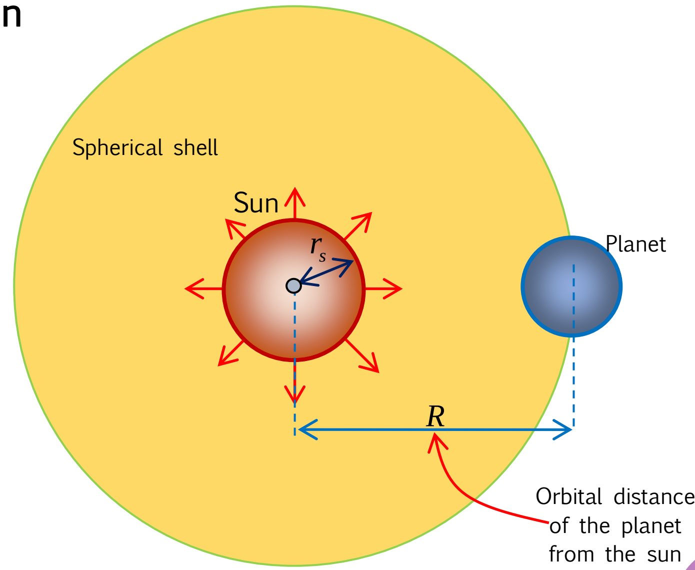
Energy from the sun



The sun maybe assumed as a spherical perfect black body. The radiations from the sun are spread out evenly over the spherical shell surrounding the sun's surface area.

The energy emitted per second (**also known as power**) by the sun at the surface temperature T_s is given by $P_{rad} = EA$

Where $E = \sigma T_s^4$ and surface area of the sun $A = 4\pi r_s^2$

$$\therefore \textit{power raddiated } (P_{rad}) = EA = 4\pi r_s^2 \sigma T_s^4$$

If we consider the radius of the sun as $7.0\times10^5 km$ and surface temperature of the sun as 6000K, then:

$$\textit{power raddiated } (P_{rad}) = EA = 4\times\left(\frac{22}{7}\right)\times(7.0\times10^8)^2\times5.7\times10^{-8}\times(6000)^4 \cong 4.47\times10^{26}W$$

Therefore every second, the Sun emits 4.47×10^{26} Joule of solar energy into the Universe.

Not all this energy falls on the surface of any planet in the galaxy every second, but instead it's spread evenly throughout the spherical shell between the sun and the planet

In order to calculate the energy a given planet receives from the sun every second, we begin first by finding the amount of solar power on a **one square meter area** of that planet, known as **solar constant**

Solar constant: the energy received per square meter per second by the planet from the sun

Alternatively; Solar constant: the power received per square meter by the planet from the sun

Mathematically; $\text{solar constant S} = \dfrac{\text{total power radiated by the sun}}{\text{area of the spherical shell}} = \dfrac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2}$

$$\boxed{\Rightarrow S = \left(\frac{r_s}{R}\right)^2 \sigma T_s^4}$$

Solar constant at the surface of the earth

Taking the sun's surface temperature as 6000K, radius of the sun as $7.0\times10^5 km$ and the radius of the earth's orbit about the sun as $1.5\times10^8 km$ then; the solar constant at the earth's surface

$$\text{Solar constant}, S = \left(\frac{r_s}{R}\right)^2 \sigma T_s^4$$

$$\text{Solar constant}, S = \left(\frac{7.0\times10^8}{1.5\times10^{11}}\right)^2 \times 5.67\times10^{-8}\times(6000)^4 = 1{,}600Wm^{-2}$$

- Thus a $1m^2$ area of the earth **theoretically** receives 1,600 joules every second.
- In practice the solar constant at the surface of the earth is less than this value owing to the fact that atmospheric gases and the ozone layer absorb some radiations from the sun before reaching the earth's surface.

Questions:

Where necessary assume the following; temperature of the sun is 6000K, radius of the sun $7.0\times10^{5}km$ and Stefan's constant $5.67\times10^{-8}wm^{-2}K^{-4}$

1. Calculate the solar constant at the surface of each of the following planets
   (i) Mercury of orbital distance $5.89\times10^{7}km$ from the sun
   (ii) Jupiter of orbital distance $7.78\times10^{8}km$ from the sun
   (iii) Neptune of orbital distance $4.47\times10^{9}km$ from the sun
   (iv) Pluto of orbital distance $5.87\times10^{9}km$ from the sun

What have you noticed about the solar constant as the distances of the planets away from the sun increase? What is the implication on the surface temperature of the planets?

