

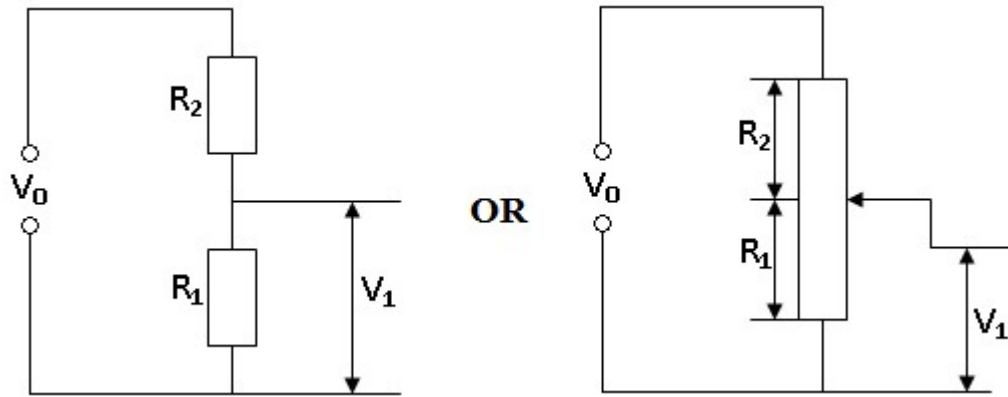
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Potential divider

Potential dividers can be used to vary the output p. d of a circuit. They can be used to produce a small p. d from a larger one.

The larger voltage V_0 (or V) is connected across two resistors in series as shown below.



Total resistance is $R = R_1 + R_2$

The current I flowing in the circuit is given by; $I = \frac{V_0}{R_1 + R_2}$

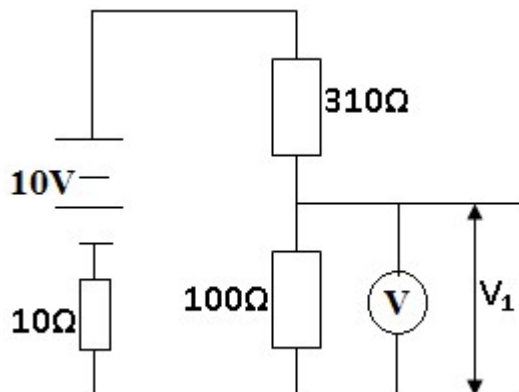
$$\text{pd across } R_1 \text{ is } V_1 = \left(\frac{R_1}{R_1 + R_2} \right) V_0$$

Hence a fraction of V_0 is obtained by use of resistors. This is a convenient way of controlling the voltage applied to a load such as a lamp.

The resistance of the load is in parallel with R_1 .

Examples

1. In the circuit below, the voltmeter V has resistance of 100Ω .



- Find the reading of the voltmeter
- What voltage would be obtained if the voltmeter was replaced by a C. R. O?

- (iii) Explain the difference in (i) and (ii) above.

Solution

- (i) Let $R_1 = 400\Omega$ and $R_2 = 100\Omega$

Effective resistance in parallel,

$$\frac{1}{R_E} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{400} + \frac{1}{100} = \frac{5}{400}$$

$$\Rightarrow R_E = 80\Omega$$

Total resistance in the circuit, $R = 80 + 310 + 10 = 400\Omega$

Total current in the circuit, $I = \frac{V}{R} = \frac{10}{400} A$

Voltmeter reading $V^1 = IR_E = \frac{10}{400} \times 80$

$$\therefore V_1 = 2V$$

- (ii) A C.R.O is an ideal voltmeter and has an infinitely high resistance. It doesn't therefore take away any current from the circuit.

Effective resistance, $R = 10 + 310 + 100 = 420\Omega$

Total current, $I = \frac{V}{R} = \frac{10}{420} A$

Voltmeter reading, $V_1 = IR = \frac{10}{420} \times 100$

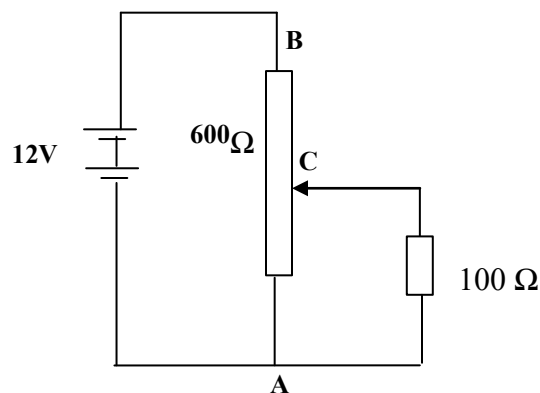
$$\therefore V_1 = 2.38V$$

2. A 12V battery is connected across a potential divider of resistance 600Ω as shown below. If the load of 100Ω is connected across the terminals A and C when the slider is half way up the divider, find:

- (i) P. d across the load
(ii) P.d across A and C when the load is removed.

Solution

- (i)



Since the slider is half way up the divider, $R_{AC} = R_{CB} = \frac{600}{2} = 300\Omega$

Effective resistance in parallel, $\frac{1}{R_E} = \frac{1}{100} + \frac{1}{300} = \frac{4}{300}$
 $\Rightarrow R_E = 75\Omega$

Total resistance in the circuit, $R = 75 + 300 = 375\Omega$

$$\frac{V}{R} = \frac{12}{375}$$

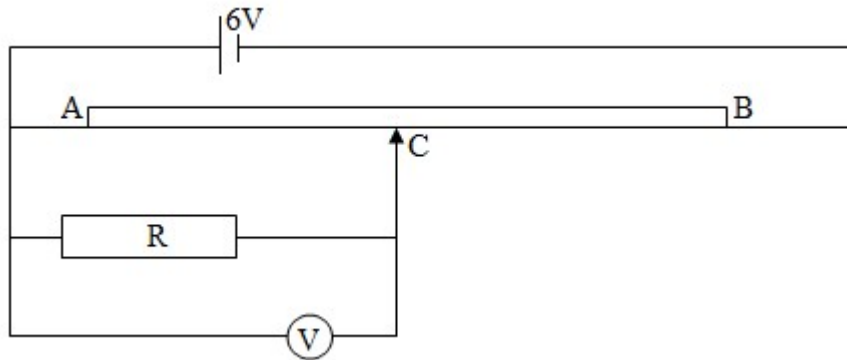
Current supplied by the battery, $I = \frac{V}{R} = \frac{12}{375} = 0.032A$

P.d across the parallel combination of resistors, $V_1 = IR_E = 0.032 \times 75 = 2.4V$

Hence the p.d across the load is **2.4V**.

(ii) when the load is removed $I = \frac{12}{600} = 0.02A$
Hence p.d across AC = $0.02 \times 300 = 6V$

3. In the figure below, wire AB is of length 1.00m and has resistance of 10Ω . If point C is the mid-point of AB, and the voltmeter reading is 2.0V, find the value of R. (6 mks)



Solution

$$V_{AC} = V_R = 2V$$

Combined resistance in parallel, $\frac{R \times R_{AC}}{R + R_{AC}} = \frac{5R}{R + 5}$

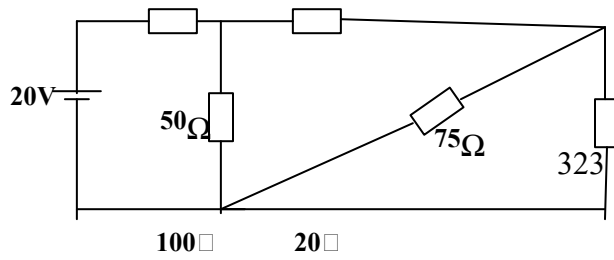
$$\text{But } V_{AC} = \frac{R_P}{R_P + R_{CB}} \times V_0$$

$$\Rightarrow 2 = \frac{\frac{5R}{R+5}}{\frac{5R}{R+5} + 5} \times 6$$

$$2 = \frac{30R}{10R + 25}$$

$$\therefore R = 5\Omega$$

Exercise



Find;

(i)

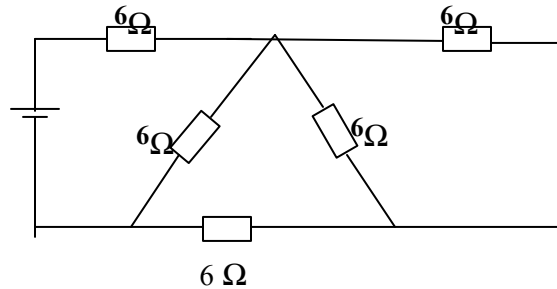
the current supplied by the dc source.

 50Ω

(ii)

power dissipated in the 20Ω resistor.

2.



Given that the internal resistance of the **50 V** battery in the circuit is 0.4Ω , find

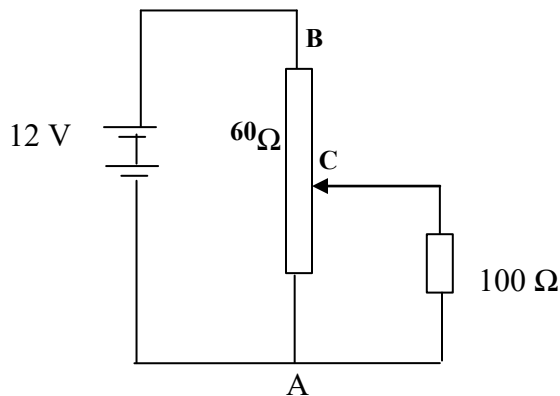
- (i) the effective resistance in the circuit
- (ii) power dissipated in the battery.

6□

3. A resistor of 500Ω and one of 200Ω are placed in series with a 6V supply. What will be the reading on a voltmeter of internal resistance 2000Ω when placed across

- (i) the 500Ω resistor
- (ii) 200Ω resistor.

4. A 12V battery is connected across a potential divider of resistance 60Ω as shown below.



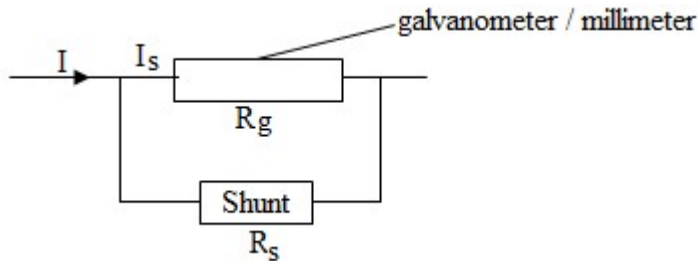
If a load of 100Ω is connected across the terminals A and B when the slider is half way up the divider, find:

- (i) the p. d across the load
- (ii) the p. d across A and C when the load is removed.

Conversion of a millimeter to an ammeter

A moving coil meter/galvanometer measures small amount of current. Thus it can only withstand small electric current when it is connected to an electric circuit.

To allow safe flow of large current in a circuit with a galvanometer, a small resistor called shunt (resistance R_s) is connected in parallel with the galvanometer as shown below.



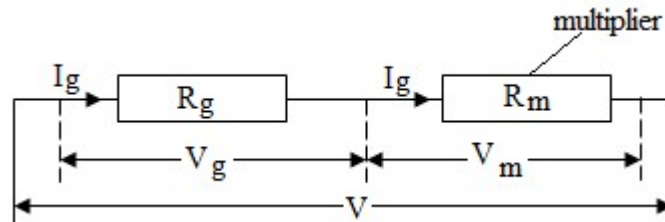
The p.d across the galvanometer = p.d across the shunt

$$V_g = V_s$$

$$I_g R_g = I_s R_s$$

Conversion of a millimeter to a voltmeter

To convert a millimeter to a voltmeter, a large resistor called the multiplier (resistance R_m) is connected in series with the millimeter as shown below.



Total voltage V is equal to the sum of the p.d across the millimeter and the p.d across the multiplier

$$\text{i.e } V = V_g + V_m$$

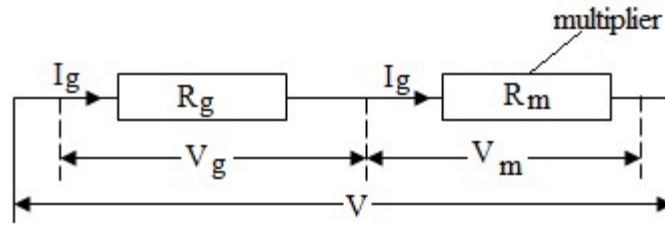
$$= I_g R_g + I_g R_m$$

$$\Rightarrow V = I_g (R_g + R_m)$$

Example

A millimeter of resistance 5Ω and full scale deflection of 50mA is to be used to measure a p.d of 50V . What should be the resistance of the multiplier?

Solution



$$\begin{aligned}
 V &= V_g + V_m \\
 &= I_g R_g + I_g R_m \\
 &= I_g (R_g + R_m)
 \end{aligned}$$

$$50 = 50 \times 10^{-3} (5 + R_m)$$

$$R_m = 995 \Omega$$

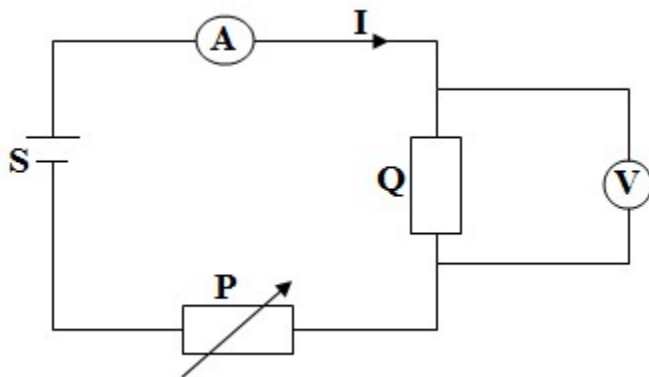
∴ The resistance of the multiplier is 995Ω

Ohm's law

It states that “Under constant physical conditions, the current flowing through an (ohmic) conductor is proportional to the potential difference across the ends.

$$V \propto I, \text{ hence } \frac{V}{I} = \text{constant}$$

Verification



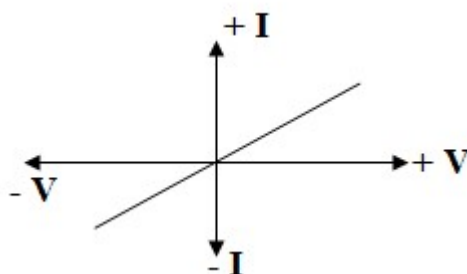
- The circuit is connected as shown above
- A suitable variable resistor or rheostat P of the same order as resistor Q used as the conductor are connected in series with an ammeter A
- By adjusting the rheostat P, current I through the circuit is varied and at each value of I, the p. d across Q is measured and noted.
- Several values of I & V are obtained and the results tabulated.
- A graph of V against I is plotted and is a straight line through the origin.
- This shows that the p. d V across the resistor Q is proportional to the current I through it, hence Ohm's law verified.

Ohmic & non-ohmic conductors

Ohm's law is obeyed by most metals (conductors), and these are called **ohmic** conductors.

If a conducting current I is reversed in direction, the p. d is also reversed but the magnitude of current is unchanged.

The characteristic ($I - V$) graph is therefore a straight line passing through the origin as shown below.



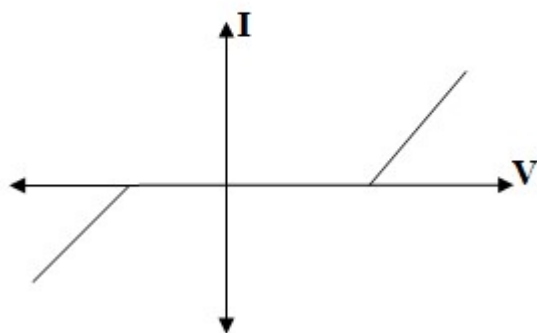
NB: An electrolyte such as CuSO_4 with Cu electrodes also obeys ohm's law

Graphs of I against V for non- Ohmic conductors

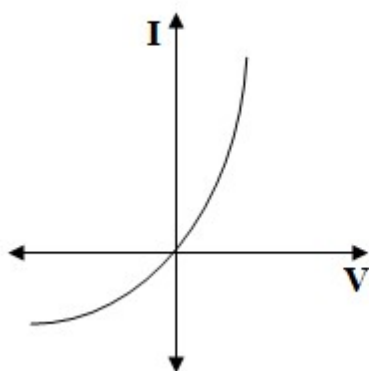
Non-ohmic conductors are those which don't obey Ohm's law.

A non-ohmic characteristic graph may be a curve instead of a line or may not pass through the origin as in the ohmic graphs.

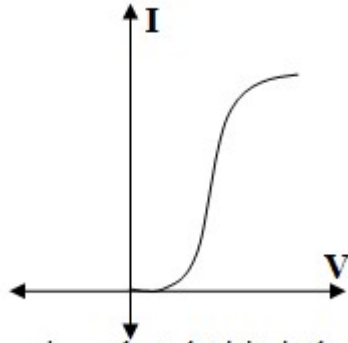
(i) Dilute sulphuric acid with platinum electrodes



(ii) Junction diode



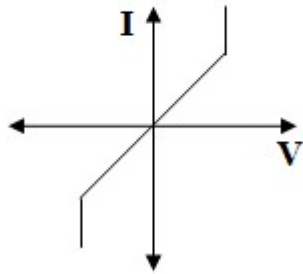
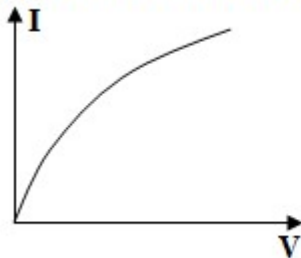
The current flows when the p.d is applied in one direction but the current is almost zero when the p.d is applied in the opposite direction

(iii) Diode valve

Doesn't conduct electricity in the reverse direction

(iv) Neon gas

obeys ohm's law upto a certain value

**(v) Filament lamp**

Qn: Describe the current versus voltage characteristics of non-ohmic conductors.

