accurate foundation plan

Chapter 6: SEQUENCE AND PATTERNS



Learning objectives

By the end of this topic,the learners should be able to

- Recognise and generate number patterns
- Explain how to generate a sequence
- Use number machines to generate a sequence
- Describe a general rule when a pattern is given
- Determine terms in a sequence

We often need to spot a pattern in order to predict what will happen next. In maths, the correct name for a pattern of numbers is called a SEQUENCE. In this topic therefore you will learn how to identify and describe general rules for patterns. You will be able to determine a term in the sequence and find the missing numbers in the sequence

6.1 Draw and Identify the Patterns

For any pattern it is important to try to spot what is happening before you can predict the next number.

Activity : Identifying the number patterns

(a) $3, 6, 9, 12, \cdots$

To obtain the next number in the sequence, we add 3 to the previous number. The numbers in this sequence are multiples of 3.

(b) $7, 14, 21, 28, \cdots$

To obtain the next number in the sequence, we add 7 to the previous number. The numbers in this sequence are multiples of 7.

(c) The table below shows the natural numbers from 1 to 100.

Multiples of 5   Multiples of 11

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

(a) The fifth multiple of 11 is 55

(b) The nineth multiple of 11 is 99

(c) The fifth multiple of 5 is 25

(d) The 12^{th} multiple of 5 is 60

(e) The 20^{th} multiple of 5 is 100

6.1 Exercise Set

1. Identify the pattern in the following sequences

   (a) $3, 6, 9, 12, \cdots$

   (b) $20, 40, 60, 80, \cdots$

   (c) $1, 2, 3, 4, 5, \cdots$

   (d) $1, 4, 9, 16, \cdots$

   (e) $3, 7, 10, 17, 27, \cdots$

   (f) $1, 3, 6, 10, 15, \cdots$

2. Write down the next 4 terms of each of these sequences:

   (a) $4, 7, 10, 13, 16, 19, \cdots$

   (b) $5, 11, 17, 23, 29, 35, \cdots$

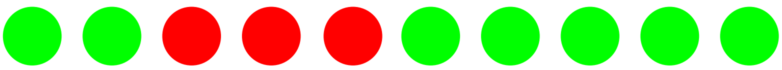
   (c) $6, 8, 11, 15, 20, 26, \cdots$

   (d) $10, 14, 20, 28, 38, \cdots$

   (e) $24, 23, 21, 18, 14, 9, \cdots$

   (f) $2, 12, 21, 29, 36, 42, \cdots$

3. Complete the sequence,by drawing the next three patterns



___ ___ ___

4. Using a number square box below ,answer the questions after the table.

Multiples of a number

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

(a) Identify the number whose multiples have been shaded yellow

(b) The 3^{rd} multiple of nine is ......

(c) The 9^{th} multiple of nine is ......

(d) The 20^{th} multiple of nine is ......

(e) The 5^{th} multiple of ...... is 60

(f) The 8^{th} multiple of ... is 56.

(g) The 400^{th} multiple of nine is ......

(h) State the multiples of 4 ,that are in the table

(i) The ......th multiple of nine is 1800

(j) The ......th multiple of nine is 2970.

5. (i) Write down the first eight multiples of 8.

   (ii) Write down the first 8 multiples of 6.

   (iii) What is the smallest number that is a multiple of both 6 and 8?

   (iv) If 48 is the nth multiple of 12, what is n?

   (v) If 96 is the nth multiple of 12, what is n ?

6. Three multiples of a number are 34, 170 and 255. What is the number?

7. Three multiples of a number are 38, 95 and 133. What is the number?

8. Four multiples of a number are 49, 77, 133 and 203. What is the number?

9. The number 24 is a multiple of 2 and a multiple of 3. What other numbers is it a multiple of?

10. Two multiples of a number have been shaded on this number square. What is the number?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

11. Two multiples of a number have been shaded on this number square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

(a) What is the number?

(b) What is the 19th multiple of this number

12. Color the numbers

Multiples of 4   Multiples of 7

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

6.2 Describing the General Rule

Activity : Finding the Next Term in the sequence

Find the next numbers in the sequences below

(a) $3, 6, 9, 12, \cdots$

To obtain the next number in the sequence, we add 3 to the previous number.

3 6 9 12 15 18 21 24

+3 +3 +3 +3 +3 +3 +3

(b) $7, 14, 21, 28, \cdots$

To obtain the next number in the sequence, we add 7 to the previous number.

7 14 21 28 35 42 49 56

+7 +7 +7 +7 +7 +7 +7

(c) $24, 21, 18, 15, \cdots$

To obtain the next number in the sequence, we subtract 3 from the previous number.

24 21 18 15 12 9 6 3

-3 -3 -3 -3 -3 -3 -3

(d) $3, 9, 27, 81, \cdots$

To obtain the next number in the sequence, we multiply 3 with the previous number.

3 9 27 81 243 729 2187 6561

×3 ×3 ×3 ×3 ×3 ×3 ×3

6.2 Exercise Set

1. Find the next three numbers(terms) in each of the following sequences

   (a) $1, 3, 5, 7, \cdots$

   (b) $2, 5, 8, 11, \cdots$

   (c) $4, 2, 0, -2, \cdots$

   (d) $1, 2, 6, 24, 120, \cdots$

   (e) $1^3, 2^3, 3^3, 4^3, \cdots$

   (f) $5, 9, 13, 17, \cdots$

   (g) $1, 2, 4, 8, \cdots$

   (h) $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \cdots$

   (i) $4, 9, 25, 49, \cdots$

   (j) $18, 30, 42, 54, 66, \cdots$

   (k) $1, \frac{-1}{2}, \frac{1}{4}, \frac{-1}{8}, \cdots$

   (l) $4.1, 4.7, 5.3, 5.9, 6.5, \cdots$

   (m) $3.42, 3.56, 3.70, 3.84, \cdots$

   (n) $10, 9.5, 9, 8.5, 8, 7.5, \cdots$

2. Fill in the missing numbers

(a) $6, 11, ..., 21, ..., \cdots$

(b) $..., ..., 41, 36, 31, 26 \cdots$

(c) $2, 4, ..., 16, 32, \cdots$

(d) $100, 81, 64, ..., 36, \cdots$

(e) $-2, ..., -8, ..., -14, \cdots$

(f) $0, 1.5, 4, ..., 12, \cdots$

(g) $1, 7, 17, ..., 49, \cdots$

6.3 Generating Number Sequence

Activity : Generating a sequence This involves using a formulae to generate sequences for given values.

EXAMPLES

What sequence do you generate by using the following formula?.Take $n = 1, 2, 3, 4, 5, \cdots$

1. $2n$
   we substitute the value of n ,in the formula given

$$\begin{aligned}
&\text{for n=1}\\
&\qquad 2n = 2 \times 1\\
&\qquad\quad = 1\\
&\text{for n=2}\\
&\qquad 2n = 2 \times 2\\
&\qquad\quad = 4\\
&\text{for n=3}\\
&\qquad 2n = 2 \times 3\\
&\qquad\quad = 6
\end{aligned}$$

   Therefore the generated sequence is $2, 4, 6, 8, 10, \cdots$

2. $8n - 5$
   we substitute the value of n ,in the formula given

$$\begin{aligned}
&\text{for n=1}\\
&\quad 8n - 5 = 8 \times 1 - 5\\
&\qquad\quad = 3\\
&\text{for n=2}\\
&\quad 8n - 5 = 8 \times 2 - 5\\
&\qquad\quad = 11\\
&\text{for n=3}\\
&\quad 8n - 5 = 8 \times 3 - 5\\
&\qquad\quad = 19
\end{aligned}$$

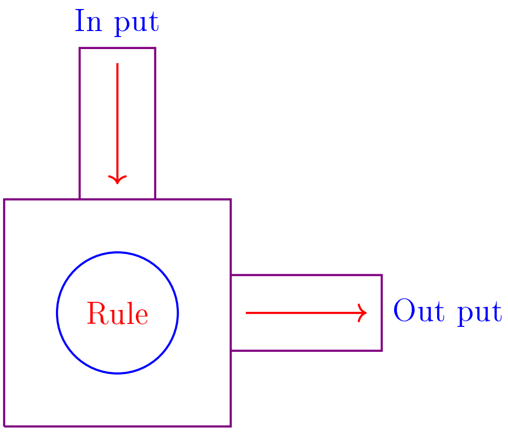
   Therefore the generated sequence is $3, 11, 19, 27, 35, \cdots$

3. $6n + 2$
   we substitute the value of n ,in the formula given

$$\begin{aligned}
&\text{for n=1}\\
&\quad 6n+2 = 6\times 1+2\\
&\qquad\quad = 8\\
&\text{for n=2}\\
&\quad 6n+2 = 6\times 2+2\\
&\qquad\quad = 14\\
&\text{for n=3}\\
&\quad 6n+2 = 6\times 3+2\\
&\qquad\quad = 20
\end{aligned}$$

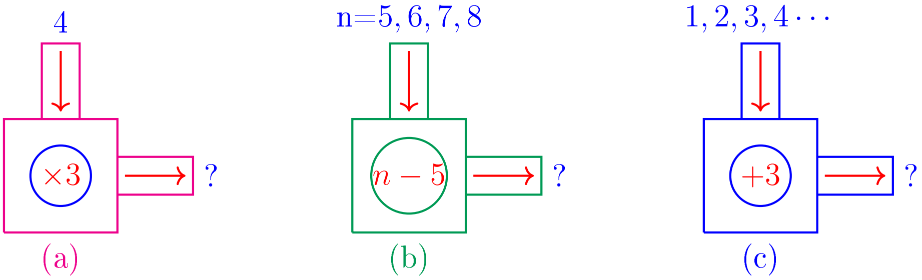
Therefore the generated sequence is $8, 14, 20, 26, 32, \cdots$

Input/Output machine
Some math problems contain a pattern,so once you find a pattern then you can make a rule that will solve the problem for a given input.Therefore we put numbers into the machine[input], and the machine uses an operation (add, subtract, multiply or divide) to give us a result[output].



EXAMPLES

What number comes out of each of these number machines?



(a)

Input	Output
4	12

Therefore the output is 12

(b)

In put	Out put
5	0
6	1
7	2
8	3

There fore the output is 0,1,2,3

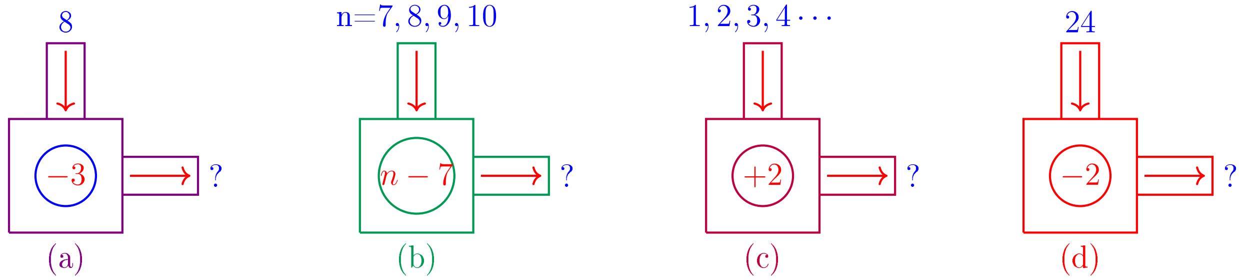
(c)

In put	Out put
1	4
2	5
3	6
4	7

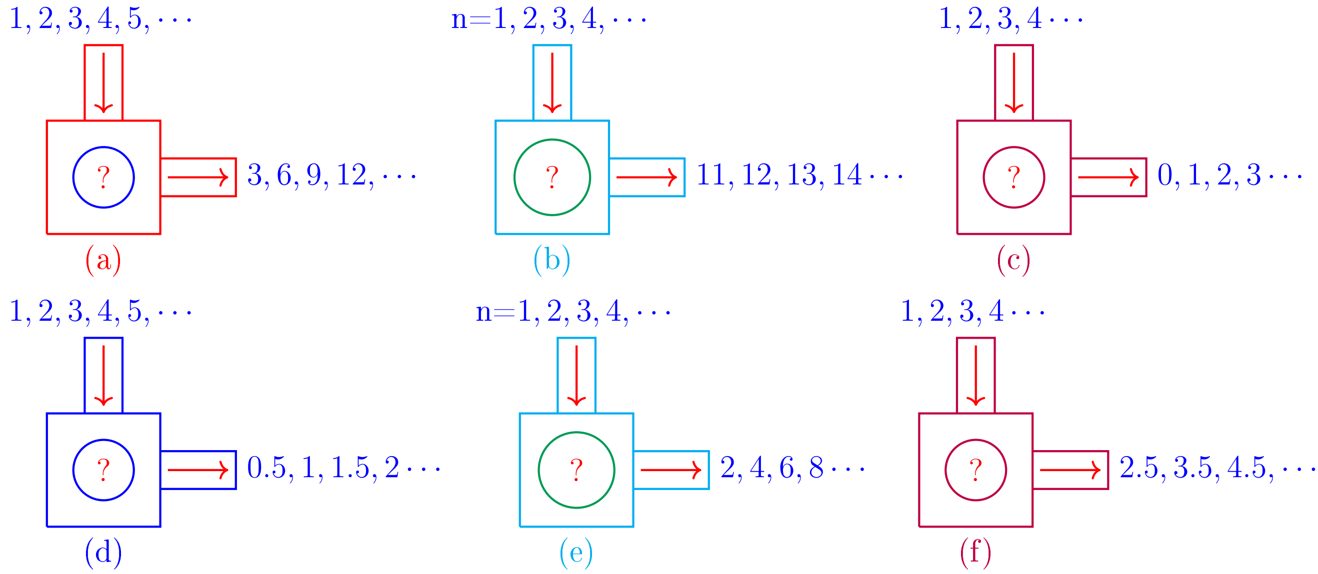
There fore the output is $4, 5, 6, 7, \cdots$

6.3 Exercise Set

1. What number comes out of each of these number machines?



2. The sequence $1, 2, 3, 4, 5, \ldots$ is put into each number machine. What does each machine do?



3. Write down the first 7 terms of the sequence given by each of these formulae.Take $n = 1, 2, 3, \cdots$

   (a) $4n - 1$

(b) $7n - 1$

(c) $8n$

(d) $9n + 3$

(e) $0.5n$

4. Taking $n = 1, 2, 3, \cdots$, what is

   (a) the 10^{th} term of the sequence 2n - 1?

   (b) the 8^{th} term of the sequence 3n ?

   (c) the 5^{th} term of the sequence 4n + 1?

   (d) the 7^{th} term of the sequence 5n + 2 ?

5. Draw a double(input/output) machine that yields the following outputs.[The formula(rule) for each sequence must be written clearly]

   (a) $1, 2, 3, 4, 5, \cdots$

   (b) $7, 14, 21, \cdots$

   (c) $2, 5, 8, 11, 14, \cdots$

   (d) $6, 11, 16, 21, 26, \cdots$

   (e) $4, 9, 14, 19, 24, \cdots$

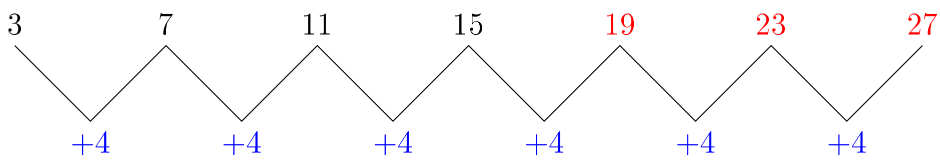
   (f) $102, 202, 302, 402, 502, \cdots$

6.4 Formulae for General Terms

It is very helpful not only to be able to write down or generate the next few terms in a sequence, but also to be able to write down any n^{th} term .for example, the 100^{th}term .Therefore this involves generating a formula using a given sequence.
Activity : Identifying the n^{th} term
EXAMPLE

1. For the sequence$3, 7, 11, 15, \cdots$Find

   (a) the next three terms.
   To obtain the next number(term) in the sequence, we add 4 to the previous number.



   (b) the 100^{th} term.

Position of term	1^{st}	2^{nd}	3^{rd}	4^{th}	10^{th}	50^{th}	100^{th}	n^{th}
Term	3	3+4	$3+(2 \times 4)$	$3+(3 \times 4)$	$3+(9 \times 4)$	$3+(49 \times 4)$	$3+(99 \times 4)$	$3+(n-1) \times 4$
Value	3	7	11	15	39	199	399	

Therefore the 100^{th} term is 399

(c) the 1000^{th} term.
To obtain the 1000^{th} term,we can base on the n^{th} term.

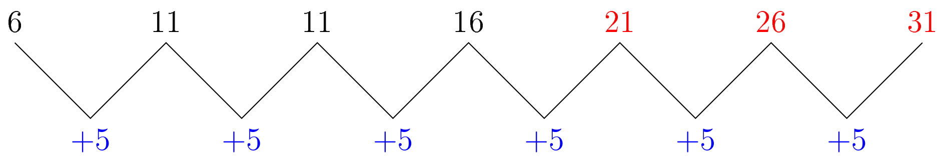
$$
\begin{aligned}
&= 3 + (n-1) \times 4 \\
&= 3 + (1000-1) \times 4 \\
&= 3 + 999 \times 4 \\
&= 3999
\end{aligned}
$$

The formula for a general term, i.e. the n^{th} term.

$$
\begin{aligned}
&= 3 + (n-1) \times 4 \\
&= 3 + 4n - 4 \\
&= 4n + 3 - 4 \\
&= 4n - 1
\end{aligned}
$$

2. For the sequence$6, 11, 16, 21 \cdots$ Find

(a) the next three terms.
To obtain the next number(term) in the sequence, we add 5 to the previous number.



(b) the 100^{th} term.

Position of term	1^{st}	2^{nd}	3^{rd}	4^{th}	10^{th}	50^{th}	100^{th}	n^{th}
Term	6	6+5	$6+(2 \times 5)$	$6+(3 \times 5)$	$6+(9 \times 5)$	$6+(49 \times 5\ )$	$6+(99 \times 5)$	$6+(n-1) \times 5$
Value	6	11	16	21	51	251	501	

Therefore the 100^{th} term is 251

(c) the 1000^{th} term.
To obtain the 1000^{th} term,we can base on the n^{th} term.

$$
\begin{aligned}
&= 6 + (n-1) \times 5 \\
&= 6 + (1000-1) \times 5 \\
&= 6 + 999 \times 5 \\
&= 5001
\end{aligned}
$$

The formula for a general term, i.e. the n^{th} term.

$$\begin{aligned} &= 6 + (n-1) \times 5 \\ &= 6 + 5n - 5 \\ &= 5n + 6 - 5 \\ &= 5n + 1 \end{aligned}$$

6.4 Exercise Set

1. Given the following sequences

   (a) $0, 1, 2, 3 \cdots$

   (b) $1, 3, 5, 7 \cdots$

   (c) $5, 10, 15, 20 \cdots$

   (d) $2, 7, 12, 17 \cdots$

   (e) $-2, -5, -8, -11 \cdots$

   Find

   (i) the next three terms in the sequences

   (ii) the 100^{th},20^{th} and 31^{st} terms for each of the sequence

   (iii) the formula for the n^{th} term of each of the sequence

2. What is the n^{th} term of the sequence $2, 5, 10, 17, 26 \cdots$?
3. Write down the first 6 multiples of 8 and the formula for the n^{th} term of the sequence .
4. What is the n^{th} term of the sequence $1, 4, 9, 16 \cdots$

Situation of Integration

There is a family in the neighbourhood of your school. The family has a rectangular compound on which they want to put up a hedge.

- Support: Physical instruments like hoes, machetes, tape measure.
- Resources: Knowledge of construction of figures like rectangles, patterns, sequences
- Task: The family requests you to plant the hedge around their rectangular compound so that it looks beautiful. Explain how you will plant the hedge, making sure that the plants at the corners of the compound are the same in terms of colour.

Chapter 7: BEARINGS

Learning objectives

By the end of this topic,the learners should be able to

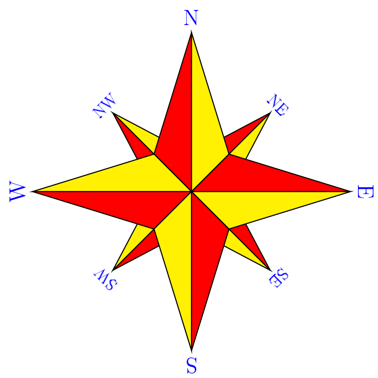
- Know the compass points
- Describe the direction of a place from a given point using compass points
- Describe the bearing of a place from a given point
- Apply bearings in real life situations
- Choose and use an appropriate scale to make an accurate drawing
- Differentiate between a sketch and a scale drawing

There are many situations in which you might need to describe your position and direction of travel. In mathematics, we use more precise ways to describe position and direction of travel and this is done by use of bearings.

Bearings have many applications in our everyday lives such as in the fields of engineering .i.e Builder architects,sailors and surveyors all use direction and angles in their work.Therefore in this topic you will learn how to tell the bearing of a point from agiven point and also determine accurately the distance between two points.

7.1 Compass directions

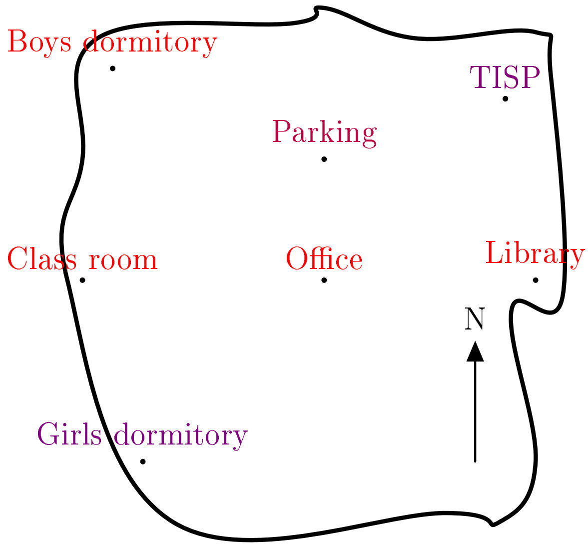
The four cardinal(main) directions are North (N), East (E), South (S), West (W). The four intercardinal (or ordinal) directions are formed by bisecting the angle of the cardinal directions: North-east(NE), South-east(SE), South-west(SW) and North-west(NW)



7.1 Exercise Set

The map below shows part of Taibah international school environment. Use it to answer the questions below.

1. What is East of the Office?



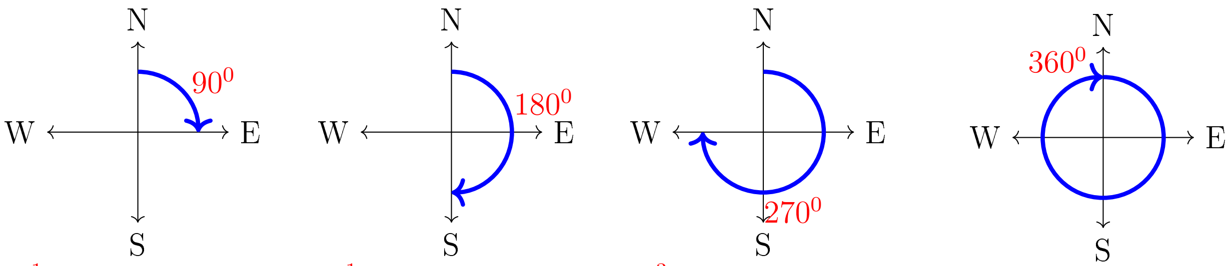
2. What is NW of office ?

3. What is SE of boys dormitory and E of classroom?

4. Draw a compass direction at the office and identify the directions of each of the places shown on the map

7.2 Angles and Turns

An Angle is a measure of rotation or turn.A turn is to rotate about a point.A turn can be described as a quarter turn,Half turn,three -quarter turn or a complete turn.This can either be done clockwise or anticlockwise.Below is how one can turn clockwise



(a):$\frac{1}{4}$ turn (b):$\frac{1}{2}$ turn (c):$\frac{3}{4}$ turn (d):1 full turn

NOTE

- Turning from N to S is 180^0 clockwise or anticlockwise.
- Turning from N to NW is 315^0 clockwise (or 45^0 anticlockwise).
- Turning from NE to E is 45^0clockwise (or 315^0 anticlockwise)
- 1 right angle $= 90^0$

7.3 Identifying the angles in relation to the compass direction

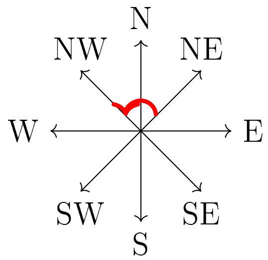
Activity : Make the following turns and in each case state the size of the angle you have turned through.

1. Turn from N to S anticlockwise
2. Turn from NE to SE clockwise
3. Turn clockwise from NE to E
4. Turn anti clockwise from NE to SW
5. Turn clockwise from E to NW
6. Turn from S to NE clockwise

EXAMPLE

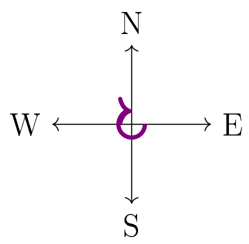
1. What angle do you turn through if you turn:

   (a) from NE to NW anticlockwise?



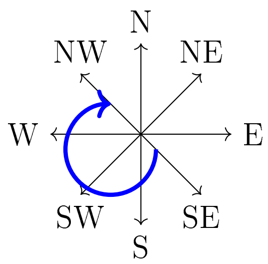
   The angle turned through is 090^0or $\frac{1}{4}$turn

   (b) from E to N clockwise?



