WAKISSHA
MARKING GUIDE
Uganda Advanced Certificate of Education
MATHEMATICS P425/1

1. $2p+2q+2r=0$ 2(i) — M1 for method

$p+2q+2r=2$ (ii) — A_1 for value of p

$p=^-2$

$\Rightarrow 2(^-2)+3r=4$ — M_1 for substitution

$3r=8$

$R=\frac{8}{3}$ — A_1 for value of **r**

$Q=^-p-r$

$=^-(^-2)+\frac{8}{3}$

$=\frac{14}{3}$ — B_1 for value of **q** **Total = 05marks**

2. $\underset{\sim}{AB}=\begin{pmatrix}5\\3\\7\end{pmatrix}-\begin{pmatrix}2\\5\\4\end{pmatrix}=\begin{pmatrix}3\\^-2\\3\end{pmatrix}$ — M_1 for AB

$\underset{\sim}{r}=\begin{pmatrix}2\\5\\7\end{pmatrix}+\lambda\begin{pmatrix}3\\^-2\\3\end{pmatrix}$ — A_1 for equation of line

$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2\\5\\4\end{pmatrix}+\lambda\begin{pmatrix}3\\^-2\\3\end{pmatrix}$ — B_1 for $\underset{\sim}{r}=\begin{pmatrix}x\\y\\z\end{pmatrix}$

$x=2+3\lambda$

$y=5-2\lambda$: $3\lambda=x-2\Rightarrow\lambda=\frac{x-2}{3}$ — M_1 for final λ in all direction

$z=4+3\lambda$

$^-2\lambda=y-5\Rightarrow\lambda=\frac{y-5}{^-2}$

$3\lambda=z-4\Rightarrow\lambda=\frac{z-4}{3}$

$\therefore\frac{x-2}{3}=\frac{y-5}{^-2}=\frac{z-4}{3}$ — A_1 for Cartesian's equation of the line

05marks

3. From $x^2+y^2-4x+6y-7=0$

$(x-2)^{-2}+(y+3)^2-2^2-3^3-7=0$ — M_1 for completing square

$(x-2)^2+(y+3)^2=20$

Centre $C_1(2,^-3)$ and $r^2,=20$

Centre $C_2(x,y)$ — A_1 for Centre and radius of given circle.

$r_2^2 = (x-1)^2 + (y-3)^2$

$\overline{C_1C_2^2} = (x-2)^2 + (y+3)^2$ — B_1 for both r_2^2 and C_1C_1

$r_1^2 + r_2^2 = \overline{C_1C_2^2}$

$20 + (x-1)^2 + (y-3)^2 = (x-2)^2 + (y-3)^2$

$20 + x^2 - 2x + 1 + y^2 - 6y + 9 = x^2 - 4x + 4 + y^2 + 6y + 9$ — M_1 for obthogal Circles

$2x + 30 - 12y + 13 = 0$

$2x - 12y + 17 = 0$ — A_1 locus of C.

05marks

4. $\tan^{-1}(2x+1) + \tan^{-1}(2x-1) = \tan^{-1}(2)$

let $\tan^{-1}(2x+1) = A, \tan^{-1}(2x-1) = B \tan^{-1}(2) = C$ — B_1

$\Rightarrow \tan A = 2x+1, \tan B = 2x-1$

and $\tan C = 2$.

$\tan(A+B) = \tan C$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = \tan C \quad M_1$$

