WAKISSHA JOINT MOCK EXAMINATIONS 2015
UGANDA ADVANCED CERTIFICATE OF EDUCATION
MARKING GUIDE
P425/1
MATHEMATICS
PAPER 1
JULY/AUGUST 2015

1. $X=5+\frac{\sqrt{3}}{2}\cos\Theta$ .... (1)		
$Y= -3\frac{\sqrt{3}}{2}\sin\Theta$ .... (2)		
From 1 $X-5 = \frac{\sqrt{3}}{2}\cos\Theta$		
$\left(\frac{2X-10}{\sqrt{3}}\right) = \cos\Theta$		
$\sqrt{3}$		
$\cos^2\Theta = \left(\frac{2X-10}{\sqrt{3}}\right)^2$		
From (2) $Y+3=\frac{\sqrt{3}}{2}\sin\Theta$		
$\sin\Theta = \frac{2(Y+3)}{\sqrt{3}}$	B1	B1 – for $Co^2\Theta$ and $\sin\Theta$
$\sin^2\Theta=\frac{(2Y-6)^2}{3}$	M1	Or $(x-5)^2 = \frac{3}{4}\cos^2\Theta$.........(1) B1 $(y+3)2 = \frac{3}{4}\sin^2\Theta$.........(2)
$\cos^2\Theta+ \sin^2\Theta=2\frac{(2X-10)^2}{3}+\frac{(2Y-6)^2}{3}$		
$(2X-10)^2 + (2Y-6)^2=3$		(1) + (2)
$4X^2-40X+100+4Y2-24Y+36=3$		$(x-5)^3 + (y+3)^2 = \frac{3}{4}$ M1
$4X^2+4Y^2-40X-24Y+133=0$		$4x2 + 4y2 - 40x - 24y + 130 = 0$ A1
$X^2+Y^2-10X+6Y-133 -0$		Is locus of a circle B1
$(X-5)^2 + (Y+3)^2=\frac{3}{4}$		The radius = $\frac{\sqrt{3}}{2}$ and centre (5, -3) B1
Locus is a circle with centre	B1	
$(5 - 3)$ and radius $\sqrt{3}$	B1	
	05	

2. 3^{2n+2}-8n-9.		
Let au= 3^{2n+2}-8n-9.		
For n=1 $a_1=3^4-8-9=64$	B1	
thus divisible.		
For n=k.		
ak=3^{2X+2}-8k-9.⇒9k = $3^{2k}.3^2$-8k-9		
9k=9($3^{2\pi}$) - 8k-9.	B1	
32k = $\frac{9k+8k+9}{9}$		
For = k+1		
9R+1 = $3^{2(k+1)+2}$-8(k+1)-9		
9R+1=$3^{2\pi+4}$-8k –8-9⇒9K+1=81(3^{4k}) -8k - 17		
Subst.(3) in (4)	B1	
81($\frac{9k+8k+9}{9}$).8k-17		
9k+1=9ak+72R+81-8K-17.		
9K+1= 9AK+64R+64.		
9K+1 = 64($\frac{9ak}{64}$+k+1)		
Ak+1=64Bfor B=$\frac{9ak}{64}$+C+1		
Since its divisible for n= π41,n=c thus this also divisible for all integral values of n.	B1	
	05	
3. 6Cos2X+777=7Sin2X using t = tan x		
$6\left(\frac{1-t^2}{1+t^2}\right)+7=7\left(\frac{2t}{1+t^2}\right)$	M1	For substitution
$6(1-t^2)+7(1+t^2)=14t.\Rightarrow t^2-14t+13=0$		
(t-1) (t-13)=0		
t=1 t=13	M1	For method
for t=1	A1	
tan X=1 X=45^0,225^0		
For t=13	A1	
tan x=13 x=85.6^0, 265^- 6^0		
Or	A1	
7Sin2x-6Cos2X=7		
R Sin(2x∝)= 7Sin2X-6CosR		
R Cos∝=7 and R Sin∝=6		
R=$\sqrt{49}+36$ = S85 tan∝=$\frac{6}{7}$		
$\sqrt{85}$ Sin(2x-∝)=7		
Sin(2x-∝)=$\frac{7}{\sqrt{85}}$		
$2x=\sin^{-1}\left(\frac{7}{85}\right)+\tan^{-1}\left(\frac{6}{7}\right)$X =		
A1A1 = 05		

