JINJA JOINT EXAMINATIONS BOARD

MOCK EXAMINATIONS 2022

P425/1 MATHEMATICS

MARKING GUIDE

$$(1+3x)^{\frac{1}{3}} = 1 + \frac{1}{3}(3x) + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)(3x)^2}{2!} + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)(3x)^3}{3!}$$ **M1**

$$= 1 + x - x^2 + \frac{5}{3}x^3 + ---$$ **A1**

$$x = \frac{1}{125}$$

$$\therefore \left(1+\frac{3}{125}\right)^{\frac{1}{3}} = 1 + \frac{1}{125} - \left(\frac{1}{125}\right)^2 + \frac{5}{3}\left(\frac{1}{125}\right)^3 ---$$ **M1**

$$\left(\frac{64 \times 2}{5^3}\right)^{\frac{1}{3}} = 1.007936853$$

$$\sqrt[3]{2} = \frac{5}{4}(1.007936853)$$ **M1**

$$= 1.25992$$

$$= 1.26$$ **A1**

05

1. $\cos\theta + \tan\theta = \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$ B1

$$= \frac{cos^2\theta + sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

$$\therefore \frac{1}{\sin\theta\cos\theta} = \frac{2}{sin^2\theta}$$

$$\sin\theta(\sin\theta - 2\cos\theta) = 0$$ B1

$\sin\theta = 0$ and $\sin\theta - 2\cos\theta = 0$ M1

$$\tan\theta = 2$$

$\theta = 0^0, 180^0\ , 360^0,$ and $63.43^0, 243.45^0$

$\therefore \theta = \ 0^0, 63.43^0, 180^0, 243.43^0, 360^0$ A1

05

2. $\int \sin(\sqrt{x})dx$

Let $t = \sqrt{x}$

$\frac{dt}{dx} = \frac{1}{2}x^{\frac{-1}{2}}$ **M1**

$\therefore\ \int sin(\sqrt{x})\ dx = \ \int \sin t\ .2\ t\ dt$ **M1**

Let $u = t$, $2\ =\ \int \sin t\ \ dt$

$\frac{du}{ut} = 1$ $2 =\ -\cos t$ **B1**

$\therefore 2\int t\ \ \sin t\ dt\ \ =\ \ [t\ \ \cos t + \ \int \cos t\ dt]\ \times 2$

$= -2t\ \ \cos t + 2\sin t + C.$ **M1**

$\Longrightarrow\ \int \sin(\sqrt{x})dx\ \ = \ -2\ (\sqrt{x})\cos(\sqrt{x}) + 2\sin(\sqrt{x}) + C$ **A1**

05

3. From $4x - 3y + 5 = 0$; $y = \frac{4}{3}x + \frac{5}{3}$

; $M_1 = \frac{4}{3}$ **B1**

Using $\tan\theta = \frac{M_1 - M_2}{1 + M_1M_2}$

$\tan 135^0 = \frac{\frac{4}{5} - m_2}{1+\frac{4}{3}(m_2)}$ **M1**

$-1\ \left(1 + \frac{4}{3}m_2\right)\ = \frac{4}{3} -\ m_2$

$m_2 =\ -7$ **B1**

At point (2, 3)

$\frac{y-3}{x-2} = -7$ **M1**

$y = -3x + 17$ **A1**

05

4. $\boldsymbol{n} = \begin{pmatrix} 2 \\ 6 \\ 5 \end{pmatrix}$ and the vectors on the plane are

$$\boldsymbol{b} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \quad \text{and} \quad \boldsymbol{c} = \begin{pmatrix} \mathbf{1} \\ \mathbf{-2} \\ \mathbf{2} \end{pmatrix}$$ **B1**

If $\boldsymbol{n}$is normal to plane, then

$\boldsymbol{n}\,.\,\boldsymbol{b} = 0$ and $\boldsymbol{n}\,.\,\boldsymbol{c} = 0$

$$\therefore \ \boldsymbol{n}\,.\,\boldsymbol{b} = \begin{pmatrix} 2 \\ 6 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = 4 + 6 - 10 = 0$$ **M1**

$$\boldsymbol{n}\,.\,\boldsymbol{c} = \begin{pmatrix} 2 \\ 6 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = 2 - 12 + 10 = 0$$ **M1**