$$\frac{(2x+1)+(2x-1)}{1-(2x+1)(2x-1)} = 2 \quad M_1$$

$$\frac{4x}{1-4x^2+1} = 2;$$

$$\frac{4x}{2-4x^2} = 2 \quad M_1$$

$2x = 2(1-2x^2)$

$2x^2 + 2x - 1 = 0$

$x = {}^{-}1 or \frac{1}{2}$ — **05marks**

Altn

4. Let $A = \tan^{-1}(2x+1) \Rightarrow \tan A = 2x+1$

$B = \tan^{-1}(2) \Rightarrow \tan B = 2$

$C = \tan(2x-1) \Rightarrow \tan C = 2x-1$ — B_1

$A = B - C$

$$\tan A = \frac{\tan B - \tan C}{1 + \tan B \tan C} \quad M_1$$

$$2x+1 = \frac{2-(2x-1)}{1+2(2x-1)} \quad M_1$$

$$2x+1 = \frac{3-2x}{1+4x-2}$$

$(2x+1)(4x-1) = 3-2x$ — M_1

$8x^2 + 2x - 1 = 3 - 2x$

$8x^2 + 4x - 4 = 0$

$2x^2 + x - 1 = 0$

$(2x+1)(x-1) = 0$

$x = \frac{^{-}1}{2}$ 0r 1 — A1

5. Let $u = x^2$ $\quad\quad\quad$ $doc = \frac{du}{2x}$

$$\int \frac{x}{1+\sin x^2}dx = \int \frac{x}{1+\sin u}.\frac{du}{2x} = \frac{1}{2}/\frac{1}{1+\sin u}$$ M1

Let $t = \tan\frac{u}{2} = du = \frac{2dt}{1+t^2}$

$$\frac{1}{2}\int \frac{1+t^2}{1+t^2+2t}.\frac{2dt}{1+t^2} = \int (1+t)^{-2}dt = \frac{^-1}{(1+t)^2} + K$$ M1

$$= \frac{^-1}{\left(1+\tan\frac{u}{2}\right)} + C = \frac{^-1}{\left(1+\tan\frac{x^2}{2}\right)^2} + C$$ A1

$$\int_0^{\sqrt{\frac{\pi}{2}}} \frac{x}{1+\sin x^2} doc = {}^-1\left[\left(\frac{1}{1+\tan\frac{x^2}{2}}\right)\right]_0^{\sqrt{\frac{\pi}{2}}}$$

$$-\left[\frac{1}{1+\tan\frac{\pi}{4}} - \frac{1}{1+\tan 0}\right]$$ M1

$$= 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$ A1 **05**

6.

	Girls	Boys
Choice	4	0
	3	1

$= {}^7C_4$ and ${}^3C_0$ or ${}^7C_3$ and ${}^3C_1$ M1

$= {}^7C_4 \times {}^3C_0 + {}^7C_3 \times {}^3C_1$ M1

$= \frac{7\times6\times5\times4!}{3\times2\times1\times4!} \times \frac{3!}{0!3!} + \frac{7\times6\times5\times4!}{4!\times3!} \times \frac{3\times2!}{2!\times1!}$

$= 35\times1 + 35\times3$

$= 35 + 105$ M1

= 140 A1 **04marks**

7. $f(x) = \frac{1}{2}m(1+2) - \frac{1}{2}m(1+x)$

$f(o) = \frac{1}{2}m1 - \frac{1}{2}m1 = 0$ B1

$f^1(x) = \frac{1x}{2}\frac{2}{1+2x} - \frac{1}{2}(1+x)$

$= \frac{1}{1+2x} - \frac{1}{2}\frac{1}{(1+x)}$

$f^1(o) = 1 - \frac{1}{2} = \frac{1}{2}$ B1

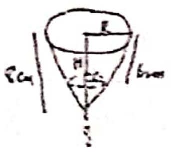
$$f''(x)=\frac{^{-}1\times 2}{(1+2x)^2}+\frac{1}{2}(1+x)^2$$

$$f(0)\frac{^{-}2}{1}+\frac{1}{2}=\frac{^{-}3}{2}$$ B1

$$\ln\sqrt{\frac{(1+2x)}{(1+x)}}=0+\frac{1}{2}x-\frac{3}{2}\frac{x^2}{2!}+\ldots$$ M1

$$=\frac{1}{2}x-\frac{3}{4}x^2$$ A1

8.



