Find $\frac{d^2y}{dx^2}$ for each of the following

1. $y = x^5$
2. $y = x^2(x+3)$
2. $y = x^3(4 - x^2)$
3. $y = \sqrt{x}\,(5 + x)$
4. $y = 6\sqrt{x}\,(x^3 - 2x + 1)$
5. $y = 6x^{\frac{1}{3}}(2x - 5)$
6. $y = 2x^{\frac{1}{4}}(x^2 - 2)$
7. $y = (x+3)(x-4)$
8. $y = (x+4)^2$
9. $y = (x+5)(2x-1)$
10. $y = 2(x-3)^2$

Find f''(x) for each of the following

11. $f(x) = 2x^2(x-1)^2$
12. $f(x) = \frac{\sqrt{x}+1}{\sqrt{x}}$

13. $f(x)=\dfrac{3x^3+5}{x^2}$

14. $f(x)=\dfrac{(x+3)(x-4)}{x}$

15. $f(x)=\dfrac{(2x-5)(x-4)}{x^3}$

16. $f(x)=\dfrac{(3x-1)^2}{2x}$

17. $f(x)=\dfrac{5x+3}{2\sqrt{x}}$

18. $f(x)=\dfrac{(x+1)^2}{\sqrt{x}}$

19. $f(x)=\dfrac{x^2+5}{3\sqrt{x}}$

20. $f(x)=\dfrac{3\sqrt{x}-7}{2\sqrt{x}}$

21. $f(x)=\dfrac{2x^2-5}{\left(3\sqrt{x}\right)^2}$

1. A particle is thrown vertically upwards and its height after t seconds is h m where $h = 25.2t - 4.9t^2$.

Find

(a) its height and velocity after 3seconds,

(b) when it is momentarily at rest,

(c) the greatest height reached,

(d) the distance moved in the 3rd second,

(e) the acceleration when $t = 2\frac{4}{7}$

2. A particle moves along the x-axis in such a way that its distance x cm from the origin after $t\ s$ is given by $x = 7t + 12t^2$.

(a) What does it travel in the 9th second?

(b) What are its velocity and acceleration at the end of the 9th second?

3. A point moves along a straight line OX so that its distance x cm from the point O at time $t\ s$ is given by the formula $x = t^3 - 6t^2 + 9t$. Find

(a) at what times and in what positions the point will have zero velocity.

(b) its acceleration at those instants

(c) its velocity when its acceleration is zero.

4. A particle moves in a straight line so that after t s, it is 5m from a fixed point O on the line where $s = t^4 + 3t^2$. Find

(a) the acceleration when $t = 1$, $t = 2$ and $t = 3s$

(b) the average acceleration between t = 1and $t = 3s$.

5. A particle moves along a straight line so that after ts, its distance from a fixed point O on the line is 5m where $s = t^3 - 3t^2 + 2t$.

(a) When is the particle at O?

(b) What is its velocity and acceleration at these times?

Exercise 6E

1. Find the gradient of each of the following curves at the points given.

(a) $y = \sqrt{x} + 2$, at $(9, 5)$

(b) $y = \frac{1}{x}$, at $\left(3, \frac{1}{3}\right)$

(c) $y = 2 - \frac{4}{x^2}$, at $(-2, 1)$

(d) $y = \frac{4 - x^3}{x^2}$, at $(-2, 3)$

2. Find the coordinates of any points on each of the following curves where the gradient is as stated.

(a) $y = x^4 + 1$, grad 32.

(b) $y = \frac{16}{x^2}$, grad 4.

(c) $y = 2x^3 - 4x^2 + 3x + 2$, grad 1

(d) $4\sqrt{x} - x$, grad 5.

3. Given that the curve with equation $y = Ax^2 + Bx$ has gradient 7 at the point $(6, 8)$, find the values of the constants A and B.

