A-LEVEL SUBSIDIARY MATH EMATICS SEMINAR QUESTIONS 2019

PURE MATHEMATICS

Qn 1: Simplify the following using the laws of indices:

(a). $16^{\left(\frac{3}{4}\right)n} \div 8^{\left(\frac{5}{3}\right)n} \times 4^{(n+1)}$.

(b). $\frac{x^{(2n+1)} \times x^{\frac{1}{2}}}{\sqrt{x^{3n}}}$.

(c). $\frac{\sqrt{(xy)} \times x^{\frac{1}{3}} \times 2y^{\frac{1}{4}}}{(x^{10}y^{9})^{\frac{1}{12}}}$.

Qn 2: Using the laws of indices, evaluate:

(a). $(0.04)^{\frac{-3}{2}}$

(b). $\frac{4^{\frac{-3}{2}}}{8^{\frac{-2}{3}}}$.

Qn 3: Show that $\frac{3\sqrt{2} - 4}{3 - 2\sqrt{2}} = \sqrt{2}$.

Qn 4: Solve the equation:

$$2^x \times 2^{x+1} = 10$$

Qn 5: Simplify:

(a). $-\log_2 \left(\frac{1}{2}\right)$.

(b). $\frac{\log 49}{\log 343}$.

Qn 6: Given that $\log_{10} 2 = 0.3010$,

(a). Show that $\log_{10} 5 = 0.6990$.

(b). Hence find the value of $\log_2 5$.

Qn 7: Show that $\log_a b \times \log_b a = 1$. Hence evaluate: $\log_3 8 \times \log_2 9$.

Qn 8: The roots of the equation $x^2 + 6x + q = 0$ are α and $\alpha - 1$.
Find the value of q.

Qn 9: The roots of the equation $2x^2 + 3x - 4 = 0$ are α and β.
Find the values of
(a). $(\alpha + 1)(\beta + 1)$.

(b). $\frac{\beta}{\alpha} + \frac{\alpha}{\beta}$.

(c). $\alpha^2 + \alpha\beta + \beta^2$.

Qn 10: The roots of the equation $2x^2 - 4x + 1 = 0$ are α and β.
Find the integral coefficients whose roots are:
(a). α^2 and β^2.

(b). $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Qn 11: The roots of the equation $x^2 + px + q = 0$ differ by 2.
Show that $p^2 = 1 + q$.

Qn 12: Find the values of a and b if $ax^4 + bx^3 - 8x^2 + 6$ has remainder $(2x + 1)$ when divided by $x^2 - 1$.

Qn 13: $(x - 1)$ and $(x + 1)$ are factors of the expression $x^3 + ax^2 + bx + c$, and it leaves a remainder of 12 when divided by $(x - 2)$. Find the values of a, b and c.

Qn 14: Given the quadratic equation $ax^2 + bx + c = 0$,
Show that $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
Hence use it to solve the equation $(2 + x)(3 - 4x) = 0$.

Qn 15: By completing squares, find the greatest value of $-7 + 12x - 3x^2$ and the value of x at which it occurs.

Qn 16: (a). Find the turning points of the curve $y = (2 + x)(1 + x)(3 - x)$.

(b). Hence distinguish the above turning points.

Qn 17: The tangent to the curve $y = ax^2 + bx + c$ at the point where $x = 2$ is parallel to the line $y = 4x$. Given that y has a maximum value of -3 where $x = 1$, find the values of a, b and c.

Qn 18: (a). Sketch the graph of the curve $y = 7 - x - x^2$.

(b). Hence calculate the area bounded between the curve and the line $y = 5$.

Question 19:

The velocity v of a point moving along a straight line is given in terms of the time, t by the formula $v = 2t^2 - 9t + 10$, the point being at the origin when $t = 0$.

(a). Find the expression in terms of t for the distance from the origin.

(b). Find the expression in terms of t for the acceleration.

(c). Show that the point is at rest twice and find its distances from the origin at those instants.

Question 20:

(a). $\int \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) dx$.

(b). Find s as a function of t given that $\frac{ds}{dt} = 6t^2 + 12t + 1$ and when $t = -2$, $s = 5$.

