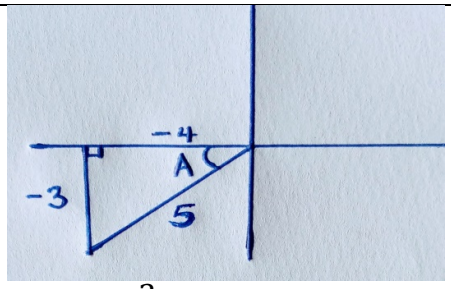
JINJA JOINT EXAMINATION BOARD

MOCK EXAMINATIONS 2022

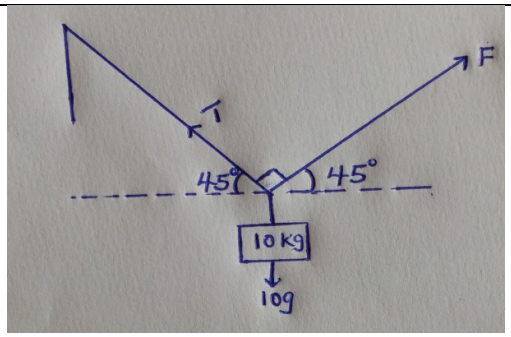
S475/1 SUB MATHEMATICS

MARKING GUIDE

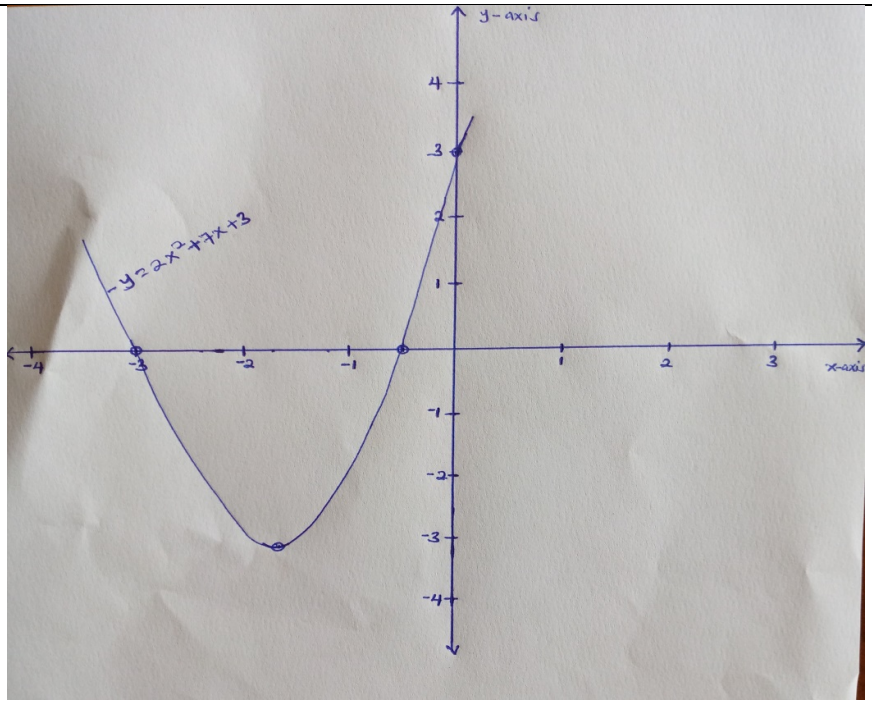
NO	SOLUTION	MARKS &COMMENT
1.	$\alpha + \beta = \frac{a^2}{5}$	
	$\frac{a^2}{5} = 5$	B_1
	$a^2 = 25$	M_1
	$a = \sqrt{25}$	
	$a = \pm 5$	A_1
	$\alpha\beta = \frac{b}{5}$	B_1
	$\frac{b}{5} = 10$	
	$\underline{\underline{\boldsymbol{b = 50}}}$	A_1
2.	$Var(x) = E(x^2) - (E(x))^2$	
	$= \int_0^2 x^2 . \frac{x^2}{4} dx - \left[\int_0^2 x . \frac{x^2}{4} dx\right]^2$	M_1
	$= \frac{1}{4}\int_0^2 x^4 dx - \left[\frac{1}{4}\int_0^2 x^3 dx\right]^2$	
	$= \frac{1}{4}\left[\frac{x^5}{5}\right]\frac{2}{0} - \left[\frac{1}{4} . \frac{x^4}{4}\right]\frac{2}{0}^2$	M_1
	$= \frac{1}{20}[2^5 - 0^5] - \left[\frac{1}{16}(2^4 - 0^4)\right]^2$	
	$= \frac{32}{20} - \left(\frac{1}{16}(16)\right)^2$	M_1
	$= \frac{32}{20} - 1^2$	M_1
	$= \frac{32}{20} - \frac{20}{20}$	
	$= \frac{12}{20}$	
	$= \boldsymbol{\frac{3}{5}}$	A_1