Resistivity (ρ)

The resistance, R of a metal wire is directly proportional to its length, l and inversely proportional to its cross sectional area, A

That is $R \propto l$ and $R \propto \frac{1}{A}$

Hence $R = \rho \frac{l}{A}$, and $\rho = R \frac{A}{l}$

where ρ is the resistivity of the material of wire

Consider a cube of the material of side 1m. Hence the area, $A = 1\text{m}^2$.

$$\text{From } R = \rho \frac{l}{A}$$

Substitute $l = 1\text{m}$, $A = 1\text{m}^2$

$$R = \rho$$

Def'n : Resistivity of a material is the **resistance** across **opposite faces** of a **cube** of a material of side **1 m**.

OR: Is the resistance of a sample of the material of length one metre and cross-sectional area of one metre squared at a constant temperature.

The unit of ρ is **ohm metre** ($\Omega \text{ m}$)

Example

A steady uniform current of 5mA flows along a metal cylinder of cross sectional area of 0.2mm^2 , length 5m and resistivity $3 \times 10^{-5} \Omega \text{ m}$. Find the p.d across the ends of the cylinder.

Question

A p.d of 4.5V is applied to the ends of a 0.69 m length of a wire of cross sectional area $6.6 \times 10^{-7} \text{m}^2$. Calculate the drift velocity of electrons across the wire. (ρ of wire is $4.3 \times 10^{-7} \Omega \text{ m}$, number of electrons per m^3 is 10^{28} and electronic charge is $1.6 \times 10^{-19} \text{C}$)

NB:

- Resistivity of a material is increased by small amount of impurities and alloys; e. g constantan has greater resistivity than its constituents like copper and nickel.
- For pure metals, resistivity increases with temperature while for semi-conductors, resistivity decreases with temperature increase.

Qns;

- Explain why the resistance of a thermistor reduces when current is passed through it.

Solution

When current is passed through a thermistor, heat is generated. As the temperature increases, the loosely bound electrons are released for conduction. Hence current increases implying that the resistance reduces.

- Explain why the resistance of a metal increases when the temperature of the metal is increased. (2 mks) **Solution**

As the temperature of a metal increases the amplitude of vibration of its atoms in its metallic lattice increases which reduces the mean free path for the conducting electrons. Therefore the number of electrons flowing per second through the metal reduces causing an increase in the resistance.

Conductivity(σ)

This is the reciprocal of resistivity.

That is; $\delta = \frac{1}{\rho}$.

Its SI unit is per ohm per meter ($\Omega^{-1}\text{m}^{-1}$)

EFFECT OF TEMPERATURE ON RESISTANCE

Temperature co-efficient of resistance (α)

The resistance of a material varies with temperature and the variation can be expressed by the temperature co-efficient of resistance, α .

If a material has a resistance R_0 at 0°C and its resistance increases by δR due to a temperature change $\delta\theta$, then α of the material is defined by the equation;

$$\alpha = \frac{\delta R}{R_0} \cdot \frac{1}{\delta\theta}$$

Def'n : Temperature co-efficient of a material is the **fractional change in the resistance** of a material / conductor at 0°C for every degree Celsius rise of temperature.

The unit of α is K^{-1} or $^\circ\text{C}^{-1}$.

If a specimen has a resistance R_θ and R_0 at θ and 0°C respectively,

$$\alpha = \frac{R_\theta - R_0}{R_0 \theta}$$

Then Rearranging gives;

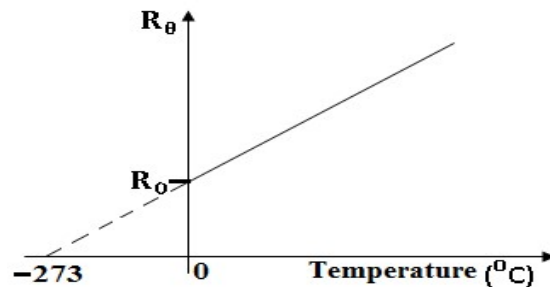
$$R_0 \theta \alpha = R_\theta - R_0$$

$$R_\theta = R_0 + R_0 \theta \alpha$$

$$\Rightarrow R_\theta = R_0(1 + \theta \alpha)$$

A graph of R_θ against θ is a straight line with an intercept of R_0 on the R_θ axis and $R_0 \alpha$ as its slope.

R_0 is the resistance of the material at 0°C and shows that the material actually exists.



If R_1 and R_2 are the resistances at θ_1^0 and θ_2^0 , then:

$$R_1 = R_0(1 + \alpha\theta_1) \dots\dots\dots(i)$$

$$R_2 = R_0(1 + \alpha\theta_2) \dots\dots\dots(ii)$$

Dividing eqn (i) by (i), we get;

$$\frac{R_1}{R_2} = \frac{1 + \alpha\theta_1}{1 + \alpha\theta_2}$$

NB: Also the expression in terms of resistivities is correct, i.e. $\rho_\theta = \rho_0(1 + \alpha\theta)$

NB: Materials whose resistance vanishes when they are cooled to temperatures near -273°C are called **super conductors**.

Qn;

(a) Explain why semi-conductors have negative temperature coefficient of resistance.

(b) Explain why metals have positive temperature coefficient of resistance.

Examples

1. The resistance of a coil of copper wire at the beginning of a heating test is 180Ω , the temperature being 20°C . At the end of the test, the resistance is 220Ω . Calculate the temperature rise of the coil. (Assume the temp. co-efficient of resistance of copper to be 0.00426K^{-1} at 0°C)

Solution

$$\text{Let } R_1 = 180\Omega, \theta_1 = 20^\circ\text{C}$$

$$R_2 = 220\Omega, \theta_2 = ?$$

Using the equation;

$$R_t = R_0(1 + \alpha\theta_t)$$

$$180 = R_0(1 + 20 \times 0.00426) \dots\dots\dots(i)$$

$$220 = R_0(1 + 0.00426 \theta) \dots\dots\dots(ii)$$

$$\frac{180}{220} = \frac{1 + 20 \times 0.00426}{1 + 0.00426 \theta}$$

$$180(1 + 0.00426 \theta) = 220(1 + 0.085)$$

$$\therefore \theta = 76.6^\circ\text{C}$$

$$\text{Increase in temperature} = 76.6 - 20$$

$$= 56.6^\circ\text{C}$$

2. A nichrome wire of length 1.0m and diameter 0.72mm at 25°C , is made into a coil. The coil is immersed in 200cm^3 of water at the same temperature and a current of 5.0A is passed through the coil for 8 minutes until when the water starts to boil at 100°C . find;
 - (i) The resistance of the coil at 25°C ,
 - (ii) The electrical energy expended assuming all of it goes into heating the water.
 - (iii) The mean temperature coefficient of nichrome, between 0°C and 100°C .
(ρ at $25^\circ\text{C} = 1.2 \times 10^{-6}\Omega\text{m}$, density of water is 1gcm^{-3})

Solution

$$(i) \quad \text{From } R = \frac{\rho l}{A},$$

$$A = \frac{\pi D^2}{4} = \pi(0.3 \times 10^{-3})^2$$

$$\text{And } l = 1\text{m}$$

$$\Rightarrow \rho_{25^\circ\text{C}} = \frac{1.2 \times 10^{-6} \times 1}{\pi(0.3 \times 10^{-3})^2} \Omega = 2.95 \Omega$$

$$(ii) \quad \text{Electrical energy} = \text{heat gained by the water}$$

$$= mc\Delta\theta$$

$$= 200 \times 10^{-3} \times 4.2 \times 10^3 (100 - 25) = 6.3 \times 10^4 \text{J}$$

$$(iii) \quad \text{Suppose the mean resistance is } \langle R \rangle$$

$$\Rightarrow I^2 \langle R \rangle t = 6.3 \times 10^4$$

$$\therefore 5^2 \times \langle R \rangle \times 8 \times 60 = 6.3 \times 10^4$$

$$\text{thus } \langle R \rangle = 5.25 \Omega$$

$$\text{But } \langle R \rangle = \frac{R_{25} + R_{100}}{2}$$

$$\text{From } R_\theta = R_0(1 + \alpha\theta), \Rightarrow \frac{R_{100}}{R_\theta} = \frac{R_0(1 + 100\alpha)}{R_0(1 + 25\alpha)} = \frac{7.55}{2.95}$$

$$\text{On simplifying, } \alpha = 0.04 \text{K}^{-1}$$

3. The temperature coefficient of resistance of two wires A & B of diameters 1.20mm and 0.80mm are 0.0004K⁻¹ and 0.0003K⁻¹ respectively. If the ratio of their resistances at 00C is 1.5, calculate;

- (i) Ratio of resistances at 1000C,
(ii) The ratio of electrical resistivities at 1000C given that they have the same length.

Solution

$$(i) \quad \text{From } R_\theta = R_0(1 + \alpha\theta), \Rightarrow R_{100A} = R_{0A}(1 + 100\alpha_A)$$

$$\dots\dots(i) \text{ and } R_{100B} = R_{0B}(1 + 100\alpha_B)$$

$$\dots\dots\dots(ii)$$

$$\frac{R_{100A}}{R_{100B}} = \left(\frac{R_{0A}}{R_{0B}} \right) \times \left(\frac{1 + 100\alpha_A}{1 + 100\alpha_B} \right) = 1.5 \times \left(\frac{1 + 100\alpha_A}{1 + 100\alpha_B} \right)$$

$$= 1.5 \times \frac{1 + (100 \times 0.0004)}{1 + (100 \times 0.0003)} = 1.51$$

$$(ii) \quad \text{From } R = \rho \frac{l}{A}, \Rightarrow R_A = \rho_A \frac{l_A}{A_A} = \rho_A \frac{l_A}{\pi(0.6 \times 10^{-3})^2} \dots\dots(i)$$

$$\text{Also } R_B = \rho_B \frac{l_B}{\pi(0.4 \times 10^{-3})^2} \dots\dots(ii)$$

$$\text{From (i) and (ii),}$$

$$\Rightarrow \frac{R_A}{R_B} = \left(\frac{\rho_A}{\rho_B} \right) \times \frac{(0.6 \times 10^{-3})^2}{(0.4 \times 10^{-3})^2} = \left(\frac{\rho_A}{\rho_B} \right) \left(\frac{0.4}{0.6} \right)^2$$

$$\text{Since } \frac{R_A}{R_B} = 1.51, \Rightarrow \rho_A = 1.51 \times \left(\frac{0.4}{0.6} \right)^2 \times \rho_B = 3.4 \rho_B$$

$$\text{Hence } \rho_A : \rho_B = 3.4 : 1$$

Electrical power & heat

Mechanism of heating (Joule's effect)

The heat produced per second in a wire by current is the measure of the energy which it liberates in a second as it flows through a wire.

The heat is produced by free electrons as they move through the metal.

On their way, they collide frequently with their atoms and at each collision, they lose some of their K. E & give to the atoms they collide.

Thus as current flows through the wire, it increases the amplitude of vibration of the metal atoms, generating heat in the wire

The energy liberated per second in any electrical device is defined as its electrical power (P).

Electrical energy, $W = VIt$,

Where **I** is the current through the device, **V** is the p. d across it and **t** is the time taken for the current **I** to pass in it.

Electrical Power $\frac{\text{electrical energy}}{\text{time}}$

$$P = \frac{W}{t} = \frac{VIt}{t}$$

Thus **P = VI**

Power is measured in watts (W) when I is in ampere and V in volts.

Other units of power are kW and MW.

From $P = VI$, since $V = IR$,

$$\Rightarrow P = I^2 R, \text{ and}$$

$$P = \frac{V^2}{R}$$

If R is the total resistance of the cable (wire) or resistor and I is the current through the cable or resistor, then:

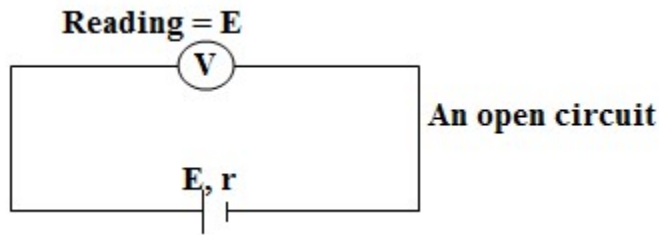
Power wasted as heat is $P = I^2 R$, and

Power delivered to the destination is $P = VI$.

Electromotive force, e.m.f (E) and internal resistance (r) of the cell

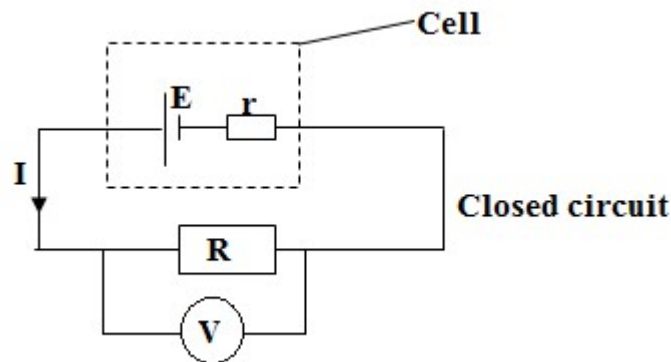
If a high resistance voltmeter is connected across the terminals of a dry cell, the meter may read about 1.5V, since practically no current flows from the cell and we say the cell is on open circuit since no current is drawn from it.

The p. d across the terminals of a cell or battery in an open circuit is equal to its electromotive force.



Internal resistance behaves as if it is a resistor in series with the battery.

When the cell is connected to an external resistor R , a steady current I flows in the circuit.



V is referred to as the terminal p.d of the battery and it is also the p.d across R .

Def'n: Terminal p.d is the p.d across any resistor or an external load connected to a cell or battery in a closed circuit.

P. d across R is given by $V = IR$ and p.d across r is $V' = Ir$.

But E.m.f, $E = \text{p.d across } R + \text{p.d across } r$

$$\Rightarrow E = IR + Ir \dots\dots\dots(i)$$

$$E = I(R + r)$$

$$\text{Hence } I = \frac{E}{(R+r)}$$

$$\text{But } V = IR$$

$$\Rightarrow E = V + Ir$$

$$\text{Thus } r = \frac{E-V}{I} \dots\dots\dots(ii)$$

Define the following

- **Electromotive force**
- **Internal resistance**
- **Terminal potential difference**

Example

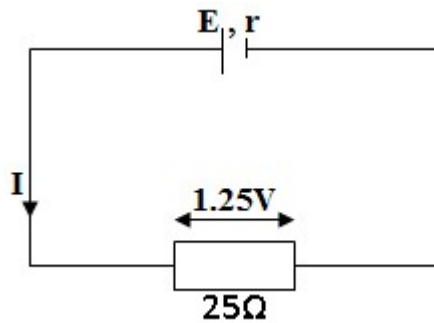
A battery 1.5V has a terminal p.d of 1.25V when a resistor of 25Ω is joined to it. Calculate the:

- (i) Current flowing in the circuit,
- (ii) Internal resistance of the cell and

- (iii) The terminal p. d when a resistor of 10Ω replaces the 25Ω resistor.

Solution

(i)



$$\begin{aligned} \text{From } V &= IR \\ \Rightarrow I &= \frac{V}{R} \\ &= 1.25/25 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad E &= V + Ir \\ \Rightarrow r &= \frac{E-V}{I} \\ r &= \frac{1.5 - 1.25}{0.05} = \frac{0.25}{0.05} \\ \therefore r &= 5\Omega \end{aligned}$$

- (iii) Replacing the 25Ω by a 10Ω resistor,

$$\text{Current } I = \frac{E}{(R+r)} = \frac{1.5}{(10+5)} = 0.1A$$

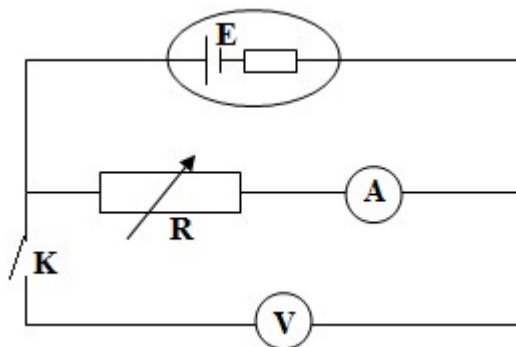
$$\text{Thus, } V = IR = 0.1 \times 10$$

$$\therefore V = 1V$$

Qn;

Describe an experiment to determine e.m.f and internal resistance of a cell using an ammeter, a resistance box and a voltmeter.

Solution

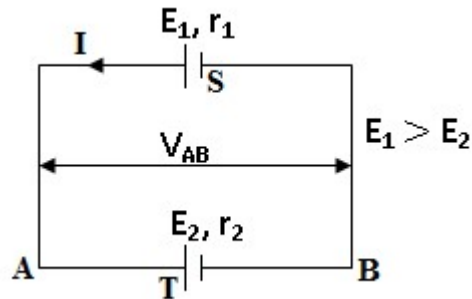


- The circuit is connected as shown above.
- Switch K is closed and the rheostat, R is adjusted to give a suitable value of I.
- The values of I and V are recorded.
- The procedure is repeated for different for various settings of R.
- A graph of V against I is plotted and its intercept on the V-axis is noted.
- This value of V gives the e.m.f of the cell.

- The slope, s is obtained and the internal resistance, r of the cell is got from $r = -s$.

TERMINAL P.D WITH CURRENT IN OPPOSITION TO e.m.f

Suppose a current is passed through a battery in opposition to its e.m.f.



A supply (S) sends current a current I through the battery (T) in opposition to its e.m.f.