   The angle turned through is 270^0or $\frac{3}{4}$turn

   (c) from SE to NW clockwise?



   The angle turned through is 180^0or $\frac{1}{2}$turn

7.2 Exercise Set

1. What angle do you turn through if you turn clockwise from:
   (a) N to E?
   (b) W to NW?
   (c) SE to NW?
   (d) NE to N?
   (e) W to NE?
   (f) S to SW?
   (g) S to SE?
   (h) SE to SW?
   (i) E to SW?

2. In what direction will you be facing if you turn:
   (a) 180^0 clockwise from NE?
   (b) 90^0 clockwise from SW?
   (c) 45^0 clockwise from N?
   (d) 225^0 clockwise from SW?
   (e) 135^0 anticlockwise from N?
   (f) 315^0 clockwise from SW?

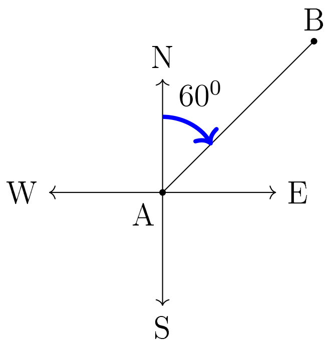
3. The sails of a windmill complete one full turn every 40 seconds.
   (a) How long does it take the sails to turn through:
      i. 180^0
      ii. 90^0
      iii. 45^0
      iv. 270^0
   (b) What angle do the sails turn through in:
      i. 30 seconds?
      ii. 15seconds?
      iii. 25 seconds?

7.4 Bearings

- Bearings are a more precise way of descibing a direction.i.e They show the direction of one-point relative to another point.
- A bearing is an angle measured clockwise from the north line
- In bearings angles are always measured from the North
- Bearings are stated using three digits. Thus $45^0$is written as 045^0
- The north line represents a bearing of 000^0
- The bearing of N130^0E means an angle of 130^0 measured from N towards E

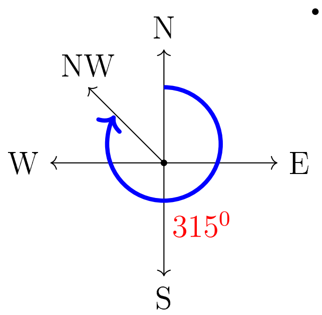
EXAMPLE

1. What is the bearing of B from A



The bearing of B from A is 060^0

2. What is the bearing of NW .



The bearing of NW is 315^0 in the clockwise direction

Activity : Estimating bearings of some places within the school compound.
Draw a sketch of your school and estimate the bearings of each building found in the School and the sports grounds.i.e offices,classrooms,labaratory,kitchen,library e.t.c
NOTE

- Your compass direction must be drawn on the administration block
- All bearings must be stated using three digits.
- All bearings are measured in a horizontal plane.

7.3 Exercise Set

1. Find the bearing of each of the following directions:

   (a) NE (b) SE (c) S (d) SW (e) E (f) W

2. Find the bearing of each of the following directions:

   (a) $N60^0E$ (b) $S60^0E$ (c) $N90^0W$ (d) $S50^0W$ (e) $N30^0W$ (f) $N45^0E$

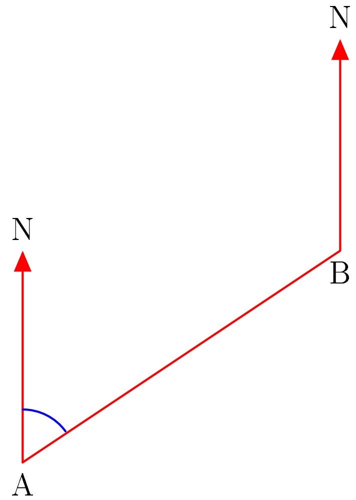
3. Draw a scale diagram to show the position of a ship which is 270 km away from a port on a bearing of 110^0.

4. The bearing of Buikwe from Lugazi in a class is 120^0.What is the bearing of Lugazi from Buikwe in the class

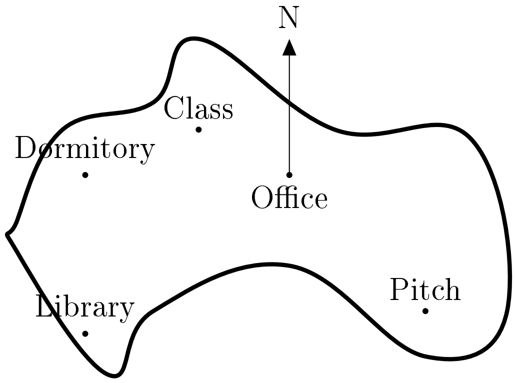
5. A plane leaves Entebbe airport on a bearing of North East to Karamoja.What is the bearing of Entebbe from Karamoja.

6. The diagram shows the positions of two ships, A and B.

   (a) What is the bearing of ship A from ship B ?
   (b) What is the bearing of ship B from ship A ?



7. The map of a school is shown below:What is the bearing from the Office, of each place shown on the map?



8. An aeroplane flies from Entebbe to mbale on a bearing of 044^0 . On what bearing should the pilot fly, to return to Entebbe from Mbale?

9. On four separate occasions, a plane leaves Entebbe airport to fly to a different destination. The bearings of these destinations from Entebbe airport are given below.

Destination	Bearings
Rwanda	205^0
Kenya	072^0
Tanzania	166^0
Nigeria	312^0

   Draw a compass direction to show the direction in which the plane flies to each destination.

10. A ship sails NW from Entebbe to take supplies to Portbell. On what bearing must it sail to return from the Portbell to the Entebbe?

11. If A is north of B, C is south east of B and on a bearing of 160^0 from A, find the bearing of:

    (a) A from B, (b) A from C, (c) C from B, (d) B from C.

7.5 Scale Drawings

Using bearings, scale drawings can be constructed to solve problems.This involves drawing accurate drawings and showing clearly the directions.
EXAMPLES

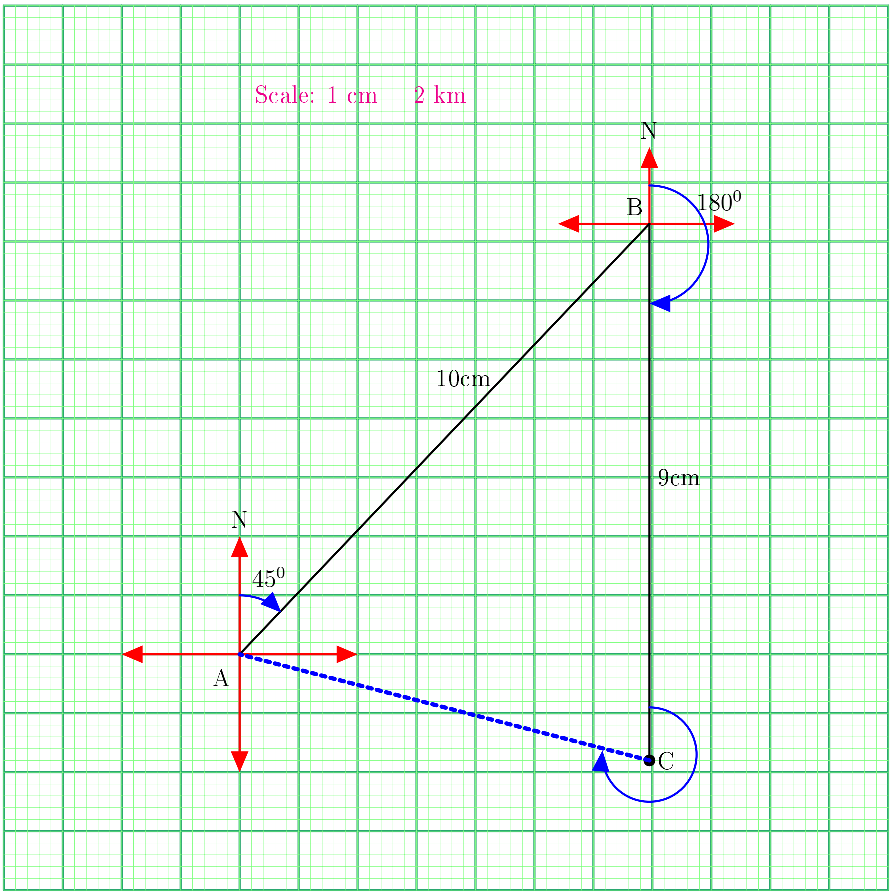
1. A ship sails 20 km North east, then 18 km south, and then stops.

   (a) Draw a scale drawing to show the routes of the ship

   (b) How far is it from its starting point when it stops?

   (c) On what bearing must it sail to return to its starting point?

   The path of the ship can be drawn using a scale of 1 cm for every 2 km, as shown in the diagram.



   (a) The distance CA can be measured on the diagram as 7.2cm which represents an actual distance of 14.4km.

   (b) The bearing of O from B can be measured as 285^0 .

7.4 Exercise Set

1. A girl walks $80m$ north and then $200m$ east.
   (a) How far is she from her starting position?
   (b) On what bearing should she walk to get back to her starting position?
2. Frank walks $300m$ NW and then walks $500m$ south and then stops.
   (a) How far is he from his starting position when he stops?
   (b) On what bearing could he have walked to go directly from his starting position to where he stopped?
3. Village A and B are such that the bearing of B from A is 060^0 . The distance between A and B is 15 km.
   (a) Represent the above information on a scale drawing.
   (b) Calculate the bearing of A from B.
4. A hot air balloon is blown 5 km NW. The wind then changes direction and the balloon is blown a further 6 km on a bearing of 300^0 before landing. How far is the balloon from its starting point when it lands?
5. A boat sets off from a point A on a bearing of 130^0 for 4 km to a point B. At B it changes direction and sails on a bearing of 240^0 to a point C, 7 km away. At point C it changes direction again and heads back to point A
   (a) Using a scale of 1 cm : 1 km, draw a scale diagram of the boat's journey
   (b) From your diagram work out:
      (i) the distance AC
      (ii) the bearing of A from C.
6. A plane flies from airport P due North for 300km to airport R. It then flies on a bearing of 295^0 for 200km to air strip Q. From there it flies on a bearing of 090^0 for 500km to air strip R
   (a) Use a scale of 1cm to represent 50km, draw an accurate diagram to show the route of the plane.
   (b) Find the distance between P and R.
7. An aeroplane flies 400 km on a bearing of 055^0 It then flies on a bearing of 300^0 , until it is due north of its starting position. How far is the aeroplane from its starting position?
8. Kaziba walks $750m$ on a bearing of 030^0 . He then walks on a bearing of 315^0 until he is due north of his starting point, and stops.
   (a) Using a scale of 1 cm to represent 100m,draw an accurate diagram to show Kaziba 's routes.
   (b) How far does he walk on the bearing of 315^0?

(c) How far is he from his starting point when he stops?

9. An aeroplane flies 200 km on a bearing of 335^0 . It then flies 100 km on a bearing of 170^0 and 400 km on 280^0 , and then lands.

   (a) How far is the aeroplane from its starting point when it lands?

   (b) On what bearing could it have flown to complete its journey directly?

Situation of Integration

Priscilla is in Kampala City and has been told to use a car to move to Lira town. She has never gone to Lira. She has been given the map of Uganda showing routes through which she can access Lira town.

- Support: Mathematical instruments, pencil, paper, pens, tracing paper and map of Uganda.

- Resources:Knowledge of construction of figures like triangles, lengths of sides of triangles, operations on numbers

- Task: Priscilla wants to use the short distance from Kampala to Lira. Explain how Priscilla can determine the shortest distance. Using the map given to her is it possible for Priscilla to use the shortest distance she has determined. Explain your answer.

Chapter 8: GENERAL AND ANGLE PROPERTIES OF GEOMETRIC FIGURES

Learning objectives

By the end of this topic,the learners should be able to

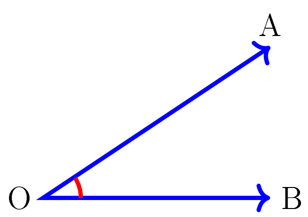
- Classify angles
- Identify different angles
- Solve problems involving angles at a point ,on a straight line,angles on a transversal and parallel lines
- Know and use the angle sum of a triangle
- State and use angle properties of polygons when solving problems

Introduction

In this topic you will study angles on the straight line, parallel lines and angle properties of polygons. Equipped with the knowledge from this topic, you will be able to solve problems related with angle properties.

8.1 Classifying angles

- An angle is the space (usually measured in degrees) between two intersecting lines or surfaces at or close to the point where they meet.In geometry, angles are measured in degrees using a protractor

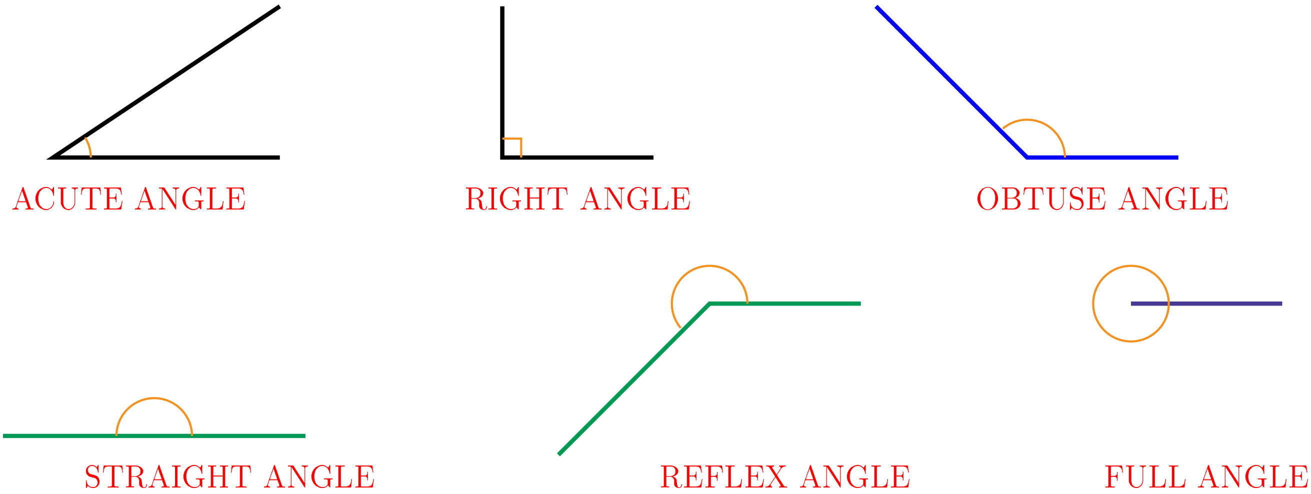


  - The above angle is described as $\angle AOB$ or $\angle BOA$ or $\angle O$

  Types of angles

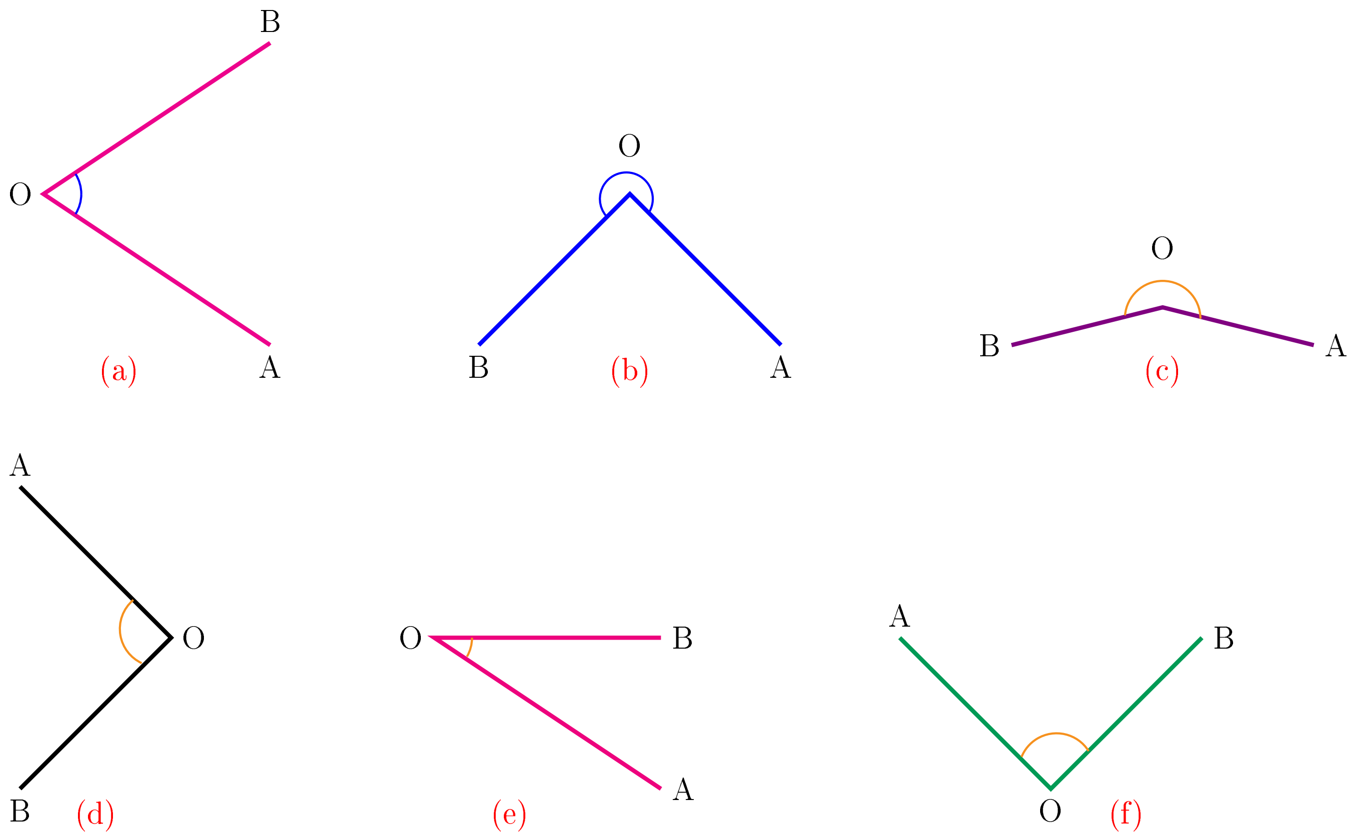
- Acute angle:Angle less than 90^0 e.g 30^0
- Right angle:An angle that is exactly 90^0
- Obtuse angle:An angle that measures between 90^0 and 180^0

- Straight angle angle that is exactly 180^0
- Reflex angle:An angle that is measures between 18^0 and 360^0
- Full angle:An angle that is equal to 360^0.



8.1 Exercise Set

State whether each of the following angles is acute, obtuse or reflex



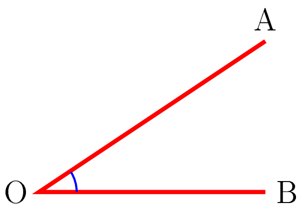
8.2 Identify Different Angles

Activity : Identifying objects that form angles.
Identify objects in your class, which make $90^0, 180^0, 360^0$
Activity : Identifying angles.
Draw two intersecting lines. Use your mathematical instruments to measure the angles formed at the intersecting point.

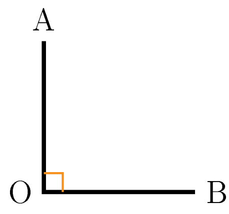
- How many angles have been formed at the point of intersection?
- What is the size of each angle formed?

8.2 Exercise Set

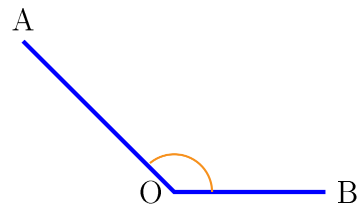
1. For each of the following angles, first estimate the angles and then measure the angle marked 0 to see how good your estimate was.



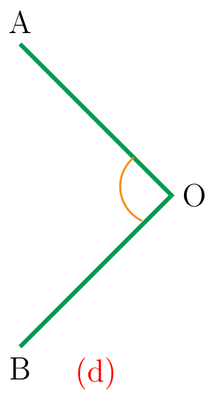
(a)



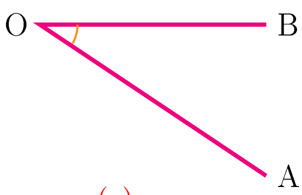
(b)



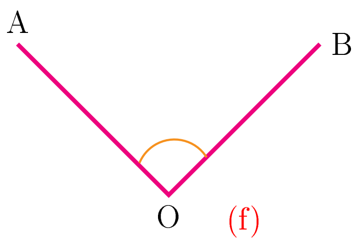
(c)



(d)



(e)



(f)

2. Draw the following angles

   (a) 20^0
   (b) 43^0
   (c) 97^0
   (d) 170^0
   (e) 200^0
   (f) 305^0
   (g) 80^0
   (h) 110^0
   (i) 330^0

3. Draw a triangle with one obtuse angle.
4. Draw a triangle with no obtuse angles.
5. Draw a four-sided shape with:

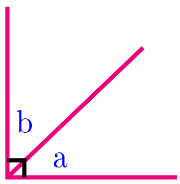
   (a) one reflex angle.

(b) two obtuse angles.

8.3 Angle Relationships

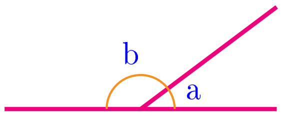
In this section we look at angle relationships and their measures.In Geometry, there are five fundamental angle pair relationships:

1. Complementary Angles:These are two positive angles whose sum is 90 degrees.



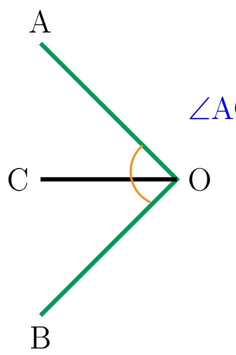
$a+b=90^0$

2. Supplementary Angles:These are two positive angles whose sum is 180 degrees.



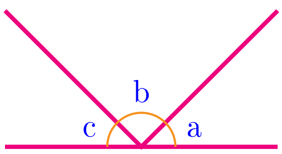
$a+b=180^0$

3. Adjacent Angles:These are two angles in a plane that have a common vertex and a common side but no common interior points.



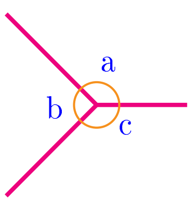
$\angle$AOC and $\angle$BOC are adjacent angles

4. Angles on a line add up to 180^0.This is because there are 180^0 in a half turn



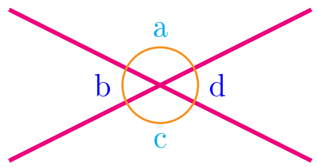
$a+b+c=180^0$

5. Angles around a point add up to 360^0.This is because there are 360^0 in a full turn



$a+b+c=360^0$

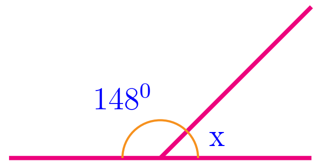
6. Vertical Angles(Vertically opposite angles):These are two nonadjacent angles formed by two intersecting lines or opposite rays.



Vertically opposite angles are equal.Thus$\angle a = \angle c$ and $\angle b = \angle d$

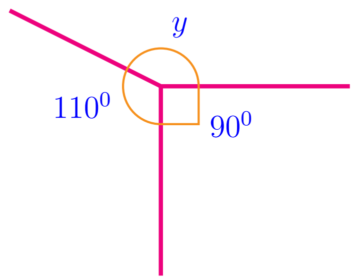
EXAMPLES

1. In the figure below, find the value of x



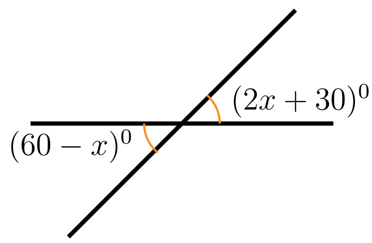
$$
\begin{aligned}
x + 148 &= 180 \\
x + 148 - 148 &= 180 - 148 \\
x &= 32^0
\end{aligned}
$$

2. In the figure below ,find the value of y



$$
\begin{aligned}
y + 110 + 90 &= 360 \\
y + 200 &= 360 \\
y + 200 - 200 &= 360 - 200 \\
y &= 160^0
\end{aligned}
$$

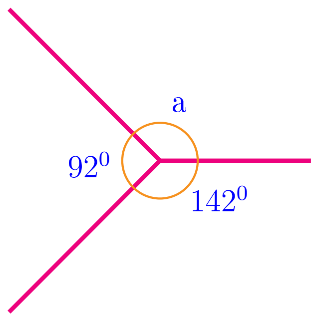
3. The figure below shows two intersecting lines.Find the value of x



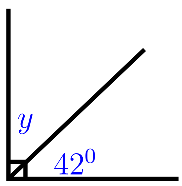
$$
\begin{aligned}
2x + 30 &= 60 - x && \text{Vertically opposite angles are equal} \\
2x + x + 30 &= 60 - x + x \\
3x + 30 &= 60 \\
3x + 30 - 30 &= 60 - 30 \\
3x &= 30 \\
\frac{3x}{3} &= \frac{30}{3} \\
\frac{\cancel{3}x}{\cancel{3}} &= \frac{\cancelto{10}{30}}{\cancelto{1}{3}} \\
x &= 10^0
\end{aligned}
$$

8.3 Exercise Set

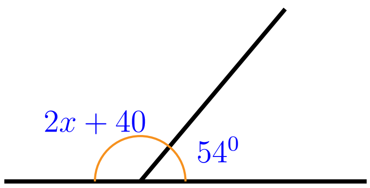
1. Two angles are supplementary. One angle measures 12^0 more than the other. Find the measures of the angles.
2. Find the value of a



3. Find the size of two complementary angles that are such that the size of one of them is four times the size of the other.
4. Find the value of y



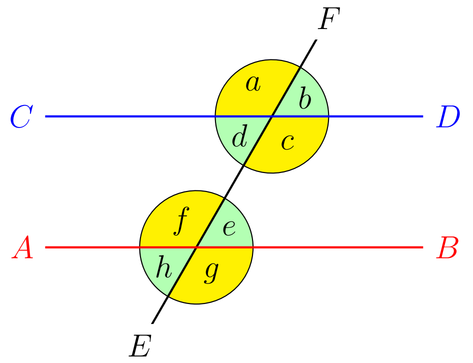
5. Find the size of the angle marked x



6. Find the value of x for which the angles $(2x + 10)^0$ and $(130 - x)^0$ are vertically opposite.

8.4 Parallel and Intersecting Lines

When a line intersects (or crosses) a pair of parallel lines, there are some simple rules that can be used to calculate unknown angles.

