GUIDE Real

P425/2
APPLIED
MATHEMATICS
PAPER 2
Jul/Aug 2018
3 hours

Mukono Kayunga MUKAJEB Joint Examination Board

MUKONO KAYUNGA JOINT MOCK EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and* **five** *questions from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All working must be shown clearly.

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

Turn over

SECTION A

1. Events A and B are such that $P(A/B) = \frac{3}{8}$, $P(A' \cap B') = \frac{1}{4}$ and $P(A) + P(B) = \frac{23}{24}$.

   Find;

   (i) $P(A \cap B)$

   (ii) $P(B/A')$

   (05 marks)

2. The table below shows the values of x and their corresponding values of $f(x)$.

x	1.8	2.3	3.1	3.9
$f(x)$	3.352	5.587	14.571	32.899

   Use linear interpolation or extrapolation to obtain the value of;

   (i) $f(2.9)$

   (ii) $f^{-1}(35.154)$

   (05 marks)

3. A particle is acted upon by two forces $F_1 = \left(-3\underset{\sim}{i} + 4\underset{\sim}{j}\right)$N and $F_2 = 75$N in the direction $24\underset{\sim}{i} - 7\underset{\sim}{j}$. Find the magnitude and direction of the resultant force. *(05 marks)*

4. A continuous random variable X is uniformly distributed in the interval (30, 45). Calculate the;

   (i) Mean of X

   (ii) $P(X > 39)$

   (05 marks)

5. A body of weight W is held in limiting equilibrium on a rough slope inclined at 60^0 to the horizontal by a force P at angle of 30^0 to the slope. The coefficient of friction being $\frac{1}{2}$, show that P = W. *(05 marks)*

6. Show graphically that the root of the equation $2x^3 - 4x - 5 = 0$ exist in the interval (1, 2). *(05 marks)*

7. A particle moving with simple harmonic motion has a speed of $2ms^{-1}$ when it is $\sqrt{2}$ m from its mean position. Given that the amplitude of its motion is 1.5m, calculate its;

   (i) velocity as it goes through the mean position.

   (ii) Time taken when it is $\frac{1}{4}$ of its amplitude from the maximum displacement.

   (05 marks)

8. The marks of 6 students in French and Biology were as follows:

French	90	60	80	54	86	70
Biology	48	72	60	78	50	65

   Calculate the rank correlation coefficient for the scores in the two tests. Comment on your results. *(05 marks)*

14. The probability density function of a continuous random variable x is represented by the equation below:

$$f(x)\begin{cases} \frac{2}{13}(x+1) & ; \quad 0 \le x \le a \\ \frac{2}{13}(5-x) & ; \quad a \le x \le 3 \\ 0 & ; \quad elsewhere \end{cases}$$

Calculate the;

a) the value of a.

b) $P(x < 2.5)$

(07 marks)

(05 marks)

15. A biased coin is thrice as likely to show heads as tails. If it is tossed 48 times, find the probability of obtaining;

a) between 30 and 40 heads.

b) at least 28 but less than 42 heads.

(12 marks)

16. A car of mass 800kg tows a trailer of mass 200kg. The constant resistance acting on the car and the trailer are 450N and R respectively. If the car has maximum speed of $54kmh^{-1}$ on the level road, with the engine at steady rate of 9.75kW, find the;

(i) Tension in the tow bar

(ii) The value of R

(iii) Acceleration of the car at a speed of $72kmh^{-1}$.

(12 marks)

END

$\frac{29}{72}$

$\frac{5a - a^2}{2} - \frac{(10a - a^2)}{2}$

SECTION B

9. The lengths (h) in inches of 40 nails were as follows.

Lengths (h)	Frequency
$3.0 \le h < 3.5$	8
$3.5 \le h < 4.0$	5
$4.0 \le h < 5.5$	12
$5.5 \le h < 6.0$	9
$6.0 \le h < 6.5$	6

a) Calculate;
   (i) The mean
   (ii) The standard deviation.

b) Display the data on a histogram and use it to estimate the mode.