3.	 $sinA = \frac{-3}{5}$ $Cos\ A = \frac{-4}{5}$ (i)$\sin 3A = 3\sin A - 4\ sin^3 A$ $=3\left(\frac{-3}{5}\right) - 4\left(\frac{-3}{5}\right)^3$ $=\frac{-9}{5} - 4\left(\frac{-27}{125}\right)$ $=\frac{-9}{5} + \frac{108}{125}$ $=\frac{-225+108}{125}$ $=\mathbf{\frac{-117}{125}}$ (i)$4cos^3A - 3cosA$ $=4\left(\frac{-4}{5}\right)^3 - 3\left(\frac{-4}{5}\right)$ $=\frac{4(-64)}{125} + \frac{12}{5}$ $=\frac{-25\ +300}{125}$ $=\mathbf{\frac{44}{125}}$	B_1 M_1 A_1 M_1 A_1
4.	$P = \frac{2}{5}, 1 - P = \frac{3}{5}, \quad n = 15$ $P(10 \le \times \le 13) = P(x = 10) + P(x = 11) + P(x = 12) + P(x = 13)$ $= \binom{15}{10}\left(\frac{2}{5}\right)^{10}\left(\frac{3}{5}\right)^5 + \binom{15}{11}\left(\frac{2}{5}\right)^{11}\left(\frac{3}{5}\right)^4 + \binom{15}{12}\left(\frac{2}{5}\right)^{12}\left(\frac{3}{5}\right)^3 + \binom{15}{13}\left(\frac{2}{5}\right)^{13}\left(\frac{3}{5}\right)^2$ $= 0.0245\ + 0.0074\ +\ 0.0016\ + 0\ .0003$ $\underline{\mathbf{=\ 0.0338}}$	B_1 M_1M_1 M_1 A_1
5.	$\log_2 x^2 - \log_x 64 = 4$ $2\log_2 x - \log_x 8^2 = 4$ $2\log_2 x - 2\log_x 8 = 4$ $\log_2 x - \log_x 8 = 2$ $\log_2 x - \frac{3\log_2 2}{\log_2 x} = 2$	M_1

	Let $\log_2 x = m$ $m - \frac{3}{m} = 2$ $m^2 - 2m - 3 = 0$ Using (1,-3) $m^2 + m - 3m - 3 = 0$ $m(m+1) - 3(m+1) = 0$ $(m-3)(m+1) = 0$ $m = 3 \; or \; m = -1$ $\log_2 x = 3, \quad \log_2 x = -1$ $\boldsymbol{x = 8}$, $\boldsymbol{x = \frac{1}{2}}$	B_1 M_1 M_1 A_1
6.	$\frac{\sum x}{n} = \frac{747 + 717 + 596 + 450 + 328 + 370}{6}$ $\frac{\sum x}{n} = \frac{1604}{3}$ $\frac{\sum x^2}{n} = \frac{747^2+717^2+596^2+450^2+328^2+370^2}{6}$ $\frac{\sum x^2}{n} = 312383$ Standard deviation = $\sqrt{\left(\frac{\sum x^2 f}{\sum f} - \left(\frac{\sum xf}{\sum f}\right)^2\right)}$ $= \sqrt{312383 - (\frac{1604}{3})^2}$ $\underline{\underline{= \mathbf{162.8329 \; (4.dp)}}}$	M_1 M_1 $M_1 \, M_1$ A_1
7.	$d = -3$ $S_{20} = 10u_2$ From $S_n = \frac{n}{2}(2a + (n-1)d) \; and \; u_n = a + (n-1)d$ $S_{20} = \frac{20}{2}(2a + 19(-3))$ $= 10(2a - 57)$ $u_2 = a + d \; = a - 3$ $10(2a - 57) = 10(a - 3)$ $2a - 57 = a - 3$ $2a \; - 57 = a \; - 3$ $\underline{\boldsymbol{a = 54}}$	B_1 B_1 M_1 M_1 A_1