$\therefore$ $\boldsymbol{n}$ is a normal vector the plane. **B1**

Equation of the plane is $2x + 6y + 5z + d = 0$

$\Longrightarrow 2x + 6y + 5z = 8$ **A1**

Or $\boldsymbol{r}\,.\begin{pmatrix} 2 \\ 6 \\ 5 \end{pmatrix} = 8$

05

5. $\log_2 x + \log_2 y = \log_2 xy = 3$

$\therefore \quad xy = 8 \ ------ \ (i)$ **B1**

$\log_4 x - \log_4 y = \log_4\left(\frac{x}{y}\right) = \frac{-1}{2}$

$\frac{x}{y} = 4^{\frac{-1}{2}}$

$y = 2x$ $--------(ii)$ **B1**

$\therefore\ 2x^2 = 8$ **M1**

$x = \pm 2$; $x = -2$ discard

$\therefore x = 2$ **A1**

And $2y = 8$

$y = 4$ **A1**

05

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6. $x^2 + 4xy + 3y^2 = 5$

Differentiating the equation with respect to x,

$2x + 4y + 4x\frac{dy}{dx} + 6y\frac{dy}{dx} = 0$ **M1**

$(4x + 4y)\frac{dy}{dx} = (2x + 4y)$

$\therefore \frac{dy}{dx} = -\frac{x+2y}{2x+3y}$ **B1**

$\frac{d^2y}{dx^2} = \frac{-(2x+3y)\left(1+2\frac{dy}{dx}\right) - (x+2y)\left(2+3\frac{dy}{dx}\right)}{(2x+3y)^2}$ **M1**

$= -\frac{-y + x\frac{dy}{dx}}{(2x+3y)^2}$

$= \frac{y - x\left(-\frac{x+2y}{2x+3y}\right)}{(2x+3y)^2}$ **M1**

$= \frac{x^2 + 4xy + 3y^2}{(2x+3y)^2}$

$= \frac{5}{(2x+3y)^3}$ **B1**

05

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7. Gradient of line $x - 2y - 4 = 0$ is $\frac{1}{2}$. **B1**

For $(y-2)^2 = x$;

$2\,(y-2)\frac{dy}{dx} = 1$

$\therefore \frac{dy}{dx} = \frac{1}{2\,(y-2)}$ **B1**

Let $A\ (x_1, y_1)$ be the point contact of the required tangent.

$\Longrightarrow \frac{1}{2\,(y_1-2)} = \frac{1}{2}$ **M1**

$y_1 = 3$

$\therefore\ (3-2)^2 = x_1$

$x_1 = 1$

Hence equation of tangent is

$y - 3 = \frac{1}{2}(x-1)$ **M1**

$x - 2y + 5 = 0$ **A1**

05

8. (a) let $a + ib$ be the square root of $15 + 8i$

$\Longrightarrow\ (a+ib)^2 = 15 + 8i$ **M1**

$(a^2 - b^2) + 2abi = 15 + 8i$

Reals:

