Understanding

SENIOR TWO MATHEMATICS

BASED ON THE NEW LOWER SECONDARY CURRICULUM

KAZIBA STEPHEN

1^{ST} EDITION 2021

UNDERSTANDING

SENIOR TWO MATHEMATICS

BASED ON THE NEW LOWER SECONDARY CURRICULUM

by

KAZIBA Stephen

Key Topics covered in the book

- *MAPPINGS AND RELATIONS*
- *VECTORS AND TRANSLATION*
- *GRAPHS*
- *NUMERICAL CONCEPT 1(Indices and logarithms)*
- *INEQUALITIES AND REGIONS*
- *ALGEBRA 2*
- *SIMILARITIES AND ENLARGEMENT*
- *CIRCLE*
- *ROTATION*
- *LENGTH AND AREA PROPERTIES OF TWO DIMENSIONAL GEOMETRICAL FIGURES*
- *NETS,AREAS AND VOLUMES OF SOLIDS*
- *NUMERICAL CONCEPT 2(INDICES ,LOGARITHMS AND SURDS)*
- *SET THEORY*

.

1st EDITION 2021

The only way to learn mathematics is to do mathematics.

To the students

This book continues to help you to learn ,enjoy ,understand and progress through mathematics in the new lower secondary curriculum.As well as a clear and concise text the book offers a wide range of learning activities and practical questions that are relevant to the mathematics you are learning.

I have included exercises at the end of each subsection that will help you to understand the work, further more an assessment is included at the end of each chapter for extra practice

Remember if you dont understand something ,ask some one/friend who can explain it to you,if you still dont understand ,ask again ,again and again.
I am going to be completing the book very soon and the complete version will be out early August 2021.

NOTE:This book is free ,and available on the internet for easy access

Chapter 0

TABLE OF CONTENTS

Chapter 1

MAPPINGS AND RELATIONS

Learning objectives

By the end of this topic,the learners should be able to

- Use arrow diagrams /mappings to represent relations and functions
- Identify domain and range of a mapping
- Draw a papygram
- Describe and distinguish between function and non function mapping
- Evaluate functions using the given domains

Introduction

In mathematics, we study relations between two sets of numbers, where members of one set are related to the other set by a rule.A relation shows a connection(relationship) between sets of values(numbers),items,events or people

Activity

- Write down as many as possible the biological relations you have with your family members.
- Write down any four relations you have among your classmates.
- Are there any students in your school that you are biologically related to? If any, write down their names and the relations you have with them.
- Write down any four mathematical relations you know.

From the activity ,you have realized that you have many relations. You have used statements such as Sylvia is **my aunt** and so on. In a statement such as 'Stephen **is the uncle of** Martin'; the phrase **"is the uncle of"** indicate that there is a biological connection between Stephen and Martin. This tells us that relation is a connection between two or more things.

Examples of mathematical relations

For a set $A = \{1, 2, 3, 4, 5, 6, 9\}$

- 2 is less than 3.
- 4 is greater than 2
- 6 is a multiple of 3.
- 2 is a factor of 6.
- 2 is half of 4.
- 9 is a square of 3

1.1 Relation

A relation is a pairing of input values with output values. Relations are also described as mappings. When we map a set of numbers onto another set of numbers, we often express the rule for the mapping using mathematical relationships instead of words.

1.1.1 Representing relations

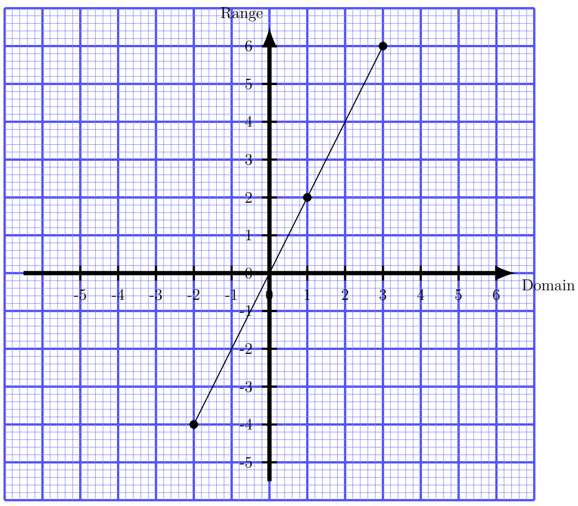
In this example, shown below, we define a relation between the set A=$\{-2, 1, 3\}$ and the set B= $\{-4, 2, 6\}$ as 'multiply by 2'. Notice that -2 is mapped onto -4, 1 onto 2,and 3 onto 6.Therefore we can represent the relation between sets A and B in the following ways.

- Ordered pairs
  Relations can be shown as a set of ordered pairs (x,y), where x is an input and y is an output(Input , output).The ordered pair preserves the directional property of the relation. .For example Daniel is a brother to Timothy.The relation is a brother and the ordered pair is (Daniel,Timothy).It is also consistent with the order of points plotted on a Cartesian Plane represented by (x, y): $(-2, -4), (1, 2)$ and $(3, 6)$

- Table
  We can also use a table to show the connection between the input ($x-$values) and output ($y-$values)

x	-2	1	3
y	-4	2	6

- Gaph
  When representing our relation on the cartesian plane ,we plot (x, y).



- Mapping diagram(Arrow diagram)
  This is shown by drawing arrows to connect members of the set A to the members of set B.We refer to the members of the set A as the input and members of the set B as the output. The direction of the arrows is always from the input to the output.

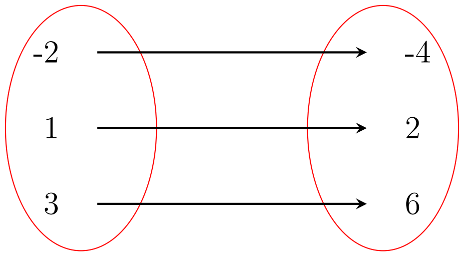


Figure 1.1: A mapping diagram(Arrow diagram)

In a mapping notation ,we describe the relationship that exists between the input and output values.i.e What has been done to the input elements to produce the output elements.For the example above ,our mapping notation is $x \longrightarrow 2x$ for $x = -2, 1, 3$

NOTE:A relation exists between two sets of numbers if we can find a rule that maps members of the first set (domain) onto members of the second set (codomain). The rule must hold for all possible pairs that are connected.

1.2 Domain and Range

- Domain(D):Is the set of input values for a relation.i.e The set of all values that the first elements in the ordered pair can take.
- Range(R):Is the set of output values for a relation.i.e The set of all values that the second elements in the ordered pair can take

Example 1.2.1

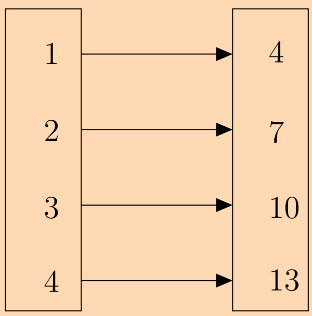
State the domain and range of the relation $\{(-2, -4), (1, 2), (3, 6)\}$

SOLUTION

- Domain=$\{-2, 1, 3\}$
- Range=$\{-4, 2, 6\}$

Example 1.2.2

Consider the relation on the arrow diagram



- Write down the ordered pairs of the mapping.
- Determine the domain and range

SOLUTION

- Ordered pairs=$\{(1,4),(2,7),(3,10),(4,13)\}$
- Domain=$\{1,2,3,4\}$
- Range=$\{4,7,10,13\}$

Example 1.2.3

For the mapping $x \longrightarrow 2x+1, x \in \{-1,0,1,2\}$.

(a) Construct a table of values

(b) State the range of the relation

(c) Represent the relation on an arrow diagram

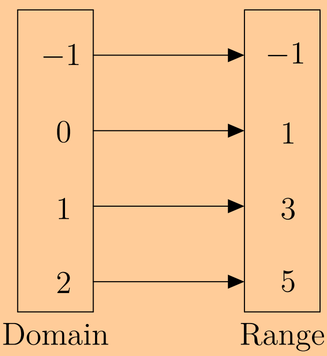
SOLUTION

(a) Table of values

x	$2x+1$
-1	-1
0	1
1	3
2	5

(b) Range=$\{-1,1,3,5\}$

(c) Arrow diagram



1.1 Exercise Set

1. State the domain and range for each of the relation shown

   (a) $\{(-3,-2),(5,8),(6,9),(7,5)\}$

   (b) $\{(-1,1),(4,11),(5,13),(1,5)\}$

   (c) $\{(-2,4),(-1,4),(0,4),(2,4)\}$

   (d) $\{(1,1),(2,4),(3,9)\}$

   (e) {(Syson,84),(Stacy,94),(Pauline,74)}

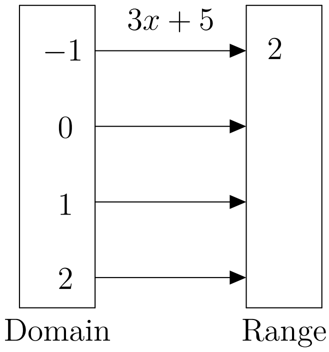
   (f) {(Museveni,1),(Robert,2),(Patrick,3)}

   (g) $\{(-1,1),(0,0),(2,4),(3,9)\}$

   (h) {(A,a),(B,b),(D,d),(E,e)}

2. Given set A = {0, 1, 2, 3, 4, 5} and the relation "multiply by 2 ", Find set B, the range.

3. Copy and complete this arrow digram.State the range for the relation



4. For the mapping $x \longrightarrow 4x$,State the missing value in each ordered pair

   (a) $(-3, -)$ (c) $(-2, -)$ (e) $(-, -4)$ (g) $(-, 12)$

   (b) $(-1, -)$ (d) $(1, -)$ (f) $(-, 8)$ (h) $(-, 16)$

5. For the mapping $x \longrightarrow 4x$,

   (a) State the missing value in each ordered pair

      (i) $\{(-2, -), (0, -), (1, -), (3, -)\}$ (ii) $\{(-, -4), (-, 4), (-, 8)\}$

   (b) Represent the relations on an arrow diagram

   (c) State the domain and range for the relations.

6. For the mapping $x \longrightarrow 2x - 4, x \in \{-3, 0, 2, 4\}$

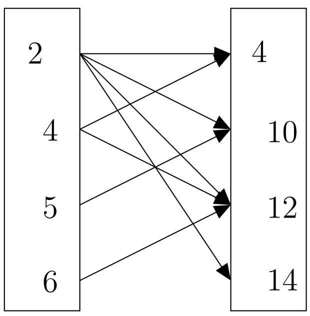
   (a) Construct the table of values for the mapping

   (b) Represent the relation on an arrow diagram

   (c) State the ordered pairs of the relation

   (d) State the range of the relation

7. Write the ordered pairs for the relation shown on the mapping diagram



8. Consider the domain A = {0, 1, 3, 4} set A is mapped onto set B by the relation x $\longrightarrow$ 2x. Find the range, the ordered pair and hence draw the graph of the relation

1.3 Mapping

Activity

- Write down set A of the first six whole numbers.
- Form another set B by squaring each number in set A above.
- Write the numbers in set A in an ellipse/rectangle and their squares in set B in a different ellipse/rectangle on the same level side by side.
- Use arrows to match the numbers in set A to their squares in set B. What would you call such matching?
- We say that squaring elements in set A maps onto set B. Your diagram showed the mapping. Set A is called the **domain**(Input) and set B is called the **range**(Output).

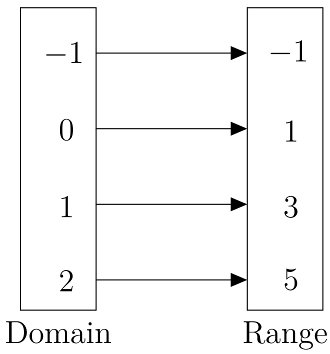
1.3.1 Types of Mappings

There are four different types of mappings:

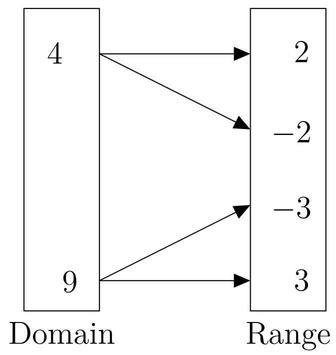
- One to One mapping:
  This is a mapping in which each element in the domain is mapped onto a unique element in the range.For example $x \longrightarrow 2x+1$, $x \in \{-1, 0, 1, 2\}$

x	-1	0	1	2
$2x+1$	-1	1	3	5

  Range=$\{-1, 1, 3, 5\}$



- One to many mapping:
  This is a mapping where an element of the domain is mapped onto more than one element of the range.For example $x \longrightarrow \sqrt{x}$,$x \in \{4, 9\}$. $\sqrt{4} = \pm 2$,$\sqrt{9} = \pm 3$.Therefore the range is $\{2, -2, 3, -3\}$



Activity

In your groups, work as pairs ,and carry out the following activity.

1. Give three names of your siblings. List the names down on a piece of paper.
2. On the domain set A, write the names of each one of you in the group.
3. On the range set B, write the six names of the siblings listed in step 1 above.
4. Match the name on the domain set A to the names of the siblings in the range set B with arrows.
5. What name would you call such a mapping?

- Many to one mapping:

Activity

Some S2 students of Taibah international school obtained the following grades:

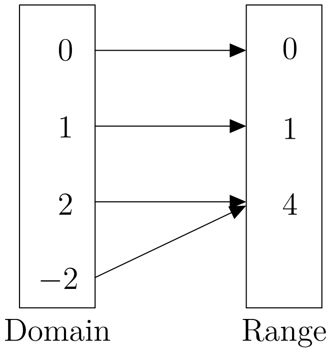
Name	Keziah	Akasha	Priscilla	Mimmi	Philemon	Stacy
Grade	D1	C3	D2	D1	C3	D1

1. Use the names as the domain and the grades as the range to show the mapping on a mapping diagram.
2. What name would you call such a mapping?
3. From the activity, you have observed that several elements in the domain set can be mapped onto one element in the range set

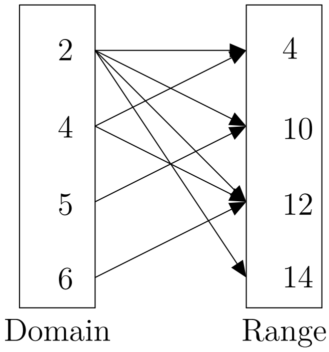
Many to one is a mapping where more than one element of the domain is mapped onto the same element of the range.i.e Two or more elements of the domain have the same range.For example $x \longrightarrow x^2$,$x \in \{0, 1, -2, 2\}$.

x	0	1	2	-2
x^2	0	1	4	4

Range=$\{0, 1, 4\}$

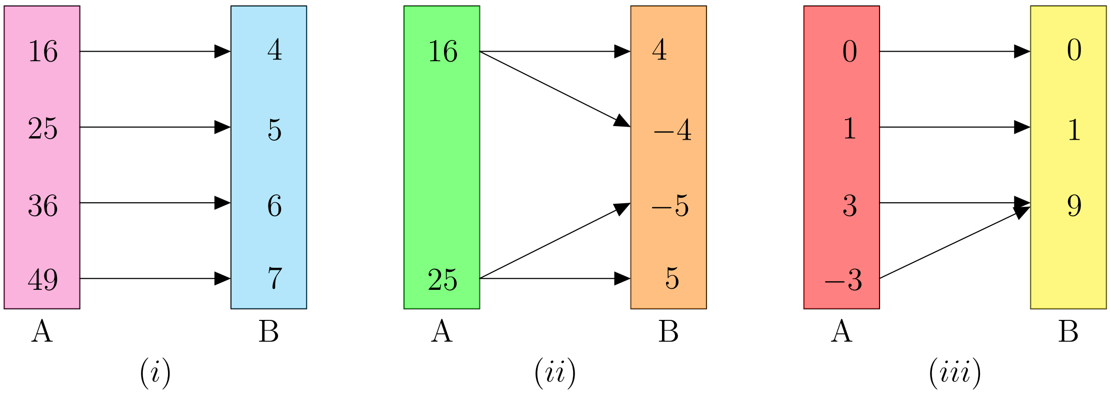


- Many to many mapping:
  This is a mapping where more than one element of the domain is mapped onto more than one element of the range.For example in the mapping **Is a multiple of** .



Example 1.3.1

For each of the examples below, state the type of mapping.



SOLUTION

(i) One to one mapping.This is because each member of B is related to only one member of A

(ii) One to many mapping.This is because members of A are mapped onto more than one element of B.

(iii) Many to one mapping.This is because more than one member of A is mapped onto the same element of B.

Example 1.3.2

A relation, A is defined by the set of ordered pairs; $(-3,2),(-2,4),(0,5),(-2,6)$

(i) List the members of the domain.

(ii) List the members of the range.

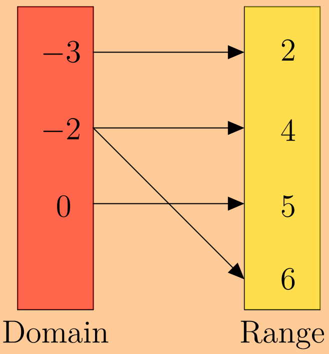
(iii) What type of mapping is A?

SOLUTION

(i) $\{-3,-2,0\}$

(ii) $\{2,4,5,6\}$

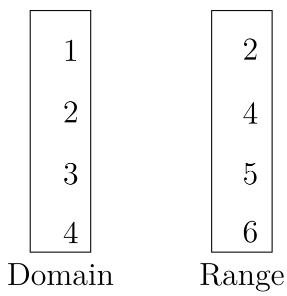
(iii) First draw a mapping digram



One to many mapping.This is because members of the domain are mapped onto more than one element of range.

1.2 Exercise Set

1. Draw a mapping diagram for $x \longrightarrow x^2$ for $x \in \{-2,-1,0,1,2\}$.
   (a) Which type of mapping is it?
   (b) State the range of the relation.
2. Copy and complete this arrow digram.



   (a) Show the relation **is a factor of**
   (b) State the type of mapping
3. Set A=$\{1,2,3,4,5\}$ is mapped onto set B by the relation **“multiply by 4”**.

(a) List the elements of set B

(b) Map A onto B.

(c) State the type of mapping

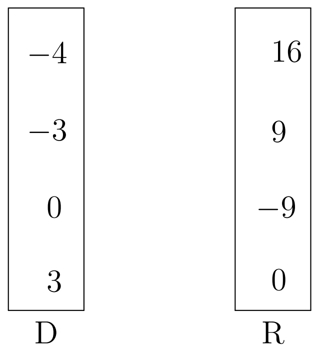
4. A set A maps onto set B by the operation **"multiply by 3 and add 1"**. The elements of set A are $\{5, 6, 7, 8, 9\}$

   (a) List the element of set B.

   (b) Map set A onto B.

   (c) What type of mapping is this?

5. Copy and complete this arrow digram.



   (a) Show the relation $x \longrightarrow x^2$

   (b) State the type of mapping

6. Some S2 students of Taibah international school obtained the following marks in a math test:

Name	Daniel	Tendo	Christopher	Hannah	Nicole	Joshua
Grade	80	78	84	80	92	84

   (a) Illustrate the mapping of marks to students.

   (b) What type of mapping is this?

7. Show the mapping of the relation **"is a factor of"** for the sets A= $\{2, 3, 4, 5, 6\}$ and B = $\{30, 32, 33, 34, 35, 36\}$.

8. Consider the domain A = {0, 1, 3, 4} set A is mapped onto set B by the relation x $\longrightarrow$ 3x. Find the range, the ordered pair and hence illustrate the relation on an arrow diagram

BE WARE!!

Do you know that **one to many** or **many to many** sexual relationships between members of the opposite sex can spread HIV/AIDS? You are advised to stay away from sex before marriage.

1.4 Papygrams

A papygram is a circular representation of relationships that exist between a given set of things. The relationship is illustrated by the help of arrows.Papygrams represent elements in a relation whose domain and range belong to the same set.

Activity

Name	Daniel	Ruth	Sylvia	Timothy
Age(years)	14	7	28	12

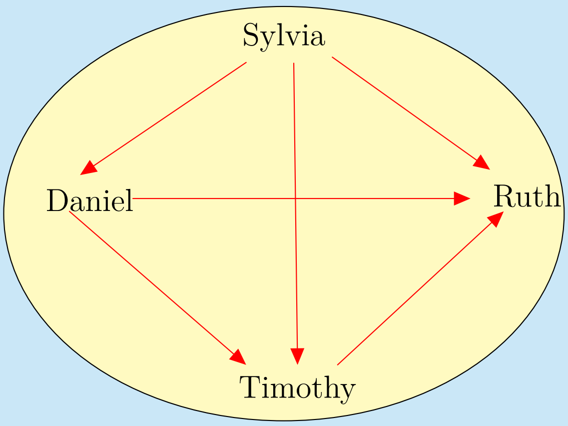
(i) Who is the oldest?

(ii) Who is Daniel older than?

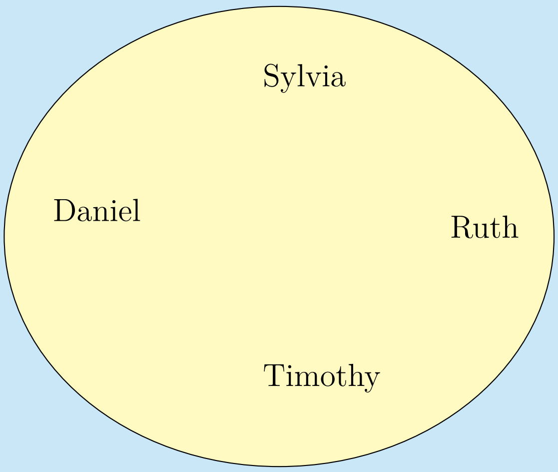
(iii) Who is Timothy older than?

(iv) Who is Sylvia older than?

(v) The relation **Is older than** for this family can be illustrated on the diagram called **Papygram** as shown below.



(vi) Copy and complete the papygram below to illustrate the relation **Is younger than**

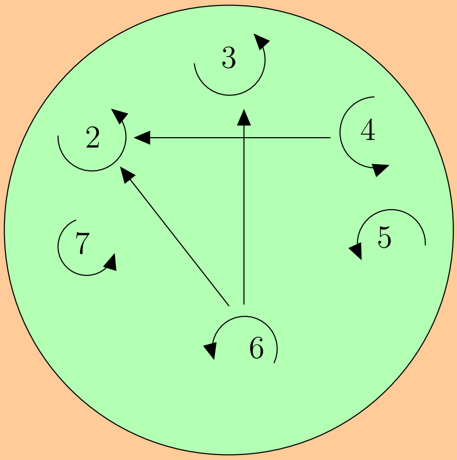


Example 1.4.1

Draw a papygram to show the relation **is a multiple of** in the set of numbers $\{2, 3, 4, 5, 6, 7\}$

SOLUTION

$M_2 = \{2,4,6\}$ $M_3 = \{3,6\}$ $M_4 = \{4\}$ $M_5 = \{5\}$ $M_6 = \{6\}$ $M_7 = \{7\}$

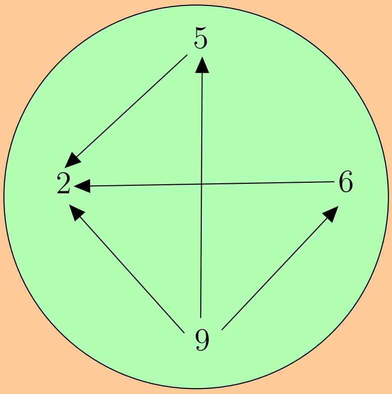


The arrow which goes back to the same number means a number is a multiple of itself.

Example 1.4.2

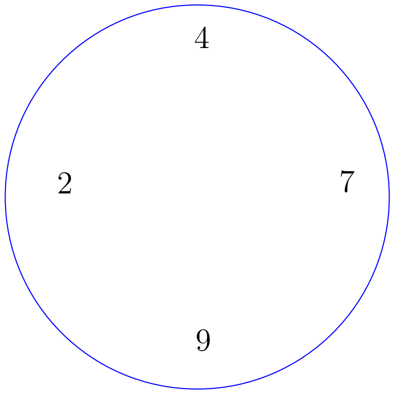
Draw a papygram to illustrate the relation **"exceeds by more than 2"** in a set$\{2,5,6,9\}$

SOLUTION



1.3 Exercise Set

1. Draw a papygram to show the relation **"is a factor of"** in the set of numbers $\{2,3,4,5,6,7\}$
2. Draw a papygram to illustrate the relation **"is less than "** in a set$\{2,4,5,6,9\}$
3. Draw a papygram showing the relation **"is a multiple of"** in the set $\{40,32,24,16,8\}$
4. Given that the set $A = \{2,3,5,6,8,10,14\}$, illustrate on papygrams the relations:
   (a) "is a factor of "
   (b) "is a prime factor of"
   (c) "Less than by 2"
5. Complete the Papygram to show the relation " **exceeds by more than 1"**.



6. The following are weights of some students in a senior two class at Taibah international School.

Name	Michael	Sandra	Shakur	Elena	Joel	Charlse
Weight(Kg)	50	68	42	60	36	38

(a) Who is Charlse heavier than?

(b) Who is Sandra heavier than?

(c) Draw a papygram to show the relation **"is heavier than"** for the group of students .

1.5 Functions

A function is a relation in which for each member of the Domain there is a single corresponding member of the Range. OR A function is a type of mapping in which every object on the domain has one and only one image in the range. Mappings, which are functions, are one to one mappings and many to one .A function involves two sets(Ordered pairs) and a rule of correspondence between them.The rule specifies how to pair the elements of one set with those in the other set.

For any mapping/relation to be a function

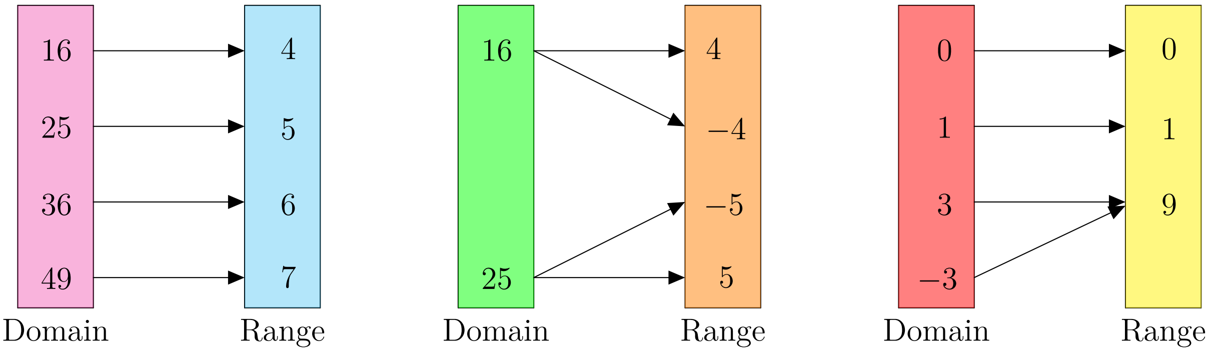
- Every member(element) of the domain is mapped onto one and only one member of the range
- An input cannot have more than one output.
- Two or more members of the domain can be mapped onto the same member of the range

DID YOU KNOW?

In the real world, we may think of a function as a mapping onto the set of sons to a corresponding set of biological mothers . Each son will be associated with one and only one mother, and two or more sons can be associated with the same mother but one son cannot be associated with two or more mothers.

Example 1.5.1

For each of the mapping diagrams below, state whether it represents a function and if not why.



(*i*) (*ii*) (*iii*)

SOLUTION

(i) This is a function because each object in the domain is mapped onto only one image in the range.This is a one to one mapping,and it meets the conditions for a mapping to be a function.

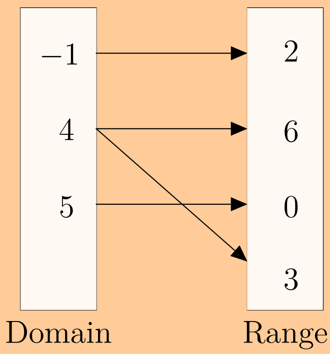
(ii) This is not a function because each object in the domain is mapped onto more than one image in the range.i.e 16 is mapped onto 4,-4 ,25 is mapped onto 5,-5.This is a one to many mapping, and so does not meet the conditions for a mapping to be a function.

(iii) This is a function because different objects in the domain are mapped onto only one image in the range.This is a many to one mapping,and it meets the conditions for a mapping to be a function.

Example 1.5.2

A mapping is defined by $\{(-1,2),(4,6),(5,0),(4,3)\}$.State whether it represents a function and if not why.