4. $|X+3a|<2|X-2a|$

$(X+3a)^2<4(x-2a)^2$

$X^2+6ax+9a2<4(X^2-4aX+4a)$

$3x^2-22ax+7a^2>0$

$(3x-a)(x-7a)>0$ A1

	$X<\frac{1}{3}a$	$\frac{1}{3}a<x<7a$	$X>7a$
3x-a	-	+	+
x-7a	-	-	+
	+	-	+

$\therefore$ The solution is $X<\frac{1}{3}a$ and $x>7a$ A1

A1 for each range of solution

5. $\int_{\sqrt{3}}^{\infty} \frac{dx}{x\sqrt{1+x^2}}$

Method 1

Let $u=\sqrt{1+x^2}$

x	$\sqrt{3}$	∞
u	2	∞

$du = \frac{1}{2}(1+x2)^{-1/2}.2x dx.$ B1

$du =\frac{dx}{\sqrt{1+x^2}} dx$

$I = \int_2^{\infty} \frac{1}{U^2-1}\frac{U}{X}\, du$

$I =\int_2^{\infty} \frac{1}{U^2-1}\, du$

Let $\frac{1}{U^2-1} =\frac{A}{u-1} +\frac{B}{u+1}$ B1

$A=\frac{1}{2}$, $B=\frac{-1}{2}$

$I=\frac{1}{2}\left[In\left(\frac{u-1}{u+1}\right)\right]_2^{\alpha}$ M1

$=\frac{1}{2}(0-in\frac{1}{3})$

$=\frac{1}{2}in3$ A1

05

Alternative method.

$$\int_{\sqrt{3}}^{\infty} \frac{dx}{x\sqrt{1+x^2}}$$

Let x=tan u

$dx= Sec^2 u du$ B1

(for only changing variablesi.e x to U because it can be worked without involving limits until toward end then we bring limits.)

$I=\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{\tan u \; secu}$ M1

$=\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{Cos\ U}{Sin\ U}\frac{1}{CosU}\, du.$

$=\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} Cosec\ u\ du$

$\left[in(Tan\frac{u}{2})\right]$ =(0-in$\frac{1}{\sqrt{3}}$ =$\frac{1}{2}$ln3	B1 A1	
6. (i-λi+4k).(2i-4j+6k)=0 4λ =-26 λ =$\frac{-13}{2}$ or -6.5 Using ratio theorem. OP=($\frac{-3}{-3+2}$)a +($\frac{-3}{-3+2}$)b 2(i-2j+4k)-2(3i-4j+6k) =3i-6j+12k-6i+8j-12k = -3i+2j	M1 M1 A1 **05**	
7. Let ar, ar^2, ar^3, be the age of the children in order and that of Pondo respectively. ar, ar^2, ar^3=140 a(i+r+r^2+r^3)=140 ………..i and a+ar =14 a(i+r) = 14………………..ii ar^2, ar^3=126 ⇒ ar^2(1+r)=126……….iii (i)/(ii)gives r^2=9 r=+-3 r=3 a(1+3)=14 a=$\frac{7}{2}$ is a root (r+1)(r+3)(r-3)=0 ⇒ r=3, r=-1, and r=-3 are roots. r=3 subset for r=3 in equation (ii) 4a=14 a=$\frac{7}{2}$ Pando's age = ar^3=$\frac{7}{2}$x3^3 =27x$\frac{7}{2}$ giving 94.5years.		

8. YCos2X $\frac{dy}{dx}$= tan x+2, at y=0, X=$\frac{\pi}{4}$ y$\frac{dy}{dx}$= $\frac{\underline{\text{tan x+2}}}{\text{Cos}^2\text{x}}$ y$\frac{dy}{dx}$=(tanx+2)sec^{2}x ydy = $\int \tan x\; sec^2xdx +$ $2sec2xdx$ $\frac{y^2}{2}$ =$\frac{1}{2}$tan^{2}x+ tan x+c At y=0, x=$\frac{\pi}{4}$ 0= $\frac{1}{2}$(tan$\frac{\pi}{4}$)^2+2tan$\frac{\pi}{4}$+c = $\frac{1}{2}$(1)^2+2(1)+c C=$\frac{-5}{2}$ $\frac{y^2}{2}$=$\frac{1}{2}$tan^{2}x+ tan x-$\frac{-5}{2}$		
9.		