Net e.m.f, $E = E_1 - E_2$

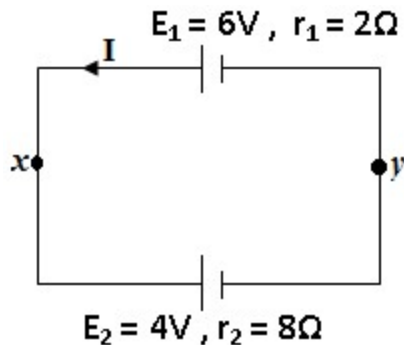
Total resistance $= r_1 + r_2$

Thus, current $I = \frac{E}{r_1 + r_2}$

Hence, $V_{AB} = E_1 - Ir_1$ or $V_{AB} = E_2 + Ir_2$

EXAMPLE

A circuit below shows two batteries in opposition to each other. One has an e.m.f $E_1 = 6V$ and internal resistance $r_1 = 2\Omega$. And the other has an e.m.f $E_2 = 4V$ and internal resistance $r_2 = 8\Omega$. Calculate the current in the circuit and the p.d across the points xy .



$$\begin{aligned}\text{Net e.m.f } E &= E_1 - E_2 \\ &= 6 - 4 \\ &= 2V\end{aligned}$$

$$\begin{aligned}\text{Total resistance} &= r_1 + r_2 = 2 + 8 \\ &= 10\Omega\end{aligned}$$

$$\text{Current } I = \frac{E}{r_1 + r_2} = \frac{2}{10} = 0.2A$$

Considering a cell of e.m.f E_2

$$V_{xy} = E_2 + Ir_2$$

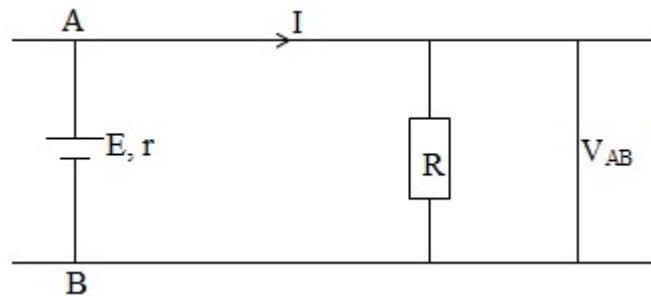
$$\begin{aligned}
 &= 4 + 0.2 \times 8 \\
 &= \mathbf{5.6V} \\
 &\mathbf{OR}
 \end{aligned}$$

Considering a cell of e.m.f E_1

$$\begin{aligned}
 V_{xy} &= E_1 - Ir_1 \\
 &= 6 - 0.2 \times 2 \\
 &= \mathbf{5.6V}
 \end{aligned}$$

Power output and efficiency

Consider the following diagram



R is the load resistance.

Total power supplied/generated by the source also called power input is given by;

$$P_{\text{input}} = EI$$

The power delivered to the external resistance is called the output power.

$$P_{\text{output}} = V_{AB}I$$

$$\text{But } V_{AB} = IR$$

$$\Rightarrow P_{\text{output}} = I^2 R$$

Power lost to the internal resistance r is $P_{\text{lost}} = I^2 r$.

Total power input = power output + power lost

$$EI = I^2 R + I^2 r$$

$$\Rightarrow E = I(R + r)$$

$$\therefore I = \frac{E}{(R + r)}$$

NB: power input = power supplied = power generated.

EFFICIENCY

The percentage ratio of power output to power input is called the **efficiency**, denoted by η .

$$\eta = \frac{P_{out}}{P_{in}}$$

$$\eta = \frac{V_{AB} I}{EI} = \frac{V_{AB}}{E}$$

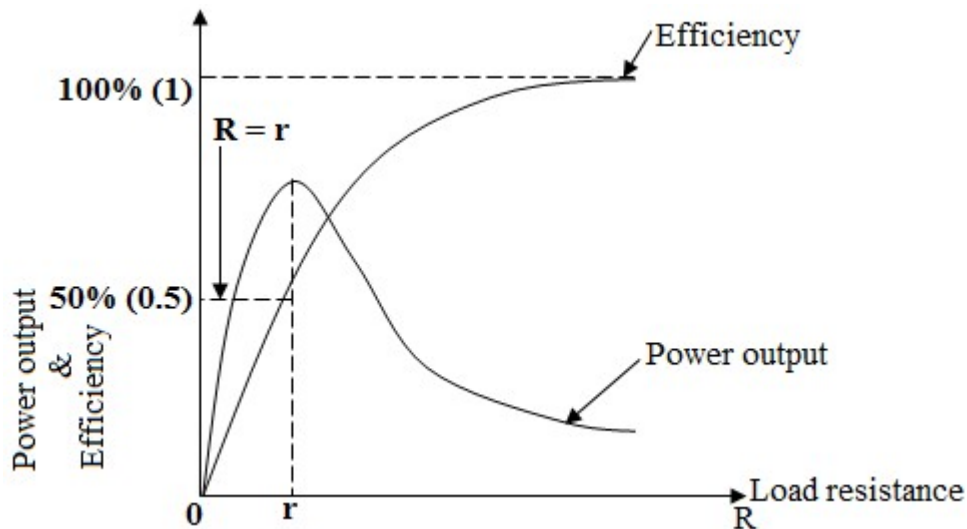
But V_{AB}

$$\therefore \eta = \frac{R}{R + r} = \frac{ER}{(R+r) IR} =$$

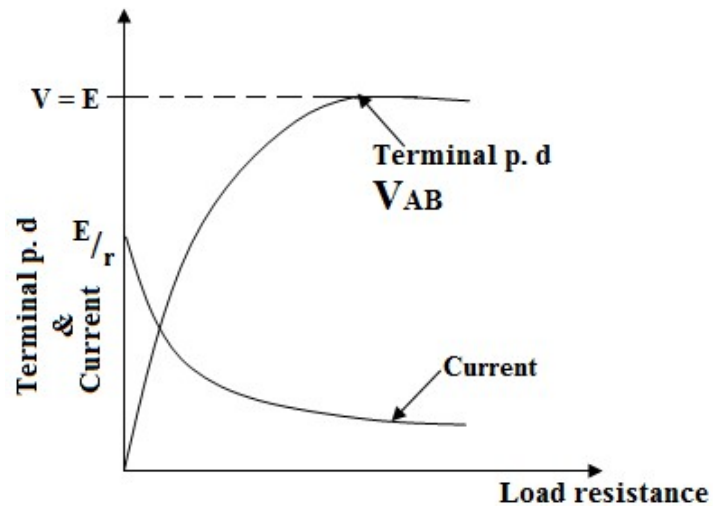
The efficiency tends to unity or 100% as the load resistance R tends to infinity. For high efficiency, the load resistance must be several times the internal resistance of the source.

When $R = r$, $\eta = \frac{R}{2R} = 0.5 = 50\%$.

Variation of efficiency and power against load resistance



Variation terminal p. d and current against load resistance.



1. A battery of e.m.f 18.0V and internal resistance 3.0Ω is connected to a resistor of resistance 8Ω . Calculate the;
- Power generated
 - Efficiency
 - If the 8Ω resistor is replaced by a variable resistor, sketch graphs to show the variation of power and efficiency with the load.

Solution

- (i) Power generated $P_{\text{gen}} = EI$

But $E = IR + Ir$.

$$\Rightarrow I = \frac{E}{R+r}$$

$$\therefore P = \frac{E^2}{R+r} = \frac{18^2}{8+3} = 24.45W$$

- (ii) $\eta = \frac{P_{\text{out}}}{P_{\text{gen}}} = \frac{IV}{IE} = \frac{V}{E} \dots \dots (i)$

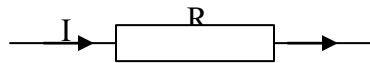
But $V = IR = \left(\frac{ER}{R+r}\right)$, since $I = \frac{E}{R+r}$

Equation (i) becomes; $\eta = \frac{R}{R+r} = \frac{8}{8+3} = 0.727$

$$\eta = 72.7\%$$

- (iii) See notes.

2. Derive the expression for the electrical energy dissipated in a resistor of resistance, R ohms carrying a current I amperes for t seconds.

Solution

Suppose a current I flows through a resistor for time t,

The total charge supplied, $Q = It \dots \dots \dots (i)$

If the p.d is V, then work done W to transfer charge Q is given by $W = QV \dots \dots \dots (*)$

But $V = IR \dots \dots \dots (ii)$

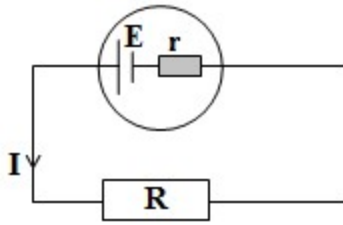
Putting (i) and (ii) in eqn *, we get; $W = It \times IR \therefore \text{Electrical energy}$

dissipated in a resistor of resistance $R\Omega$, $W = I^2 R t$

Maximum power output theorem

Qn: A battery of e.m.f E and internal resistance r is connected to a resistor of variable resistance R. Obtain the expression for maximum power dissipated in the resistor. (5 mks)

Solution



From $power_{output} = I^2 R$ and $I = \frac{E}{R+r}$

$$\Rightarrow P_{output} = \frac{E^2 R}{(R+r)^2}$$

Maximum power output (P_{max}) occurs when $R = r$.

$$\Rightarrow P_{max} = \frac{E^2 R}{(R + R)^2} = \frac{E^2 R}{4R^2}$$

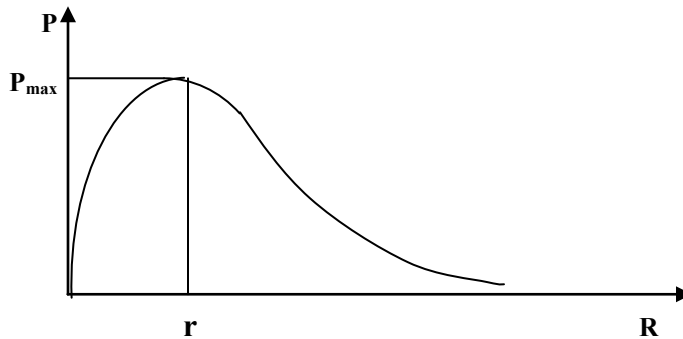
$$\therefore P_{max} = \frac{E^2}{4R} \text{ or } \frac{E^2}{4r}$$

TASK, Show by differential approach that maximum power output occurs when $R = r$

(Hint use quotient rule and differentiate the expression for power output with respect to external resistance i.e

$\frac{d(\text{power output})}{d(\text{external resistance})}$ or simply $\frac{dp_{output}}{dR}$, and later for maximum power $\frac{dp_{output}}{dR} = 0$)

If E and r are constant and a graph of power output P against load resistance R is plotted, it is found to have a maximum value when $R = r$.



As R tends to zero, P tends to zero

As R tends to infinity, P tends to zero.

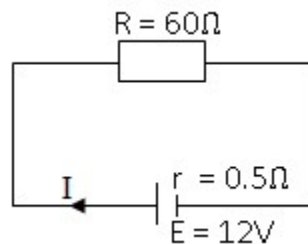
Qn: A battery of e.m.f 12V and internal resistance 0.5Ω is connected across a 60Ω load.

Calculate:

- (i) The rate of energy conversion in the battery,
- (ii) The rate of dispersion of electrical energy in a resistor,
- (iii) Comment on the difference between (i) and (ii),
- (iv) Sketch a graph showing variation of power output from the circuit and load resistance.

Solution.

(i)



$$P = EI$$

$$= I(R + r)$$

$$I = \frac{E}{(R + r)}$$

$$P = \frac{E^2}{(R + r)}$$

$$= \frac{12^2}{(60 + 0.5)}$$

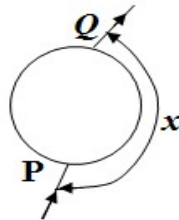
$$\therefore \text{Power input } P = 2.38\text{W}$$

$$(ii) \quad \text{Power output} = \frac{E^2 R}{(R + r)^2} = \frac{12^2 \times 60}{60.5^2} = 2.36\text{W}$$

- (iii) The power output is less than the power input because a lot of energy is wasted in overcoming the internal resistance of the battery.
- (iv) **See notes**

Qn:

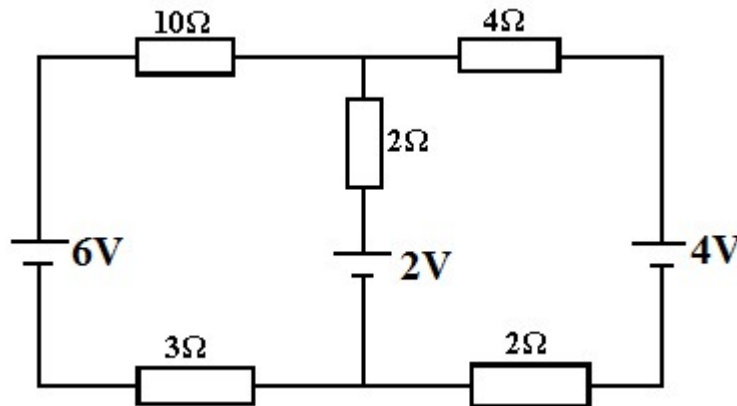
- (a) (i) State ohm's law. (1 mk)
- (ii) Describe an experiment which verifies ohm's law. (4 mks)
- (b) (i) What is meant by e.m.f and internal resistance of a battery? (2 mks)
- (ii) A wire of diameter d , length l and resistivity ρ forms a circular loop. A current enters and leaves the loop at points P and Q respectively as shown in the figure below.



Show that the resistance R of the wire is given by the expression:

$$R = \frac{4\rho x(l-x)}{\pi d^2 l} \quad (4 \text{ mks})$$

(c)



In the circuit diagram above;

- (i) Find the values of the current through the 10Ω , 3Ω and 4Ω resistors. (7 mks)
- (ii) Calculate the power dissipated in the 5Ω resistor. (2 mks)

Exercise

1. When a 10Ω resistor is connected across the terminals of the cell of e.m.f E and internal resistance, r , a current of 0.1A flows through the resistor. If the 10Ω is replaced with a

3 Ω resistor, the current increases to 0.24A. Find E and r.

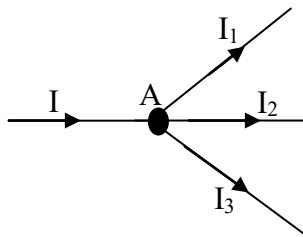
2. A voltmeter with resistance 20 k Ω is connected across the power supply and gives a reading of 44V. Another voltmeter with a resistance of 50 k Ω connected across the same supply gives a reading of 50V. Find the e.m.f of the power supply.

Kirchoff's laws of circuit networks

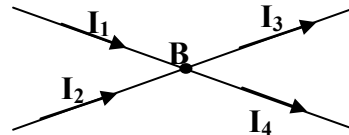
Law I

The sum of current entering any junction of a point is equal to the sum of current leaving that junction. **Or**

The algebraic sum of currents at any junction or point is equal to zero (0). That is $\sum I = 0$. Here, I is +ve when entering any junction and – ve when leaving the junction.



At point A, $I = I_1 + I_2 + I_3$



At junction B, $I_1 + I_2 = I_3 + I_4$

Also, $I_1 + I_2 - I_3 - I_4 = 0$

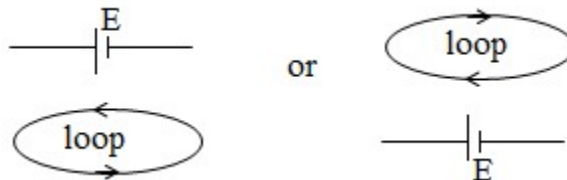
Or $\sum_1^4 I_n = 0$

Law II

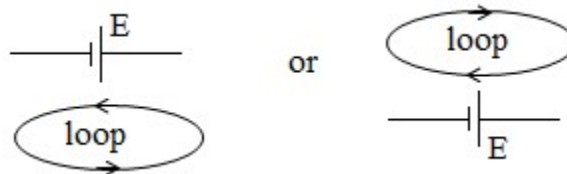
In any closed loop the algebraic sum of potential drops is equal to the algebraic sum of the e.m.f. $\sum IR = \sum E, \sum V = \sum E$.

Sign allocation

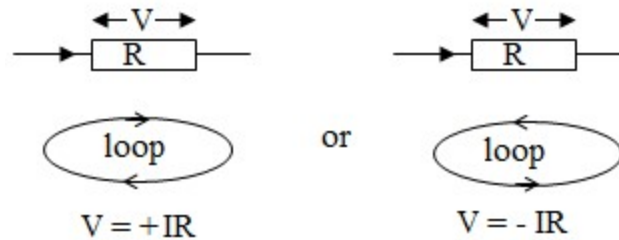
E is +ve if the loop moves from the –ve terminal of the cell to the +ve terminal.



E is –ve if the loop moves from the +ve terminal to the –ve terminal.

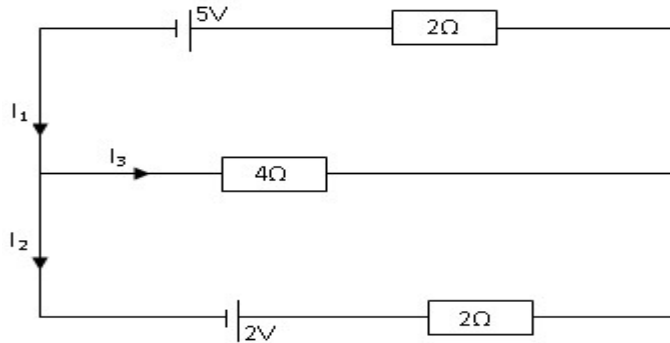


The p. d V is +ve if the current flows in the direction of the loop and –ve if the p. d is in the reverse direction.