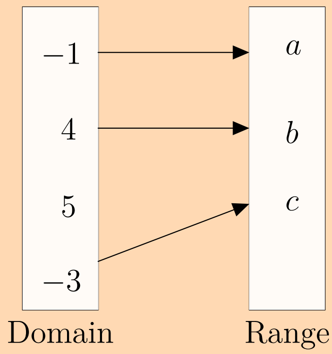
SOLUTION



This is not a function because an object in the domain is mapped onto more than one image in the range.i.e 4 is mapped onto 6,and 3 .This is a one to many mapping.

Example 1.5.3

State whether the mapping diagram below represents a function and if not why.

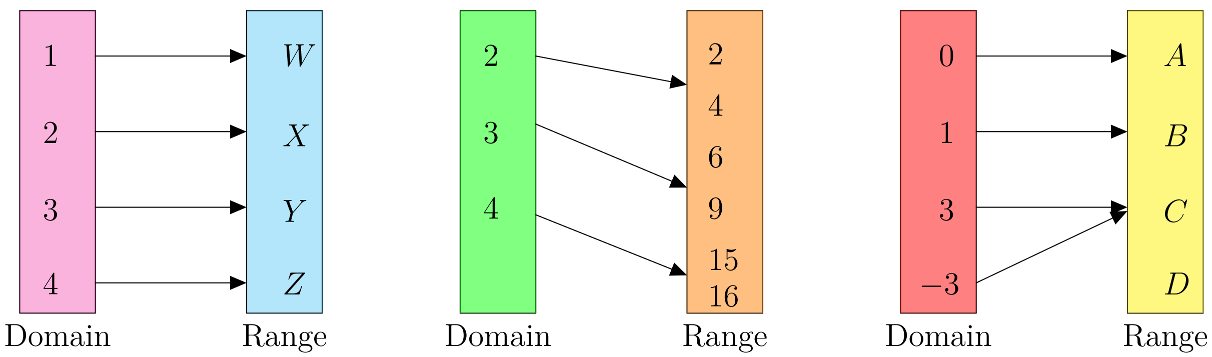


SOLUTION

This is not a function because at least one object in the domain is not associated to any image in the range.i.e 5 is not assigned to an element in the range.

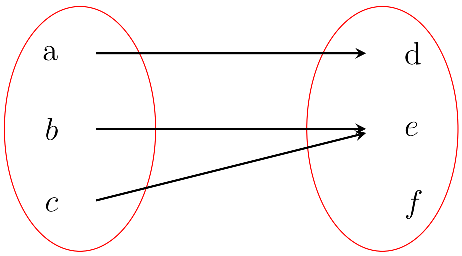
1.4 Exercise Set

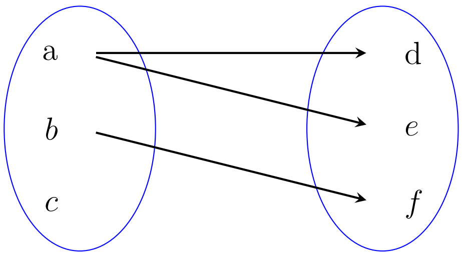
1. A mapping is defined by $\{(-1,1),(4,11),(5,13),(1,5)\}$.
   (a) Represent the mapping on an arrow diagram
   (b) Which type of mapping is it?
   (c) Is the mapping a function?
2. For the mapping $x \longrightarrow 2x-4, x \in \{-3,0,2,4\}$
   (a) Represent the relation on an arrow diagram
   (b) State the type of mapping
   (c) Is the mapping a function?
3. For each of the mapping diagrams below, state whether it represents a function and if not why.

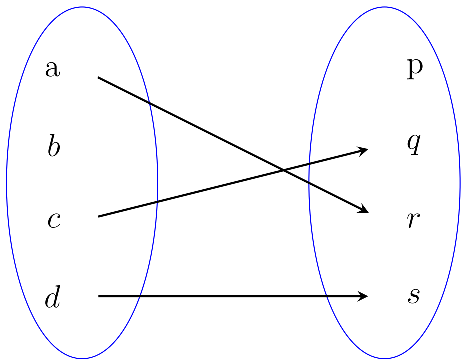


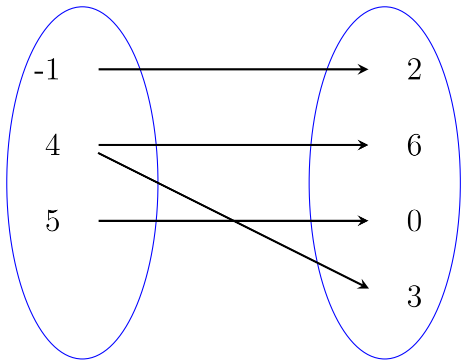
(i) (ii) (iii)

4. Do these mapping diagrams represent functions ? Give reasons for your answers.

(a)


(b)


(c)


(d)


1.5.1 Function Notation

We can describe a function using mathematical notation, written as **f(x)**.We read **f(x)** as "f of x" or "f at x" since **f(x)** gives the value of f at x.The notation **f(x)** represents the second element in the ordered pair that has x as its first element.The ordered pair can be represented as $(x, f(x))$.Often we replace **f(x)** by y and write $y = f(x)$.For example $f(x) = 2x + 3$ may be written as $y = 2x + 3$

Mappings that represent functions can be written as follows;

- $x \xrightarrow{\text{f}} 2x + 3$:Read as the function f maps x onto $2x + 3$
- $f : x \longrightarrow 2x + 3$: Read as a function f such that x is mapped onto $2x + 3$
- $f(x) = 2x+3$:An algebraic formula that gives the values of the Range given particular values of x in the domain.

Example 1.5.1.1

Given the function $f(x) = 2x + 3$, find

(a) $f(0)$

(b) $f(1)$

(c) $f(-1)$

(d) $f\left(\frac{1}{2}\right)$

SOLUTION

(a) $f(0)$

$f(0)$ means we Substitute 0 for x in the function

$$\begin{aligned} f(x) &= 2x+3 \\ f(0) &= 2\times 0+3 && \text{Substituting 0 for x} \\ f(0) &= 0+3 \\ f(0) &= 3 \end{aligned}$$

(b) $f(1)$

$f(0)$ means we Substitute 1 for x in the function

$$\begin{aligned} f(x) &= 2x+3 \\ f(1) &= 2\times 1+3 && \text{Substituting 1 for x} \\ f(1) &= 2+3 \\ f(1) &= 5 \end{aligned}$$

(c) $f(-1)$

$f(0)$ means we Substitute 0 for x in the function

$$\begin{aligned} f(x) &= 2x+3 \\ f(-1) &= 2\times -1+3 && \text{Substituting } -1 \text{ for x} \\ f(-1) &= -2+3 \\ f(-1) &= 1 \end{aligned}$$

(d) $f\left(\frac{1}{2}\right)$

$$\begin{aligned} f(x) &= 2x+3 \\ f\left(\frac{1}{2}\right) &= 2\times\frac{1}{2}+3 && \text{Substituting } \frac{1}{2} \text{ for x} \\ &= \cancelto{1}{2}\times\frac{1}{\cancelto{1}{2}}+3 \\ &= 1+3 \\ &= 4 \end{aligned}$$

Example 1.5.1.2

Given that $f: x \longrightarrow 6x-2$ .Find

(a) $f(0)$ (b) $f(2)$ (c) $f(-4)$ (d) $f\left(\frac{1}{2}\right)$

SOLUTION

$f: x \longrightarrow 6x-2$ can be written as $f(x)=6x-2$

(a) $f(0)$

$$\begin{aligned} f(x) &= 6x-2 \\ f(0) &= 6\times 0-2 \\ f(0) &= 0-2 \\ f(0) &= -2 \end{aligned}$$

(b) $f(2)$

$$f(x) = 6x - 2$$
$$f(2) = 6 \times 2 - 2$$
$$f(2) = 12 - 2$$
$$f(2) = 10$$

(c) $f(-4)$

$$f(x) = 6x - 2$$
$$f(-4) = 6 \times -4 - 2$$
$$f(-4) = -24 - 2$$
$$f(-4) = -26$$

(d) $f\left(\frac{1}{2}\right)$

$$f(x) = 6x - 2$$
$$f\left(\frac{1}{2}\right) = 6 \times \frac{1}{2} - 2$$
$$f\left(\frac{1}{2}\right) = \cancel{6}^{3} \times \frac{1}{\cancel{2}_{1}} - 2$$
$$f\left(\frac{1}{2}\right) = 3 - 2$$
$$f\left(\frac{1}{2}\right) = 1$$

Example 1.5.1.3

Given that $g(x) = 2x + 3$, find the range $g(x)$,given the domain $\{-1, 1, 4, 5\}$

SOLUTION

The range=$\{g(-1), g(1), g(4), g(5)\}$.

(a) $g(-1)$

$$g(x) = 2x + 3$$
$$g(-1) = 2 \times -1 + 3$$
$$g(-1) = -2 + 3$$
$$g(-1) = 1$$

(b) $g(1)$

$$g(x) = 2x + 3$$
$$g(1) = 2 \times 1 + 3$$
$$g(1) = 2 + 3$$
$$g(1) = 5$$

(c) $g(4)$

$$g(x) = 2x + 3$$
$$g(4) = 2 \times 4 + 3$$
$$g(4) = 8 + 3$$
$$g(4) = 11$$

(d) $g(5)$

$$g(x) = 2x + 3$$
$$g(5) = 2 \times 5 + 3$$
$$g(5) = 10 + 3$$
$$g(5) = 13$$

The Range =$\{1, 5, 11, 13\}$

Example 1.5.1.4

Given that $h(y) = y^2 + 3y$, find $h(2)$

SOLUTION

$$\begin{aligned} h(y) &= y^2 + 3y \\ h(2) &= 2^2 + 3y \\ h(2) &= 2 \times 2 + 3 \times 2 \\ h(2) &= 4 + 6 \\ h(2) &= 10 \end{aligned}$$

Example 1.5.1.5

The function $f(x) = bx^2 - 2$ and $f(2) = 18$. Find:

(a) the value of b

(b) $f(2)$

(c) $f(0)$

SOLUTION

(a)

$$\begin{aligned} f(x) &= bx^2 - 2 \\ f(2) &= 18 \\ f(2) &= b \times 2^2 - 2 \\ f(2) &= 4b - 2 \\ 4b - 2 &= 18 \\ 4b - 2 + 2 &= 18 + 2 \\ 4b &= 20 \\ \frac{4b}{4} &= \frac{20}{4} \\ \frac{\cancel{4}b}{\cancel{4}} &= \frac{\cancelto{5}{20}}{\cancelto{1}{4}} \\ b &= 5 \end{aligned}$$

(b) $f(2)$

$$\begin{aligned} f(x) &= 5x^2 - 2 \\ f(2) &= 5 \times 2^2 - 2 \\ f(2) &= 5 \times 4 - 2 \\ f(2) &= 20 - 2 \\ f(2) &= 18 \end{aligned}$$

(c) $f(0)$

$$\begin{aligned} f(x) &= 5x^2 - 2 \\ f(0) &= 5 \times 0^2 - 2 \\ f(2) &= 5 \times 0 - 2 \\ f(2) &= 0 - 2 \\ f(2) &= -2 \end{aligned}$$

Example 1.5.1.6

If $f(x) = \frac{1}{2x-10}$,find the value of :

(i) $f(6)$

(ii) x for which $f(x)$ is undefined

SOLUTION

(i) $f(6)$

$$f(x) = \frac{1}{2x-10}$$
$$f(6) = \frac{1}{(2\times 6)-10}$$
$$f(6) = \frac{1}{12-10}$$
$$f(6) = \frac{1}{2}$$

(ii) x for which $f(x)$ is undefined

A function is undefined or meaningless if its denominator part is equal to zero.

$$2x-10=0$$
$$2x-10+10=0+10$$
$$2x=10$$
$$\frac{2x}{2}=\frac{10}{2}$$
$$\frac{\cancel{2}x}{\cancel{2}}=\frac{\cancelto{5}{10}}{\cancelto{1}{2}}$$
$$x=5$$

1.5 Exercise Set

1. Given that $h(x)=6x+2$, Find

   (a) $h(-4)$ (b) $h(0)$ (c) $h(1)$ (d) $h(3)$

2. The function $f(y)=4y^2+3y$.Find

   (a) $f(0)$ (b) $f(-2)$ (c) $f(1)$ (d) $f(0.5)$ (e) $f(\frac{1}{3})$ (f) $f(-1)$ (g) $f(7)$ (h) $f(4)$

3. Given that $g : x \longrightarrow \frac{1}{3}x-4$, find

   (a) $f(0)$ (b) $f(9)$ (c) $f(-18)$ (d) $f\left(\frac{1}{2}\right)$

4. Determine the range corresponding to the domain $\{-1,1,4,5\}$ for the function $g(x)=2x+3$.
5. Given that $g(x)=5(x^2+4)$.Find

   (a) $g(-4)$ (b) $g(0)$ (c) $g(1)$ (d) $g(3)$

6. Given that $h(x)=6x+4$, find the range $h(x)$,given the domain $\{-2,0,3,7\}$
7. The function $f(x)=4x-2$ and $f(x)=6$.Find the value of x

8. Given $h(x) = 5x + 4$, If $f(a) = 14$, find the value of a.
9. Given that $f(x) = \frac{x-4}{5}$.Find

   (a) $f(24)$ (b) $f(-6)$ (c) $f(5)$ (d) $f(-16)$

10. If f(x) $= 2 - \frac{1}{2}x$ has the domain $\{-2, 0, 2, 4, 6\}$, find the range.
11. If h(y)= $\frac{y^2-1}{y+1}$, find:

    (a) $h(0)$ (b) $h(2)$ (c) $h(-4)$ (d) $h(-5)$

12. The function $h(x) = \frac{1}{2}x^2 + p$ and $h(-2) = 6$. Find the value of p.

Summary

1. Relation—It is a connection between two or more numbers or things.
2. Domain—It is the set of input values for a mapping
3. Range—It is the output set in mapping.
4. Papygram—It is a circular representation of relationships that exist between a given set of things.
5. Function—It is a relation in which for each member of the Domain there is a single corresponding member of the Range.

ASSESSMENT

1. Given that B=$\{(2, 1), (4, 5), (6, 7), (8, 9)\}$.Determine

   (a) the domain and range

   (b) Whether B is a function and if not why.

   (c) the type of mapping

2. Given that $A = \{2, 5, 6, 8, 9, 10, 12, 13\}$,illustrate on papygrams the relations:

   (i) **"Greater than by 3"**

   (ii) **" Factor of "**

3. Ethan,Joel,Daniel and Martin like the following types of foods:matooke,rice,meat and matooke respectively.

   (a) List the elements of the domain and range of the relation **"likes"**

   (b) Draw an arrow diagram to illustrate the relation

   (c) Identify the type of mapping

4. Given that $f(x) = 4x - 3$,find

   (i) $f(2)$ (ii) $f(-1)$ (iii) $f\left(\frac{1}{2}\right)$

5. Consider the function $f(x) = 2x$. If the domain is $\{0, 1, 2, 3, 4\}$ .Find the range of values. Hence, draw the arrow diagram.

6. Given that $h(y) = 4y - 2$,find the value of

   (i) $h(-2)$

   (ii) y when $h(y) = 6$

7. The function is defined as $f : x \longrightarrow 3 - 2x$.Determine the range if the domain is $\{0, 1, 2, 3\}$.

8. If $g(x) = qx + 3$ and $g(5) = 23$,Find the value of

   (i) q (ii) $g(0)$ (iii) $g(-5)$

9. (a) Given the set $\{2, 4, 6\}$ draw a papygram to show the relation **"is the smallest prime factor of"**

   (b) For the mapping $x \longrightarrow 4x + 5$,find the domain when the range is $\{1, 13\}$

   (c) The function $f(x) = 17 - 5x$ and $g(x) = 3x - 7$ .Find the value of x such that $f(x) = g(x)$

10. Set A maps onto set B by the operation “divide by 2 add 3".If set A = $\{4, 8, 12, 16, 20, 24\}$

    (a) list the elements of set B.

    (b) Find the ordered pairs.

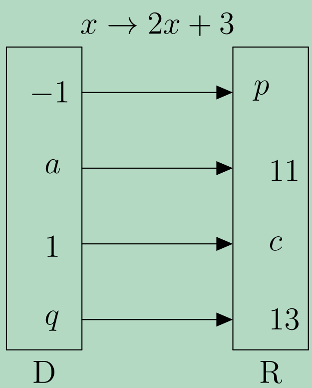
    (c) Draw the graph of the relation.

11. If $f(x) = \frac{5x}{x^2-9}$,find t :

    (i) $f(2)$

    (ii) the values of x for which $f(x)$ is meaningless

12. Find the unknown values in the arrow diagram for the mapping $x \longrightarrow 2x + 3$



End

Chapter 2

VECTORS AND TRANSLATION

Learning objectives

By the end of this topic,the learners should be able to

- Define translation with a vector
- Identify scalars and vectors
- Use vector notation
- Represent vectors both single and combined geometrically

Introduction

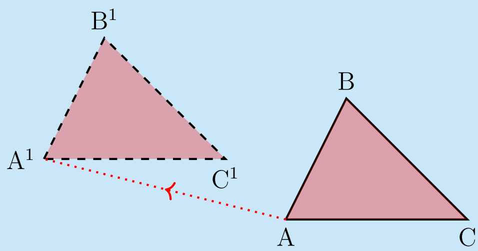
In chapter 10 of Book one ,we saw how objects can be reflected on different mirror lines.In this chapter we will look at another type of transformation.

2.1 Translations

A translation is a transformation that moves every point in a figure the same distance in the same direction.Translation deals with movement of an object to a new position.This can be in form of moving a shape up, down or from side to side but it does not change its appearance in any other way.

Activity

1. Trace the figures below on a tracing paper



2. Draw the line segment joining A to A^1as shown above.
3. Slide the tracing using line AA^1 as a guide line, to ensure that A moves onto A^1 in a straight line.(You can as well make a cut out)

4. When A coincides with A^1, stop the slide. What do you notice about the positions of B and C?
5. What do you notice about the new position of the triangle ABC? What can you say about the two triangles?

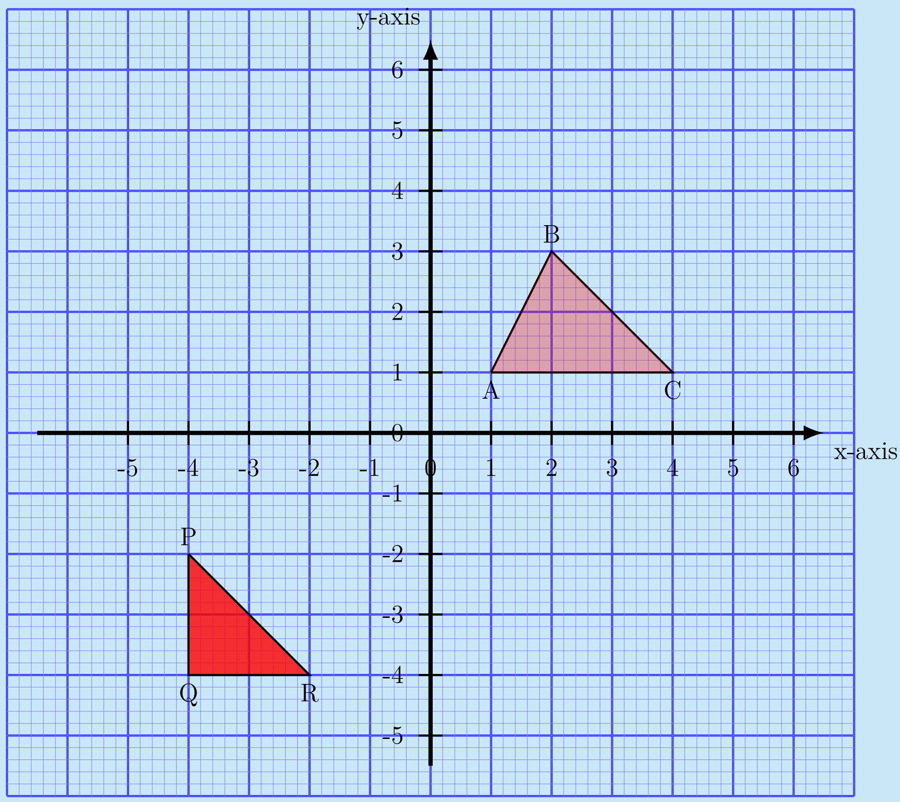
From the activity we notice that each point on triangle ABC has moved the **same distance** and in the **same direction**. The process that moves triangle ABC onto triangle $A^1B^1C^1$ is called **translation**.

Properties of Translation

1. All the points on the object move the same distance.
2. All the points move in the same direction.
3. The object and the image are identical(same) and they face the same direction.Hence, they are directly congruent(same shape,size,and angles)
4. A translation is fully defined by stating the distance and direction that each point moves.

2.1.1 Translation in the Cartesian plane

Activity



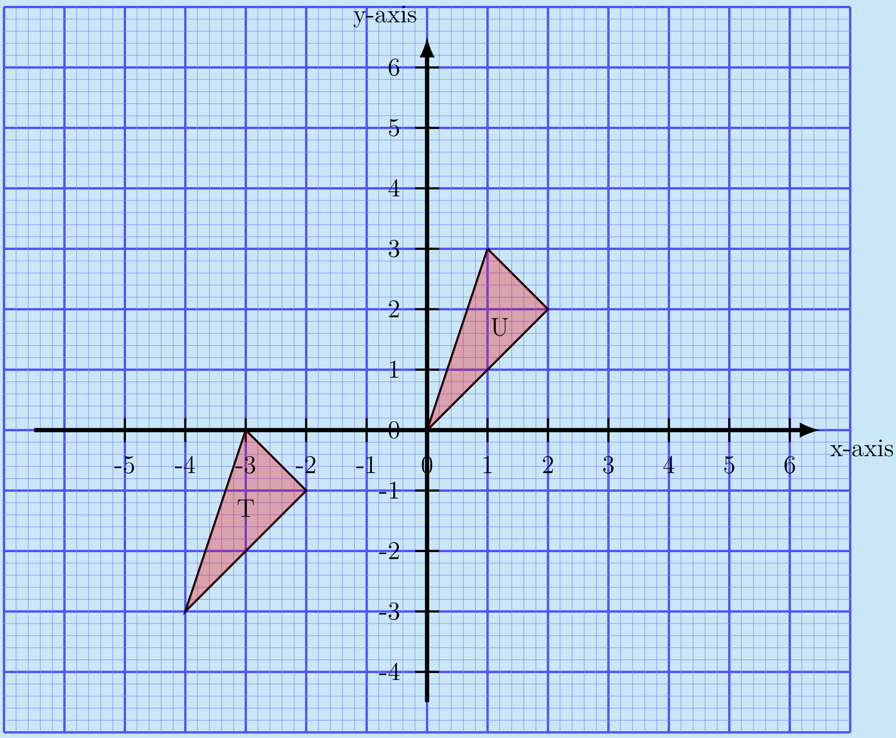
1. Move the vertex A of triangle ABC ,4 steps to the left and 2 steps up.Name the new position A^1
2. Write the coordinates of the image A^1

3. Move the vertex B and C of triangle ABC ,4 steps to the left and 1 step up.Name the new positions B^1 and C^1 respectively.
4. Join A^1,B^1 and C^1 to form the image of triangle ABC.
5. Move each vertex of triangle PQR ,3 steps in the x -direction and -1 step in the y direction.
6. Write the coordinates of the triangle $P^1Q^1R^1$ formed

NOTE

- If each point moves distance **a** in the x-direction(horizontal distance) and distance **b** in the y-direction(vertical distance),we use the 'vector' notation $\binom{a}{b}$ to describe the translation.
- A translation T=$\binom{a}{b}$ means that an object is moved a distance **a** in the x-direction and a distance **b** in the y-direction
- From the previous activity ,the Translation of triangle ABC is $\binom{-4}{2}$
- A translation T=$\binom{a}{b}$ moves point A(x, y) to a new position $A^1(x + a, y + b)$ Thus Translation + object = image .

Activity



1. What is the vector that translates T to U
2. What is the vector that translates U to T

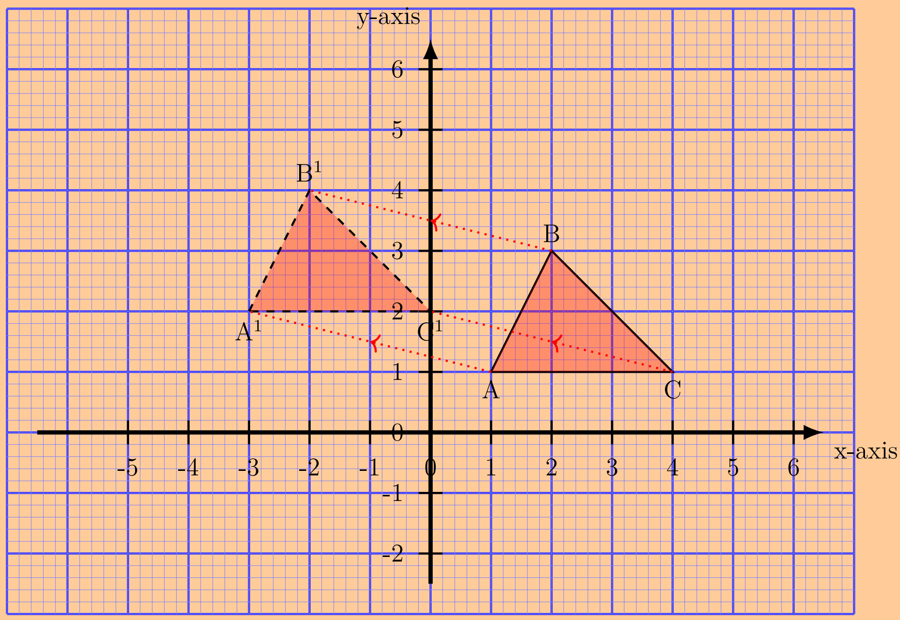
Example 2.1

(a) Draw the triangle ABC with corners at the points with coordinates (1,1),(2,3) and (4,1) respectively.

(b) The triangle is translated along the vector $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$. Draw the new triangle obtained by the translation.

SOLUTION

For this translation each point should be moved 4 steps to the left(-4 spaces in the x - direction) and 1 step up(1 space in the y- direction) .



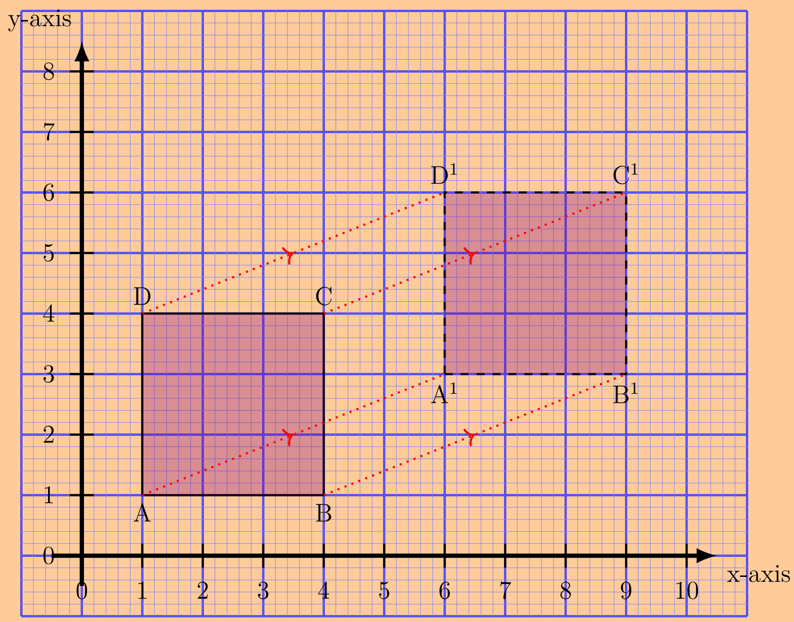
Example 2.2

(a) Draw the square ABCD with vertices A(1,1),B(4,1) , C(4,4) and D(1,4)respectively.

(b) The square is translated by the displacement vector $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$. Draw the new triangle obtained by the translation and write down the coordinates for the images.

SOLUTION

For this translation each point should be moved 5 steps to the left(5 spaces in the x -direction) and 2 step up(2 spaces in the y- direction) .



The coordinates of the images are $A^1(6,3)$,$B^1(9,3)$,$C^1(9,6)$ and $D^1(6,6)$

Example 2.3

A translation $T=\binom{5}{2}$ maps the points $P(3, 7)$ onto the point P^1. Find the coordinates of P^1.