$\therefore\ a^2 - b^2 = 15$ --------(1) **B1**

Im: $2ab = 8$ ----------(2) **B1**

$a = \frac{4}{6}$

$\therefore\ \frac{16}{b^2} - b^2 = 15$ **M1**

$b^4 + 15b^2 - 16 = 0$

$(b^2+16)(b^2-1) = 0$ **M1**

$\therefore\ b^2 = -16\ or\ b^2 - 1$

but $b^2 \neq -16$ since $b \neq R$

so $b^2 = 1 \quad \Longrightarrow\ b = \pm 1$

and $a = \pm 4$

A1 **A1**

$\therefore$ The square roots are $4 + i$ and $-4 - i$

(b) If $Z = x + iy$

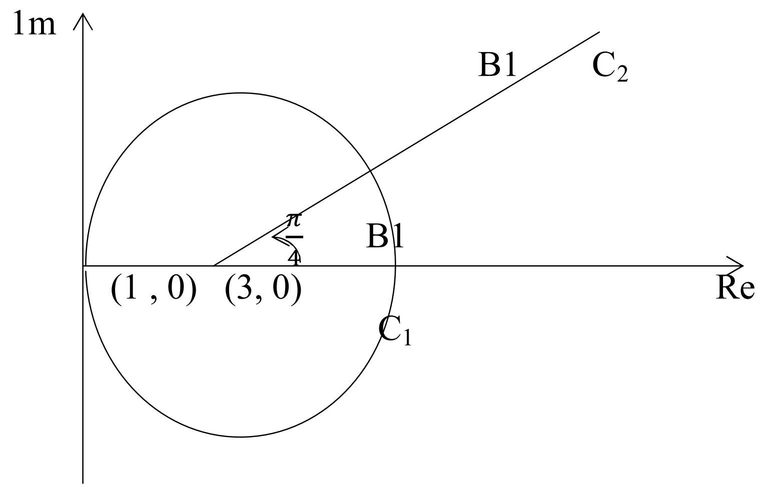
$|z - 3| = |(x-3) + iy| = 3$

$= \sqrt{(x-3)^2 + y^2} = 3$ **M1**

: $(x-3)^2 + y^2 = 9$ **B1**

Locus of C1 is a circle centre (3, 0) and radius 3. **B1**

And $\text{Arg}(z-1) = \frac{\pi}{4}$, is a half line starting at point (1, 0) on the real axis making an angle of $\frac{\pi}{4}$ **B1**



A1

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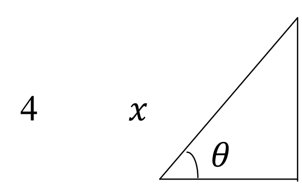
9. (a)



Let $x = 4sin\theta$

$dx = 4\ cos\theta\ d\theta$

$\sqrt{16 - x^2}$ **M1**

$$\therefore \int \frac{\sqrt{16-x^2}}{x^2}\ dx = \int \frac{(16 - 1\ \ sin^2\theta)^{\frac{1}{2}}}{16\ sin^2\theta}.410\theta\ d\theta \quad \textbf{M1}$$

$$= \int cot^2\theta\ d\theta$$

$= \int (cosec^2\theta - 1)\ d\theta$ **B1**

$= -\cot\theta - \theta + c$ **M1**

$= -\sqrt{\frac{16-x^2}{x}} - sin^{-1}\left(\frac{x}{4}\right) + c$**A1**

(b)

x	t
1	$\frac{\pi}{6}$
$\sqrt{3}$	$\frac{\pi}{3}$

B1M1

$dx = 2cost\ dt$

$\therefore \int_1^{\sqrt{3}} \frac{x+3}{\sqrt{4-x^2}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{2\sin t + 3}{\sqrt{4-4sin^2 t}} \ .\ 2\ \cos t\ dt$**M1**

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (2\sin t + 3)\ dt$

$= [-2\cos t + 3t]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ **M1**

$= \left[\left(-2\cos\frac{\pi}{3} + 3\left(\frac{\pi}{3}\right)\right)\right] - \left[-2\cos\frac{\pi}{6} + 3\left(\frac{\pi}{6}\right)\right]$**M1**

$= \left[\left(-2 \times \frac{1}{2} + \pi\right) + \sqrt{3} - \frac{\pi}{2}\right]$

$= \frac{\pi}{2} + \sqrt{3} - 1$ **B1**

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M1 M1

$$10.\tan(\theta+60^0)\tan(\theta-60^0)=\left(\frac{\tan\theta+\tan 60^0}{1-\tan\theta\tan 60^0}\right)\left(\frac{\tan\theta-\tan 60^0}{1+\tan\theta\tan 6\ \ ^0}\right)$$

$$=\frac{tan^2\theta-\left(\sqrt{3}\right)^2}{(1)^2-\left(\sqrt{3}\,\text{ta}\quad\right)^2}\qquad \textbf{M1}$$

$$=\frac{tan^2\theta\ -\ 3}{1-\ 3\ tan^2\theta}\qquad \textbf{B1}$$

For $\tan(\theta+\ 60^0)\tan(\theta-\ 60^0)=4sec^2\theta-3$

$$\frac{tan^2\theta-3}{1\ -\ 3\ tan^2\ \theta}\ =\ 4sec^2\theta\ -3\qquad \textbf{M1}$$

$$\frac{sec^2\theta\ -\ 4}{4-\ sec^2\theta}\ =\ 4sec^2\theta\ -3\textbf{M1}$$

$$3sec^4\theta\ \ -6sec^2\theta\ +2\ =0\qquad \textbf{B1}$$

$$sec^2\theta\ =\ \frac{6\pm\ \sqrt{(-6)^2-4\times3\times2}}{2\ \times\ 3}\qquad \textbf{M1}$$

$$=\ 1.5774\ \ \text{or}\ 0.4226$$

$$\therefore sec\theta\ =\ \pm1.2559\ \ \text{or}\ sec\theta=\ \pm0.6501$$

$$\cos\theta\ =\ \pm0.7962\ \ \textbf{B1}\ \ \text{or}\cos\theta=\ \pm1.5382\textbf{B1}$$