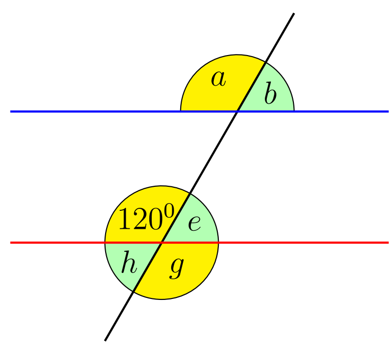


- Line AB and CD are parallel.
- EF is called a transversal line

- The line cutting across parallel lines is called a transversal.
- $\angle b = \angle e$,$\angle d = \angle h$,$\angle a = \angle f$,$\angle c = \angle g$.These are called Corresponding angles.
- $\angle d = \angle e$ and $\angle f = \angle c$.These are called Alternating angles .
- $\angle f = \angle g$,$\angle e = \angle h$,$\angle a = \angle c$,$\angle d = \angle b$.These are called Vertically opposite angles .
- $h + f = 180^0$:because adjacent angles on a straight line add up to 180^0 . These are called supplementary angles.
- Co-interior angles lie inside the parallel lines on the same side of the transversal.e.g $\angle f$ and $\angle d$ are co-interior angles
- Co-interior angles add up to 180^0 ,thus $f + d = 180^0$

EXAMPLES

1. Find the size of the unknown angles in the figure below.



To find h

$$h + 120 = 180 \quad \text{Angles on a straight line}$$
$$h + 120 - 120 = 180 - 120$$
$$h = 60^0$$

To find e

$$e = h \quad \text{Vertically opposite angles are equal}$$
$$e = 60^0$$

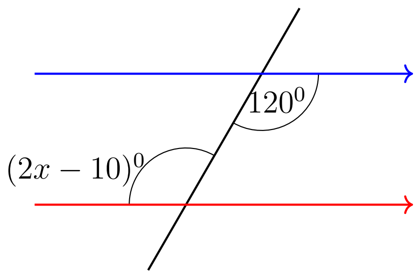
To find a

$$a = f \quad \text{Corresponding angles are equal}$$
$$a = 120^0$$

To find b

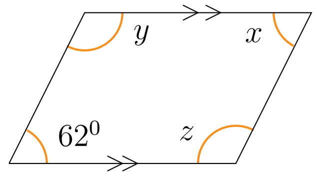
$$b = e \quad \text{Corresponding angles are equal}$$
$$b = 60^0$$

2. Find the size of the unknown angles in the figure below.



$$2x - 10 = 120 \quad \text{Alternating angles are equal}$$
$$2x - 10 + 10 = 120 + 10$$
$$2x = 130$$
$$\frac{2x}{2} = \frac{130}{2}$$
$$\frac{\cancel{2}x}{\cancel{2}} = \frac{\cancel{130}^{65}}{\cancel{2}_{1}}$$
$$x = 65^0$$

3. Find the size of the unknown angles in the parallelogram shown in this diagram:



To find z

$$z + 62 = 180^0 \qquad \text{supplementary angles}$$
$$z + 62 - 62 = 180 - 62$$
$$z = 118^0$$

To find x

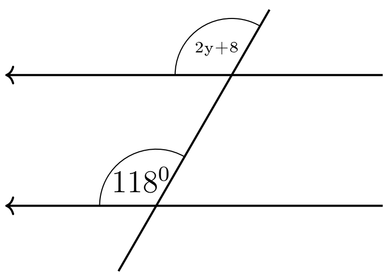
$$x + 62 = 180^0 \qquad \text{supplementary angles}$$
$$x + 62 - 62 = 180 - 62$$
$$x = 118^0$$

To find y

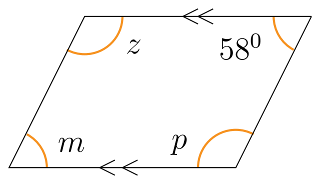
$$y + 118 = 180^0 \qquad \text{supplementary angles}$$
$$x + 118 - 118 = 180 - 118$$
$$x = 62^0$$

8.4 Exercise Set

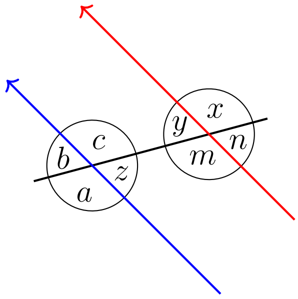
1. Find the value of y



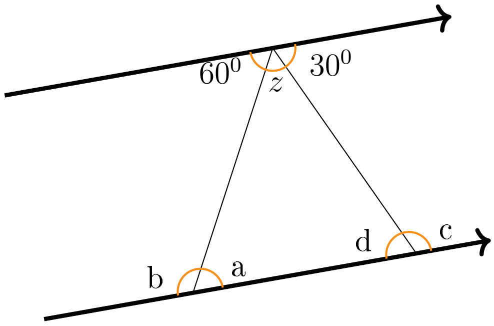
2. Find the size of the three unknown angles in the parallelogram .



3. One angle in a parallelogram measures 40^0 . What is the size of each of the other three angles?
4. Which angles in the diagram are the same size as $\angle a$ and $\angle b$



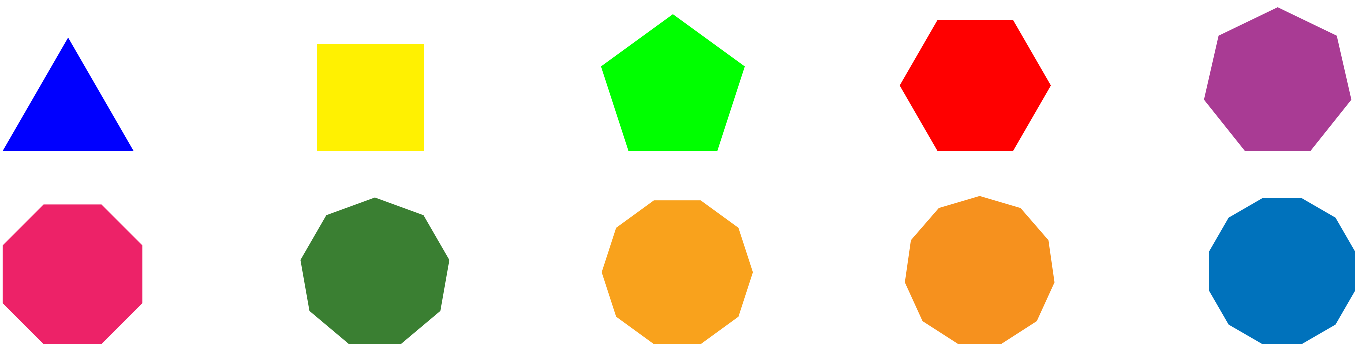
5. Find the sizes of the unknown angles marked with letters in the diagram:



6. One angle in a rhombus measures 133^0 . What is the size of each of the other three angles?
7. One angle in a parallelogram measures 60^0 . What is the size of each of the other three angles?

Activity : Identifying the polygons

- Find the number of sides of different polygons and their corresponding names.
- Determine the size of each interior and exterior angles of the regular polygons.
- Determine the sum of the angles in the regular polygons



NOTE

1. In any polygon with n- sides, the following properties apply
   - Interior angle sum $=(n-2)180^0$

- Exterior angle sum $= 360^0$.
- Each interior angle + each exterior angle $=180^0$.
- Number of diagonals $=\frac{n(n-3)}{2}$

2. In a regular polygon with n- sides, the following properties apply
   - One interior angle$=\frac{(n-2)180^0}{n}$
   - One exterior angle $=\frac{360^0}{n}$

EXAMPLES

1. Find the interior angle sum of a pentagon.

A pentagon has 5 sides.

$$\begin{aligned}\text{Interior angle sum} &= (n-2)180^0\\ &= (5-2)180^0\\ &= 3\times 180^0\\ &= 540^0\end{aligned}$$

2. Find the size of the interior angle of a regular pentagon.

A pentagon has 5 sides.

$$\begin{aligned}\text{One interior angle} &= \frac{(n-2)180^0}{n}\\ &= \frac{(5-2)180^0}{5}\\ &= \frac{3\times 180^0}{5}\\ &= \frac{540}{5}\\ &= 108^0\end{aligned}$$

3. Find the size of each exterior angle of a regular pentagon.

A pentagon has 5 sides.

$$\begin{aligned}\text{One exterior angle} &= \frac{360^0}{n}\\ &= \frac{360}{5}\\ &= 72^0\end{aligned}$$

4. Find the number of sides of a polygon whose interior angle sum is 1260^0

$$\begin{aligned}
\text{Interior angle sum} &= (n-2)180^0 \\
1260^0 &= (n-2)180^0 \\
1260 &= n \times 180 - 2 \times 180 \\
1260 &= 180n - 360 \\
1260 + 360 &= 180n - 360 + 360 \\
180n &= 1620 \\
\frac{180n}{180} &= \frac{1620}{180} \\
\frac{\cancel{180}n}{\cancel{180}} &= \frac{\cancel{1620}^{\,9}}{\cancel{180}^{\,1}} \\
n &= 9
\end{aligned}$$

8.5 Exercise Set

1. Find the sum of the interior angles of a polygon with 22 sides.
2. The interior angle of a regular polygon is 162^0 . How many sides does the polygon have ?
3. The interior angle sum of a regular polygon is 1800^0 . How many sides has the polygon?.Name the polygon
4. Find the interior angle sum of a decagon.
5. Find the size of each interior angle of a regular hexagon.
6. Find the number of sides of a regular polygon whose each interior angle is 135^0
7. Find the size of each exterior angle of a regular octagon.
8. If the vertices of a regular hexagon are joined to the centre of the hexagon, what is the size of each of the six angles at the centre? Use your answer to construct a regular hexagon ABCDEF of side 3cm. Start with a circle of radius 3cm. Measure the length of the diagonal AC.

Activity of Integration

The table below shows the Covid 19 active cases discovered in some districts of uganda in a year 2020.

District	Buikwe	Elegu	Busia	Mutukula	Malaba
Active cases	24	350	150	120	56

- Support: Mathematical instruments, pencil, paper, pens,set
- Resources:Knowledge of construction of figures like circle and the knowledge of measuring angles
- Task: The ministry of health was asked to represent the information above on pie chart. As a senior one learner help the ministry to solve the challenge.

Chapter 9: DATA COLLECTION AND PRESENTATION

Learning objectives
By the end of this topic,the learners should be able to

- Understand the differences between types of data
- Collect simple data from the local environment using tally chart
- Represent data using bar chart,pie chart and line graphs
- Interpret represented data

In this topic, you will learn different types of data, data collection methods, presentation and analysis.

9.1 Types of Data

The term data refers to groups of information that represent the qualitative or quantitative attributes of a variable or set of variables.There are two main types of data:

- Qualitative data
- Quantitative data
- Qualitative data is data that is not given numerically and is used to characterize objects or observations.
  - Qualitative observations relate to qualities and involve descriptions of how something looks, feels, smells, taste, texture ,colour. For example, "the car is yellow", "the fumes are pungent", the "leaf is smooth".
  - Qualitative observations are often subjective. That is, they can be interpreted differently by different people. For example, you might describe the colour of a stone or the sound of a bird quite differently to someone else.
  - Other examples include favourite colour, place of birth, favourite food, type of car.
- Quantitative data is numerical and can be counted, quantified,measured and mathematically analyzed.
  - They can be described with numerical values and units of measurement, and include things like mass, temperature or speed.For example, the "dog weighs 16.5 kg", "the air temperature is "16^0C", "the train is travelling at $80ms^{-1}$".
  - Quantitative observations are objective. That is, different people should make the same observation. For example, if you measured the height of the classroom door, you should get the same answer (or a very similar answer) as someone else in your class.

There are two types of quantitative data:

- Discrete data can only take specific numeric values e. g. shoe size, number of brothers, number of cars in a car park.
- Continuous data can take any numerical value e.g. height, mass, length,temperature

EXAMPLE
Identify which of the following terms best describes each of the information by coloring.Give reason for your response

Qualitative data   Continuous Quantitative Data   Discrete Quantitative Data

Age	Temperature	Height	Mass
School life	Birth place	Ten cars	Number of cars

SOLUTION

Age	Temperature	Height	Shoe size
School life	Birth place	Number of cars	Mass

- Age is discrete quantitative, becauses it is given as a whole number.
- Temperature is Continuous quantitative ,because it can take any value and is measureable with units

9.1 Exercise Set

1. State whether each of the following variables is qualitative, discrete or continuous.
   (a) the number of goals scored in Premier league soccer matches on a Saturday
   (b) the colour of children's eyes in a class
   (c) the circumference of apples collected from a tree
   (d) the type of vehicle seen on a road at rush hour.
   (e) Tr stephen's phone number
2. Which of the following would give
   - qualitative data
   - discrète quantitative data
   - continuous quantitative data

(a) Favourite football team
(b) Colour of car
(c) Price of chocolate bars
(d) Amount of pocket money
(e) Distance from home to school
(f) Number of sweets in a jar
(g) Number of pets
(h) Mass of crisps in a packet.
(i) Number of cars
(j) Weight

3. Mr Kaziba starts to make a database for his lesson.

Name	Age	Primary school	Transport to school	Height	Reading glasses
Daniel	12	Taibah	Bus	145cm	no
Sheba	11	st stephen	Car	152cm	no
Priscilla	12	st john	Bike	158cm	no
Philemon	11	Hilltop		164cm	yes
Edgar	11	Taibah	walk	155cm	no
Mimmi	12	Taibah	car	168cm	no
Cooper		Green hill	Bus	166cm	no

(a) What is missing from Mr Kaziba's database?
(b) Which columns in the database contain quantitative data?
(c) Which columns in the database contain qualitative data?
(d) Write down what Mr kaziba would put in his database if you joined his class.

4. The table below shows a database that has no entries.

Name	Age	Favourite color	Favourite subject	Favourite Tv show	Favourite sports

(a) Collect data from 10 of your classmates to complete the data base. State whether each column contains:
- qualitative data.
- continuous quantitative data.
- or discrete quantitative data.

(b) Answer the following questions:
(i) What is the most popular TV show?
(ii) Who is the oldest?
(iii) What is the favourite sports for the youngest person?
(iv) What is the favourite subject for the oldest person?

(v) What is the favourite color for the youngest person?

(c) Write 3 more questions you could answer using your database and write the answers to them.

9.2 Collecting Data

In this section, you will learn how data is collected, organized and interpreted, using a tally chart and then displayed using:

- Pictograms
- Bar charts
- Line graph
- Piecharts

Activity :
Identify the means of transport each learner uses to come to school.

- Identify how many of you in class use the same means of transport.
- Which means of transport is used by the majority?
- Which one is the least used means of transport?

EXAMPLE

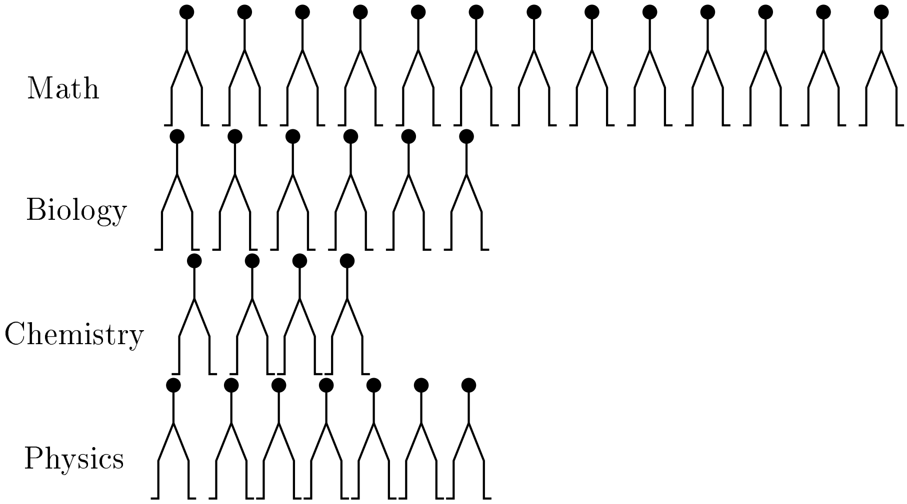
1. The learners in a class were asked there Favourite science subjects.

Subject	Tally	Frequency
Math	$\cancel{////}$ $\cancel{////}$ ///	13
Biology	$\cancel{////}$ /	6
Chemistry	////	4
Physics	$\cancel{////}$//	7
		$\sum f = 30$

Illustrate this data using:

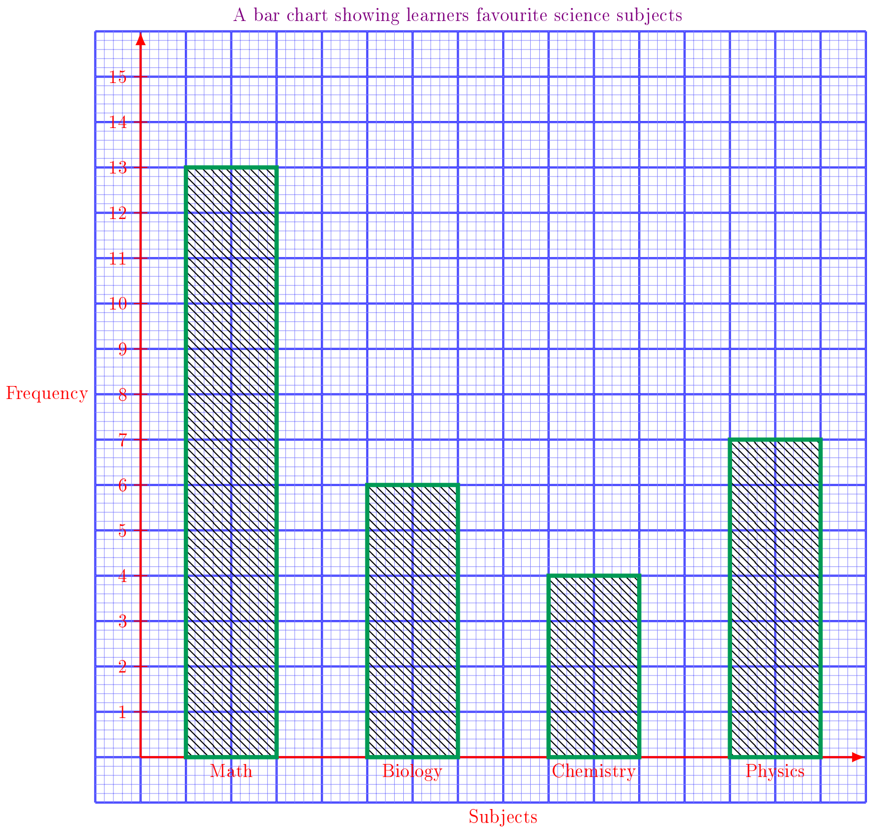
- A pictogram
- Bar chart(graph)
- Line graph
- Pie chart

(a) A pictogram
A pictogram is a simple way to organise data in which each object is represented by a picture of itself.

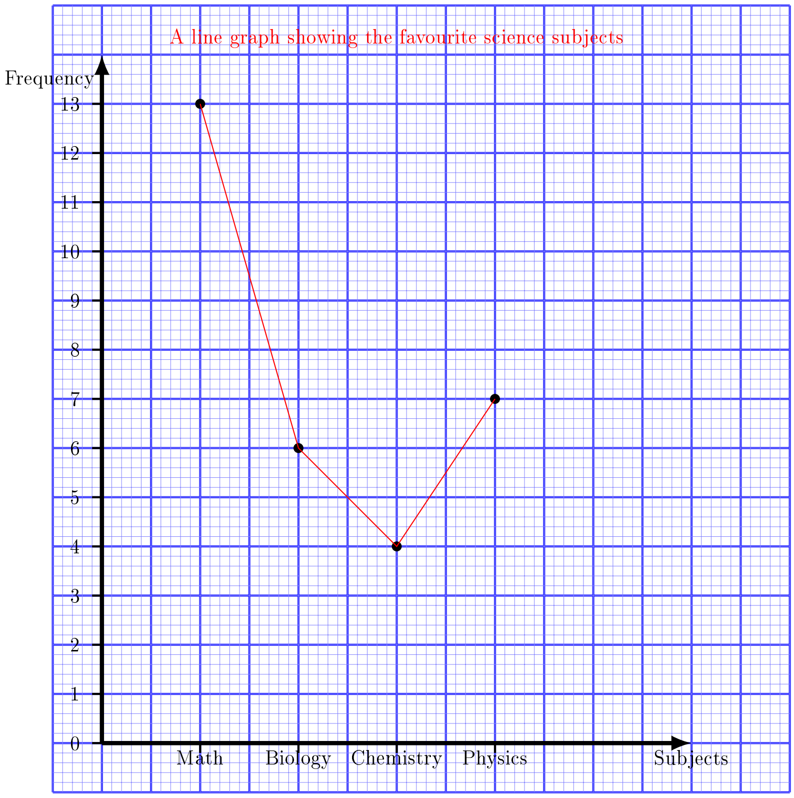


NOTE:You can have a scale when representing a pictogram

(b) A bar chart
A bar chart or bar graph is a chart or graph that presents categorical data with rectangular bars with heights or lengths proportional to the values that they represent.



(c) Line graph A line graph displays data that changes continuously over periods of time.

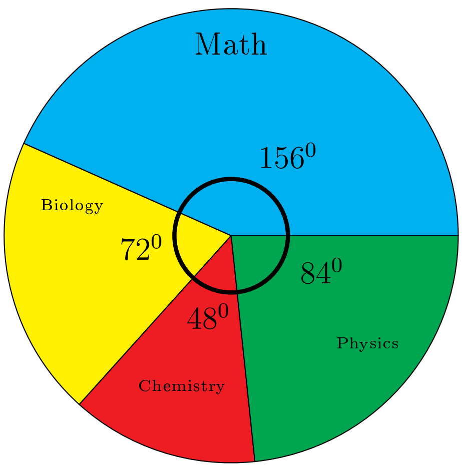


(d) A pie chart

A pie chart is a circular statistical graphic, which is divided into slices to illustrate numerical proportion. In a pie chart, the arc length of each slice, is proportional to the quantity it represents.

To illustrate the data with a pie chart, You need to first find out the angle for each learner.

Subject	Frequency	Calculation	Angle
Math	13	$\frac{13}{30} \times 360$	156^0
Biology	6	$\frac{6}{30} \times 360$	72^0
Chemistry	4	$\frac{4}{30} \times 360$	84^0
Physics	7	$\frac{7}{30} \times 360$	84^0
	$\sum f = 30$		Total$= 360^0$



From the data we can see that:

- The Favourite subject for the learners is Mathematics. (This is called the mode.)
- The least liked subject is chemistry.

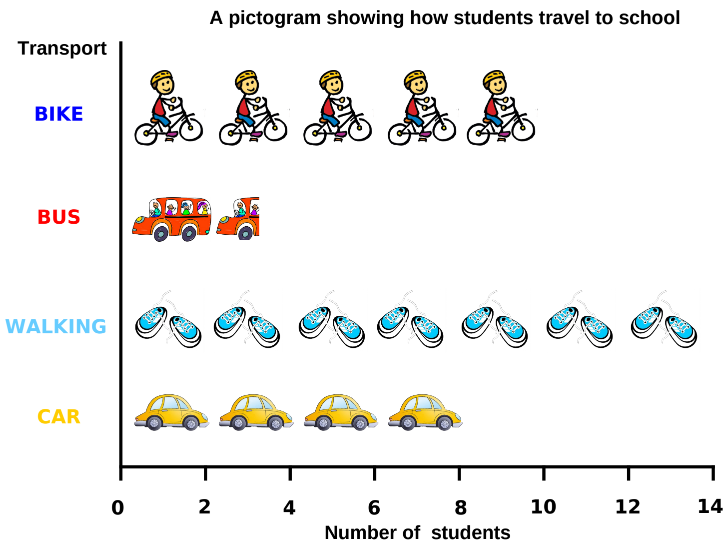
2. Students at Masaka sss were asked how they travel to school.The findings were summarized in the tally chart below.