$$\frac{H}{R}=\frac{8}{4}\Rightarrow H=2R$$

h =2r B1

t =2r

r =3cm

$$V=\frac{1}{3}\pi r^2 h=\frac{2}{3}\pi r^3 \qquad \frac{dV}{dr}=2\pi r^2$$ M1

$$S=\pi r^2 \qquad \frac{ds}{dr}=2\pi r$$ M1

$$\frac{ds}{dt}=\frac{dV}{dt}\times\frac{dr}{dv}\times\frac{ds}{dr}$$

$$0.05\times\frac{1}{2\pi r^2}\times 2\pi r$$ M1

$$=\frac{0.05}{r}$$

$$\frac{ds}{dt}\Big|r=3=\frac{0.06}{3}=0.02cm^2s^{-1}$$ A1 05marks

9. (a) $$\frac{(x-2)}{(x+1)}-\frac{(x+1)}{(x+3)}\geq 0$$ M1 for making R.H S = o

$$\frac{(x-2)(x+3)-(x+1)(x+1)}{(x+1)(x+3)}\geq 0$$ M1 for bring under same L.C.M

$$\frac{x^2+x-6-x^2-2x-1}{(x+1)(x+3)}\geq 0$$

$$\frac{^{-}x-7}{(x+1)(x+3)}\geq 0$$ A1

For the table M1

For the table including equal sign M1

	$x\angle^{-}7$	$x=^{-}$	$^{-}7\angle x\angle^{-}3$	$x=^{-}3$	$^{-}3<x<^{-}1$	$x=^{-}1$	$x>^{-}1$
$^{-}(x+7)$	+ve	0	-ve	-ve	-ve	-ve	-ve
$x+1$ $x+3$	-ve -ve	-ve -ve	-ve -ve	-ve 0	-ve 0	0 +ve	+ve +ve
$(x+1)$	+ve	+ve	+ve		-ve		+ve

$(x+3)$				0		0	
$\dfrac{^-x-7}{(x+1)(x+3)}$	+ve	0	-ve	$^-\infty$	+ve	$^-\infty$	-ve

Solution set $\{x : x \le 0, ^-3 < x < ^-1\}$ A1 06

(b) Given P= 3,000,000 B1 for R =1.125

N= 6 r=0.125 R=1.125 B1 for n = 6

$$A = \frac{PR(R^n - 1)}{R-1} = 3{,}000{,}000\frac{(1.125)(1.125^6 - 1)}{1.125 - 1} \text{ M1}$$

M1

M1 A1

= Shs 27,736,736.30 06

10.(a) $\underset{\sim}{r} = \begin{pmatrix}3\\1\\2\end{pmatrix} + \lambda\begin{pmatrix}2\\^-1\\1\end{pmatrix}$ M1A1

$2x - y + z = 4$ is eqn of the plane

$x = 3 + 2\lambda, y = 1 - \lambda$ and $Z = 2 + \lambda$

$= 2(3+2\lambda) - (1-\lambda) = 4$ M1

$6+4\lambda-1+\lambda+2+\lambda$

$6\lambda+7=4$

$\lambda = \frac{-1}{2}$ A1

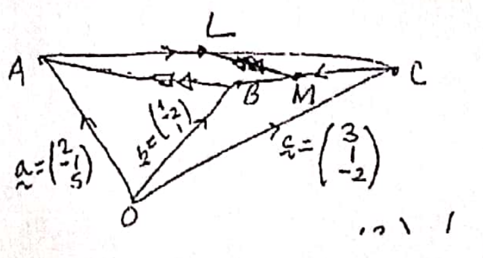
$x = 3 + 2\left(\frac{^-1}{2}\right) = 2$

$y = 1 - \left(\frac{^-1}{2}\right) = \frac{3}{2}$ M1

$z = 2 - \frac{^-1}{2} = \frac{3}{2}$

$\therefore$ they meet at $(2, \frac{3}{2}, \frac{3}{2}$ A1 06

(b)



