UGANDA TEACHERS EDUCATION CONSULT (UTEC)
SUBSIDIARY MATHEMATICS S475/1 2022
MARKING GUIDE

QN	SOLUTION	MKS	REMARKS
1.	$(\log_2 x)^2 + \log_2 x^2 + 1 = 0$		
	$(\log_2 x)^2 + 2\log_2 x + 1 = 0$	B1	for $\log_2 x^2 = 2\log_2 x$
	$y^2 + 2y + 1 = 0$; $y = \log_2 x$		
	$(y+1)^2 = 0$ or equivalent	M1	Correct method used.
	$y + 1 = 0$	A1	for $y = -1$
	$y = -1$		
	$\Rightarrow \log_2 x = -1$	M1	
	$x = 2^{-1} = \frac{1}{2}$	A1	
		05	
2.	i) $P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$	B1	
	$P(A) + P(B) - P(A \cap B)$		
	$= \frac{1}{5} + \frac{1}{2} - \frac{1}{5} \times \frac{1}{2}$	M1	Correct substitution
	$= 3/5$ $P(A \cap B) = P(A) \times P(B)$	A1	Accept 0.6
	iii) $P(A \cup B) = P(A) + P(B)$	B1	Correct substitution
	$= \frac{1}{5} + \frac{1}{2}$		
	$= \frac{7}{10}$	A1	Accept 0.7.

QN	SOLUTION	MKS	REMARKS
3.	$y = (x-1)(x^2-2)$		
	$y = x^3 - x^2 - 2x + 2$	B1	Expansion
	$\frac{dy}{dx} = 3x^2 - 2x - 2$	M1	Correct differentiation
	$\frac{dy}{dx}(x=2) = 3(2^2) - 2(2) - 2$	M1	Substitution
	$= 12 - 4 - 2$	M1	Simplification
	$= 6$	A1	CAO
		05	
4.	$\cos\theta = \sin\theta$		
	$\frac{\cos\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta}$	M1	Dividing by $\cos\theta$
	$1 = \tan\theta$	M1	Simplification
	$\theta = \tan^{-1} 1$	M1	Simplification
	$\theta = 45^\circ, 225^\circ$	A1 A1	A1 for each angle Correct
		05	
5.		B1	for both R_x and R_y Columns Correct
		B1	for d^2 Column

X	Y	Rx	Ry	d	d^2
15	75	1	10	10	1	-9	9	81
22	70	2	9	8	3	-6	6	36
25	72	3	8	9	2	-6	6	36
28	60	4	7	7	4	-3	3	9
31	35	5	6	3	8	2	-2	4
33	57	6	5	6	5	0	0	0
36	30	7	4	1.5	9.5	5.5	-5.5	30.25
39	55	8	3	5	6	3	-3	9
42	50	9	2	4	7	5	-5	25
44	30	10	1	1.5	9.5	8.5	-8.5	72.25

$\Sigma d^2 = 302.5$

QN	SOLUTION	MKS	REMARKS
	$r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$		
	$r_s = 1 - \frac{6 \times 302.5}{10(10^2-1)}$	M1	
	$r_s = 1 - 1.8333$		
	$r_s = -0.8333$	A1	
	These indicate a high negative relationship btn the ranks.	B1	
		05	
6	$\|M\| = n(n-1) - 2(3-n)$	M1	
	$\|M\| = n^2 + n - 6$	A1	
	$\Rightarrow 0 = n^2 + n - 6$	M1	Equating $\|M\|$ to 0.
	$0 = (n-2)(n+3)$	M1	for any method used
	$\Rightarrow n = 2$ or $n = -3$	A1	for values correct
		05	
7.	i) $0.1 + 0.2 + a + 0.2 + a = 1$	M1	
	$2a + 0.5 = 1$		
	$2a = 0.5$		
	$a = 0.25$	A1	

