P425/1
PURE MATHEMATICS
Paper 1
July/August 2018
3 hours

WAKISSHA JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- *Answer* **all** *the eight questions in section* **A** *and any* **five** *questions from section* **B.**
- *Any additional question(s) answered will not be marked.*
- *Show* **all** *necessary working clearly.*
- *Begin each answer on a fresh page of paper.*
- *Silent, non programmable scientific calculators and mathematical tables with a list of formulae may be used.*

Turn Over

SECTION A (40 MARKS)

Answer **all** *questions in this section*

1. Use raw echelon reduction to solve the following equations simultaneously

$3x = z - 2y$

$3y = x + 2z + 1$

$3z = 2x - 2y + 3$ (05marks)

2. Find the solution set for the inequality.

$\left|\frac{x^2 - 4}{x}\right| \le 3$ (05marks)

3. Find the coordinates of the points of intersection of the lines.

$\frac{x+5}{2} = \frac{y+4}{2} = \frac{z+9}{4}$ and $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-2}{1}$ (05marks)

4. Express $2\cos x - 5\sin^2 x$ in the form $a(\cos x + b)^2 + c$, hence find the minimum valve of the expression. (05marks)

5. Find the equation of the tangent to the circle $(x-2)^2 + (y-3)^2 = 16$, at the point $[(2+4\cos\theta),(3+4\sin\theta)]$. (05marks)

6. Evaluate $\int_2^3 \frac{dx}{x^2 - 4x + 13}$. (05marks)

7. , Differentiate $y = x\text{In}x$ from first principles. (05marks)

8. How many teams of 6 players can be formed from a group of 7 boys and 5girls if
   (i) Each team should have at least 3 boys and a girl?
   (ii) Each team contains almost 3 girls. (05marks)

SECTION B (60 MARKS)

Answer any ***five*** *questions from this section.*

9. (a) Given that $y = (\sec x + \tan x)^2$, show that $\cos x \frac{d^2y}{dx^2} - 2\,{}^{dy}\!/_{dx} = 2y\tan x$.

(05marks)

(b) Prove that in triangle ABC, $\frac{b-c}{b+c} = \frac{\sin B - \sin C}{\sin B + \sin C}$, deduce that $\tan\frac{1}{2}(B-C) = \frac{b-c}{b+c}\tan\frac{1}{2}(B+C)$ and hence find B and C given that A = 40^0, b = 4.4cm and C = 2cm.

(07marks)

10. If the curve $y=\frac{x^2-4x+4}{x+1}$,

(i) Show that the curve is restricted, state the region and hence investigate the nature of the turning points.

(ii) Determine the equations of asymptotes to the curve.

(iii) Sketch the curve. (12marks)

11. (a) Find the equation of the plane passing through the origin and parallel to the lines.

$\frac{x+2}{3}=\frac{y-1}{4}=\frac{z+1}{5}$ and $\frac{x-3}{4}=\frac{y-2}{^-5}=\frac{z+1}{1}$ (06 marks)

(b) Show that the line $\frac{x-2}{2}=\frac{y-2}{^-1}=\frac{z-3}{3}$ and plane $\underset{\sim}{r}.\begin{pmatrix}4\\ ^-1\\ ^-3\end{pmatrix}=4$ are parallel

and find the perpendicular distance of the line from the plane. (06 marks)

12. (a) Prove by induction that $4^{n+3}-3n-10$ is divisible by 3 for all positive integral values of n. (06 marks)

(b) Use Demoivres theorem to prove that the complex number $\left(\sqrt{3}+i\right)^n+\left(\sqrt{3}-i\right)^n$ is always real and hence find the value of the expression when n = 6. (06 marks)

13. (a) Prove that $\log(x+y)=\log x+\frac{1}{2}\log\left[1+\frac{2y}{x}+\left(\frac{y}{x}\right)^2\right]$ (06 marks)

(b) Nansubuga opened up a bank account with Shs 50, 000. She deposits the same amount every year and makes no withdrawals. After how many years will she accumulate more than one million shillings on her account if the bank offers a 5% compound interest per annum? (06 marks)