8.	 $T\cos 45^0 = Fcos45^0$ $T = F \ldots\ldots\ldots\ldots\ldots (i)$ $T\sin 45^o + F\sin 45^0 = 10g$……(ii) $T\ \sin 45^o + T\sin 45^0 = 10g$ $2T\sin 45^0 = 10g$ $2T\sin 45^0 = 10g$ $\frac{T}{\sqrt{2}} = 5g$ $\boldsymbol{T = 5\sqrt{2}gN}$ $\boldsymbol{F=5\sqrt{2}gN}$	B_1 B_1 M_1 M_1 A_1
9.	i) $\frac{dy}{dx} = 4x + 7$ $\int dy = \int (4x+7)dx$ $y = 2x^2 + 7x + c$ $At\ (1,12)$ $12 = 2(1)^2 + 7(1) + c$ $12 = 9 + c$ $c = 3$ $\boldsymbol{\therefore y = 2x^2 + 7x + 3}$ ii) At turning point, $\frac{dy}{dx} = 0$ $4x + 7 = 0$ $x = \frac{-7}{4}$ $For\ x = \frac{-7}{4}$ $y = 2\left(\frac{-7}{4}\right)^2 + 7\left(\frac{-7}{4}\right) + 3$ $y = 2\frac{(49)}{16} - \frac{49}{4} + 3$	B_1 M_1 M_1 A_1 M_1 A_1 B_1 M_1

	$y = \frac{49}{8} - \frac{49}{4} + 3$ $y = \frac{49 - 98 + 24}{8}$ $y = \frac{-25}{8}$ $\therefore$ *the turning point is* $\left(\frac{-7}{4}, \frac{-25}{8}\right)$ *from* $\frac{dy}{dx} = 4x + 7$ $\frac{d^2y}{dx^2} = 4$ $\boldsymbol{since}\ \boldsymbol{\frac{d^2y}{dx^2}} > 0$, *then the turning point is a minimum point.*	M_1 A_1 B_1
	i) Finding intercepts For x-intercept, $y = 0$ $2x^2 + 7x + 3 = 0$ $x = \frac{-7 \pm \sqrt{7^2 - 4(2)(3)}}{2(2)}$ $x = \frac{-7 \pm \sqrt{25}}{4}$ $x = \frac{-7 \pm 5}{4}$ $x = \frac{-1}{2}, x = -3$ *the x − intercepts are* $\left(\frac{-1}{2}, 0\right)$ and $(-3,0)$ *For y − intercept* $x = 0$ $y = 2(0)^2 + 7(0) + 3$ $y = 3$ $\therefore$ $\boldsymbol{the\ y - intercept\ is}$ $(\mathbf{0}, \mathbf{3})$	M_1 A_1 B_1 B_1



10.