QN	SOLUTION	MKS	REMARKS
7(ii)	Mean($\bar{x}$) $= 0\times0.1+1\times0.2+2\times0.25+3\times0.2+4\times0.25$	M1	
	$= 0+0.2+0.5+0.6+1$	M1	
	$= 1+1.3$		
	$= 2.3$	A1	
		05	
8(i)	$u = 0\,ms^{-1}$, $a = 12\,ms^{-2}$, $t = 24s$.		
	$V = u+at$	B1	
	$V = 0+12\times24$	M1	
	$V = 288\,ms^{-1}$	A1	
(ii)	$S = ut+\frac{1}{2}at^2$		
	$S = 0\times24+\frac{1}{2}\times12\times24^2$	M1	
	$S = 3516m$	A1	
		05	

QN	SOLUTION						MKS	REMARKS
9)	TIME	f	x	fx	x^2	fx^2		
	70-75	8	72.5	580	5256.25	42,050	B1	for x Column
	75-80	20	77.5	1550	6006.25	120,125	M1	for fx Column
	80-85	26	82.5	2145	6806.25	176,962.5	A1	for $\Sigma fx = 8415$
	85-90	30	87.5	2625	7656.25	229,687.5	B1	for x^2 Column
	90-95	9	92.5	832.5	8556.25	77,006.25	M1	for fx^2 Column
	95-100	7	97.5	682.5	9506.25	66,543.75	A1	for $\Sigma fx^2 = 712,375$
		100		8415		712,375		

i) Mean $(\bar{x}) = \frac{8415}{100}$ — M1 Correct substitution

$= 84.15$ — A1 CAO

ii) Standard deviation

$= \sqrt{\frac{712375}{100} - 84.15^2}$ — M1 Correct substitution

$= \sqrt{7123.75 - 7081.2225}$ — M1 Simplification

$= \sqrt{42.5275}$

$= 6.5213$ — A1 CAO

QN	SOLUTION	MKS	REMARKS
b)	Both axes correctly labelled	B1	
	Six bars correctly drawn	B1	
	Process of determining the mode on a graph	B1	
	Mode = 85.8 ± 0.2	B1	Accept 85.6 — 86.0
QN		15	
10	a)(i) $f(x) = 2x^3+5x^2+ax+b$		
	$x=-1, f(-1) = -2+5-a+b = 0$		
	$\Rightarrow b-a = -3$ ——— i)	M1	Correct equation
	$x=2, f(2) = 2(2^3)+5(2)^2+2a+b=36$		
	$\Rightarrow 2a+b = 0$ ——— (ii)	M1	Correct equation
	i) -(ii); $-3a = -3$	M1	Correct method used
	$a = 1$	A1	
	From (1), $b = -3+1 = -2$	A1	
	(ii) $f(x) = 2x^3+5x^2+x-2$		
	$x+1 \overline{)\,2x^3+5x^2+x-2}$ $\quad 2x^2+3x-2$		
	$2x^3+2x^2$	M1	Correct used of long division
	$3x^2+x-2$		
	$3x^2+3x$		
	$-2x-2$		
	$-2x-2$		
	$0 \quad 0$		

QN	SOLUTION	MKS	REMARKS
	$\Rightarrow f(x) = (x+1)(2x^2+3x-2)$	M1	
	$= (x+1)(x+2)(2x-1)$	A1	
b)	$x^2 - 3x + 5 = 0$		
	$\alpha+\beta = 3, \quad \alpha\beta = 5$	B1	for both correct
	Sum of the roots $= \alpha^3 - \alpha^2 + \beta^3 - \beta^2$		
	$= (\alpha^3+\beta^3) - (\alpha^2+\beta^2)$		
	$= (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) - (\alpha+\beta)^2 + 2\alpha\beta$		
	$= 3^3 - 3(5)(3) - (3)^2 + 2(5)$	M1	
	$= 27 - 45 - 9 + 10$		
	$= -17$	A1	
	Product of the roots		
	$= (\alpha^3 - \alpha^2)(\beta^3 - \beta^2)$		
	$= (\alpha\beta)^3 - \alpha^3\beta^2 - \alpha^2\beta^3 + (\alpha\beta)^2$		
	$= (\alpha\beta)^3 - \alpha^2\beta^2(\alpha+\beta) + (\alpha\beta)^2$		
	$= (\alpha\beta)^2(\alpha\beta - (\alpha+\beta) + 1)$		