(12 marks)

10. a) Use the trapezium rule with 6 ordinates to estimate, to 3 decimal places the value of the integral $\int_1^3 \frac{x}{1+x^2}\,dx$.

b) Obtain the exact value of the integral in a) above. Hence calculate the percentage error in your estimation.

(12 marks)

11. A ball is projected from the top of a vertical cliff 36m high with a speed of 40ms^{-1} at an angle of elevation θ. The ball passing the highest point, P which is 12m above the point of projection after 2 seconds.

a) Find the value of θ.

b) The horizontal from the foot of the cliff where the ball lands.

c) Find the speed and direction of the ball as it hits the ground.

(12 marks)

12. a) The mass M and velocity V of a car were estimated with error ΔM and ΔV respectively. Show that the maximum relative error in the kinetic energy is $\left|\frac{\Delta M}{M}\right| + 2\left|\frac{\Delta V}{V}\right|$. *(07 marks)*

b) Find the range with in which the exact value of $\frac{4.25}{3.152 - 2.4}$ lies.

(05 marks)

13. A square ABCD of side 4m has forces of magnitude 8N, 3N, 4N 2N and $2\sqrt{2}$N acting along AB, CB, DA, CD and BD respectively. Taking AB and AD as x and y axes respectively,

a) Find the distance from A where the line of action of the resultant crosses AB.

(07 marks)

b) When a force P is introduced, the system reduces to a couple. Find the magnitude of P. *(05 marks)*

GUIDE APPLIED MATHS

1. $P(A|B) = \frac{3}{8}$

$P(A' \cap B') = \frac{1}{4}$

$P(A) + P(B) = \frac{23}{24}$

From $P(A' \cap B') = P(A \cup B)'$

$P(A \cup B) = 1 - \frac{1}{4} = \frac{3}{4}$ B1

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{3}{4} - \frac{23}{24} = -P(A \cap B)$ M1

$-\frac{5}{24} = -P(A \cap B)$

$\therefore P(A \cap B) = \frac{5}{24}$ A1

(ii) $P(B|A') = \dfrac{P(A' \cap B)}{P(A')} = \dfrac{P(B) - P(A \cap B)}{P(A')}$

$= \dfrac{\frac{P(A \cap B)}{P(A|B)} - P(A \cap B)}{P(A')}$; $P(B) = \frac{5}{24} / \frac{3}{8} = \frac{5}{24} \times \frac{8}{3}$

$= \frac{5}{9}$

$P(A) = \frac{23}{24} - \frac{5}{9} = \frac{69 - 40}{72} = \frac{29}{72}$

$= \dfrac{\frac{5}{9} - \frac{5}{24}}{\frac{43}{72}} = \frac{25}{72} \times \frac{72}{43}$ B1

$= \frac{25}{43}$. A1

05

2.

2.3	2.9	3.1
5.587	y_0	14.571

B1

$$\frac{y_0 - 5.587}{2.9 - 2.3} = \frac{14.571 - y_0}{3.1 - 2.9} \quad B1$$

$$\frac{y_0 - 5.587}{0.6} = \frac{14.571 - y_0}{0.2}$$

$0.8 y_0 = 9.86$

$y_0 = 12.325$ A1

(ii)

3.1	3.9	x_0
14.571	32.899	35.154

$$\frac{32.899 - 14.571}{3.9 - 3.1} = \frac{35.154 - 32.899}{x_0 - 3.9}$$

$$\frac{18.328}{0.8} = \frac{2.255}{x_0 - 3.9} \quad B1$$

$1.804 = 18.328 x_0 - 71.4792$

$x_0 = 3.998$

$\overline{05}$

$\therefore \; x_0 = 4.0$ A1

3. $\underset{\sim}{F_1} = -3i + 4j$

$$F_2 = \frac{75\,(24i - 7j)}{|24i - 7j|} = \frac{75}{25}(24i - 7j) = 3(24i - 7j) \quad B1$$