Question 21:

(a). Find the first term of a geometric series that has a common ratio of $\frac{3}{5}$ and a sum to infinity of 8.

(b). The first term of an A.P is -12 and the last term is 40. If the sum of the progression is 196, find:

(i). the number of terms,

(ii). the common difference.

Question 22:

(a). The first term of a G.P is 16 and the fifth term is 9. What is the value of the seventh term?

(b). The sum of the first terms of a G.P is 9 and the sum to infinity of the G.P is 25. If the G.P has a positive common ration r, find:

(i). r,

(ii). the first term.

Question 23:

(a). Find the possible values x can take given that

$$\boldsymbol{A} = \begin{pmatrix} x^2 & 3 \\ 1 & 3x \end{pmatrix},\ \boldsymbol{B} = \begin{pmatrix} 3 & 6 \\ 2 & x \end{pmatrix} \text{ and } \boldsymbol{AB} = \boldsymbol{BA}.$$

(b). If $\boldsymbol{A} = \begin{pmatrix} 3 & 2 \\ -4 & 1 \end{pmatrix}$, find the values of m and n given that $\boldsymbol{A^2} = m\boldsymbol{A} + n\boldsymbol{I}$ where $\boldsymbol{I}$ is a 2×2 identity matrix.

(c). Solve the following equations using the matrix method.

$$x - 3y = 3$$
$$5x - 9y - 11 = 0$$

Question 24:

(a). Find the inverse of the matrix $\boldsymbol{M}$ where $\boldsymbol{M} = \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}$ and hence solve the matrix equation $\boldsymbol{MX} = \boldsymbol{C}$ in which $\boldsymbol{X} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

(b). Find the values of x for which the matrix $\begin{pmatrix} x-2 & 1 \\ 2 & x-3 \end{pmatrix}$ has no inverse.

(c). Calculate the inverse of $\boldsymbol{M^{-1}}$ of the matrix $\boldsymbol{M} = \begin{pmatrix} x & x-1 \\ y & y \end{pmatrix}$, where $y \neq 0$.

Find the values of x and y such that $\boldsymbol{M^{-1}} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$.

Question 26:

Solve the following equations for all values of θ from $0°$ to $360°$.

(i). $\sin(\theta + 20)° = \frac{-\sqrt{3}}{2}$

(ii). $4 - \sin\theta = 4\cos^2\theta$.

(iii). $3\tan^2\theta + 5 = 7\sec\theta$.

Question 27:

If $\cos\theta = \frac{-8}{7}$ and θ is obtuse, find without using tables or calculators, the value of

(i). $\sin\theta$,

(ii). $\cot\theta$.

Question 28:

If $\tan\theta = \frac{7}{24}$ and θ is reflex, find without using tables or calculators, the value of

(i). $\sec\theta$,

(ii). $\sin\theta$.

Question 29:

If $\sin\theta = \frac{3}{5}$, find without using tables or calculator, the values of

(i). $\cos\theta$,

(ii). $\tan\theta$.

Question 30:

Prove the following identities:

(a). $\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$.

(b). $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$.

(c). $\frac{1-\cos^2\theta}{\sec^2\theta-1} = 1 - \sin^2\theta$.

Question 31:

If $\sin A = \frac{3}{5}$ and $\sin B = \frac{5}{13}$, where A and B are acute angles, find the values of

(a). $\sin(A + B)$

(b). $\cos(A + B)$

(c). $\tan(A + B)$

Question 32:

If $\sin(x - \alpha) = \cos(x + \alpha)$, prove that $\tan x = 1$.

Question 33:

If $\sin A = \frac{4}{5}$ and $\cos B = \frac{12}{13}$, where A is obtuse and B is acute, find the values of

(a). $\sin(A - B)$

(b). $\tan(A - B)$

(c). $\tan(A + B)$.

Question 34:

(a). The points E, F and G have position vectors $\left(2\underset{\sim}{\boldsymbol{i}} + 2\underset{\sim}{\boldsymbol{j}}\right)$, $\left(\underset{\sim}{\boldsymbol{i}} + 6\underset{\sim}{\boldsymbol{j}}\right)$ and $\left(-7\underset{\sim}{\boldsymbol{i}} + 4\underset{\sim}{\boldsymbol{j}}\right)$. Show that the triangle EFG is right angled at F.