SOLUTION

$$\text{Image} = \text{Translation} + \text{Object}$$
$$P^1 = \begin{pmatrix}5\\2\end{pmatrix} + \begin{pmatrix}3\\7\end{pmatrix}$$
$$P^1 = \begin{pmatrix}5+3\\2+7\end{pmatrix}$$
$$P^1 = \begin{pmatrix}8\\9\end{pmatrix}$$

The coordinate of the image is $P^1(8,9)$

Example 2.4

A triangle with vertices $A(1, 1)$ $B(2, 3)$ and $C(4, 1)$ is mapped onto its image by a translation $T=\binom{-4}{1}$ Find the coordinates of the image of the triangle ABC.

SOLUTION

Image = Translation + Object

$$A^1 = T + A$$
$$A^1 = \begin{pmatrix}-4\\1\end{pmatrix} + \begin{pmatrix}1\\1\end{pmatrix}$$
$$A^1 = \begin{pmatrix}-4+1\\1+1\end{pmatrix}$$
$$A^1 = \begin{pmatrix}-3\\2\end{pmatrix}$$

$$B^1 = T + B$$
$$B^1 = \begin{pmatrix}-4\\1\end{pmatrix} + \begin{pmatrix}2\\3\end{pmatrix}$$
$$B^1 = \begin{pmatrix}-4+2\\1+3\end{pmatrix}$$
$$B^1 = \begin{pmatrix}-2\\4\end{pmatrix}$$

$$C^1 = T + A$$
$$C^1 = \begin{pmatrix}-4\\1\end{pmatrix} + \begin{pmatrix}4\\1\end{pmatrix}$$
$$C^1 = \begin{pmatrix}-4+4\\1+1\end{pmatrix}$$
$$C^1 = \begin{pmatrix}0\\2\end{pmatrix}$$

The coordinates of the image of the triangle are $A^1(-3, 2)$,$B^1(-2, 4)$,$C^1(0, 2)$

Example 2.5

A translation $T=\binom{-4}{1}$ maps point P onto $P^1(0, 2)$. Find the coordinates of point P

SOLUTION

$$\text{Translation} + \text{Object} = \text{Image}$$
$$\text{T} + \text{P} = \text{P}^1$$
$$\begin{pmatrix} -4 \\ 1 \end{pmatrix} + \text{P} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$
$$\text{P} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} - \begin{pmatrix} -4 \\ 1 \end{pmatrix}$$
$$\text{P} = \begin{pmatrix} 0 - -4 \\ 2 - 1 \end{pmatrix}$$
$$\text{P} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

The coordinate of the object is P(4,1)

Example 2.6

The diagram below shows the shapes A, B, C and D . Along what vector would you translate:

(a) A to B

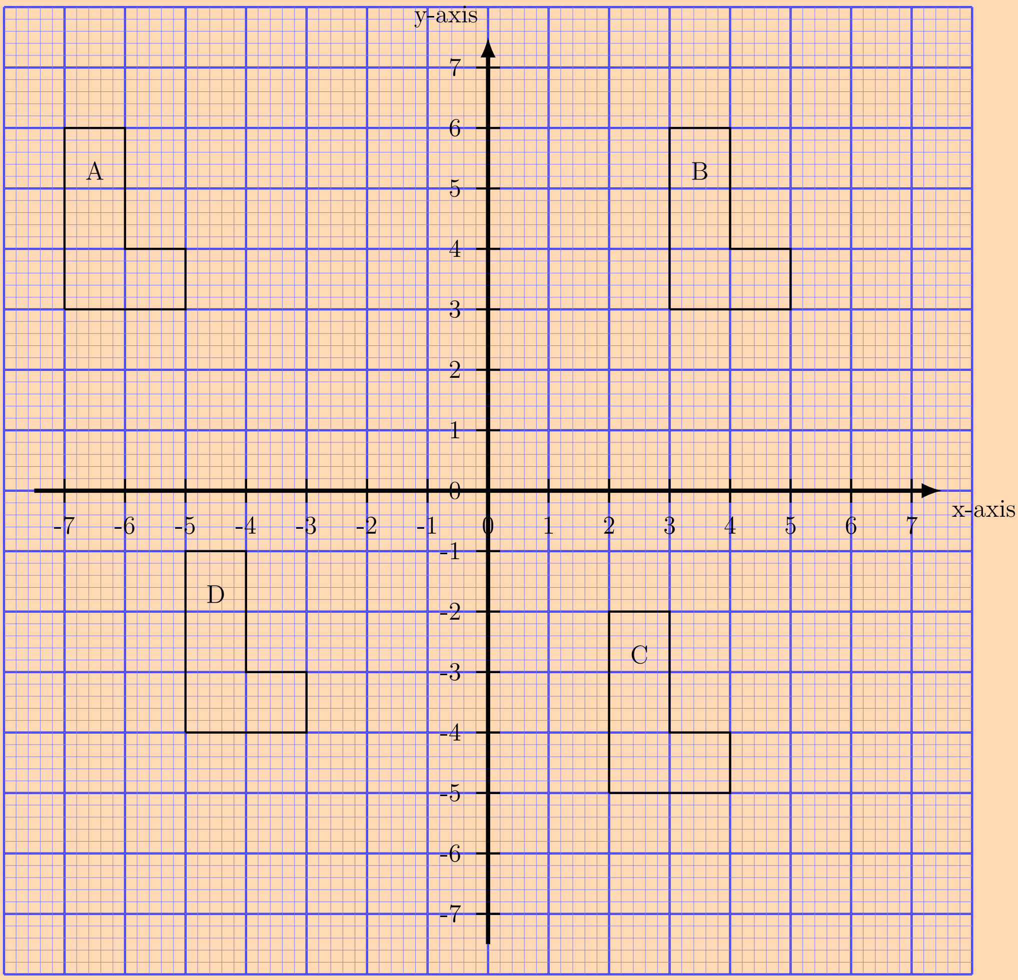
(b) A to C

(c) D to B

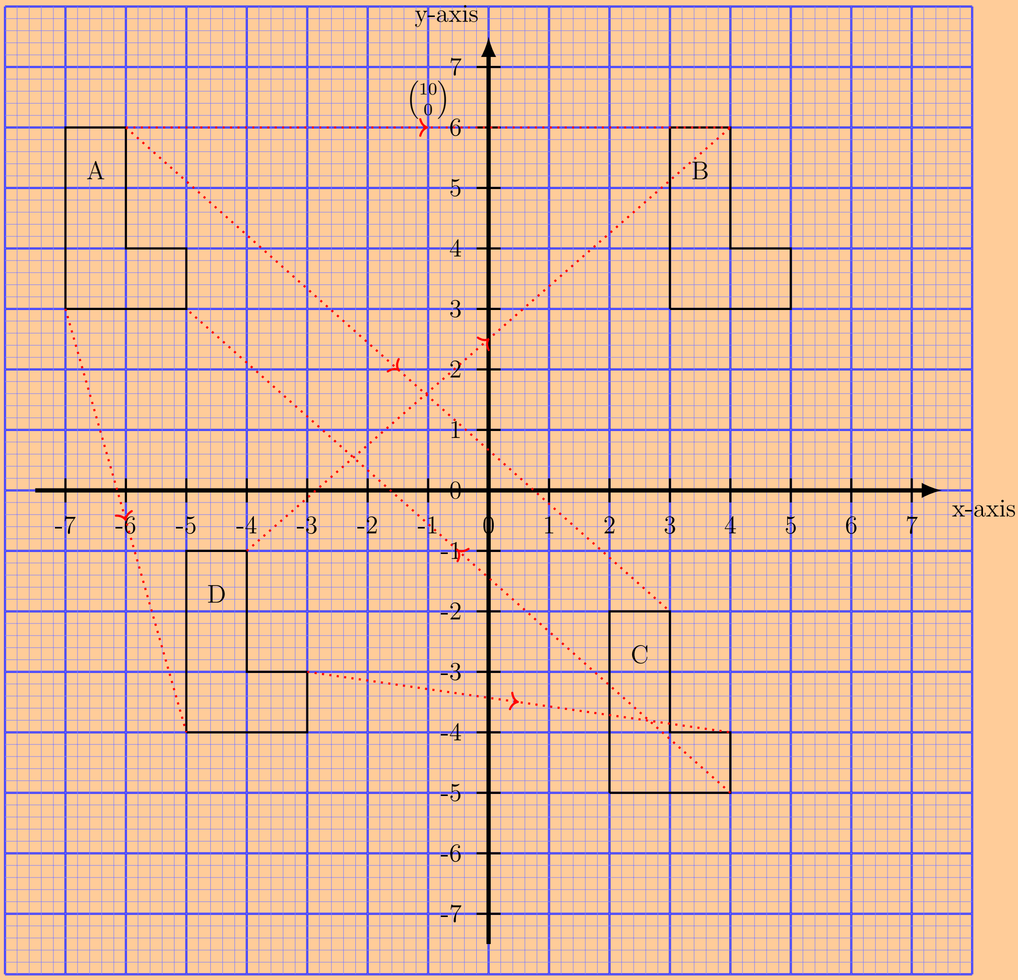
(d) C to A

(e) D to C

(f) A to D



SOLUTION



(a) A to B.
10 steps to the right and 0 step up.
$$\begin{pmatrix}10\\0\end{pmatrix}$$

(b) A to C
9 steps to the right and 8 steps down.
$$\begin{pmatrix}9\\-8\end{pmatrix}$$

(c) D to B
8 steps to the right and 7 steps up.
$$\begin{pmatrix}8\\7\end{pmatrix}$$

(d) C to A
9 steps to the left and 8 steps up.
$$\begin{pmatrix}-9\\8\end{pmatrix}$$

(e) D to C
7 steps to the right and 1 step down.
$$\begin{pmatrix}7\\-1\end{pmatrix}$$

(f) A to D
2 steps to the right and 7 steps down.
$$\begin{pmatrix}2\\-7\end{pmatrix}$$

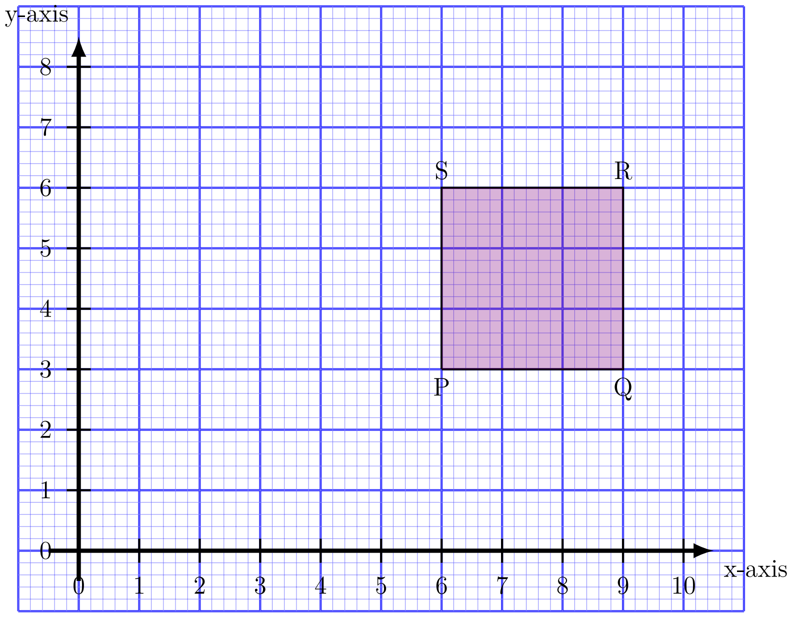
2.1 Exercise Set

1. (a) Draw the triangle which has corners at the points with coordinates (5, 3), (9, 3) and (7, 6).

   (b) Translate the triangle along the vector $\binom{-2}{-1}$

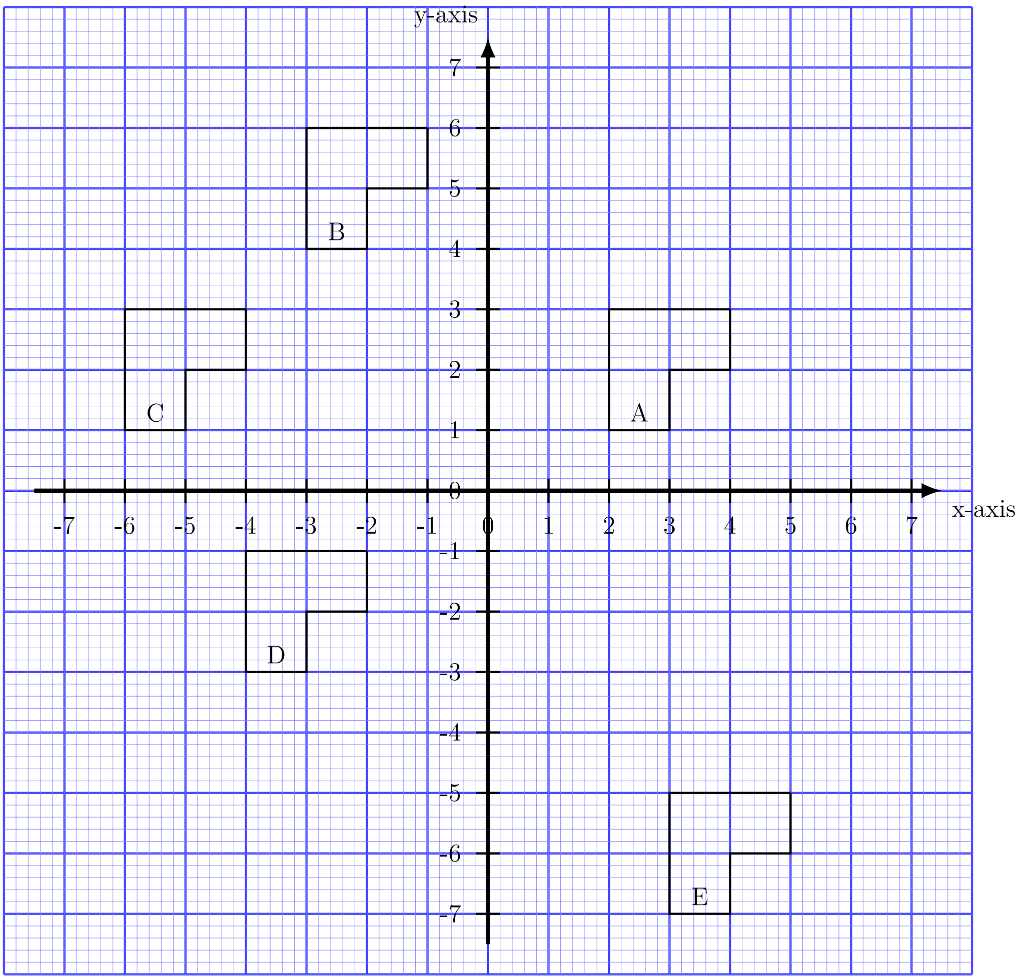
   (c) Write down the coordinates of the corners of the translated triangle.

2. The square PQRS is translated by the vector $\binom{-2}{-3}$.Copy the diagram and show on it the square $P^1Q^1R^1S^1$ under this translation hence write the coordinates for the images of the square.



3. The following diagram shows the shape A which is translated to give the shapes B, C, D and E:Write down the vector that describes the translation from:

(a) A to B	(d) A to E	(d) B to A
(b) A to C	(e) E to A	(e) C to A
(c) A to D	(f) E to D	(f) E to B



4. (a) Join the points with coordinates (1, 1), (2, 3) and (5, 4) to form a triangle. Label this triangle A.

   (b) Translate the triangle A along the vector:

   (i) $\binom{2}{1}$,to obtain triangle B

   (ii) $\binom{1}{3}$,to obtain triangle C

   (iii) $\binom{2}{-5}$,to obtain triangle D

   (iv) $\binom{-1}{-2}$,to obtain triangle E

   (v) $\binom{0}{-1}$,to obtain triangle F

   (vi) $\binom{2}{0}$,to obtain triangle G

5. (a) Draw the triangle, A, that has corners at the points with coordinates $(-7, -2)$, $(-5, -5)$ and $(-4, -2)$.

   (b) Translate this shape along the vector $\binom{4}{8}$ to obtain B.

   (c) Describe the translation that would take B to A

6. Triangle ABC has vertices A (0, 0),B (5, 1) and C (1, 3). Find the coordinates of the points A^1, B^1 and C^1,the images of A, B and C respectively, under a translation with displacement vector $\binom{2}{5}$

7. The image of a rectangle ABCD has vertices A^1, B^1, C^1 and D^1 at the points (5, 10), (9, 10) (9, 8), (5, 8) respectively. If the translation vector is $\binom{2}{5}$, find the coordinates of the object rectangle.

8. A parallelogram has corners at the points A, B, C and D. The points A, B and C have coordinates (1, 2), (2, 5) and (5, 3) respectively.
   (a) Draw the parallelogram.
   (b) State the coordinates of the fourth corner, D.
   (c) Describe the translation that moves A B onto D C.
   (d) Describe the translation that moves A D onto B C.
9. The shape A has corners at the points with coordinates $(4, 2)(4, -1)(6, -3)$ and $(6, 0)$.
   (a) What is this shape?
   (b) The shape is translated along the vector $\binom{4}{-2}$ to give shape B and then shape B is translated along the vector $\binom{-5}{2}$ to give C.Draw A, B and C.
   (c) What translation would take A straight to C ?
10. A translation T maps point P(2, 5) onto P^1(3,2) Find the image of Q(5, 7) under translation T

2.2 Vectors and Scalars

Activity

1. A tourist arrived at Entebbe international airport and is to visit the source of the nile in jinja town.
   (a) What two aspects of the journey must he know?
   (b) What is the name of the quantity that has these two aspects?
2. In pairs, roughly estimate the following :
   (a) The distance between your home and the nearest shopping centre.
   (b) The direction of your home from the nearest shopping centre.
   (c) How did you estimate the direction in (b) above?
3. The distance from Kampala to Jinja is about 80km. Stephen drove from kampala to Jinja and back.
   (a) What is the total distance covered by Stephen?
   (b) What is the total displacement?
   (c) Are your answers for part(a) and (b) the same .If yes why and if they are different explain the cause.

From the activity we have noticed that vectors are described using length and direction.

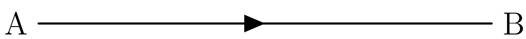
Summary

1. A vector is a physical quantity that is described by magnitude(size or length) and direction.
2. Examples of vectors include.displacement,velocity,force,acceleration etc
3. A scalar is a physical quantity that is described by only magnitude(size or length)
4. Examples of scalars include.distance,speed,time,etc

2.2.1 Notation and representation of vectors

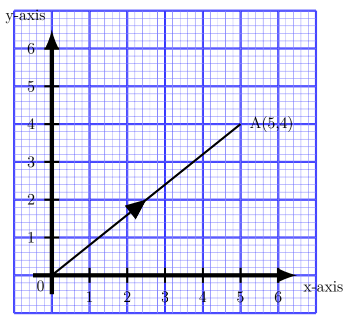
- Geometrically, a vector is represented using a directed line segment, whose length is proportional to the magnitude(size,length) of the vector and with an arrow indicating the direction.The direction of the vector is from its tail to its head.

- In mathematics Vectors are denoted in any of the following ways.
  - Capital letters with over right arrows e.g $\overrightarrow{AB}, \overrightarrow{CD}$.
  - Bold letters e.g $\mathbf{AB}, \mathbf{CD}, \mathbf{a}$
  - underaccent (Tilde)e.g $\underset{\sim}{a}, \underset{\sim}{b}$
- $\overrightarrow{AB}$ means a vector starting at A ending at B.So the arrow points towards B



2.2.2 Position Vector

This is a vector that defines/describes the position of a point with reference to the origin.e.g $\overrightarrow{OA} = \binom{x}{y}$ is the position vector of point A(x,y).

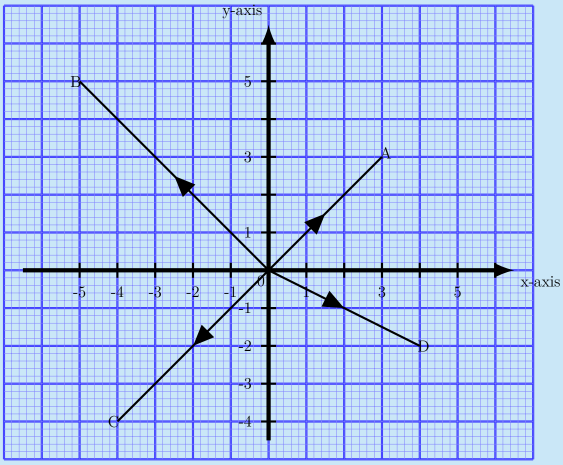


From the origin, A is 5 units in the x- direction and 4 units in the y -direction. Thus, A has coordinates (5 , 4) and $\overrightarrow{OA}$ has a position vector $\overrightarrow{OA} = \binom{5}{4}$

NOTE:All position vectors have O as their initial point.

Activity

In pairs write the position vectors of point A,B,C and D

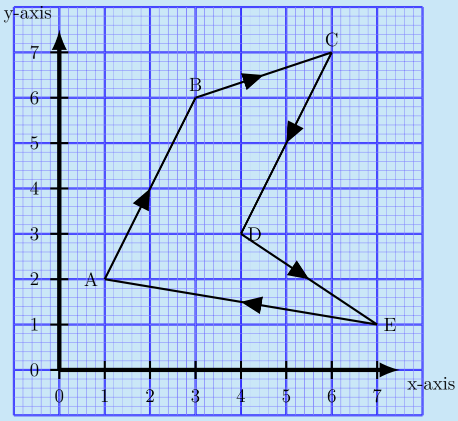


2.2.3 Column Vector

Column vectors are vectors that a written in a column form representing a change in x and y directions.i.e $\binom{x}{y}$ for example.$\binom{-4}{1}$,$\binom{5}{2}$

Activity

Describe each translation using a column vector

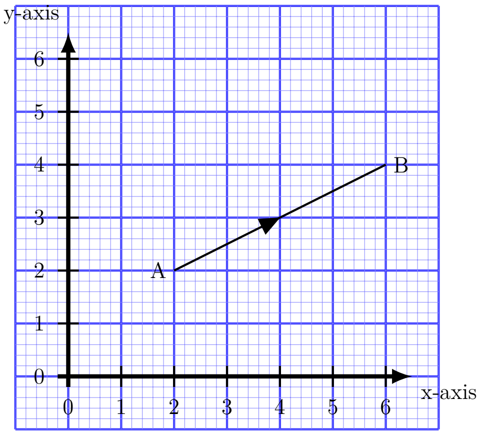


1. $\overrightarrow{AB}$
2. $\overrightarrow{BC}$
3. $\overrightarrow{CD}$
4. $\overrightarrow{DE}$
5. $\overrightarrow{EA}$
6. $\overrightarrow{AE}$
7. $\overrightarrow{DA}$
8. $\overrightarrow{CA}$
9. $\overrightarrow{DB}$

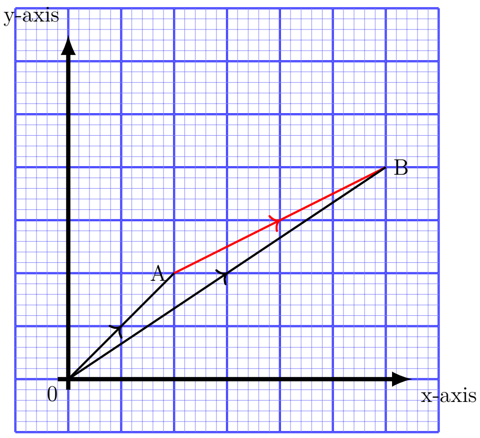
2.2.4 Displacement Vector

A displacement vector defines the distance that is moved in a specified direction.A displacement vector is represented by a directed line segment and doesnot start from the origin.e.g **AB**,**PQ**,**CD**

Expressing displacement vectors in terms of position vectors



AB is the displacement vector between A and B



$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$
$$\overrightarrow{OA} - \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$\boxed{\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}}$$

2.2.5 Operations on vectors

Addition of vectors

- To add two vectors we add the corresponding numbers
- If **a**=$\binom{x_1}{y_1}$ and **b**=$\binom{x_2}{y_2}$,then

$$\mathbf{a} + \mathbf{b} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$$
$$\mathbf{a} + \mathbf{b} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix}$$

Example 2.7

If **a**=$\binom{3}{4}$ and **b**=$\binom{4}{6}$,find **a+b**

SOLUTION

$$\mathbf{a}+\mathbf{b}=\begin{pmatrix}3\\4\end{pmatrix}+\begin{pmatrix}4\\6\end{pmatrix}$$
$$\mathbf{a}+\mathbf{b}=\begin{pmatrix}3+4\\4+6\end{pmatrix}$$
$$\mathbf{a}+\mathbf{b}=\begin{pmatrix}7\\10\end{pmatrix}$$

Subtraction of vectors

- To subtract two vectors we subtract the corresponding numbers
- If $\mathbf{a}=\begin{pmatrix}x_1\\y_1\end{pmatrix}$ and $\mathbf{b}=\begin{pmatrix}x_2\\y_2\end{pmatrix}$,then

$$\mathbf{a}-\mathbf{b}=\begin{pmatrix}x_1\\y_1\end{pmatrix}-\begin{pmatrix}x_2\\y_2\end{pmatrix}$$
$$\mathbf{a}-\mathbf{b}=\begin{pmatrix}x_1-x_2\\y_1-y_2\end{pmatrix}$$

Example 2.8

If $\mathbf{a}=\begin{pmatrix}3\\4\end{pmatrix}$ and $\mathbf{b}=\begin{pmatrix}4\\6\end{pmatrix}$,find $\mathbf{a}-\mathbf{b}$

SOLUTION

$$\mathbf{a}-\mathbf{b}=\begin{pmatrix}3\\4\end{pmatrix}-\begin{pmatrix}4\\6\end{pmatrix}$$
$$\mathbf{a}-\mathbf{b}=\begin{pmatrix}3-4\\4-6\end{pmatrix}$$
$$\mathbf{a}-\mathbf{b}=\begin{pmatrix}-1\\-2\end{pmatrix}$$

Scalar Multiplication and Division of vectors

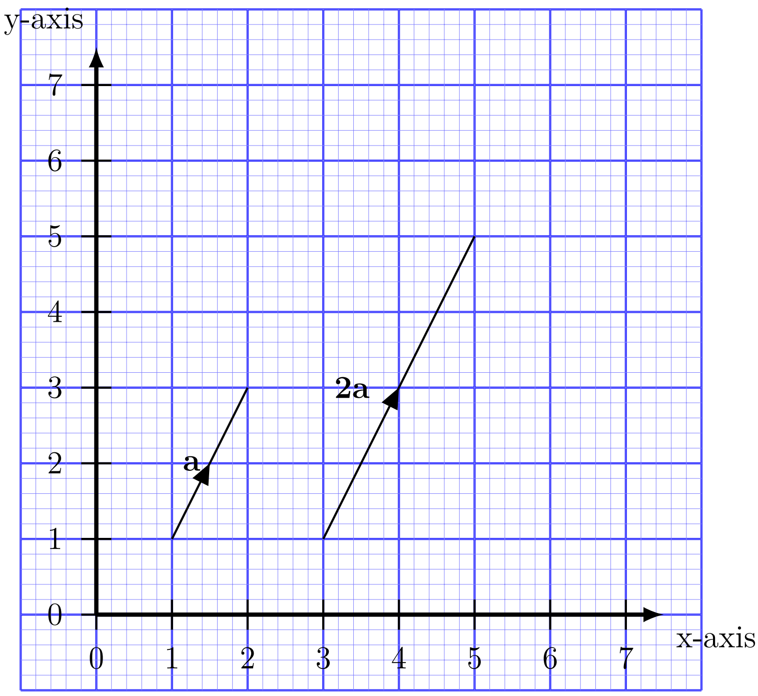
- A scalar k multiplied by vector $\mathbf{a}=\begin{pmatrix}x\\y\end{pmatrix}$ is treated as follows:

$$\mathbf{ka}=k\begin{pmatrix}x\\y\end{pmatrix}$$
$$\mathbf{ka}=\begin{pmatrix}kx\\ky\end{pmatrix}$$

- A scalar k divided by vector $\mathbf{a}=\begin{pmatrix}x\\y\end{pmatrix}$ is treated as follows:

$$\frac{1}{k}\mathbf{a}=\frac{1}{k}\begin{pmatrix}x\\y\end{pmatrix}$$
$$\frac{1}{k}\mathbf{a}=\begin{pmatrix}\frac{x}{k}\\\frac{y}{k}\end{pmatrix}$$

- For $\mathbf{a}=\begin{pmatrix}1\\2\end{pmatrix}$,we can represent **2a** on a diagram as shown below.