4. A curve whose equation is $y = \frac{a}{x} + c$, passes through the point $(3, 9)$ with gradient 5. Find the values of the constants a and b

5. Find the equation of the tangent to each of the following curves at the point indicated by the given value of x

(a) $y = 2x^3 - 1$, where $x = 1$,

(b) $y = 5 - \frac{8}{x^2}$, where $x = -2$

(c) $y = x(x^2 - 3)$, where $x = 2$

(d) $y = \frac{x + 5}{\sqrt{x}}$, where $x = 25$

6. Find the equation of the normal to each of the following curves at the point indicated by the given value of x

(a) $y = \frac{6}{x}$, where $x = 3$

(b) $y = x^3 + 2x^2 - 3$, where $x = -2$

(c) $y = \sqrt{x}(x^2 - 2)$ where $x = 1$

(d) $y = 3\sqrt{x} + \frac{1}{\sqrt{x}}$, where $x = \frac{1}{4}$

7. Find the equation of the normal to the curve $y = x^3 - 8$ at the point where the curve cuts the x-axis

8. The two tangents to the curve $y = x^2$ at the points where $y = 9$, intersect at the point P. find the coordinates of P.

1. Find the coordinates of the stationary points on each of the following curves, and determine their nature.

(a) $y = x^2 - 2x + 5$

(b) $y = 3(x + 3)(2x - 1)$

(c) $y = 3 + 15x - 6x^2 - x^3$

(d) $y = x^4 - 14x^2 + 24x - 10$

2. Find the coordinates of the stationary points on each of the following and determine their nature

(a) $y = x + \dfrac{1}{x}$ (b) $y = \dfrac{1}{x} - \dfrac{3}{x^2}$

(c) $y = \frac{2}{x^3} - \frac{1}{x^2}$ (d) $y = \frac{2 - x^3}{x^4}$

3. If $S = 4r^2 - 10r + 7$, find the minimum value of S and the value of r which gives this minimum value.
4. If $V = 30t - 6t^2$, find the maximum value of V and the value of t for which it occurs.
5. If $V = 4rx + 2r^2$ and $3r + x = 5$, find the maximum value V and the values of r and x that give this maximum value.
6. A rectangular enclosure is formed by using 1200m fencing. Find the greatest possible area that can be enclosed in this way and the corresponding dimensions of the rectangular enclosure.
7. An open metal tank with a square base is made from $12m^2$ of sheet metal. Find the length of the side of the base for the volume of the tank to be a maximum and find this volume.

1. Given that $y = x^4$, show that

$$\frac{4y}{3}\left(\frac{d^2y}{dx^2}\right) - \left(\frac{dy}{dx}\right)^2 = 0$$

2. Given that $y = x^3 - 4x^2 + 5x - 2$,

(a) find $\frac{dy}{dx}$.

(b) P is the point on the curve where $x = 3$

(i) Calculate the y coordinate of P.

(ii) Calculate the gradient at P

(iii) Find the equation of the tangent at P

(iv) Find the equation of the normal at P

(v) Find the values of x for which the curve has gradient of 5

Answers:

(a) $3x^2 - 8x + 5$

(b)(i) 4 (ii) 8 (iii) $y = 8x - 20$ (iv) $x + 8y = 35$

(v) 0, $2\frac{2}{3}$

3. (a) Given $y = 5x^3 - 2x^2 + 1$, find $\frac{dy}{dx}$.

(b) Hence find the exact values of x at which the graph of $y = 5x^3 - 2x^2 + 1$ has turning points.

Answers

(a) $15x^2 - 4x$ (b) $0, \frac{4}{15}$

4. Given the function $y = x^3 - 12x + 5$,

(a) Find $\frac{dy}{dx}$.