Discard

$\theta = 37.32^0, \; 142.23^0, \; 217.23^0, \; 322.74^0$ **M1A1**

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11.(a) direction vector for L_1, $d_1 = \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix}$

Direction vector of x-axis $d_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$\therefore \cos\theta = \frac{\begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} \circ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}{\sqrt{2^2 + 2^2 + (-3)^2} \circ \sqrt{1^2 + 0^2 + 0^2}}$$ **M1 M1**

$\theta = 60.98^0$ **A1**

(b) (i) for L_2 : $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 & + & as \\ 6 & + & 4s \\ 2 & + & 9s \end{pmatrix}$

At A : $\begin{pmatrix} 2 \\ -2 \\ b \end{pmatrix} = \begin{pmatrix} 4 & + & as \\ 6 & + & 4s \\ 2 & + & 9s \end{pmatrix}$ **M1**

Taking coefficient of **j**;

$-2 = 6 + 4s$

$s \ = \ -2$ **B1**

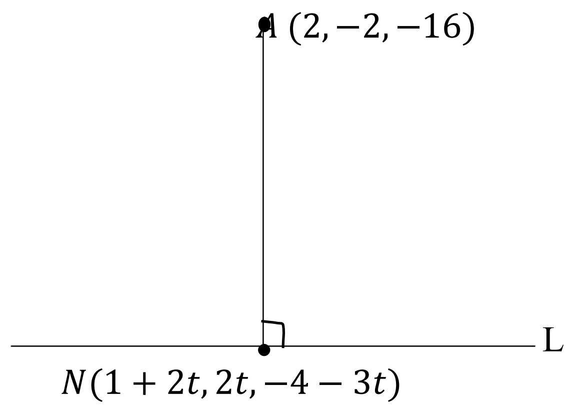
For $\boldsymbol{l}$: $2 = 4 + a\,(-2)$

$a \ = \ +1$ **A1**

For $\boldsymbol{k}$; $b = 2 + 9\,(-2)$

$= \ -16$ **A1**

(b) (ii)



$$\boldsymbol{b} = \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix}$$

AN $= \begin{pmatrix} 1 + 2t \\ 2t \\ -4 - 3t \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ -16 \end{pmatrix} = \begin{pmatrix} -1 & + & 2t \\ 2 & + & 2t \\ 12 & - & 3t \end{pmatrix}$ **B1**

AN . b = 0

$$\begin{pmatrix} -1 & + & 2t \\ 2 & + & 2t \\ 12 & - & 3t \end{pmatrix} . \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} = 0$$ **M1**

$17t = 34$

$t = 2$ **B1**

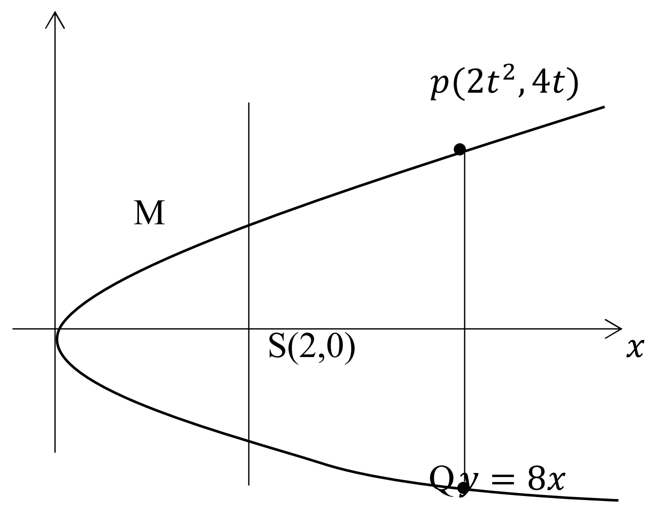
$\therefore \boldsymbol{AN} \qquad = \begin{pmatrix} 3 \\ 6 \\ 6 \end{pmatrix}$

$|\boldsymbol{AN}| \qquad = \sqrt{(3)^2 + 6^2 + 6^2}$ **M1**

$= 9$ units **A1**

12

12. y



(a) solving simultaneously equation of normal and curve.