Note: A scalar is just a number(constant value)

Example 2.9

If $\mathbf{a}=\begin{pmatrix}4\\6\end{pmatrix}$ and $\mathbf{b}=\begin{pmatrix}2\\-3\end{pmatrix}$,find

(a) $2\mathbf{a}$ (b) $\frac{1}{2}\mathbf{a}$ (c) $2\mathbf{a}+3\mathbf{b}$

SOLUTION

(a) $2\mathbf{a}$

$$\mathbf{2a} = 2\begin{pmatrix}4\\6\end{pmatrix}$$

$$\mathbf{2a} = \begin{pmatrix}2\times 4\\2\times 6\end{pmatrix}$$

$$\mathbf{2a} = \begin{pmatrix}8\\12\end{pmatrix}$$

(b) $\frac{1}{2}\mathbf{a}$

$$\frac{1}{2}\mathbf{a} = \frac{1}{2}\begin{pmatrix}4\\6\end{pmatrix}$$

$$\frac{1}{2}\mathbf{a} = \begin{pmatrix}\frac{4}{2}\\\frac{6}{2}\end{pmatrix}$$

$$\frac{1}{2}\mathbf{a} = \begin{pmatrix}2\\3\end{pmatrix}$$

(c) $2\mathbf{a}+3\mathbf{b}$

$$\begin{aligned}
2\mathbf{a}+3\mathbf{b} &= 2\begin{pmatrix}4\\6\end{pmatrix}+3\begin{pmatrix}2\\-3\end{pmatrix}\\
&= \begin{pmatrix}2\times 4\\2\times 6\end{pmatrix}+\begin{pmatrix}3\times 2\\3\times -3\end{pmatrix}\\
&= \begin{pmatrix}8\\12\end{pmatrix}+\begin{pmatrix}6\\-9\end{pmatrix}\\
&= \begin{pmatrix}8+6\\12+-9\end{pmatrix}\\
2\mathbf{a}+3\mathbf{b} &= \begin{pmatrix}14\\3\end{pmatrix}
\end{aligned}$$

Magnitude(modulus,length) of a vector

Activity

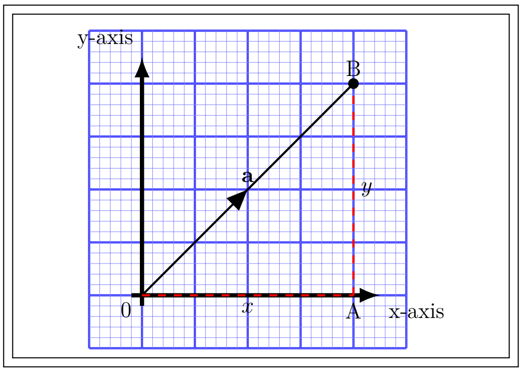
An insect moved from a point A, 10cm due East to point B. At point B,it turned north and moved 8cm to point C . However, there is a direct route from A to C.

(a) Sketch the diagram showing the insect's movement.

(b) Find the shortest distance from A to C.

(c) If the distance from A to B is x units and the distance from B to C is y units. Express the distance A to C in terms of x and y.

- Consider the vector vector $\mathbf{OB}=\binom{x}{y}$



$OB^2 = OA^2 + AC^2$ Pythagoras theorem

$OB^2 = x^2 + y^2$

$OB = \sqrt{x^2+y^2}$

The magnitude or length or modulus of the vector $\mathbf{OB}=\binom{x}{y}$ is $|\mathbf{OB}| = \sqrt{x^2+y^2}$

Example 2.10

If $\mathbf{a}=\binom{3}{4}$ and $\mathbf{b}=\binom{3}{6}$,find

(a) $|\mathbf{a}|$

(b) $|\mathbf{a}-\mathbf{b}|$

(c) $|4\mathbf{b}|$

SOLUTION

(a) $|\mathbf{a}|$

$$\begin{aligned}|\mathbf{a}| &= \sqrt{x^2+y^2}\\ &= \sqrt{3^2+4^2}\\ &= \sqrt{9+16}\\ &= \sqrt{25}\\ |\mathbf{a}| &= 5\text{units}\end{aligned}$$

(b) $|\mathbf{a}-\mathbf{b}|$

$$\begin{aligned}\mathbf{a}-\mathbf{b} &= \begin{pmatrix}3\\4\end{pmatrix} - \begin{pmatrix}3\\6\end{pmatrix}\\ \mathbf{a}-\mathbf{b} &= \begin{pmatrix}3-3\\4-6\end{pmatrix}\\ \mathbf{a}-\mathbf{b} &= \begin{pmatrix}0\\-2\end{pmatrix}\\ |\mathbf{a}-\mathbf{b}| &= \sqrt{x^2+y^2}\\ &= \sqrt{0^2+(-2)^2}\\ &= \sqrt{0+4}\\ &= \sqrt{4}\\ |\mathbf{a}-\mathbf{b}| &= 2\text{units}\end{aligned}$$

(c) $|4\mathbf{b}|$

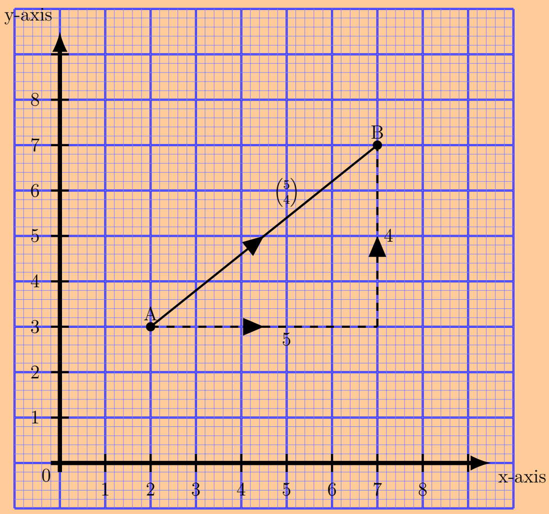
$$\begin{aligned}4\mathbf{b} &= 4\begin{pmatrix}3\\6\end{pmatrix}\\ &= \begin{pmatrix}4\times3\\4\times6\end{pmatrix}\\ 4\mathbf{b} &= \begin{pmatrix}12\\24\end{pmatrix}\\ |4\mathbf{b}| &= \sqrt{x^2+y^2}\\ &= \sqrt{12^2+24^2}\\ &= \sqrt{144+576}\\ &= \sqrt{720}\\ |4\mathbf{b}| &= 26.8328\text{units}\end{aligned}$$

Example 2.11

(a) Plot the points A (2, 3) and B(7,4) and show vector **AB** .

(b) Write down the column vector **AB** .

SOLUTION



There fore $\mathbf{AB} = \binom{5}{4}$

Example 2.12

Given $\mathbf{r}=\binom{-3}{4}$,find

(a) $|\mathbf{r}|$

(b) $|2\mathbf{r}|$

(c) $2|\mathbf{r}|$

SOLUTION

(a) $|\mathbf{r}|$

$$\begin{aligned}|\mathbf{r}| &= \sqrt{x^2+y^2}\\ &= \sqrt{(-3)^2+4^2}\\ &= \sqrt{9+16}\\ &= \sqrt{25}\\ |\mathbf{r}| &= 5\text{units}\end{aligned}$$

(b) $|2\mathbf{r}|$

$$\begin{aligned}2\mathbf{r} &= 2\begin{pmatrix}-3\\4\end{pmatrix}\\ &= \begin{pmatrix}2\times -3\\2\times 4\end{pmatrix}\\ &= \begin{pmatrix}-6\\8\end{pmatrix}\\ |2\mathbf{r}| &= \sqrt{x^2+y^2}\\ &= \sqrt{(-6)^2+8^2}\\ &= \sqrt{36+64}\\ &= \sqrt{100}\\ |2\mathbf{r}| &= 10\text{units}\end{aligned}$$

(c) $2|\mathbf{r}|$

$$\begin{aligned}2|\mathbf{r}| &= 2\sqrt{(-3)^2+4^2}\\ &= 2\sqrt{9+16}\\ &= 2\sqrt{25}\\ 2|\mathbf{r}| &= 2\times 5\\ 2|\mathbf{r}| &= 10\text{units}\end{aligned}$$

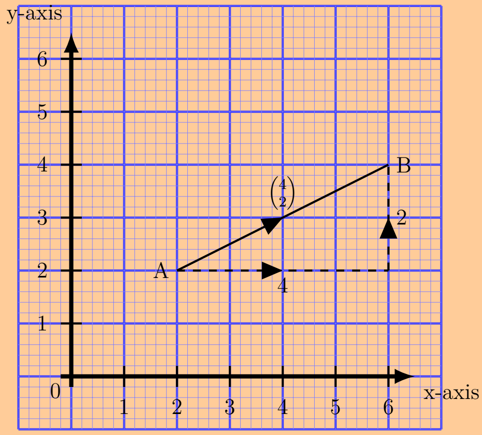
NOTE:$|2r| = 2|r|$,therefore in general, $|kr| = k|r|$.

Example 2.13

Given the points A(2, 2) and B(6, 4), find the column vector AB

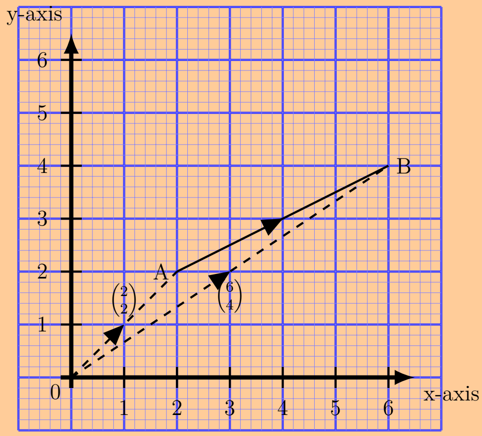
SOLUTION

Method 1



There fore $\mathbf{AB} = \binom{4}{2}$

Method 2



$$\mathbf{AB} = \mathbf{OB} - \mathbf{OA}$$

$$\mathbf{AB} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix} 6-2 \\ 4-2 \end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

Example 2.14

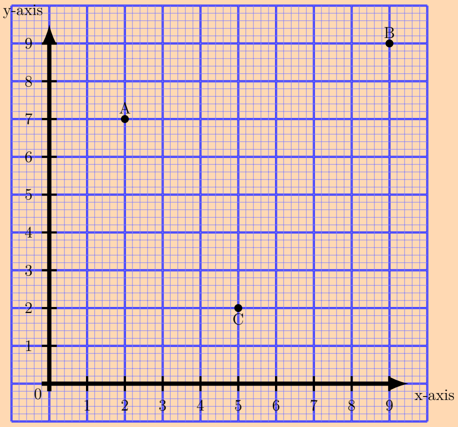
Write each of the following vectors in the form $\binom{a}{b}$

(a) $\overrightarrow{AB}$

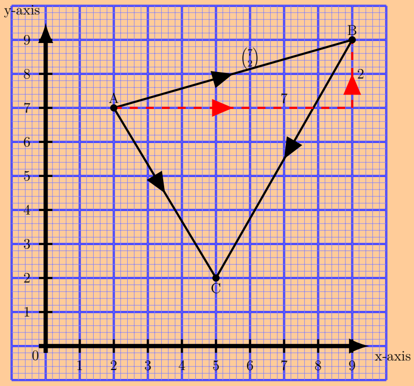
(b) $\overrightarrow{BC}$

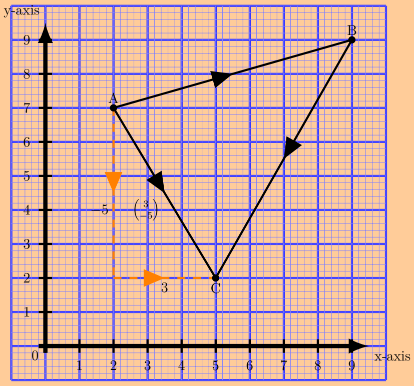
(c) $\overrightarrow{AC}$

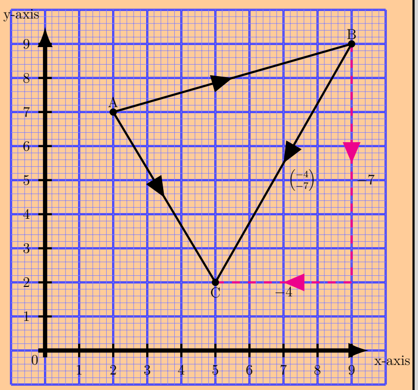
HINT :Draw a line to join the points



SOLUTION







(a) $\overrightarrow{AB} = \binom{7}{2}$

(b) $\overrightarrow{BC} = \binom{-4}{-7}$

(c) $\overrightarrow{AC} = \binom{3}{-5}$

Example 2.15

If $\mathbf{OA}=\binom{3}{4}$ and $\mathbf{OB}=\binom{4}{6}$,find the column vector **AB**

SOLUTION

$$\mathbf{AB} = \mathbf{OB} - \mathbf{OA}$$

$$\mathbf{AB} = \begin{pmatrix}4\\6\end{pmatrix} - \begin{pmatrix}3\\4\end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix}4-3\\6-4\end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix}1\\2\end{pmatrix}$$

Example 2.16

Given that vector$\mathbf{PQ}=\binom{2}{6}$ and $\mathbf{OP}=\binom{-5}{1}$,find the column vector **OQ**

SOLUTION

$$\mathbf{PQ} = \mathbf{OQ} - \mathbf{OP}$$
$$\mathbf{OQ} = \mathbf{PQ} + \mathbf{OP}$$
$$\mathbf{OQ} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} + \begin{pmatrix} -5 \\ 1 \end{pmatrix}$$
$$\mathbf{OQ} = \begin{pmatrix} 2 + -5 \\ 6 + 1 \end{pmatrix}$$
$$\mathbf{OQ} = \begin{pmatrix} 2 - 5 \\ 6 + 1 \end{pmatrix}$$
$$\mathbf{OQ} = \begin{pmatrix} -3 \\ 7 \end{pmatrix}$$

Example 2.17

Given that vector$\mathbf{OF}$=$\binom{3}{8}$ and $\mathbf{OG}$=$\binom{-1}{-4}$,find the column vector

(a) $\mathbf{FG}$ (b) $|\mathbf{FG}|$ (c) $\mathbf{GF}$ (d) $|\mathbf{GF}|$

SOLUTION

(a) $\mathbf{FG}$

$$\mathbf{FG} = \mathbf{OG} - \mathbf{OF}$$
$$\mathbf{FG} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} - \begin{pmatrix} 3 \\ 8 \end{pmatrix}$$
$$\mathbf{FG} = \begin{pmatrix} -1 - 3 \\ -4 - 8 \end{pmatrix}$$
$$\mathbf{FG} = \begin{pmatrix} -4 \\ -12 \end{pmatrix}$$

(b) $|\mathbf{FG}|$

$$\begin{aligned} |\mathbf{FG}| &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-4)^2 + (-12)^2} \\ &= \sqrt{16 + 144} \\ &= \sqrt{160} \\ |\mathbf{FG}| &= 12.649\text{units} \end{aligned}$$

(c) $\mathbf{GF}$

$$\mathbf{GF} = \mathbf{OF} - \mathbf{OG}$$
$$\mathbf{GF} = \begin{pmatrix} 3 \\ 8 \end{pmatrix} - \begin{pmatrix} -1 \\ -4 \end{pmatrix}$$
$$\mathbf{GF} = \begin{pmatrix} 3 - -1 \\ 8 - -4 \end{pmatrix}$$
$$\mathbf{GF} = \begin{pmatrix} 3 + 1 \\ 8 + 4 \end{pmatrix}$$
$$\mathbf{GF} = \begin{pmatrix} 4 \\ 12 \end{pmatrix}$$

(d) $|\mathbf{GF}|$

$$\begin{aligned} |\mathbf{GF}| &= \sqrt{x^2 + y^2} \\ &= \sqrt{4^2 + 12^2} \\ &= \sqrt{16 + 144} \\ &= \sqrt{160} \\ |\mathbf{GF}| &= 12.649\text{units} \end{aligned}$$

NOTE:In vectors $\mathbf{FG} \neq \mathbf{GF}$

Example 2.18

If $\mathbf{a}$=$\binom{2}{1}$ and $\mathbf{b}$=$\binom{3}{-4}$,solve the equations below to find the column vector x

(a) $\mathbf{a} + \mathbf{x} = \mathbf{b}$ (b) $\mathbf{2x} + \mathbf{a} = \mathbf{b}$

SOLUTION

(a) $\mathbf{a} + \mathbf{x} = \mathbf{b}$

$$a + x = b$$
$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} + x = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$
$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} + x = \begin{pmatrix} 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$
$$x = \begin{pmatrix} 3-2 \\ -4-1 \end{pmatrix}$$
$$x = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$$

(b) $\mathbf{2x} + \mathbf{a} = \mathbf{b}$

$$2x + a = b$$
$$2x + \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$
$$2x + \begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$
$$2x = \begin{pmatrix} 3-2 \\ -4-1 \end{pmatrix}$$
$$2x = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$$
$$\frac{2x}{2} = \frac{1}{2}\begin{pmatrix} 1 \\ -5 \end{pmatrix}$$
$$\frac{\cancel{2}x}{\cancel{2}} = \begin{pmatrix} \frac{1}{2} \\ \frac{-5}{2} \end{pmatrix}$$
$$x = \begin{pmatrix} 0.5 \\ -2.5 \end{pmatrix}$$

2.2 Exercise Set

1. State the position vectors of the following points.

   (a) A$(2, 3)$ (b) B$(-6, 8)$ (c) P$(-3, -4)$ (d) Q$(2, 3)$

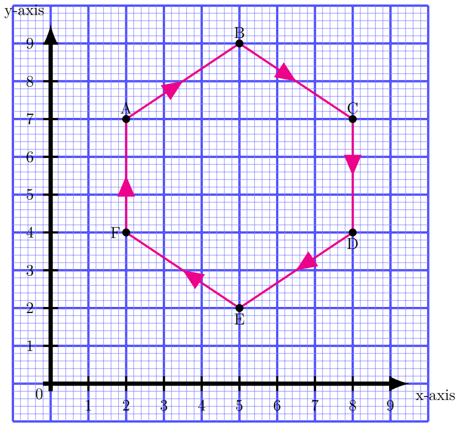
2. Plot the positions of the points A, B, C, D, E and F relative to a point O if:

   (a) $\overrightarrow{OA} = \binom{3}{2}$ (c) $\overrightarrow{AC} = \binom{-3}{2}$ (e) $\overrightarrow{CE} = \binom{-4}{-1}$

   (b) $\overrightarrow{OB} = \binom{5}{-2}$ (d) $\overrightarrow{BD} = \binom{-2}{-2}$ (f) $\overrightarrow{DF} = \binom{1}{4}$

   Write the vector EF as a column vector.

3. Using the diagram below describe the following translations using column vectors .



(a) $\overrightarrow{AB}$
(b) $\overrightarrow{AC}$
(c) $\overrightarrow{DE}$
(d) $\overrightarrow{BE}$
(e) $\overrightarrow{EB}$
(f) $\overrightarrow{AD}$
(g) $\overrightarrow{CD}$
(h) $\overrightarrow{DC}$
(i) $\overrightarrow{BC}$
(j) $\overrightarrow{FB}$
(k) $\overrightarrow{EF}$
(l) $\overrightarrow{BA}$

4. A displacement starts from A$(3,3)$ and ends at B(-2,4).Find the column vector **AB**

5. Given the points A$(2,3)$ and B$(4,-1)$.Find
   (a) the position vectors for each of the points
   (b) the column vector **AB**
   (c) the column vector **BA**

6. If $\mathbf{a}=\binom{1}{3}$,$\mathbf{b}=\binom{4}{-1}$ and $\mathbf{c}=\binom{-2}{-2}$,solve the equations below to find the column vector d
   (a) $\mathbf{d}-\mathbf{c}=\mathbf{a}$
   (b) $\mathbf{d}+\mathbf{b}=\mathbf{c}$
   (c) $\mathbf{a}-\mathbf{2d}=\mathbf{4c}$
   (d) $\mathbf{3a}+\mathbf{2d}=\mathbf{c}$
   (e) $\mathbf{3a}+\mathbf{2d}=\mathbf{4b}$
   (f) $\mathbf{3b}+\mathbf{2d}=\mathbf{c}$

7. Given that vector**OP**$=\binom{5}{3}$ and **OP**$=\binom{-3}{-2}$,find the column vector **PQ**

8. If **OM**$=\binom{1}{3}$,**ON**$=\binom{4}{-1}$ and **OP**$=\binom{-2}{-2}$,Find the column vector
   (a) **MN**
   (b) **MP**
   (c) **NM**
   (d) **MN**

9. Given that vector**OA**$=\binom{4}{-3}$ and **AB**$=\binom{2}{5}$,find the position vector **OB**

10. If $\mathbf{a}=\binom{4}{7}$,$\mathbf{b}=\binom{3}{-5}$ and $\mathbf{c}=\binom{0}{4}$,find
   (a) $\mathbf{a}+\mathbf{b}$
   (b) $\mathbf{a}+\mathbf{b}+\mathbf{c}$
   (c) $\mathbf{a}-\mathbf{b}$
   (d) $\mathbf{b}-\mathbf{a}$
   (e) $\mathbf{c}-\mathbf{a}$
   (f) $\mathbf{3a}$
   (g) $-\mathbf{2b}$
   (h) $\mathbf{4c}$
   (i) $|\mathbf{2a}+\mathbf{3b}|$
   (j) $|\mathbf{2b}-\mathbf{a}|$
   (k) $|\mathbf{5c}-\mathbf{3a}|$

11. Given that $\underset{\sim}{a}=\binom{4}{-5}$ and $\underset{\sim}{b}=3\underset{\sim}{a}$,find $|\underset{\sim}{a}+\underset{\sim}{b}|$

12. Given the points A(4, 1) and B(12, 16), find the:
    (a) column vector **AB**
    (b) length of **AB**
13. Given the vectors $\overrightarrow{OP} = \binom{3}{7}$ and $\overrightarrow{PQ} = \binom{5}{-1}$ .Find the
    (a) column vector $\overrightarrow{OQ}$
    (b) the length of $|\overrightarrow{OQ}|$
14. Given the points P(-2,3) and Q(3,6) ,find the coordintaes of R ,if $\overrightarrow{OR} = 3\overrightarrow{OP} + \frac{1}{3}\overrightarrow{OQ}$
15. Given the points A(3,4) and B(9,2) .Find the coordinates of T ,if $\overrightarrow{OT} = 3\overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$

Chapter Summary

1. Translation$-$ This is a transformation that moves every point in a figure the same distance in the same direction
2. A translation T=$\binom{a}{b}$ means that an object is moved a distance **a** in the $x-$direction and a distance **b** in the $y-$direction
3. Translation + object = image
4. Vector$-$This is a physical quantity that has magnitude(size or length) and direction.e.g displacement,velocity
5. A scalar $-$This is a physical quantity that has magnitude(size or length)only.e.g distance ,speed
6. When a displacement vector is written as **AB**=$\binom{x}{y}$ is called a Column vector
7. Column Vector describes the movement of an object in both the x-direction and the y-direction
8. A Null vector has no magnitude and direction. It is denoted as **O** or $\overrightarrow{O}$.
9. All position vector have **O** as their initial position.
10. If **a**=$\binom{x_1}{y_1}$ and **b**=$\binom{x_2}{y_2}$,then $\mathbf{a} + \mathbf{b} = \binom{x_1+x_2}{y_1+y_2}$
11. If **a**=$\binom{x_1}{y_1}$ and **b**=$\binom{x_2}{y_2}$,then $\mathbf{a} - \mathbf{b} = \binom{x_1-x_2}{y_1-y_2}$
12. When a vector **a**=$\binom{x}{y}$ is multiplied by a scalar k, we obtain $\binom{kx}{ky}$
13. The magnitude of the column vector **a**=$\binom{x}{y}$ is given by $|a| = \sqrt{x^2 + y^2}$
14. The vectors **AB** and **BA** are equal in length but opposite in direction i.e $\mathbf{BA} = -\mathbf{AB}$
15. $\mathbf{AB} \neq \mathbf{BA}$
16. When a vector is multiplied by a position scalar, its direction does not change. However, when a vector is multiplied by a negative scalar, its direction is reversed.

ASSESSMENT

1. A translation T=$\binom{3}{4}$ maps the points A(2, 5) onto the point P^1 . Find the coordinates of P^1
2. If **r**=$\binom{1}{3}$,**s**=$\binom{3}{-4}$ and **t**=$\binom{2}{4}$,find

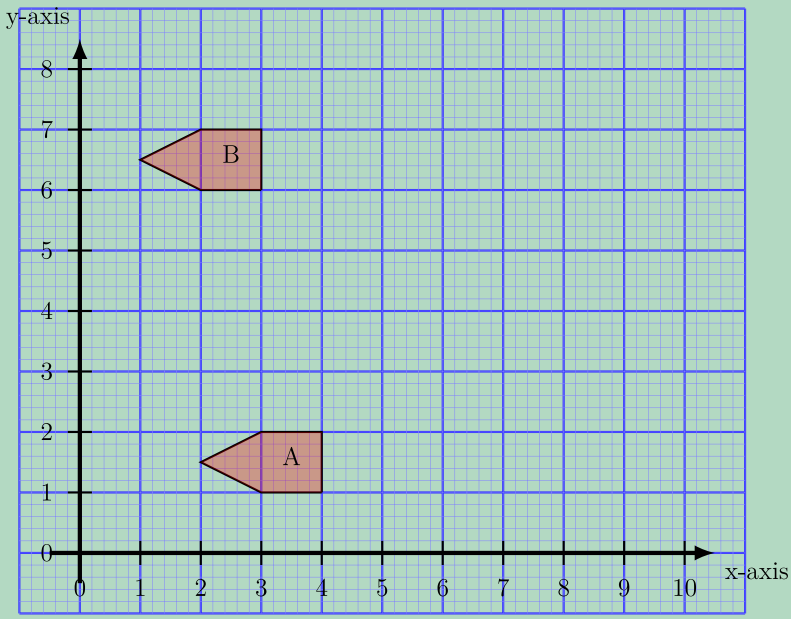
(a) $\mathbf{r}+\mathbf{s}$

(b) $\mathbf{s}-\mathbf{t}$

(c) $|\mathbf{2r}|$

(d) $|\mathbf{3s}+\mathbf{2t}|$

3. Write down the translation vector that would take A to B.



4. Given that $\overrightarrow{OB}=\binom{1}{9}$ and $\overrightarrow{AB}=\binom{-2}{5}$.Find the:
   (a) coordinates of A
   (b) $|OA|$
5. Triangle ABC has vertices A $(-3,0)$,B $(0,3)$ and C $(3,0)$. Find the coordinates of the points A^1, B^1 and C^1,the images of A, B and C respectively, under a translation with displacement vector $\binom{2}{3}$
6. Find the translation that maps point K(2, 6) onto K^1(3,8)
7. Given the points A$(-2,3)$ and B$(3,6)$, find the coordinates of C, if $\mathbf{OC}=\mathbf{3OA}+\frac{\mathbf{1}}{\mathbf{3}}\mathbf{OB}$
8. Given the vectors $\overrightarrow{OP}=\binom{3}{7}$ and $\overrightarrow{PQ}=\binom{5}{-1}$ .Find the
   (a) column vector $\overrightarrow{OQ}$
   (b) the length of $|\overrightarrow{OQ}|$
9. Given that $\underset{\sim}{a}=\binom{-2}{-9}$,$\underset{\sim}{b}=\binom{7}{2}$,and m=$\underset{\sim}{a}+2\underset{\sim}{b}$
10. Given the vectors $\underset{\sim}{a}=\binom{-2}{7}$,$\underset{\sim}{b}=\binom{8}{11}$ and $\underset{\sim}{c}=\binom{4}{-5}$.Find the length of $\underset{\sim}{a}+2\underset{\sim}{b}+\underset{\sim}{c}$

End

Chapter 3

GRAPHS

Learning objectives

By the end of this topic,the learners should be able to

- Tabulate values from given relations
- Plot and draw lines through given points
- Choose and uses appropriate scale to draw graphs
- Draw distance -time and speed time graphs
- Read , interpret and estimate distance ,time and speed on graphs.

3.1 Linear Relations

Activity

1. Copy and complete the table below using the relation $p = 2n + 1$

n	0	1	2	3	4	5
p	1					

2. (a) Plot the points with coordinates:$(-2,-1)$,$(-1,0)$,$(0,1)$,$(1, 2)$, $(2, 3)$, $(3, 4)$,and $(4, 5)$ on a graph paper.

   (b) Join the points to form a line

   (c) Express the relation as an equation(write the relation between x and y values)

From the activity ,we can express relations in any of the following ways.