Scanned with CamScanner

$$BA = OA - OB = \begin{pmatrix}2\\-1\\5\end{pmatrix} - \begin{pmatrix}1\\-2\\1\end{pmatrix} = \begin{pmatrix}1\\1\\4\end{pmatrix} \quad B1$$

$$OL = OA + \frac{1}{2}AC = \begin{pmatrix}2\\-1\\5\end{pmatrix} + \frac{1}{2}\left[\begin{pmatrix}3\\1\\-2\end{pmatrix}\right] - \begin{pmatrix}2\\-1\\5\end{pmatrix} = \frac{2+\frac{1}{2}}{-1+1} = \begin{pmatrix}\frac{5}{2}\\0\\\frac{3}{2}\end{pmatrix} \quad B1$$

$$\frac{1}{2}BC = \begin{pmatrix}1\\-2\\1\end{pmatrix} + \frac{1}{2}\begin{bmatrix}3\\1\\-2\end{bmatrix} - \begin{pmatrix}1\\-2\\1\end{pmatrix} = \begin{pmatrix}1+1\\-2+3/2\\1-\frac{3}{2}\end{pmatrix} = \begin{pmatrix}2\\\frac{-1}{2}\\\frac{-1}{2}\end{pmatrix} \quad B1$$

$$ML = OL - OM = \begin{pmatrix}\frac{5}{2}\\0\\\frac{3}{2}\end{pmatrix} - \begin{pmatrix}2\\\frac{-1}{2}\\-\frac{1}{2}\end{pmatrix} = \begin{pmatrix}\frac{1}{2}\\\frac{1}{2}\\2\end{pmatrix} \quad B1$$

$$2ML = 2\begin{pmatrix}\frac{1}{2}\\\frac{1}{2}\\2\end{pmatrix} = \begin{pmatrix}1\\1\\4\end{pmatrix} \quad B1$$

$\therefore BA = 2ML$ B1

11. (a) $\cos 3\theta + \cos\theta + \cos 2\theta = 0$

$2\cos 2\theta\cos\theta + \cos 2\theta = 0$ M1

$\cos 2\theta\,(2\cos\theta + 1) = 0$ M1

$\cos 2\theta = 0$ or $\cos\theta = \frac{-1}{2}$ $\theta = 120^0$ A1

$2\theta = 90^0, 270^0$ M1

$\theta = 45^0, 135^0$

$\therefore \theta\, 45^0, 120^0, 135^0$ A1 **05**

(b) $\sin 3\theta = \sin(2\theta + \theta)$ M1

$= \sin 2\theta\cos\theta + \sin\theta(1 - 2\sin^2\theta = 2\sin\theta\cos^2\theta + \sin\theta(\cos^2\theta - \sin^2\theta)$ M1

$2\sin\theta(1 - \sin^2\theta + \sin\theta(1 - 2\sin\theta)$ M1

$2\sin\theta - 2\sin 3\theta + \sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$ A1

From $6x-8x^3=1$

$$3x-4x^3=\frac{1}{2}$$

If $x=\sin\theta$ then $\sin 3\theta=\frac{1}{2}$ B1

$3\theta=\sin^{-1}\left(\frac{1}{2}\right)$

$=30^0,150^0,390^0,510^0,750^0$ M1

$\theta=10^0,50^0,130^0,170^0,250^0$

$x=\sin\theta=0.766,-0.940$ A1

07

12. Vertical a symptoles $x-9=0$ or $x-1=0$ A1

$x=9$ and $x=1$ A1

Horizontal asymplote $y=\dfrac{x^2-6x+9}{x^2-10x+9}$

$$\frac{\frac{x^2}{x^2}-\frac{6x}{x^2}+\frac{9}{x^2}}{\frac{x^2}{x^2}-\frac{10x}{x^2}+\frac{9}{x^2}}$$ M1