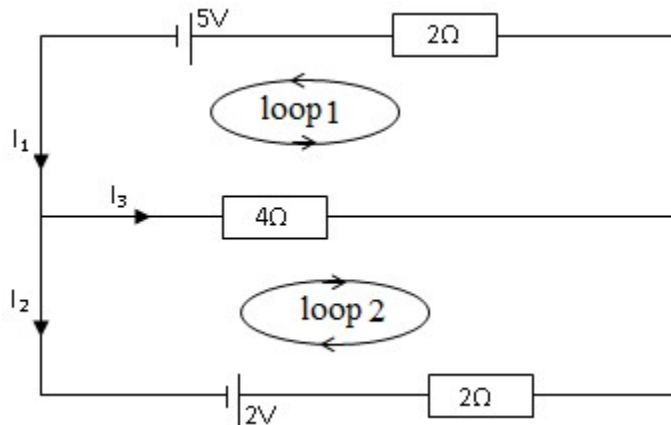


Examples

1. Use Kirchhoff's laws to find currents I_1 , I_2 and I_3 in the circuit below.



Solution



At the junction, $I_1 = I_2 + I_3$ (i)

Considering;

Loop 1,

$$2I_1 + 4I_3 = -5 \text{(ii)}$$

Loop 2,

$$4I_3 - 2I_2 = -2$$

$$\Rightarrow 2I_3 - I_2 = -1 \text{(iii)}$$

From eqn (ii), $2I_1 + 4I_3 = -5$

$$\Rightarrow I_1 = \frac{-5 - 4I_3}{2} \text{(iv)}$$

From (iii), $2I_3 - I_2 = -1$

$$\Rightarrow I_2 = 2I_3 + 1 \text{(v)}$$

Sub eqn (iv) & (v) in (i), we get;

$$I_1 = I_2 + I_3$$

$$\frac{-5-4I_3}{2} = 2I_3 + 1 + I_3$$

$$\begin{aligned} -5 - 4I_3 &= 2(2I_3 + 1 + I_3) \\ &= 4I_3 + 2 + 2I_3 \end{aligned}$$

$$\Rightarrow -10I_3 = 7$$

$$\therefore I_3$$

I_3 in Putting value of

$$I_2 = 2I_3 + 1$$

$$I_2 = 2(-0.7) + 1$$

$$I_2 = -0.4A$$

Putting value of I_3 in eqn (iv), we get;

$$I_1 = \frac{-5 - 4I_3}{2}$$

$$I_1 = \frac{-5 - 4(-0.7)}{2}$$

$$I_1 = -1.1A$$

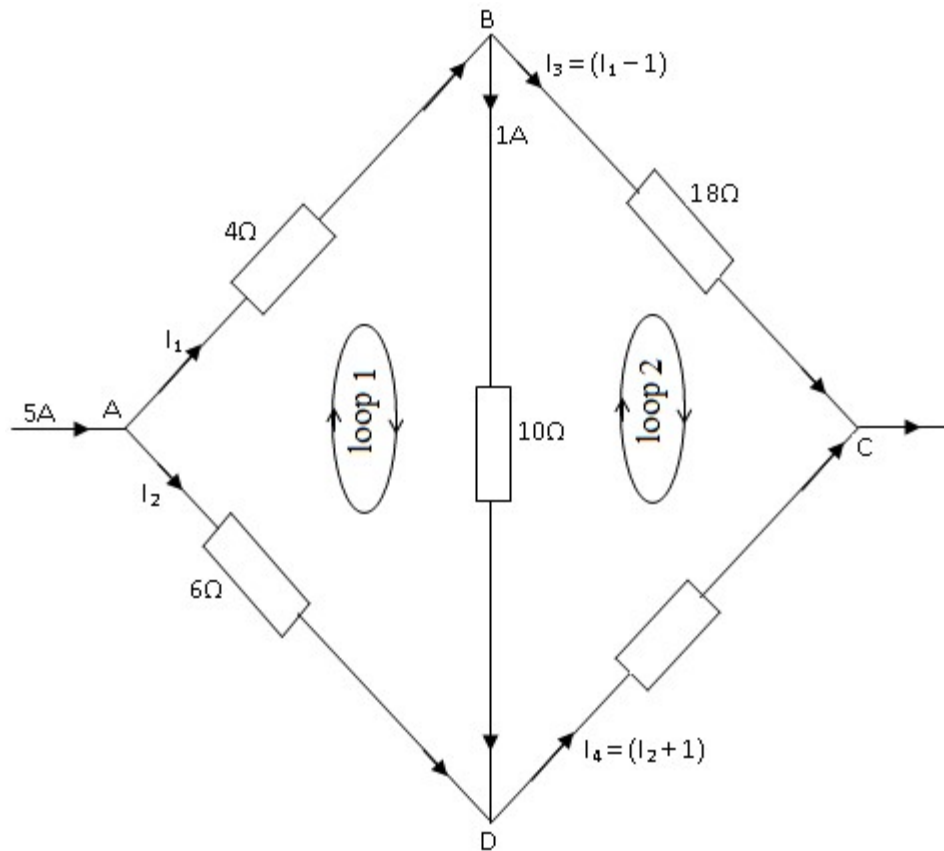
Thus, $I_1 = -1.1A$, $I_2 = -0.4A$ and $I_3 = -0.7A$

2. from the diagram above, find the

- (i) Current through BC,
- (ii) Value of R,
- (iii) P. d between A & C.

Solution

- (i)



$$I_1 + I_2 = 5 \dots\dots\dots(i)$$

Considering;

Loop 1

$$4I_1 + 10 - 6I_2 = 0$$

$$\Rightarrow 4I_1 - 6I_2 = -10 \dots\dots\dots(ii)$$

Loop 2,

$$-10 - R(I_2 + 1) + 18(I_1 - 1) = 0$$

$$\Rightarrow 18I_1 - I_2R - R = 28 \dots\dots\dots(iii)$$

On solving the equations gives;

$$I_1 = 2A \text{ and}$$

$$I_2 = 3A$$

Thus, current through BC is $I_1 - 1$

$$= 2 - 1$$

$$= 1A$$

(ii) Value of R.

From eqn (iii),

$$18I_1 - I_2R - R = 28$$

$$36 - 3R - R = 28$$

$$4R = 8$$

$$\therefore R = 2\Omega$$

(iii) P. d across A & C,

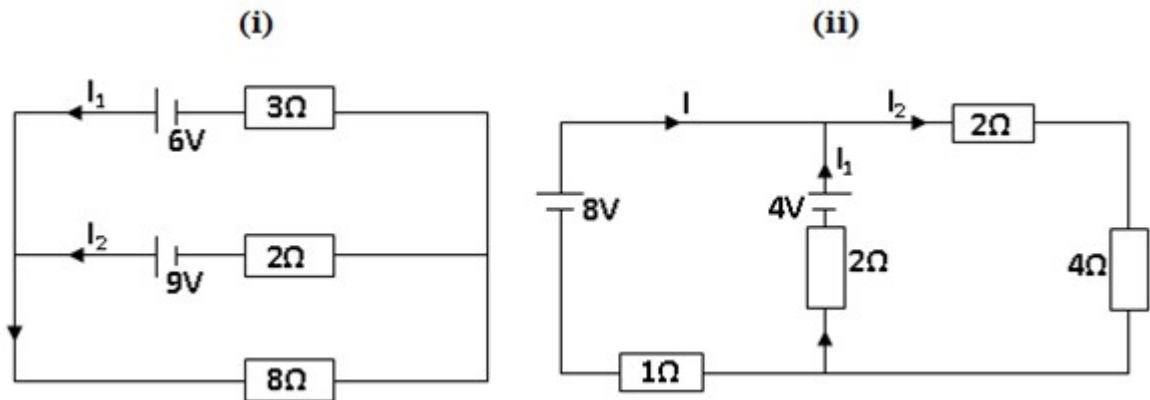
$$\begin{aligned} V_{AC} &= V_{AB} + V_{BC} \\ &= 4I_1 + 18I_3 \\ &= (4 \times 2) + (18 \times 1) \\ &= 8 + 18 \\ &= 26V \end{aligned}$$

Alternatively, $V_{AC} = V_{AD} + V_{DC}$ or $V_{AB} + V_{BD} + V_{DC}$.

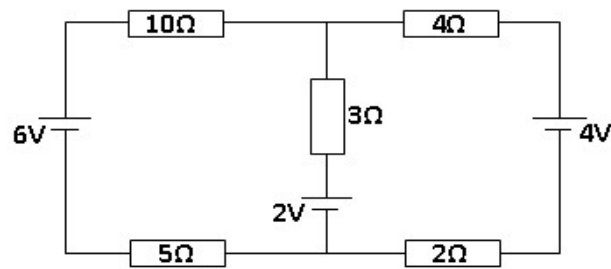
NB: When a negative value of current is obtained, it indicates that current is flowing in a direction opposite to that shown in the diagram. I.e. it is flowing against the e.m.f of the battery.

Trial questions

1. Find the value of currents I , I_1 & I_2 as show in the figure below.



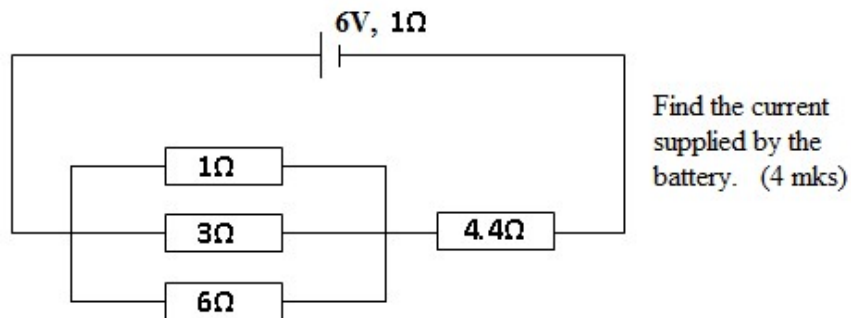
2. (a) State ohm's law.
- (b) Sketch the current – voltage characteristics (graphs) for very small voltage ranges of;
 - (i) a piece of metallic wire,
 - (ii) two electrodes immersed in an electrolyte
- (c) Give two devices to which ohm's law doesn't apply and sketch their current – voltage graphs.
- (d) Draw a circuit diagram to show how the value of a high resistance can be determined using a low resistance meter and a high voltmeter. Justify your connection of the meter and voltmeter.
- (e)



In the circuit diagram above, find;

- (i) the values of current through 10Ω , 3Ω and 4Ω resistors. (7 mks)
- (ii) the power dissipated in the 5Ω resistor. (4 mks)

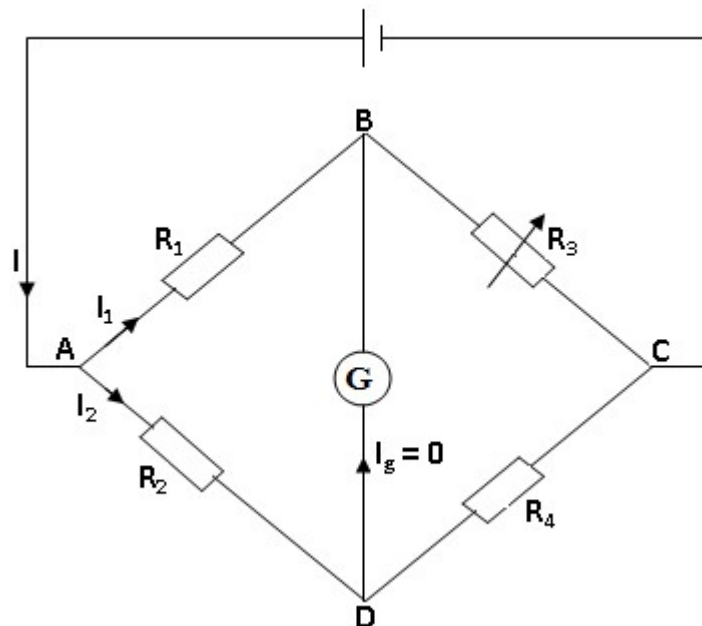
3.



Wheatstone bridge

A Wheatstone bridge circuit provides a more accurate method of measure an unknown resistance accurately.

The circuit is as shown below.



- Four resistors are joined as shown above, one of them being an unknown resistor whose resistance has to be measured.
- A sensitive galvanometer is connected between B & D so as to show balance and a cell which provides a steady current is also included in the circuit.
- If R_1 is the unknown resistor, R_2 & R_4 known, then R_3 is adjustable adjusted until no current flows through the galvanometer G, i.e G shows no deflection. The bridge circuit is then said to be balanced.

In this condition it can be shown that $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

PROOF OF BALANCE CONDITION.

At balance no current flows through G, therefore the p.d across BD is zero.

\Rightarrow p.d across AB = p.d across AD,

Current through R_1 = current through R_3

Hence, $I_1 R_1 = I_2 R_2$ (i)

Similarly, p.d across BC = p.d across DC and the same current I_2 flows through R_4 .

Thus $I_1 R_3 = I_2 R_4$ (ii)

Dividing the two equations, we get;

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

The above equation is the condition for a Wheatstone bridge to balance.

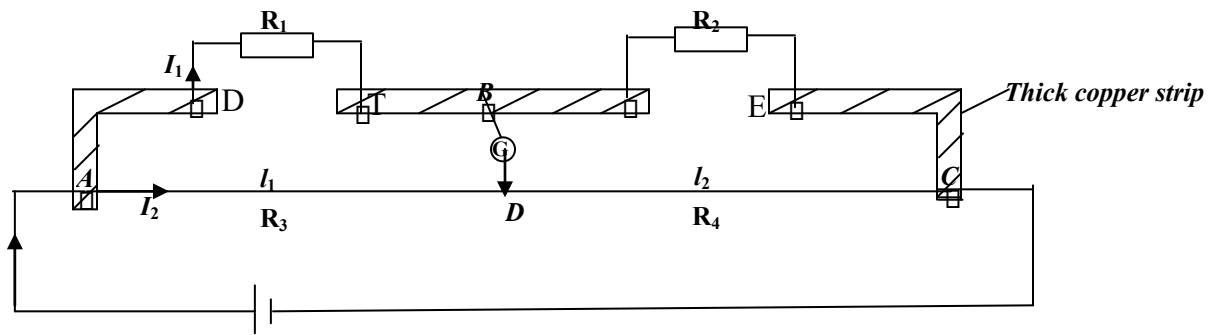
The relationship is true even when the resistors are interchanged.

Questions

1. Derive the condition for a Wheatstone bridge to be balanced. (4 mks)
2. In a Wheatstone bridge, the ratio arms R_1 and R_2 are approximately equal. When $R_3 = 500\Omega$, the bridge is balanced. On interchanging R_1 and R_2 , the value of R_3 for balancing is 505Ω . Find the value of R_4 and the ratio $R_1 : R_2$. (502.5Ω , 1:1.005)

Simple metre bridge

It is a form of a Wheatstone bridge with a resistance wire of uniform cross section area mounted on a metre rule. It is used to measure resistance, resistivity and temperature coefficient of resistance. It is a practical form of a Wheatstone bridge



- The resistors R_3 & R_4 constitute of a wire AC of uniform cross section usually 1m long made of an alloy such as constantan.
- The ratio of R_3 to R_4 is altered by changing the position on the wire of the movable contact or jockey D.
- The other arm of the bridge contains the unknown resistor R_1 and a standard resistor R_2 .
- Thick copper strips of low resistance connect the various parts. The position of D is adjusted until there is null or zero deflection on G.

The connections are checked by placing D at A and then at C. deflections must be opposite. By trial and error, the balance point can be obtained and the balance length l_1 & l_2 are measured.

Proof for balance condition.

Suppose at balance current I_1 flows through R_1 & R_2 and current I_2 flows through AC

P.d across R_1 = P.d across AD

$$I_1 R_1 = I_2 R_3 \dots \dots (i)$$

P.d across R_2 = P.d across DC

$$I_1 R_2 = I_2 R_4 \dots \dots (ii)$$

Dividing the two equations, we get;

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$\text{From } R = \rho \frac{l}{A}$$

$$R_3 = \rho \frac{l_1}{A} \text{ and } R_4 = \rho \frac{l_2}{A}$$

$$\text{Substituting back gives } \frac{R_3}{R_4} = \frac{l_1}{l_2}$$

Where A is the cross sectional area of the uniform wire of the metre bridge, ρ is the resistivity of the material of wire, l_1 is the balance length of the metre bridge from the left hand side. Where l_2 is the balance length of the metre bridge from the right hand side.

Hence

The above equation is the condition for a metre bridge to balance.

If the length of the wire is 100cm then $l_2 = (100 - l_1)\text{cm}$

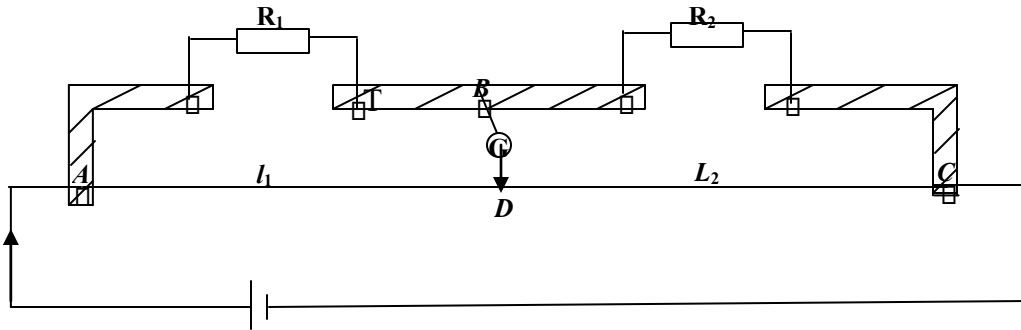
NB: The balance point should be close to the middle because:

- ✓ If it is at the end such that l_1 or l_2 is small, then the resistance of the end connections AD and EC will not be negligible vis-à-vis the resistance of the lengths l_1 and l_2 .
Hence the relation $\frac{R_2}{R_1} = \frac{l_2}{l_1}$ will be invalid.
- ✓ The percentage error in reading l_1 & l_2 is high if their values are small. So by setting R_1 so that the balance point is near the middle, the degree of accuracy is improved.
- ✓ The bridge is more sensitive near the middle since the unbalanced current is larger per mm change in position.