- Using word e.g The distance traveled (**d**) by students equals to the product of speed(**u**) and the time (**t**)
- Using a formula or an equation e.g $d = ut$
- Using a table of values
- Using a graph

Example 3.1

Copy and complete the table below using the relation $d = 2t$

t	0	1	2	3	4	5
d						

SOLUTION

for t=0
$$d = 2t$$
$$d = 2 \times 0$$
$$d = 0$$

for t=1
$$d = 2t$$
$$d = 2 \times 1$$
$$d = 2$$

for t=2
$$d = 2t$$
$$d = 2 \times 2$$
$$d = 4$$

for t=3
$$d = 2t$$
$$d = 2 \times 3$$
$$d = 6$$

t	0	1	2	3	4	5
d	0	2	4	6	8	10

3.1 Exercise Set

Copy and complete the tables using the given relations

1. $d = 5t$

t	0	1	2	3	4	5
d	0					

2. $s = \frac{1}{2}t^2$

t	0	2	4	6	8	10
s	0					

3. $y = 2x - 5$

x	-2	-1	0	1	2	3
y	-9					

4. $v = u - 5$

v	-2	-1	0	1	2	3
u						8

5. $p = 3q - 2$

p	-8	-5	-2	1	4	7
q	-2					

6. $a = 5b^2$

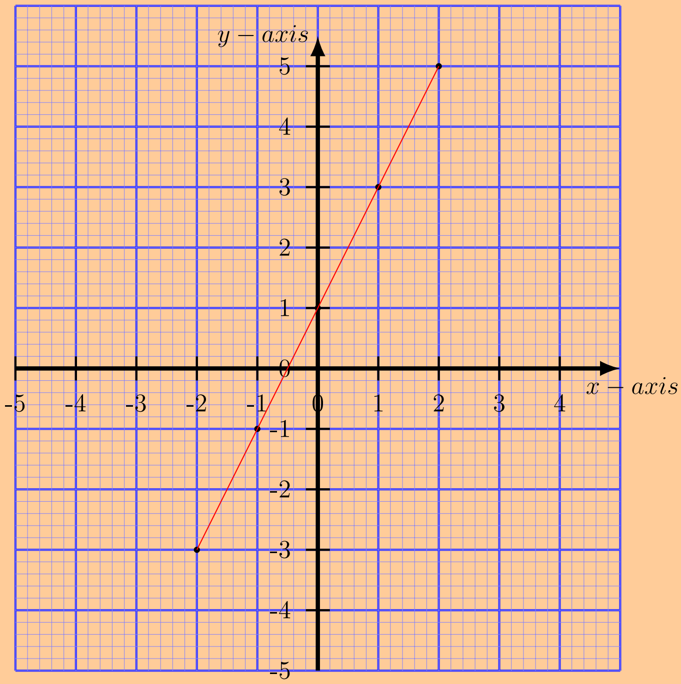
b	-8	-5	-2	1	4	7
a						

3.2 Plotting and drawing lines

Example 3.2

Plot the following points on a graph paper $(-2,-3),(-1,-1),(0,1)(1,3)$ and (2, 5),hence join them to form a straight line.

SOLUTION

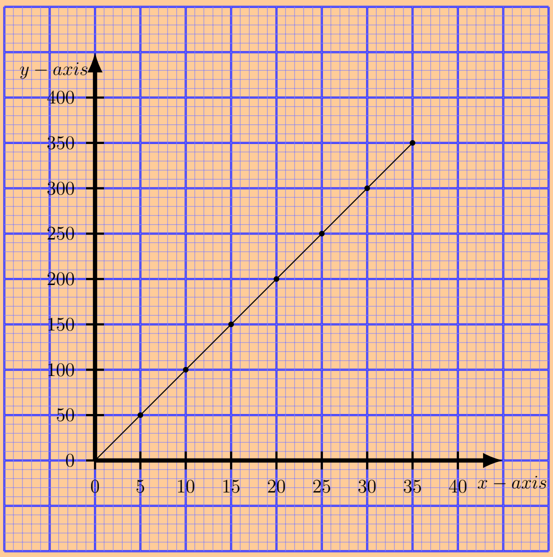


Example 3.3

Plot the following points on the axes:(0,0) (5, 50), (10,100), (15,150), (20,200),(25,250) ,(30, 300), and (35,350),Join them to form a line.

SOLUTION

You realise that on the horizontal axis(x- values) there are 5 units for each space and On the vertical axis (y- values)there are 50 units for each space



3.2 Exercise Set

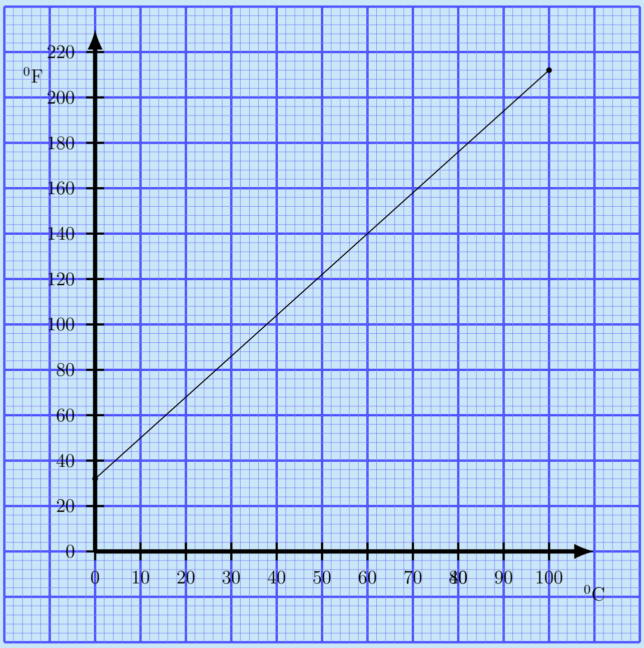
1. Plot the points (0, 5), (2, 3), (4, 1) and (5, 0). Draw a straight line through them.
2. The points $(-3, -4), (-1, -2)$, (1, 0), and (4, 3) lie on a straight line. Plot these points and draw a straight line through them.
3. Plot the points with coordinates
   (a) (2, 6), (3, 5), (4, 4) and (7, 1)and draw a straight line through them.
   (b) On the same set of axes, plot the points with coordinates (0, 1), (1, 2), (3, 4) and (5, 6) and draw a straight line through them.
4. Plot the points with coordinates (0, 3), (1, 5), (2, 7), (3, 9), and (4, 11) and draw a straight line through them.
5. Plot the following points with coordinates (0,0) (3, 30), (6,60), (9,90), (12,120),(15,150) ,(18, 180), and (21,210),Join them to form a line.

3.3 Conversion Graphs

A conversion graph is used to change one unit into another. This could be changing between ^{0}C and ^{0}F, pounds,dollars to a foreign currency.

Activity

The graph below shows the relationship between centigrade and fahrenheit scales.



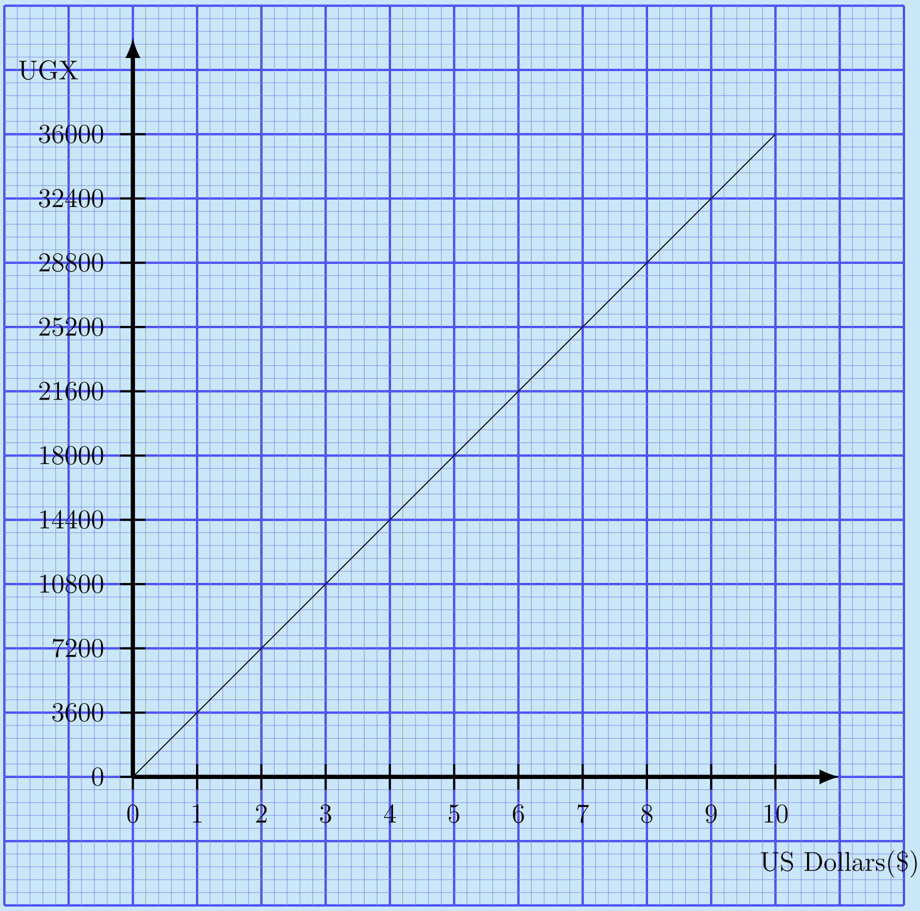
1. Use the graph to make an approximate conversion of the following temperatures to ^{0}F

   (a) 50^0C (b) 64^0C (c) 24^0C (d) 70^0C (e) 10^0C

2. Use the graph to make an approximate conversion of the following temperatures to ^{0}C

   (a) 180^0F (b) 176^0F (c) 158^0F (d) 20^0F (e) 30^0F

Activity



1. Use the graph to
   (a) Convert $3 to UGX
   (b) Convert UGX18000 to US dollars($)
   (c) Find the difference in UGX between $4 and UGX 28800
   (d) Calculate the balance from UGX 57600,when you spend $7.Give your answer in $

2. Jessica went to the shop with $15 and bought ,2 loaves of bread,2 black books and 3 kilogrammes of sugar. The Unit costs for the items are estimated in the table below.

Item	Bread	Book	Sugar
Us dollars($)	1	2	1

   Use the graph to
   (a) Find the total cost of all items in UGX
   (b) How many US dollars did Jessica remain with?

3.4 Travel Graphs

Travel graphs are representations of journeys of objects over a duration of time.

Calculating Speed, Distance and Time

- Speed :This is the change of distance with time
  - Speed=$\frac{\text{Distance}}{\text{Time}}$
- Average Speed :This is the ratio of the total distance travelled to the total time taken
  - Average Speed=$\frac{\text{Total Distance Travelled}}{\text{Total Time taken}}$
- Distance=Speed × Time
- Speed=$\frac{\text{Distance}}{\text{Time}}$
- Time=$\frac{\text{Distance}}{\text{speed}}$
- The speed at any time is called the instantaneous speed.
- The following table lists units in common use for speed,distance and time and their abbreviations:

Distance	Time	Speed	Units for speed
kilometres	hours	kilometres per hour	kmh^{-1}
metres	seconds	metres per second	ms^{-1}

- Time in minutes can be converted into hours and vice versa as follows:
  - Minutes to hours
    Convert 30 minutes to hours
    $$\begin{aligned}30\text{minutes} &= \frac{30}{60}\\ &= \frac{1}{2}\text{hours}\end{aligned}$$
  - Hours to minutes
    Convert $\frac{1}{2}$hours to minutes
    $$\begin{aligned}\frac{1}{2}\text{hours} &= \frac{1}{2}\times 60\\ &= 30\text{minutes}\end{aligned}$$
- Speed in kmh^{-1} can be converted into ms^{-1} as follows:
  Convert 72kmh^{-1} to ms^{-1}
  $$\begin{aligned}72\text{kmh}^{-1} &= \frac{72\times 1000}{3600}\\ &= 20\text{ms}^{-1}\end{aligned}$$
- If a car travels at a constant speed of 72kmh^{-1} it means that the car covers 72km after every one hour

Example 3.4

A body travels a distance of 60metres in 4 seconds.Calculate its speed.

SOLUTION

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$
$$\text{Speed} = \frac{60}{4}$$
$$\text{Speed} = 15\text{ms}^{-1}$$

Example 3.5

A car travels 75km at a constant speed of 15kmh^{-1} How long does the journey take?

SOLUTION

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$
$$\text{Time} = \frac{75}{15}$$
$$\text{Time} = \frac{\cancelto{5}{75}}{\cancelto{1}{15}}$$
$$\text{Time} = 5 \text{ hours}$$

Example 3.6

A car travels with a constant speed of 72kmh^{-1} How far can it travel in 2 hours?

SOLUTION

$$\text{Distance} = \text{Speed} \times \text{Time}$$
$$\text{Distance} = 72 \times 2$$
$$\text{Distance} = 144 \text{ km}$$

Example 3.7

A car toy travels with a constant speed of 2ms^{-1} How far can it travel in 2 minutes?

SOLUTION

Convert to common units i.e minutes to seconds

$$\text{Distance} = \text{Speed} \times \text{Time}$$
$$\text{Distance} = 2 \times (2 \times 60)$$
$$\text{Distance} = 240 \text{ m}$$

Example 3.8

A train travels with a constant speed of 12kmh^{-1} How far can it travel in 120 minutes?

SOLUTION

Convert to common units i.e minutes to hours

$$\text{Distance} = \text{Speed} \times \text{Time}$$
$$\text{Distance} = 12 \times \frac{120}{60}$$
$$\text{Distance} = 12 \times \frac{\cancelto{2}{120}}{\cancelto{1}{60}}$$
$$\text{Distance} = 12 \times 2$$
$$\text{Distance} = 24 \text{ km}$$

Example 3.9

A car travels 40km in 30minutes. Find its average speed

SOLUTION

Convert to common units i.e minutes to hours

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Time taken}}$$
$$\text{Average Speed} = 40 \div \frac{30}{60}$$
$$\text{Average Speed} = 40 \times \frac{60}{30}$$
$$\text{Average Speed} = 40 \times \frac{\cancelto{2}{60}}{\cancelto{1}{30}}$$
$$\text{Average Speed} = 40 \times 2$$
$$\text{Average Speed} = 80\text{kmh}^{-1}$$

Example 3.10

A car travels for 5 hours with a constant speed 85kmh^{-1} and then travels for 3 hours with a constant speed of 69kmh^{-1}.

SOLUTION

(a) Calculate the total distance covered by the car

$$\text{Distance} = \text{Speed} \times \text{Time}$$
$$\text{d}_1 = 85 \times 5$$
$$\text{d}_1 = 425\text{km}$$

$$\text{Distance} = \text{Speed} \times \text{Time}$$
$$\text{d}_2 = 69 \times 3$$
$$\text{d}_2 = 207\text{km}$$

$$\text{Total distance} = d_1 + d_2$$
$$\text{Total distance} = 425 + 207$$
$$\text{Total distance} = 632\text{km}$$

(b) Find its average speed

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$
$$\text{Average speed} = \frac{632}{8}$$
$$\text{Average speed} = \frac{\cancel{632}^{79}}{\cancel{8}^{1}}$$
$$\text{Average speed} = 79\text{kmh}^{-1}$$

3.3 Exercise Set

1. Convert the following speeds to kmh^{-1}:

   (a) 66ms^{-1} (b) 8ms^{-1} (c) 72ms^{-1} (d) 25ms^{-1}

2. Calculate the distance that you would travel if you drove for:

   (a) 3 hours at 20kmh^{-1}
   (b) $\frac{1}{2}$hour at 76kmh^{-1}
   (c) 6 hours at 60kmh^{-1}
   (d) 45 minutes at 60kmh^{-1}
   (e) 8 minutes at 60ms^{-1}
   (f) 90 minutes at 45ms^{-1}

3. How long does it take to travel:

   (a) 120m at 40ms^{-1}
   (b) 240m at 60ms^{-1}
   (c) 390km at 60kmh^{-1}
   (d) 200cm at 0.4ms^{-1}
   (e) 5km at 70ms^{-1}
   (f) 385m at 70ms^{-1}

4. Find the average speed of an object moving

   (a) 30m in 6 s
   (b) 70km in 2hr
   (c) 50km in 2.5hr
   (d) 48m in 12s
   (e) 400km in 2hr 30 min
   (f) 110km in 2hr 12 min

5. A car travels 300km in 5 hours. Calculate the average speed of the car in

   (a) kmh^{-1}

   (b) ms^{-1}

6. A car travels 97.5km with a constant speed of 65kmh^{-1} and then travels 60km with a constant speed of 80kmh^{-1}. Find its average speed

7. Hannah and Lewis leave their home at the same time. Hannah has 60m to travel and drives at 40ms^{-1}. Lewis has 80m to travel and also drives at 40ms^{-1}.

   (a) How long does Hannah's journey take?

   (b) How much longer does Lewis spend driving than Hannah?

8. A car travels for 40 minutes with a constant speed of 84kmh^{-1}. Find the speed of another car which takes 48 minutes to travel the same distance

9. Kiprotich can run long distances at 4 metres per second. How far can he run in:

   (a) 30 seconds (b) 5 minutes (c) $\frac{1}{4}$hour (d) $2\frac{1}{2}$hours

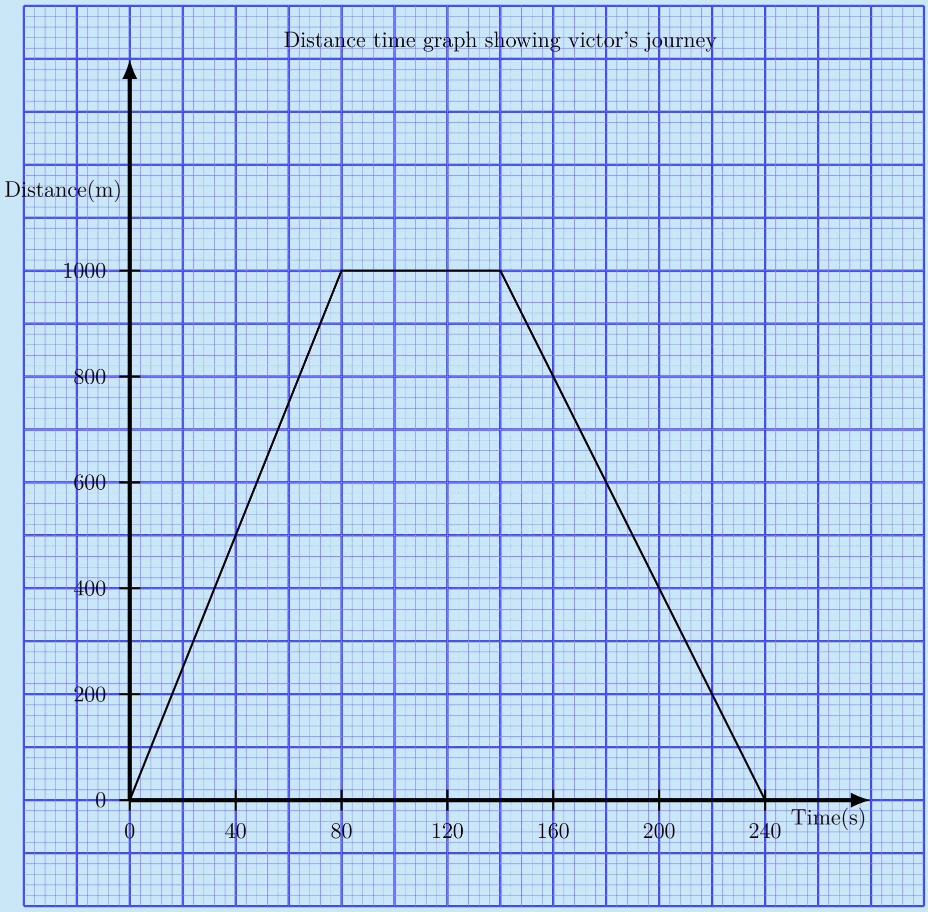
10. A car travels 200m in 3 hours and 20 minutes. Calculate the average speed of the car in ms^{-1}.

3.5 Distance-Time Graphs

- When drawing travel graphs, time is always on the horizontal axis.

Activity

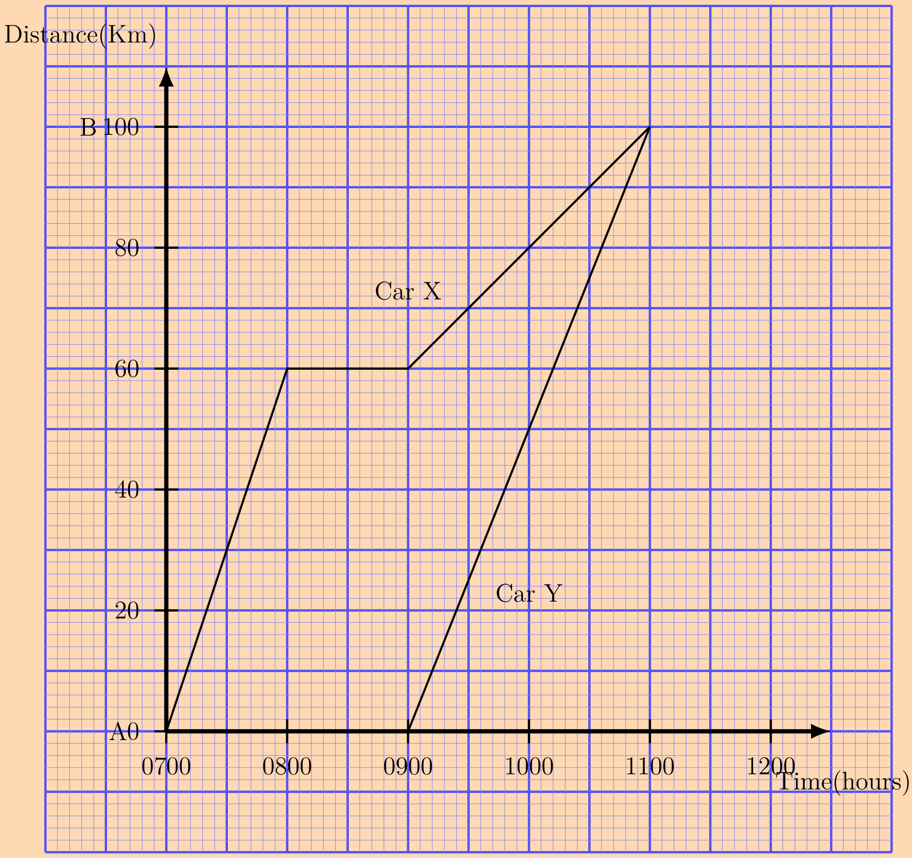
The distance time graph below shows Victor's journey from the class to the school canteen.



1. How far did Victor walk to reach the school's canteen.
2. How long did Victor take to purchase an item at the canteen?
3. How long did it take Victor to walk from the canteen back to class?
4. Calculate Victor's speed from the canteen to class
5. Calculate Victor's average speed for the whole journey.
6. How many minutes did Victor use for the whole journey
7. Draw the speed time graph for Victor's journey

Example 3.11

The distance-time graph below describes the journey of two cars X and Y between two stations, A and B .Car X and Car Y both reach point B 100km from A at 1100



(a) Calculate the speed of Car X between 0700 and 0800

(b) Calculate the speed of Car Y between 0900 and 1100

(c) Explain what is happening to Car X between 0800 and 0900

SOLUTION

(a)

$$\begin{aligned}\text{Speed} &= \frac{\text{Distance}}{\text{Time}}\\ &= \frac{60}{1}\\ \text{Speed} &= 60\text{kmh}^{-1}\end{aligned}$$

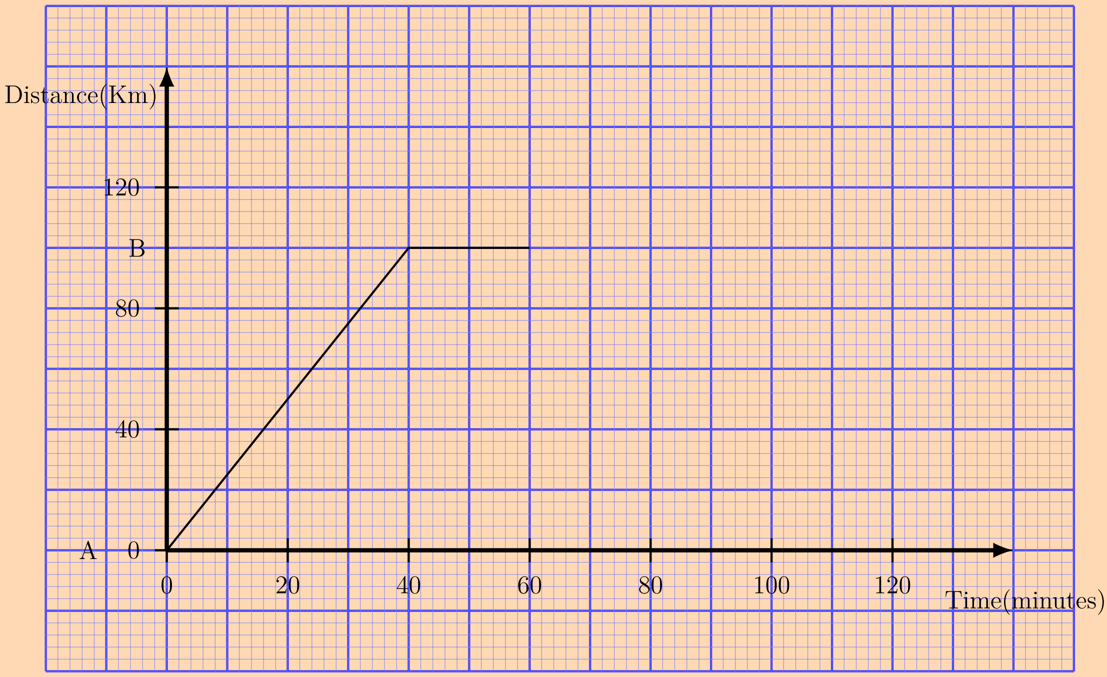
(b)

$$\begin{aligned}\text{Speed} &= \frac{\text{Distance}}{\text{Time}}\\ &= \frac{100}{2}\\ &= \frac{\cancel{100}^{50}}{\cancel{2}^{1}}\\ \text{Speed} &= 50\text{kmh}^{-1}\end{aligned}$$

(c) Car X is stationary .i.e it is at rest for 1 hour.Therefore no distance has been travelled

Example 3.12

The displacement-time graph below describes the journey of a train between two train stations, A and B



(a) For how many minutes was the train at rest at B?

(b) Determine the average speed of the train, in kmh^{-1} on its journey from A to B.

(c) The train continued its journey away from stations A and B to another station C, which is 50 km from B. The average speed on this journey was 60 kmh^{-1}. Calculate the time, in minutes, taken for the train to travel from B to C.

(d) Draw the line segment which represents the journey of the train from B to C.