(b) Find the coordinates of the stationary points on the curve

Exercise

1. Given that curve $y = x(x-1)$
   (a) Find the intercepts
   (b) Find the turning point and distinguish it
   (c) Sketch the curve
2. Given $y = (x-3)^2 + 2$
   (a) Find the intercepts
   (b) Find the turning point and distinguish it.
   (c) Sketch the curve

3. Given $y = x^2 - 6x + 8$
   (a) Find the intercepts
   (b) Find the turning point and distinguish it.
   (c) Sketch the curve

4. Given $y = 2x^2 - 4x + 5$
   (a) Find the intercepts
   (b) Find the turning point and distinguish it.
   (c) Sketch the curve

5. Given $y = x^2 + 2x - 15$
   (a) Find the intercepts
   (b) Find the turning point and distinguish it.
   (c) Sketch the curve
6. Given $y = -x^2 + 4x + 5$
   (a) Find the intercepts

(b) Find the turning point and distinguish it.
(c) Sketch the curve

7. Given $y = 8 + 2x - x^2$
   (a) Find the intercepts
   (b) Find the turning point and distinguish it.
   (c) Sketch the curve
8. Given $y = 20 + 6x - x^2$
   (a) Find the intercepts
   (b) Find the turning point and distinguish it.
   (c) Sketch the curve
9. Given $y = 7 - 3x - 4x^2$
   (a) Find the intercepts
   (b) Find the turning point and distinguish it.
   (c) Sketch the curve

10. Given $y = 14 + x - 3x^2$
   (a) Find the intercepts
   (b) Find the turning point and distinguish it.
   (c) Sketch the curve

Exercise 7A

1. Integrate each of the following with respect x

   (a) x^4 (b) $4x^2$ (c) $10x^5$ (d) 5 (e) $\frac{1}{3}x^4$

   (f) $\frac{2}{3}x^3$ (g) $-\frac{1}{4}x^2$

2. Find each of the following integrals

   (a) $\int x^{-4}dx$ (b) $\int 2x^{-3}dx$ (c) $\int -5x^{-4}dx$

   (d) $\int -\frac{1}{x^5}dx$ (e) $\int \frac{3}{x^2}dx$ (f) $\int \frac{3}{2x^4}dx$

3. Integrate each of the following functions with respect to x

   (a) $f(x) = -3\sqrt{x}$ (b) $f(x) = \frac{4}{\sqrt[3]{x}}$

   (c) $f(x) = \frac{3}{7\sqrt{x}}$ (d) $f(x) = \frac{6}{5\sqrt[3]{x}}$

4. Integrate each of the following with respect to x

   (a) $x^2 - 5x + 3$ (b) $x^9 + 2x^5$ (c) $x^6 - 3x + 3$

   (d) $3x^4 - \frac{2}{x^3}$ (e) $2x^6 + \frac{8}{x^5}$

5. Find $\int y\,dx$ for each of the following

(a) $y = x^2(x+5)$ (b) $y = \sqrt{x}(x+3)$

(c) $y = 3\sqrt{x}(x^2 - x + 1)$ (d) $y = x(x-1)^2$

6. Find the following definite integrals

(a) $\int_0^2 x^2\,dx$ (b) $\int_0^3 4x^3\,dx$ (c) $\int_2^3 (6x^2 - 1)\,dx$

(d) $\int_1^2 \frac{4}{x^3}\,dx$ (e) $\int_{-2}^{-1} \frac{2}{x^3}\,dx$

7. Find the following definite integrals

(a) $\int (\sin x + 2\cos x)\,dx$ (b) $\int (\sin 2x - \cos 3x)\,dx$

(c) $\int (\cos 2x + \sin x)\,dx$ (d) $\int (\cos 4x - \sin 5x)\,dx$

1. Find the area enclosed by the curve

   $y = x^2 + 3x$, the lines $x = 2$, $x = 5$ and the x-axis

2. Find the area enclosed above the x-axis and below the curve $y = 16 - x^2$

3. The line $y = 3x + 1$ and the curve $y = x^2 + 3$ meet at points P and Q.
   (a) Calculate the coordinates of P and Q
   (b) Sketch the line and the curve on the same set of axes
   (c) Calculate the area bounded by the curve and the line