Athletes	Time spent in 200m	Time spent in 400m	R_x	R_y	D	D^2
A	64	150	4	5	-1	1
B	69	154	6.5	6	0.5	0.25
C	56	133	1	1	0	0
D	72	169	8	8	0	0
E	57	140	2.5	2	0.5	0.25
F	68	159	5	7	-2	4
G	57	146	2.5	4	-1.5	2.25
H	69	143	6.5	3	3.5	12.25

B_1
B_1
B_1

	$\sum D^2 = 20$

$$\rho = 1 - \frac{6\sum D^2}{n(n^2-1)} \quad M_1$$

$$\rho = 1 - \frac{6\times 20}{8(8^2-1)} \quad M_1$$

$$\rho = 0.7610(4.dp) \quad A_1 \; B_1$$

There is a high position relationship between x & y

$M(\bar{x}, \bar{y})$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{512}{8}$$

$$\bar{x} = 64$$

$$\bar{y} = \frac{\sum y}{n}$$

$$= \frac{1194}{8} \quad M_1$$

$$\bar{y} = 149.25$$

$M(64,149.25)$

$M_L(\overline{x_L}, \overline{y_L})$

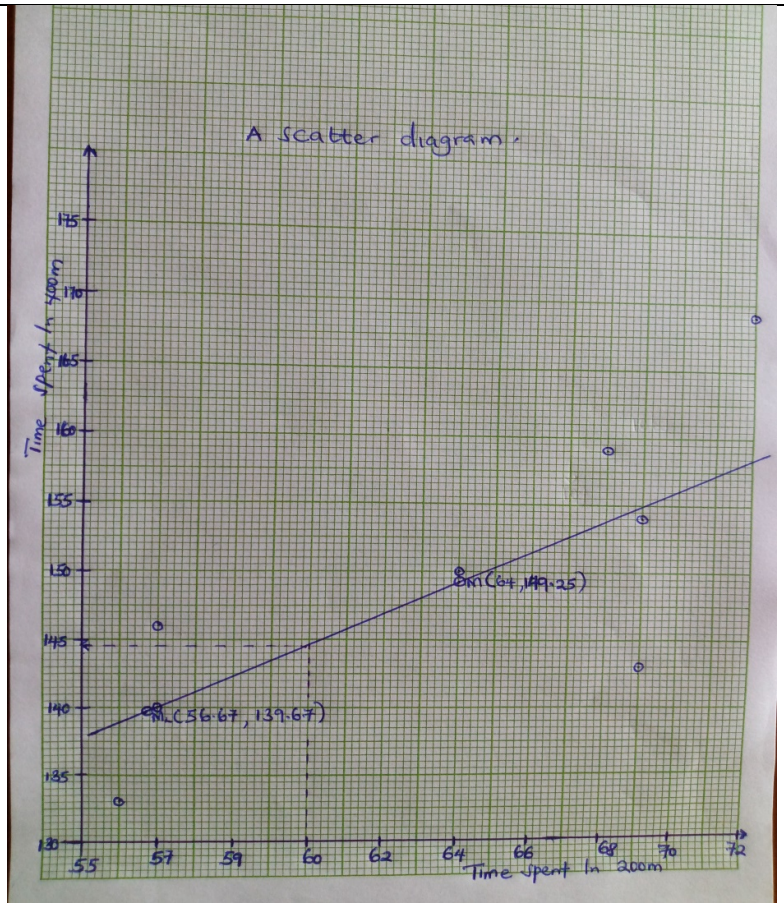
$$\overline{x_L} = \frac{170}{3}$$

$$\overline{x_L} = 56.67$$

$$\overline{y_L} = \frac{419}{3}$$

$$\overline{y_L} = 139.67$$

$M_L(56.67, 139.67)$ $B_1\,B_1$



B_1
B_1
B_1

B_1 - Line of best fit

B_1

$iii)$ $144.5\ seonds$

11.

$\boldsymbol{a}.\boldsymbol{b} = 0$

$\begin{pmatrix} \beta \\ 2+\beta \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix} = 0$ M_1

$-\beta + 6 + 3\beta = 0$ M_1

$2\beta + 6 = 0$

$2\beta = -6$

$\underline{\boldsymbol{\beta = -3}}$ A_1

bi) $\overrightarrow{3OB} - \overrightarrow{2OA} + \overrightarrow{OC} = 3\begin{pmatrix} 4 \\ -1 \end{pmatrix} - 2\begin{pmatrix} -3 \\ 5 \end{pmatrix} + \begin{pmatrix} 6 \\ 7 \end{pmatrix}$