2. The solar constant at the earth's surface is $1.4kWm^{-2}$. What is the solar constant on a planet whose orbital radius about the sun is eight times the orbital radius of the earth from the sun?

Solution

Let orbital radius of earth $R_E = R$ then orbital radius of the planet $R_p = 8R$ and $S_E = 1.4\times10^{3}Wm^{-2}$

From $\text{Solar constant, } S = \left(\frac{r_s}{R}\right)^2 \sigma T_s^4$

For the earth: $S_E = \left(\frac{r_s}{R}\right)^2 \sigma T_s^4 \; . \; . \; . \; .$ (i)

For the planet: $S_p = \left(\frac{r_s}{8R}\right)^2 \sigma T_s^4 \; . \; . \; .$ (2)

Equation $(2)\div(1)$ and simplify

$$S_p = \left(\frac{1}{8}\right)^2 \times S_E = 21.875Wm^{-2}$$

3. Calculate the power received from the sun by a solar panel of area 18000cm^2 placed at the surface of the earth

Solution

First obtain the solar constant at the earth's surface

$$\text{Solar constant, } S = \left(\frac{r_s}{R}\right)^2 \sigma T_s^4$$

$$S = \left(\frac{7.0\times10^{8}}{1.5\times10^{11}}\right)^2 \times 5.67\times10^{-8}\times(6000)^4 = 1{,}600Wm^{-2}$$

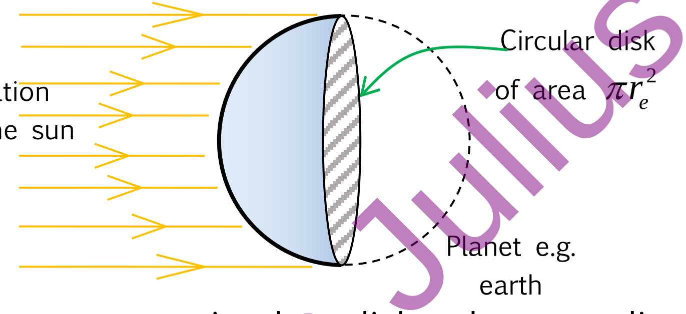
Area of solar panel $A_p = 1.8m^2$

Total power received by the solar panel therefor $P = S\times A_p = 2{,}880W$

4. A hot spherical sphere of diameter 28cm and surface temperature 450K has surface emissivity of 0.8. Find the concentration of the radiant energy per second per unit area from the sphere, at a region 1.2m away from the sphere. **Ans:** $25.32Wm^{-2}$

Equilibrium temperature of planets

At any instant, only half of the earth's surface is facing the sun and the radiation from the sun does not fall normally at every part of the earth's surface



Therefore the earth presents a circular disk whose radius is equal to the radius of the earth, to the sun

The area of the earth's surface that's normal to the radiation from the sun is πr_e^2 where r_e the radius of the earth is

$$\begin{aligned}
\left(\begin{array}{c}\textit{Thus the total energy per second } (\textit{power}) \\ \textit{received by the earth}\end{array}\right) &= (\textit{solar consntant}) \times \left(\begin{array}{c}\textit{circular area of} \\ \textit{the planet}\end{array}\right) \\
&= S \times A \\
&= \left(\frac{r_s}{R}\right)^2 \sigma T_s^4 \times \pi r_e^2 \ldots\ldots\ldots\ldots\ldots\ldots (i)
\end{aligned}$$

If the planet is assumed to be a perfect blackbody at surface temperature T_e then

$$\textit{The power radiated by the planet} = 4\pi r_e^2 \sigma T_e^4 \ldots\ldots\ldots\ldots\ldots (ii)$$

At radiative equilibrium;

$$\text{The power radiated by the planet} = \text{total power received by the earth}$$

$$4\pi r_e^2 \sigma T_e^4 = \left(\frac{r_s}{R}\right)^2 \sigma T_s^4 \times \pi r_e^2$$

$$4T_e^4 = \left(\frac{r_s}{R}\right)^2 T_s^4$$

Illustration: Taking the sun's surface temperature as 6000K, radius of the sun as $7.0\times10^5 km$ and the radius of the earth's orbit about the sun as $1.5\times10^9 km$; then the surface temperature of the earth can be estimated as follows;

$$4T_e^4 = \left(\frac{r_s}{R}\right)^2 T_s^4 \Rightarrow T_e = T_s \times \sqrt{\left(\frac{r_s}{2R}\right)} = 289.8K \simeq 290K$$

This value of temperature 290K ($\simeq 17^0 C$) is lower than the current global temperature of the earth (which is approximately 300K), because of the effect of the global warming.