SOLUTION

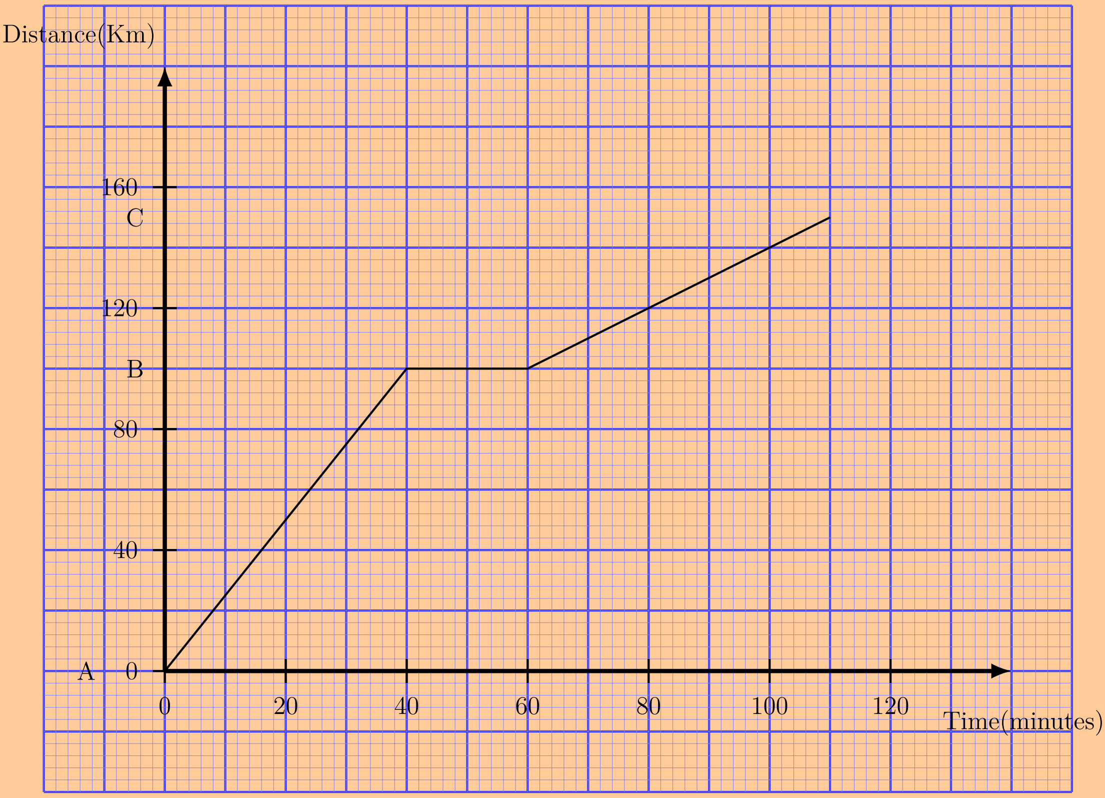
(a) 10 minutes

(b)

$$
\begin{aligned}
\text{Average speed} &= \frac{\text{Total distance}}{\text{Total time}} \\
&= 100 \div \frac{40}{60} && \text{Time in hours} \\
&= 100 \times \frac{60}{40} \\
&= \frac{6000}{40} \\
\text{Average speed} &= 150\text{kmh}^{-1}
\end{aligned}
$$

(c)

$$\begin{aligned}
\text{Time} &= \frac{\text{Total distance}}{\text{Speed}} \\
&= \frac{50}{60} \\
\text{Time} &= \frac{5}{6}\text{hr} \\
\text{Time} &= \frac{5}{6} \times 60 && \text{Converting time in minutes} \\
\text{Time} &= 50\text{minutes}
\end{aligned}$$



(d)

Example 3.13

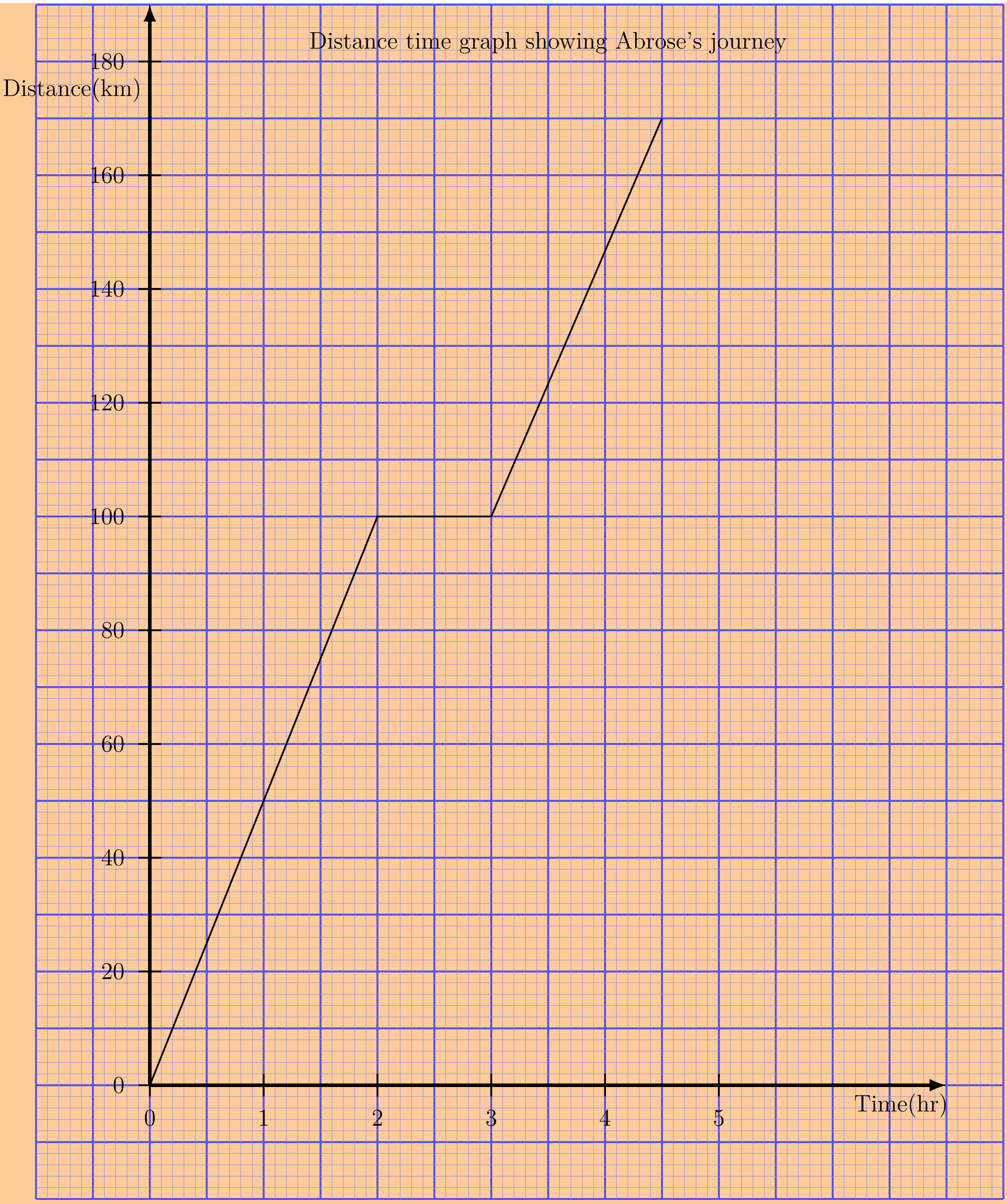
On a journey, Abrose drives at 50kmh^{-1} for 2 hours, rests for 1 hour and then drives another 70km in $1\frac{1}{2}$ hours.Draw a distance-time graph to illustrate Abrose's journey.

SOLUTION

Distance-time graph,we plot distance against time

$$\begin{aligned}
\text{Distance} &= \text{Speed} \times \text{Time} \\
&= 50 \times 2 \\
&= 100\text{km}
\end{aligned}$$

Distance(km)	0	100	100	170
Time(hr)	0	2	3	$4\frac{1}{2}$



Example 3.14

The distance from Lira to kampala is 380km.A bus leaves Lira at 7 : 30am and travels non stop to Kampala at 60kmh^{-1}.At 8 : 50am a pajero car leaves Kampala and travels towards Lira at a steady speed of 120kmh^{-1}.On the same axes draw a distance time gaph showing the journey of both vehicles,Hence or otherwise determine when and at what distance from Lira they met. (Use a scale of 2cm to represent 50km and 2cm to represent 1 hour)

SOLUTION

Table showing the distance travelled by the bus from Lira to Kampala.

60kmh^{-1} means that the bus covers a distance of 60km in every hour

Time(hrs)	7:30	8:30	9:30	10:30	11:30	12:30	13:30	14:30
Distance	0	60	120	180	240	300	360	420

Table showing the distance travelled by the Pajero car from kampala to Lira.
120kmh^{-1} means that the Pajero covers a distance of 120km in every hour

Time(hrs)	8:50	9:50	10:50	11:50
Distance(km)	380	260	140	20

The Bus and Pajero met at 10:31am at a distance of 196km from Lira

Example 3.15

Kampala and Jinja are 110 km apart. A bus, leaves Jinja at 9.00 a.m. and travels towards Kampala at a steady speed of 44kmh^{-1}. At the same time, a motorcycle , leaves Kampala and travels towards Jinja at a constant speed of 25 kmh^{-1}

(a) On the same axes draw, distance-time graphs for the Bus and motorcycle.

(b) From the graph determine:

(i) determine when and at what distance from Kampala they met,

(ii) the distance the bus had traveled before meeting the motorcycle

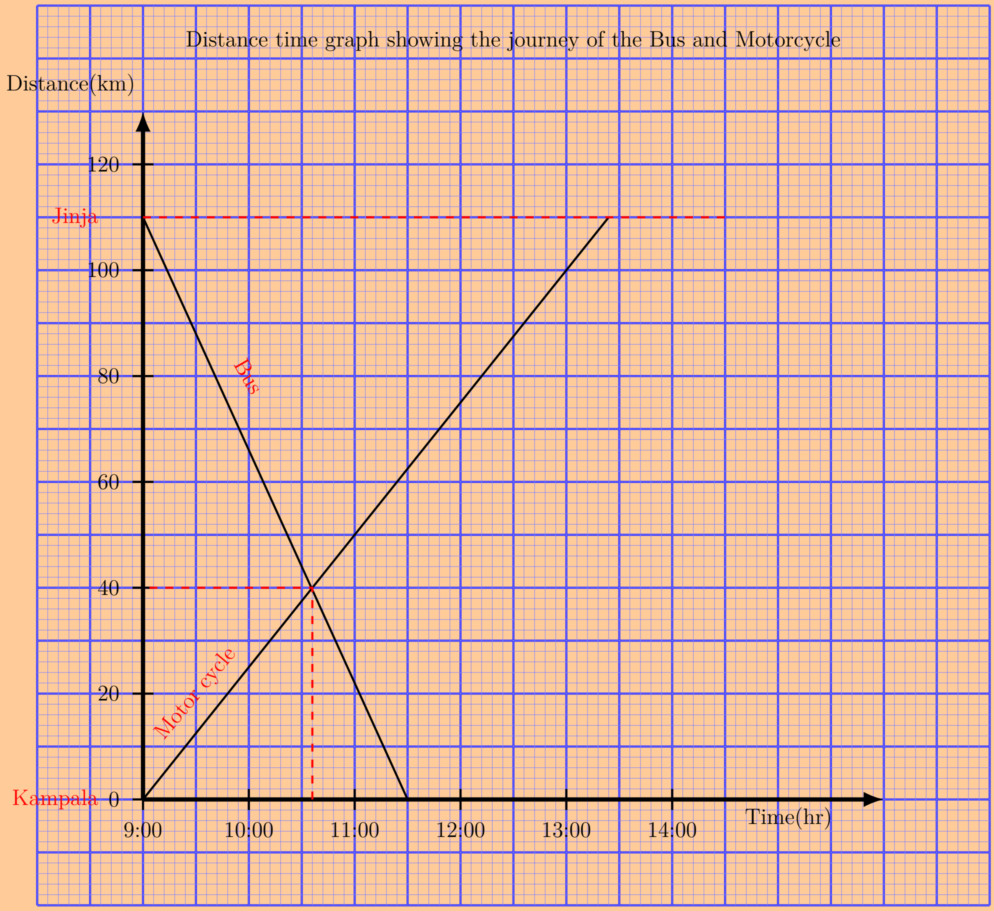
SOLUTION

(a) Table showing the distance travelled by the bus from Jinja to Kampala.
$44kmh^{-1}$ means that the bus covers a distance of 44km in every hour

Time(hrs)	9:00	10:00	11:00
Distance	110	66	22

Table showing the distance travelled by the Motor cycle from kampala to Jinja.
$25kmh^{-1}$ means that the Motor cycle covers a distance of 25km in every hour

Time(hrs)	9:00	10:00	11:00	12:00	13:00	14:00
Distance	0	25	50	75	100	125



(b) (i) The Bus and Motorcycle met at 10:36am at a distance of 40km from kampala
On the Time axis every small square box contains 6minutes,therefore
10:00+(6×6)=10:36

(ii) The bus had travelled a distance of 70km i.e110 − 40=70km

3.4 Exercise Set

1. On a journey, Sheeba drives 200km in 4 hours,rests for 1 hour and drives another 100km in 2 hours.Draw a distance-time graph for Sheeba's journey.

2. Mimmi cycles for 1.5 hours at $10kmh^{-1}$ .She stops for an hour then travels for a further 15km in 1 hour.Draw a distance time graph of Mimmi's journey

3. A car travels at a speed of $60kmh^{-1}$ for 1 hour.It stops for 30 minutes,then continues at a constant speed of $80kmh^{-1}$ for a further 1.5 hours.Draw a distance time graph for this journey

4. Shakur walks 420m from his house to a shop in 7 minutes. He spends 5 minutes at the shop and then walks home in 6 minutes.

   (a) Draw a distance-time graph for Shakur's shopping trip.

   (b) Calculate the speed at which Shakur walks on each part of the journey.

5. Cheptegei completes a 10 000 m race. He runs the first 2000 m at 5 ms^{-1}, the next 7400 m at $4ms^{-1}$ and the last 600 m at $6ms^{-1}$.

   (a) Draw a distance-time graph for Cheptegei's race.

   (b) How long does he take to complete the race?

6. A helicopter leaves Entebbe at 10:00.It flies for 45 minutes at $80kmh^{-1}$. It lands for 30 minutes and then flies a further 65 kilometres in 30 minutes. The helicopter then immediately returns to its base in Entebbe, flying at $100kmh^{-1}$. Draw a distance-time graph to show the journey.

7. Towns P and Q are 500km apart. At 8:15 am a car left P for Q traveling at a steady speed of $60kmh^{-1}$. Two and a half hours later, a bus left P for Q along the same road at a steady speed of $100kmh^{-1}$

   (a) On the same axes show the journeys of the two vehicles (Use a scale of 2cm to represent 50km and 2cm to represent 1 hour)

   (b) Use your graphs to find the:

      i. distance of the car from Q when the bus took off

      ii. time and distance from P where the bus overtook the car

      iii. difference in the times of arrival of the two vehicles

8. A lorry sets off at 7:00am from station A to station B,360km away.It travelled at a constant speed of $50kmh^{-1}$ for 2 hours.The lorry then stopped for 1 hour.It then proceeded at a steady speed for 4 hours to station B.A mini bus left station B at 8:00am for station A and moved non stop for $4\frac{1}{2}$hours.

   (a) Using a scale 2cm to represent 40km on the vertical axis and 2cm to represent 1 hour on the horizontal axis,draw on the same axes ,the distance time graphs for the lorry and mini bus.

   (b) Use your graph to find the ;

(i) time when the two vehicles met

(ii) distance from A where they met

(iii) average speed for the mini bus

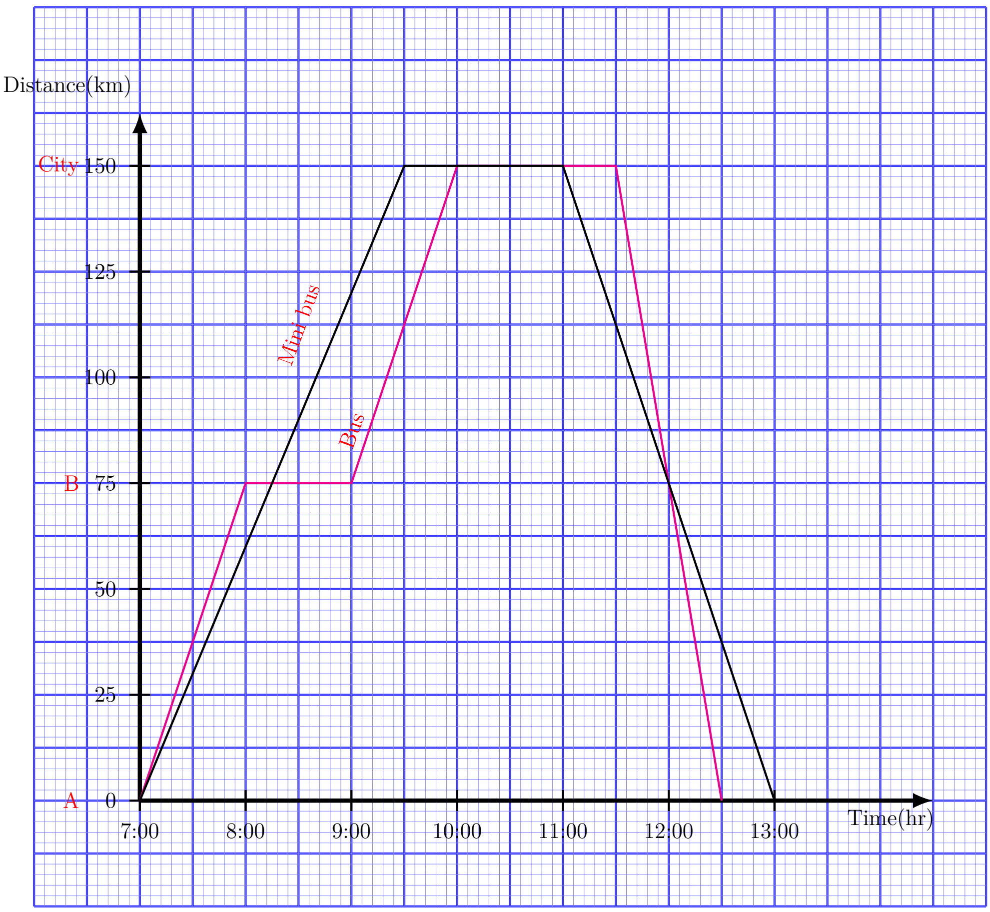
9. A cyclist sets off from town A at 4:00am at a speed of 20kmh^{-1} to go to town B,100km away.A motorist also sets off from town A at 7:30am at a speed of 100kmh^{-1} to got to town B.Find the

(a) distance from town A when the motorist over takes the cyclist

(b) time when the motorist over takes the cyclist

(c) time the cyclist reached town B

10. Two vehicles ,a bus and a minibus leave town A to a city 150km a way.The bus makes a stop over in town B to load more passengers.Both spent some time in the city reloading passengers before returning to to town A.



(a) Fow how long did the bus stay in town B

(b) At what time did the mini bus overtake the bus on their way to the city

(c) At what time did the bus overtake the mini bus on the way back to town A

(d) Calculate the average speed of the bus and the mini bus between town A and the city

(e) Calculate the average speed of the bus for the whole journey.

11. Lugazi is 45km from kampala.Kintu set off at 0815 hours from kampala riding a bicycle at 15kmh^{-1}.Kintu's bicycle broke down at 0915 hours and he was delayed for 45 minutes.He then walked back to kampala and reached at 1230 hours.Ojok set off at 0915 hours from kampala,riding a bicycle and reached Lugazi at 1200 hours.

   (a) On the same axes,draw the graphs showing the journeys of kintu and ojok

   (b) Use your graph in (a) to find

      (i) how far fro kampala kintu was when his bicycle broke down

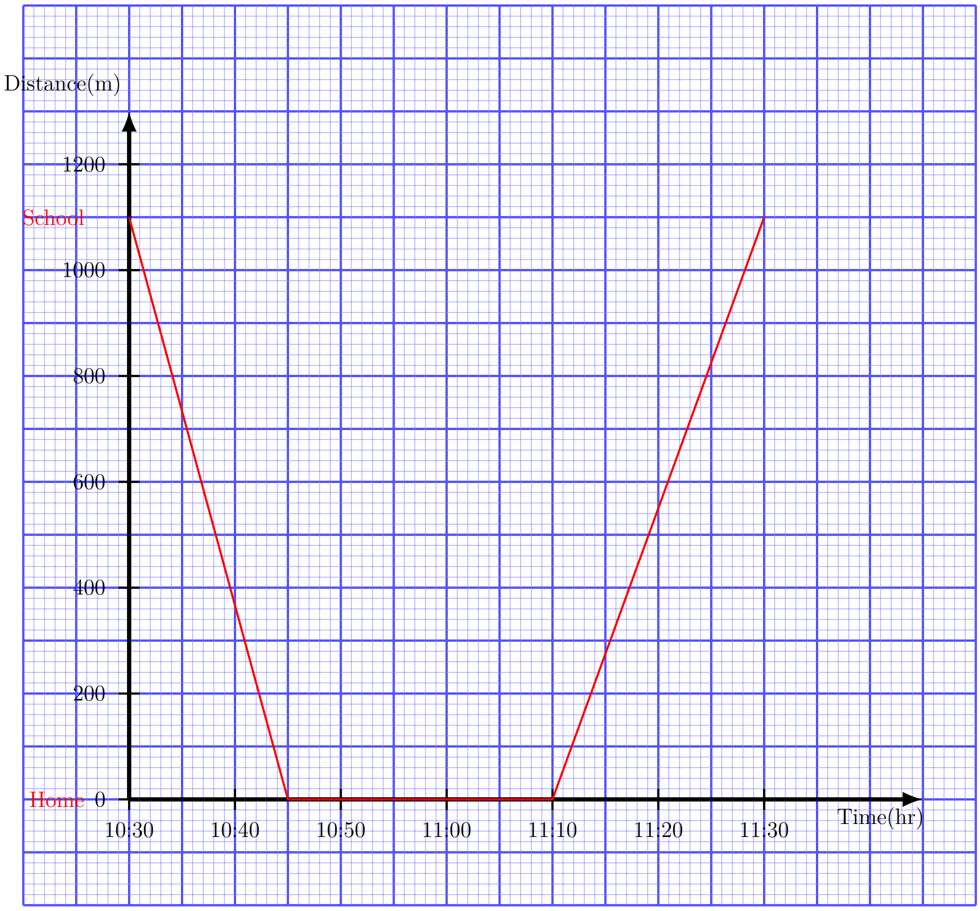
      (ii) the speed at which kintu walked back to kampala

      (iii) Ojok's average speed

      (iv) the time when the two men met

      (v) the distance from kampala where the two men met

12. After morning school, Mike walks home from school to have his Break. The distance-time graph below describes his journey on one day,showing his distance from home.



   (a) How far is Mike's home from school?

   (b) How long does it take Mike to walk home?

   (c) At what speed does he walk on the way home? Give your answer in ms^{-1}.

   (d) How long does Mike spend at home?

13. Mbarara and Jinja are 360km apart. At 7:30 am a car left Jinja for Mbarara traveling at a steady speed of 80kmh^{-1} At the same time a bus left Mbarara for Jinja at an average speed of 100kmh^{-1}
    (a) On the same axes show the journeys of the two vehicles (Use a scale of 2cm to represent 50km and 2cm to represent 1 hour)
    (b) Use your graphs to find the:
        i. time when the two vehicles met
        ii. distance from Mbarara to where the two vehicles met
        iii. difference in the times of arrival of the two vehicles
14. Joshua was sent to a school garden about 2km from the class on foot.He set off at 3:30pm for a 70 minutes journey.20 minutes later, Daniel was also set to the garden riding a bicycle at a speed of 6kmh^{-1}.If Daniel spent 20 minutes in the garden and returned to class there after.
    (a) On the same axes show that Daniel met Joshua twice on the way before Joshua got to the garden.(Use a scale of 4cm to represent 1km and 6cm to represent 1 hour)
    (b) Calculate Joshua's average speed
    (c) At what times did Daniel meet Joshua
    (d) How far from class was Joshua when Daniel passed him the first time.
15. A and B are two places 6km apart.Starting at 8:20am ,Martin walks from A to B at 4kmh^{-1} .He spends 40 minutes at B and then walks back to A at 3kmh^{-1}.
    (a) On the same axes ,draw a distance time graph for Martin's journey from A to B and back to A.(Use a scale of 1cm to represent 1km and 3cm to represent 1 hour)
    (b) At what time did Martin reach B
    (c) At what time did he set off from B
    (d) How far from B was he at 11:30 am
    (e) When did Martin arrive at A

Chapter Summary

1. When graphing we often use a table to find the unknown values .
2. Distance time graphs ,we plot distance(on vertical axis) against time(on hoizontal axis)
3. Speed(Velocity) time graphs we plot speed(on vertical axis) against time(on hoizontal axis)
4. Distance =Speed×Time
5. When calculating ,we must use the same units.i.e km with hours, metres with seconds

ASSESSMENT

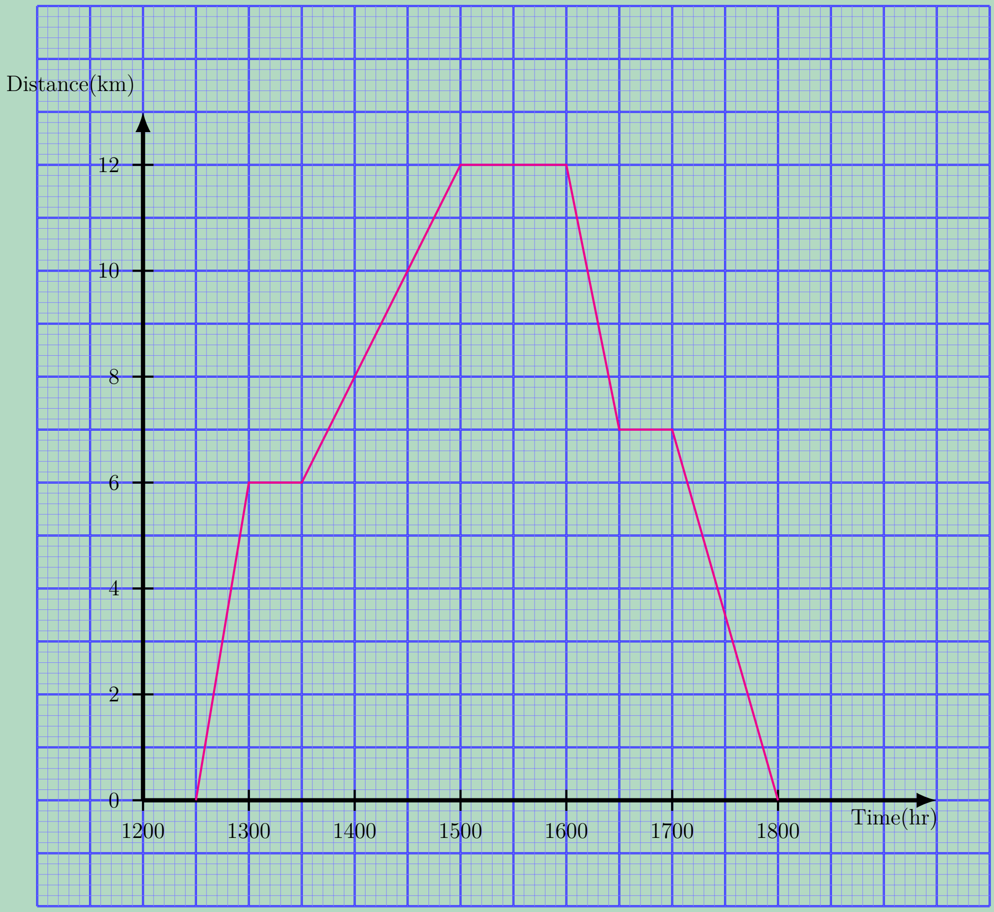
1. Copy and complete the table below using the relation $y = 3x + 5$,hence plot the points on the graph

x	-3	-2	-1	0	1	2
y						

2. Plot the points with coordinates (0, 3), (1, 5), (2, 7), (3, 9), and (4, 11) and draw a straight line through them.

3. If a car travels at a speed of 10ms^{-1} for 3 minutes, calculate the distance it travels.

4. Hannah visited her friend Sheilah and then returned home. The travel graph shows some information about Hannah's journey.



   (a) Write down the time that Hannah started her journey.

   (b) Find her distance from home when she stopped for the first rest.

   (c) How long did hannah rest ,on her way to Sheilah's home

   (d) How many minutes did Hannah spend at Sheilah's home

   (e) On her way back home ,Hannah met Purity and rested for some minutes for a chat.How long did it last.

   (f) Calculate the total distance travelled by Hannah

5. A car travels 97.5km with a constant speed of 65kmh^{-1} and then travels 60km with a constant speed of 80kmh^{-1}. Find its average speed

6. A plane takes off at 1625 for the 3200km journey from Entebbe to South Africa.If the plane flies at an average speed of 600kmh^{-1},when will it land in South Africa

7. A car travels at 60kmh^{-1} for 1 hour.The driver then takes a 30 minute break.After her break,she continues at 80kmh^{-1} for 90 minutes.
   (a) Draw a distance time graph for her journey
   (b) Calculate the total distance travelled

8. An aeroplane(UR360) leaves Entebbe air port (E) at 1800 and flies to air port Q 150km a way .It flies at a steady speed of 75kmh^{-1}.At 1820 another aeroplane (UR321) leaves air port Q for Entebbe(E) at a steady speed of 100kmh^{-1}
   (a) On the same axes draw a distance time graph to show both journeys
   (b) From the graph estimate the time at which the aeroplanes pass each other
   (c) Which aeroplane arrives at its destination first

9. Town P is 300km from town Q. A lorry left town P for Q at 7:30am and travelled at a steady speed of 80kmh^{-1} At the same time, a bus left town Q for town P and travelled at a steady speed of 120kmh^{-1}
   (a) On the same axes show the journeys of the two vehicles (Use a scale of 2cm to represent 50km and 2cm to represent 1 hour)
   (b) Use your graph, to find the time and distance from Q to where the two vehicles met

10. Draw the graphs for the following lines using the range of -3 to $^{+}3$

    (a) $y = -2x + 4$ (b) $y = 4 + x$ (c) $x = y + 1$

11. Stacy drives from Soroti to Kampala in stages:

BUS STATION	DISTANCE(km)	ARRIVE	DEPART
Soroti			2030
Kumi	50	2200	2215
Mbale	56	2330	2400
Iganga	107	0130	0145
Jinja	39	0205	0255
Kampala	84	0400	

   (a) How long does it take stacy to drive from Soroti to Kumi
   (b) How many hours did stacy take to drive from Iganga to Jinja
   (c) Calculate her average speed for each stage of her journey.