14. (a) $y = me^{3x} + ne^{-3x}$ is a particular solution to differential equation, where m and n are constants. Find the differential equation and state its order. (05marks)

(b) A spherical bubble evaporates at a rate proportional to its surface area. If half of it evaporates in 2 hours, when will the bubble disappear? (07marks)

15. Evaluate $\int_3^4\frac{2x+1}{(x-2)(x+1)^2}dx$. Give your answer corrects to 3 decimal places.

(12marks)

16. (a) Given that $x=\theta-\sin\theta, y=1-\cos\theta$, show that $y^2\frac{d^2y}{dx^2}+1=0$ (06 marks)

(b) Using Maclaurin's theorem, determine the first two non-zero terms of the series for f(x) = $\tan^{-1}(2x)$ and hence evaluate $\int_0^1\tan^{-1}(2x)dx$. (06 marks)

END

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Turn Over

SECTION A (40 MARKS)

Answer **all** *questions in this section*

1. Show that the parametric equations $x = 5 + \frac{\sqrt{3}}{2}\cos\theta$, $y = -3 + \frac{\sqrt{3}}{2}\sin\theta$ represent a circle find the radius and centre of the circle. (05 marks)

2. Prove by induction that $3^{2n+2} - 8n - 9$ is divisible by 64 for all positive intregral values of n. (05 marks)

3. Solve $6\cos 2x + 7 = 7\sin 2x$ for $0^0 \leq x \leq 360^0$ (05 marks)

4. Solve the inequality $|x + 3a| < 2|x - 2a|$ (05 marks)

5. Show that $\int_{\sqrt{3}}^{\infty} \frac{dx}{x\sqrt{1+x^2}}\, dx = \frac{1}{2} In3$ (05 marks)

6. (i) Given that the vectors $\underset{\sim}{\hat{i}} - \lambda\underset{\sim}{\hat{j}} + 4\underset{\sim}{\hat{k}}$ and $2\underset{\sim}{\hat{i}} + 4\underset{\sim}{\hat{j}} + 6\underset{\sim}{\hat{k}}$ are perpendicular, determine the value of λ.

   (ii) The position vectors of points A and B are given by $\underset{\sim}{a} = \underset{\sim}{\hat{i}} - 2\underset{\sim}{\hat{j}} + 4\underset{\sim}{\hat{k}}$ and $\underset{\sim}{b} = 3\underset{\sim}{\hat{i}} - 4\underset{\sim}{\hat{j}} + 6\underset{\sim}{\hat{k}}$.

   Determine the position vector of a point P which divides the line segment AB externally in the ratio 2:3.

7. Mr. Pondo's age and his three children are in a geometrical progression, the sum of their ages is 140 years, the sum of the ages of the last two children is 14 years, find Mr. Pondo's age. (05 marks)

8. Solve the differential equation.

   $y\text{Cos}^2 x \frac{dy}{dx} = \tan x + 2$, given that y = O when $x = \frac{\pi}{4}$ (05 marks)

SECTION B (60 MARKS)

Answer any **five** questions from this section. **All** questions carry equal marks.

9. (a) Show that the lines $\underset{\sim}{r} = 5\underset{\sim}{\hat{i}} + 3\underset{\sim}{\hat{j}} - 5\underset{\sim}{k} + \mu(\underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{k})$ and $\frac{x-7}{3} = \frac{y+1}{-2} = \frac{Z+4}{-2}$ intersect and find the coordinates of the point of intersection. (08 marks)

(b) A plane is at a distance of $\sqrt{11}$ units from the origin. If the line passing through the points A(4,-9,3) and B(6, -7,9) is perpendicular to the plane, find the cartesian equation of the plane. (04 marks)

10. (a) Shade in separate argand diagrams the regions represented by the inequalities given below.