10. (a)(i) Let Z=x+iy $\lvert Z-i\rvert<3$ $\lvert x+(y-1)i\rvert<3$ $X^2+(y-1)^2<3^2$ This is a circle with centre (0,1) and radius r=3 (ii)This $\pi\leq \arg(z,2) \leq \pi$ This is a region of half lines from (2,0) between $\frac{1}{3}\pi$	B1B1 B1	B1 for stating centre (0,1) B1 for stating $r \leq 3$ For correct sketch and shading
(b) Z =X+IY Re $\underline{Z+i)}= 0$ $Z+2$ Re $\frac{x+(y+1)i}{(x+2)+yi}$ Re $\frac{x+(y+1)i\ (x+2)-yi}{(x+2)+yi\ (x+2)-yi}=0$ Re $\frac{x(x+2)-xyi+i(x+2)(y+1)+y(y+1)}{(x+2)^2+y^2}=0$ $\frac{X(x+2)+y(y+1)}{(x+2)^2+y^2}$ $X^2+2x+y^2+y=0$ $(x+1)^2-1+(y+\frac{1}{2})^2-\frac{1}{4}=0$ $(x+1)^2+(y+\frac{1}{2})^2=\frac{5}{4}$ Z=X+iy … on a circle of centre $(-1,\frac{1}{2})$ and radius $R=\frac{\sqrt{5}}{2}$		

11. $Sin30=Sin(2\Theta+\Theta)$ $= Sin2\Theta Cos\Theta + Cos2\Theta Sin\Theta$ $(2Sin\Theta Cos\Theta Cos\Theta)+(1-2Sin^2\Theta)sin\Theta$ $2Sin\Theta(1-Sin2\Theta)+(1-2Sin^2\Theta)Sin\Theta$ $2Sin\Theta-2Sin^3\Theta+Sin\Theta-2Sin^3\Theta$ $Sin^3\Theta = 3Sin\Theta-4Sin^3\Theta$ Let $Sin3\Theta = \frac{p}{q}$ $\Rightarrow\frac{p}{q}=3Sin\Theta-4\ Sin^{-3}\Theta$ $4q\ Sin^{-3}\Theta-3qSin\Theta+P=0$ If $Sin\Theta= x$ $\Rightarrow 4qx^3-3qx+p=8x^3-6x-1=0$ Q=2 and P=-1 $Sin3\Theta\ =-\frac{1}{2}$		
$3\Theta = Sin^{-1}(-\frac{1}{2})$ $210^0,330^0,570^0,690^0,930^0$ $Q=70^0,110^0,190^0,230^0,310^0$ X=Sin700=0.9397. X= Sin1900=-0.1736 X=Sin2300=-0.7660		
12. (a) Y2=4X-8 Can be written as Y2=4(x-2) =4.1(x-2) =4ax X= x-2 a=1 $Y^2=4(x-2)$ is the image of $y^2=4x$ under translation vector $\binom{2}{0}$ New focus = $\binom{1}{0}\binom{2}{0}\binom{3}{0}$ Focus (3,0) New direction = -1+2 X=1		

(b) At P(ap2,2ap) X=ap2 y=2ap $\frac{dx}{dp}$=2aP $\frac{dy}{dp}$=2a. $\frac{dy}{dx}=\frac{2a}{2ap}\,\frac{1}{p}$ Gradient of tangent at P $=\frac{1}{p}$ Gradient of tangent at Q=$\frac{1}{p}$ Equation of tangent at P $\frac{y-2aP}{x-9p2}=\frac{1}{p}$ x-py+ap2=0……….1 Equation of tangent at Q. x-qy+aq^2=0………..2 solving equations 1 and 2 -py+qy+aq^2-aq^2 =0 (q-p)y+a(p-q)(p-q) =0 Y=a(p+q) Substitute for y in equation 1 X=Pa(P+q)-ap^2 =apq. R is (apq, a(P+q) If R lies on 2x+a=0 Then R satisfies thus equations. 2(apq)+a=0 2pq+1=0 Pq - $\frac{1}{2}$		

