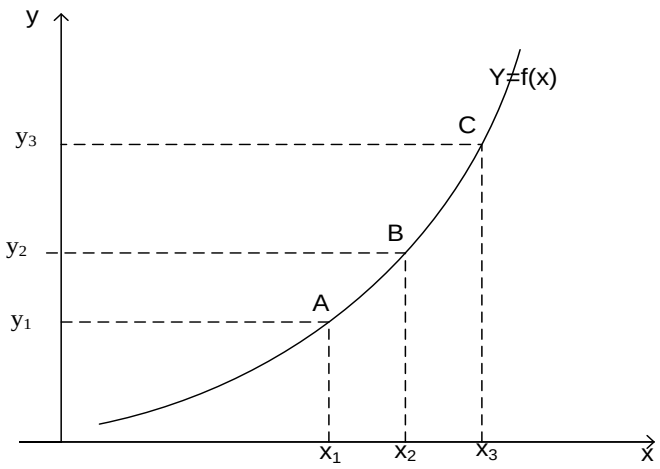
Topic 6: LINEAR INTERPOLATION AND EXTRAPOLATION

This is used to estimate values of a function by use of gradient method. If we are to locate a value which is not tabulated but lies within or outside two successive values x_1 and x_2, we can use gradient approach to determine the unknown. Consider a function $y = f(x)$ with three close points; $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$. If line AC is approximated to be a straight line then from the graph below the method of gradient can be used.



By equating correct gradients we get $\frac{y_3 - y_1}{x_3 - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

If the unknown point is B then, the method is referred to as linear interpolation and if the unknown point is A or C then we can use the same equation but the method now becomes linear extrapolation.

Example: The table below shows the delivery charges by a bus company as shown below

Mass(g)	200	400	600
Charges(shs)	700	1,200	3,000

Using linear interpolation or extrapolation, find the:

a) Delivery charges of a parcel weighing 352g
b) Mass of a parcel whose delivery charge is shs 3,300

Soln:

a) Since the unknown point lies between two points the method is linear interpolation. It is always advisable to start with the unknown since it becomes easy to simplify.

200	352	400
700	x	1200

$$\frac{x-700}{352-200}=\frac{1200-700}{400-200}$$

$x = shs1080$. The delivery charge is shs1080

b) Here we shall use linear extrapolation. It is always advisable to start with the unknown since it becomes easy to simplify.

400	600	y
1,200	3,000	3,300

$$\frac{y-400}{3{,}300-1200}=\frac{600-400}{3{,}000-1{,}200}$$

$$y = 633\frac{1}{3}g$$

Example: The table below shows values of a function f(x) as a set of points.

X	1.8	2.0	2.2	2.4
f(x)	0.532	0.484	0.436	0.384

Use linear interpolation to estimate the value of:

a) f(2.08)
b) x corresponding to f(x)=0.5

soln

a)

2.0	2.08	2.2

0.484	y	0.436

$$\frac{y-0.484}{2.08-2.0}=\frac{0.436-0.484}{2.2-2.0} \qquad y=0.4648$$

b)

1.8	x	2.0
0.532	0.5	0.484

$$\frac{x-1.8}{0.5-0.532}=\frac{2.0-1.8}{0.484-0.532} \qquad x=1.933$$

ASSIGNMENT 6 .1.9

Linear interpolation and Extrapolation

1. Given the table below:

x	0	10	20	30
y	6.6	2.9	-0.1	-2.9

Use linear interpolation to find:

a) y when x is 16

b) x when y is -1

2. The bus stages along Jinja-Kampala are 10km apart. An express bus travelling between two towns only stops at these stages, except in case of an emergency when it is permitted to stop at a point between two stages. The fares up to the first, second, third and fourth stage from Jinja are 1,100/=, 1,500/=, 1,850/= and 2,000/= respectively. On a certain day, a passenger paid to travel from Jinja to the fourth stage, but fell sick and had to be left at a health centre, 33km from Jinja.
   a) Given that he was refunded money for the part of the journey he had not travelled, find the approximate amount of money he received?
   b) Another person who had only 1,650/= was allowed to board, but would be left at a point worth his money, how far from Jinja was he left?

3. The table below is an extract from the table of $Cosx$

80^0	0^0	10’	20’	30’	40’	50’

$Cosx$	0.1736	0.1708	0.1679	0.1650	0.1622	0.1593

Determine;

i) $Cosx80^0 36'$

ii) $Cos^{-1}0.1685$

4. The table below shows values of $\sqrt{x}$ forgiven values of x.

x	0.12	0.13	0.14	0.15
$\sqrt{x}$	0.3464	0.3606	0.3742	0.3873

Using linear interpolation or extrapolation, find

a) $\sqrt{0.135}$

b) The value of x corresponding to $\sqrt{x} = 0.403$