QN	SOLUTION	MKS	REMARKS
	$= 5^2(5-3+1)$	M1	
	$= 25 \times 3$		
	$= 75$	A1	
	$\Rightarrow x^2 - (-17)x + 75 = 0$	M1	
	$x^2 + 17x + 75 = 0$	A1	
		15	

11.

2018 (P_0)	2020 (P_n)	$\frac{P_n}{P_0} \times 100$ (P)	W	PW
2200	2800	127.273 B1	4	509.092
1200	2000	166.667 B1	5	833.335
4000	3600	90 B1	2	180.000
3600	4000	111.111 B1	6	666.666
1000	1500	150 B1	7	1050
$\Sigma P_0 = 12000$	$\Sigma P_n = 13900$	$\Sigma P = 645.051$	$\Sigma W = 24$	$\Sigma PW = 3239.093$

B1	for ΣP_0	
B1	for ΣP_n	
B1	for 127.273	
B1	for 166.667	
B1	for 90	
B1	for 111.111	
B1	for 150	
M1	for PW Column	
A1	for $\Sigma PW = 3239.093$	

(i) Simple aggregate price index

$$= \frac{13900}{12000} \times 100$$

$$= 115.8\bar{3}$$

M1 for Correct Substitution

A1 CAO

(ii) Simple average price index

$$= \frac{\Sigma P}{n}$$

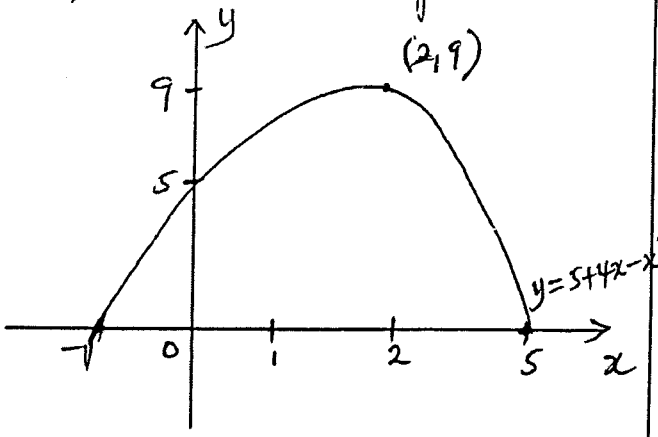
$$= \frac{645.051}{5}$$

$$= 129.0102 \simeq 129.01$$

M1 for Correct Substitution

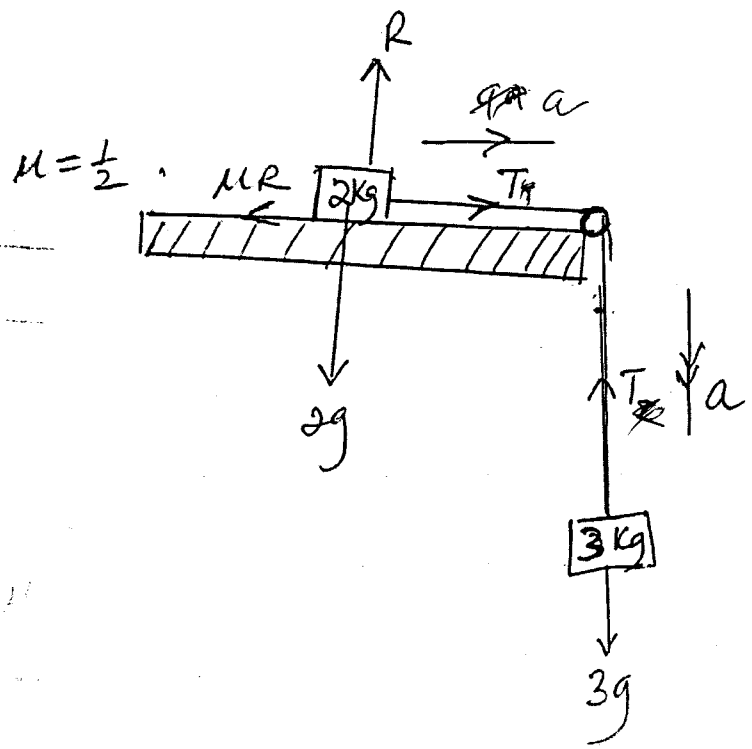
A1 CAO.