Mid-point of PQ is $m\left(\frac{ap^2+aq^2}{2},\frac{2ap+2aq}{2}\right)$. M $\frac{9}{2}$(p^2+q^2), 9(p+q) At M: X=$\frac{9}{2}$(p^2+q^2), y=a(p+q) p^2+q^2 =$\frac{2x}{9}$……………. 3 P+q = $\frac{y}{9}$……………… 4 From 3 $\frac{2x}{9}$= (p+q)2-2pq Pq =- $\frac{1}{2}$ $\frac{2x}{9}$ = (P+q)^2+1…….. 5 Substitution equation 4 in 5 $\frac{2x}{9}$ =($\frac{y}{9}$)^2+1 2ax =y^2+a^2 Y^2=2ax-a^2		

13. (a)

$\sqrt{x} - 3 + \sqrt{2x+1} = \sqrt{3x+4}$

Squaring both sides.

$((x-3)+2\sqrt{(x-3)(2x+1)}+(2x+1)=3x+4$

$\Rightarrow\sqrt{(x-3)(2x+1)}=3.$

2x2-5x-12=0

(x+4)(2x+3)=0

X=4 or x=$\frac{-3}{4}$

Checking.

When x=4 , LHS=RHS

When x=$\frac{-3}{2}$, LHS=RHS

X=4 is only solution.

13(b).

Let the first term of an AP be a and the common difference be d let also the first term of a GP be b and the common ratio be r.

AP= a+(a+d)+(a+2d)+……….+(a+(n-1)d)

G.P= $b+br+br^2+br^3+\ldots\ldots+ br^{n-1}$

⇒Sum of the first terms;

a+b=57………………i

⇒Sum of second terms.

a+d+br = 94

at r=2.

A+d+2b=94………………..ii

Sum of the third term

a+2d+br2=171

at r=2.

a+2d+4b=171………………iii

14. (a) let y= $\frac{2x^2}{x^2+1}$ Dy = $\frac{2(x+h)^2}{\left[(x+h)^2+1\right]}-\frac{2x^2}{x^2+1}$ $\frac{2(x+h)^2}{\left[(x+h)^2+1\right]}-\frac{(x^2+1)-2x^2\left[(x+h)^2\right]+1}{\left[x^2+1\right]}$ $\frac{\left[2(x^2 2xh+h^2)+1\right]\left[x^2+1\right]-2^2\left[x^2+2xh+h^{2+1}\right]}{\left[(x+h)^2+1\right]\left[x^2+1\right]}$ $\frac{\left[2x^2+4xh+2h^2+1\right]\left[x^2+1\right]-2x^2\left[x^2+2xh+h^2+1\right]}{\left[(x+h)^2+1\right]\left[x^2+1\right]}$ $\frac{2x^4+2x^2+4x^3h+4xh+2x^2h^2+2h^2-2x4-4x^3h-2x^2h^2}{\left[(x+h)^2+1\right]\left[x^2+1\right]}$ $\frac{4xh+2h^2}{\left[(x+h)^2+1\right]\left[x^2+1\right]}$ $\frac{dy}{dx}=\frac{4x+2h}{\left[(x+h)^2+1\right]\left[x^2+1\right]}$		
$\frac{dy}{dx}=\frac{4x}{(x^2+1)(x^2+1)}$ $\frac{dy}{dx}=\frac{4x}{(x^2+1)^2}$		

(b) $X^2+6x+34=(x+3)^2+25$ $25\left[1+\left(\frac{x+3}{5}\right)^2\right]$ Let $\frac{x+3}{5}=\tan\theta$ $\frac{1}{5}=\sec^2\theta\frac{dQ}{DX}$ When x=3 ,tanϴ=0 ϴ=$\tan^{-1}(0) = 0$ X=2 tanϴ=$\frac{5}{5}$ ϴ = $\tan^{-1}(1)$ =$\frac{\pi}{4}$ $\int_{-3}^{2}\frac{dx}{x^2+6x+34}=\int_0^{\pi/4}\frac{1}{25}\left(\frac{1}{1+\tan^2\theta}\right)5\sec^2\theta dx$ $=\frac{1}{5}\int_0^{\pi/4}d\theta$ $=\frac{1}{5}[\theta]_0^{\pi/4}$ $=\frac{\pi}{20}$		
15. (a) $\frac{dy}{dx}$ = y+ $\tan\left(\frac{y}{x}\right)$ using y=ux From y=ux $\frac{dy}{dx}$ =u$\frac{d(x)}{dx}+x\frac{d(u)}{dx}$ $\frac{dy}{dx}=u+x\frac{du}{dx}$ $\left(\text{u+x}\frac{\text{du}}{\text{dx}}\right)$ = ux +tan(u) xu+x$^2\frac{dy}{dx}dx=x4+\tan u$		