(i) $|z - i| < 3$ (03 marks)

(ii) $\frac{1}{3}\pi \leq arg(z-2) \leq \pi$ (03 marks)

(b) (a) Given that $z = x + iy$, show that when $\text{Re}\left(\frac{z+i}{z+2}\right) = 0$, the point (x, y) lies on a circle and hence find the centre and radius of the circle.

(06 marks)

11. (a) Show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

Hence solve the equation $8x^3 - 6x - 1 = 0$ (07 marks)

(b) By expressing $5\cos x + 8\sin x$ in the form $R\cos(x + \beta)$, where R is a constant and β is an acute angle, solve $5\cos x + 8\sin x = 7$ for $0^0 \leq x \leq 360^0$. (05 marks)

12. (a) Find the focus, directrix and length of the latus rectum for the parabola $y^2 = 4x - 8$. (04 marks)

(b) The tangents at points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ to the parabola $y^2 = 4ax$ meet at a point R. Find the coordinates of R.
If R lies on the line $2x + a = 0$, find the equation of the locus of the mid-point of PQ. (08 marks)

13. (a) Solve the equation $\sqrt{(x-3)} + \sqrt{(2x+1)} = \sqrt{(3x+4)}$

(b) By expressing the equations $x + 2y + z = 8,\ 2x - y + 3z = 9$ and $3x + 4y - z = 8$ in raw echelon form, solve the equations. (07 marks)

14. (a) Differentiate $\frac{2x^2}{x^2+1}$ from the first principles with respect to x. (05 marks)

(b) Evaluate $\int_{-3}^{2} \frac{dx}{x^2 + 6x + 34}$ (07 marks)

Turn Over

15. (a) Solve the differential equation.

$x\frac{dy}{dx} = y + \tan\left(\frac{y}{x}\right)$, using the substitution of y = ux (05 marks)

(b) It has been discovered that number of accidents decreases with the number of traffic police men deployed along Entebbe road and is directly proportional to the number of police men deployed on the road. If the number of accidents decreased from 7 to 1 when the number of police men increased from 2 to 4 on the road.

Find the number of police men that should be deployed on the road in order to stump out accident along the road completely. (07 marks)

16. (a) Expand $(2 - x)^{10}$ up to the term in x^3. Hence use the expression to find $(1.98)^{10}$ correct to 3 decimal places. (05 marks)

(b) Find the first three non-zero of the Maclaurin's series expansion for $f(x) = \tan^{-1}(x)$ and hence evaluate $\int_0^1 tan^{-1}(x)dx$. (07 marks)

END

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Turn Over

SECTION A (40 MARKS)

Answer **all** *questions in this section*

1. Solve the inequality $(0.7)^{-3t} < 4.2$ giving your answer correct to four significant figures. (05marks)

2. Evaluate $\int_0^1 \frac{3x-1}{x^2+3x+2}\,dx$ giving your answer to four decimal places. (05marks)

3. Solve the equation $4\cos x - 2\cos 2x = 1$ for $0 \le x \le \pi$ (05marks)

4. Differentiate $3x^2 + \cos 2x$ from first principles. (05marks)

5. Sketch the parabola $y^2 + 8y - 4x + 12 = 0$ showing clearly the focus and directrix. (05marks)

6. (i) Given the vectors $\mathbf{i} - \lambda\mathbf{j} + 4\mathbf{k}$ and $2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$. are perpendicular, determine the value of λ. (02marks)

   (ii) The position vectors of points A and B are given by $\mathbf{a} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$.
   Determine the position vector of a point P which divides the line segment AB externally in the ratio 2:3. (03marks)

7. Find the Cartesian equation of the curve whose polar equation is $r^2 = a(\sec 2\theta + 1)$. (05marks)

8. Use the substitution in $y = vx$ where v is a function of x to obtain the general solution of the differential equation $xy\frac{dy}{dx} = 2x^2 + y^2$. (05marks)

SECTION B (60MARKS)

Answer **five** *questions from this section.*

9. (a) Shade in separate Argand diagrams the unwanted regions represented by the inequalities given below.