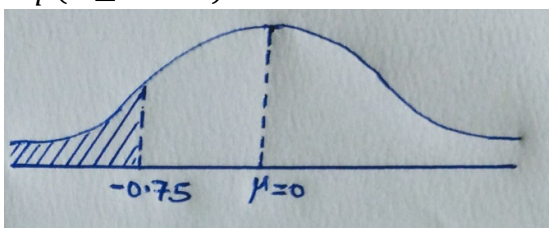
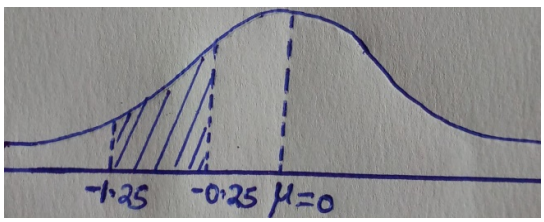
$= \begin{pmatrix} 12 \\ -3 \end{pmatrix} - \begin{pmatrix} -6 \\ 10 \end{pmatrix} + \begin{pmatrix} 6 \\ 7 \end{pmatrix}$ M_1

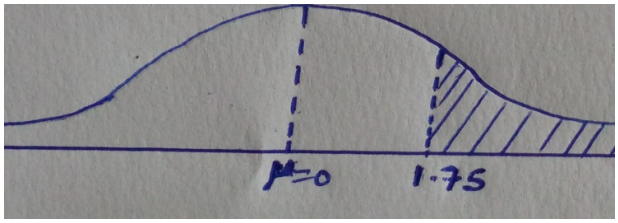
$= \begin{pmatrix} 12 - -6 + 6 \\ -3 - 10 + 7 \end{pmatrix}$ M_1

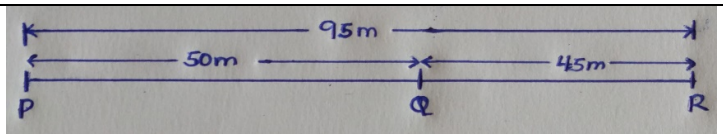
$= \begin{pmatrix} 12 + 6 + 6 \\ -13 + 7 \end{pmatrix}$ M_1

$= \begin{pmatrix} 24 \\ -6 \end{pmatrix}$ M_1

$\underline{\therefore \boldsymbol{\overrightarrow{3OB} - \overrightarrow{2OA} + \overrightarrow{OC} = 24i - 6j}}$ A_1

	ii) $\left\|\overrightarrow{30B} - \overrightarrow{20A} + \overrightarrow{OC}\right\| = \sqrt{24^2 + (-6)^2}$ $= \sqrt{576 + 36}$ $= \sqrt{612}$ $= 24.74 units (2.dp)$	M_1 A_1
	iii) $\overrightarrow{OA}.\overrightarrow{0C} = \left\|\overrightarrow{0A}\right\|\overrightarrow{0C}\|\cos\theta$ $\begin{pmatrix}-3\\5\end{pmatrix}.\begin{pmatrix}6\\7\end{pmatrix} = \sqrt{(-3)^2 + 5^2}\sqrt{6^2 + 7^2}\cos\theta$ $-18 + 35 = \sqrt{34}\sqrt{85}\cos\theta$ $17 = \sqrt{2890}\cos\theta$ $\cos\theta = \frac{17}{\sqrt{28901}}$ $\theta = cos^{-1}\left(\frac{17}{\sqrt{28901}}\right)$ $\underline{\underline{\boldsymbol{\theta = 79.52^0}}}$	$M_1M_1M_1$ M_1 A_1
12.	Let x demote the height of babies i) $P(\times \leq 42) = P\left(Z \leq \frac{42-45}{\sqrt{16}}\right)$ $= p(z \leq -0.75)$  $P(Z \leq -0.75) = 0.5 - P\ (-0.75 \leq Z \leq 0$ $= 0.5 - P(0 \leq Z \leq 0.75)$ $= 0.5 - 02734$ $\underline{\underline{\boldsymbol{= 0.2266}}}$ ii) $P(40 < x < 44)$ $P\left(\frac{40-45}{\sqrt{16}} < z < \frac{44-45}{\sqrt{16}}\right)$ $P(-1.25 < Z < -0.25)$  	B_1 M_1 M_1 A_1 B_1 B_1