Method of travel	Tally	Frequency
Bike	~~////~~ ~~////~~	10
Bus	///	3
Walking	~~////~~ ~~////~~ ////	14
Car	~~////~~///	8
		$\sum f = 35$

Illustrate this data using:

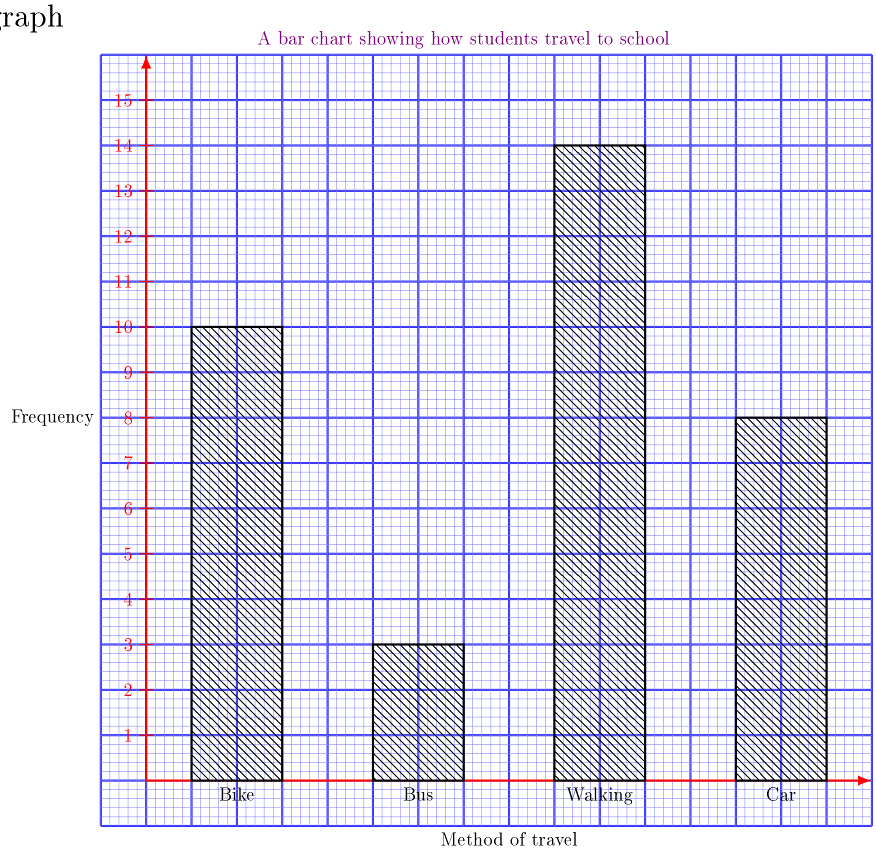
(a) A pictogram (b) Bar chart(graph) (c) Pie chart

(a) Pictogram



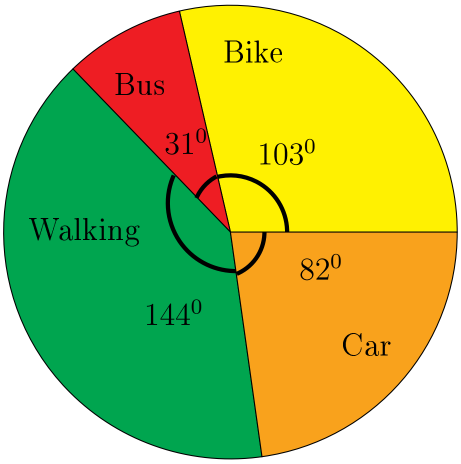
NOTE:Each picture represents 2 students, and half represents 1 student.

(b) Bar graph



(c) Pie chart

Method of travel	Frequency	Calculation	Angle
Bike	10	$\frac{10}{35} \times 360$	103^0
Bus	3	$\frac{3}{35} \times 360$	31^0
Walking	14	$\frac{14}{35} \times 360$	144^0
Car	8	$\frac{8}{35} \times 360$	82^0
	$\sum f = 35$		Total$= 360^0$



From the data we can see that:

- The most common way of getting to school is by Walking. (This is called the mode.)
- The least popular way of getting to school is by bus

9.3 Hypothesis

A hypothesis is a prediction based on an observation.OR an hypothesis is an idea that you want to investigate to see if it is true or false. For example, you might think that most people in your school get there by bus. You could investigate this using a survey. A tally chart can be used to record your data..During Interpreting Results and Making Conclusions, an hypothesis can either be accepted or rejected basing on the observations made.
Activity : Testing an hypothesis
"More students in my class love mathematics than any other subject."

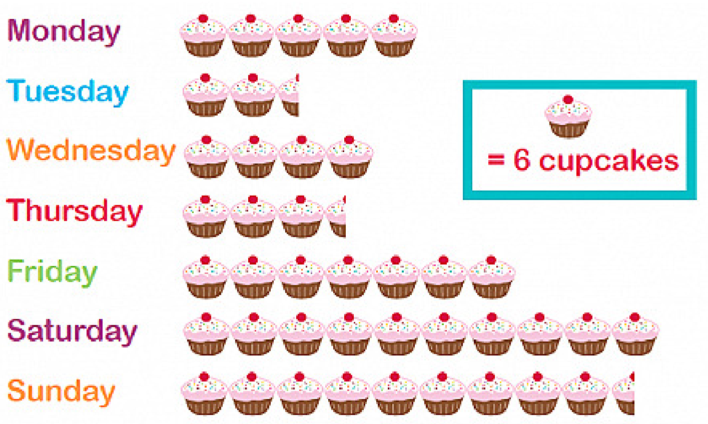
- Collect data to test this hypothesis.
- Present your data on a tally chart.
- Represent your findings using a pie chart
- Was the original hypothesis correct?

HINT:In the survey , make a survey form and ask your classmates their opnions.A table can have two columns as below

Student Name	Favourite subject

9.2 Exercise Set

1. The pictogram shows the number of customers who take cup cakes in a week.



   (a) How many customers took cupcakes on thursday
   (b) How many customers took cupcakes on tuesday
   (c) How many more customers took cupcakes on monday than friday
   (d) How many customers were included in the survey
   (e) Represent the information above on a bar chart
   (f) What is the mode?

2. The students of senior one at Taibah international school were asked to give the clubs they support in the english premier league.The results are given in the tally chart below:

Club	Tally	Frequency
Liverpool Fc	$\cancel{////}$ $\cancel{////}$ $\cancel{////}$ ///	
Man city Fc	///	
Arsenal Fc	$\cancel{////}$ $\cancel{////}$ /	
Chelsea Fc	///	
Marchester united Fc	$\cancel{////}$	
		$\sum f =$

   (a) Copy and complete the table by filling in the frequencies.

   (b) Represent the data on a bar chart.

   (c) Draw a pictogram for this data.

   (d) Which club has the highest number of supporters.

   (e) Copy and complete the following table and draw a pie chart

Club	Frequency	Calculation	Angle
Liverpool Fc			
Man city Fc			
Chelsea Fc	3	$\frac{3}{40} \times 360$	27^0
Arsenal Fc			
Marchester united Fc			
	$\sum f =$		Total $= 360^0$

3. On a particular day 40 new active cases for covid 19 were discovered and the Patients were admitted to different hospitals as shown below.

Hospital	Mulago	Entebbe	Mbale	Masaka
Number of patients	15	9	4	12

   (a) Which hospital admitted the highest number of patients

   (b) What is the mode

   (c) Represent the above information on a bar chart and on a pie chart

4. The total number of goals scored in each of the Premier League matches one Saturday were:

$$0 \quad 1 \quad 4 \quad 0 \quad 4 \quad 5 \quad 4 \quad 2 \quad 3$$

   (a) Illustrate these data on a pie chart.

   (b) Which number of goals was the most common?

5. A school conducted a survey to know the favourite sports of the students. The table below shows the results of this survey.

Sport	Number of students
Swimming	30
Basket ball	16
Foot ball	24
Chess	12
Table Tenis	8
Bad minton	10

   (a) Draw a bar graph representing the sports and the total number of students.

   (b) Calculate the range of the graph.

   (c) Which sport is the most preferred one?

   (d) What is the mode?

6. Do you think Novida and Fanta will be the most popular soft drinks in your class?

   (a) Carry out a favourite soft drink survey for your class. Present the results in a bar chart and state which flavour is the mode.

   (b) Was your hypothesis correct?

7. A survey was carried out at the bank of uganda. State whether each of the following variables is discrete or continuous.

   (a) The number of people entering the bank per hour.

   (b) The time it takes to serve each person by the cashier.

   (c) The number of people creating bank accounts.

   (d) The total amount paid by each customer.

8. Complete the table by naming the type of data formed by each of the stated measurements. The first one has been completed for you.

Measurement	Type of data
Height of Rose Trees	Continuous
Number of brothers	
Length of shoe laces	
Number of pages in library books	
Temperature of a patient	

9. "Most children in my class are from western uganda."

   (a) Collect data to test this hypothesis.

   (b) Present your data in a suitable diagram.

   (c) Was the original hypothesis correct?

10. In term one students complained about the high rate of theft of the students properties. The security guard then ran a high profile campaign encouraging students to improve their personal security.
    (a) State a hypothesis that should be investigated to test the effectiveness of their campaign.
    (b) Collect suitable data from your class.
    (c) Present your data using a suitable diagram.
    (d) Was the hypothesis correct?
11. Alexadra finds out the favourite sports for members of her class. She works out the angles in the list shown below for a pie chart. Draw the pie chart.

SPORT	ANGLE
Foot ball	130^0
Table Tennis	10^0
Net ball	70^0
Badminton	17^0
chess	32^0
Swimming	80^0
Hockey	21^0

12. The pupils in Mr Stephen's class take a maths test and get scores out of 10, which are listed below:

3	7	6	2	5	9	10	8	7	1	8	4	3	5	6
7	8	7	6	5	3	6	9	8	7	5	9	6	7	8

    (a) Construct a tally chart for the data
    (b) Represent this data using a pie chart and a line graph

Activity of Integration

The Games Master at your school wants to buy football boots for the three teams in the school. The three teams are the under 18 years, under 16 years and the under 14 years. The Games Master does not know the foot size for each of the players.

- Support: pens, paper, tape measure, team members
- Resources:Knowledge of tabulation, of tallying, of approximation, of central measures and of collection of suitable data.
- Task: The total number of players for the three teams is 54. The Games Master wants to know the size of the boots for each player and the number of pairs for each size.
  Explain how the Games Master will get the required data and how to determine the total cost for buying the football boots for the 54 players. Is there another way of getting the required data other than what you have explained above?

Chapter 10: REFLECTION

Learning objectives

By the end of this topic,the learners should be able to

- Identify lines of symmetry for different figures
- Reflect shapes and objects
- Apply reflection in the cartesian plane

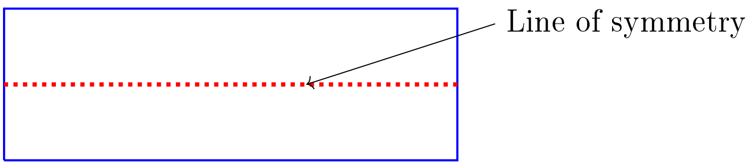
In this section we look at line symmetry and reflections of simple shapes, in horizontal, vertical and sloping lines. In a reflection, a point will move to a new position that will be the same distance from the mirror line, but on the other side. These distances will always be measured at right angles to the mirror line.

10.1 Identify Lines of Symmetry for Different Figures

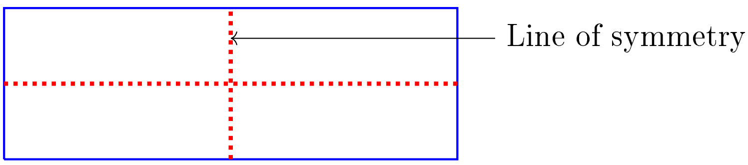
Line of Symmetry is defined as the axis or imaginary line that passes through the center of the shape or object and divides it into identical halves. In otherwords a Line of Symmetry divides a figure into two parts such that when the figure is folded along the line,the two parts of the figure coincide(match).The line of symmetry is also called the mirror line or axis of symmetry.

Activity : Identifying the lines of symmetry for a rectangle

- Take a rectangular sheet of paper
- Fold it once lengthwise, so that one half fits exactly over the other half and crease the edges



- Now open it, and again fold it once along its width.



- The rectangle has two lines of symmetry

Activity : Identifying lines of symmetry

One of the two set squares in your geometry set has angle of measure 30^0, 60^0, and 90^0.

1. Take two such identical set squares

2. Place them side by side to form a 'kite'
3. Look at both sides of the fold line. Are they the same size and shape?
4. How many lines of symmetry does the shape have?

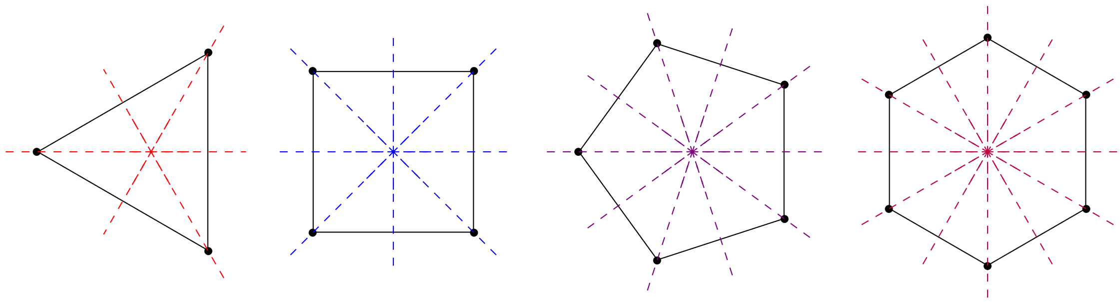
10.1 Exercise Set

1. Copy the following alphabetic letters and draw in all their lines of symmetry.

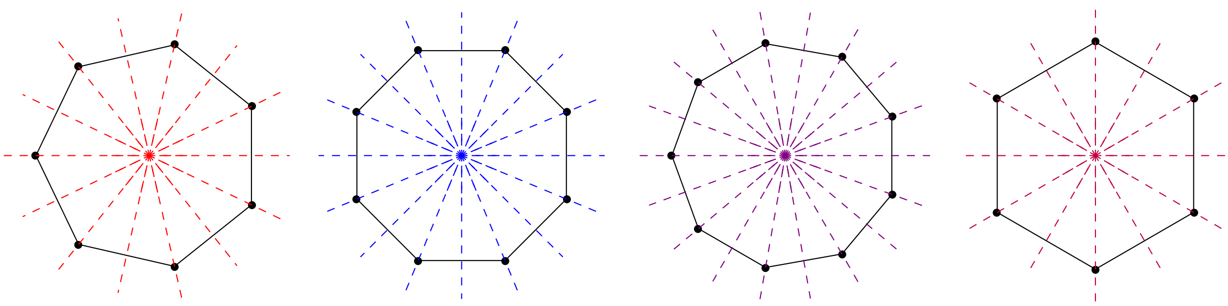
A B C D E F H I K L M O R S W X Z

2. Draw a square on a tracing paper. Fold it to find the lines of symmetry. How many lines of symmetry does a square have?
3. Find the number of lines of symmetry of
   (a) a semi circle
   (b) an equilateral triangle
   (c) a rhombus
   (d) atrapezium
   (e) Isosceles trapezoid
   (f) Isosceles triangle
4. Using the figures below ,copy and complete the table.

Name of the figure	Number of sides	Lines of symmetry

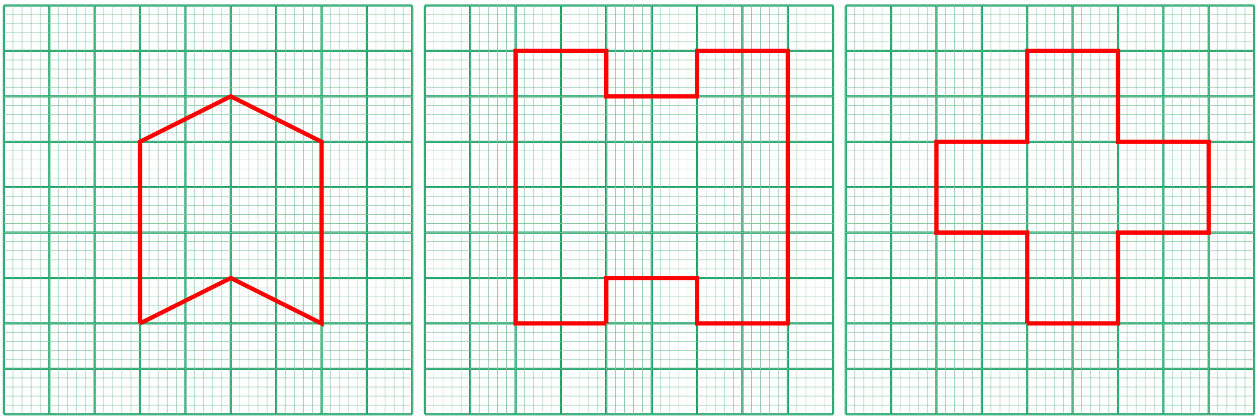


(a) (b) (c) (d)



(e) (f) (g) (h)

5. Draw in the lines of symmetry for each of the following shapes:

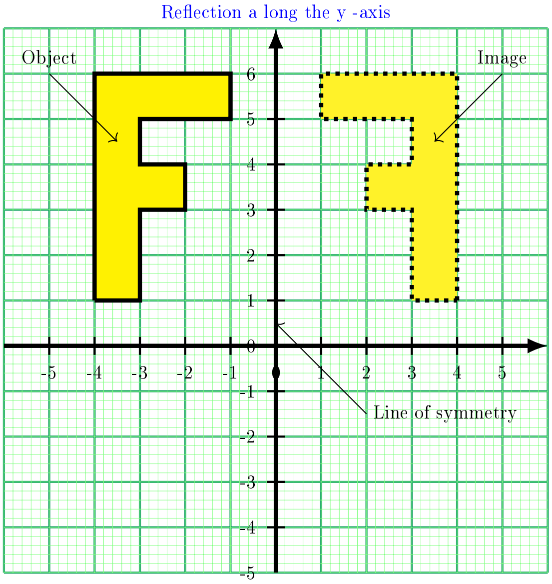


(a) (b) (c)

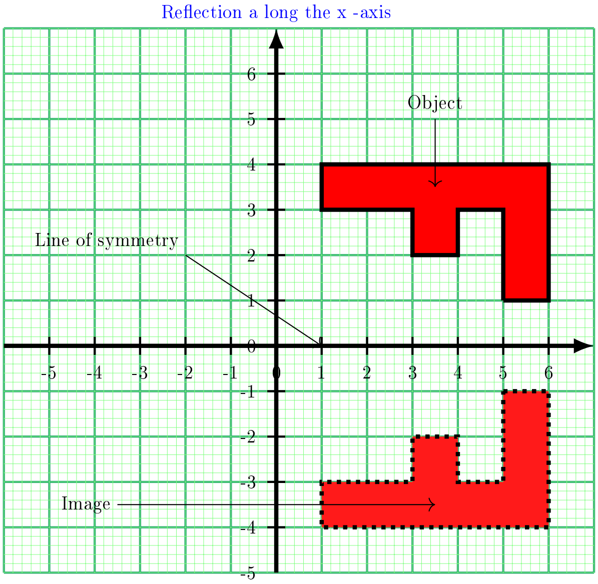
10.2 Reflection in the Cartesian Plane

- A reflection can be thought of as folding or "flipping" an object over the line of reflection.
- The original object is called the pre-image, and the reflection is called the image.
- Object(pre-image) is the initial figure (shape) formed before reflection has taken place.
- Image is the figure (shape) obtained when an object has undergone a reflection.
- The image is usually labeled using a prime symbol, such as A^1, B^1, C^1.
- The image is formed using a mirror line
- A mirror line is a line of symmetry from where reflection of object takes place.
- An object and its reflection have the same shape and size, but the figures face in opposite directions. The objects appear as if they are mirror reflections, with right and left reversed.For example the mirror image of the letter p for reflection with respect to a vertical axis would look like q. Its image by reflection in a horizontal axis would look like b.
- The image is as far behind the mirror as the object is in front of it.i.e The distance of the image and the object from the mirror line must be equal.

Reflection a long the y -axis



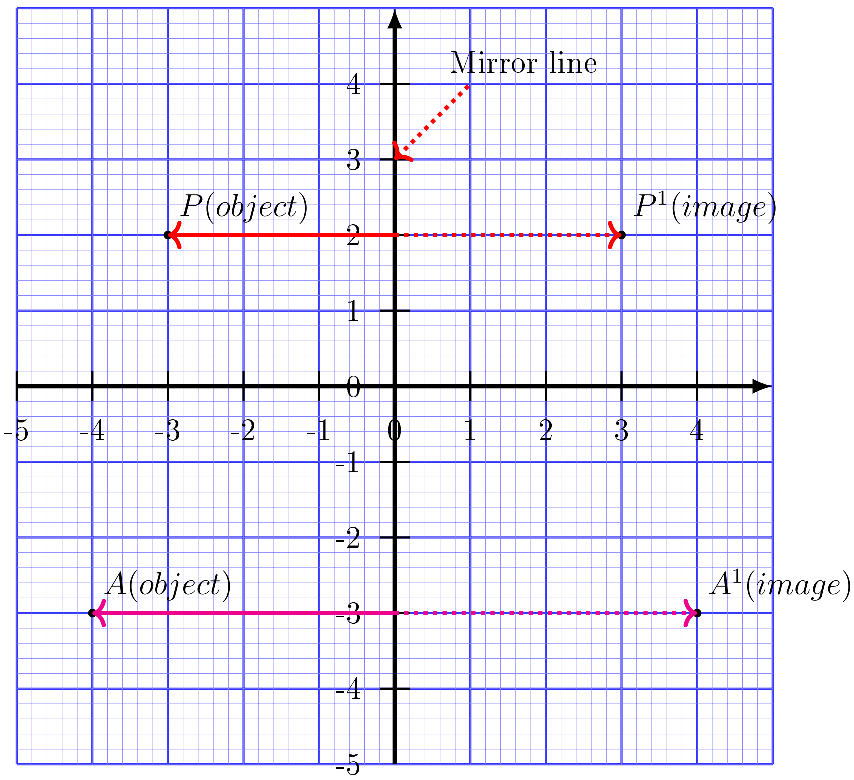
Reflection a long the x -axis



Activity : Reflecting in a Cartesian plane

1. Plot the points A (-3, 1), B(-1, 1) and C (-1, 3) on squared graph paper.
   (a) A mirror is placed on the x- axis. Where would the images of the tree points be? What are the coordinates of the image points A^1 ,B^1 and C^1?
   (b) A mirror is placed on the y- axis. Where would the images of the tree points be? What are the coordinates of the image points A^1 ,B^1 and C^1?
   (c) Draw another pair of axes. Plot the same points again. Take the line y = 2 as the mirror line. Where would the images of the three points be? What are the coordinates of the new image points A^1 ,B^1 and C^1?
   (d) Draw another pair of axes. Draw the line x = 4. Plot the points (1, -3). Using the line x = 4 as the mirror line, find the image of the point (1, -3).
1. Find the Coordinates of the image of the points, P(-3,2) and A(-4,-3) for a reflection a long the y-axis

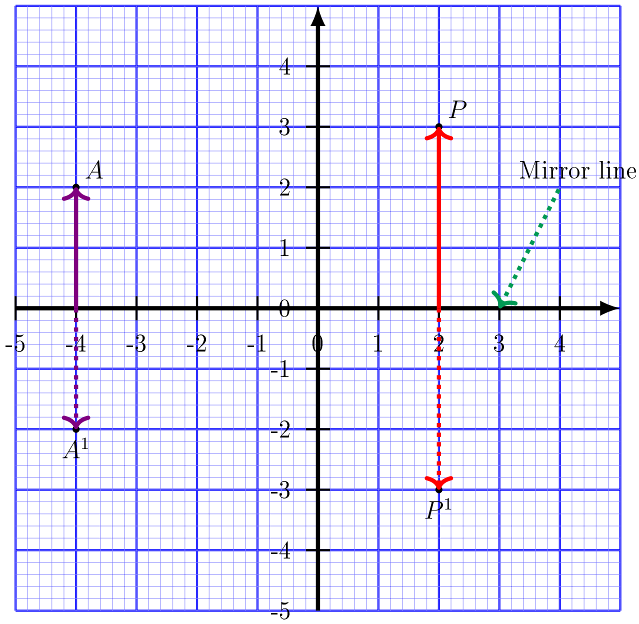
Reflection a long the y -axis



The images are $P^1(3,2)$ and $A^1(4,-3)$

2. Find the Coordinates of the image of the points, P(2,3) and A(-4,2) for a reflection a long the x-axis

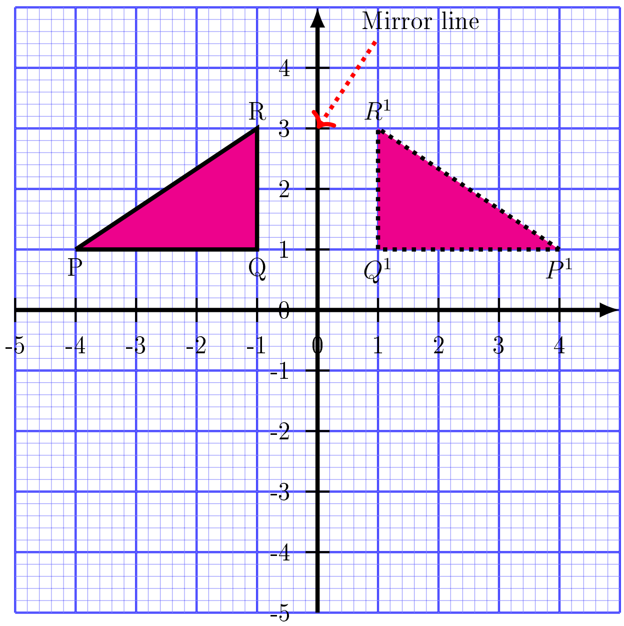
Reflection a long the x -axis



The images are P^1(2,-3) and A^1(-4,-2)

3. Find the image of triangle PQR with coordinates P (-4, 1), Q (-1, 1) and R (-1, 3) after a reflection along the y–axis.