   (i) $|z - i| < 3$ (03marks)

   (ii) $\frac{\pi}{3} \le \arg(Z - 2) \le \pi$ (03marks)

   (b) Given that $z = x + iy$, show that when $\left(\frac{z+i}{z+2}\right)$ is purely imaginary, z describes locus of circle. Find the center and radius of the circle. (06marks)

10. (a) Given that $\cos\beta = \frac{2ab}{a^2+b^2}$, Find the values of $\tan\beta$ and $\sin\beta$. (04marks)

    (b) In a triangle ABC, prove that $\tan B \cot C = \frac{a^2+b^2-c^2}{a^2-b^2+c^2}$ (04marks)

    (c) Find all the sides of a triangle ABC whose area is 1008m^2 and $a = 65\text{cm}$, $b + c = 97\text{cm}$. (04marks)

11. (a) If $a = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{b} = 2i + \mathbf{j} - \mathbf{k}$. Given that $\mathbf{c} = \lambda\mathbf{a} + \mu\mathbf{b}$ and that c and **a** are perpendicular. Find the ratio of λ to μ. (04marks)

(b) The lines $\underset{\sim}{r}_1$ and $\underset{\sim}{r}_2$ have equations $\underset{\sim}{r}_1 = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\underset{\sim}{r}_2 = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ respectively.

(i) Prove that $\underset{\sim}{r}_1$ and $\underset{\sim}{r}_2$ intersect and hence find the point of intersection. (05marks)

(ii) Determine the Cartesian equation of a plane containing $\underset{\sim}{r}_1$ and $\underset{\sim}{r}_2$. (03marks)

12. (a) Show that $\int \frac{1+cosx}{1+sinx}\, dx = \tan x - \frac{2}{1-\tan^2\frac{x}{2}} + In\,(1+\sin x) + c.$ (06marks)

(b) Evaluate $\int_0^{\pi/4} x\cos 2x, dx.$ (03marks)

(c) Find $\int_o^{\sqrt{a}} \sqrt{a-x^2}\; dx.$ (03marks)

13. (a) Find the length of the Latus rectum for a standard parabola $y^2 = 4ax$, and hence find the length of the Latus rectum of $y^2 - 4y - 20 = 8x$ (05marks)

(b) Show that the tangents drawn from the end points of a focal chord joining the points $T_1(at_1^2, 2at_1)$ and $T_2(at_2^2, 2at_2)$ intersect at 90^0 at the directrix. (07marks)

14. (a) The sum to infinity of the terms of a G.P with common ratio r is S. The sum to infinity of cubes of the terms is $3S^3$. Determine the value of r. Hence find S given that sum of the first four terms is $\frac{45}{8}$. (06marks)

(b) A city tycoon invested £100,000 in a business at a compound interest of 8% per annum.
Determine,
(i) the amount after ten years
(ii) the time, corrected to the nearest year it takes to reach more than £300,000.
(06marks)

15. A curve is given by $y = \frac{2(x-2)\ (x+2)}{2x-5}$

(i) Determine the turning points on the curve and hence find the range of values of y for which the curve is undefined.

(ii) Determine the asymptotes to the curve.

(iii) Sketch the curve.

(12marks)

16. a) Solve the differential equation,

$\frac{dy}{dx} + y = e^{-x} cos \frac{1}{2} x$ given that y = -1 when x = 0. (05marks)

b) The rate of growth of a disease causing virus increases at a rate proportional to the number of virus present in the body.

If the number increases from 1000 to 2000 in 1 hour.

(i) How many virus will be present after 1½ hours?

(ii) How long will it take the number of virus in the body to be 4000?