	$P(-1.25 < Z < -0.25) = P(-1.25 < Z < 0) - P(-0.25 < Z < 0)$ $= P(0 < Z < 1.25) - P(O < Z < 0.25)$ $= 0.3944 - 0.0987$ $=0.2957$ iii) $P(x > 52)$ $P\left(z > \frac{52-4}{\sqrt{16}}\right)$ $P(z > 1.75)$  $P(z > 1.75) = 0.5 - P(0 < z < 1.75)$ $=0.5 - 0.4599$ $=0.0401$ The approximate number of babies out of 150 babies with height more than 52cm is 0.0401 x150 $= 6.015$ $\simeq$6 babies	M_1 M_1 A_1 B_1 M_1 A_1 M_1 A_1
13.	i) price relative $=\frac{P_{2021}}{P_{2016}} \times 100$ For sugar $= \frac{3600}{3000} \times 100$ $=120$ The price of sugar increased by 20% For maize flour $=\frac{1700}{2200} \times 100$ $=77.27$ The price of maize flour reduced by 23.27% For beans $=\frac{2500}{2900} \times 100$ $=86.21$ The price of beans decreased by 13.79%	A_1 A_1

	For soap $= \frac{4900}{3500} \times 100$ $=140$	A_1
	The price of soap increased by 40%	A_1
	For rice $= \frac{1950}{2400} \times 100$ $=81.25$	
	The price of rice reduced by 18.75%	A_1
	ii) Simple aggregate price index $= \frac{\sum P_{2021}}{\sum P_{2016}} \times 100$	
	$= \frac{3600+1700+2500+4900+1950}{3000+2200+2900+3500+240} \times 100$	M_1
	$= \frac{14650}{14000} \times 100$	M_1
	$=104.64$	M_1 A_1
	The price of items increased by 4.64%	B_1
	i) Value index $= \frac{\sum P_{2021} \cdot Q_{2021}}{\sum P_{2016} \cdot Q_{2016}} \times 100$	
	$= \frac{3600 \times 26 + 17 \times 70 + 2500 \times 30 + 4900 \times 15 + 1950 \times 80}{3000 \times 20 + 2200 \times 50 + 2900 \times 25 + 3500 \times 10 + 2400 \times 60} \times 100$	M_1M_1
	$= \frac{517100}{421500} \times 100$	M_1
	$=122.68$	A_1
	The prices of the items increased by 22.86%	B_1
14.		
	a) Average speed $= \frac{distance\ covered}{time\ taken}$	M_1
	Average speed $= \frac{45}{6}$	M_1
	$=7.5ms^{-1}$	A_1
	b) Average speed from P to Q $= \frac{50}{t_{PQ}}$	
	$8 = \frac{50}{t_{PQ}}$	
	$t_{PQ} = \frac{50}{8}$	M_1
	$=6.25s$	
	Average speed from P to R $= \frac{95}{6.25+4}$ $= \frac{95}{10.25}$	M_1

	$=9.268ms^{-1}$	A_1
	c) Average speed for the whole journey $=\frac{PR+RQ}{6.25+4+\ RQ}$	
	$\frac{6}{1}=\frac{95+45}{10.25+t_{RQ}}$	M_1
	$6=\frac{140}{1025+tRQ}$	M_1
	$61.6+6t_{RQ}=140$	M_1
	$6t_{RQ}$=78.5	
	t_{RQ} =13.08 seconds	A_1
	d) Average velocity = $\frac{displacement\ PQ}{Total\ time\ taken}$	
	$=\frac{50}{6.25+4+13.08}$	M_1
	$\underline{\underline{=\mathbf{2.14ms^{-1}}}}$	M_1 M_1 A_1