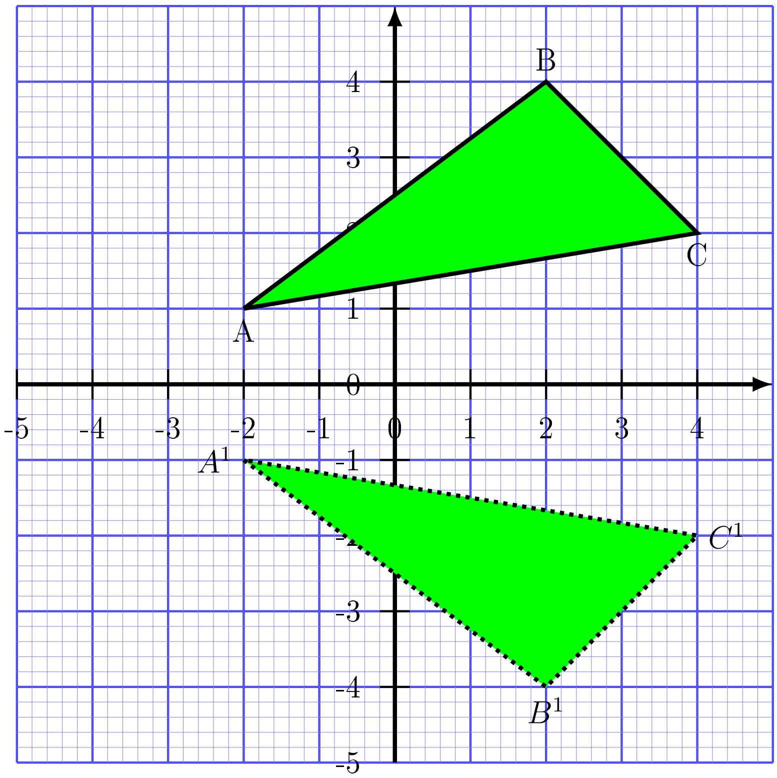
Reflection a long the y -axis



The images are P^1(4,1) , Q^1(1,1) and R^1(1,3)

4. Find the image of triangle ABC with coordinates A (-2, 1), B (2, 4) and C (4, 2) after a reflection along the x–axis.

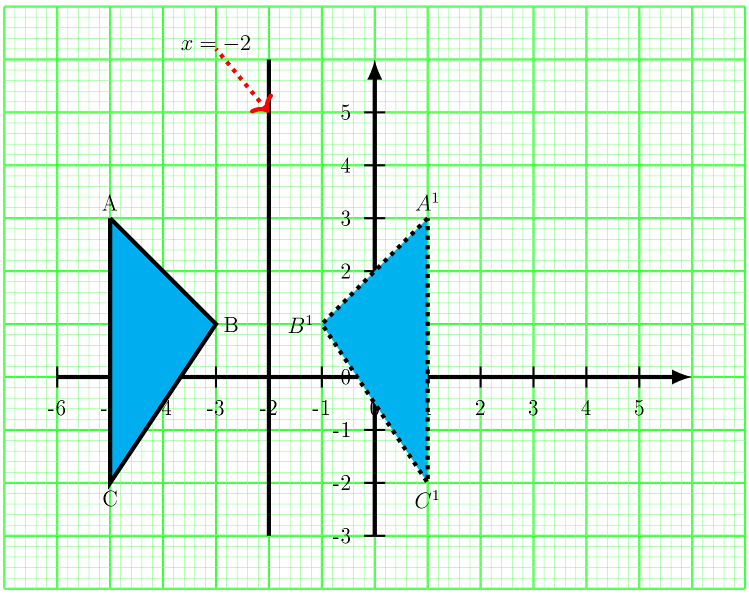
Reflection a long the x -axis



The images are A^1(-2,-1) , B^1(2,-4) and C^1(4,-2)

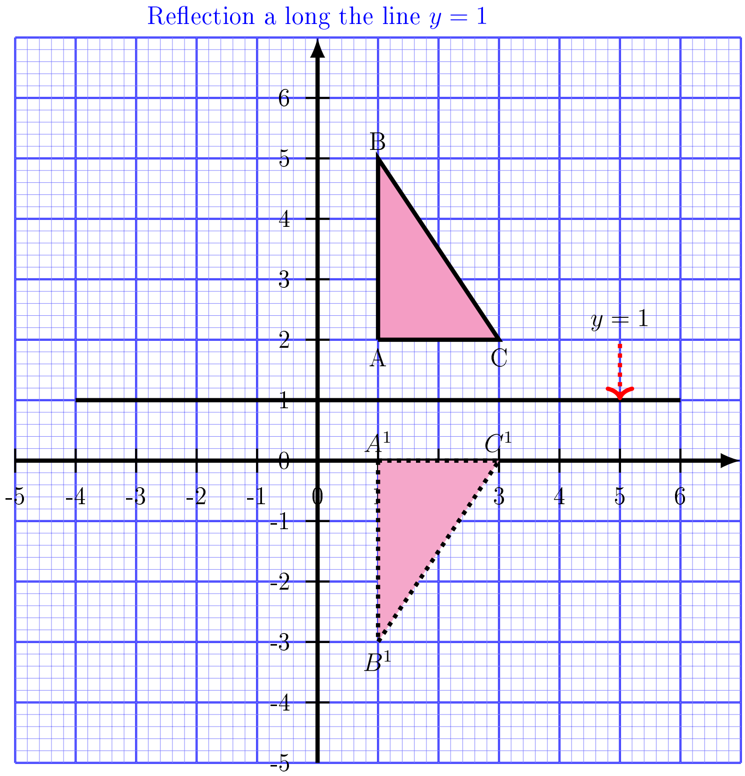
5. Find the image of triangle PQR with A (-5, 3), B (-3, 1) and C (-5, -2) after a reflection along the line $x = -2$

Reflection a long the line $x = -2$



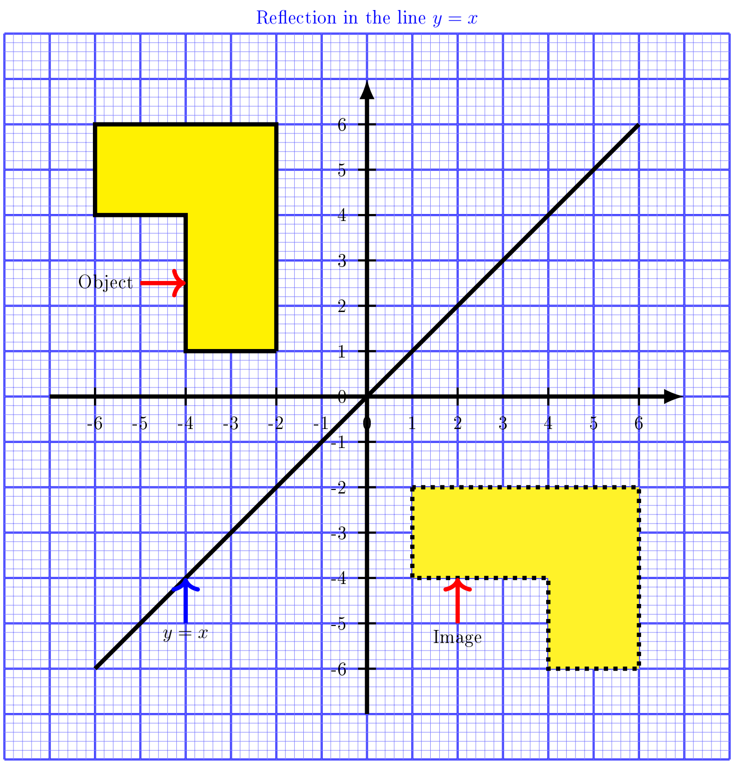
The images are A^1(1,3) , B^1(-1,1) and C^1(1,-2)

6. Find the image of triangle PQR with A (1, 2), B (1, 5) and C (3, 2) after a reflection along the line $y = 1$



The images are A^1(1,0) , B^1(1,-3) and C^1(3,0)

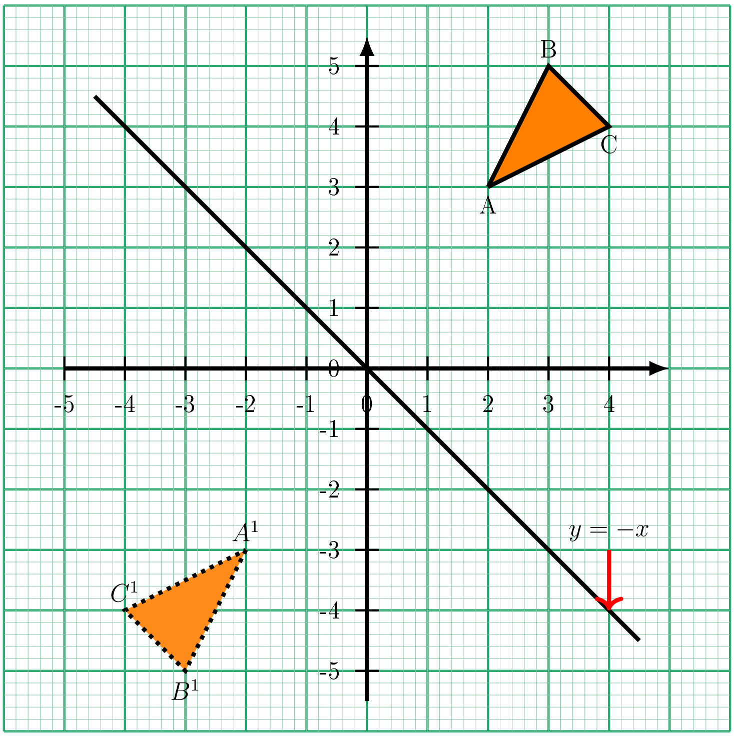
7. Find the image of the figure with points (-2,1),(-2,6),(-6,6),(-6,4),(-4,4),(-4,1) after a reflection along the line $y = x$



The images are (1,-2) , (1,-4),(4,-4),(4,-6),(6,-6) and (6,-2)

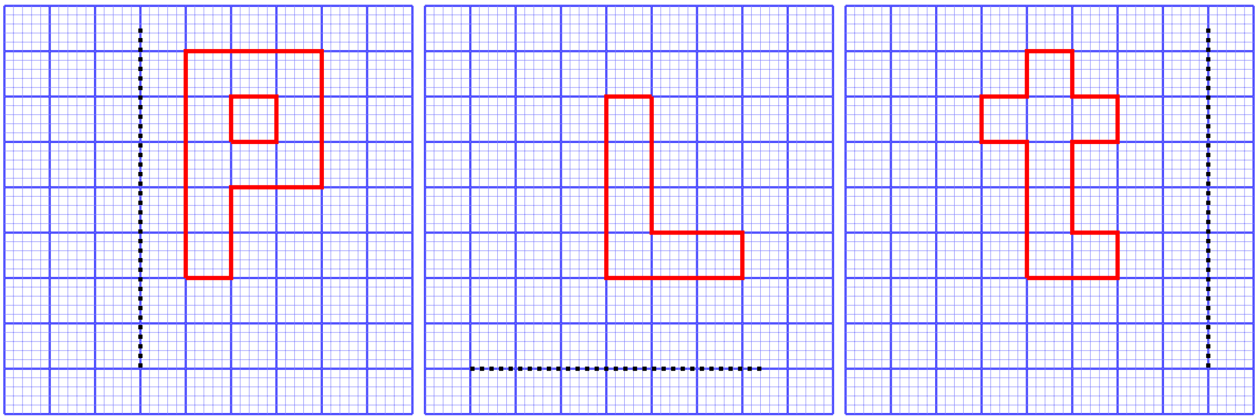
8. Find the image of triangle ABC with A (2, 3), B (3, 5) and C (4, 4) after a reflection along the line $y = -x$
The images are A^1(-2,-3) , B^1(-3,-5) and C^1(-4,-4)

Reflection in the line $y = -x$



10.2 Exercise Set

1. Draw the reflection of each of the following shapes in the line given:



(a) (b) (c)

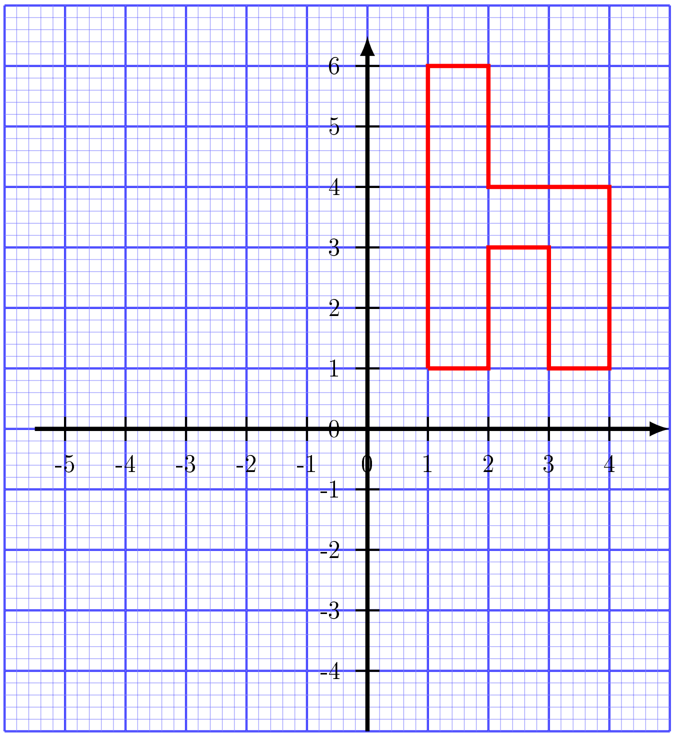
2. Find the image of the point (2, 5) under reflection in the y axis.

3. After a point has been reflected in the x axis, its image is at (3,2). Find the coordinates of the object point.

4. The points A(4, 2) , B(1, 3) and C(1,-2)are reflected in the line y = 4. Find the coordinates of the images A^1 , B^1 and C^1 .

5. Find the image of ABCD with A (0, 0), B (1, 0), C (1, 1) and D (0, 1) being reflected along the x–axis.

6. (a) Copy the following diagram:



(b) Reflect the shape in the lines $x = -4$ and $y = -3$.

7. Find A and B the images of A and B respectively under the reflection in the y–axis with A (1, 3) and B (3, 7).

8. Find the images of a triangle with vertices P(1, 4), Q(3, 2) and R(5, 3) after a reflection in the line y = 0

9. A triangle with vertices P(2, 3), Q(5, 4) and R(5, 6) is mapped onto its image by a reflection in the line x = 0 .Find the coordinates of its images.

10. (a) Draw the triangle which has corners at the points with coordinates A(1, 4), B(1, 7) and C(3, 5).

    (b) Reflect this shape in the line $y = x$ and state the coordinates of the corners of the reflected shape.

    (c) Reflect the original triangle in the line $y = -x$ and state the coordinates of the corners of the reflected shape.

11. Draw the triangle with corners at the points with coordinates P(1, 3), Q(1, 8) and R(6, 8). Reflect this triangle in the following lines:

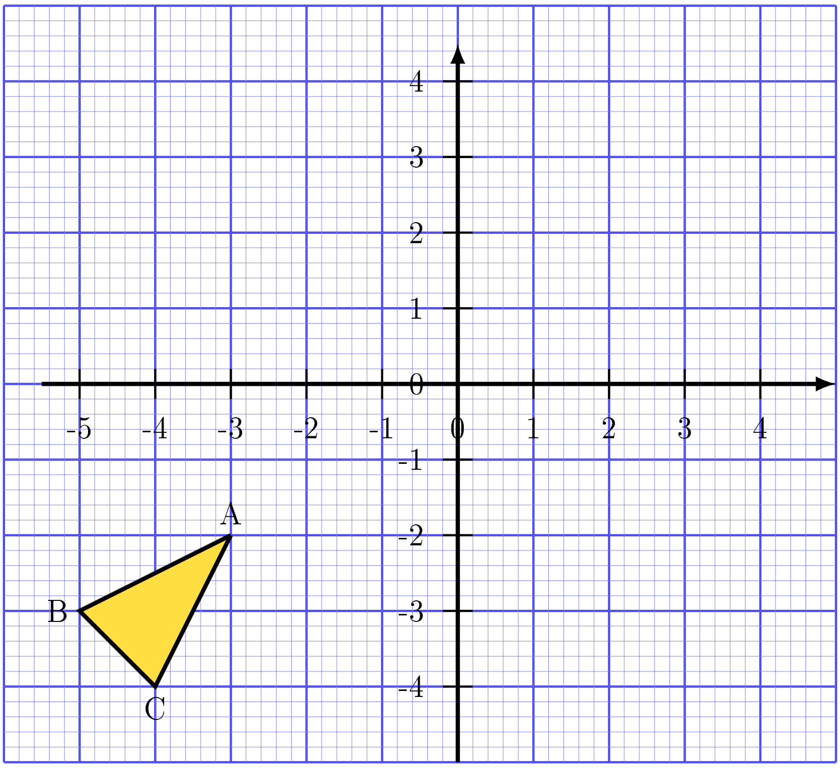
    (a) $x = 0$ (b) $y = 0$ (c) $y = x$ (d) $y = -x$

12. (a) Draw the triangle that has corners at the points with coordinates Q(1, 1), R(4, 7) and S(2, 5).

    (b) Reflect the triangle in the lines:

    (i) $x = 8$ (ii) $x = -1$ (iii) $y = -2$ (iv) $y = 4$

13. Reflect the triangle in the graph in any quadrant and write down the coordinates of its image.



(a) Write down the coordinates of the images of triangle ABC

(b) What is the equation for the mirror line?

Activity of Integration

One of your relatives wants to make a hair salon. He approaches you for help.As a senior one graduate draw a plan of how you can help your relative make his salon shop be up to date

- Support: Interior plan of the shop
- Resources:knowledge of reflection
- Task: Advice the barber on how he can organize the salon such that the customers are able to get a clear view of themselves with their images not distorted.

Chapter 11: EQUATIONS OF LINES AND CURVES

Learning objectives

By the end of this topic,the learners should be able to

- Form linear equations with given points
- Draw the graph of a line given its equation
- Differentiate between a line and a curve

In this topic you will learn how to form linear equations with given points, draw graphs for the given linear equations and differentiate between a line and a curve.

11.1 Forming Linear Equations with given points

Under this we shall find out the equation of the straight line ,given some points that lie a long the line.

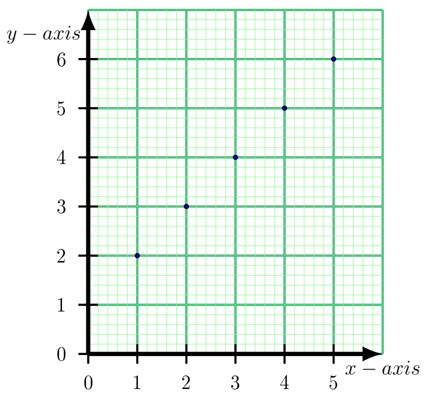
Activity:Writing the equation of a straight line

- Plot the points (3,0),(3,1),(3,2),(3,3) and (3,4) on the squared graph paper.
- What do you notice about these plotted points.
- Give the coordinates of four more points which belong to this set.
- What is the x coordinate of any point which belongs to this set.
- What is the equation of the line on which the points lie.

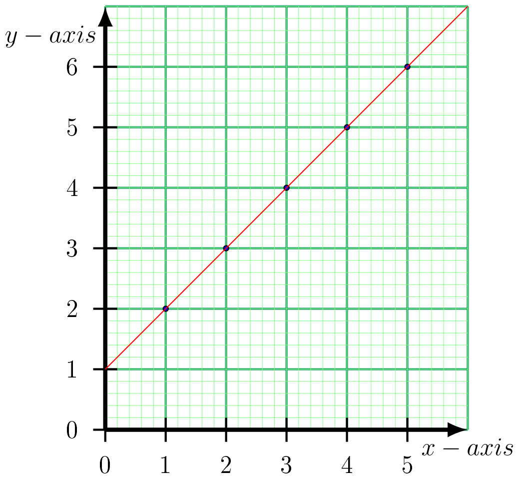
EXAMPLE

1. Plot the points with coordinates:

   (a) (1, 2), (2, 3), (3, 4), (4, 5) and (5, 6).



(b) Draw a straight line through these points.



(c) Describe how the x- and y-coordinates of these points are related.
The y-coordinate is always one more than the x-coordinate, so we can write $y = x + 1$.

(d) Write down two more points that lie on the line.
The other coordinates which lie a long the line are (0,1),(6,7)

11.1 Exercise Set

1. The following points lie on a line.Write down the equation of the line and give two more points which belong to each set of points.

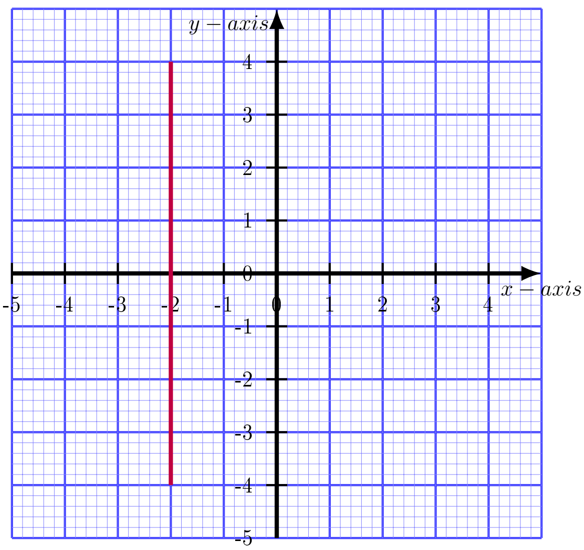
   (a) $(1, -2), (1, 0), (1, 2), (1, 5)$

   (b) $(-4, -2), (0, -2), (2, -2)$

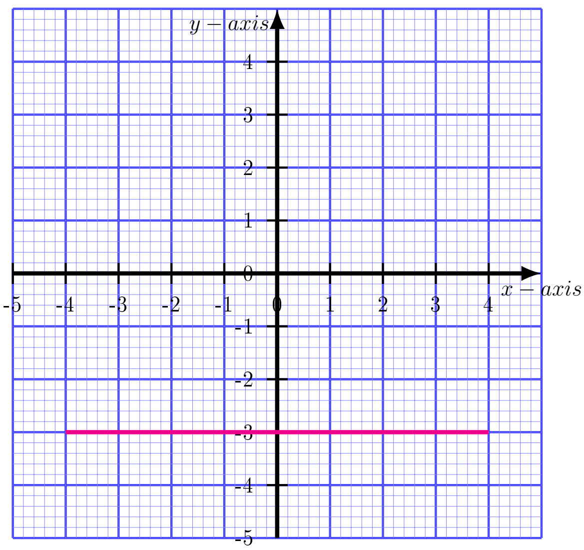
   (c) $(0, 4), (0, 2), (0, 0), (0, -2)$

   (d) $(-4, 2), (-4, 0), (-4, -2)$

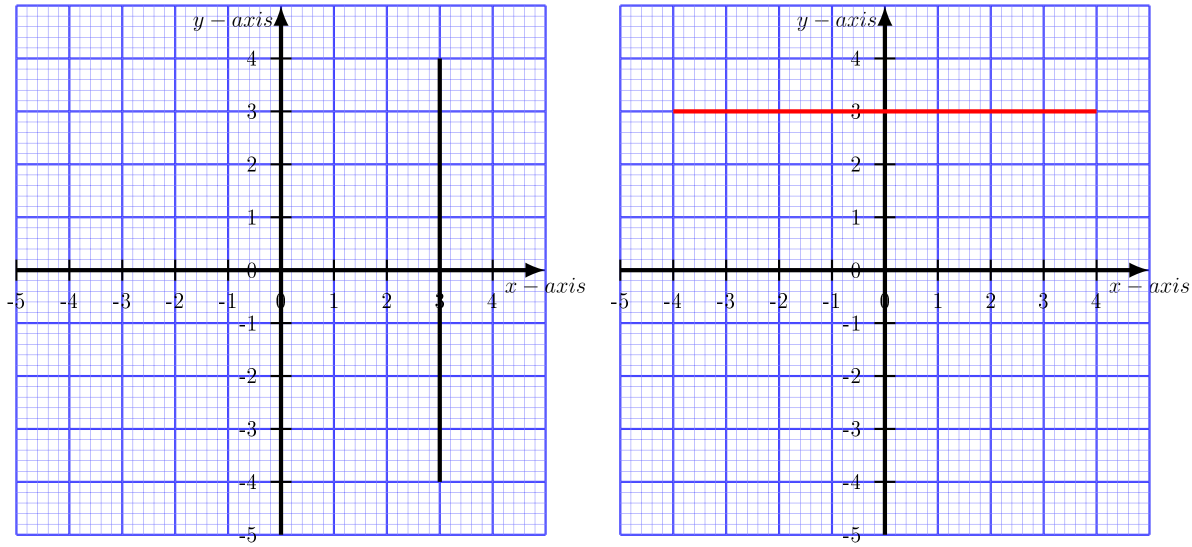
2. Write down the equation of each of the lines below



(a)



(b)



(c)

(d)

3. Plot the points with coordinates:
   (a) (0, 0), (1, 3), (3, 9) and (5, 15).
   (b) Draw a straight line through these points
   (c) Write down the coordinates of two other points on this line.
   (d) Write down the equation of the line drawn
4. Plot the points with coordinates:
   (a) (0, 4), (1, 5), (3, 7) and (5, 9)..
   (b) Draw a straight line through these points
   (c) Write down the coordinates of three other points on this line.
   (d) Write down the equation of the line drawn
   (e) On the same graph , plot the points with coordinates (1, 8), (2, 7), (5, 4) and (7, 2) and draw a straight line through them.
   (f) What is the relationship between the two lines
5. Plot the points with coordinates
   (a) (2, 6), (3, 5), (4, 4) and (7, 1)and draw a straight line through them.
   (b) On the same set of axes, plot the points with coordinates (0, 1), (1, 2), (3, 4) and (5, 6) and draw a straight line through them.
   (c) What is the relationship between the two lines
6. Plot the points with coordinates
   (a) (1, 1), (2, 2), (4, 4) and (5, 5) and draw a straight line through them.
   (b) Write down the coordinates of two other points on the line.
   (c) Describe the relationship between the x- and y-coordinates.