(07marks)

END

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Turn Over

SECTION A (40 MARKS)

Answer all questions in this section

1. Solve the equation $\frac{16^x-4^x}{4^x+2^x} = 5(2^x) - 8$. *(05 marks)*

2. Given that $sin2x = \sqrt{3}cos^2x, solve\ forx\ for - 180 \leq x \leq 180°$. (05 marks)

3. Evaluate $\int_0^1 \frac{3x-1}{x^2+3x+2}\,dx$ giving your answer to three significant figures. *(05 marks)*

4. Prove that the tangent to the parabola $y^2 = 4ax$ at the point $P(ap^2, 2ap)$ is $y = \frac{x}{p} + ap$. *(05 marks)*

5. Find the equation of a plane parallel to the vectors $\mathbf{r}_1 = \mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ and $\mathbf{r}_2 = 4\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ which also passes through a point with position vector $(2\mathbf{i} + 3\mathbf{k})$. *(05 marks)*

6. Given that $p = \log_2 3\ and\ q = \log_4 5, show\ that\ \log_{45} 2 = \frac{1}{2(p+q)}$. *(05 marks)*

7. Show that $\frac{d}{dx}(tan^{-1}x^x) = \frac{(1+1nx)x^x}{1+x^{2x}}$. *(05 marks)*

8. Given that $y = Ae^{3x} + Be^{-2x}$, show that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$ *(05 marks)*

SECTION B (60 Marks)

Answer any **five** *questions from this section*

All *questions carry equal marks.*

9. (a) Find the first four terms of the expansion of $(1-8x)^{\frac{1}{2}}$ in ascending powers of x. Hence by putting $x = \frac{1}{100}$, obtain the value of $\sqrt{23}$ correct to five significant figures. *(06 marks)*

   (b) Given that $S_\infty = 10$ and $U_1 = 7.5$ where U_1 is the first term of the G.P and S_∞ is the sum to infinity where $|r| < 1$. Find,
   (i) the common ratio r
   (ii) Deduce ratio of U_2 to U_5 *(06 marks)*

10. (a) Show that $\tan^{-1}x + \cot^{-1}x = {}^{\pi}/_{2}$ *(05 marks)*

   (b) If $\sin3\theta = p\ and\ sin^2\theta = \frac{3}{4} - q$. Prove that $p^2 + 16q^3 = 12q^2$. *(07 marks)*

11. (a) Find the acute angle between the vectors $a = 2i + k$ and $b = 3i + 4j + 5k$. *(04 marks)*

(b) In a triangle OAB, OA = a, OB = b. A point L is on the side AB and M on the side OB. OL and AM meet at S. $\overline{AS} = \overline{SM}$ and OS = $\frac{3}{4}$ OL,

Given that OM = XOB and AL = yAB, express the vectors.

(i) AM and OS in terms of a, b and x

(ii) OL and OS in terms of a, b and y

Hence find the value of x and y *(08 marks)*

12. (a) If $y = \sqrt{(1 + sinx)}$, show that $\frac{dy}{dx} = \frac{1}{2}\sqrt{(1 - sinx)}$ *(05 marks)*

(b) Integrate $x\tan^{-1} x$ with respect to x. *(07 marks)*

13. Given the curve $y = \frac{4x-10}{x^2-4}$.

(i) Find the range of values of y within which the curve does not lie,

(ii) Determine the stationary points of the curve.

(iii) State the equations of asymptotes and sketch the curve. *(12 marks)*

14. (a) Given parabola whose equation is $y^2 - 4x = 4(y-2)$.
Find the vertex, focus and directrix of the parabola.

(b) If the line $cy + x + d = 0$ is a tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then show that $d^2 = a^2 + b^2c^2$.
Hence determine the equations of the four common tangents to the ellipse $4x^2 + 14y^2 = 56$ and $3x^2 + 23y^2 = 69$. *(07 marks)*

15. (a) Use the substitution $y = x + \frac{1}{x}$ to solve the equation
$2x^4 - 9x^3 + 14x^2 - 9x + 2 = 0$ *(07 marks)*

(b) The expression $Py^2 + qy + r$ is divisible by y - 1, has remainder 2 when divided by y + 1 and has remainder 8 when divided by y – 2.
Find the values of p, q and r. *(05 marks)*

16. (a) Find the general solution of the equation $2x\frac{dy}{dx} - (2y + 1)(x + 1) = 0$ *(04 marks)*

