P425/1
PURE MATHEMATICS
Paper 1
Jul. / Aug. 2022
3 hours

''Together for Mathematics''

SECONDARY MATHEMATICS TEACHERS' ASSOCIATION

SMATA JOINT MOCK EXAMINATIONS 2022

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** the ***eight*** questions in Section **A** and **five** questions from Section **B**.

Any additional question(s) answered will **not** be marked.

All working **must** be shown clearly.

Begin each answer on a **fresh** sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40 MARKS)

*Answer **all** questions in this Section.*

1. Solve the equation $\cos 5\theta + \cos 4\theta = \sin 5\theta + \sin 4\theta$ for $0^0 \le \theta \le 180^0$.

(5 marks)

2. Calculate the perpendicular distance of the point $T(2, -1, 2)$ to the line $\frac{x-3}{2} = y = \frac{2-z}{3}$. ***(5 marks)***

3. Solve the equation $\sqrt{\frac{x-1}{2x}} - 3\sqrt{\frac{2x}{x-1}} = 2$. ***(5 marks)***

4. A point P moves so that its distances from two points $A(-2, 0)$ and $B(8,6)$ are in the ratio $AP : PB = 3 : 2$. Show that the locus of P is a circle. ***(5 marks)***

5. Solve the differential equation: $xy\frac{dy}{dx} = Inx$, given that $y = 2$ when $x = e$. ***(5 marks)***

6. Find $\frac{dy}{dx}$ if $y = \ln\left(\frac{e^{-x}(x-2)}{x+2}\right)$. ***(5 marks)***

7. If the n^{th} term of an arithmetic progression (A.P) is $\frac{3n-1}{6}$, deduce that the sum of the first n terms of the progression is $\frac{n}{12}(3n+1)$.

(5 marks)

8. Find the volume of the solid generated when the area bounded between the curve $y = x\sin x$, the x- axis from $x=0$ to $x=\frac{\pi}{4}$ is rotated through one complete revolution about the $x-$ axis.

(5 marks)

SECTION B: (60 MARKS)

Answer only ***five*** questions in this section.

9. (a) Find the values of p and q given that $\frac{p}{2+3i} + \frac{qi}{5-i} = \frac{11+20i}{13}$.

(6 marks)

(b) Describe the locus defined by $\left|\frac{z+2-i}{z+1}\right| = 2$. ***(6 marks)***

10. (a) Using calculus of small changes, find $\sqrt[4]{78}$ correct to **two** decimal places. ***(5 marks)***

(b) A rectangular box has a square cross-section and the sum of its length and the perimeter of this cross-section is 2 m. If the length of the box is x m,

(i) show that its volume V is given by $V = \frac{x(2-x)^2}{16}$.

(ii) find the value of x for which the volume is a maximum.

(12 marks)

11. (a) In a triangle OAB, E divides $\mathbf{OA}$ in the ratio $6:1$. D divides $\mathbf{AB}$ in the ratio $1:2$. Point C is on $\mathbf{OB}$ produced such that $\mathbf{OC}:\mathbf{OB} = 3:2$ Given that $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$, find the ratio $\mathbf{ED}:\mathbf{DC}$. ***(5 marks)***

(b) Determine the

(i) co-ordinates of the point of intersection of the lines

$$\frac{x-1}{1} = \frac{y-1}{2} = \frac{z+2}{3} \text{ and } \frac{x+2}{2} = \frac{y-5}{-1} = \frac{z+3}{2}.$$

(ii) Cartesian equation of the plane containing the lines in

(7 marks)

12. (a) Solve the equations $\begin{array}{l} x + 2y = 2 \\ x^3 + 8y^3 = 56 \end{array}$ ***(7 marks)***

(b) Use the binomial theorem to expand $\frac{1-x}{\sqrt{1+2x}}$ as far as the term in x^3.

(5 marks)

13. (a) Find the equations of the tangents from the point $(16, 17)$ to the parabola $y^2 = 4x$. ***(5 marks)***

(b) (i) Find the equation of the normal to the parabola $y^2 = 4ax$ at the point $P(ap^2, 2ap)$.

(ii) The normal in (b) (i) cuts the x- axis at the point G. Determine the locus of the mid-point of PG. ***(7 marks)***

14. Evaluate $\int_2^5 \frac{1+9x-2x^2}{(2x-3)(1+4x^2)}dx$. ***(12 marks)***

15. (a) Show that if $2\sin(x+\alpha) = \cos(x-\alpha)$, then $\tan x = \frac{1-2\tan\alpha}{2-\tan\alpha}$. Hence solve the equation $2\sin(x+20^0) = \cos(x-20^0)$ for $0^0 \leq x \leq 360^0$.

(7 marks)

(b) Given that $\tan\phi = \frac{a}{b}$, prove that $\frac{a}{a+b} = \frac{\sin\phi}{\sqrt{2}\sin\left(\phi+\frac{\pi}{4}\right)}$. ***(5 marks)***

16. (a) By letting $u = 5^x$, show that $\int \frac{5^x}{5^x+1}dx = \log_5 A(5^x+1)$. ***(5 marks)***

(b) The velocity of a particle at a distance x from a fixed point in a line at a time t, is given by $4 - \frac{x}{t}$. It is known that $x = 10$ when $t = 1$. Find the value of t when $x = 8$. ***(7 marks)***

END