7. The points (1, 3), (2, 4), (3, 5) and (5, 7) lie on a straight line.
   (a) Plot these points and draw a straight line through them.
   (b) Write down the coordinates of four other points on the line.
   (c) Write down the equation of the line.
8. The points (– 3, – 4), (– 1, – 2), (1, 0), and (4, 3) lie on a straight line.
   (a) Plot these points and draw a straight line through them.
   (b) Write down the coordinates of two other points on the line.
   (c) Write down the equation of the line.
9. Find the equations of the lines on which the following sets of points lie:
   (a) (0,0),(5,5),(8,8),(-1,-1)
   (b) (0,-4),(2,0),(4,4)
   (c) (-2,-1),(-2,1),(0,3)
   (d) (0,3),(3,0),(1,2),(2,1)
10. Find the equation of atleast five straight lines that pass through the point (2,2)

11.2 Plotting Graphs Given Their Equations

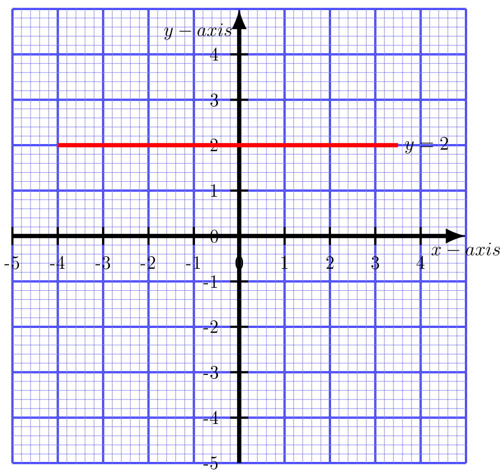
Often, to get an idea of the behavior of an equation, we will make a picture that represents the solutions to the equations called a graph

- When graphing we often use a table to find the unknown values .
- All linear equations yield straight lines when plotted.
- The x -axis is the same as the line $y = 0$
- The y -axis is the same as the line $x = 0$

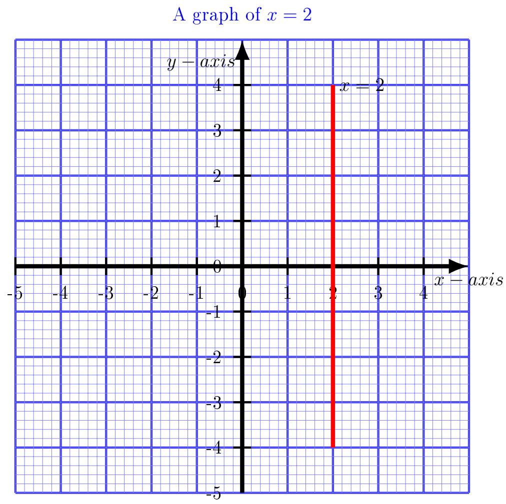
EXAMPLE

1. Draw the graph of $y = 2$

A graph of $y = 2$



2. Draw the graph of $x = 2$



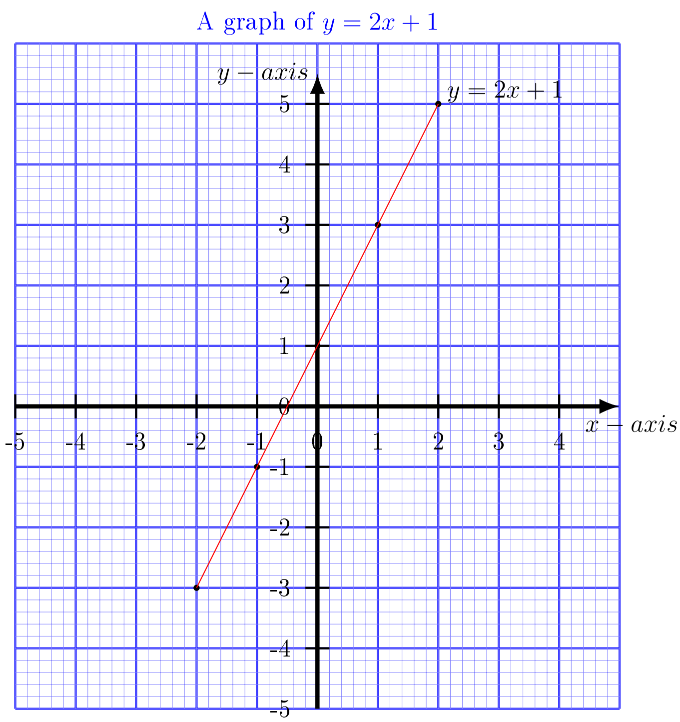
3. Complete the table below for $y = 2x + 1$.

x	-2	-1	0	1	2
y					

(b) Use the information in the table to plot the graph with equation y = 2 x + 1 .

x	-2	-1	0	1	2
y	-3	-1	1	3	5

The points y $(-2, -3)$, $(-1, -1)$, $(0, 1)$ $(1, 3)$ and $(2, 5)$ can then be plotted, and a straight line drawn through these points



4. Draw the graph of $y = 2x + 3$ for $-4 \leq x \leq 4$
The y values are obtained by substituting for the value of x in the equation.For example when $x = -4$.

for $x = -4$
$$y = 2x + 3$$
$$y = (2 \times -4) + 3$$
$$y = -5$$

for $x = -3$
$$y = 2x + 3$$
$$y = (2 \times -3) + 3$$
$$y = -3$$

$x = -2$
$$y = 2x + 3$$
$$y = (2 \times -2) + 3$$
$$y = -1$$

$x = 0$
$$y = 2x + 3$$
$$y = (2 \times 0) + 3$$
$$y = 3$$

for $x = 1$
$$y = 2x + 3$$
$$y = (2 \times 1) + 3$$
$$y = 5$$

for $x = 2$
$$y = 2x + 3$$
$$y = (2 \times 2) + 3$$
$$y = 7$$

$x = 3$
$$y = 2x + 3$$
$$y = (2 \times 3) + 3$$
$$y = 9$$

$x = 4$
$$y = 2x + 3$$
$$y = (2 \times 4) + 3$$
$$y = 11$$

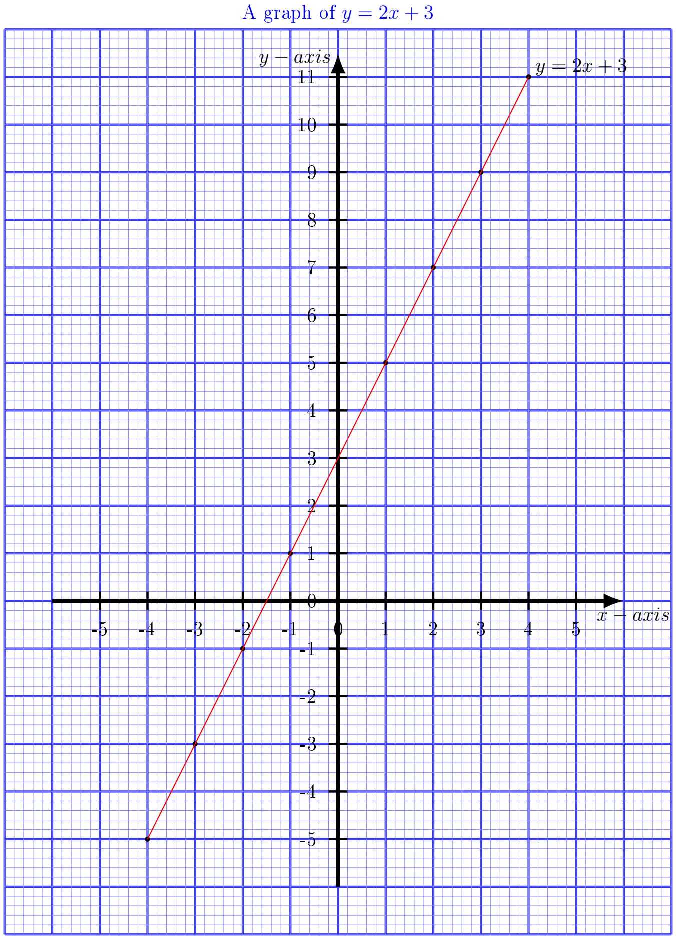
The coordinates are $(-4, -5), (-3, -3), (-2, -1), (-1, 1), (0, 3), (1, 5), (2, 7), (3, 9), (4, 11)$

Using the second approach of the table.

x	-4	-3	-2	-1	0	1	2	3	4
y	-5	-3	-1	1	3	5	7	9	11

The coordinates are $(-4, -5), (-3, -3), (-2, -1), (-1, 1), (0, 3), (1, 5), (2, 7), (3, 9), (4, 11)$

A graph of $y = 2x + 3$



11.2 Exercise Set

1. Draw the graphs for the following lines

   (a) $y = -2$
   (b) $x = -4$
   (c) $y = 0$
   (d) $x = 5$
   (e) $x = 0$
   (f) $y = 4$

2. Draw the graphs for the following lines using the range of -3 to $^+3$

   (a) $y = -2x + 4$
   (b) $y = 4 + x$
   (c) $y = x - 1$
   (d) $y = 5x - 4$
   (e) $y = 2x + 1$
   (f) $y = 4 - x$
   (g) $y = 3x + 1$
   (h) $y = -2x - 2$
   (i) $y = -1 - 6x$
   (j) $y = 4 - 2x$
   (k) $y = x$
   (l) $y = -x$
   (m) $y = \frac{1}{2}x + 1$
   (n) $y = x - 4$
   (o) $y = x + 1$

3. (a) Complete the table below for $y = x + 3$.

x	-2	-1	0	1	2
y					

   (b) Draw the line with equation $y = x + 3$.

4. (a) Complete the table below for $y = x - 2$. .

x	-3	-2	-1	0	1	2	3
y				-2			

   (b) Draw the line with equation $y = x - 2$. .

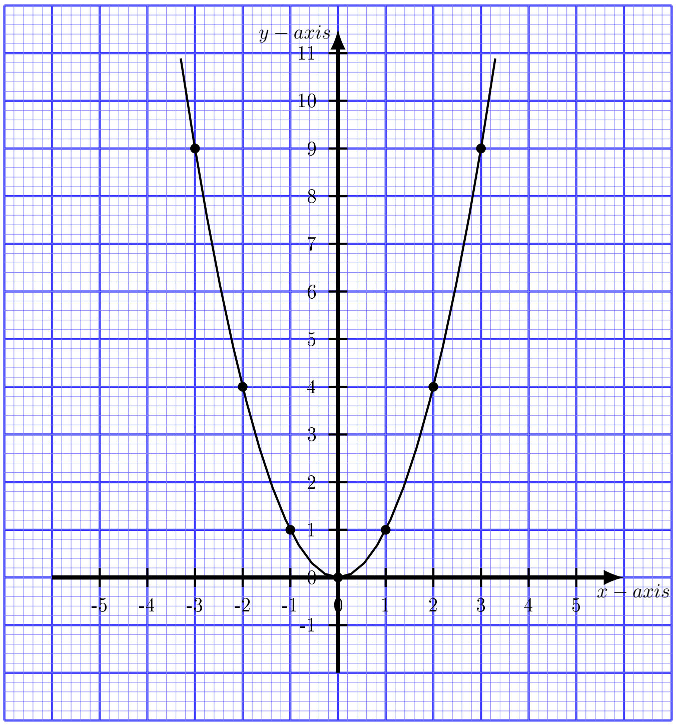
5. Determine the coordinates of the point where the lines $y = x + 3$ and $y = 7 - x$ cross.

11.3 Curves

In this section we look at a brief introduction about curves

Activity: Plotting a curve

- Plot the following coordinates (-3,9),(-2,4),(-1,1),(0,0),(1,1),(2,4),(3,9) on a squared graph paper
- what do you notice about the plotted points
- Join the points using a free hand to form a smooth curve



11.3 Exercise Set

1. On a squared paper ,plot the following points and join them with a smooth curve (1,12),(2,6),(3,4),(4,3),(5,2.4),(6,2).
2. Plot the following coordinates (-3,6),(-2,0),(-1,-4),(0,-6),(1,-6),(2,-4),(3,0),(4,6) on a squared graph paperand join them with a smooth curve
3. Plot the following coordinates (0,5),(1,0),(2,-3),(3,-4),(4,-3),(5,0),(6,5) on a squared graph paper and join them with a smooth curve

Activity of Integration

A glass mart company in uganda wants to make a stained glass window.As a senior one student help them to come up with a good glass window.

- Support: pens, graph paper, pencil,ruler and colors
- Resources:Knowledge of graphing linear equations
- Task:By plotting the following lines $y = 2, x = -5, y = x, y = -2x + 4, y = -x - 4, y = x + 6, y = -3$ on the same graph paper,Design a stained glass window .Shade using any 5 colours

Chapter 12: ALGEBRA 1

Learning objectives

By the end of this topic,the learners should be able to

- Use letters to represent numbers
- Write statements in algebraic form
- Simplify algebraic expressions
- Evaluate algebraic expressions by substituting numerical values
- Manipulate simple algebraic equations in one variable and solve them

In this topic you will learn the basic algebraic skills, how to form linear equations and draw graphs for the given linear equations. The logical reasoning abilities developed during this topic will promote deeper critical thinking and problem-solving prowess that will serve learners throughout a lifetime

12.1 Fundamental Algebraic Skills

This section looks at some skills you will need as you start to learn about algebra. It starts with some work on evaluation and substitution,removing brackets,collecting like terms , then moves on to work with formulae.

- In algebra letters such as a,b,c,x,y e.t.c. are called variables.This is because they keep on varying and they represent unknown numbers.Variables are place-holders for numbers
- An expression is a combination of numbers and variables with out an equal sign e.g $5y, 2x + 5a - 2$
- A term is a number or the product of a number and variables raised to powers.For example in an expression such as $5y + 3x + 5$,the terms are 5y,3x,5
- In an expression every term is connected to the sign which is directly to the left of it.i.e it can be $+, -, \times, \div$
- $\frac{x}{y}$ means x dividend by y $(x \div y)$
- xy means “x times y ”$(x \times y)$.
- ab is the same as ba
- $a + b$ is the same as $b + a$
- $a - b$ is the same as $-b + a$
- The term $-y$ means $-1y$, x means $1x$

12.1.1 Substituting numbers for letters

- In algebra, we use a letter(variable) to represent an unknown number.
- Substitute numbers in place of letters to evaluate the given expressions.

EXAMPLE

1. Given that $a = 3, b = 8, c = 4$,and $d = 2$, Calculate:

   (a) $a + b$

   $$\begin{aligned} a + b &= 3 + 8 \\ &= 11 \end{aligned}$$

   (b) $a + b + c$

   $$\begin{aligned} a + b + c &= 3 + 8 + 4 \\ &= 15 \end{aligned}$$

   (c) $b - d$

   $$\begin{aligned} b - d &= 8 - 2 \\ &= 6 \end{aligned}$$

   (d) $2a + b + 2d$

   $$\begin{aligned} 2a + b + 2d &= (2 \times 3) + 8 + (2 \times 2) \\ &= 6 + 8 + 4 \\ &= 18 \end{aligned}$$

   (e) $3c - 2d + a$

   $$\begin{aligned} 3c - 2d + a &= (3 \times 4) - (2 \times 2) + 3 \\ &= 12 - 4 + 3 \\ &= 11 \end{aligned}$$

   (f) bcd

   $$\begin{aligned} bcd &= 8 \times 4 \times 2 \\ &= 64 \end{aligned}$$

2. Find the value of x when $3x + 9$ when $x = -3$

   $$\begin{aligned} 3x + 9 &= (3 \times -3) + 9 \\ &= -9 + 9 \\ &= 0 \end{aligned}$$

3. Find the value of $8 + 3y - t$ when $y = 2$ and $t = 3$

   $$\begin{aligned} 8 + 3y - t &= 8 + (3 \times 2) - 3 \\ &= 8 + 6 - 3 \\ &= 1 \end{aligned}$$

12.1.2 Collecting like terms

To be able to collect like terms, there are 3 things we need to know.A term can come in three forms:

- A number by itself
- A letter by itself
- A combination of letters and numbers

Therefore like terms have the same combination of letters and numbers.

- Terms with the same variables raised to exactly the same powers are called like terms.
- In collecting like terms, the expression is re-arranged so that like(same) terms are next to each other.
- The products ab and ba are the same. Thus ab and ba are like terms.
- To add or subtract terms with the same letter, we add or subtract the numbers like usual and just put the letter back on the end.

EXAMPLES

1. Simplify the following expressions:

(a) $2a + 3a + a$

$$\begin{aligned} &= 2a + 3a + a \\ &= 6a \end{aligned}$$

(c) $4x + 9y - 3x + 5y$

$$\begin{aligned} 4x + 9y - 3x + 5y &= 4x - 3x + 9y + 5y \\ &= x + 14y \end{aligned}$$

(b) $a + 2b + 4a - b$

$$\begin{aligned} a + 2b + 4a - b &= a + 4a + 2b - b \\ &= 5a + b \end{aligned}$$

(d) $12f + 3f - 13f$

$$\begin{aligned} 12f + 3f - 13f &= 15f - 13f \\ &= 2f \end{aligned}$$

12.1.3 Simplification of brackets

In order to simplify mathematical expressions it is frequently necessary to ‘remove brackets’. This means to rewrite an expression which includes bracketed terms in an equivalent form, but without any brackets.This process of rewriting an expression to remove brackets is usually referred to as expanding brackets.

- Brackets are removed(opened)by multiplying each term inside the bracket by the quantity(variable)outside the bracket.
- If the sign in front of the bracket is negative, the signs inside the brackets change according to the operation.

EXAMPLES

1. Remove the brackets and simplify the following expressions:

(a) $2(3x+2)$

$$\begin{aligned} 2(3x+2) &= 2\times 3x + 2\times 2 \\ &= 6x+4 \end{aligned}$$

(b) $-5(a-b)$

$$\begin{aligned} -5(a-b) &= -5\times a - -5\times b \\ &= -5a - -5b \\ &= -5a+5b \end{aligned}$$

(c) $-4(3+x)$

$$\begin{aligned} -4(3+x) &= -4\times 3 + -4\times x \\ &= -12 + -4x \\ &= -12 - 4x \end{aligned}$$

(d) $7(a-2b)$

$$\begin{aligned} 7(a-2b) &= 7\times a - 7\times 2b) \\ &= 7a - 14b \end{aligned}$$

2. Simplify the expression $3(4x-2)+8(2x+3)$

$$\begin{aligned} 3(4x-2)+8(2x+3) &= 3\times 4x - 3\times 2 + 8\times 2x + 8\times 3 \\ &= 12x - 6 + 16x + 24 \\ &= 12x + 16x - 6 + 24 \\ &= 28x + 18 \end{aligned}$$

3. Simplify the expression $4(2x+6)-2(3x+3)$

$$\begin{aligned} 4(2x+6)-2(3x+3) &= 4\times 2x + 4\times 6 - (2\times 3x + 2\times 3) \\ &= 8x + 24 - (6x+6) \\ &= 8x + 24 - 6x - 6 \\ &= 8x - 6x + 24 - 6 \\ &= 2x + 18 \end{aligned}$$

12.1 Exercise Set

1. Work out the value of these terms if $x=4, y=5$ and $z=-2$

(a) $2x+y$

(b) $4z-5x+2$

(c) $\frac{5x}{y}$

(d) $\frac{2z+4y}{x}$

(e) $-2x-4z+x$

(f) $x+4z-y$

2. If x = 10, find the value:

(a) $\frac{x}{2}$

(b) $\frac{4x-10}{x}$

(c) $\frac{5x}{2}+5$

(d) $\frac{x+4}{2}$

3. Use the formula $v = u + at$ to find v, when $u = 20, a = -2$ and $t = 7$.

4. If $a = 7, b = 5, c = -3$ and $d = 4$, calculate the value of:

   (a) $2(a + b)$
   (b) $4(a - b)$
   (c) $6(a - d)$
   (d) $2(a + c)$
   (e) $2a(b + c)$
   (f) $c(d - 2)$
   (g) $a(2b - c)$
   (h) $d(2a - 3b)$
   (i) $d(b + a)$
   (j) $c(b - a)$
   (k) $d(a + 2b - c)$
   (l) $c(3d + 2a - b)$

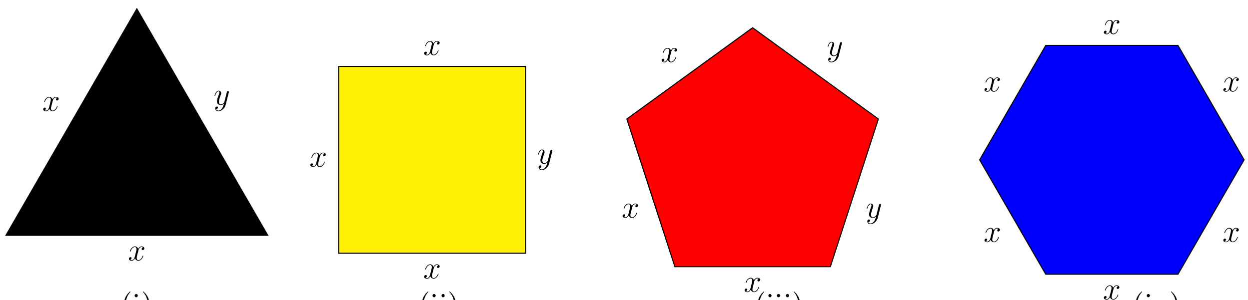
5. Simplify the expressions below, where possible.

   (a) $7a + 5b + 2a - 6b$
   (b) $p - 5q + 3p - q$
   (c) $3x - 4y - 2x + 6y$
   (d) $2a + 4c - 6a + 3c$
   (e) $2ab + 4c + 4ab - 2)$
   (f) $8t - 6t + 7s - 2s$
   (g) $11m + 3n - 5p + 2q - 2n + 9q - 8m + 14p$
   (h) $4p + 6pq - 2q + 8p - 11qp + 10q$
   (i) $a^2 + 2b^2 + 3a^2$

6. Expand the brackets and simplify.

   (a) $8(3 + 2y)$
   (b) $-(-3 - 2x)$
   (c) $7(-a + b)$
   (d) $-7(-x + y)$
   (e) $x(x + 1)$
   (f) $15(x + y)$
   (g) $-(3 - 2x)$
   (h) $11(m + 3n)$
   (i) $5(x + 3) - 2(x + 4)$
   (j) $2(a - b) + 3(a + b)$
   (k) $4(2x - 3y) - 3(x - y)$
   (l) $5(p + 2q) + 7(2p - q)$
   (m) $5[4(y-4)+15]-2(5y-3)$
   (n) $5(x + 2) + 2(x - 3)$
   (o) $2(7 + 5x) + 4(x + 6)$
   (p) $3(2x + 7) + 2(x - 5)$
   (q) $3(2x + 4) + 6(x - 1)$
   (r) $2p(q + r) - p(3q - 2r)$

7. Complete the formula for the perimeter of each of the shapes below.Your answers must be simplified.



(i) (ii) (iii) (iv)

12.2 Function Machines

In this section we look at how to find the input and output of function machines. Function machines take a number as INPUT and give another number as OUTPUT.

$$\text{INPUT} \longrightarrow \boxed{\text{FUNCTION MACHINE}} \longrightarrow \text{OUTPUT}$$

Activity:Function machine activity

Calculate the output of the function machine below.

$$3 \longrightarrow \boxed{+5} \longrightarrow ?$$

To work out the output, we need to take the input and add it to 5. This gives an output of 8.