Qn: (a) Draw a labeled diagram of a metre bridge and derive the condition for the balance. (5 mks)

(b) Explain why the balance point should be close to the middle of the wire. (2 mks)

End errors or end corrections



- Owing to imperfect electrical contacts at A and C, contacts have small resistance which affect the accuracy of the results.
- But the effect on the accuracy becomes less significant if the balance point is near the 50cm mark.
- For accuracy, the contact resistances have to be considered. The contact resistances at A and C are equivalent to extra lengths e_1 and e_2 of the slide wire. e_1 and e_2 are called end corrections or end errors.
- Hence end errors have to be added to the balance lengths to account for the resistances at the contacts at the ends of the slide wire.
- At balance, $\frac{R_1}{R_2} = \frac{l_1 + e_1}{l_2 + e_2}$ (i)

When R_1 and R_2 are interchanged

$$\frac{R_2}{R_1} = \frac{l'_1 + e_1}{l'_2 + e_2}$$

Where l'_1 and l'_2 are new balance lengths from the left hand side and right hand side respectively.

Solve equations (i) and (ii) simultaneously to obtain e_1 and e_2 .

Example

When resistors of 3Ω and 5Ω are connected in the LHG and RHG of a metre bridge respectively, a balance point is obtained at 37.4cm from LHS. When the resistors are interchanged, the balance point is 62.8cm from the LHS. The resistance of the slide wire is 10Ω . Calculate the end corrections and resistance of the contacts at LHS and RHS.

Solution

$$\frac{3}{5} = \frac{37.4 + e_1}{62.6 + e_2} \dots\dots\dots(i)$$

After interchanging them

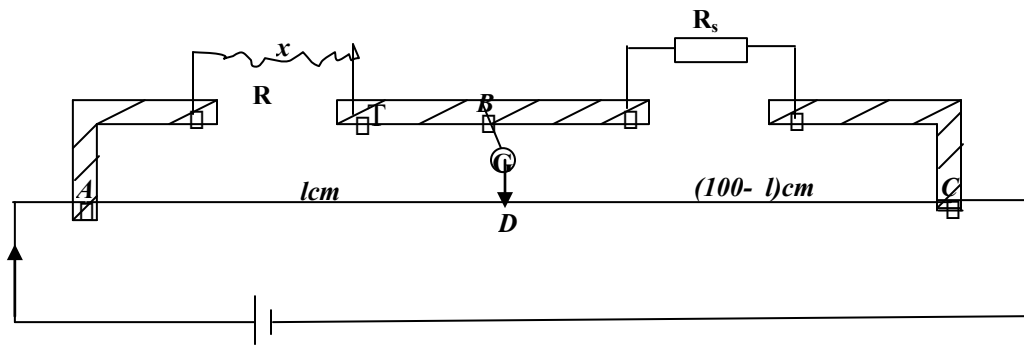
$$\frac{5}{3} = \frac{62.8 + e_1}{37.2 + e_2} \dots\dots\dots (ii)$$

Solving equations (i) and (ii) simultaneously, we obtain $e_1 = 0.7\text{cm}$ and $e_2 = 0.9\text{cm}$.

Resistances from zero end $r_1 = 0.07\ \Omega$, on the right end $r_2 = 0.09\ \Omega$.

Measurement of resistivity of a wire using a metre bridge

Qn: Describe an experiment to determine the electrical resistivity of a material in the form of a wire using a metre bridge. (7 mks)



- The circuit is connected as shown above
- A specimen wire of known length say x is connected in the left hand gap of a metre bridge with a standard resistor, R_s in the right hand gap of the bridge.
- The slider is moved along AC until the galvanometer indicates zero deflection.
- At this point the metre bridge is balanced. The balance length l from the left hand side of the metre bridge is measured.
- The experiment is repeated for different values of x and the corresponding balance lengths l are measured and recorded. Suppose R_x is the resistance of the wire of length x ,

$$\text{At balance, } \frac{R_x}{R_s} = \frac{l}{100-l}.$$

$$\Rightarrow R_x = \frac{R_s l}{100-l}$$

- A graph of R_x against x is plotted and its slope S determined.
- The resistivity of the wire, ρ is then obtained from; $\rho = SA$, Where A is the area of the specimen wire.

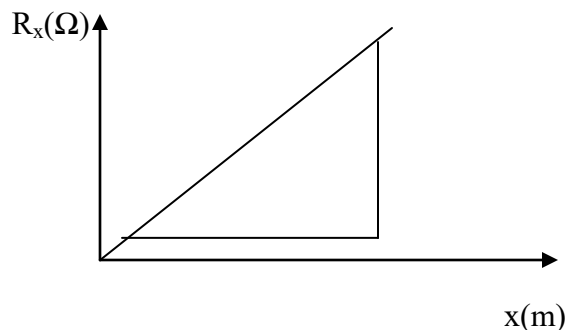
Theory of experiment (not necessary in the exam, but no penalty)

$$\text{At balance, } \frac{R_x}{R_s} = \frac{l}{100-l}.$$

$$\Rightarrow R_x = \frac{R_s l}{100-l}$$

$$\text{But } R_x = \frac{\rho x}{A}$$

Thus a graph of R_x against l_x is plotted



The slope of the graph is determined and is equal to $S = \frac{R_x}{x} = \frac{\rho}{A}$
 $\Rightarrow \rho = SA$

But $A = \frac{\pi d^2}{4}$, where d is the diameter of the wire, measured by a micrometer screw gauge.

Hence the resistivity of the wire, $\rho = S \frac{\pi d^2}{4}$

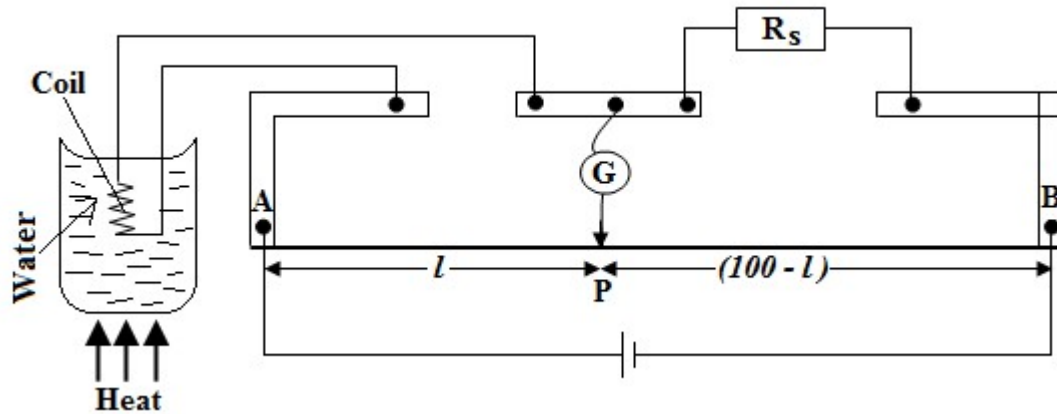
When a graph of R_x is plotted against x , a straight line passing through the origin is obtained. This means that $R_x \propto x$, i.e the resistance is proportional to the length of a wire.

NB: The above experiment can be used to describe an experiment to determine the relationship between the resistance and the length of a wire:

Temperature coefficient (α) of a metal wire

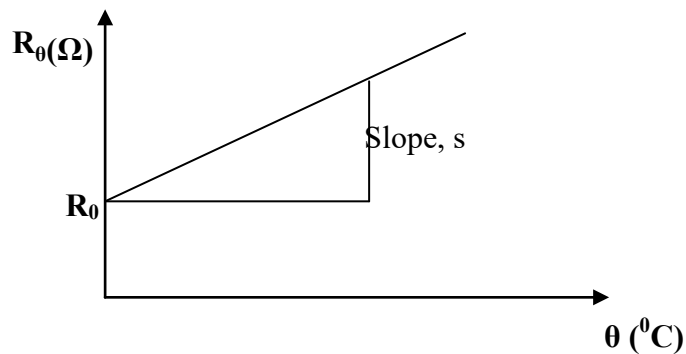
The mean temperature coefficient of a material can be defined as the fractional increase in resistance at 0°C for every $^\circ\text{C}$ increase in temperature.

Experiment to determine temperature coefficient of resistance using the metre bridge



- A specimen wire is made into a coil, and its ends connected to the left hand gap of the bridge.
- The coil is immersed/lowered in a water bath and heated. The water is stirred so as to uniformly distribute the heat. The temperature θ is then measured and recorded.
- With the switch closed, a jockey is tapped on the slide wire until a point when the galvanometer shows no deflection.
- The resistance, R_θ corresponding to the temperature,

The temperature coefficient of resistance, α is then calculated from $\alpha = \frac{s}{R_0}$,
Where R_0 is the intercept on the R_θ axis



Examples

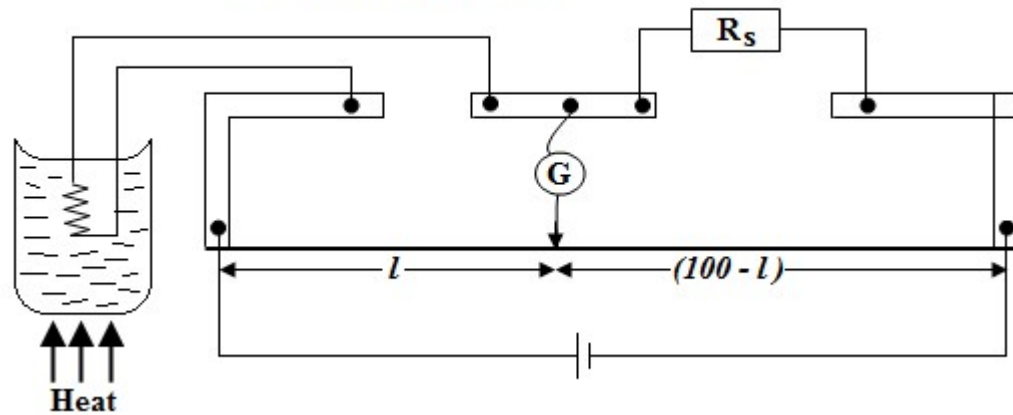
- 2.(i) Define temperature coefficient of resistance. (1 mk)
- (ii) When a coil X is connected across the Left hand gap of a metre bridge and heated to a temperature of 30°C , the balance point is found to be 51.5cm from the left hand side of the slide wire. When the temperature is raised to 100°C , the balance point is 54.6cm from the left hand side. Find the temperature coefficient of resistance of X.

Solution

- (i) See notes

(ii)

Sketch for the explanation

For $\theta = 30^\circ\text{C}$,

At balance point

$$\frac{X_{30}}{R_s} = \frac{51.5}{100 - 51.5} = 1.062 \dots \dots \dots \text{(i)}$$

At 100°C ,

$$\frac{X_{100}}{R_s} = \frac{54.6}{100 - 54.6} = 1.203 \dots \dots \dots \text{(ii)}$$

From (i) and (ii)

$$\Rightarrow \frac{X_{30}}{X_{100}} = \frac{1.062}{1.203} = 0.883 \dots \dots \dots \text{(iii)}$$

From $R_\theta = R_0(1 + \alpha\theta)$, $\Rightarrow R_{100} = R_0(1 + 100\alpha)$ and $R_{30} = R_0(1 + 30\alpha)$

$$\therefore \frac{X_{30}}{X_{100}} = \frac{(1 + 30\alpha)}{(1 + 100\alpha)} \dots \dots \dots \text{(iv)}$$

From (iii) and (iv),

$$\Rightarrow \frac{(1 + 30\alpha)}{(1 + 100\alpha)} = 0.883$$

$$\therefore \alpha = 2.0 \times 10^{-3} \text{K}^{-1}$$

Note:

- The Wheatstone bridge can measure accurately resistances from 1Ω to $10^6\Omega$.
- It cannot be used to measure resistances less than 1Ω because the contact resistances (resistance of the connecting wires) become comparable to the test resistances.
- The bridge cannot be used to measure accurately the resistance above $10^6\Omega$ because the galvanometer becomes insensitive. Very high resistances may allow very small currents or none to pass through.

Exercise

1. Two resistance coils P and Q are placed in the gaps of a metre bridge. A balance point is found when the movable contact touches the bridge wire at a distance of 35.5cm from the end joined to end P. When the coil Q is shunted with a resistance of 10Ω , the balance point is moved through a distance of 15.5cm. Find the values of the resistances P and Q.

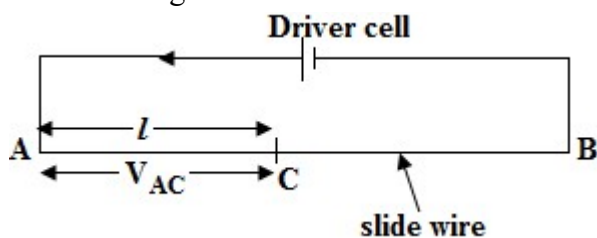
2. In a metre bridge when a resistance in left gap is 2Ω and unknown resistance in right gap, the balance point is obtained from the zero end at 40cm on the bridge wire. On shunting the unknown resistance with 2Ω , find the shift of the balance point on the bridge wire. (22.5cm)
3. With a certain resistance in the left gap of a slide wire, the balancing point is obtained when a resistance of 10Ω is taken out from the resistance box. On increasing the resistance from the resistance box by 12.5Ω , the balancing point shifts by 20cm. Find the value of unknown resistance. (15Ω)

Potentiometers

A potentiometer is an arrangement which measures p.d accurately. It can be adopted to measure current and resistance.

Principle of a potentiometer

A Potentiometer consists of a uniform resistance wire AB of definite length (usually 1m long) mounted on a metre rule. A driver cell (usually called an accumulator) is used to maintain a steady current through the wire.



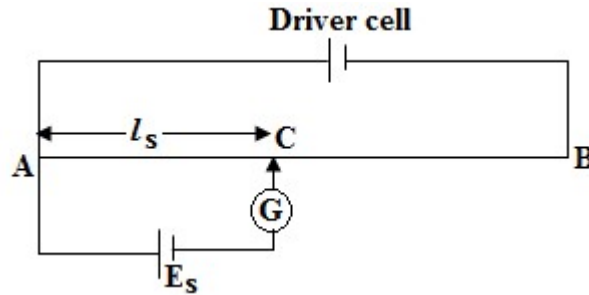
Since the wire is uniform, its resistance per centimeter is also constant. Thus the p. d between end A of the wire and any point C on it is proportional to the length l of the wire between A & C. The unknown p.d is compared with the p.d across the slide wire of the potentiometer.

N.B

The **principle** of the **potentiometer** is that: For a wire having uniform area of cross section and uniform composition, the potential drop is directly proportional to the length of wire. The above **principle** is valid when **potentiometer** is used in comparing the EMF of two cells.

(This is simply because the cross-section area is considered the same, and also the resistance per unit length is considered to be the same)

Calibration/ standardization of a potentiometer

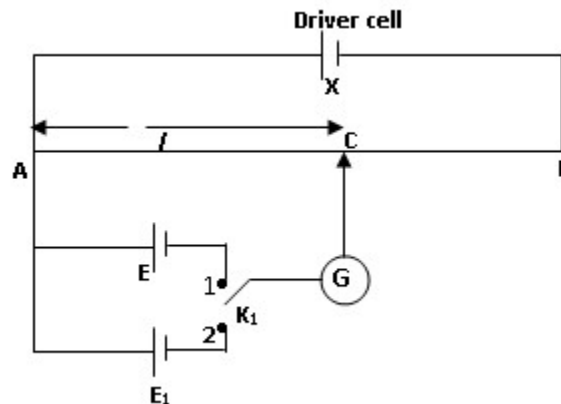


- With a standard cell of e.m.f, E_s in the circuit, the jockey (sliding contact) is moved along AB to find the balance length l_s when the galvanometer shows no deflection.
- At balance, the p.d across AC = the e.m.f, E_s ,
- But the p.d across AC, $V_{AC} \propto \text{balance length, } l_s$ i.e. $V_{AC} = kl_s$

$$\Rightarrow E_s = kl_s.$$

Thus, the calibration constant $k = \frac{E_s}{l_s}$ from which any potential difference across the slide wire can be obtained

Measuring e.m.f / comparing e.m.f using a potentiometer.



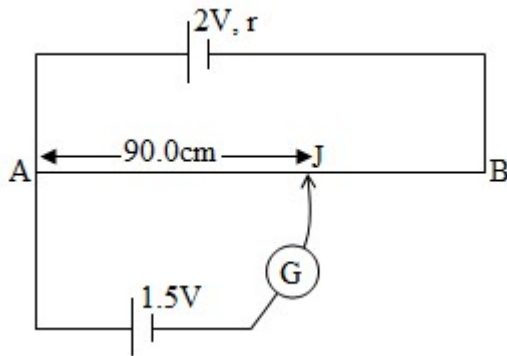
- Switch K_1 is connected to position 1 and the sliding contact (jockey) is moved along the wire AB until the galvanometer indicates zero deflection.
- The corresponding balance length l is measured.
- At balance, $E = kl \dots \dots \dots (i)$
- K_1 is then connected to position 2, the jockey is moved along the slide wire AB until the galvanometer indicates zero deflection, and the corresponding balance length l_1 is measured.
- At balance, $E_1 = kl_1 \dots \dots \dots (ii)$
- Dividing the two equations, gives $\frac{E}{E_1} = \frac{l}{l_1}$

- If one of the cells is a standard cell whose e.m.f is known, then the e.m.f of the other cell can be calculated accurately.