(b). If the angle between the vectors $\underset{\sim}{\boldsymbol{c}} = a\underset{\sim}{\boldsymbol{i}} + 2\underset{\sim}{\boldsymbol{j}}$ and $\underset{\sim}{\boldsymbol{d}} = 3\underset{\sim}{\boldsymbol{i}} + \underset{\sim}{\boldsymbol{j}}$ is $45°$, find the possible values of a.

Question 35:

(a). The points A, B, C and D have position vectors $\left(5\underset{\sim}{\boldsymbol{i}} + \underset{\sim}{\boldsymbol{j}}\right)$, $\left(-3\underset{\sim}{\boldsymbol{i}} + 2\underset{\sim}{\boldsymbol{j}}\right)$, $\left(-3\underset{\sim}{\boldsymbol{i}} - 3\underset{\sim}{\boldsymbol{j}}\right)$ and $\left(\underset{\sim}{\boldsymbol{i}} - 6\underset{\sim}{\boldsymbol{j}}\right)$. Show that AC is perpendicular to BD.

(b). Find a vector that is of magnitude 2 units and is parallel to $\left(4\underset{\sim}{\boldsymbol{i}} - 3\underset{\sim}{\boldsymbol{j}}\right)$.

(c). The point $\boldsymbol{A}$ has position vector $\left(3\underset{\sim}{\boldsymbol{i}} + \underset{\sim}{\boldsymbol{j}}\right)$ and point $\boldsymbol{B}$ has position vector $\left(10\underset{\sim}{\boldsymbol{i}} + \underset{\sim}{\boldsymbol{j}}\right)$. Find the position vector of the point which divides $\boldsymbol{AB}$ in the ratio $3:4$.

Question 36:

The points A, B and C have position vectors $\underset{\sim}{\boldsymbol{a}}$, $\underset{\sim}{\boldsymbol{b}}$ and $\underset{\sim}{\boldsymbol{c}}$ respectively referred to an origin O.

(a). Given that the point X lies on AB produced so that $AB:BX = 2:1$, find x, the position vector of X in terms of $\underset{\sim}{\boldsymbol{a}}$ and $\underset{\sim}{\boldsymbol{b}}$.

(b). If Y lies on BC between B and C so that $BY:YC = 1:3$, find y, the position vector of Y in terms of $\underset{\sim}{\boldsymbol{b}}$ and $\underset{\sim}{\boldsymbol{c}}$.

(c). Given that Z is the midpoint of AC, show that X, Y and Z are collinear.

(d). Calculate $XY:YZ$.

STATISTICS AND PROBABILITY

Question 1:
The table below represents the weights of 50 patients who visited a certain health centre in May 2019.

Weights (kg)	Percentage frequency (%)
40 – 50	10
50 – 60	16
60 – 70	24
70 – 80	30
80 – 90	14
90 – 100	6

(a). Calculate the:
- (i). mean weight,
- (ii). modal weight,
- (iii). number of patients who weigh between 55 kg and 85 kg.

(b). Draw a cumulative frequency curve and use it to estimate the:
- (i). median weight,
- (ii). interquartile range of weight.

Question 2:
The table below shows the number of pupils who failed to hand in the Maths homework each day and the minutes of yoga the teacher does to calm himself down.

Pupils missing homework (X)	3	5	2	10	2	0	4	8	15	6	1	4
Minutes of yoga (Y)	10	12	9	25	8	3	15	20	26	10	7	10

(a). Draw a scatter diagram and line of best fit to show the information.

(b). (i). If 7 pupils forget to hand in their homework, how many minutes of yoga might the teacher do?
- (ii). If the teacher did 28 minutes of yoga, how many pupils might have forgotten their homework?

(c). Calculate the rank correlation coefficient and comment on the level of significance.

Question 3:
A construction company uses five items of materials for a particular construction job. The table below represents the prices in shillings for each material used in the job for the year 2017 and 2019.