Qn	SOLUTION	MKS	REMARKS
11.	(iv) Weighted average price index $= \frac{\Sigma PW}{\Sigma W}$		
	$= \frac{3239.093}{24}$	M1	Correct Substitution
	$= 134.9622$	A1	CAO
		15	
12	a) i) $\frac{dy}{dx} = 2(2-x)$		
	$dy = 2(2-x)\,dx$		$dy = 4 - 2x \cdot dx$
	$\int dy = 2\int (2-x)\,dx$		$\int dy = \int 4 - 2x \cdot dx$
	$y = 2(2x - \frac{1}{2}x^2) + c$	M1	$y = 4x - x^2 + c$
	At $(1,8)$ $8 = 2((2\times1) - \frac{1}{2}\times1^2) + c$	M1	$8 = 4\times2 - 2^2 + c$
	$8 = 4 - 1 + c$		$8 = 8 - 4 + c$
	$\Rightarrow c = 5$	A1	$c = 5$
	$\Leftrightarrow y = 4x - x^2 + 5$	A1	$\Leftrightarrow y = 4x - x^2 + 5$
	(ii) when $x=0$, $y=5$, $(0,5)$	B1	
	when $y=0$, $x^2 - 4x - 5 = 0$		
	$(x-5)(x+1) = 0$		
	$x=5$ or $x=-1$		
	$(5,0)$; $(-1,0)$		

QN	SOLUTION	MKS	REMARKS
	$\therefore (5,0)\ ;\ (-1,0)$	B1 B1	B for each point correct.
	(iii) At a turning point, $\frac{dy}{dx}=0$		
	$\Rightarrow 2(2-x)=0$ | $4-2x=0$	M1	Equating to zero
	$x=2$ | $x=2$		
	$\Rightarrow y=4(2)-2^2+5=9$	A1	for both values of x and y
	$\frac{d^2y}{dx^2}=-2$ (max)	B1	
	$\therefore (2,9)$ maximum point	A1	Correct turning point
		B1 B1	B1 for plotting indicating the turning point and intercepts.
	$\text{Area}=\int_{-1}^{5}(5+4x-x^2)\,dx$	M1	Correct integration
	$=\left[5x+2x^2-\frac{1}{3}x^3\right]_{-1}^{5}$		
	$=\left(25+50-\frac{125}{3}\right)-\left(-5+2+\frac{1}{3}\right)$	M1	Correct substitution
	$=75-\frac{125}{3}+3\frac{1}{3}$		
	$=36$ sq. units	A1	CAO
		15	

QN	SOLUTION	MKS	REMARKS
13a)	$X \sim N(120; 16)$		
	(i) $P(X < 125) = P\left(Z < \frac{125-120}{4}\right)$	M1	Correct Conversion
	$= P(Z < 1.25)$		
	$= 0.5 + P(0 < Z < 1.25)$	M1	Simplification
	$= 0.5 + 0.3944$		
	$= 0.8944$ (tab)	A1	CAO
	(ii) $P(X > 128) = P\left(Z > \frac{128-120}{4}\right)$	M1	
	$= P(Z > 2)$		
	$= 0.5 - P(0 < Z < 2)$		
	$= 0.5 - 0.4772$	M1	
	$= 0.0228$ (Tab)	A1	
	(iii) $P(118 < X < 127) = P\left(\frac{118-120}{4} < Z < \frac{127-120}{4}\right)$	M1	
	$= P(-0.5 < Z < 1.75)$		
	$= P(0 < Z < 0.5) + P(0 < Z < 1.75)$	M1	
	$= 0.1915 + 0.4599$		
	$= 0.6514$ (Tab)	A1	