12.2 Exercise Set

1. Calculate the output of each of these function machines:

(a) $-4 \longrightarrow \boxed{\times 5} \longrightarrow ?$

(b) $12 \longrightarrow \boxed{-5} \longrightarrow ?$

(c) $24 \longrightarrow \boxed{\div 2} \longrightarrow ?$

(d) $5 \longrightarrow \boxed{+5} \longrightarrow ?$

2. Fill in the output on each of these function machines

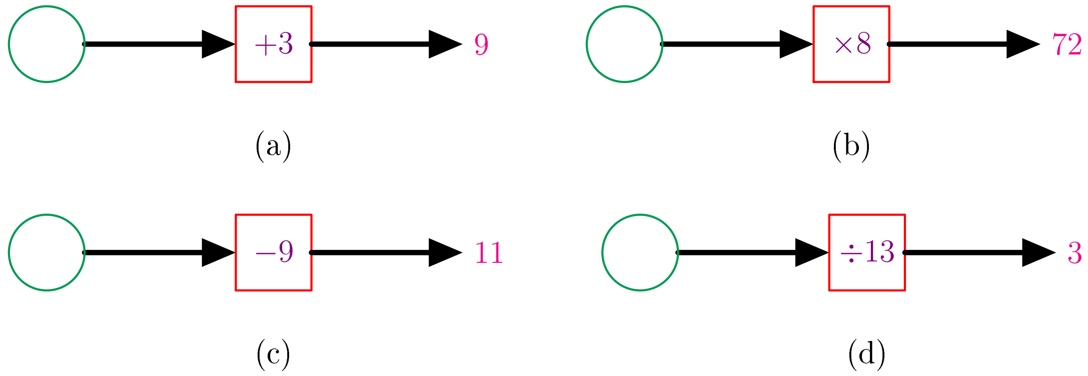
(a) $-2 \longrightarrow \boxed{+10} \longrightarrow \bigcirc$

(b) $9 \longrightarrow \boxed{\times 4} \longrightarrow \bigcirc$

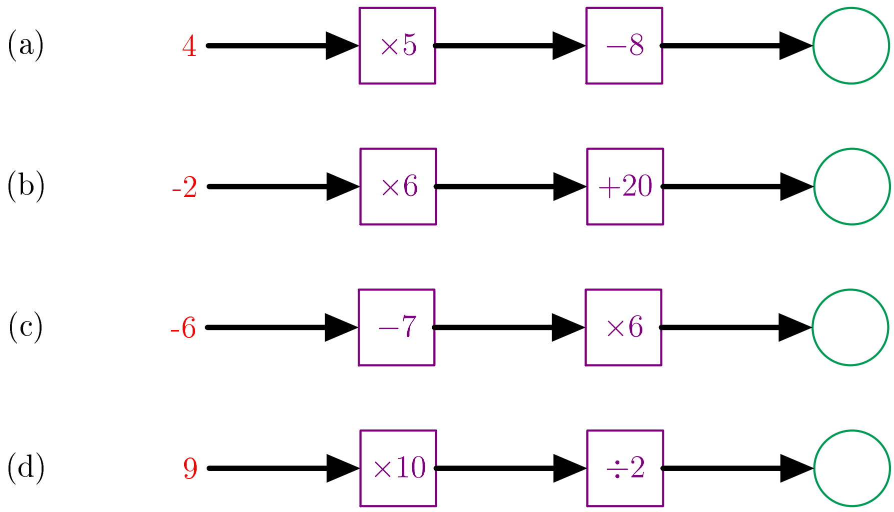
(c) $-2 \longrightarrow \boxed{-66} \longrightarrow \bigcirc$

(d) $24 \longrightarrow \boxed{\div 3} \longrightarrow \bigcirc$

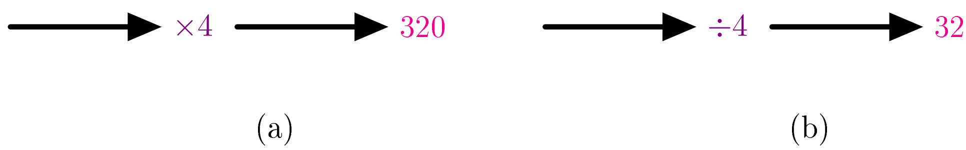
3. Fill in the input on each of these function machines.



4. Fill in the output on each of these double function machines.



5. What is the input of each of these function machines



6. A number is added to 10, and then 6 is subtracted to get 40. What is the number?

7. A number is multiplied by 10, and then 6 is added to get 36. What is the number?

8. Hannah asks his teacher, Tr stephen, how old he is. Tr stephen replies that if you double her age, add 14 and then divide by 3, you get 12.

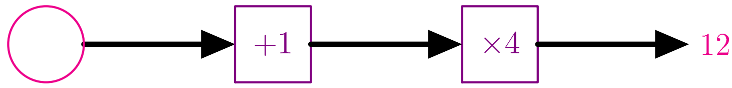
9. Alexadra is given her pocket money. She puts half in the bank and then spends UGX 30000 in one shop and UGX50,00O in another shop.She goes home with UGX 2400. How much pocket money was she given?

10. A bus leaves uganda to kenya with its maximum number of passengers from the bus station. At the first stop, half of the passengers get off. At the next stop 8 people get on and at the next stop 18 people get off. There are now 24 people on the bus. How many passengers were on the bus when it left the bus station?

11. Stephen transplanted a tomato plant. In the first week it doubles its height. In the second week it grows 8 cm. In the third week it grows 5 cm. What was the height of the plant when stephen transplanted it if it is now 35 cm in height?

12. Fill in the input on each of these double function machines.

(a)



(b)

(c)

(d)

12.3 Solving Linear Equations

- An equation is a mathematical statement that asserts the equivalence of two expressions.In other words an equation is a statement that two expressions have the same value.For example $2y + 5 = 9$is an equation
- A solution is a value that can be substituted for a variable to make an equation true.
- An unknown value is a variable in an equation that needs to be solved for.For example $2y + 5 = 9$,the unknown is y
- Solving the equation for the variable is the process of finding the solution of an equation
- Linear equations are equations which only contain number terms like 3 and 6 and single variables with no power such as $2x$ and $-4y$ .For example $2x + 5 = 2, 5 + y = 10$.Note, the powers to the variables for linear equations are raised to power one only.
- An equation will always contain an equal sign with an expression on each side.

Steps for solving linear equations

- Simplify each side of the equation if needed by distributing or collecting like terms
- Move variables to one side of the equation by using the opposite operation of addition and subtraction.i.e When moving any term from one side of the equation to the other, its sign changes

- Isolate the variable by applying the opposite operation to each side
  - First use the opposite operation of addition or subtraction
  - Secondly use the opposite operation of division or multiplication

NOTE:

- Whatever you do to one side of an equation, you must also do the same to the other side".
- If the equation contains brackets, first remove the brackets and then workout
- if the equation contains fractions, multiply each term by the LCM of the denominators to remove the fractions

EXAMPLES

1. Solve for x in the equation $x + 5 = 8$:

$$\begin{aligned} x + 5 &= 8 \\ x + 5 - 5 &= 8 - 5 \\ x &= 3 \end{aligned}$$

Subtract 5 from both sides of the equation

2. Solve for x in the equation $x - 5 = 7$:

$$\begin{aligned} x - 5 &= 7 \\ x - 5 + 5 &= 7 + 5 \\ x &= 12 \end{aligned}$$

Add 5 to both sides of the equation

3. Solve for the value of y in these equations

   (a) $4y = 12$

$$4y = 12$$

$$\frac{4y}{4} = \frac{12}{4}$$

Divide through out the equation by 4

$$\frac{\cancel{4}y}{\cancel{4}} = \frac{\cancel{12}^{3}}{\cancel{4}^{1}}$$

Simplify

$$y = 3$$

(b) $4y - 5 = 15$

$$\begin{aligned} 4y - 5 &= 15 \\ 4y - 5 + 5 &= 15 + 5 && \text{Add 5 to both sides of the equation} \\ 4y &= 20 \\ \frac{4y}{4} &= \frac{20}{4} && \text{Divide through out the equation by 4} \\ \frac{\cancel{4}y}{\cancel{4}} &= \frac{\cancelto{5}{20}}{\cancelto{1}{4}} && \text{Simplify} \\ y &= 5 \end{aligned}$$

(c) $3 - 2y = 7$

$$\begin{aligned} 3 - 2y &= 7 \\ 3 - 3 - 2y &= 7 - 3 && \text{Subtract 3 from both sides of the equation} \\ -2y &= 4 \\ \frac{-2y}{-2} &= \frac{4}{-2} && \text{Divide through out the equation by -2} \\ \frac{\cancel{-2}y}{\cancel{-2}} &= \frac{\cancelto{4}{-2}}{\cancelto{1}{-2}} && \text{Simplify} \\ y &= -2 \end{aligned}$$

(d) $5y - 6 = 6y - 5$

$$\begin{aligned} 5y - 6 &= 6y - 5 && \text{Collect like terms} \\ 5y - 6 + 6 &= 6y - 5 + 6 && \text{Add 6 to both sides of the equation} \\ 5y &= 6y + 1 \\ 5y - 6y &= 6y - 6y + 1 && \text{Subtact 6y from both sides of the equation} \\ -y &= 1 \\ \frac{-y}{-1} &= \frac{1}{-1} && \text{Divide through out the equation by -1} \\ \frac{\cancel{-}y}{\cancel{-1}} &= \frac{\cancelto{-1}{1}}{\cancelto{1}{-1}} && \text{Simplify} \\ y &= -1 \end{aligned}$$

12.3.1 Solving word problems on linear equations

In solving word problems on linear equation:

- Understand the problem. During this step, become comfortable with the problem. Some ways of doing this are to:

- Read and re read the problem.
- Choose a variable to represent the unknown.
- Construct a drawing whenever possible.
- Propose a solution and check. Pay careful attention to how you check your pro posed solution. This will help when writing an equation to model the problem.

- Translate the problem into an equation.
- Solve the equation.
- Interprete the results: Check the proposed solution in the stated problem and state your conclusion.

EXAMPLES

1. I think of a number, add 8 and the answer is 10.

   (a) Form a linear equation with the variable x

   Let the number be x

   $$x + 8 = 10$$

   (b) Find the number.

   $$x + 8 = 10$$
   $$x + 8 - 8 = 10 - 8$$
   $$x = 2$$

2. You ask a friend to think of a number. He then multiplies it by 5 and subtracts 7. He gets the answer 43

   (a) Use this information to write down an equation for x, the unknown number.
   Let the number be x

$$5x - 7 = 43$$

(b) Solve your equation for x

$$\begin{aligned} 5x - 7 &= 43 \\ 5x - 7 + 7 &= 43 + 7 \\ 5x &= 50 \\ \frac{5x}{5} &= \frac{50}{5} \qquad \text{Divide through out the equation by 5} \\ \frac{\cancel{5}x}{\cancel{5}} &= \frac{\cancel{50}^{10}}{\cancel{5}^{1}} \\ x &= 10 \end{aligned}$$

The number therefore is 10

12.3 Exercise Set

1. Solve fot the unknown in each of these equations

   (a) $x + 7 = 18$
   (b) $x - 14 = 28$
   (c) $x - 2 = -44$
   (d) $y - 9 = 1$
   (e) $-2 = t - 56$
   (f) $-17 = x + 3$
   (g) $r - 8.6 = -8.1$
   (h) $t - 9.2 = -6.8$
   (i) $7x + 2x = 8x - 3$
   (j) $3n + 2n = 7 + 4n$
   (k) $2x = 18$
   (l) $3x = 36$

2. Solve for x in these equations

   (a) $5x - 6 = 6x - 5$
   (b) $8x + 2 - 6x = 3 + x - 10$
   (c) $4x - 11 - x = 2 + 2x - 20$
   (d) $2x + 4 = 14$
   (e) $3x + 7 = 25$
   (f) $4x + 2 = 22$
   (g) $6x - 4 = 26$
   (h) $11x - 4 = 29$
   (i) $7x + 2x = 8x - 3$
   (j) $6x + 7 = 31$
   (k) $5x - 8 = 37$
   (l) $2x + 4 = 36$
   (m) $3(9x - 9) + 4 = 58$
   (n) $2(8x - 10) + 3 = 15$
   (o) $4(10x + 8) + 5 = 197$

3. Solve for x in these equations

   (a) $5x - 6 = 6x - 5$
   (b) $8x + 2 - 6x = 3 + x - 10$
   (c) $4x - 11 - x = 2 + 2x - 20$
   (d) $2x + 4 = 14$
   (e) $3x + 7 = 25$
   (f) $4x + 2 = 22$
   (g) $6x - 4 = 26$
   (h) $11x - 4 = 29$
   (i) $7x + 2x = 8x - 3$
   (j) $6x + 7 = 31$
   (k) $5x - 8 = 37$
   (l) $2x + 4 = 36$
   (m) $3(9x - 9) + 4 = 58$
   (n) $2(8x - 10) + 3 = 15$
   (o) $4(10x + 8) + 5 = 197$

4. Solve these equations, giving your answers as fractions or mixed numbers

(a) $7x - 6 = 5$
(b) $8x + 3 - 6x = -10$
(c) $4x = 22$
(d) $6x + 4 = 14$
(e) $3x + 7 = 26$
(f) $7x + 2 = 22$
(g) $6x = 26$
(h) $8x + 2 = 5$
(i) $2x = 3$
(j) $6x + 7 = 30$
(k) $9x = 1$
(l) $8x + 8 = 9$

5. Solve these equations

   (a) $\frac{5x}{4} = 10$
   (b) $\frac{y}{2} + y = 6$
   (c) $\frac{4}{5} + x = \frac{1}{5}$
   (d) $\frac{t}{20} = \frac{1}{10}$
   (e) $\frac{1}{3} = \frac{2}{3} + x$
   (f) $\frac{x}{2} - 3 = \frac{x}{3}$

6. Solve for y in the following equations:

   (a) $4(2y + 3) = 31 - 3(y - 1)$
   (b) $15(y - 7) - 3(y - 9) + 5(y + 6) = 0$
   (c) $10 - 2(y - 4) = 2(y - 1) - 6y$
   (d) $5(y - 1) = 3(2y - 5) - (1 - 3y)$

7. A man is 24 years older than his son. In two years time, his age will be twice the age of his son. Find the present age of the son

8. I think of a number, multiply by 6, take away 5. The answer is the same if I multiply it by 4 and then add 9.

   (a) Form a linear equation in terms of x
   (b) Find the number x

9. The sum of two consecutive odd numbers is 52. .

   (a) Form a linear equation in terms of t
   (b) Find the number t

10. The sum of the ages of 5 children born at intervals of 3 years each is 50 years. Find the age of the youngest child

11. I think of a number, take away three and then divide by 4. The answer is 3.

    (a) Form a linear equation in terms of y
    (b) Find the number y

12. I think of a number, multiply by 3, add 4. The answer is the same, if I add 10 to the number.

    (a) Form a linear equation in terms of k
    (b) Find the number k

13. Twice the sum of a number and 4 is the same as four times the number, decreased by 12. Find the number

14. Find two numbers such that one exceeds the other by 8 and their sum is 24

15. The sum of three consecutive multiples of 13 is 195. Find these multiples

Chapter 13: BUSINESS ARITHMETIC

Learning objectives

By the end of this topic,the learners should be able to

- Describe and calculate profit,loss,commission,interest insurance and discount.
- Express profit or loss as a percentage
- Solve simple interest problems

In this topic, you will learn how to calculate loss,profit and expressing them as percentages.Further more we shall learn about discount,simple interest ,commission and simple interest.

13.1 Profit and Loss

Buying and selling is part of any trade.The goods we use at home are bought from shops, markets and supermarkets. People who sell to us also buy from other wholesalers and sell them to us at a higher price. The extra money the goods are sold for is the profit. If the goods are sold at a lower price than the price at which they were bought, the difference is the loss.

TERMS USED

Cost price (C.P) is the price at which an item is purchased or The price at which the goods are bought

Selling price (S.P) is the price at which an item is sold

PROFIT= Selling price − Cost price	*LOSS*= Cost price − Selling price

EXAMPLES

1. A dress bought for UGX 15,000 was sold for UGX 20,000.

   (a) What was the cost price?
   UGX 15,000

   (b) What was the selling price?
   UGX 20,000

   (c) Calculate the profit.

$$\begin{aligned}\text{Profit } &= \text{Selling price-Cost price}\\ &= 20,000 - 15,000\\ &= \text{UGX } 5000\end{aligned}$$

2. John bought a radio at 10,000 UGX and sold it to his brother at 4,000 UGX.
   (a) What was the cost price?
   10,000UGX
   (b) What was the selling price?
   4,000UGX
   (c) Calculate the loss.

$$\begin{aligned} \text{Loss} =& \text{Cost price-Selling price} \\ =& 10,000 - 4,000 \\ =& \text{6000UGX} \end{aligned}$$

13.1 Exercise Set

1. Priscilla bought a radio at 60,000 UGX and sold it to his brother at 55,000 UGX. Calculate the profit or loss made on this item.
2. A crate of soda has 24 bottles. A shopkeeper bought it from the wholesale shop at 18,500 UGX. He sold each bottle at 1000 UGX.
   (a) What was the cost price?
   (b) What was the selling price?
   (c) Calculate the profit.
3. A trader bought a plot of land at Shs 4.8 million and during the Covid 19 crisis he decided to sell it at Shs 3.8 million.
   (a) What was the cost price?
   (b) What was the selling price?
   (c) Calculate the loss.
4. A goat which costs shs 50,000 was sold for shs 48,000.Find the Loss

13.2 Percentage Profit and Loss

We can express the profit or loss as a percentage using the formulae below:

1. Percentage profit $= \frac{\text{Profit}}{\text{Cost price}} \times 100$
2. Percentage loss $= \frac{\text{Loss}}{\text{Cost price}} \times 100$

3. A profit of 20% means the selling price is 120% of the cost price

$$\text{S.P} = \frac{120}{100} \times C.P$$

4. A loss of 20% means the selling price is 80% of the cost price

$$\text{S.P} = \frac{80}{100} \times C.P$$

EXAMPLES

1. A trader bought a radio at Shs 16,000 and sold it at Shs 20,000. Find his:

   (a) profits .

$$\begin{aligned} \text{Profit} &= \text{S.P} - \text{C.P} \\ &= 20,000 - 16000 \\ \text{Profit} &= \text{Shs}4000 \end{aligned}$$

   (b) percentage profit

$$\begin{aligned} \text{Percentage Profit} &= \frac{\text{Profit}}{\text{Cost price}} \times 100 \\ &= \frac{4000}{20,000} \times 100 \\ &= \frac{\cancel{4000}^{20}}{\cancel{20,000}} \times \cancel{100} \\ &= 20\% \end{aligned}$$

2. A trader bought a car at Shs 6 million and sold it at Shs 5 million. Find his:

   (a) loss

$$\begin{aligned} \text{Loss} &= \text{C.P} - \text{S.P} \\ &= 6000000 - 5000000 \\ \text{Loss} &= \text{Shs}1,000,000 \end{aligned}$$

   (b) percentage loss

$$\begin{aligned} \text{Percentage Loss} &= \frac{\text{Loss}}{\text{Cost price}} \times 100 \\ &= \frac{1000000}{6000000} \times 100 \\ &= \frac{1\cancel{000000}}{6\cancel{000000}} \times 100 \\ &= 16.67\% \end{aligned}$$

3. A book is bought at Shs 1,500 and sold at a profit of 40%. Find the selling price.

   A profit of 40% means the selling price is 140% of the cost pricei.e $100 + 40 = 140\%$

$$
\begin{aligned}
\text{S.P} &= \frac{140}{100} \times C.P \\
&= \frac{140}{100} \times 1500 \\
&= \frac{140}{\cancel{100}} \times 15\cancel{00} \\
&= \text{Shs } 2100
\end{aligned}
$$

4. A trader sold an item at Shs 7,000 and made a loss of $12\frac{1}{2}\%$ Find the cost price A Loss of $12\frac{1}{2}\%$ means the selling price is 87.5% of the cost pricei.e $100 - 12\frac{1}{2} = 87\frac{1}{2}\%$ or 87.5%

$$
\begin{aligned}
\text{S.P} &= \frac{87.5}{100} \times C.P \\
&= \frac{87.5}{\cancel{100}} \times 70\cancel{00} \\
&= \text{Shs } 6125
\end{aligned}
$$

13.2 Exercise Set

1. A radio is bought at Shs 15,000 and sold at Shs 16,200. Find
   (a) Profit
   (b) the percentage profit
2. A trader bought a motorcycle at Shs 4 million and sold it at Shs 3.5 million. Find his:
   (a) loss
   (b) percentage loss
3. A plot of land is bought at Shs 5 million and sold at a loss of 20%. Find the selling price
4. A company selling newspapers spends UGX 15,00 to produce a copy of the newspaper and sells it at UGX 2,000. On a given day, the company produced 2000 copies and managed to sell 1000 copies only.
   (a) Did the company make a profit or loss on that day?
   (b) Calculate the percentage profit/ loss for the day.

13.3 Discount

In the areas of competition, shops find ways of encouraging customers to buy. One way of encouraging customers is through reducing an amount from the usual marked price. This reduction in price is called Discount.

- Discount is a reduction in the selling price of an item
- Sale price = original price − discount
- Percentage discount = $\frac{\text{Discount}}{\text{Original price}} \times 100$
- A discount of 20% means the sale price is 80% of the original price

EXAMPLES

1. Kaziba paid shs.2800 for T-shirt in a sale, while the price tag shows shs.3000.
   (a) What is the Original price
   shs.3000
   (b) What is the sale price
   shs.2800
   (c) Find the discount

$$\begin{aligned}\text{Discount} &= \text{ original price } - \text{Sale price}\\ &= 3000 - 2800\\ \text{Discount} &= \text{Shs}200\end{aligned}$$

2. Sheila paid shs.28125 for T-shirt in a sale, while the price tag shows shs.31250.
   (a) What is the Original price
   shs.31250
   (b) What is the sale price
   shs.28125
   (c) Find the discount

$$\begin{aligned}\text{Discount} &= \text{ original price } - \text{Sale price}\\ &= 31250 - 28125\\ \text{Discount} &= \text{Shs}3125\end{aligned}$$

   (d) Find the percentage discount

$$\begin{aligned}\text{Percentage discount} &= \frac{\text{Discount}}{\text{Original price}} \times 100\\ &= \frac{3125}{31250} \times 100\\ &= \frac{\cancel{3125}}{\cancel{3125}\cancel{0}} \times 10\cancel{0}\\ &= 10\%\end{aligned}$$

3. An item costing Shs 18,000 was sold at Shs 16,000. Find the:

(a) the discount

$$\begin{aligned} \text{Discount} &= \text{ original price } - \text{Sale price} \\ &= 18000 - 16000 \\ \text{Discount} &= \text{Shs}2000 \end{aligned}$$

(b) percentage discount

$$\begin{aligned} \text{Percentage discount} &= \frac{\text{Discount}}{\text{Original price}} \times 100 \\ &= \frac{2000}{18000} \times 100 \\ &= 11.1\% \end{aligned}$$

4. An item costing Shs 60,000 was sold at a discount of 20% . Find:

   (a) how much was paid for it A discount of 20% means the sale price is 80% of the original price

$$\begin{aligned} \text{Sale price} &= \frac{80}{100} \times \text{Original price} \\ &= \frac{80}{100} \times 60,000 \\ &= \text{Shs}48,000 \end{aligned}$$

   (b) cash value of the discount

$$\begin{aligned} \text{Cash value of the discount } &= \frac{20}{100} \times \text{Original price} \\ &= \frac{20}{100} \times 60,000 \\ &= \text{Shs}12,000 \end{aligned}$$

13.3 Exercise Set

1. Find the percentage discount allowed when an item costing Shs 60,000 is sold at Shs 48,000
2. Stephen paid shs.200,000 for a phone in a sale, while the price tag shows shs.252000. Find
   (a) the discount
   (b) Percentage discount
3. Find the percentage discount allowed when an item costing Shs 2000,000 is sold at Shs 1800,000
4. A bicycle priced Shs 200,000 was sold at a discount of 15%. Find:

   (a) how much was paid for it
   (b) cash value of the discount

5. Sarah buys a dress for cash whose marked price is shillings 50,000. A shopkeeper offers 10% discount for cash payments.
   (a) How much is the discount?
   (b) How much does she actually pay for the dress?

6. The marked price of a watch is 46,500. The shopkeeper offers an off-season discount of 18% on it. Find its selling price.
7. The price of a sweater was slashed from 9600 shillings to 8160 shillings by a shopkeeper in a rainy season. Find the rate of discount given by him.
8. . Find the percentage discount being given on a shirt whose selling price is 54,600 shillings after deducting a discount of 10,400 on its marked price.

13.4 Commission

Agents and sales people who sell goods on behalf of some body else are usually paid a commission.

- Commission is a reward to the sales agent based on the level of sales
- Commission is usually a percentage of the value of goods sold

EXAMPLES

1. A sales agent gets a commission of 15% for selling goods. Find his commission for sales worth Shs 600,000

$$\begin{aligned}\text{Commission} &= \frac{15}{100} \times 600,000 \\ &= \text{Shs}90,000\end{aligned}$$

2. An agent receives a commission of 4% on goods sold for shs 70,000. Find her commission.

$$\begin{aligned}\text{Commission} &= \frac{4}{100} \times 70,000 \\ &= \text{Shs}2800\end{aligned}$$

13.4 Exercise Set

1. A sales agent gets a commission of 12% for selling goods. Find his commission for sales worth Shs 6,000,000
2. An agent receives a commission of 5% on goods sold for shs 50,000. Find her commission.