$\underset{\sim}{F} = \underset{\sim}{F_1} + \underset{\sim}{F_2} = (-3i + 4j) + (72i - 21j) = 69i - 17j$

$X \sim R(30, 45)$

$f(x) = \begin{cases} \frac{1}{15}, & 30 \le x \le 45 \\ 0, & \text{otherwise} \end{cases}$ B1

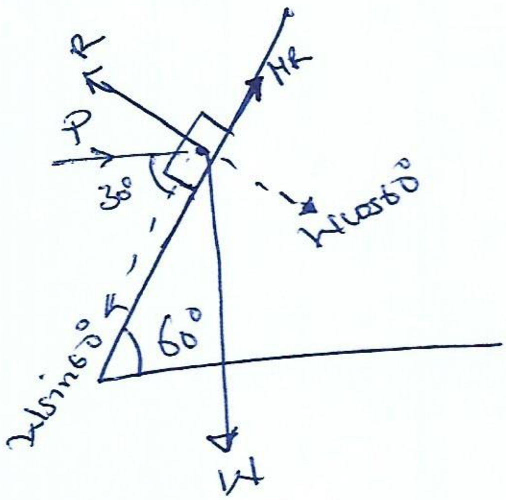
$E(X) = \int_{30}^{45} \frac{1}{15} x \, dx$ B1 $= \left. \frac{x^2}{30} \right|_{30}^{45} = \frac{45^2 - 30^2}{30} = \frac{1125}{30}$

$= 37.5$ A1

(ii) $P(X > 39) = \int_{39}^{45} \frac{1}{15} \, dx$ B1 $= \left. \frac{x}{15} \right|_{39}^{45} = \frac{45 - 39}{15} = 0.4$ A1

$\underline{\overline{05}}$

5.



B1

$\rightarrow \quad P\cos 30 + \mu R = W \sin 60$ B1

$\uparrow \quad R = W\cos 60 + P \sin 30$ B1

$P\cos 30 + \frac{1}{2}(W\cos 60 + P\sin 30) = W\sin 60$

$\frac{\sqrt{3}}{2} P + \frac{1}{2}\left(W\left(\tfrac{1}{2}\right) + \frac{1}{2}P\right) = \frac{\sqrt{3}}{2} W$ M1

$\frac{1}{4}(W + P) = \frac{\sqrt{3}}{2}(W - P)$

$2W + 2P = 4\sqrt{3}W - 4\sqrt{3}P$

$(2 + 4\sqrt{3})P = (4\sqrt{3} - 2)W$

$P = \frac{(4\sqrt{3} - 2)W}{} = \frac{(4\sqrt{3} - 2)^2 W}{} = (2\sqrt{3} - 1)^2 W$

6. $f(x) = 2x^3 - 4x - 5$

$f(1) = 2 - 4 - 5 = -7$ graph 05

$f(2) = 16 - 8 - 5 = 3$

$f(1) \cdot f(2) = -21 < 0$.

7. $V = 2 m/s$

$S = \sqrt{2} m$.

$A = 1.5 m$.

at mean position; $V = max$.

$$V_{max} = \omega A = \frac{2\pi}{T} A = \left(\frac{2\pi}{T}\right) 1.5 \quad B1$$

but

$$V^2 = \omega^2 (A^2 - x^2)$$

$$4 = \omega^2 (1.5^2 - 2)$$

$$\omega = \sqrt{\frac{4}{2.25 - 2}}$$

$$\omega = \sqrt{\frac{4}{0.25}}$$

$$\omega = \sqrt{\frac{400}{25}}$$

$$\omega = \frac{20}{5} = 4 \quad B1$$

$$V_{max} = 4 \times 1.5 = 6 m/s. \quad A1$$

$$X = A \sin \omega t$$

$$\frac{1}{4}(1.5) = 1.5 \sin \omega t.$$

$$\omega t = \sin^{-1}\left(\frac{1}{4}\right) \quad B1 = \frac{14.5\pi}{\ldots} \quad A1$$