Qn:

1. Describe with the aid of a circuit diagram, how a slide wire potentiometer can be used to measure e.m.f of a cell. (4 mks)
2. An accumulator of e.m.f 2.0V is connected across a uniform wire of length 1.0m and resistance 8.0Ω . A cell of e.m.f 1.5V is connected in series with a galvanometer and connected across a length, l of the slide wire. The galvanometer shows no deflection when l is 90.0cm. Find the internal resistance of the accumulator. (4 mks)

Solution



$$R_{AJ} = \frac{90}{100} \times 8 = 7.2\Omega$$

From $E = IR + Ir$

$$\Rightarrow 2 = I(r + 8) \dots \dots \dots (i)$$

But $V_{AB} = IR_{AB} = 1.5V$

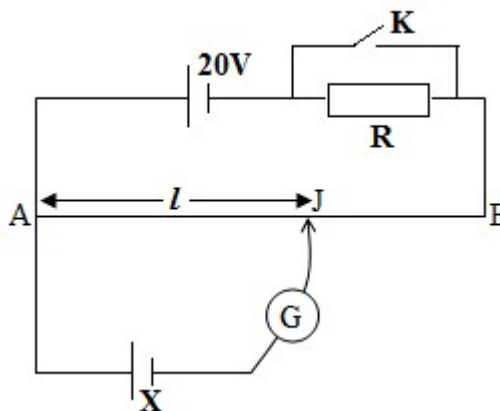
$$\Rightarrow 7.2I = 1.5$$

$$I = 0.208A \dots \dots (ii)$$

Putting (ii) into (i), we get;

$$r = \left(\frac{2}{0.208} - 8 \right) = 1.62\Omega$$

3. A cell of e.m.f X is balanced on the potentiometer as shown in the circuit below.



When the switch K is closed, cell X is balanced at 60.0cm mark from end A. When K is open, the new balance length l is 90.0cm. If the resistance of the potentiometer wire AB is 8Ω , find;

- (i) The e.m.f of cell X
- (ii) The value of the unknown resistor R
- (i) When K is closed, $l = 60.0\text{cm}$ Potentiometer

Solution

$$\text{Current, } I = \frac{20}{8}\text{A}$$

$$\text{Resistance per cm} = \frac{8}{100}\Omega\text{cm}^{-1}$$

$$\text{P.d per cm, } k = \frac{20}{8} \times \frac{8}{100} = 0.2V\text{cm}^{-1}$$

$$\text{E.m.f of X} = \text{p.d across AJ}$$

$$= kl$$

$$= 0.2 \times 60$$

$$= \mathbf{12V}$$

- (ii) When K is open, $l = 90.0\text{cm}$

$$\text{Total resistance} = R_{AB} + R$$

$$= (8 + R)\Omega$$

$$\text{Current } I = \frac{20}{8+R}$$

$$\text{p.d per cm} = \frac{8}{100} \times \frac{20}{8+R} = \frac{8}{5(8+R)}V\text{cm}^{-1}$$

At balance point when K is open,

$$\text{e.m.f of X} = \text{p.d across AJ}$$

$$12 = kl$$

$$12 = \frac{8}{5(8+R)} \times 90$$

$$12(8 + R) = 8 \times 18$$

$$8 + R = \frac{8 \times 18}{12}$$

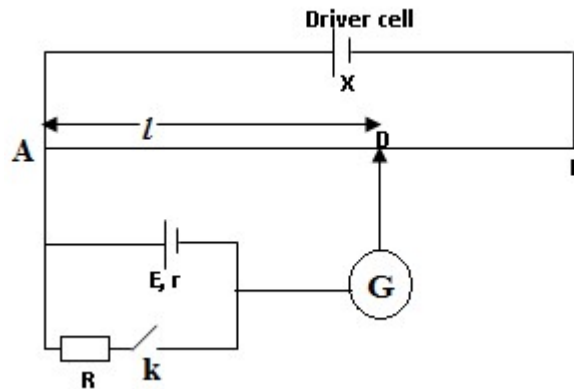
$$\mathbf{R = 4\Omega}$$

Qn;

In a potentiometer, a cell of e.m.f x gave a balance length of a cm and another cell of e.m.f y gave a balance length of b cm. when the cells are connected in series, a balance length of c cm was obtained. It was also discovered that $a + b \neq c$. Show that the true ratio $\frac{x}{y} = \frac{c-b}{c-a}$

(04 marks)

Measuring internal resistance using potentiometer



With K open, a jockey is moved along wire AB until the galvanometer shows zero deflection. The balance length l_s is measured and recorded.

$$\Rightarrow E = kl_s \dots \dots \dots (i)$$

Switch K is then closed with a known resistor R connected to the cell as shown above. The balance point where the galvanometer indicates zero deflection is again obtained. The corresponding balance length l is measured and recorded.

$$\text{At balance, p.d across R, } V = kl \dots \dots \dots (ii)$$

Equation (i) divided by (ii),

$$\frac{E}{V} = \frac{l_s}{l}$$

But $E = I(R + r)$ and $V = IR$

$$\Rightarrow \frac{E}{V} = \frac{I(R+r)}{IR} = \frac{R+r}{R}$$

$$\text{Thus, } \frac{R+r}{R} = \frac{l_s}{l}$$

$$R + r = \frac{l_s}{l} \times R$$

$$r = \frac{l_s R}{l} - R$$

$$\therefore r = \left(\frac{l_s}{l} - 1 \right) R$$

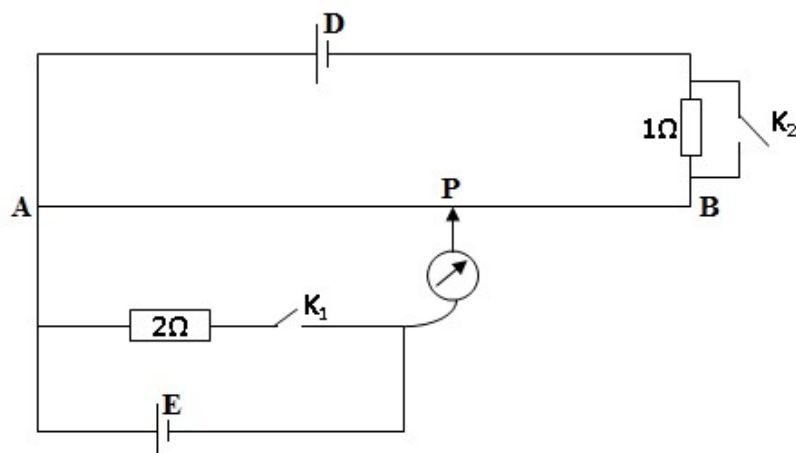
Hence r can be calculated.

NB: A better way is to obtain several values of l using different values of R .

A graph of $\frac{l_s}{l}$ against $\frac{1}{R}$ is then plotted and a straight line whose gradient is equal to r .

Examples

1.

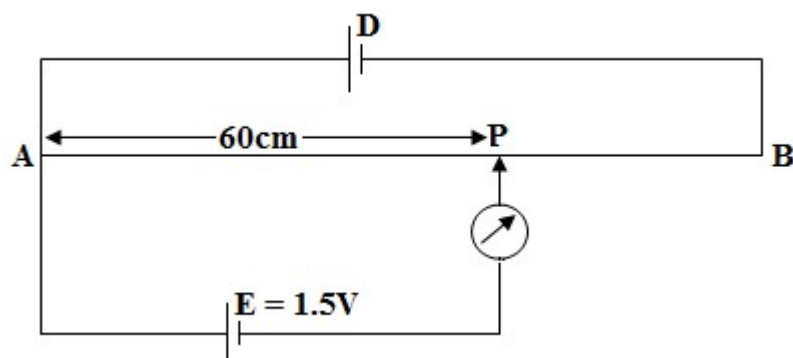


In the above figure, AB is a uniform resistance wire of resistance 4Ω and length 100cm. E is a cell of e.m.f 1.5V, D is a driver cell of negligible internal resistance. When switch k_2 is closed and k_1 opened, the balance length AP is 60cm. when both k_1 and k_2 are closed, the balance length is 35cm. find;

- The internal resistance of E,
- The balance length when k_1 is closed and k_2 is open.
- Explain what happens when the e.m.f of cell E is greater than that of D and k_2 is closed while k_1 is open.

Solution

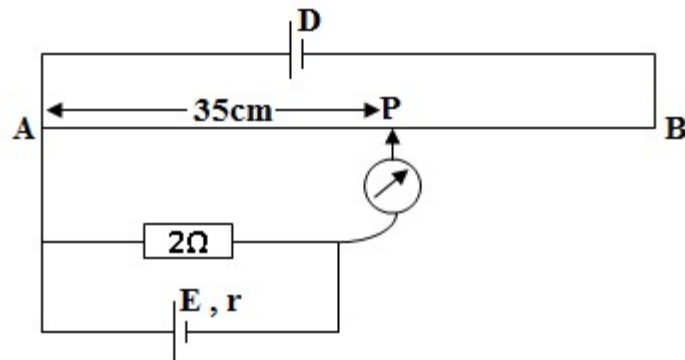
(i)



$E = k \cdot 60$, where k is the p.d per cm

$$1.5 = 60k$$

$$\Rightarrow k = \frac{1}{40} \text{ V cm}^{-1}$$



p.d across the 2Ω resistor, $V = kl$
 $= \frac{1}{40} \times 35$
 $= 0.875V$

From $V = IR$, $I = \frac{V}{R} = \frac{0.875}{2} = 0.4375A$

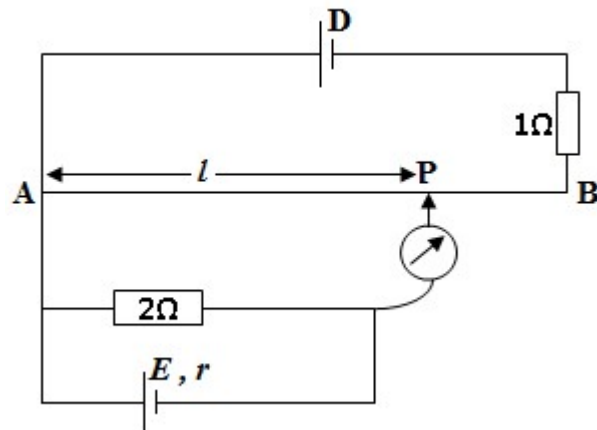
Using $E = I(R + r)$,

$$r = \frac{E}{I} - R$$

$$= \frac{1.5}{0.4375} - 2 = 1.43\Omega$$

$\therefore r = 1.43\Omega$

(ii)



E.m.f of D = p.d per cm x length of the wire
 $= \frac{1}{40} \times 100 = 2.5V$

Considering the upper circuit,

$$I(4 + 1) = 2.5$$

\Rightarrow Current $I = 0.5A$

p.d across the resistance wire, $V_{AB} = I.R_{AB} = 0.5 \times 4 = 2V$

New p. per cm, $k = \frac{2}{100} = \frac{1}{50} Vcm^{-1}$

But p.d across the 2Ω resistor = $0.875V$

$\Rightarrow 0.875 = \frac{1}{50} \times l$

$$\therefore l = 43.75\text{cm}$$

- (iii) There will be no balance point because the e.m.f of the cell E will be greater than the p.d across the length AB of the wire. So at any point you place the jockey along AB, current will flow in the direction of the e.m.f of E.

2. A dry cell gives a balance length of 84.8cm on a potentiometer wire. When a resistor of resistance 15Ω is connected across the terminals of the cell, a balance length of 75.0cm is obtained. Find the internal resistance of the cell.

Solution

Let $l = 84.8\text{cm}$, $l' = 75.0\text{cm}$.

$$\Rightarrow E = kl \text{ and } V = kl'$$

$$\text{Thus, } \frac{V}{E} = \frac{l'}{l} \dots \dots (i)$$

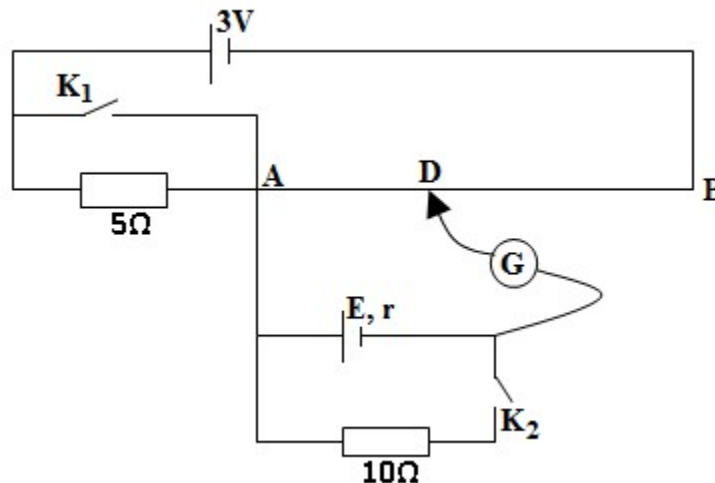
$$\text{From } I = \frac{E}{R+r} \text{ and } V = IR, \Rightarrow \frac{V}{E} = \frac{R}{R+r} \dots \dots (ii)$$

from (i) and (ii),

$$\frac{l'}{l} = \frac{R}{R+r}$$

$$\Rightarrow r = \left(\frac{l'}{l} - 1 \right) R = \left(\frac{84.8}{75} - 1 \right) \times 15\Omega = 1.96\Omega$$

3.



The circuit above shows a uniform slide wire AB of length 100cm and resistance of 15Ω . The wire is connected in series with a resistor of resistance 5Ω across a 3V battery of negligible internal resistance. A cell of e.m.f E, and internal resistance r is connected as shown. With switches K_1 and K_2 open, the galvanometer G shows no deflection when AD is 75.0cm. With K_1 open and K_2 closed, the galvanometer shows no deflection when AD is 65.0cm. Find the;

- value of e.m.f, E,
- internal resistance, r,
- balance length when K_1 is closed and K_2 is open

Solution

- (i) the p.d across slide wire $AB = \left(\frac{15}{15+5}\right) \times 3$
 \Rightarrow p.d per cm of the wire $= \left(\frac{15}{15+5}\right) \times \frac{3}{100} = 0.0225 V cm^{-1}$
 When K_1 & K_2 are open, we are balancing the e.m.f, E , of the cell
 $\Rightarrow E = \text{p.d per cm} \times AD = 0.0225 \times 75$
 $\therefore E = 1.6875 V$
- (ii) For K_1 open & K_2 closed, we are measuring the p.d across the 10Ω resistor.
 \Rightarrow p.d, $V = \text{p.d per cm} \times 65 = 0.0225 \times 65 = 1.4625 V$
 But $V = IR \dots\dots(i)$

And $E = I(r + R)$(ii) where r is the internal resistance of the cell.

From (i) and (ii)

$$\Rightarrow \frac{V}{E} = \frac{R}{R+r}$$

$$\therefore \frac{1.4625}{1.6875} = \frac{10}{10+r}$$

$$\Rightarrow r = \left(\frac{10 \times 1.6875}{1.4625} - 10 \right) = \mathbf{1.538\Omega}$$

(iii) For K_1 closed and K_2 open, p.d per cm = 3/100

So since $E = 1.6875V$

From p.d = p.d per cm x length l

$$\Rightarrow 1.6875 = \frac{3}{100} \times l$$

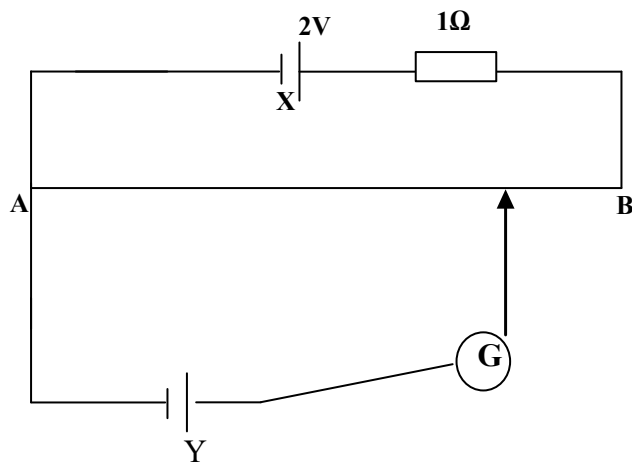
$$\therefore l = \mathbf{56.25cm}$$

4. AB is a uniform wire of length 1m and resistance 4Ω . X is battery of e.m.f 2V and negligible internal resistance. Y has e.m.f 1.5V.

(i) Find the balance length of AD when the switch S is open.

(ii) If the balance length is 75cm, when the switch S is closed, find the internal resistance of Y.

(iii)



- (i) $E_Y = kl$
 $k = IR_{/cm}$
 where, $R_{/cm} = \frac{4}{100} \Omega cm^{-1}$ and $I = \frac{2}{1+4} = \frac{2}{5} A$
 Hence, $k = \frac{2}{5} \times \frac{4}{100} = \frac{8}{500} V cm^{-1}$
 $\Rightarrow l = \frac{E_Y}{k} = \frac{500}{8} \times 8$
 $\therefore l = 93.75 cm$
- (ii) When S is closed

$$\frac{E_Y}{V} = \frac{l}{l_1} = \frac{93.75}{75}$$

$$\Rightarrow V = \frac{E_Y \times 75}{93.75} = \frac{1.5 \times 75}{93.75} = 1.2V$$

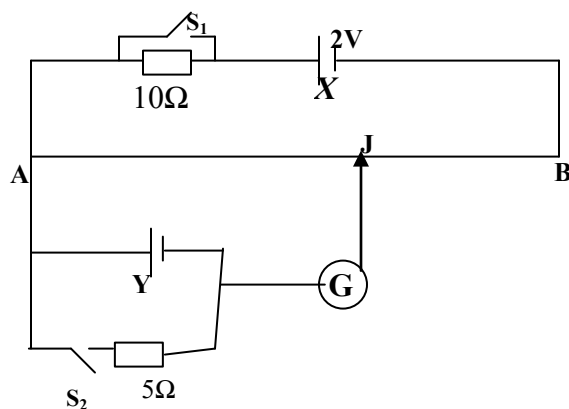
$$\text{But } V = \frac{E_Y R}{R+r} = \frac{1.5 \times 2}{2+r}$$

$$\Rightarrow 1.2 = \frac{3}{2+r}$$

$$\therefore r = 0.5 \Omega$$

EXERCISES

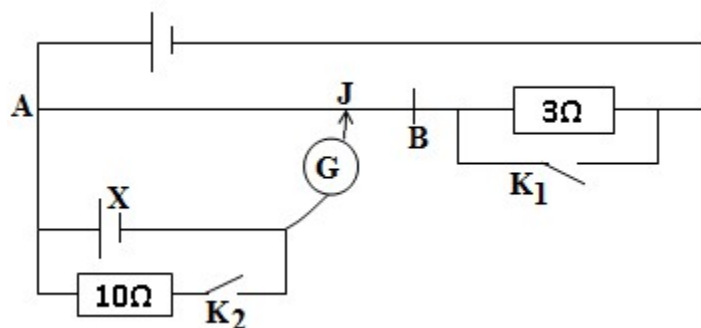
1.