Materials	Price in 2017	Price in 2019	Weight
Sand	10,500	18,424	3
Cement	16,750	18,424	2
Water	1,250	1,500	4
Lime	8,200	10,250	1
Paste	3,750	5,250	1

Given that 2017 is assigned as an index of 100,
(i). Construct the unweighted price relative index for each material.
(ii). construct the simple average price relative for the material.
(iii). construct the weighted aggregate index for the materials.

Question 4:
The table below shows the sales in units of each of the four quarters for the years 2003, 2004 and 2005.

Years	Quarters			
	1^{st}	2^{nd}	3^{rd}	4^{th}
2003	600	840	420	720
2004	640	860	420	740
2005	670	900	430	760

(a). Calculate the four point moving averages.
(b). On the same axes, plot graphs of sales and the moving averages against time. Comment on the general trend of the sales for the three years' period.
(c). Use your graph to estimate the sales in:
    (i). the 4^{th} quarter of 2002.
    (ii). the 1^{st} quarter of 2006.

Question 5:

(a). A die and a coin are tossed once, what is the probability of getting a five on a die and a head on a coin?

(b). A student who wishes to join a higher institution of learning after A-level has the following choices with their corresponding probabilities.

Institution	**Probability**
M.U.B.S	04
Mbale	0.2
Nkozi	0.1
Nkumba	0.2
U.C.U	0.1

(i). Find the probability that he joins either M.U.B.S or Nkumba.
(ii). State, with a reason, the type of events above.

(c). In a certain school, there are 50 Senior five students of which 10 are girls; and 30 Senior six students of which 5 are girls. A student is to be picked at random to represent the school in a mathematics contest.
(i). Find the probability that the student chosen is a girl.
(ii). If the selected student is a girl, what is the probability that she is in Senior six?

Question 6:
In how many ways can the letters of the word MISSISSIPI be arranged:
(i). without restriction.
(ii). if the I's are kept together.
(iii). if the I's are separated.
(iv). if all the I's are the end of every word.
(v). if every word starts with M and ends with P.

Question 7:
An organizing committee of 4 people is to be chosen from 3 women and 4 men to organize a school end of year party. Determine the probability that the school with consist of at most 2 women.

Question 8:

A discrete random variable X has a probability density function $f(x)$, defined by

$$f(x) = \begin{cases} kx^3 & ; \quad x = 0, 1, 2, 3, \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

Where k is a constant.

Determine :

(i). the value of k,

(ii). $P((1 \leq X < 3)/X > 0)$,

(iii). $E(X)$,

(iv). $Var\ (X)$

Question 9:

In a certain clinic, 20% of the patients who are treated with a certain drug often develop complications. If a random sample of 50 patients are treated with this particular drug, find the mean, variance and standard deviation of patients who will develop the complication.

Question 10:

A continuous random variable X has a probability density function $f(x)$, defined by

$$f(x) = \begin{cases} k(x^2 - 1) & ; \quad 0 \leq x \leq 2, \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

Where k is a constant.

Determine the:

(i). value of k,

(ii). $P(X > 1)$,

(iii). mean value of X,

(iv). variance of X.

Question 11:

X is a random variable of a random experiment following a normal distribution with mean 40 and standard deviation 8.5. Determine:

(i). $P(X < 42.5)$,

(ii). $P(X > 31.5)$,

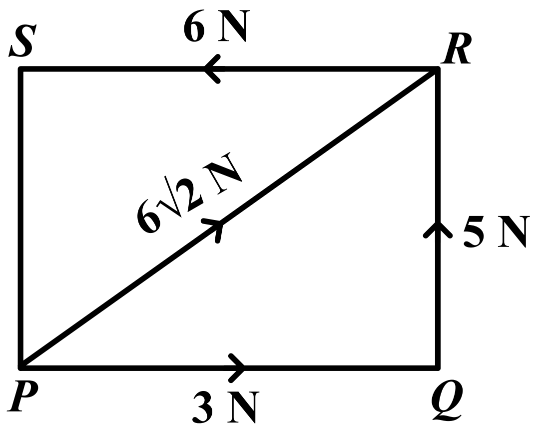
(iii). $P(31.5 < X < 36.8)$.

MECHANICS

Question 1:
A car initially moving at a speed of 80 m s^{-1} decelerates uniformly and attains a velocity of 40 m s^{-1} for 20s and comes to rest in the next 30 s. Sketch a velocity – time graph and use it to calculate the average velocity.