QN	SOLUTION	MKS	REMARKS
13b) (i)	$\int_{-\infty}^{\infty} f(x)dx = 1$		
	$\frac{k}{3}\int_{1}^{3} x\,dx = 1$	M1	
	$\frac{k}{3}\left[\frac{1}{2}x^2\right]_1^3 = 1$	M1	Correct integration
	$\frac{9k}{6} - \frac{k}{6} = 1$		
	$8k = 6$		
	$k = 3/4$	A1	Accept $k = 0.75$
(ii)	$E(x) = \frac{1}{4}\int_{1}^{3} x \cdot x\,dx$	M1	
	$= \frac{1}{4}\int_{1}^{3} x^2 dx$		
	$= \frac{1}{4}\left[\frac{1}{3}x^3\right]_1^3$	M1	
	$= \frac{1}{4}\left(\frac{1}{3}\cdot 27 - \frac{1}{3}\right)$		
	$= \frac{1}{4} \times \frac{26}{3}$		
	$= \frac{13}{6}$	A1	
		15	

QN	SOLUTION	MKS	REMARKS
14.			
QN 14		B1	Correct force diagram.

a) For 2kg mass:

$$T - \mu R = 2a \quad \text{(ma)} \quad \text{——(i)}$$

$\Rightarrow T - 2g \times \frac{1}{2} = 2a$; $R = 2g$, $\mu = \frac{1}{2}$ M1

$T - g = 2a$ ——(ii) M1

For 3kg mass:

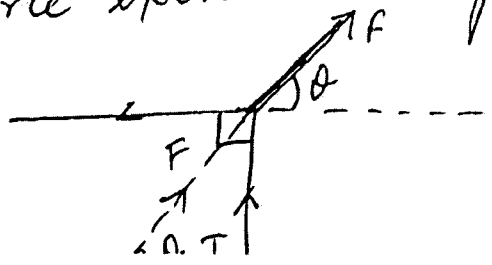
$3g - T = 3a$ ——(iii) M1

(ii) + (iii), $2g = 5a$ M1

$\Rightarrow a = \frac{2g}{5}$

$a = \frac{2}{5} \times 9.8 = 3.92 ms^{-2}$ A1 Reject $4 ms^{-2}$

QN	SOLUTION	MKS	REMARKS
	From (ii), $T - 9.8 = 2(3.92)$		
	$\Rightarrow T = 17.64N$	A1	
b)	The system snaps when $t = 35$.		
	$v = ut + at$		
	$V_1 = 0 + 3.92 \times 3 = 11.76 ms^{-1}$	M1	
	$S_1 = ut_1 + \frac{1}{2}at_1^2$		
	$S_1 = \frac{1}{2} \times 3.92 \times 3^2$		
	$S_1 = 17.64 m$	M1	
	$\Rightarrow v^2 = u^2 - 2aS_2$		
	$0 = 11.76^2 - 2 \times 3.92 S_2$		
	$\Rightarrow S_2 = 17.64m$	M1	
	Total distance Covered $= 17.64 \times 2$	~~A1~~	
	$= 35.28m$	A1	
c)	Force exerted on the pulley		



Q/N	SOLUTION	MKS	REMARKS
c)	$F = \sqrt{T^2+T^2}$		
	$F = \sqrt{2T^2}$		
	$F = \sqrt{2\times 17.64^2}$	M1	
	$F = 24.9467N$	A1	
	Direction: $\theta = \tan^{-1}(1)$		
	$\theta = 45°$	A1	
	The force exerted on the pulley is of magnitude 24.9467N acting at 45° to the horizontal	B1	
		15	

END