4. The line $y = 9 - x$ and the curve $y = x^2 - 2x + 3$ meet at points A and B.
   (a) Find the coordinates of A and B
   (b) Sketch the line and the curve on the same set of axes
   (c) Calculate the area bounded by the curve and the line

5. Find the area enclosed between the curves $y = 2x^2 - 7$ and $y = 5 - x^2$

6. The curve $y = x^2 - 2x$ cuts the x-axis at the points O and P, and meets the line $y = 2x$ at points O and Q
   (a) Calculate the coordinates of P and Q
   (b) Find the area enclosed between the line and the curve

Answer
(i) (0, 0), (5, 10) (ii) $20\frac{5}{6}$

Sample examination questions

1. On the same axes, sketch the curves $y = x(x+2)$ and $y = x(4-x)$.
Find the area enclosed by the two curves

Answer: $\frac{1}{3}$

2. Given the function $f(x) = \dfrac{(3x^2+1)}{x^2}$

   (a) Express f(x) in the form $Ax^2 + B + \dfrac{C}{x^2}$, where the constants A, B and C are to found

   (b) $\int f(x)dx$

   (c) Hence evaluate $\int_1^2 f(x)dx$.

   Answer

   (a) $9x^2 + 6 + \dfrac{1}{x^2}$

   (b) $9x^2 + 6 + \dfrac{1}{x^2}$

   (c) $27\frac{1}{2}$

3. (a) Sketch the curve $y = (x-2)(x-3)$, showing where it crosses the x-axis. Hence find the area enclosed below the x-axis and above the curve.
   (b) Sketch the curve with equation $y = (x-2)^2$ .Calculate the area of the region bounded by the curve, the x and y - axes
   (a)Area $= \frac{1}{6}$ (b) Area $= 2\frac{2}{3}$

4. (a) Sketch on the same diagram, the curves $y = x^2 - 5x$ and $y = 3 - x^2$, and find their points of intersection.
   (b) Find the area of the region bounded by the two curves.
   Answer: $14\frac{7}{24}$

5. (i) Find the coordinates of the points of intersection of the line $y = 2x$ with the curve $y = x^2 - 3x$
   (ii) Sketch the line and the curve for the domain $0 \leq x \leq 6$.
   (iii) Calculate the area enclosed between the line and the curve

Exercise 9A

1. Find the general solution to each of the following differential equations

(a) $\frac{dy}{dx}=\frac{x-1}{y}$ (b) $\frac{dy}{dx}=\frac{x^2+1}{y}$ (c) $y^2\frac{dy}{dx}=x-3$

(d) $\frac{dy}{dx}=\frac{x^2-1}{2y}$ (e) $\cos y\frac{dy}{dx}=\sin x$

2. Find the particular solution to each of the following differential equations

(a) $\frac{dy}{dx}=4-3x^2$, $y=5$ at $x=1$

(b) $\frac{dy}{dx}=\frac{x+1}{y}$, $y=3$ at $x=-2$

(c) $(y-3)\frac{dy}{dx}=x+3$, $y=4$ at $x=0$

(d) $3y^2\frac{dy}{dx}+2x=1$, $y=2$ at $x=4$

(e) $x^3\frac{dy}{dx}=2y^2$, $y=4$ at $x=2$

1. The rate of change of y with respect to x varies directly as the square of x.
   Form a differential equation from the above information and solve it

2. The rate of change of v with respect to t is directly proportional to t.
   Write a differential equation to model the above statement and solve it.

3. The rate of change of v with respect to t is inversely proportional to t.
   Solve the differential equation.

4. The gradient of a curve at the point P(x,y) is given by the expression $\frac{x-1}{y}$. Given that the curve passes through the point (0, 2). Find the particular differential equation from the above information

5. The gradient of a curve at the point P(x,y) is given by the expression $\frac{x^2}{y}$.Form a differential equation to model the above information