$$\frac{1-\frac{6}{x}+\frac{9}{x^2}}{1-\frac{10}{x}+\frac{9}{x^2}}$$

As x and y =1 A1

Intercepts S

When $x=0$ $y=\dfrac{(^-3)^2}{(^-9)(^-1)}=1$

When $y=0$ $0=(x-3)\Rightarrow x=3$

Hence intercepts are (3,0) and (0,1) B1

Determine turning points

$$\frac{dy}{dx}=\frac{2(x-3)(x-9)(x-1)-(x-3)^2(2x-10)}{[(x-9)(x-1)]^2}$$ M1

For turning pt $\frac{dy}{dx}=0$

$(x-3)[x^2-10x+9-(2x^2-16x+30)]$

$(x-3)[x^2-10x+9-2x^2 16x-30]$

$(x-3)[^-x^2+6x^-21]$

$(x-3)[x^2-6x+21]$

$x=3$

$y(3)=\dfrac{(3,3)^0}{(^-6)(2)}=0$ M1

Turning pt is $(3,0)$

	L	3	R
$\frac{dy}{dx}$	+ve	o	-ve

Max M1

But $\frac{dy}{dx}\frac{(x-3)(^-x^2+6x-21)}{(x^2-10x+9)^2}$

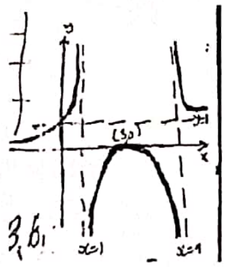
Hence (3,0) is max turning pt $x=2$ $^-1(^-4+12-21)$ A1

$x=4$ $1(^-16+24-21)$

Investigation

	$x\angle 1$	$1\angle x\angle 3$	$3\angle x\angle 9$	$x>9$
$(x-3)^2$	+ve	+ve	+ve	+ve
$(x-9)$	-ve	-ve	-ve	+ve
$(x-1)$	-ve	+ve	+ve	+ve
$(x-9)(x-11)$	+ve	-ve	-ve	+ve
y	+ve	-ve	-ve	+ve

B₁ B₁ **12**



13. $$\frac{x^3+4x^2-5x-4}{(x-2)^2(1+x^2)}=\frac{A}{(x-2)}+\frac{B}{(x-2)^2}+\frac{Cx+D}{1+x^2}$$

$$\Rightarrow x^3+4x^2-5x-4=A(x-2)(x-1+x^2)+(Cx+D)(x-2)^2$$

When $x=0$

$^-4=^-2A+B+4D$

$^-6=^-2A+4D$

$A-2D=3$

Coff x3

1= A+C

When $x=1$

1+4-5-4 =A(⁻1)(2)+B(2)+(C+D)(⁻1)²

⁻4 = ⁻2A +2B+C+D

8 = 2A-C-D

vv + vvv 2A-C-D =8

A+ C 1

3A - D =9

2(vvvv) -V 6A-2D =18

A - 2D = 3

5A =15

A =3 B1

C = ⁻2 B1

D = 0 B1

$$\frac{x^3+4x^2-5x+4}{(x-2)^2(1+x^2)} = \frac{3}{x-2} + \frac{2}{(x-2)^2} + \frac{-2}{1+x^2}$$ A1

$$\int_3^5 f(x)dx = 3m(x-2)\Big|_3^5 + \left[\frac{-2}{x-2}\right]_3^5 - 2\left[\tan^{-1}(x)\right]_3^5$$ M1 A1

$$3[(3) - m1] - 2\left(\frac{1}{3} - \frac{1}{1}\right) - 2\left[\tan^{-1}(5) - \tan^{-1}(3)\right]$$ M1