3. A salesman gets a fixed salary of£2000 per month and a commission of 2% on sale. If total sale for the month of April was £30,000, find his total salary for that month?

4. Joan makes a commission of 2% when a house is sold by his company. How much money will Joan make as a commission if her company sells the house for 300,000,000 shillings?

5. Mike makes a commission of 10% on each TV set sold at store. Each TV costs £120. How much money will he make as commission if the store sells 25 TV sets?

13.5 Simple interest

If you borrow money from a bank or other financial institution, the bank charges for the use of the money.This charge is called interest usually denoted by **(I)** .The interest is usually calculated as a Percentage Rate usually denoted by (R). Interest also depends on the length of Time (T) that the money is borrowed or invested for.

- The amount(money) borrowed or lent is called the principal(P)
- The reward to the lender is called interest
- In solving simple interest problems, the following relations apply:
  (i) Simple interest $= \frac{\text{Principal} \times \text{Rate} \times \text{time}}{100}$
  (ii) Amount = principal + interest

EXAMPLES

1. Find the simple interest on Shs 25,000 for 3 years at a rate of 8% per annum

$$\begin{aligned}\text{Simple interest} &= \frac{\text{Principal} \times \text{Rate} \times \text{time}}{100} \\ &= \frac{25000 \times 3 \times 8}{100} \\ &= \text{Shs}6000\end{aligned}$$

METHOD 2: (STEP BY STEP METHOD)

$$\begin{aligned}\text{First year interest} &= \frac{8}{100} \times 25,000 \\ &= \text{Shs}2,000 \\ \text{Second year interest} &= 2000 + 2000 \\ &= 4000 \\ \text{Third year interest} &= 2000 + 4000 \\ &= \text{Shs}6000\end{aligned}$$

2. Find the amount to which Shs 80,000 accumulates in 9 months at a simple interest rate of

15% per annum

$$\begin{aligned}\text{Simple interest} &= \frac{\text{Principal} \times \text{Rate} \times \text{time}}{100}\\ &= \frac{80000 \times \frac{9}{12} \times 15}{100}\\ &= \text{Shs}9000\end{aligned}$$

$$\begin{aligned}\text{Amount} &= \text{principal} + \text{interest}\\ &= 80,000 + 9000\\ &= \text{Shs}89000\end{aligned}$$

13.5 Exercise Set

1. Find the simple interest on sh. 10,000 for 2 years at 4% per annum.
2. Find the simple interest on sh. 25 000 for 3.5 years at 18% per annum.
3. Find how long will it take for a sum of Shs 80,000 to yield an interest of Shs 12,000 at a rate of 5% per annum simple interest
4. Find the principal that yields a simple interest of Shs 27,000 in 9 years at a rate of 6% per annum
5. A man borrowed Shs 15.6 million from a bank at a simple interest rate of 15% per annum. He has to repay the loan within 2 years in equal weekly instalments. Calculate the:
   (a) interest he paid to the bank
   (b) total amount to be paid
6. How much interest is earned on 5,000,000 at 4% for seven years?
7. Jane borrowed 2,250,000 shillings from the bank for eight years at an interest rate of 6%. How much interest will she pay?
8. If you put 624,000 shillings into a savings account that earns 5%, how much money will you have at the end of four years?

13.6 Insurance

Insurance is a means of protection from financial loss. It is a form of risk management. An entity which provides insurance is known as an insurer, insurance company, insurance carrier or underwriter. A person or entity who buys insurance is known as an insured or as a policyholder. The amount of money charged by the insurer to the policyholder for the coverage set forth in the insurance policy is called the premium. There are many different types of insurance policies .Below are a few of the main types of insurance

1. Life insurance
2. Burial insurance
3. Motor vehicle insurance
4. Property insurance
5. Pecuniary insurance
6. Marine insurance
7. Accident insurance
8. Liability insurance

Activity of Integration

- Support: pens, paper, internet
- Resources:Knowledge of profit,loss ,insurance ,discount and commission.
- Task:As a senior one student,come up with a business idea,which you can use to generate pocket money.Show how you will generate profits,curb lossess and safe guard your business from any risk.

Chapter 14: TIME AND TIME TABLES

Learning objectives

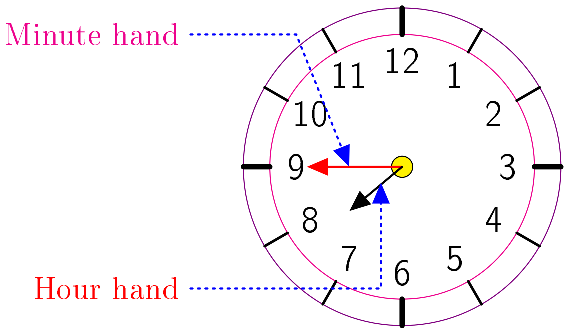
By the end of this topic,the learners should be able to

- Identify and use units of time
- Use and interpret different representations of time
- Apply understanding of time in a range of relevant real life contexts

In this topic, you will learn various units of time, such as minutes, seconds, hours, day, week, month, year. You will be able to understand and apply time in a range of relevant real-life contexts.Further more you will also learn how to make and read time tables

14.1 Telling the Time

- *On a clockface the large hand of the clock points to the minutes*
- On a clockface the small hand on the clock points to the hours

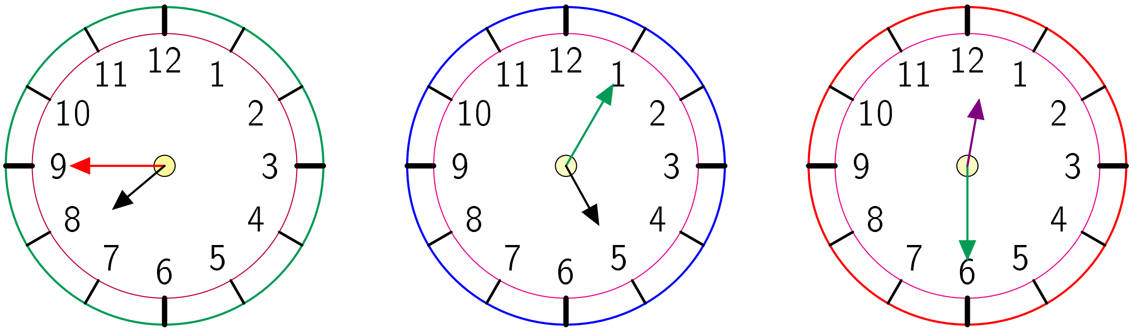


Recall

$$\text{One hour} = 60 \text{ minutes}$$
$$\text{Three quarters of an hour} = 45 \text{ minutes}$$
$$\text{Half an hour} = 30 \text{ minutes}$$
$$\text{Quarter of an hour} = 15 \text{ minutes}$$
$$\text{One minute} = 60 \text{ seconds}$$
$$\text{One hour} = 3600 \text{ seconds}$$

EXAMPLES

1. Write each of the times shown on these clocks:



(a) (b) (c)

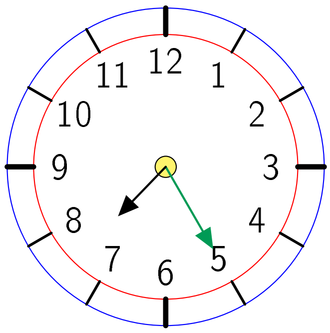
	Time in words	Time in digits
(a)	Quarter to eight o'clock OR Forty five minutes past seven o'clock	7 : 45
(b)	Five minutes past five o'clock	5 : 05
(c)	Thirty minutes past twelve o'clock OR Thirty minutes to one o'clock	12 : 30

2. Write each time using digits and show the position of the hands on a clockface:

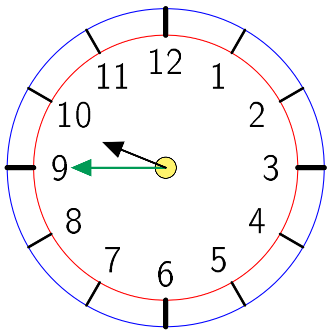
   (a) twenty five minutes past seven o'clock

   (b) quarter to ten o'clock.

(a) *Twenty five minutes past seven o'clock using digits is* 7 : 25



(b) *Quarter to ten o'clock using digits is* 9 : 45



14.1 Exercise Set

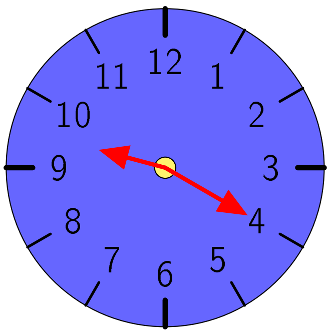
1. Draw these times on clock faces:

(a) Twenty minutes past two o'clock
(b) Twenty minutes to two o'clock
(c) Ten minutes to nine o'clock
(d) Ten minutes past nine o'clock
(e) Quarter to seven
(f) Half past ten
(g) Twenty five minutes past six o'clock
(h) Twenty five minutes to six o'clock

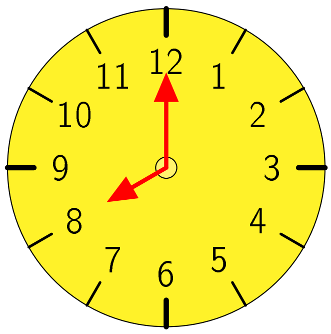
2. Draw these times on clock faces:

(a) 5 : 00
(b) 8 : 25
(c) 3 : 55
(d) 7 : 20
(e) 11 : 45
(f) 2 : 10
(g) 7 : 05
(h) 1 : 50
(i) 6 : 55

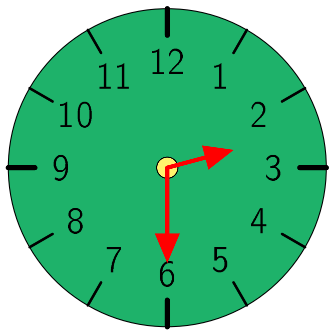
3. Write the times shown on each of these clocks in words and digits.



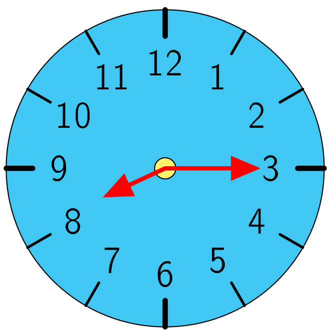
(a)



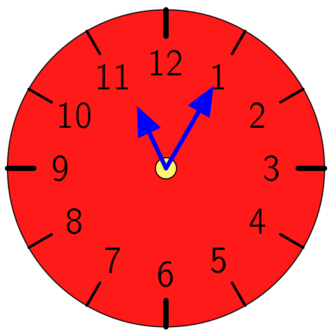
(b)



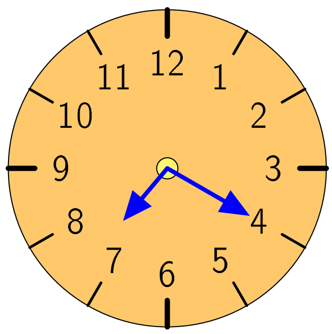
(c)



(d)



(e)



(f)

4. Write these times in words:

(a) 5 : 00
(b) 8 : 30
(c) 3 : 55
(d) 7 : 20
(e) 11 : 45
(f) 2 : 10
(g) 7 : 05
(h) 1 : 50
(i) 8 : 55

5. Write these times using digits:

(a) Twenty minutes past two o'clock
(b) Twenty minutes to two o'clock
(c) Ten minutes to nine o'clock
(d) Ten minutes past nine o'clock
(e) Quarter to seven
(f) Half past ten
(g) Twenty five minutes past six o'clock
(h) Twenty five minutes to six o'clock
(i) Twenty five to nine
(j) Twenty to three

6. Alexadra looks at her watch and sees that the time is 7 : 55.
   (a) Write this time in words.
   (b) What will be the time 10 minutes later?
7. Ethan looks at his watch and sees that the time is 1 : 45.
   (a) Write this time in words.
   (b) What will be the time 15 minutes later?
8. A bus leaves school at five minutes past five and Ethan gets off 20 minutes later.
   (a) What is the time when Ethan gets off the bus in digits?
   (b) Write your answer in words
   (c) Represent your answer on a clock face

14.2 12−hour and 24−hour Clocks

- The 12−hour clock notation uses am and pm to indicate morning and afternoon respectively.
- *am* is the time from 12 midnight and before 12 noon.
- *pm* is the time from 12 midday and before 12 midnight.
- The 24−hour clock uses the numbers 00 : 00 to 23 : 59 (midnight is 00 : 00).
- In the 24 hour clock, there are no 'am' and 'pm' labels.
- The hours start at zero and go throught to 23.
- All 24 hour clock times should be written hh:mm or hh:mm:ss, where h is the hour, m is the minute and s is the seconds
- Sometimes the colons are omitted bewteen the hours and minutes.
- The minutes and seconds never change when changing between 24 hour and 12 hour times.

14.2.1 Converting from 12 hour times to 24 hour clock

The following simple steps will help you change a 12 hour time to a 24 hour time.

- If the hour is exactly 12pm, then simply remove the 'pm' label.e.g 12 : 07pm = 12:07 or 1207
- If the hour is 12am, then change it to 00.
- If the hour is a pm time, then simply add 12 to the hour.
- The hour in a.m does not change. If it is less than 10, just add a zero before the digit.

EXAMPLES.

1. Write these times in 24-hour clock time:

   (a) 3.06 a.m.

   3.06a.m = 03 : 06 or 0306 The hour is in am,and is less than 10, then add a zero before the d

   (b) 11 : 32am
   In 24−hour is 1130 or 11 : 30

   (c) 12 : 30am
   In 24−hour is 0030 or 00 : 30

   (d) 3 : 06pm

$$\begin{array}{r} 3:06 \\ +12:00 \\ \hline 15:06 \\ \hline \end{array}$$

   Therefore 3 : 06pm is 15:06 or 1506

   (e) 8 : 14pm

$$\begin{array}{r} {}^{1}8:14 \\ +12:00 \\ \hline 20:14 \\ \hline \end{array}$$

   Therefore 8 : 14pm is 20:14 or 2014

14.2 Exercise Set

1. Convert these times to 24-hour clock times:

(a) 5 : 00 a.m	(e) 11 : 45 a.m	(i) 8 : 55 p.m
(b) 8 : 30 a.m	(f) 2 : 10 p.m	(j) 7 : 13 p.m
(c) 3 : 55 p.m	(g) 7 : 05 p.m	(k) 9 : 50 p.m
(d) 7 : 20 p.m	(h) 1 : 50 a.m	(l) 4 : 59 p.m

2. Write these times in 24-hour clock time:

(a) Quarter to nine in the evening

(b) Ten minutes to miday

(c) Quarter to eight in the morning

(d) Ten minutes to midnight

(e) Ten past nine in the evening

(f) Five to seven in the morning

(g) Quarter past five in the afternoon

(h) Half past two in the afternoon

3. Taibah international school runs the following daily programm.

Activity	Time
Break fast	7:00am
Registration	8:00am
Lesson 1	8:30am
Lesson 2	9:30am
Break fast	10:30am
Lesson 3	11:30am
Lesson 4	12:30pm
Lunch	1:30pm
Lesson 5	2:30pm
Lesson 6	3:30pm
Sports	4:40pm
Supper	6:10pm
Evening preps	7:15pm

(a) Convert the time for school activities to 24-hour clock time

(b) Write them in words

4. A school bus leaves at 0030. Write this time in words.

14.2.2 Converting from 24 hour times to 12 hour times

The following simple steps will help you change a 24 hour time to a 12 hour time with 'am' and 'pm'.

- If the hour is exactly 12, then simply label it as a pm time.
- If the hour is 00, then change it to 12 and label it as an am time.
- If the hour is greater than 12, then simply subtract 12 from the hour and label it as a pm time.
- If the hour is less than 10, simply label it as an am time and take away any leading zeros.

EXAMPLES.

1. Write these times using 'a.m.' or 'p.m':

(a) 0742.
In $12-$hour is $7:42$a.m

(b) 0845am
In 12−hour is 8 : 45a.m

(c) 15 : 06

$$\begin{array}{r} 15:06 \\ -12:00 \\ \hline 3:06 \\ \hline \end{array}$$

Therefore 1506 is 3 : 06pm

(d) 2014

$$\begin{array}{rrl} \overset{1}{\cancel{2}} & 10\ \cancel{0} & :14 \\ -\ 1 & 2 & :00 \\ \hline & 8 & :14 \\ \hline \end{array}$$

Therefore 2014 is 8 : 14pm

14.3 Exercise Set

1. Write these 24-hour clock times in 12-hour clock times, using 'a.m.' or 'p.m

   (a) 1808
   (b) 0345
   (c) 2224
   (d) 1204
   (e) 0822
   (f) 1842
   (g) 1330
   (h) 1440
   (i) 2305
   (j) 1735
   (k) 1605
   (l) 0342
   (m) 1430
   (n) 1555
   (o) 0745
   (p) 0835

2. Insert 'a.m.' or 'p.m.' into each sentence so that it makes sense:

   (a) Frank woke up at 6:45
   (b) John came home from school at 3:55.
   (c) Philemon started his night shift at 10:15
   (d) Grace ate her lunch at 11:45.
   (e) James went to bed at 8:55.
   (f) Hannah cooked a meal at 5:15.

3. The premier league games were scheduled as follows.

Match	Time
Liverpool fc vs wolves	1900
Arsenal fc vs spurs	2100
Chelsea vs Hull city	2315
Marchester united vs Everton	2245
Marchester city vs Southampton	2030
Burnley vs Aston villa	2020

   (a) Convert the time for the match to 12-hour clock time
   (b) Write them in words

4. Stephen leaves home at 0900 and returns 7 hours later. Write the time that Stephen gets home in 12-hour clock time using 'a.m.' or 'p.m.'

14.3 Units of time

A unit of time or midst unit is any particular time interval, used as a standard way of measuring or expressing duration

1 hour	60 minutes
1 minute	60 seconds
1 day	24 hours
1 week	7 days
1 year	365 days or 366 days in a leap year
1 year	12 months
February	28 days (29 days in a leap year)
April	30 days
May	31 days
June	30 days
July	31 days

NOTE
[2]A leap year (also known as an intercalary year or bissextile year) is a calendar year that contains an additional day added to keep the calendar year synchronized with the astronomical year or seasonal year.Each leap year has 366 days instead of 365, by extending February to 29 days
EXAMPLES

1. How many hours are there in April?

$$\begin{aligned} \text{Number of hours in April} &= 30 \times 24 \\ &= 720\text{hours} \end{aligned}$$

2. How many hours are there in a week

$$\begin{aligned} \text{Number of hours in a week} &= 7 \times 24 \\ &= 168\text{hours} \end{aligned}$$

3. How many seconds are there in a day

$$\begin{aligned} \text{Number of seconds in a day} &= \text{Number of hours in a day} \times \text{Number of seconds} \\ &= 24 \times 60 \\ &= 1440\text{seconds} \end{aligned}$$

4. 25 February is a Friday. What will be the date on the next Friday

Friday	25
Saturday	26
Sunday	27
Monday	28
Tuesday	1
Wednesday	2
Thursday	3
Friday	4

(a) if it is not a leap year.
So the next Friday will be 4 March.
Method 2

$25 + 7 = 32$	Add the number of days in a week (7)
$32 - 28 = 4$	Subtract the number of days for the month of februaryin a common year(28)

So the next Friday will be 4 March.

(b) if it is a leap year?

$25 + 7 = 32$	Add the number of days in a week (7)
$32 - 29 = 3$	Subtract the number of days for the month of february in a leap year(29)

So, in a leap year, the next Friday will be 3 March.

14.4 Exercise Set

1. How many hours are there in a week?
2. How many hours are there in:
   (a) September
   (b) February (2 answers needed).
   (c) one year (2 answers needed).
3. How many minutes are there in:
   (a) Two days
   (b) One week
   (c) Fortnight
   (d) Three weeks
4. How many seconds are there in:
   (a) Two hours
   (b) One day
   (c) Month of january
   (d) In 90 minutes
5. If 25^{th} March is a Friday, what will be the date on the following Friday?

6. Hannah goes on holiday on Monday 20th June. She returns 14 days later.On what date does she return from her holiday

7. If 3rd October is a Monday:

   (a) what day of the week will 1st November be.

   (b) what will be the date of the first Monday in November?

8. Hannah goes to the bank every Tuesday. The last time she went was on Tuesday 20th October.

   (a) What will be the dates of her next 2 visits to the bank?

   (b) On the second Tuesday in November she is ill and goes to the bank on Wednesday instead. What is the date of that Wednesday?

9. This year Stephen's birthday is on a Saturday in June. What day will his birthday be on next year if:

   (a) next year is a leap year.

   (b) next year is not a leap year?

10. In 2010, Christmas Day was on a saturday. Name the day of the week for Christmas Day in:

    (a) 2019
    (b) in you birth year
    (c) 2008
    (d) 2014
    (e) 2009
    (f) 2011

14.4 Timetables

In this section we consider how to extract information from timetables.

14.5 Exercise Set

1. Use the train timetable below to answer these questions:

Kampala	depart	1903	1915	1930	2000
Mukono	depart	1935	1945	2003	2030
Lugazi	arrive	2009	2015	2036	2103
Buikwe	arrive	2022	-	-	2250
Jinja	arrive	-	2051	2117	-
Busia	arrive	-	-	2257	0106

   (a) If you catch the 1915 from kampala, at what time would you arrive in Lugazi?

   (b) If you catch the 1935 from Mukono, at what time would you arrive in Buikwe?

   (c) Akasha arrives in Jinja at 2117. At what time did he leave Kampala?

(d) Christine catches the 2250 at Buikwe. At what time does she arrive in Busia?

2. The table below gives the timetable for a bus that runs from kampala bus park to Mbarara:

Kampala	depart	0857
Busega	depart	0930
Mpigi	arrive	1657
Masaka	arrive	1723
Mbarara	arrive	1842

(a) At what time does the bus leave kampala?

(b) At what time does the bus arrive at Mbarara?

(c) Where does the bus arrive at 1657?

(d) Bah arrives at kampala at five past nine. Can he catch the bus?

3. The Journey from Kabale (Uganda) to Kigali (Rwanda) takes $2\frac{1}{2}$hours. The time in Uganda is 1 hour ahead of Rwanda.

(a) If you leave Kabale at 10 : 00, what will be the local time when you arrive in Kigali?

(b) If you leave Kigali at 17 : 45, what will be the local time when you arrive in Kabale?

4. The Journey from Uganda) to America takes $10\frac{1}{4}$hours. The time in Uganda is 8 hours ahead of America.

(a) If you leave Entebbe at 9 : 00, what will be the local time when you arrive in America?

(b) If you leave America at 17 : 45, what will be the local time when you arrive in Entebbe?

5. Jean earns UGX 4,000 per hour on weekdays, UGX 4,500 per hour on Saturdays and UGX 6,000 per hour on Sundays.

Day	Hours worked for
Monday	4
Tuesday	3
Wednesday	2
Thursday	8
Friday	7
Saturday	4
Sunday	5

(a) How much money did Jean earn on Saturday?

(b) How much money did Jean earn on Sunday?

(c) How much money did Jean earn that week?

6. Ethiopian air lines has the the schedule below

(a) At what time does the Aeroplane leave Entebbe on wednesday?

Flight number	Frequency	Departure air port	Departure time	Arrival air port	Arrival time	Sub fleet
ET0760	mon,wed	Entebbe	21:45	Kenya	3:45	ET0761
ET0750	mon,fri	Entebbe	0340	Rwanda	0420	ET0763
ET0765	Tue,thur	Entebbe	0630	Tanzania	0835	ET0767
ET0760	sun,sat	Entebbe	0945	Qatar	2330	ET0768

(b) At what time does the Aeroplane arrive at Rwanda air port?

(c) Where does the Aeroplane arrive at 2330 ?

(d) How long does it take to fly from Entebbe to Tanzania?

(e) How long does it take to fly from Entebbe to Qatar

7. As of January 2020, Uganda Airlines operates flights to the following destinations

Flt No	Frequency	Departure air port	Departure time	Arrival air port	Arrival time
UR360	Mon,Wed	EBB	1545	BJM	1600
UR361	Frid,Sun	BJM	1955	EBB	2210
UR204	Mon,Tue,Wed,Thu,Fri,Sat,Sun	EBB	2005	NBO	2120
UR203	Sat ,Sun	NBO	0945	EBB	1100
UR122	Mon	EBB	0700	JUB	0825
UR121	Tue,Wed,Thu,Fri	JUB	1150	EBB	1315
UR320	Sat,Sun	EBB	1145	DAR	1335
UR321	Tue,Thu	DAR	1635	EBB	1825

(a) Calculate the time taken for the aeroplanes to travel from one air port to the other,fro all the flight numbers.

(b) At what time does the Aeroplane arrive at NBI?

(c) Where does the Aeroplane arrive at 1335 ?

(d) What is the furthest airport from Entebbe?

(e) What is the nearest airport from any Entebbe?

Activity of Integration

Secondary schools have morning and evening preps .The morning preps start at 5:00am and end at 6:30am,While the evening preps start at 6:15pm and end at 9:15pm.Each science subject must have 2hours on the time table

- Support: Mathematical instruments, pencil, paper, pens,colors
- Resources:Knowledge of time tables,time,subjects offered
- Task:As a senior one student ,design a personal time table ,clearly indicating the time given to each subject offered and the day of the week.

Key Words

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