In the circuit above, X has negligible internal resistance, length AB is 100cm and resistance of AB is 50Ω . When S_1 and S_2 are open, the balance length $AJ = 90cm$. When S_2 is closed and S_1 open, the balance length $AJ = 75cm$. Find

- (i) the e.m.f of cell y (1.5V)
- (ii) Internal resistance of cell Y. (1Ω)
- (iii) the balance length when S_1 and S are both closed

2. In the figure below, a uniform wire of length 1.2m, diameter 0.24mm and resistance 8Ω .



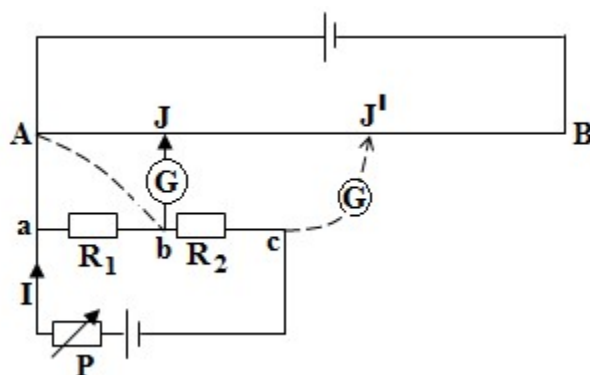
When K_1 is closed and K_2 open, the galvanometer shows no deflection when $AJ = 62.5\text{cm}$.

When

K_1 is open and K_2 closed, G shows no deflection when $AJ = 83.3\text{cm}$. Find;

- resistivity of the potentiometer wire,
- e.m.f of the cell X ,
- internal resistance of cell X ,
- balance length when both K_1 and K_2 are closed
- balance length when both K_1 & K_2 are open

Comparison of resistance using a potentiometer



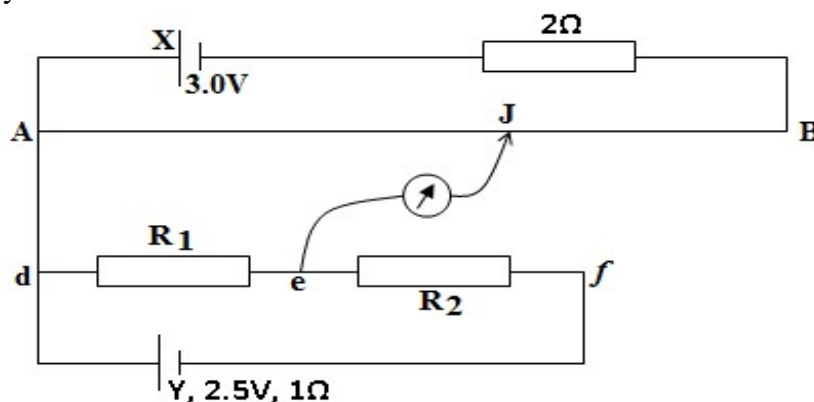
- The two resistors to be compared are connected in series so that the same current flows through them.
- With the galvanometer at **a** and **b**, the balance length $AJ = l_1$ is measured and recorded.
- Hence $IR_1 = kl_1$ (i)
- Connections at **a** and **b** are removed and replaced by those at **b** and **c**(dotted lines).
- The new balance length, l_2 , is measured and recorded.
- Hence $IR_2 = kl_2$(ii)
- Equation (i) divided by (ii)

$$\frac{R_1}{R_2} = \frac{l_1}{l_2}$$

- Thus if one of the resistances is standard, the other one can be calculated using the above ratio/expression.

Example

In the figure below X is an accumulator of e.m.f 3V and negligible internal resistance connected in series with a 2Ω resistor and a slide wire AB of length 1m and resistance 8Ω . A cell Y of e.m.f 2.5V and internal resistance of 1Ω is connected in series with the resistances R_1 & R_2 . Balance lengths of 42.8 cm and 75.0cm are obtained when the galvanometer is connected at e and f respectively.



Find;

- The current flowing through R_1 ,
- The resistances R_1 and R_2 .

Solution

$$(i) \quad \text{Potentiometer current, } I = \frac{3}{2+8} = \frac{3}{10} A$$

$$\text{Resistance per cm} = \frac{8}{100} = \frac{2}{25} \Omega \text{cm}^{-1}$$

$$\text{p.d / cm, } k = \frac{3}{10} \times \frac{2}{25} = \frac{3}{125} V \text{cm}^{-1}$$

$$\text{When G is at e, } l = 42.8 \text{ cm}$$

$$\Rightarrow \text{p.d across } R_1 = \text{p.d across AJ}$$

$$I_1 R_1 = k l$$

$$= \frac{3}{125} \times 42.8 = 1.0272$$

$$\Rightarrow I_1 R_1 = 1.0272 \dots \dots \dots (i)$$

$$\text{When G is at f, } l = 75 \text{ cm}$$

$$\text{p.d across } R_1 \text{ and } R_2 = \text{p.d across AJ}$$

$$I_1 (R_1 + R_2) = \frac{3}{125} \times 75$$

$$\text{Let } R_1 + R_2 = h \Rightarrow h I_1$$

$$= 1.8 \dots \dots \dots (ii)$$

$$\text{Also, p.d across } R_1 \& R_2 = I_1 (R_1 + R_2 + 1) = 2.5$$

$$\Rightarrow I_1 (h + 1) = 2.5$$

$$I^1 = \frac{2.5}{1+h} \dots \dots (iii)$$

Putting eqn (iii) in (ii),

$$\frac{2.5}{1+h} \times h = 1.8$$

On simplifying, $h = 2.57\Omega$

Putting the value of h in eqn (iii),

$$I^1 = \frac{2.5}{1+h} = \frac{2.5}{1+2.57}$$

$$\therefore I_1 = 0.7A$$

(i) From eqn (i),

$$I_1 R_1 = 1.0272$$

$$0.7 R_1 = 1.0272$$

$$R_1 = 1.0272/0.7$$

$$\therefore R_1 = 1.47\Omega$$

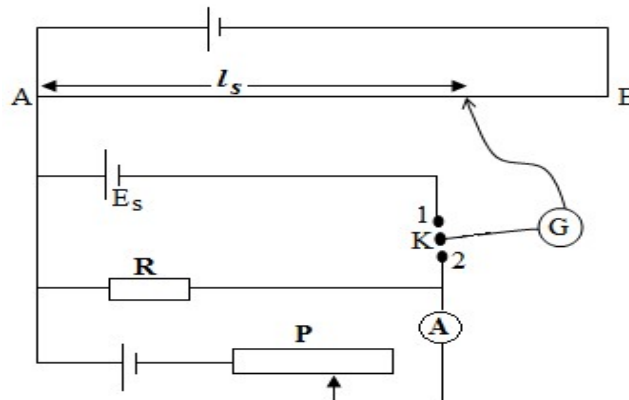
From $R_1 + R_2 = h$

$$1.47 + R_2 = 2.57$$

$$\therefore R_2 = 1.1\Omega$$

Calibration of ammeter using potentiometer (measurement of current)

- Switch K is first connected to 1 so as to get the balance length, l_s .
- So the p.d per cm of the wire, $k = \frac{E_s}{l_s}$.
- K is then connected to 2 and the potential divider P is adjusted to give a suitable reading I_r on the ammeter.
- The balance length l on AB is found and recorded.



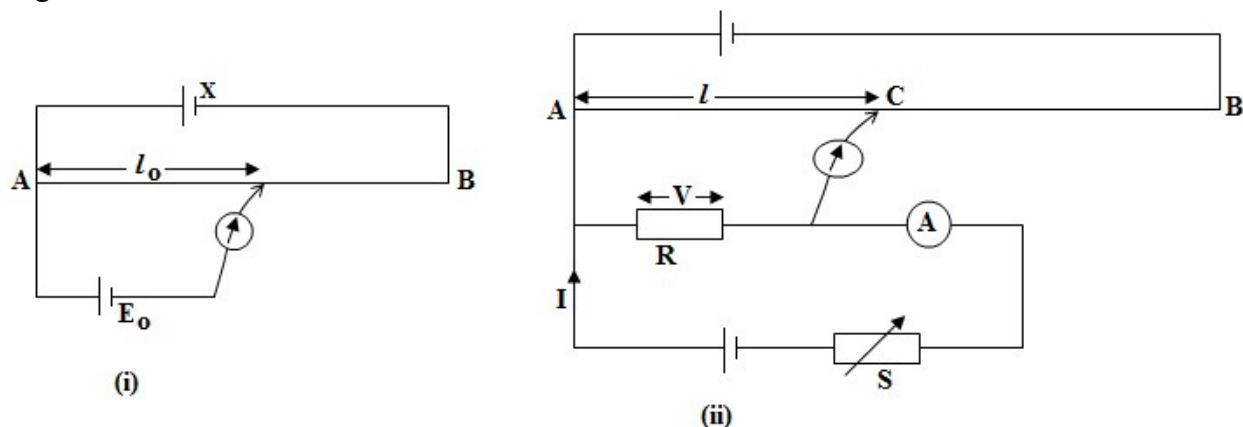
- The procedure is repeated for different values of I_r which are got by adjusting P.
- The results are tabulated including values of current through ammeter, $I_A = \frac{E_s}{l_s} \times \frac{l}{R}$.

- A graph of I_A against I_r is plotted and this gives the calibration curve for the ammeters.

Alternatively;

The potentiometer wire is first calibrated using a standard cell so that the p.d per cm is known.

Fig. 1



- The balance length of the e.m.f E_0 is obtained & the p.d per cm, $k = \frac{E_0}{l_0}$ (i)
- The circuit is then connected as in Fig. (ii) where a suitable standard resistor R is in series with the ammeter A to be calibrated.
- The rheostat S is adjusted until the required ammeter reading is obtained & this current I recorded as the observed current (I_{obs}).
- The p.d V across the terminals of R is balanced on the potentiometer wire. Suppose it gives a balance length, l .

- From the calibration using a standard cell, $V = kl = \frac{E_0}{l_0} l$

$$\Rightarrow \frac{V}{E_0} = \frac{l}{l_0}$$

- The true current I_m that flows through R is then calculated from;

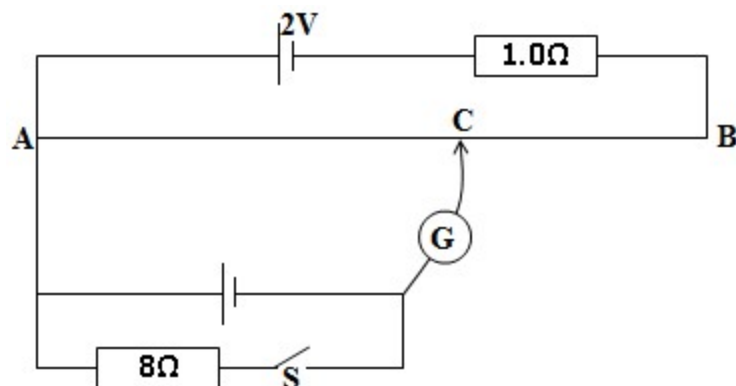
$$I_m = \frac{V}{R}$$

- I_m is compared with the observed value I_{obs} & the difference in error is corrected by adding or subtracting from I_{obs} .

NB: The resistances of the wires connecting the potential terminals at the points & through the potential circuit don't affect the result because at the balance point, the current through them is zero.

Example

1.



In the above figure, AB = 1.0m long and has resistance of 4Ω .

- (i) When S is open, the balance length AC = 88.5cm. Find the e.m.f of the cell.
- (ii) When S is closed, the balance length AC = 82.5cm. Calculate the internal resistance of the cell.

Solution

- (i) When S is open, total resistance = $1 + 4 = 5\Omega$

Current in the potentiometer wire, $I = \frac{2}{5}\text{A}$

$$\text{Resistance per cm} = \frac{R_{AB}}{l_{AB}} = \frac{4}{100} = \frac{1}{25} \Omega \text{cm}^{-1}$$

P. d per cm, $k = \text{resistance per cm} \times \text{current}$

$$\Rightarrow k = \frac{1}{25} \times \frac{2}{5} = \frac{2}{125} \text{Vcm}^{-1}$$

E.m.f of the cell equals p.d across AC

$$E = kl_{AC}$$

$$= \frac{2}{125} \times 88.5 = 1.416\text{V}$$

Thus e.m.f of the cell $E = 1.42\text{V}$

- (ii) When S is closed,

P.d across the 8Ω resistor, V , equals p.d across AC

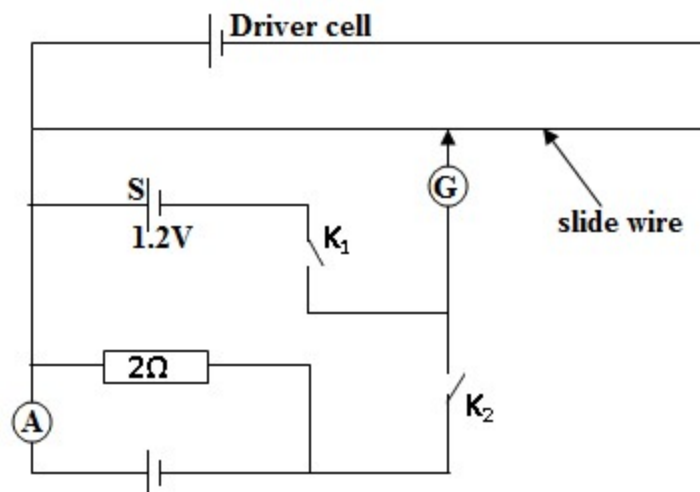
$$V = kl_{AC} = \frac{2}{125} \times 82.5 = 1.32\text{V}$$

$$\text{From } V = IR, I = \frac{V}{R} = \frac{1.32}{8} = 0.165\text{A}$$

Also, $E = I(R + r)$, where r is the internal resistance of the cell.

$$1.42 = 0.165(8 + r)$$

$$\therefore r = 0.61\Omega$$

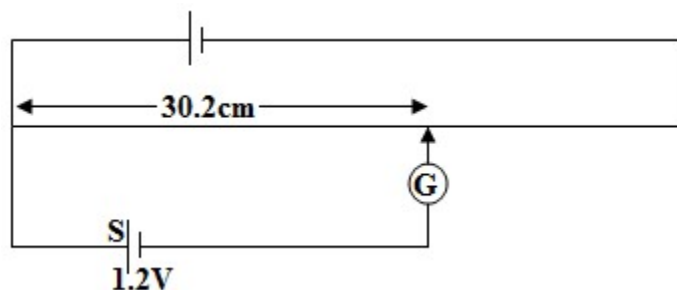


When K_1 is closed & K_2 open balance point = 30.2cm.

When K_1 is open & K_2 closed, balance point = 26.8cm and the ammeter A reads 0.4A. Calculate the percentage in error in the ammeter reading.

Solution

When K_1 is closed & K_2 open,



$$E = k l$$

$$\Rightarrow k = \frac{1.2}{30.2} \text{ Vcm}^{-1}$$

When K_1 is open & K_2 closed,

P.d across the 2Ω resistor, V , equals to p.d across the balance length ($l = 26.8\text{cm}$)

$$V = \frac{1.2}{30.2} \times 26.8 = 1.065\text{V}$$

But $V = IR$

$$\Rightarrow 1.065 = 2I_m$$

True current, $I_m = 0.532\text{A}$

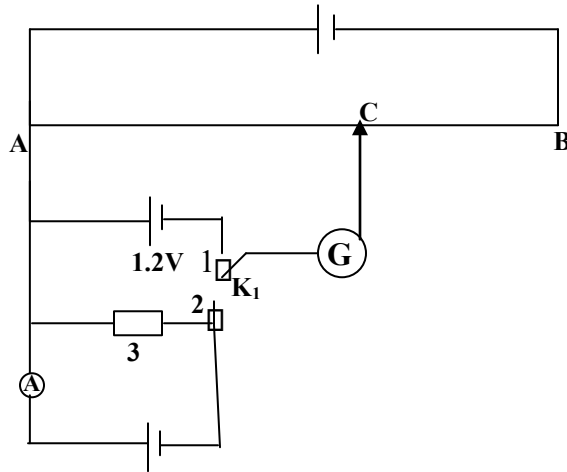
Observed current, $I_{obs} = 0.4\text{A}$

Error = $0.532 - 0.4 = 0.132\text{A}$

$$\begin{aligned} \text{Percentage error} &= \frac{\text{Error}}{\text{True current}} \times 100\% \\ &= \frac{0.132}{0.532} \times 100\% \end{aligned}$$

$$= 24.81\%$$

3. In the circuit below, when K_1 is connected to position 1, the balance length $AC = 30.2\text{cm}$. When K_1 is connected to position 2, the balance length $AC = 26.8\text{cm}$ and the ammeter reading is 0.4A . Find the percentage error in the ammeter reading.



