STANDARD HIGH SCHOOL ZZANA

Uganda Advanced Certificate of Education

Pure Mathematics

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in Section* **A** *and* **five** *questions from Section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *working* **must** *be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent non – programmable scientific calculators and Mathematical tables with a list of formulae may be used.

Turn Over

*Attempt **all** questions in this section.*

1. Solve the equations;

   $\sqrt{x+2} - \sqrt{x-3} = \sqrt{x-6}$ and hence verify your answer. (05 marks)

2. Show that $\frac{Sin6\theta - Cos6\theta + 1}{Sin6\theta + Cos6\theta + 1} = tan3\theta$. (05 marks)

3. The sides of an equilateral triangle increases at a rate of $3cms^{-1}$ without the angles changing. Find the rate of increase of area when the side is $10cm$. (05 marks)

4. Determine the value of r if;

   $8C_r = 70$ and $8P_r = 1680$. (05 marks)

5. Prove that; (05 marks)

$$\int_0^{\frac{1}{3}} \frac{6tan^{-1}(3x)}{5 + 45x^2} \, dx = \frac{\pi^2}{80}$$

6. Find the Cartesian equation of a plane passing through the midpoint of $P(-1,0,5)$ and $Q(7,-4,1)$ which is perpendicular to the line $\frac{x-6}{7} = -2 - y = \frac{z-5}{2}$.

   (05 marks)

7. Determine the length of tangent drawn from a point $(5,-3)$ to the circle

   $x^2 + y^2 - 4y = 12$. (05 marks)

8. Solve the differential equation $\frac{dy}{dx} = e^{2x} + 5y$ given that y $= -3 \, at \, x = 0$.

   (05 marks)

SECTION B: (60 marks)

Answer any **five** *questions from this section.* **All** *questions carry* **equal** *marks.*

9. (a) Show that $i - 1$ is a root to the equation
    $Z^4 - 2Z^3 - Z^2 + 2Z + 10 = 0$ and hence find all other roots to the
    equation; . $Z^4 - 2Z^3 - Z^2 + 2Z + 10 = 0$ (06 marks)

    (b) Given that the complex numbers Z and its conjugate $\bar{Z}$ satisfies the
    equations; $|Z + 3i - 2| = 2$, $Arg\ (Z + 2\bar{Z} + 5i) = \frac{\pi}{4}$
    Find the complex number Z.   (06 marks)

10. (a) Solve the equation; $8\ Cos^4\theta - 5Cos\ 2\theta = 2$ for $0^o \leq \theta \leq 360^o$.
                                                                       (07 marks)
    (b) If A, B and C are angles of a triangle, prove that;

    $Sin^2A + Sin^2B + Sin^2C - 2 = 2CosACosBCosC$ (05 marks)

11. Two lines L_1 and L_2 are given by $\frac{x+1}{5} = \frac{y+2}{-1} = Z - 6$ and $\frac{x-6}{2} = y - 3 = \frac{Z}{5}$

    respectively. L_1 meets $y = 0$ at A, while L_2 meets $Z = -5$ at B.
    (a) Find the;

        (i)   coordinates of A and B,

        (ii) Cartesian equation of line AB.
                                                                       (07 marks)

    (b) Determine the angle between the plane $5x + 7y - Z = 11$ and line in (a) (ii)
    above;   (05 marks)

12. (a) Find $\int \frac{dx}{1-sinx}$
                                                                       (06 marks)
    (b) Given that $y = e^{-x}Cos(x + \theta)$ where θ is a constant, show that;

    $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$.   (06 marks)

Turn Over

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13. (a) Find the equation of normal chord to the rectangular hyperbola $xy = 4$ at a point $\left(2t, \frac{2}{t}\right)$. (05 marks)

(b) If the normal chord to the hyperbola above drawn from a point $A(4, 1)$ meets the hyperbola again at point B, find length of the chord AB. (07 marks)

14. (a) Prove that the curve $y = \frac{x - 1}{2x^2 - 5x - 3}$ has no turning points. (04 marks)

(b) Sketch the curve $y = \frac{x - 1}{2x^2 - 5x - 3}$. (08 marks)

15. (a) Expand $\sqrt[3]{\left(\frac{1-2x}{1+x}\right)}$ in ascending powers of x upto x^3 and hence using $x = \frac{1}{7}$ estimate $\sqrt[3]{5}$ correct to three significant figures. (08 marks)

(b) Find the term independent of x in the expansion;
$\left(3x^2 - \frac{1}{2x}\right)^6$. (04 marks)

16. The temperature of a hot liquid drops at a rate which is directly proportional to the difference between its temperature θ and that of the surrounding. At $4:35Pm$, the temperature of a hot liquid was 95^oC and that of the surrounding was 30^oC. At $4:37Pm$ the temperature of the liquid droped to 89^oC. The temperature of the surrounding is constant;

(a) Show that $\theta = 30 + 65e^{-Kt}$ where K is constant and θ is temperature after time t. (05 marks)

(b) Find the;
  (i) temperature of the liquid in the next 4 minutes,
  (ii) time when the temperature of the liquid is 60^oC.
(07 marks)

END