Worked examples:

1. The earth receives energy from the sun at the rate of $1.4\text{x}10^3 Wm^{-2}$. If the ratio of the earth earth's orbit to the suns radius is 216, calculate the surface temperature of the sun.

Approach:

$$\left(\begin{matrix}\text{solar} \\ \text{constant } S\end{matrix}\right) = \frac{\text{total power radiated by the sun}}{\text{area of the spherical shell}}$$

$$= \left(\frac{r_s}{R}\right)^2 \times T_s^4$$

$$\Leftrightarrow T_s = \left[\frac{S}{\sigma}\left(\frac{R}{r_s}\right)^2\right]^{\frac{1}{4}} = \left[\frac{(216)^2 \times 1.4\times10^3}{5.67\times10^{-8}}\right]^{\frac{1}{4}}$$

$$\therefore T_s = 5839.3K$$

2. The average distance of Pluto from the sun is about 40 times that of the Earth from the sun. If the sun radiates as a black body at 6000K, and is $1.5\text{x}10^{11}$m from the Earth, calculate the surface temperature of Pluto (radius of the sun $7.0\times10^5 km$ and Stefan's constant $5.67\times10^{-8} wm^{-2}K^{-4}$)

Solution:

Orbital radius of Pluto

$$R_P = 10\times1.5\times10^{11} = 1.5\times10^{12} m$$

Solar constant at Pluto

$$S = \left(\frac{r_s}{R}\right)^2 \delta T_s^4 = \left(\frac{7.0\times10^8}{1.5\times10^{12}}\right)^2 \times 5.67\times10^{-8} \times (6000)^4$$

$$\therefore S = 16.00 Wm^{-2}$$

Total power received by Pluto

$$P_{ab} = S \times \pi r_p^2 \ldots\ldots\ldots\ldots\ldots (i)$$

If Pluto radiates as a perfect black bod, then;

$$P_{rad} = 4\pi r_p^2 \sigma T_p^4 \ldots\ldots\ldots\ldots\ldots (ii)$$

From (1) and (2)

$$4\pi r_p^2 \sigma T_p^4 = S \times \pi r_p^2 \qquad \Leftrightarrow T_p = \left(\frac{S}{4\sigma}\right)^{\frac{1}{4}}$$

$$T_p = \left(\frac{16}{4 \times 5.67 \times 10^{-8}}\right)^{\frac{1}{4}} = 91.65K$$

3. The flux of solar energy incident on earth's surface is $1.36 \times 10^3 Wm^{-2}$ Calculate:

(i) the temperature of the surface of the sun,

(ii) the total power emitted by the sun,

(iii) the rate of loss of mass by the sun

Solution:

(i) From formula of solar constant

$$S = \left(\frac{r_s}{R}\right)^2 \delta T_s^4 \qquad \Rightarrow T_s = \left[\frac{S}{\delta} \times \left(\frac{R}{r_s}\right)^2\right]^{\frac{1}{4}}$$

$$\Leftrightarrow T_s = \left[\frac{1.36 \times 10^3}{5.67 \times 10^{-8}} \times \left(\frac{1.5 \times 10^8}{7.0 \times 10^5}\right)^2\right]^{\frac{1}{4}}$$

$$\therefore T_s = 5760.84K$$

(ii) From the definition of solar constant

$$\text{solar constant S} = \frac{\text{total power radiated by the sun}}{\text{area of the spherical shell}}$$

$$\Rightarrow \begin{pmatrix}\text{total power radiated} \\ \text{by the sun}\end{pmatrix} = \begin{pmatrix}\text{solar} \\ \text{constant S}\end{pmatrix} \times \begin{pmatrix}\text{area of the} \\ \text{spherical shell}\end{pmatrix}$$

$$= S \times 4\pi R^2$$

$$= 1.36 \times 10^3 \times 4 \times 3.142 \times \left(1.5 \times 10^{11}\right)^2$$

$$= 3.85 \times 10^{26} W$$

(iii) Applying Einstein's mass energy relation $E = mc^2$ (refer to modern physics for details of this relation)

$$\frac{dE}{dt} = \frac{dm}{dt} \times c^2 \qquad \Rightarrow \frac{dm}{dt} = \frac{1}{c^2} \times \frac{dE}{dt}$$

But $c = 3.0 \times 10^8 ms^{-1}$ and $\frac{dE}{dt}$ is power

$$\therefore \frac{dE}{dt} = 3.85 \times 10^{26} W$$

From above

$$\frac{dm}{dt} = \frac{1}{\left(3.0 \times 10^8\right)^2} \times 3.85 \times 10^{26} = 4.27 \times 10^9 kgS^{-1}$$