(b) A man starts to climb a mountain whose height is 1000m above its foot. He notices that the rate at which the temperature drops with height is directly proportional to the height. The temperature is 16^0C at the foot and drops to - 9^0C at the top of the mountain. Find the height at which the temperature reaches the freezing point of water (^{0}C). *(08 marks)*

END

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INSTRUCTIONS:

- *Attempt all questions in section A, and five from section B.*
- *All working must be shown clearly.*
- *Mathematical tables with list of formulae and squared paper are provided.*

SECTION A (40 MARKS)

1. Solve the equation: $\log_7 49x + \log_{49} 7x = 7$ (5 marks)

2. Given that $y = x + a$ is a tangent to the curve $y = ax^2 + bx + c$ at the point (2, 4). Find the values of the constants a, b and c. (5 marks)

3. Prove the identity: $2\cot {}^{A}/_{2} - \sin A = \cot^2 A/_2 \sin A$.

(5 marks)

4. Prove that : $\dfrac{d}{dx}\left[\log_2 x + \log_x 2\right] = \dfrac{(In2x)(In\frac{x}{2})}{(In2^x)(Inx)^2}$

(5 marks)

5. Find the principal arguments of the number.

$(i - 1)(-8)$ (5 marks)

6. Show that $\displaystyle\int_{\sqrt{3}}^{\infty} \frac{dx}{x\sqrt{1+x^2}} = \frac{1}{2} In3$

(5 marks)

7. Find the eccentricity, the coordinates of the focii and the equation of the directrices of the ellipse.

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$ (5 marks)

8. A, P, Q, B are points in a straight line such that AP: PB = 1:4 , AQ: QB = 3: 1

Find (i) AP: PQ: QB

(ii) AQ: QP. (5 marks)

SECTION B (60 MARKS)

9. A polynomial $P(x)$ leaves a remainder 6 when divided by $x^2 - 4$, and a remainder $x + a$ when divided by $x^2 - x - 2$

Find (a) The value of the constant a,

(b) The remainder when $P(x)$ is divided by

(i) $x^2 + 3x + 2$

(ii) $x^3 + x^2 - 4x - 4$. (12 marks)

10. Given the curve $y = \dfrac{2x^2 - 8}{2x - 5}$

(i) Find the equations of the asymptotes.

(ii) Find the stationary points, hence or otherwise obtain the range of values of y within which the curve does not lie.

(iii) Sketch the curve. (12marks)

11. (a) Solve the equation:

$$\sqrt{x - 3} + \sqrt{2x + 1} = \sqrt{3x + 4}$$

(6 marks)

(b) Given that $y = \ln(2 - e^x)$, show that $\dfrac{d^2y}{dx^2} + \left[\dfrac{dy}{dx}\right]^2 = \dfrac{dy}{dx}$,

hence find the first three non – vanishing terms of the Maclaurin's expansion of $y(x)$.

(6 marks)

12. (i) Find the coordinates of the point of intersection of the line

$\dfrac{x-2}{3} = \dfrac{3-y}{2} = z + 1$ and the plane $2x + y + 3z = 11$

(ii) Determine the angle between the line and the plane in (i) above. (12 marks)

12. A boy starts to sip a 900ml soda from a bottle at a rate of $10cm^3$ per minute. Given that his rate of consumption is inversely proportional to the square root of the volume of soda remaining at any time, find the time he takes to empty the bottle.

(12 marks)

13. (a) Solve the equation: $\sin^3 x = \sin 2x$ for $0^\circ \leq x \leq 360^\circ$ (6 marks)

(b) Given that A,B,C are angles of a triangle, show that

$\sin(A - B) + \cos(A - B)\tan C = \sin 2B \sec C$. (6 marks)

14. (a) Use small changes to estimate

$$\sqrt{0.041}$$

(6 marks)

(b) Find the equation of the normal to the curve $y^2(y - 3x) = 3 - x^2$ at the point (-1, 2)

(6 marks)

16 (a) Evaluate

$$\int_9^{25} \frac{dx}{\sqrt{x} - \sqrt{x-9}}$$

(5 marks)