Question 2:
Forces of magnitude $2\sqrt{2}$ N, 4 N and 6 N, act on a body at angles 40°, 240° and 330° with the positive x −axis. Draw a clear force diagram and use it to find the magnitude of the resultant force and its direction to the x −axis.

Question 3:
PQRS is a square, forces of 3N, 5N, 6N, and $6\sqrt{2}$ N act along PQ, QR, RS, and PR respectively.



Calculate the resultant force of the system.

Question 4:
Two particles A and B of masses 3 kg and 5 kg are connected by a fine string passing over a smooth fixed pulley. Find:
(i). their common acceleration.
(ii). the tension in the string.

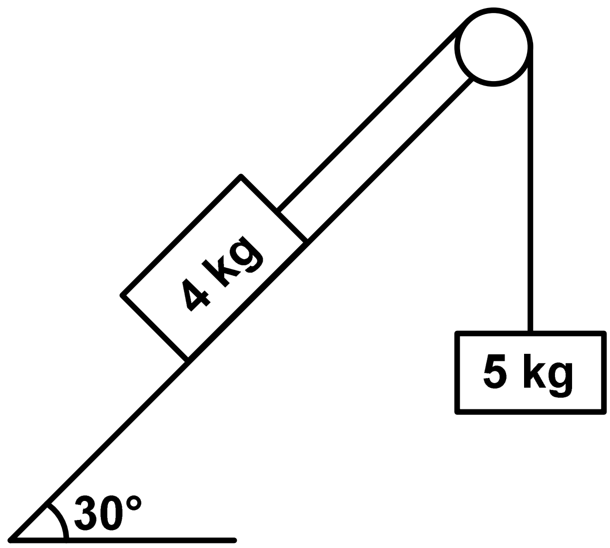
Question 5:

Body of mass 8 kg rests on a rough horizontal table and is connected to another body of mass 3 kg by a light inextensible string passing over a smooth pulley fixed at the edge of the table. The 3 kg mass hangs freely. The coefficient of friction between the 8 kg mass and the table is 0.3. The system is released from rest. Find the

(i). acceleration of the system.
(ii). tension in the string.
(iii). distance the 3 kg mass covers after 2 seconds.

Question 6:

The diagram below shows two masses of 4 kg and 5 kg attached to each other by a light inextensible string which passes over a smooth pulley fixed at the top of the wedge.



(a). Find the coefficient of friction between the 4 kg mass and the wedge if the system is in limiting equilibrium.
(b). Calculate the acceleration of the system if the 5 kg mass is increased by 1 kg.

Question 7:
A particle of mass 3 kg slides from rest down a rough plane inclined at 30° to the horizontal. Given that the coefficient of friction between the particle and the plane is 0.3, find the:
(i). normal reaction of the plane
(ii). frictional force exerted on the body
(iii). acceleration of the particle
(iv). how far it travels in 4 seconds.

Question 8:
A force of magnitude 40 N acts on a body causing it to change its velocity from 4 m s^{-1} to 10 m s^{-1} after 5 seconds. Find the work done by the force.

Question 9:
A lorry of mass 200 kg travels against friction resistance of 2600 N for 50 m along a level road at a constant speed of 45 km h^{-1}. Calculate the:
(i). work done by the engine,
(ii). power at which the engine is working.

Question 10:
A heavy truck of mass 10 tonnes has an engine which exerts a maximum power of 20 kW. The truck moves on a level ground against a constant resistance force F newton.
(a). If the truck attains a maximum speed of 57.6 km h^{-1}, find the value of F.
(b). Given that the truck ascends an incline of 1 in 100, find its maximum speed. (Assume resistance remains the same as in (a) above)

Question 11:
A rough plane is inclined at an angle $\sin^{-1}\left(\frac{3}{5}\right)$ to the horizontal.
A body of mass 20 kg is released from rest at the top of the plane to slide down the whole length of the plane in 5 s. If the coefficient of friction between the plane and the body is 0.5, calculate the:
(a). length and height of the inclined plane,
(b). loss in potential energy of the mass during the slide down period,
(c). velocity at the time the mass is at the bottom end of the inclined plane.

END