4. A blackened platinum sphere of area 0.20 cm^2 is placed at a distance of 200 cm from a white – hot iron sphere of diameter 1.0 cm, so that the radiation falls normally on the strip. The radiation causes the temperature, and hence the resistance of the strip to increase. It is found that the same increase in resistance can be produced under similar conditions, but in the absence of radiation, when a current of 3.0 mA is passed through the platinum strip with a p.d of 24mV across its ends. Estimate the temperature of the iron sphere. (Take $\delta = 5.7\times10^{-8}wm^{-2}K^{-4}$)

Solution

$R = 200cm$

$r_s = 0.5cm$

$A = 0.20cm^2$

$T_s = ??$

$$\begin{pmatrix} Power\ in\ 1m^2 \\ area\ at\ the\ strip \end{pmatrix} = \frac{\text{total power radiated by the sphere}}{\text{area of the sphere of radius R}} = \frac{4\pi r_s^2\sigma T_s^4}{4\pi R^2}$$

$$\Rightarrow S = \left(\frac{r_s}{R}\right)^2 \sigma T_s^4 = \left(\frac{0.5}{200}\right)^2 \times 5.7\times10^{-8}\times T_s^4$$

$$\therefore S = 3.5625\times10^{-13}\times T_s^4$$

Total power received by the platinum strip

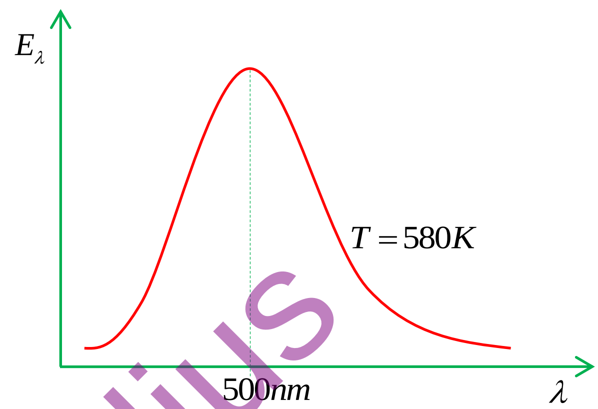
$$P = 3.5625\times10^{-13}\times T_s^4\times0.2\times10^{-4} = 7.125\times10^{-18}\times T_s^4$$

Electrical power $P = IV$

$$T_s = \left(\frac{IV}{7.125\times10^{-18}}\right)^{\frac{1}{4}} = \left(\frac{3.0\times10^{-3}\times24\times10^{-3}}{7.125\times10^{-18}}\right)^{\frac{1}{4}}$$

$$\therefore T_s = 1,782.94K$$

5. The diagram shows how E_λ the energy radiated per unit area per second per unit wavelength, varies with wavelength λ for radiation from the sun's surface



Use the graph above to find the maximum wavelengths λ_{max} which 'corresponds to the curve with peaks of;

(i) Radiation in the suns core temperature of 1.65×10^4K

(ii) Radiation in the interstellar space when temperature is 3.8K

Solution

When $\lambda_1 = 500\times10^{-9}m$ $T_1 = 580K$ and

(i) When $\lambda_2 = ??$ $T_2 = 1.65\times10^4 K$

From $\lambda_{max}T = constant \Rightarrow \lambda_1T_1 = \lambda_2T_2$

$$\Rightarrow \lambda_2 = \frac{\lambda_1T_1}{T_2} = \frac{500\times10^{-9}\times580}{1.65\times10^4} = 1.76\times10^{-8}m$$

(ii) When $\lambda_2 = ??$ $T_2 = 3.8K$

$$\Rightarrow \lambda_2 = \frac{\lambda_1T_1}{T_2} = \frac{500\times10^{-9}\times580}{3.8} = 7.63\times10^{-5}m$$

6. A roof measures 20 m by 50 m and is blackened. If the temperature of the sun's surface is 6000K, Stefan's constant $5.72\times10^{-8}wm^{-2}K^{-4}$, the radius of the sun $7.8\times10^{8}m$ and the distance of the sun from the earth $1.5\times10^{11}m$, calculate how much solar energy is incident on the roof per minute, assuming that half is lost in passing through the earth's atmosphere, the roof being normal to the sun's rays