12. The relationship between the temperature in degrees celsius(^0C) and Farenheit(^0F) is given in the table

Degrees Celsius(^0C)	-40	-20	0	20	40	60	80	100
Degrees Farenheit(^0F)	-40	-4	32	68	104	140	176	212

Use a suitable scale draw a graph of Degrees Farenheit(^0F) against Degrees Celsius(^0C)

(a) Use the graph to convert the following temperatures to ^0C

i. 20^0F ii. 98^0F iii. 25^0F iv. 158^0F

(b) Use the graph to convert the following temperatures to ^0F

i. 24^0C ii. 37^0C iii. 25^0C iv. 4^0C

End

Chapter 4

NUMERICAL CONCEPT 1(INDICES AND LOGARITHMS)

Learning objectives

By the end of this topic,the learners should be able to

- Give approximate answers to calculations
- Write numbers to a given number of significant figures
- Differentiate between significant figures and decimal places
- Express numbers in standard form
- Identify base number and index
- State and apply the laws of indices in calculations
- Use a calculator to find powers and roots
- State and apply the laws of logarithms in calculations

4.1 Approximation

4.1.1 Rounding off

- To round off whole numbers, find the place value of the rounding digit and look at the digit just to the right of it. if the digit is less than 5(e.g 4,3,2,1,0),do not change the rounding digit but change all the digits to the right of the rounding digit to zero. If the digit is greater than or equal to 5(e.g 5,6,7,8,9), add one to the rounding digit and change all the digits to the right of the rounding digit to zero.
- To round off decimals, find the place value of the rounding digit and look the digit right of the digit, if it is less than 5(e.g 4,3,2,1,0), do not change the rounding digit but drop all digits to the right of it. If that digit is more or equal to 5(e.g 5,6,7,8,9), add one to the rounding digit and drop all digits to the right of it

Example 4.1
Round to the nearest ten

(a) 52 (b) 56 (c) 524 (d) 528

SOLUTION

(a)
$$\begin{array}{ccc} & 5 & 2 \\ + & 0 & \\ \hline & 5 & 0 \\ \hline \end{array}$$

(b)
$$\begin{array}{ccc} & 5 & 6 \\ + & 1 & \\ \hline & 6 & 0 \\ \hline \end{array}$$

(c)
$$\begin{array}{cccc} & 5 & 2 & 4 \\ + & & 0 & \\ \hline & 5 & 2 & 0 \\ \hline \end{array}$$

(d)
$$\begin{array}{cccc} & 5 & 2 & 8 \\ + & & 1 & \\ \hline & 5 & 3 & 0 \\ \hline \end{array}$$

Example 4.2
Round to the nearest hundred

(a) 528 (b) 563 (c) 5328 (d) 7356

SOLUTION

(a)
$$\begin{array}{cccc} & 5 & 2 & 8 \\ + & 0 & & \\ \hline & 5 & 0 & 0 \\ \hline \end{array}$$

(b)
$$\begin{array}{cccc} & 5 & 6 & 3 \\ + & 1 & & \\ \hline & 6 & 0 & 0 \\ \hline \end{array}$$

(c)
$$\begin{array}{ccccc} & 5 & 3 & 2 & 8 \\ + & & 0 & & \\ \hline & 5 & 3 & 0 & 0 \\ \hline \end{array}$$

(d)
$$\begin{array}{ccccc} & 7 & 3 & 5 & 6 \\ + & & 1 & & \\ \hline & 7 & 4 & 0 & 0 \\ \hline \end{array}$$

4.1 Exercise Set

1. Round off the numbers to the nearest number indicated in the bracket

   (a) 489(10) (c) 6489(1000) (e) 48999(100) (g) 45234489(10000)

   (b) 489(100) (d) 949(10) (f) 125789(10000) (h) 9989967(1000)

2. A company was reported to have made a profit of sh 93 678 563 .New vision gave the figure to the nearest 1000,000 and Daily monitor to the nearest 10000.What was the difference between the rounded off figures

3. Taibah international school paid sh 1155800 for the online lessons.Round off to the nearest 10000

4. The ministry of Health has recorded the following covid 19 cases in some districts.

District	Buikwe	Kampala	Wakiso	Elegu
Covid 19 cases	49	395	256	299

   (a) Write the cases for each district to the nearest 10

   (b) Obtain the total number of Covid 19 cases and write them to the nearest 100

4.1.2 Decimal Places

- Look at the first digit after the decimal point if rounding to one decimal place or the second digit for two decimal places
- Draw a vertical line to the right of the place value digit that is required
- Look at the next digit
- If it's 5 or more, increase the previous digit by one
- If it's 4 or less, keep the previous digit the same
- Remove any numbers to the right of the line

Example 4.3

Write the following numbers to the decimal places(dp) indicated in the bracket

(a) 12.451(2dp) (b) 556.4567(3dp) (c) 0.45623(2dp) (d) 0.2432(1dp)

SOLUTION

(a) 12.45(2dp) (b) 556.457(3dp) (c) 0.456(3dp) (d) 0.2(1dp)

4.2 Exercise Set

1. Write the numbers correct to one decimal place(1d.p)

   (a) 0.99 (c) 0.556 (e) 19.53 (g) 0.095

   (b) 556.899 (d) 0.212 (f) 9.111 (h) 19.53

2. Write the numbers correct to two decimal places (2d.p)

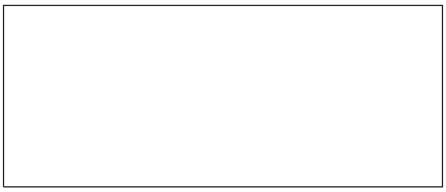
   (a) 5.391 (c) 0.0724 (e) 9.015 (g) 0.46666

   (b) 0.414 (d) 8.0062 (f) 88.044 (h) 11.0482

3. Complete the following table by collecting to a given number of decimal places

Digit	0 dp	1dp	2 dp	3dp	4 dp
0.3567856					
8.3257					
0.687432					
7.2301					
0.562789					
9.2346					
7.4466					
0.002489					
56.987					

4. Use a ruler to measure the dimensions of the rectangle below



   (a) Write down the length and width in cm correct to 1d.p

   (b) Work out the area of the rectangle and give your answer to 1d.p

4.1.3 Significant figures

Are digits in a number that are known with certainty plus the first digit that is uncertain.The Significant Figures of a number have meaning in reference to a measured or specified value. Correctly accounting for Significant Figures is paramount while performing arithmetic so that the resulting answers accurately represent numbers that have computational significance or value.

Rules governng significant figures

1. All non zero digits in a number are significant e.g 4362(4 S.F), 1241(4 S.F),1.26(3 S.F),1.2(2 S.F)
2. All zeroes occuring between non zero digits are significant.They are also known as trapped zeroes.e.g 1.004(4 S.F),1002(4S.F)
3. Law of trailing zeroes:All zeroes to the right of the last non zero digit are
   (a) Significant if they are not as a result of rounding off.e.g 720(3 S.F)
   (b) Not significant if they are obtained as aresult of rounding off e.g 6259 to 3S.F is 6260(3S.F).so this zero is not significant
4. Zeroes before a non zero digit are not significant.They are known as leading zeroes.e.g 0.06(1S.F),0.000000054(2 S.F).

Example 4.4

Write the following numbers to the significant figures(sf) indicated in the bracket

(a) 12.471(3sf) (b) 12600(2sf) (c) 0.45623(1sf) (d) 12.0058(4sf)

SOLUTION

(a) 12.5(3sf) (b) 13000(2sf) (c) 0.5(1sf) (d) 12.01(4sf)

4.3 Exercise Set

1. Write 5894 to 3 significant figure.
2. Write the numbers correct to four significant figures

   (a) 486.82 (b) 600.36 (c) 3.88888 (d) 4.04053 (e) 587.55 (f) 0.071542 (g) 588852 (h) 2345.44

3. Complete the following table by writing to the required number of significant figures

Digit	1s.f	2s.f	3s.f	4s.f	5s.f
0.78667					
8.3257					
0.687432					
7.04423					
1.999978					
9.2346					
26.5673					
0.00248992					
56.987					

4.2 Indices

4.2.1 Index Notation

Activity

1. Write the following numbers as products of their prime numbers.

   (a) 4 (b) 8 (c) 9 (d) 16 (e) 25 (f) 27 (g) 125 (h) 2401

2. Express the factors of the numbers in simple form.

From the activity considering the number 8. Writing it as a product of its prime numbers we get $8 = 2 \times 2 \times 2$,We notice that in this format, the factor 2 is repeated 3 times.i.e 2^3.The raised numeral is called **an index** (plural indices), power or exponent. Representing a number in this short form is known as **index notation.**

- Index refers to the power to which a number is raised
- 3^4 is read as 3 raised to power 4
- 3 is known as the **base** and 4 is known as the **index(power)**

Example 4.5

Write the following expanded forms in index form:

(a) $3 \times 3 \times 3$

(b) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

(c) $5 \times 5 \times 5 \times 5$

(d) $7 \times 7 \times 3 \times 3 \times 3$

SOLUTION

(a) 3^3 (b) 2^7 (c) 5^4 (d) $7^2 \times 3^3$

Example 4.6

Calculate the value of

(a) 3^2 (b) 7^3 (c) 10^3 (d) $7^2 \times 3^3$

SOLUTION

(a) 3^2

$$\begin{aligned} 3^2 &= 3 \times 3 \\ &= 9 \end{aligned}$$

(b) 7^3

$$\begin{aligned} 7^3 &= 7 \times 7 \times 7 \\ &= 343 \end{aligned}$$

(c) 10^3

$$\begin{aligned} 10^3 &= 10 \times 10 \times 10 \\ &= 1000 \end{aligned}$$

(d) $7^2 \times 3^3$

$$\begin{aligned} 7^2 \times 3^3 &= 7 \times 7 \times 3 \times 3 \times 3 \\ &= 1323 \end{aligned}$$

Example 4.7

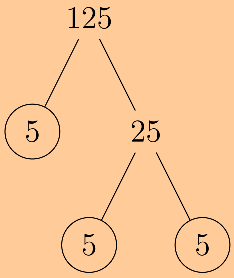
Write the following numbers in index form

(a) 125

(b) 32

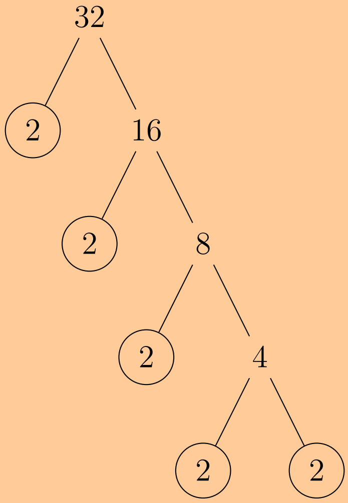
SOLUTION

(a) 125



$$125 = 5 \times 5 \times 5$$
$$= 5^3$$

(b) 32



$$32 = 2 \times 2 \times 2 \times 2 \times 2$$
$$= 2^5$$

4.4 Exercise Set

1. Calculate:

   (a) 10^2 (b) 2^3 (c) 5^3 (d) 3^2

2. Write the following expanded forms in index form:

   (a) $19 \times 19 \times 19$
   (b) $11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11$
   (c) $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$
   (d) $7 \times 7 \times 7 \times 7 \times 7$

3. Write each of the following index forms in expanded form:

   (a) 10^5 (b) 9^3 (c) 5^5 (d) 2^2

4. Write the following numbers in index form

   (a) 81
   (b) 625
   (c) 10000
   (d) 216
   (e) 49
   (f) 256
   (g) 169
   (h) 243

5. Calculate:

   (a) $3^2 + 3^3$
   (b) $3^2 \times 3^3$
   (c) $10^3 + 2^5$
   (d) $5^2 - 2^2$
   (e) $2^3 \times 3^4$
   (f) $3^2 \times 4^3 \times 2^4$
   (g) $4^3 \times 6^3$
   (h) $2^2 \times 2^0$

6. Writing your answers in index form, calculate:

   (a) $10^3 \times 10^2$ (b) $3^2 \times 3^3$ (c) $10^5 \div 10^3$ (d) $4^5 \div 4^2$

4.2.2 Laws of Indices

When working with numbers involving indices there are three fundamental laws which can be applied .These are

. Multiplication Law

1. $a^m \times a^n = a^{m+n}$

. Quotient (or Division) Law

2. $a^m \div a^n = a^{m-n}$

. Power Law

3. $(a^m)^n = a^{mn}$

Example 4.8

Simplify

(a) $3^2 \times 3^4$ (b) $7^5 \div 7^2$ (c) $10^3 \times 10^2 \times 10^4$ (d) $(11^2)^3$

SOLUTION

(a) $3^2 \times 3^4$

$$\begin{aligned} 3^2 \times 3^4 &= 3^{2+4} \\ &= 3^6 \end{aligned}$$

(b) $7^5 \div 7^2$

$$\begin{aligned} 7^5 \div 7^2 &= 7^{5-2} \\ &= 7^3 \end{aligned}$$

(c) $10^3 \times 10^2 \times 10^4$

$$\begin{aligned} 10^3 \times 10^2 \times 10^4 &= 10^{3+2+4} \\ &= 10^9 \end{aligned}$$

(d) $(11^2)^3$

$$\begin{aligned} (11^2)^3 &= 11^{2\times 3} \\ &= 11^6 \end{aligned}$$

Example 4.9

Simplify

(a) $a^2 \times a^5 \times a^3$ (b) $y^6 \div y^3$ (c) $b^3 \times b^2 \times b^4 \times b^2$ (d) $(y^4)^{\frac{1}{2}}$

SOLUTION

(a) $a^2 \times a^5 \times a^3$

$$\begin{aligned} a^2 \times a^5 \times a^3 &= a^{2+5+3} \\ &= a^{10} \end{aligned}$$

(b) $y^6 \div y^3$

$$\begin{aligned} y^6 \div y^3 &= y^{6-3} \\ &= y^3 \end{aligned}$$

(c) $b^3 \times b^2 \times b^4 \times b^2$

$$\begin{aligned} b^3 \times b^2 \times b^4 \times b^2 &= b^{3+2+4+2} \\ &= b^{11} \end{aligned}$$

(d) $(y^4)^{\frac{1}{2}}$

$$\begin{aligned} (y^4)^{\frac{1}{2}} &= y^{4\times\frac{1}{2}} \\ &= y^{\cancelto{2}{4}\times\frac{1}{\cancelto{1}{2}}} \\ &= y^2 \end{aligned}$$

Example 4.10

Simplify

(a) $\frac{2^2\times 2^4}{2^3}$

(b) $\frac{6^3\times 6^2\times 6^4}{6^2\times 6^3}$

SOLUTION

(a) $\frac{2^2\times 2^4}{2^3}$

$$\begin{aligned}\frac{2^2\times 2^4}{2^3} &= \frac{2^{2+4}}{2^3}\\ &= \frac{2^6}{2^3}\\ &= 2^{6-3}\\ &= 2^3\end{aligned}$$

(b) $\frac{6^5\times 6^6\times 6^4}{6^2\times 6^3}$

$$\begin{aligned}\frac{6^5\times 6^6\times 6^4}{6^2\times 6^3} &= \frac{6^{5+6+4}}{6^{2+3}}\\ &= \frac{6^{15}}{6^5}\\ &= 6^{15-5}\\ &= 6^{10}\end{aligned}$$

4.5 Exercise Set

1. Simplify the following using indices

   (a) $3^2\times 3^5$
   (b) $9^5\times 9^4$
   (c) $5^2\times 5^3\times 5^5$
   (d) $4^5\times 5^2\times 4^6\times 5^3$
   (e) $2^4\times 5^7\times 5^3\times 6^2\times 6^6$
   (f) $19^2\times 19^4\times 19^2\ 10\times 10$
   (g) $7^2\times 7^3\times 7^1\times 7^5\times 7^4$
   (h) $a^2\times a^3\times a^8$
   (i) $y^2\times y^7$

2. Simplify the following using indices

   (a) $3^5\div 3^3$
   (b) $9^5\div 9^4$
   (c) $5^7\div 5^3$
   (d) $4^5\div 4^2$
   (e) $\frac{6^5}{6^4}$
   (f) $\frac{3^{12}}{3^7}$
   (g) $\frac{19^7}{19^3}$
   (h) $y^9\div y^7$

3. Simplify the following using indices

   (a) $(3^3)^3$
   (b) $(9^5)^4$
   (c) $(5^7)^3$
   (d) $(4^5)^{\frac{1}{5}}$

4. Simplify the following

   (a) $\frac{y^2\times y^5\times y^3}{y^4}$
   (b) $\frac{5^6\times 5^7}{5^3\times 5^9}$
   (c) $\frac{4^4\times 2^8\times 4^3}{2^7\times 4^6}$
   (d) $\frac{(6^2)^3\times 6^2}{6^7}$
   (e) $(2^6\times 2^8)^2$
   (f) $\left(\frac{3^2\times 3^5}{3^3}\right)^2$
   (g) $(x^4y^3)^3$
   (h) $8a^4\div 4a^2$
   (i) $(x^3)^3\div x^2$
   (j) $\frac{a^7\times(ab^4)^3}{b^3}$
   (k) $\frac{2^y\times 8^{y-1}}{16^{y-1}}$
   (l) $\frac{4^y\times 8^{y-2}}{32^{y-1}}$
   (m) $\frac{5^y\times 125^{y+1}}{625^{y+1}}$
   (n) $\left(\frac{6^2\times 6^8}{6^3}\right)^4$
   (o) $\left(\frac{7^8}{7^2\times 7^3}\right)^5$
   (p) $\left(\frac{3^2\times 9}{3^3}\right)^4$

5. Simplify the following using indices

(a) $3y^2 \times 4y^4$ (b) $25x^4 \div 5x^2$ (c) $x^5 \times 2x^4$ (d) $y^5 \div y^-2$

4.2.3 The Zero index

Activity

1. Using the law of indices, find the solution of the following:

   (a) $5^4 \div 5^4$ (b) $a^3 \div a^3$ (c) $\frac{3^2}{3^2}$

2. What is your observations

- Any non zero number raised to power 0 is equal to 1
- a^0=1
- For example $10^0 = 1$,$1000000^0 = 1$,$(ab)^0 = 1$,$c^0 = 1$,$4^0 = 1$

4.6 Exercise Set

1. Calculate

   (a) $3^0 + 4^0$
   (b) $6^0 \times 8^0$
   (c) $2^0 - 4^0$
   (d) $5^0 \div 15^0$
   (e) $b^0 + z^0$
   (f) $1000000^0 \div 234^0$

2. Simplify

   (a) $5^3 \times 5^0$
   (b) $7^3 \times 7^{-3}$
   (c) $(10^0)^2$
   (d) $6^2 \times 6^{-2}$
   (e) $\frac{2^4x^2y^3}{2\times2\times2\times2\times y^2\times y\times x\times x}$
   (f) $\frac{z^2\times x^4\times b^2}{x^2z\times b^2zx^2}$

4.2.4 Negative Indices

Activity

1. Using the quotient law of indices, solve the following:

   (a) $2^3 \div 2^2$ (b) $2^2 \div 2^3$ (c) $a^3 \div a^4$

2. Express $\frac{2^2}{2^3}$ in a standard form,what relationship exists with the answer obtained in number 2.

From the activity we can observe that

$$\frac{a^2}{a^3} = \frac{a \times a}{a \times a \times a} = \frac{1}{a}$$

Also on applying the law of indices

$$\frac{a^2}{a^3} = a^{2-3} = a^{-1}$$

Therefore

$$a^{-1} = \frac{1}{a}$$

In general

$$a^{-n} = \frac{1}{a^n}$$

For positive integers of n

Example 4.11

Calculate, leaving your answers as fractions:

(a) 7^{-2}

(b) $3^{-1} + 4^{-1}$

SOLUTION

(a) 7^{-2}

$$7^{-2} = \frac{1}{7^2} = \frac{1}{7 \times 7} = \frac{1}{49}$$

(b) $3^{-1} + 4^{-1}$

$$3^{-1} + 4^{-1} = \frac{1}{3^1} + \frac{1}{4^1} = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

Example 4.12

Simplify:$(5^{-2})^3$

SOLUTION

$$(5^{-2})^3 = 5^{(-2 \times 3)} = 5^{-6} = \frac{1}{5^6}$$

Example 4.13

Simplify:

(a) $\frac{y^3}{y^4}$

(b) $\frac{2^2}{2^5}$

(c) $\frac{2^5}{2^{-5}}$

(d) $\left(\frac{3}{2}\right)^{-2}$

SOLUTION

(a) $\frac{y^3}{y^4}$

$$\begin{aligned}\frac{y^3}{y^4} &= y^{3-4}\\ &= y^{-1}\\ &= \frac{1}{y^1}\\ &= \frac{1}{y}\end{aligned}$$

(b) $\frac{2^2}{2^5}$

$$\begin{aligned}\frac{2^2}{2^5} &= 2^{2-5}\\ &= 2^{-3}\\ &= \frac{1}{2^3}\\ &= \frac{1}{2\times 2\times 2}\\ &= \frac{1}{8}\end{aligned}$$

(c) $\frac{2^5}{2^{-5}}$

$$\begin{aligned}\frac{2^5}{2^{-5}} &= 2^{5--5}\\ &= 2^{10}\\ &= 1024\end{aligned}$$

(d) $\left(\frac{3}{2}\right)^{-2}$

$$\begin{aligned}\left(\frac{3}{2}\right)^{-2} &= \frac{1}{\left(\frac{3}{2}\right)^2}\\ &= \frac{1}{\left(\frac{9}{4}\right)}\\ &= 1\div\frac{9}{4}\\ &= 1\times\frac{4}{9}\\ &= \frac{4}{9}\end{aligned}$$

Example 4.14

Simplify: $\frac{225\times 5^{-2}}{64\times\frac{1}{4^2}}$

SOLUTION

$$\begin{aligned}\frac{125\times 5^{-2}}{64\times\frac{1}{4^2}} &= \frac{5^3\times 5^{-2}}{4^3\times 4^{-2}}\\ &= \frac{5^{3+-2}}{4^{3+-2}}\\ &= \frac{5^1}{4^1}\\ &= \frac{5}{4}\end{aligned}$$

4.7 Exercise Set

1. With out using a calculator ,evaluate the following

(a) 3^{-2}

(b) 64×2^{-4}

(c) 100×10^{-2}

(d) 5×10^{-3}

(e) 100×10^{-1}

(f) 36×6^{-3}

(g) $\frac{3}{2^{-2}}$

(h) $\frac{5}{4^{-2}}$

(i) $\frac{8^{-6}}{8^{-8}}$

(j) $(a^{-2})^3$

(k) $(b^{-4})^{-2}$

(l) $\left(\frac{1}{3}\right)^{-2}$

2. Simplify the following

(a) $2x^5y^3 \div 4x^{-1}y$
(b) $81 \times 10^6 \div (9 \times 10^{-4})$
(c) $(y^3)^2 \div y^13$
(d) $(a^5)^3 \times a^{-5}$
(e) $b^{12} \times b^{-2} \div b^{-5}$
(f) $2a^{-5} \times a$
(g) $\frac{8^{-4} \times 4^{-4}}{4^{-2} \times 8^{-4}}$
(h) $2^{-3} \times 4^5 + 3^2 \times 3^{-4}$
(i) $6^{-1} \div 6^{-4}$

4.2.5 Same power Law

When the index of the terms is the same then the following rules should be applied:

- $$\begin{aligned} a^n \times b^n &= (a \times b)^n \\ &= (ab)^n \end{aligned}$$

- $$a^n \div b^n = (a \div b)^n$$
  OR
  $$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

- $$(a^m b^n)^p = a^{mp} b^{np}$$

- Any number raised with a negative power, is the reciprocal of the number with a positive power.
  $$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

- $$ax^m \times bx^n = (a \times b)x^{m+n}$$

- $$ax^m \div bx^n = (a \div b)x^{m-n}$$

Example 4.15

Evaluate

(a) $4^2 \times 3^2$
(b) $15^3 \div 5^3$
(c) $(2xy)^5$
(d) $\left(\frac{2}{3}\right)^{-2}$

SOLUTION

(a) $4^2 \times 3^2$

$$4^2 \times 3^2 = (4 \times 3)^2 = 12^2 = 144$$

(b) $15^3 \div 5^3$

$$15^3 \div 5^3 = (15 \div 5)^3 = 3^3 = 27$$

(c) $(2xy)^5$

$$(2xy)^5 = 2^5x^5y^5 = 32x^5y^5$$

(d) $\left(\frac{2}{3}\right)^{-2}$

$$\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$$

4.8 Exercise Set

1. Simplify each of the following

(a) $\left(\frac{b^2}{c^4}\right)^3$ (b) $(5a^3bc^4)^2$ (c) $(5x^4)^2$ (d) $(4y^3)^2$ (e) $(3a^2)^3$ (f) $7x^4 \times 2x$ (g) $5n^3 \times n^4$ (h) $6y^2 \times 2$ (i) $(5^4)^3$ (j) $2a^2 \times 3a^3$ (k) $\left(\frac{2y}{y^3}\right)^4$

2. Simplify each of the following

(a) $3y^2 \times 4y^3$ (b) $4a^3 \times 5a^3$ (c) $12b^3 \div 3b^5$ (d) $7y^6 \times 8y^{-2}$ (e) $2a \times 5a$ (f) $49b^5 \div 7b^3$ (g) $6y^4 \div y^{-2}$ (h) $\frac{16x^3}{4x^{-2}}$

4.2.6 Fractional Power (or Root) Law

- The square root of any number is that number raised to power a half.

$$\sqrt{a} = a^{\frac{1}{2}}$$

- A number raised to the power of **one -nth** is equal to the **nth** root of the same number

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

- In general

$$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$

Example 4.16

Find the value of the following index forms:

(a) $49^{\frac{1}{2}}$ (b) $8^{\frac{2}{3}}$ (c) $125^{\frac{1}{3}}$ (d) $\left(\frac{16}{81}\right)^{\frac{-3}{4}}$

Use a calculator to find the roots of the numbers

SOLUTION

(a) $49^{\frac{1}{2}}$

$$\begin{aligned} 49^{\frac{1}{2}} &= \sqrt{49} \\ &= \pm 7 \end{aligned}$$

(b) $8^{\frac{2}{3}}$

$$\begin{aligned} 8^{\frac{2}{3}} &= (\sqrt[3]{8})^2 \\ &= (2)^2 \\ &= 4 \end{aligned}$$

(c) $125^{\frac{1}{3}}$

$$\begin{aligned} 125^{\frac{1}{3}} &= \sqrt[3]{125} \\ &= 5 \end{aligned}$$

(d) $\left(\frac{16}{81}\right)^{\frac{-3}{4}}$

$$\begin{aligned} \left(\frac{16}{81}\right)^{\frac{-3}{4}} &= \left(\frac{81}{16}\right)^{\frac{3}{4}} \\ &= \left(\frac{81^{\frac{3}{4}}}{16^{\frac{3}{4}}}\right) \\ &= \frac{\left(\sqrt[4]{81}\right)^3}{\left(\sqrt[4]{16}\right)^3} \\ &= \frac{3^3}{2^3} \\ &= \frac{27}{8} \end{aligned}$$

Example 4.17

Simplify the following:

(a) $3a^{\frac{1}{2}} \times 4a^{\frac{2}{3}}$

(b) $\frac{1}{2} - \left(\frac{25}{4}\right)^{\frac{-1}{2}}$

SOLUTION

(a) $3a^{\frac{1}{2}} \times 4a^{\frac{2}{3}}$

$$\begin{aligned} 3a^{\frac{1}{2}} \times 4a^{\frac{2}{3}} &= (3 \times 4)a^{\frac{1}{2}+\frac{2}{3}} \\ &= 12a^{\frac{7}{6}} \end{aligned}$$

(b) $\frac{1}{2} - \left(\frac{25}{4}\right)^{\frac{-1}{2}}$

$$\begin{aligned} \frac{1}{2} - \left(\frac{25}{4}\right)^{\frac{-1}{2}} &= \frac{1}{2} - \left(\frac{4}{25}\right)^{\frac{1}{2}} \\ &= \frac{1}{2} - \left(\frac{4^{\frac{1}{2}}}{25^{\frac{1}{2}}}\right) \\ &= \frac{1}{2} - \left(\frac{\sqrt{4}}{\sqrt{25}}\right) \\ &= \frac{1}{2} - \frac{2}{5} \\ &= \frac{1}{10} \end{aligned}$$

Example 4.18

Simplify the following:$16^{\frac{-1}{2}} + 32^{\frac{-1}{5}} - \left(\frac{8}{125}\right)^{\frac{2}{3}}$