$$= 3m3 - 2\left(\frac{-2}{3}\right) - 2(0.035081)$$

3.295837-1.333333- 0.070162

=1.8923 A1

12marks

14. (a) $\frac{dy}{dx} + \frac{2}{x}y = x$

$$P = e^{2\int\frac{1}{x}dx} = e^{2/n x^2} = e^{m x^2} = x^2$$ B1

$$\int d(x^2y) = \int (x^2)x\,dx$$ M1

$$x^2y = \frac{x^4}{4} + C$$ A1

$$y = \frac{x^2}{4} + \frac{C}{x^2}$$

$$\Rightarrow 1 = \frac{1}{4} + C \Rightarrow C = \frac{3}{4}$$ M1

$$y = \frac{x^2}{4} + \frac{3}{4x^2}$$ A1

$$4x^2y = x^2 + 3$$

05 marks

14. (b) $\int_{79}^{63}\frac{d\theta}{\theta} = -\int_0^{10} k\,dt$ M1

$$m\theta\Big|_{79}^{63} = -k[t]_0^{10}$$ A1

$$m\left(\frac{79}{63}\right) = 10K$$ M1

$$K = \frac{1}{10}m\frac{79}{63}$$ A1

$$\int_{79}^{49}\frac{d\theta}{\theta} = -\int_0^t \frac{1}{10}m\left(\frac{79}{63}\right)dt$$

$$m\frac{79}{49} = \frac{1}{10}m\left(\frac{79}{63}\right)t$$ M1

$$t = \frac{10m\left(\frac{79}{49}\right)}{m\left(\frac{79}{63}\right)}$$ M1

$= \mathbf{21.11\ minutes}$ A1

07 marks

Altn

14. (b) $\frac{d\theta}{dt} = K\theta$ B1

$\int \frac{d\theta}{\theta} = \int {}^{-}kdt$ M1

$m\theta = {}^{-}kt + C$ A1

T =100, t=0 θ=100-21 =79

$m79 = C$ B1

T =84 t =10 θ =84-21 =63

$m63 = {}^{-}10k + m79$

$10k = m\left(\frac{79}{63}\right)$

$k = \frac{1}{10}m\left(\frac{79}{63}\right)$ B1

$\Rightarrow m\theta = {}^{-}\frac{1}{10}m\left(\frac{79}{63}\right)t + m79$

T =70 $\theta = 70 - 21 = 49$

$m49 = \frac{{}^{-}1}{10}m\left(\frac{79}{63}\right)t + m79$ M1

$\frac{1}{10}m\left(\frac{79}{63}\right)t = m\frac{79}{49}$

$t = \frac{10m\left(\frac{79}{49}\right)}{m\left(\frac{79}{63}\right)}$

= **21.11 minutes** **A1**

15. (a) If z =I is a root, then z =1 +I should be a root

Sum of roots = 2

Product $(1+i)(1-i) = 1 - I^2 = 2$ M_1

Z^2-2Z+2 should be a factor. A_1

$$
\begin{array}{r}
z^2 - 2z - 3 \\
z^2 2z + 2\overline{\smash{)}z^4 - 4z^3 + 3z^2 + 2z - 6} \\
\underline{z^4 - 2z^3 + 2z^2} \\
{}^{-}2z^3 + z^2 + 2z - 6 \\
\underline{{}^{-}2z^3 + 4z^2 - 4z} \\
{}^{-}3z^2 + 6z - 6 \\
{}^{-}3z^2 + 6z - 6 \\
0 \quad 0 \quad 0
\end{array}
$$

M_1

A_1

Since the remainder is zero, hence 1-i a root

but $z^4 - 4z^3 + 3z^2 + 2z - 6 = (z^2 - 2z + 2)(z^2 - 2z - 3) = 0$

$z^2 - 2z + 2 = 0$

$Z = 1 + i$ or $z^2 - 2z - 3 = 0$

$(Z-3)(Z+1) = 0$ M1

$Z = 3$ or $^-1$

Other roots are $1 + i$, 3 and $^-1$ A1

06

15. (b) $W = \left(1 + i\sqrt{3}\right)^2$

$|W| = \left(1 + i\sqrt{3}\right)^2 = \left(\sqrt{1+3}\right)^2 = 4$

$\text{Arg } W = 2\tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = 120^0$ M1

$W = 4\left(\cos 120^0 + i\sin 120^0\right)$ A1

$W = 4\left[Cos\left(120 + 360k\right)^0 + i\sin\left(120 + 360k\right)^0\right] k = 0, 1, 2$