When K_1 is in position 1, $1.2 = 30.2k \dots\dots\dots(i)$

When K_1 is in position 2, $V = 26.8k \dots\dots\dots(ii)$

$$V = \frac{1.2}{30.2} \times 26.8 = 1.065\text{V}$$

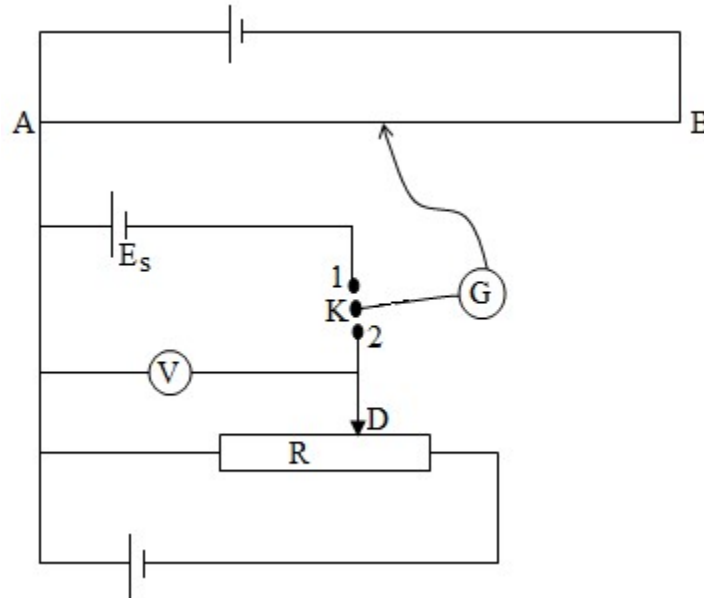
$$\text{But } V = IR = 3I$$

$$\text{Hence } I = \frac{1.065}{3} = 0.355\text{A}$$

$$\text{Error} = 0.4 - 0.355 = 0.045\text{A}$$

$$\% \text{error} = \frac{0.045}{0.355} \times 100 = 12.7\%$$

Qn; A 1Ω resistor is in series with an ammeter **m** in a circuit. The p.d across the resistor is balanced by a length of 60cm on a potentiometer wire. A standard cell of e.m.f 1.02V is balanced by a length of 50cm . If **m** reads 1.1A , what is the error in the reading? **(0.124A)**

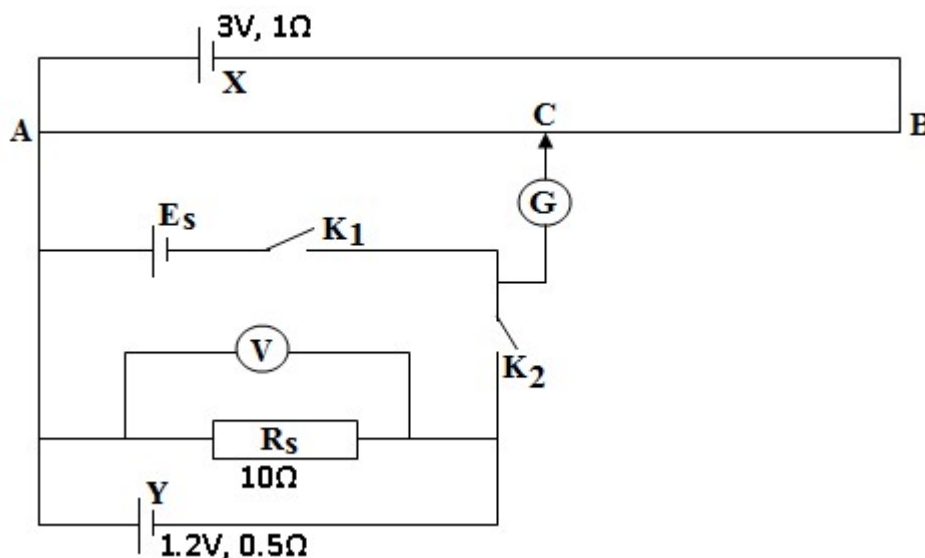
Calibration of voltmeter (measurement of voltage)

- A standard cell E_s is first used to find the p.d per cm of the wire by connecting switch K to 1.
- The jockey is tapped along AB until a balance length l_s is got.
- \Rightarrow P.d per cm, $k = \frac{E_s}{l_s}$.
- The switch K is connected to 2 and different voltages V are applied to the voltmeter by adjusting the rheostat R. the corresponding balance length l is obtained.
- Then $V_m = \frac{E_s}{l_s} \times 100$. Where V_m is the true value of the p.d across the terminals of the voltmeter.
- The reading V_{obs} of the voltmeter is got.
- The procedure is repeated for other values of V_m and the results are tabulated including values of $V_{obs} - V_m$

A graph of $V_{obs} - V_m$ against V_{obs} is plotted and constitutes the correction curve or calibration curve when the meter is being used.

Qn: In the figure below, AB is a uniform resistance wire of length 1m and resistance 4Ω . X is a driver cell of e.m.f 3V and internal resistance 1Ω and E_s is a standard cell. R_s is a standard

resistor of resistance 10Ω which is connected in series with cell Y of e.m.f 1.2V and internal resistance 0.5Ω .



With switch K_1 closed and K_2 open, the balance length, AC is 60cm while the voltmeter reading is 1.14V . With switch K_1 open and K_2 closed, the balance length is 80cm. calculate the;

- e.m.f E_s of the standard cell
- Percentage error in the voltmeter reading.

Solution

- Current I through $AB = \frac{3}{1+4} = 0.6\text{A}$
 P.d across AB , $V_{AB} = IR_{AB} = 0.6 \times 4 = 2.4\text{V}$
 Hence p.d per cm of AB , $K = \frac{2.4}{100}\text{Vcm}^{-1}$

$$\begin{aligned}\text{Thus e.m.f, } E_s &= \text{p.d per cm} \times \text{balance length} \\ &= \frac{2.4}{100} \times 60 = 1.44\text{V}\end{aligned}$$

- NB: voltmeter reading is the p.d across R_s

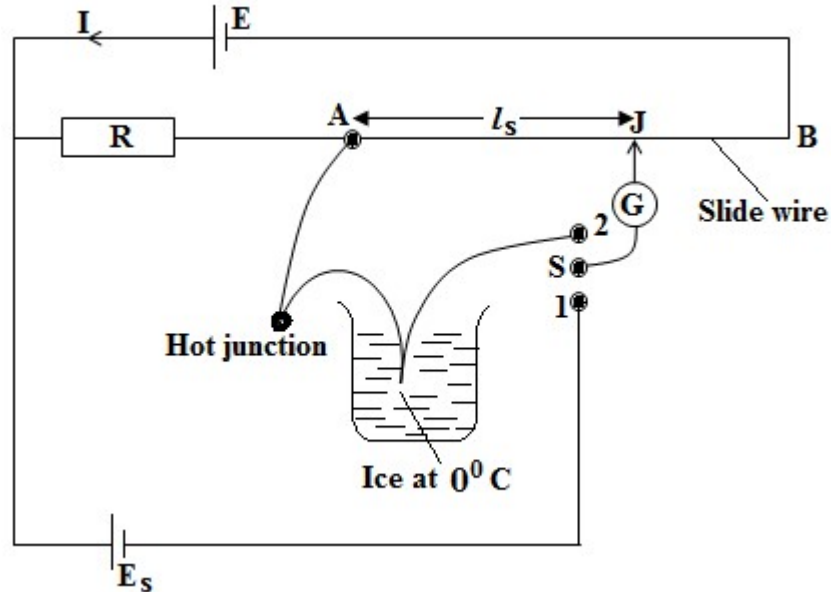
Observed reading, $V_{obs} = 1.14\text{V}$

The true reading across the voltmeter terminals, $V_m = \frac{2.4}{100} \times 80 = 1.92\text{V}$

Percentage error in reading = $\left(\frac{V_m - V_{obs}}{V_{obs}} \times 100\right)\%$

$$\therefore \text{percentage error} = \frac{1.92 - 1.14}{1.92} \times 100\% = 40.6\%$$

Measurement of small e.m.f (e.g. E.m.f of a thermocouple)



The circuit is connected as shown above. An approximately high resistance R is connected in series with the potentiometer wire so that an appreciable length is obtained, since the e.m.f of thermocouple is of the order of few millivolts.

The p.d per cm of the wire is obtained using a standard cell E_s by connecting switch S to 1. The jockey is tapped along AB until a balance length l_s is got.

At balance point, $E_s = \text{p.d across } R + \text{p.d across } AJ$

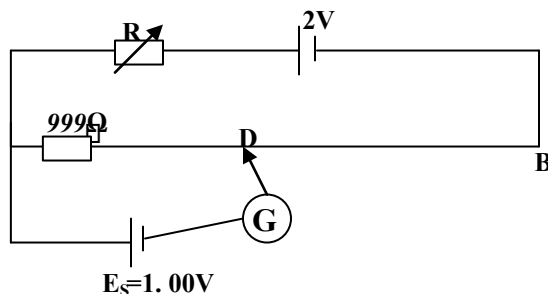
$$\Rightarrow E_s = IR + kl_s, \text{ where } k \text{ is the p.d per cm.}$$

Switch S is now connected to 2 and the new balance length l measured & recorded.

The e.m.f of the thermocouple is calculated from $E = kl$.

Examples

1.



The slide wire has length 100cm and resistance 10Ω . The galvanometer shows no deflection when $AD = 10\text{cm}$. Find

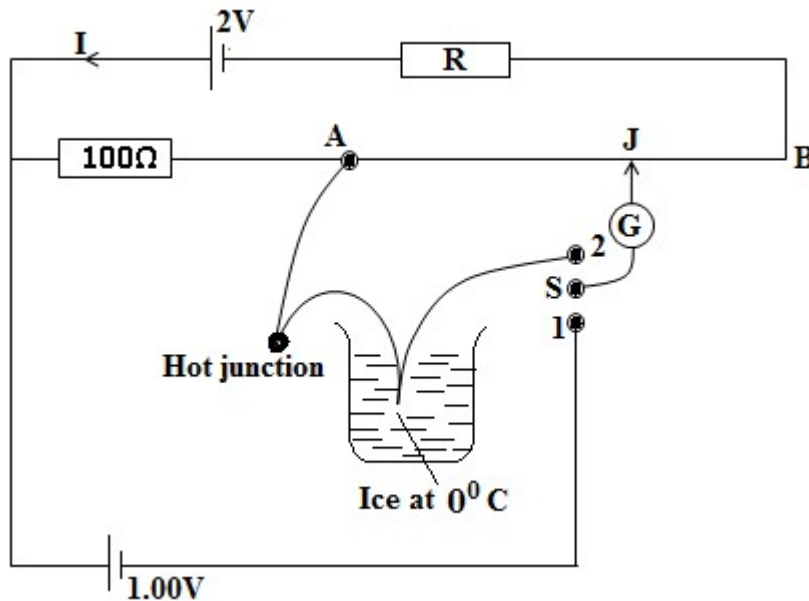
- (i) the current flowing in the driver cell

- (ii) the value of R .
 (iii) the e.m.f of thermocouple which balanced by a length of 60cm of the slide AB.

Solution

- (i) $E_S = 999I + kl_S$
 $k = I \times \frac{10}{100} = \frac{I}{10}$
 $\Rightarrow E_S = 999I + \frac{10 \times I}{10} = 1000I$
 But $E_S = 1.00V$
 $\Rightarrow 1000I = 1$
 $\therefore I = \frac{1}{1000} A$
- (ii) e.m.f of the driver cell, $2 = I(999 + 10 + R) = \frac{1}{1000} (999 + R + 10)$
 $2000 = 1009 + R$
 $\therefore R = 991\Omega$
- (iii) e.m.f of the thermocouple, $\varepsilon = kl = \frac{1}{10} \times \frac{1}{1000} \times 60 = 6mV$

2. In the figure below, AB is a uniform wire of length 1.0m and resistance of 2.00Ω .



When switch S is connected to 1, the balance length is 90.0cm. With S connected to 2, the balance length is 45cm.

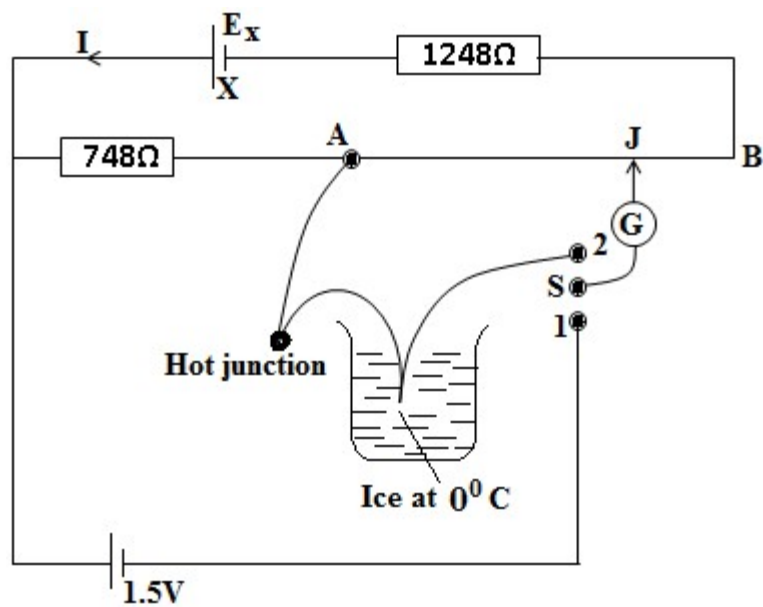
- (i) Find the e.m.f of the thermocouple.
 (ii) What is the value of R if the driver cell has negligible internal resistance?

Solution

- (i) When S connected to 1,
 $l = 90.0\text{cm}$ $AB = 100\text{cm}$
 $R_{AB} = 2.0\Omega$
 $\rho = \frac{R_{AB}}{AB} = \frac{2}{100} = \frac{1}{50}\Omega\text{cm}^{-1}$
 $R_{AJ} = \rho x l = \frac{1}{50} \times 90 = \frac{9}{5}\Omega$
 At balance, $E_s = \text{p.d across R} + \text{p.d across AJ}$
 $\Rightarrow E_s = I(R + R_{AJ})$
 $I = \frac{E_s}{R + R_{AJ}} = \frac{1}{100 + \frac{9}{5}} = 9.82 \times 10^{-3}A$
 From $V_{AB} = IR_{AB}$
 $\Rightarrow V_{AB} = 9.82 \times 10^{-3} \times 2 = 0.0196V$
 p.d per cm, $k = \frac{V_{AB}}{AB} = \frac{0.0196}{100} = 1.96 \times 10^{-4}V\text{cm}^{-1}$
 When S is connected to 2, $l = 45\text{cm}$
 e.m.f of the thermocouple, $\varepsilon = kl$
 $= 1.96 \times 10^{-4} \times 45$
 $= 8.82 \times 10^{-3}V$

$$\therefore \varepsilon = 8.82mV$$

- (ii) Considering the upper circuit,
 Total resistance $R_T = R + 100 + R_{AB}$
 $\Rightarrow I(R_T = R + 100 + R_{AB}) = 2$
 $9.82 \times 10^{-3}(100 + 2 + 2) = 2$
 On simplifying, **$R = 101.7\Omega$**



In the circuit above X is an accumulator of negligible internal resistance. AB is a uniform wire of length 1.0m, diameter 4×10^{-4} mm and resistivity $1.26 \times 10^{-6} \Omega\text{m}$. G is a galvanometer connected to a sliding contact D. when switch S is thrown to position 1, G shows no deflection when $AJ = 80.0\text{cm}$. When S is thrown to position 2, G shows no deflection when $AJ = 40\text{cm}$. find;

- The resistance of wire AB,
- The e.m.f of cell X, E_x
- The e.m.f of the thermocouple.

Solution

- Length of AB, $l = 1\text{m}$,
Diameter, $D = 4 \times 10^{-4}\text{mm} = 4 \times 10^{-7}\text{m}$

Resistivity, $\rho = 1.26 \times 10^{-6} \Omega\text{m}$

From $R = \frac{\rho l}{A}$, where $A = \frac{\pi D^2}{4}$

$$\Rightarrow R_{AB} = \frac{4\rho l}{\pi D^2}$$

$$= \frac{4 \times 1.26 \times 10^{-6} \times 1}{\pi (4 \times 10^{-7})^2}$$

$$\therefore R_{AB} = 10\Omega$$

- When S is connected to position 1, $l = 80\text{cm}$
 $1.5 = 748I + kl$, where $k = \text{p.d per cm}$

$$= I \cdot \frac{R_{AB}}{l_{AB}}$$

$$k = \frac{10}{100} I = \frac{1}{10} I$$

$$\Rightarrow 1.5 = 748I + \frac{1}{10} \times 80$$

$$15 = 7480I + 80I$$

$$I = \frac{15}{7560} A$$

$$\text{But } E_x = I(748 + 1248 + 10)$$

$$\Rightarrow E_x = \frac{15}{7560} \times 2006$$

$$\therefore E_x = 4V$$

(iii) When S is connected to position 2, $l = 40\text{cm}$

E.m.f of the thermocouple, $\varepsilon = \text{p.d across AJ}$

$$= k \times 40$$

$$= \frac{15}{7560} \times \frac{1}{10} \times 40 = 0.008V$$

$$\therefore \varepsilon = 8mV$$

Qn;

A potentiometer wire of length 1m and resistance 1Ω is used to measure an e.m.f of the order **mV**. A battery of e.m.f 2V and negligible internal resistance is used as a driver cell. Calculate the resistance to be in series with potentiometer so as to obtain a potential drop of 5mV across the wire. **(399 Ω)**

Advantages of potentiometer over a moving coil voltmeter

- (i) The potentiometer is more accurate because it does not draw current from the circuit whose p.d it is meant to measure. The potentiometer can be considered to be a voltmeter with an infinitely high resistance which is an ideal voltmeter.
- (ii) The potentiometer method is a null method. The accuracy of the potentiometer does not depend on the accuracy of the galvanometer but only on its sensitivity. The accuracy of the result is not affected by the fault accuracy of the galvanometer.

Disadvantages of potentiometer over a moving coil voltmeter

- (i) It does not give direct reading since there may be errors.
- (ii) It requires a skilled person
- (iii) It is slow in operation.

UNEB QUESTIONS

2019, 2018, 2013, 2011, 2010

ROGER MUNCASTER (POTENTIOMETER)

F48, F49, F50, F51, F52, F53, F54, F56, F55, F57

ROGER MUNCASTER (METER BRIDGE AND WHEATSTONE BRIDGE)

F41, F42, F43, F44, F45.