(b) The area bounded by the curve $y = \ln x$, the x – axis and the line $x = e$ is rotated about the x – axis through 360^0. Show that the volume generated is equal to $e - 2$. (7 marks)

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SECTION A (40 MARKS)

1. Solve the equation: $\log_7 49x + \log_{49} 7x = 7$ (5 marks)

2. Given that $y = x + a$ is a tangent to the curve $y = ax^2 + bx + c$ at the point (2, 4). Find the values of the constants a, b and c. (5 marks)

3. Prove the identity: $2\cot {}^{A}/_{2} - \sin A = \cot^2 A/_2 \sin A$.

(5 marks)

4. Prove that : $\frac{d}{dx}[\log_2 x + \log_x 2] = \frac{(In2x)(In\frac{x}{2})}{(In2^x)(Inx)^2}$

(5 marks)

5. Find the principal arguments of the number.

$(i - 1)(-8)^{\frac{1}{3}}$ (5 marks)

6. Show that $\int_{\sqrt{3}}^{\infty} \frac{dx}{x\sqrt{1+x^2}} = \frac{1}{2} In3$

(5 marks)

7. Find the eccentricity, the coordinates of the focii and the equation of the directrices of the ellipse.

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$ (5 marks)

8. A, P, Q, B are points in a straight line such that AP: PB = 1:4 , AQ: QB = 3: 1

Find (i) AP: PQ: QB
(ii) AQ: QP. (5 marks)

SECTION B (60 MARKS)

9. A polynomial $P(x)$ leaves a remainder 6 when divided by $x^2 - 4$, and a remainder $x + a$ when divided by $x^2 - x - 2$

Find (a) The value of the constant a,

(b) The remainder when $P(x)$ is divided by

(i) $x^2 + 3x + 2$

(ii) $x^3 + x^2 - 4x - 4$. (12 marks)

10. Given the curve $y = \frac{2x^2 - 8}{2x - 5}$

(i) Find the equations of the asymptotes.

(ii) Find the stationary points, hence or otherwise obtain the range of values of y within which the curve does not lie.

(iii) Sketch the curve. (12marks)

11. (a) Solve the equation:

$$\sqrt{x-3} + \sqrt{2x+1} = \sqrt{3x+4}$$

(6 marks)

(b) Given that $y = \ln(2 - e^x)$, show that $\frac{d^2y}{dx^2} + \left[\frac{dy}{dx}\right]^2 = \frac{dy}{dx}$,

hence find the first three non – vanishing terms of the Maclaurin's expansion of $y(x)$.

(6 marks)

12. (i) Find the coordinates of the point of intersection of the line

$\frac{x-2}{3} = \frac{3-y}{2} = z + 1$ and the plane $2x + y + 3z = 11$

(ii) Determine the angle between the line and the plane in (i) above. (12 marks)

12. A boy starts to sip a 900ml soda from a bottle at a rate of $10cm^3$ per minute. Given that his rate of consumption is inversely proportional to the square root of the volume of soda remaining at any time, find the time he takes to empty the bottle.

(12 marks)

13. (a) Solve the equation: $\text{Sin}^3 x = \sin 2x$ for $0^\circ \le x \le 360^\circ$ (6 marks)

(b)Given that A,B,C are angles of a triangle, show that
$\text{Sin}(A - B) + \text{Cos}(A - B)\tan C = \text{Sin } 2B \text{ Sec } C.$ (6 marks)

14. (a) Use small changes to estimate

$$\sqrt{0.041}$$

(6 marks)

(b) Find the equation of the normal to the curve $y^2(y - 3x) = 3 - x^2$ at the point $(-1, 2)$

(6 marks)

16 (a) Evaluate

$$\int_9^{25} \frac{dx}{\sqrt{x} - \sqrt{x-9}}$$

(5 marks)

(b) The area bounded by the curve $y = \ln x$, the x – axis and the line $x = e$ is rotated about the x – axis through 360^0. Show that the volume generated is equal to $e - 2$. (7 marks)

END