$y + t\left(\frac{y^2}{8}\right) = 4t + 2t^3$ **M1**

$\Longrightarrow \quad ty^2 + 8y - 32t - 16t^3 = 0$ **B1**

Let y_1 and y_2 be the roots of the quadratic equation;

$\Longrightarrow y_1 + y_2 = \frac{-8}{t}$

$\therefore \; 4t + y_2 = \frac{-8}{t}$ **M1**

$y_2 = \frac{-8}{t} - 4t$

From $y^2 = 8x$

$$x = \frac{\left(\frac{-8}{t} - 4t\right)^2}{8}$$
$$= \frac{\left(t^2 + 2\right)^2}{t^2}$$

$\therefore \quad Q\left[\frac{\left(t^2+2\right)^2}{t^2} \quad , \quad -\left(\frac{8+4t^2}{t}\right)\right]$ **B1**

$\therefore \quad \overline{PQ}^2 = \left[2t^2 - \frac{\left(t^2+2\right)^2}{t^2}\right]^2 + \left[4t + \frac{8+4t^2}{t}\right]^2$ **M1**

$$= \left[\frac{\left(8+8t^2\right)^2}{t^4}\right] + \left[\frac{\left(8+8t^2\right)^2}{t^2}\right]$$

$$= (8+8t^2)^2\left[\frac{1}{t^4} + \frac{1}{t^2}\right]$$

$$= (8+8t^2)^2 \ . \ t^2\,\frac{(1+t^2)}{t^4 \ . \ t^2}$$

$\overline{PQ} = \frac{8\left(1+t^2\right) \ . \ (1+t^2)^{\frac{1}{2}}}{t^2}$ **M1**

$\therefore \ \overline{PQ} = \frac{8\ (1+t^2)^{\frac{3}{2}}}{t^2}$ **B1**

(c) grad of normal is $-t$

$\therefore$ equation of line SM;

$$\frac{y-0}{x-2} = -t$$

$y = -xt + 2t \ ------(1)$ **B1**

But equation of tangent at P is

$$y = \frac{1}{t}x + 2t \ ------- (2)$$

At M, (1) = (2)

$$-xt - \frac{1}{t}x = 0$$

$$\therefore \ x\left(t + \frac{1}{t}\right) = 0$$

$$\therefore x = 0 \text{ since} \left(t + \frac{1}{t}\right) \neq 0$$

$$\Longrightarrow \ y = 2t$$

$\therefore \ M\ (0, 2t)$ **B1**

$$\overline{SM} = \sqrt{(0-2)^2 + (2t - 0)}$$

$$= 2(1 \times t^2)^{\frac{1}{2}}$$ **B1**

But $5\overline{SM} = \overline{PQ}$

$$\Longrightarrow 5 \times 2(1 + t^2)^{\frac{1}{2}} = \frac{8(1+t^2)^{\frac{3}{2}}}{t^2}$$ **M1**

$$t^2 = 4$$ **B1**

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13. (a) $\propto + \beta = -p$; $\propto \beta = q$ **B1**

(i) $\propto^3 + \beta^3 = (\propto + \beta)(\propto^2 - \propto p + \beta^2)$ **M1**

$= (\propto + \beta)[(\propto + \beta)^2 - 3 \propto \beta]$

$-p[(-p)^2 - 3q]$

$= 3pq - p^3$ **A1**

(ii) $(\propto - \beta^2)(\beta - \propto^2) = \propto \beta - \propto^3 + \propto^2 \beta^2$

$= q - [3pq - p^3] + q^2$ **M1**

$= q^2 + q + p^3 - 3pq$ **A1**

(iii) If $\propto = \beta^2$

$\Longrightarrow (\beta^2 - \beta^2)(\beta - \beta^4) = q^2 + q + p^3 - 3rq$

$\therefore p^3 - 3pq + q^2 + q = 0$ **B1**

(b) $\propto + \propto r + \propto r^2 = 8400$; $\propto r^2 = 4800$

$\frac{\propto(1 + r + r^2)}{\propto r^2} = \frac{8400}{4800}$ dividing the equation **M1**

$4(1 + r + r^2) = 7r^2$

$3r^2 - 4r - 4 = 0$

$(3r + 2)(r - 2) = 0$ **M1**

$\therefore r = \frac{-2}{3}$ or 2.