SOLUTION

$$\begin{aligned}16^{\frac{-1}{2}}+32^{\frac{-1}{5}}-\left(\frac{8}{125}\right)^{\frac{2}{3}} &= \left(\frac{1}{16}\right)^{\frac{1}{2}}+\left(\frac{1}{32}\right)^{\frac{1}{5}}-\left(\frac{8^{\frac{2}{3}}}{125^{\frac{2}{3}}}\right)\\ &= \frac{1}{16^{\frac{1}{2}}}+\frac{1}{32^{\frac{1}{5}}}-\frac{\left(\sqrt[3]{8}\right)^2}{\left(\sqrt[3]{125}\right)^2}\\ &= \frac{1}{\sqrt{16}}+\frac{1}{\sqrt[5]{32}}-\frac{2^2}{5^2}\\ &= \frac{1}{4}+\frac{1}{2}-\frac{4}{25}\\ &= \frac{3}{4}-\frac{4}{25}\\ &= \frac{59}{100}\end{aligned}$$

4.9 Exercise Set

1. Evaluate each of the following:

   (a) $27^{\frac{1}{3}}$ (b) $81^{\frac{1}{4}}$ (c) $16^{\frac{1}{4}}$ (d) $32^{\frac{3}{5}}$ (e) $729^{\frac{2}{3}}$ (f) $216^{\frac{2}{3}}$

2. Given that $a = 36$ and $b = 64$,evaluate

   (a) $3a^{\frac{1}{2}}$ (b) $4a^{\frac{1}{2}} - 3b^{\frac{1}{3}}$ (c) $2b^{\frac{-2}{3}} \div 6a^{\frac{-3}{2}}$

3. Calculate:

   (a) $\left(\frac{4\times8}{2}\right)^{\frac{1}{2}}$ (b) $\left(\frac{125\times5}{25}\right)^{\frac{1}{2}}$ (c) $\left(\frac{27\times9}{3}\right)^{\frac{1}{4}}$ (d) $\left(\frac{625}{5}\right)^{\frac{-1}{3}}$

4. Simplify

   (a) $(x^9)^{\frac{1}{3}}$ (b) $(a^{10})^{\frac{-1}{2}}$ (c) $\frac{a^{\frac{1}{3}}}{a}$ (d) $\frac{a^{\frac{1}{3}}}{a^{\frac{-1}{2}}}$

5. Without using a calculator, evaluate the expression $\sqrt[3]{\frac{125}{1000}}$

6. Calculate the value of

   (a) $\frac{y\times y^{\frac{-1}{2}}}{y^{\frac{1}{3}}}$ when $y = 64$

   (b) $\frac{p\div p^{\frac{-1}{2}}}{p^{\frac{1}{3}}}$ when $p = 64$

   (c) $\sqrt{\frac{x}{y}}$ when $x = 64^{\frac{2}{3}}$ and $y = 3^{-2}$

   (d) $(y-1)^{\frac{5}{2}} + (y+6)^{\frac{1}{2}} + 5y^0$.When $y = 10$

7. Simplify

   (a) $(27)^{\frac{-1}{3}}(25)^{\frac{1}{2}} - 5(0.008)^{\frac{1}{3}}$

   (b) $32^{\frac{1}{5}} \times 25^{\frac{1}{2}} \times 27^{\frac{1}{3}}$

   (c) $\left(\frac{16}{81}\right)^{\frac{1}{4}} - \left(\frac{8}{27}\right)^{\frac{2}{3}}$

4.2.7 Solving equations involving indices

- If $a^m = a^n$ then $m = n$ then Similarly, if $a^m = b^m$, then $a = b$, provided both the bases are positive numbers.
- To solve equations involving indices,express both sides of the equation with a common base and simplify

Example 4.19

Solve for x in the following equations:

(a) $4^x = 16$ (b) $2^x = \frac{1}{8}$ (c) $81^x = 3$ (d) $2^{2x+1} = 32$

SOLUTION

(a) $4^x = 16$

$$\begin{aligned} 4^x &= 16 \\ (2^2)^x &= 2^4 \\ 2^{2x} &= 2^4 \\ 2x &= 4 \\ \frac{2x}{2} &= \frac{4}{2} \\ x &= 2 \end{aligned}$$

(b) $2^x = \frac{1}{8}$

$$\begin{aligned} 2^x &= \frac{1}{8} \\ 2^x &= \frac{1}{2^3} \\ 2^x &= 2^{-3} \\ x &= -3 \end{aligned}$$

(c) $81^x = 3$

$$\begin{aligned} 81^x &= 3 \\ (3^4)^x &= 3 \\ 3^{4x} &= 3^1 \\ 4x &= 1 \\ \frac{4x}{4} &= \frac{1}{4} \\ x &= \frac{1}{4} \end{aligned}$$

(d) $2^{2x+1} = 32$

$$\begin{aligned} 2^{2x+1} &= 32 \\ 2^{2x+1} &= 2^5 \\ 2x+1 &= 5 \\ 2x+1-1 &= 5-1 \\ 2x &= 4 \\ \frac{2x}{2} &= \frac{4}{2} \\ x &= 2 \end{aligned}$$

4.10 Exercise Set

1. Solve for the value of x

(a) $2^x = 32$
(b) $4^{x+1} = 8^x$
(c) $81^x = 1$
(d) $5^x = \frac{1}{25}$
(e) $128^x = 2$
(f) $a^4 \div a^12 = a^x$
(g) $3^{x+1} = 9^x$
(h) $2^{3x+2} = 4^{x+5}$
(i) $3^{2x-1} = 243$
(j) $5^{x+3} = 25^x$
(k) $2^{x+1} = 4^x$
(l) $x^2 = 9$
(m) $10x^3 = 640$
(n) $3^x = 3$
(o) $10^x = 0.1$
(p) $10^x = 1000$
(q) $7^x = 1$
(r) $5^x = 25$
(s) $4^x = \frac{1}{4}$

2. Solve for the value of y :$4 \times 2^{3y-3} = 2^{-y}$

3. Solve for the value of n
   (a) $5^{2n+3} = 125^{n+5}$
   (b) $0.5^n = 8$

4.3 Standard Form

- Standard form is a convenient way of writing very large or very small numbers in terms of powerss of 10
- Standard form is also known as Scientific notation or Standard Index Form
- In standard form, numbers are written as $a \times 10^{\,n}$ where $1 \leq a < 10$ and n is an integer.
- To express a number in standard form, we shift the decimal point until the digit part a is between 1 and 10. This digit a has a decimal point placed after the first digit.
- The power part (10^n) shows how many places to move the decimal point
- The rules of indices apply to calculations in standard form

Example 4.20

Write the following numbers in standard form

(a) 82000 (b) 40000 (c) 3000000 (d) 1230

SOLUTION

(a) 8.2×10^4 (b) 4×10^4 (c) 3×10^6 (d) 1.23×10^3

Example 4.21

Write the following numbers in standard form

(a) 0.063 (b) 0.0000567 (c) 0.0004 (d) 0.000621

SOLUTION

(a) 6.3×10^{-2} (b) 5.67×10^{-5} (c) 4×10^{-4} (d) 6.21×10^{-4}

Example 4.22

Express the following numbers in standard form:

(a) 234×10^3 (b) 7680×10^{-7} (c) 34.5×10^{-4} (d) 0.0876×10^5

SOLUTION

(a) 234×10^3

$$\begin{aligned} 234 \times 10^3 &= 2.34 \times 10^2 \times 10^3 \\ &= 2.34 \times 10^{2+3} \\ &= 2.34 \times 10^5 \end{aligned}$$

(b) 7680×10^{-7}

$$\begin{aligned} 7680 \times 10^{-7} &= 7.68 \times 10^3 \times 10^{-7} \\ &= 7.68 \times 10^{3-7} \\ &= 7.68 \times 10^{-4} \end{aligned}$$

(c) 34.5×10^{-4}

$$\begin{aligned} 34.5 \times 10^{-4} &= 3.45 \times 10^1 \times 10^{-4} \\ &= 3.45 \times 10^{1-4} \\ &= 3.45 \times 10^{-3} \end{aligned}$$

(d) 0.0876×10^5

$$\begin{aligned} 0.0876 \times 10^5 &= 8.76 \times 10^{-2} \times 10^5 \\ &= 8.76 \times 10^{-2+5} \\ &= 8.76 \times 10^3 \end{aligned}$$

Example 4.23

By expressing each of the numbers in standard form, evaluate the following:

(a) 0.0003×0.002

(b) 80000×0.0005

(c) $\frac{800}{0.004}$

(d) 0.02×0.0015

(e) $\frac{0.009 \times 8000}{0.002 \times 0.3}$

(f) $\frac{0.81}{0.0027}$

SOLUTION

(a)

$$\begin{aligned} 0.0003 \times 0.002 &= 3 \times 10^{-4} \times 2 \times 10^{-3} \\ &= (3 \times 2) \times 10^{-4+-3} \\ &= 6 \times 10^{-7} \end{aligned}$$

(b)

$$\begin{aligned} 80000 \times 0.0005 &= 8 \times 10^4 \times 5 \times 10^{-4} \\ &= (8 \times 5) \times 10^{4-4} \\ &= 40 \times 10^0 \\ &= 4 \times 10^1 \times 10^0 \\ &= 4 \times 10^{0+1} \\ &= 4 \times 10^1 \end{aligned}$$

(c)

$$\begin{aligned} \frac{800}{0.004} &= \frac{8 \times 10^2}{4 \times 10^{-3}} \\ &= 2 \times 10^{2--3} \\ &= 2 \times 10^5 \end{aligned}$$

(d)

$$\begin{aligned} 0.02 \times 0.0015 &= 2 \times 10^{-2} \times 1.5 \times 10^{-3} \\ &= (2 \times 1.5) \times 10^{-2-3} \\ &= 3 \times 10^{-5} \end{aligned}$$

(e)

$$\begin{aligned} \frac{0.009 \times 8000}{0.002 \times 0.3} &= \frac{9 \times 10^{-3} \times 8 \times 10^3}{2 \times 10^{-3} \times 3 \times 10^{-1}} \\ &= \frac{(9 \times 8) \times 10^{-3+3}}{(2 \times 3) \times 10^{-3+-1}} \\ &= \frac{72 \times 10^0}{6 \times 10^{-4}} \\ &= (72 \div 6) \times 10^{0--4} \\ &= 12 \times 10^4 \\ &= 12 \times 10^4 \\ &= 1.2 \times 10^1 \times 10^4 \\ &= 1.2 \times 10^5 \end{aligned}$$

(f)

$$\begin{aligned} \frac{0.81}{0.0027} &= \frac{8.1 \times 10^{-1}}{2.7 \times 10^{-3}} \\ &= (8.1 \div 2.7) \times 10^{-1--3} \\ &= 3 \times 10^2 \end{aligned}$$

Example 4.24

Without using a calculator, determine:

(a) $(4 \times 10^3) \times (2 \times 10^4)$

(b) $(9 \times 10^5) \div (3 \times 10^{-3})$

SOLUTION

(a)

$$\begin{aligned}(4 \times 10^3) \times (2 \times 10^4) &= (4 \times 2) \times 10^{3+4} \\ &= 8 \times 10^7\end{aligned}$$

(b)

$$\begin{aligned}(9 \times 10^5) \div (3 \times 10^{-3}) &= (9 \div 3) \times 10^{(5--3)} \\ &= 3 \times 10^8\end{aligned}$$

Example 4.25

Without using a calculator, determine:

(a) $(4 \times 10^3) + (2 \times 10^4)$

(b) $(9 \times 10^5) - (3 \times 10^3)$

SOLUTION

(a)

$$\begin{aligned}(4 \times 10^3) + (2 \times 10^4) &= 4000 + 20000 \\ &= 24000 \\ &= 2.4 \times 10^4\end{aligned}$$

Method 2.Change the indices such that they have the same index

$$\begin{aligned}(4 \times 10^3) + (2 \times 10^4) &= (4 \times 10^3) + (20 \times 10^3) \\ &= (4 + 20) \times 10^3 \\ &= 24 \times 10^3 \\ &= 2.4 \times 10^4\end{aligned}$$

(b)

$$\begin{aligned}(9 \times 10^5) - (3 \times 10^3) &= 900000 - 3000 \\ &= 897000 \\ &= 8.97 \times 10^5\end{aligned}$$

Method 2.Change the indices such that they have the same index

$$\begin{aligned}(9 \times 10^5) - (3 \times 10^3) &= (900 \times 10^3) - (3 \times 10^3) \\ &= (900 - 3) \times 10^3 \\ &= 897 \times 10^3 \\ &= 8.97 \times 10^5\end{aligned}$$

4.11 Exercise Set

1. Write each of the following numbers in standard form:

(a) 4000
(b) 9000
(c) 6 000 000
(d) 87 000
(e) 870 000
(f) 24 000 000 000
(g) 81 900 000 000
(h) 7 35.234

2. Write each of the following numbers in standard form:

(a) 0.05
(b) 210.00856
(c) 0.5682
(d) 0.00004356
(e) 124.00688
(f) 0.00000098
(g) 245.12
(h) 0.00000000000023

3. Without using a calculator, evaluate the following give your answer in standard form:

(a) $(5 \times 10^5) \times (2 \times 10^{-5})$
(b) $(9 \times 10^5) \times (4 \times 10^3)$
(c) $(8 \times 10^6) \div (4 \times 10^{-3})$
(d) $(6 \times 10^2) \div (3 \times 10^3)$
(e) $(9.8 \times 10^3) + (2.5 \times 10^3)$
(f) $(2 \times 10^3) + (3 \times 10^3)$
(g) $(5 \times 10^5) - (2 \times 10^2)$
(h) $(7 \times 10^2) - (5 \times 10^1)$
(i) $4000 \times 8000 \div 640$
(j) $(6.5 \times 10^8) \div (1.3 \times 10^4) \times (5 \times 10^3)$

4. If $x = 3 \times 10^3$ and $y = 2 \times 10^2$,work out the value of

(a) xy
(b) $x + y$
(c) $x - y$
(d) $x \div y$

5. Without using a calculator or tables, evaluate: $\frac{0.0035}{0.07 \times 0.2}$

6. Without using a calculator or tables, evaluate: $\frac{0.42 \times 0.35 \times 0.0015}{0.049 \times 0.003}$

7. The radius of the earth is 6.4×10^6m. Giving your answers in standard form, correct to 3 significant figures, calculate the circumference of the earth .Take $\pi = 314 \times 10^{-2}$

4.4 Logarithms

- Logarithm is a derived term from two Greek words, namely: **logos (expression)** and **arithmos (number)** . Thus, logarithm is a technique of expressing numbers.
- Logarithm is a system of evaluating multiplication, division, powers and roots by appropriately converting them to addition and subtraction.
- The logarithm of **b** to the base **a** is written as $\log_a b$
- A logarithm to the base of 10 is called a **common logarithm** .It is written as $\log_{10} b$ or $\log b$

4.4.1 Rules of Logarithm

1. Addition-Product Law
$$\log_c a + \log_c b = \log_c ab$$
2. Subtraction-Quotient Law
$$\log_c a - \log_c b = \log_c \left(\frac{a}{b}\right)$$
3. Power Law
$$\log_c a^m = m \log_c a$$
4. (Logarithm to the) Same Base Law
$$\log_c c = 1$$
5. Unity Law (or Log of Unity Law)
$$\log_c 1 = 0$$
6. $\log_c a = \log_c b$ implies a=b
7. $\log_c a = b$ implies $a = c^b$

Example 4.26

Without using a calculator simplify:

(a) $\log_{10} 10$ (c) $\log_3 3$ (e) $\log_{10} 1000$

(b) $\log_{10} 100$ (d) $\log_3 81$ (f) $\log_2 64$

SOLUTION

(a)
$$\log_{10} 10 = 1$$

(c)
$$\log_3 3 = 1$$

(e)
$$\begin{aligned}\log_{10} 1000 &= \log_{10} 10^3\\ &= 3\log_{10} 10\\ &= 3 \times 1\\ &= 3\end{aligned}$$

(b)
$$\begin{aligned}\log_{10} 100 &= \log_{10} 10^2\\ &= 2\log_{10} 10\\ &= 2 \times 1\\ &= 2\end{aligned}$$

(d)
$$\begin{aligned}\log_3 81 &= \log_3 3^4\\ &= 4\log_3 3\\ &= 4 \times 1\\ &= 4\end{aligned}$$

(f)
$$\begin{aligned}\log_2 64 &= \log_2 2^6\\ &= 6\log_2 2\\ &= 6 \times 1\\ &= 6\end{aligned}$$

Example 4.27

Without using a calculator simplify:

(a) $\log_4 \frac{1}{4}$ (b) $\frac{1}{3}\log_3 729$

SOLUTION

(a)

$$\begin{aligned}\log_4 \frac{1}{4} &= \log_4 4^{-1}\\ &= -1\log_4 4\\ &= -1\times 1\\ &= -1\end{aligned}$$

(b)

$$\begin{aligned}\frac{1}{3}\log_3 729 &= \frac{1}{3}\log_3^6\\ &= \left(\frac{1}{3}\times 6\right)\log_3 3\\ &= 2\times 1\\ &= 2\end{aligned}$$

Example 4.28

Without using a calculator simplify:

(a) $\log_{10} 5+\log_{10} 2$ (b) $\log_2 4+\log_2 8$ (c) $\log_3 9+\log_3 3$ (d) $\log_5 5+3\log_5 5$

SOLUTION

(a)

$$\begin{aligned}\log_{10} 5+\log_{10} 2 &= \log_{10}(5\times 2)\\ &= \log_{10}(10)\\ &= 1\end{aligned}$$

(b)

$$\begin{aligned}\log_2 4+\log_2 8 &= \log_2(4\times 8)\\ &= \log_2 32\\ &= \log_2 2^5\\ &= 5\log_2 2\\ &= 5\end{aligned}$$

(c)

$$\begin{aligned}\log_3 9+\log_3 3 &= \log_3(9\times 3)\\ &= \log_3 27\\ &= \log_3 3^3\\ &= 3\log_3 3\\ &= 3\end{aligned}$$

(d)

$$\begin{aligned}\log_5 5+3\log_5 5 &= \log_5 5+\log_5 5^3\\ \log_5 5+3\log_5 5 &= \log_5(5\times 125)\\ &= \log_5 625\\ &= \log_5 5^4\\ &= 4\log_5 5\\ &= 4\end{aligned}$$

Example 4.29

Without using a calculator simplify:

(a) $\log_{10} 10000-\log_{10} 100$ (b) $\log_4 64-\log_4 4$

SOLUTION

(a)

$$\begin{aligned}\log_{10} 10000 - \log_{10} 100 &= \log_{10}\left(\frac{10000}{100}\right)\\ &= \log_{10} 100\\ &= \log_{10} 10^2\\ &= 2\log_{10} 10\\ &= 2\end{aligned}$$

(b)

$$\begin{aligned}\log_4 64 - \log_4 4 &= \log_4\left(\frac{64}{4}\right)\\ &= \log_4 16\\ &= \log_4 4^2\\ &= 2\log_4 4\\ &= 2\end{aligned}$$

Example 4.30

Without using a calculator simplify:

(a) $\log_{10} 40 + \log_{10} 50 - \log_{10} 20$

(b) $\log_2 18 + \log_2 32 - 2\log_2 6$

SOLUTION

(a)

$$\begin{aligned}\log_{10} 40 + \log_{10} 50 - \log_{10} 20 &= \log_{10}(40 \times 50) - \log_{10} 20 \qquad \text{Apply BODM}\mathbf{A}\text{S}\\ &= \log_{10}(2000) - \log_{10} 20\\ &= \log_{10}\left(\frac{2000}{20}\right)\\ &= \log_{10} 100\\ &= \log_{10} 10^2\\ &= 2\log_{10} 10\\ &= 2\end{aligned}$$

(b)

$$\begin{aligned}\log_2 18 + \log_2 32 - 2\log_2 6 &= \log_2(18 \times 32) - \log_2 6^2\\ &= \log_2 576 - \log_2 36\\ &= \log_2\left(\frac{576}{36}\right)\\ &= \log_2 16\\ &= \log_2 2^4\\ &= 4\log_2 2\\ &= 4\end{aligned}$$

4.12 Exercise Set

1. Without using a calculator simplify:

(a) $\log_{10} 10000$

(b) $\log_8 64$

(c) $\log_2 128$

(d) $\log_3 27$

(e) $\log_5 \frac{1}{125}$

(f) $4\log_9 \frac{1}{9}$

(g) $\frac{1}{5}\log_5 625$

(h) $\log_4 \frac{1}{256}$

(i) $\log 100000$

(j) $\log 1000000$

2. Without using a calculator simplify:

   (a) $\log_2 32 + 2\log_2 8$
   (b) $\log_{10} 25 + \log_{10} 4$
   (c) $\log_{10} 20 + \frac{1}{3}\log_{10} 125$
   (d) $\log_5 625 - \log_2 25$
   (e) $\log_4 256 - 2\log_4 8$
   (f) $\log_4 80 - \log_4 5$
   (g) $\log_6 12 + \log_6 18$
   (h) $\log_2 56 - \frac{1}{2}\log_2 49$

3. Without using a calculator simplify:

   (a) $\log_{10} 40 + \log_{10} 50 - \log_{10} 20$
   (b) $\log 75 + 2\log 2 - \log 3$
   (c) $\log_2 18 + \log_2 32 - 2\log_6$
   (d) $2\log_5 5 + \frac{1}{2}\log_5 81 - \log_5 45$

4. Express the following as a single logarithm:

   (a) $\log_{10} 25 + \log_{10} 5$
   (b) $\log_{10} 84 - \log_{10} 12$
   (c) $\log_3 56 - \log_3 27$
   (d) $2\log_{10} 8 + \frac{1}{3}\log_{10} 125 - 1$

4.4.2 Using Logarithms for calculation

Rules of logarithms apply when we are multiplying or dividing numbers.

Example 4.31

Use logarithms to calculate 38×145

SOLUTION

x	Standard form	$\log_{10} x$
38	3.8×10^1	1.5798
145	1.45×10^2	+2.1614
5511	5.511×10^3	3.7412

Therefore $38 \times 145 \approx 5511$

Example 4.32

Use logarithms to evaluate 356×43.6

SOLUTION

x	Standard form	$\log_{10} x$
356	3.56×10^2	2.5514
43.6	4.36×10^1	+ 1.6395
15520	1.552×10^4	4.1909

Therefore $356 \times 43.6 \approx 15520$

Example 4.33

Use logarithms to evaluate 0.0417×0.00928

SOLUTION

x	Standard form	$\log_{10} x$
0.0417	4.17×10^{-2}	$\bar{2}.6201$
0.00928	9.28×10^{-3}	$+\bar{3}.9675$
0.0003869	3.869×10^{-4}	$\bar{4}.5876$

Therefore $0.0417 \times 0.00928 \approx 0.0003869$

Example 4.34

Use logarithms to evaluate $8.62 \div 3.457$

SOLUTION

x	Standard form	$\log_{10} x$
8.62	8.62×10^{0}	0.9355
3.457	3.457×10^{0}	-0.5387
2.49	2.49×10^{0}	0.3968

Therefore $8.62 \div 3.457 \approx 2.49$

Example 4.35

Use logarithms to evaluate $4565 \div 98.88$

SOLUTION

x	Standard form	$\log_{10} x$
4565	4.565×10^{3}	3.6594
98.88	9.888×10^{1}	-1.9951
46.16	4.616×10^{1}	1.6643

Therefore $4565 \div 98.88 \approx 46.16$

4.13 Exercise Set

1. Evaluate using logarithms

   (a) 36.5×480.2
   (b) 8.21×516.4
   (c) 245×22.34
   (d) 12.9×3.135
   (e) $56.34 \div 5.86$
   (f) $24.46 \div 13.34$

2. Evaluate using logarithms

   (a) 75.6×0.8563
   (b) 0.0075×98
   (c) $234.2 \div 12.34$
   (d) 6.26×45.678
   (e) $1200 \div 12$
   (f) $63.74 \div 8.46$

Chapter Summary

1. During rounding off, if the digit to the right is $0, 1, 2, 3, 4$ the digit being rounded to remains the same i.e Rounded down
2. During rounding off,if the digit to the right is $5, 6, 7, 8, 9$ the digit being rounded to increases by one i.e Rounded up
3. In standard form, numbers are written as $a \times 10^{n}$ where $1 \leq a < 10$ and n is an integer.
4. The following rules apply to indices:

LAW	Example
$a^m \times a^n = a^{m+n}$	$2^3 \times 2^4 = 2^7$
$a^m \div a^n = a^{m-n}$	$2^3 \div 2^4 = 2^{-1}$
$(a^m)^n = a^{mn}$	$(2^3)^4 = 2^12$
a^0=1	$2021^0 = 1$
$a^{-n} = \frac{1}{a^n}$	$2^{-3} = \frac{1}{8}$
$a^n \times b^n = (ab)^n$	$2^3 \times 3^3 = 6^3$

5. The following rules apply to logarithm:

LAW	Example
$\log_c a + \log_c b = \log_c ab$	$\log_3 4 + \log_3 5 = \log_3 20$
$\log_c a - \log_c b = \log_c(\frac{a}{b})$	$\log_3 10 - \log_3 2 = \log_3 5$
$\log_c a^m = m \log_c a$	$\log_3 6^5 = 5 \log_3 6$
$\log_c c = 1$	$\log_4 4 = 1$
$\log_c 1 = 0$	$\log_3 1 = 0$
$\log_c a = \log_c b$ implies a=b	$\log_2 x = \log_2 3$ implies x=3
$\log_c a = b$ implies $a = c^b$	$\log_2 x = 3$ implies $x = 2^3$
$\log_{10} a$ is sometimes written as $\log a$	$\log_{10} 6$ is the same as $\log 6$

ASSESSMENT

1. During the 2021 national elections ,Museveni obtained 5,300,831 votes.Round off to the nearest 1000
2. Round the following to the specified degree of accuracy.

   (a) 0.00621968 to 4 d.p
   (b) 3.95451 to 3 d.p
   (c) 2174.12 to 1 d.p
   (d) 460.762 to 2 d.p
   (e) 0.571859 to 3 d.p
   (f) 1.066564 to 2 d.p

3. Round the following to the specified degree of accuracy.

   (a) 0.7862 to 1 significant figure.
   (b) 9371 to 2 significant figures.
   (c) 2.3608 to 3 significant figures.
   (d) 401 to 1 significant figure.
   (e) 0.00967947 to 3 significant figures.
   (f) 376.92 to 2 significant figures.

4. Write the following expanded forms in index form:

   (a) $5 \times 5 \times 5 \times 5 \times 5$

   (b) $3 \times 3 \times 3 \times 3 \times 3 \times 3$

5. Simplify $\frac{2.4\times10^2}{6.0\times10^{-3}}$.Give your answer in standard form

6. With out using mathematical tables or calculator,find the value of

   (a) $2\log_{10}5 + \log_{10}4 - \log_{10}0.1$

   (b) $\log_3 9 + \log_3 21 - \log_7$

   (c) $\log 120 - 2\log 6 + \frac{1}{3}\log 27$

   (d) $\log 400 + \log 500 - \log 200 + 1$

7. Simplify

   (a) $\frac{5^{x+1}\times5^{-2x}}{5^{-x}}$

   (b) $\frac{27^{\frac{1}{3}}\times16^{\frac{3}{4}}}{9^{\frac{1}{2}}}$

   (c) $\frac{3^n\times9^{n-2}}{27^{n-1}}$

   (d) $\left(\frac{8}{27}\right)^{\frac{-2}{3}}$

8. Evaluate $\frac{\left(y^2\right)^{\frac{1}{6}}}{(9x)^{\frac{1}{2}}}$ when x=16 and y=8

9. Solve for x in $32^{\frac{3}{5}} \div x^{\frac{1}{2}} = 2$

10. Use logarithms to work out the following

    (a) 67.44×34.5

    (b) $57.4 \div 24.5$

End

UNDERSTANDING SENIOR TWO MATHEMATICS

This new edition of understanding senior two mathematics is a continuation of book one .It covers all the 13 topics as designed in the new lower secondary curriculum.

The aim of the author has been to provide a simple and direct approach to understanding mathematics by all learners by themselves.Ample opportunity is given for practice in exercises at the end of the sections.

Other books ,pamphlets and articles by the same author

- O level physics practical workbook Revised edition 2021.
- Understanding senior one mathematics 1st edition 2020.
- O level physics definitions,experiments and laws 2020.
- O level mathematics revision questions (S.4 and S.3).
- A level mathematics revision questions (Paper one and two).
- Making of biomass briquettes using sugar residues and banana peelings

AUTHOR'S CONTACTS

kazibastephen4@gmail.com

0703822752/0787698238(Whatsapp)/0787430783