$\overline{05}$

R_1	R_2	$d = R_1 - R_2$	d^2
1	6	-5	25
5	2	3	9
3	4	-1	1
6	1	5	25
2	5	-3	9
4 B1	3 B1	1	1
			$\Sigma d^2 = 70$ B1

$$\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 70}{6(36-1)}$$ B1

$$= 1 - \frac{420}{6 \times 35}$$

$$= 1 - 2$$

$$\rho = -1$$

Significant correlation at both 5% and 1% A1

05

h	f	c	f·d	x	fx	fx^2
3.0 — <3.5	8	0.5	16	3.25	26	84.5
3.5 — <4.0	5	0.5	10	3.75	18.75	70.3125
4.0 — <5.5	12	1.5	8	4.75	57	270.75
5.5 — <6.0	9	0.5	18	5.75	51.75	297.5625
6.0 — <6.5	6	0.5	12	6.25	37.5	234.375
			B1	B1	B1	B1
					$\Sigma fx = 191$	$\Sigma fx^2 = 957.5$

$$\bar{x} = \frac{191}{40} = 4.775 \text{ Inches}$$ B1

$$S.D = \sqrt{\frac{957.5}{40} - \left(\frac{191}{40}\right)^2}$$ B1

$$= \sqrt{1.136875}$$

$$S.D = \underline{1.066 \text{ Inches}}$$ A1

$\overline{OS}$ on a histog

$\frac{12}{12}$

Histogram on the graph.

$$\int_1^3 \frac{x}{1+x^2}\,dx$$

$$h = \frac{3-1}{6-1} = \frac{2}{5} = 0.4 \quad B1$$

n	x_n	$y_n = \frac{x_n}{x_n^2+1}$	
0	1	0.5000	
1	1.4		0.4730
2	1.8		0.4245
3	2.2		0.3767
4	2.6		0.3351
5	3.0	0.3000 B1	B1
6		0.8000	1.6093

$$\int_1^3 \frac{x}{1+x^2}\,dx = \frac{1}{2}(0.4)\left(0.8000 + 2(1.6093)\right)] \quad B1$$

$$= 0.8030 \quad A1$$

$$= 0.803 \ (3DP) \quad A1$$

$$\text{Actual} = \int_1^3 \frac{x}{1+x^2}\,dx = \frac{1}{2}\ln(1+x^2)\Big|_1^3 = \frac{1}{2}(\ln 10 - \ln 2)$$