But $r > 0$;

$\therefore r = 2$ **B1**

From $\propto r^2 = 4800$

$\propto = \frac{4800}{4}$

$= 1200$

$\therefore \propto r = 2 \times 1200$

$= 2400$ **A1**

$\therefore$ the prices of other items are 1200/= and shs 2400/= **A1**

12

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14. (a) Given the equation $x^3 + y^3 = 3xy$

Differentiating both sides with respect to x,

$3x^2 + 3y^2\frac{dy}{dx} = 3x\frac{dy}{dx} + 3y$ (1) **M1 M1**

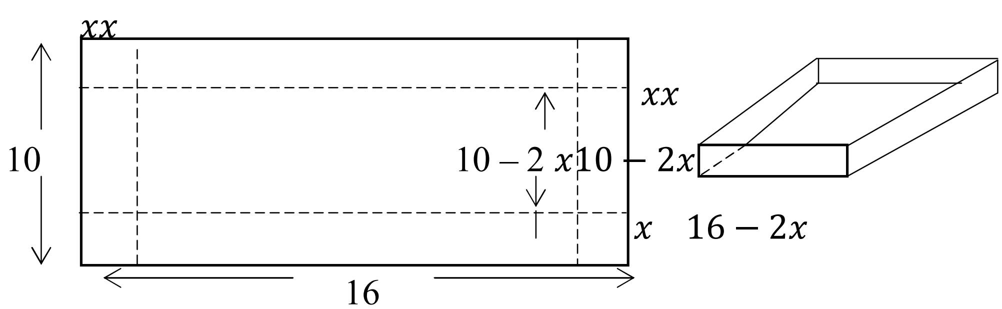
$\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$ **B1**

At point $\left(\frac{3}{2}, \frac{3}{2}\right)$,

Gradient, $\frac{dy}{dx} = \frac{\frac{3}{2} - \left(\frac{3}{2}\right)^2}{\left(\frac{3}{2}\right)^2 - \frac{3}{2}}$ **M1**

$= -1$ **A1**

(b)



$\therefore$ volume, $v = x(10 - 2x)(16 - 2x)$ **M1**

$= 160x - 52x^2 + 4x^3$

$\frac{dy}{dx} = 160 - 104x + 12x^2$ **M1**

$\therefore 160 - 104x + 12x^2 = 0$ **M1**

$(3x - 20)(x - 2) = 0$ **M1**

$: x = \frac{20}{3}$ or $x = 2$

The length of the side of the square must be less than half the side of the rectangle

$\therefore x = 6\frac{2}{3}$ is not a possible solution.

$\Longrightarrow x = 2$

For maximum value, $\frac{d^2v}{dx^2} < 0$

$\frac{d^2v}{dx^2} = -104 + 24x$

$= -104 + 24x$

$= -56 < 0$ **B1**

$\therefore x = 2$ **A1**

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15.(a) $\frac{dy}{dx} - \frac{2}{x}y = x^2 Inx, x > 0$

$R = e^{\int \frac{2}{x}dx} = e^{-2Inx} = x^{-2}$ **B1**

$\therefore x^{-2}\frac{dy}{dx} - 2x^{-3}y = Inx$

$x^{-2}y = \int Inx\, dx$

Let $u = Inx$ $\quad \frac{dv}{dx} = \int Idx$

$\frac{du}{dx} = \frac{1}{x} \qquad v = x$

$\int Inx\, dx = x\, Inx - \int x . \frac{1}{x} dx$ **M1**

$= xInx - x + c$

$\therefore x^{-2}y = x\ (Inx - 1) + c$ **B1**

When $x = 1$ and $y = 2$

$\Longrightarrow 2 = -1 + c$ **B1**

$c = 3$

$\therefore y = x^3(Inx - 1) + 3x^2$ **A1**

(b) (i) $\frac{dT}{dt} \quad \alpha \ (T - \ \theta)$

$\frac{dT}{dt} \ = \ -K(T-\theta)$ **A1**

(ii) $\therefore \int \frac{1}{T-\theta} \ dT \quad = \ -K \int dt$

$In \ (T - \ \theta) = \ -Kt + c$ **M1**

$T - \theta = Ae^{-k}$

$T = \ \theta e^{-kt}$ **B1**

At $t = 0s$, $T = 100^0C \ ; t = 600s, T = 84^0C, \theta = 25^0C$

$\therefore 100 \ = 21 \ + Ae^{-k(10)}$

$A = 79$ **B1**

And 84 $= 21 + 79 \ e^{-600k}$

$K = 0.00038$ **B1**

$\therefore T = 21 + 79e^{-0.00038}$ **A1**

When $t = 21$ minutes $= 1260sec$.

(iii) $\Longrightarrow T = 21 + 79e^{-0.00038(1260)}$ **M1**

$\therefore T \ = \ 70^0C$ **A1**

12

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