$x^2\frac{du}{dx}=\tan u$ separating variables X^2du = tan dx $\frac{du}{\tan u}=\frac{dx}{x^2}$ $\frac{1}{\tan u}du=\frac{dx}{x^2}$ $\frac{\cos u}{\sin u}du=x^{-2dx}$ $\Rightarrow\int\frac{\cos udu}{\sin u}=\int^{x^2dx}$	

In sin u $=\frac{-1}{x}$+c But from y=ux $u = {}^{y}\!/\!_{x}$ $\Rightarrow$ in $\sin\left({}^{y}\!/\!_{x}\right) = \frac{-1}{x} + c$ Or $In \cos {}^{y}\!/\!_{x} + \frac{1}{x} + A = 0$	
(b) Let A represent the number of accidents and P represent the number of police deployed. $\frac{dA}{dp} = -kp$. dA=-Kpdp. SdA =-KSpdp. A=-kSpdp A=$\frac{-kp^2}{2} + c$ At A=7,P=2. 7=-k$\frac{4}{2}$+c. 7 =-2k+c……………. 1 At a=1 , P=4 1=-k$\frac{k16}{2} + c$ 1 =-8k+c…………11 K=1, c=9.	

So the equation connecting A and P is A=$\frac{-p^2}{2} + 9$ A= $\frac{-p^2}{2} + 9$ 2A= -p^2+(9X2) P^2 = (2X9)-2A P2=18-2A.	
(i) When there is no policeman P=0. P2 = 18-2A O = 18-2A $\frac{2A}{2} = \frac{18}{2}$ A = 9	

There are 9 accidents if no policeman is deployed.

(ii) To completely stump out accidents ie A=0

From P2= 18-2A. A=o

P2=18

P=$\sqrt{18}$ = 4.24 =5

5 policemen are required.

16. $k = \dfrac{5t}{16+\left(\frac{t}{a}\right)^2}$

$$k = \frac{5t}{1+\frac{t^2}{a^2}}$$

$$k = \frac{5a^2t}{a^2+t^2}$$

$$\frac{dk}{dt} = \frac{\left(a^2+t^2\right)-5a^2-5a^2t\left(2t\right)}{\left(a^2+t^2\right)^2}$$

But at maximum concentration

$\dfrac{dk}{dt}$=0

$5t^2a^2+5a^4-10a^2t^2=0$

At t=6

$180a^2+5a^4-360a^2=0$

$5a^2(a^2-36)=0$

Either 5a2=0 , a=0

Or a2-36=o, a=$\pm$ 6

A=$\pm$ 6

(b)

Volume of a cone.

$\dfrac{1}{3}\pi r^2 h]$

$$v = \frac{1}{3\sqrt{3}}\pi r^3$$

Given $\dfrac{dv}{dv} = \dfrac{\pi r^2}{\sqrt{3}}$

Required $\dfrac{dv}{dt} = \dfrac{dv}{dv}\,\dfrac{dv}{dt}$

$$\frac{dr}{dt} = \frac{\sqrt{3}}{\pi}.\frac{dv}{t}$$

From $\frac{dv}{dt}$=9m/s When v=$\frac{6}{60}$ t=1 $v = \frac{6}{60}.20\sqrt{3}$ t=$20\sqrt{3}$ V=$3\sqrt{3}$ From V=$\frac{1}{3\sqrt{3}}\pi r^3$ $3\sqrt{3} = \frac{1}{3\sqrt{3}}\pi r^3$ $\pi r^3 = 27$ $r = \frac{3}{\pi^{\frac{1}{3}}}$ $\frac{dr}{dt} = \frac{\sqrt{3}}{\pi\left(\frac{3}{\pi^{1/3}}\right)^2}\frac{dv}{dt}$ $\frac{dr}{dt} = \frac{\sqrt{3}}{\pi^{1/3}}$		