Solution

$$\text{Solar constant, } S=\frac{\varepsilon 4\pi r_s^2\sigma T_s^4}{4\pi R^2}=\left(\frac{r_s}{R}\right)^2\varepsilon\sigma T_s^4$$

$$S=\left(\frac{7.8\times10^{8}}{1.5\times10^{11}}\right)^2\times0.5\times5.72\times10^{-8}\times(6000)^4$$

$$\therefore S=1.00\times10^{3}Wm^{-2}$$

Area of the panel $A=20\times50=1000m^2$

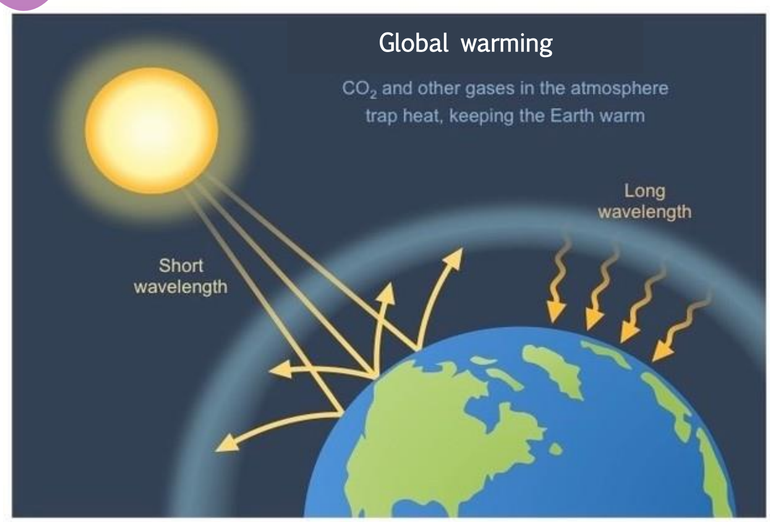
Total energy received per second by the panel $P=S\times A=1\times10^{6}Js^{-1}$

Total energy received per second by the panel

$$P=1\times10^{6}\times60J\min^{-1}=6.0\times10^{7}J\min^{-1}$$

The greenhouse effect

- A greenhouse is made of glass. Glass has the property of being able to transmit visible light and absorb ultraviolet and infrared radiations
- Thermal radiation from the sun enters the greenhouse and is absorbed by the plants and the soil.
- The plants and the soil emit thermal radiation of long wavelength, i.e. infrared.
- These wavelengths are not transmitted by the glass. Hence the greenhouse warms up.



It's important to note the following

— The interior of a car parked in the sun with its windows closed warms up easily due the greenhouse effect. This effect can be minimized by using reflective material known as sun visor when parking the car under direct sunlight. The sun visor will reflect off the incident radiation without being absorbed by the interior of the car that would in turn produce long wavelength infra-red to warm the inside of the car

— The temperature of the earth (world) is slowly rising due to greenhouse effect. This is called **global warming**

Global warming

- The energy received from the sun is mainly in the range of the ultraviolet, visible light and short wavelength infrared because of the high temperature of the surface of the earth.
- These radiations pass through to the surface of the earth and therefore being absorbed by the earth.
- On the other hand the earth being cooler source emits long wavelength radiations I.e. infrared.
- These radiations are absorbed by water vapour and carbon dioxide in the earth's atmosphere.
- The atmosphere therefore radiates most of this energy back to the earth, resulting into the temperature of the earth being higher than it would otherwise be. This temperature rise of the earth is referred to as **global warming**
- Fumes emitted from factories and motor vehicles have increased greatly the amount of carbon dioxide in the atmosphere. This has resulted in a steady rise in the temperature of the earth

Sample questions

1. (a) Sate the Stefan-Boltzmann law of blackbody radiation.

(b) Consider the sun to be a sphere of radius 7.0×10^8m whose surface temperature is 5900K.

(i) Find the solar power incident on an area of $1m^2$ at the top of the Earth's atmosphere if this is a distance of 1.5×10^{11}m, from the sun. Assume that the sun radiates as a black body.

(ii) Explain why the solar power incident on $1m^2$ of the Earth's surface is less than the calculated value in (b) (i) above.

(c) Explain briefly the greenhouse effect and its relation to global warming.

2. (a)(i) State the laws of Black Body Radiation.

(ii) Sketch the variation of intensity with wavelength in a black body for three different temperatures.

(b) (i)What is a perfectly black body?

(ii) How can a perfectly black body be approximated in reality?

(c) The energy intensity received by a spherical planet from a star is 1.4×10^3 Wm^{-2}. The Star is of radius 7.0×10^5 km and is 14.0×10^7km from the planet.

(i) Calculate the surface temperature of the star.

(ii) State any assumptions you have made in (c)(i) above.

(d) (i) What is convection?

(ii) Explain the occurrence of land and sea breeze.