$Z = W^{1/3} = 4^{1/3}\left[Cos\left(\frac{120 + 360k}{3}\right)^0 + i\sin\left(\frac{120 + 360k}{3}\right)^0\right] k = 0, 1$ B1

When $k = 0$

$Z = 4^{1/3}\left(\cos 40^0 + i\sin 40^0\right)$

$= 1.587401\left(0.766044 + 0.642787i\right)$

$1.2160 + 1.0203i$ B1

When $k = 1$

$Z = 1.587401\left(Cos 160^0 + i\sin 160^0\right)$

$= {}^-1.4917 + 0.5429i$ B1

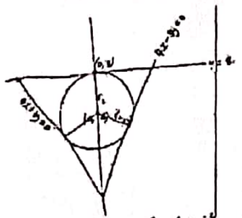
When $k = 2$

$Z = 1.587401\left(Cos 280^0 + i\sin 280^0\right)$

$0.2756 - 1.5634i$ B1

06marks

16. (a)



$r_1 = (8 - c) \Rightarrow r_1^2 = (8 - c)^2$ B1

$r_2 = \frac{4(0) - 3(c)}{\sqrt{4^2 + (^-3)^2}} = \frac{^+3c}{\sqrt{25}} = \frac{3c}{5}$ B1

$r_1^2 = r^2$

$(8 - c)^2 = \left(\frac{3c}{5}\right)^2$ B1

$64 - 16c^2 + c^2 = \frac{9c^2}{25}$

$1600 - 400c + 25c^2 - 9c^2 = 0$

$16c^2 - 400c^2 + 1600 = 0$

$4c^2 - 100c + 400 = 0$

$C^2 - 25c + 100 = 0$

when $C = 20$

$r^2 = 144$

when $C = 20$

$r^2 = 144$

$C = \frac{25 \pm \sqrt{25^2 - 4 \times 100}}{2}$ M1

$\frac{15+25}{2} or \frac{25-15}{5}$

C= 20 or 5 A1

A1

C = 5 $r^2 = 9$ B1

$x^2+(y-20) = 144$

$x^2+y -40y+400-144=0$

$X^2+y^2-40y+256=0$

$X^2+(y-5)^2 = 9$

$X^2+y^2-10y+16=0$

16 (a). $a^2(mx+c)^2 + b^2x^2 - a^2b^2 = 0$

$a^2m^2x^2 + 2mca^2 + c^2a^2 + b^2x^2 - a^2b^2 = 0$

$(a^2m^2 + b^2)x^2 + (2mca^2)x + (c^2a^2 - a^2b^2) = 0$

For tangency

$(2mca^2)^2 = 4(a^2m^2 + b^2)(a^2c^2 - a^2b^2)$ B1

$4m^2c^2a^4 = 4(a^4m^2c^2 - a^4m^2b^2 + a^2c^2b^2 - a^2b^4)$

$m^2c^2a^4 = a^4m^2c^2 - a^4m^2b^2 + a^2c^2b^2 - a^2b^4$

$a^4m^2b^2 + a^2b^4 = a^2b^2c^2$ dividing through by a^2b^2

$a^4m^2 + b^2 = c^2$ A1

In an ellipse $4x^2 + 14y^2 = 56$ $a^2 = 14$ and $b^2 = 4$

$\Rightarrow 14m^2 + 4 = C^2$ _______________(i) M1 for (i)

In an ellipse $3x^2 + 23y^2c69$ $a^2 = 23$ and $b^2 = 3$

$\Rightarrow 23m^2 + 3 = C^2$ _______________(ii) M1 for (ii)

(i)-(ii)

$^-9m^2 + 1 = 0$ $m = \pm\frac{1}{3}$ M1

$\Rightarrow 14\left(\frac{1}{9}\right) + 4 = c^2$

$\frac{14}{9} + 4 = c^2$

$\frac{50}{9} = c^2 \Rightarrow C = \pm\frac{\sqrt{50}}{3}$

$y = \pm\frac{1}{3}x$ $\pm\frac{1}{3}\sqrt{50}$ A1

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END