B1 B1

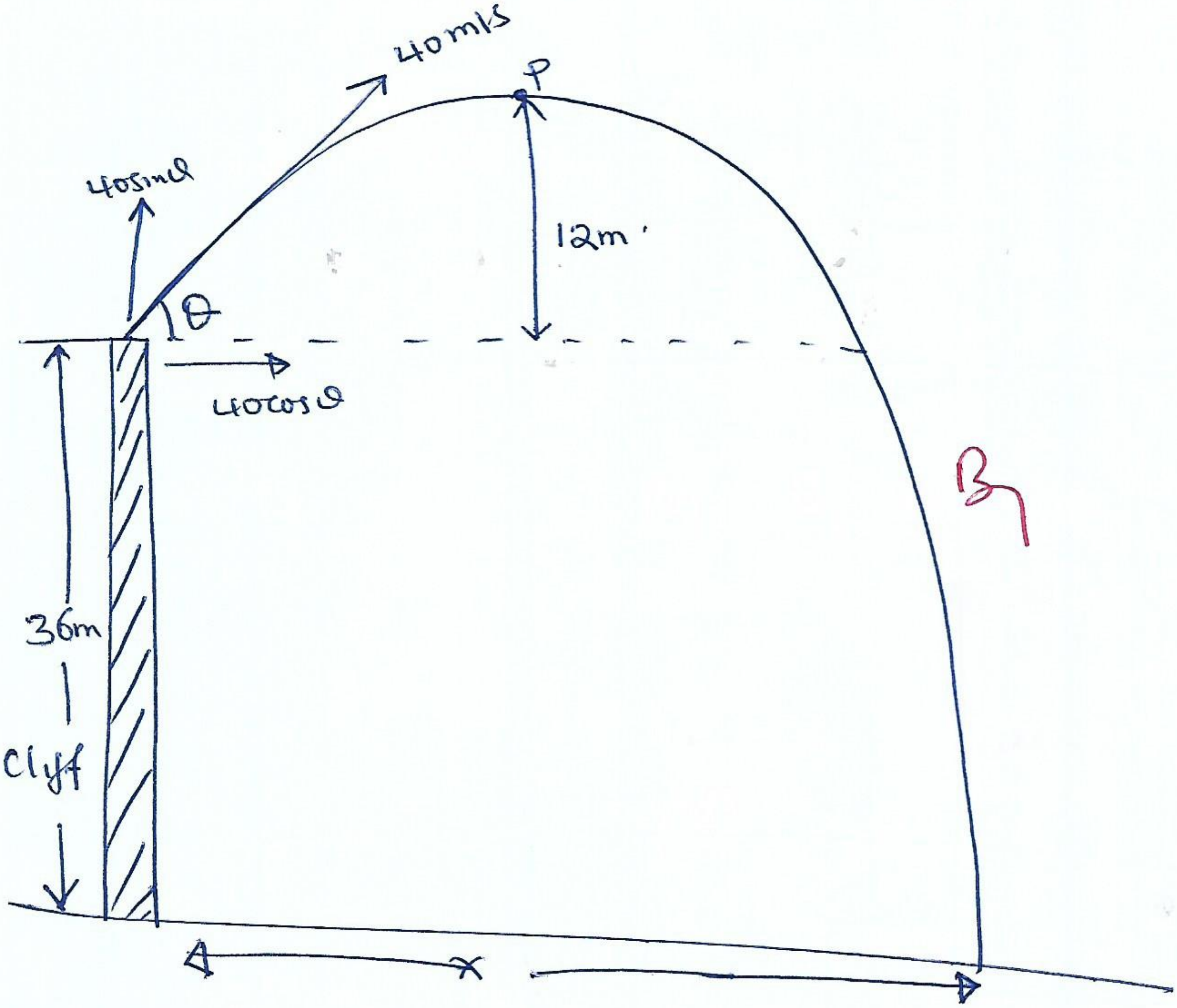
$$= 0.8047 \quad M1$$

$$\simeq 0.805 \ (3DP) \quad A1$$

$$\% = \frac{|0.805 - 0.803|}{0.805} \times 100 = 0.248\,\% \quad B1 \quad A1$$

12

11.



$$H = \frac{U^2 \sin^2\theta}{2g}$$

$$12 = \frac{40^2 \sin^2\theta}{2g}$$ B1

$$\sin^2\theta = \frac{12 \times 2g}{40^2}$$

$$\sin^2\theta = 0.147$$

$$\theta = 22.5^\circ$$ A1

$$12 = (40\sin\theta)(2) - \tfrac{1}{2} \times 9.8 \times 4$$

$$\sin\theta = 0.395$$

$$\theta = 23.3^\circ.$$

12/12

$$-36 = (U\sin\theta)\,t - \tfrac{1}{2} g t^2$$

$$4.9t^2 - 15.3t - 36 = 0.$$ B1

$$t = 15.3 \pm \sqrt{15.3^2 \ldots}$$

$x = 40\cos 22.5 \times 4.69 = 173.3\,m.$ B1 A1

(c) $V_x = 40\cos 22.5 = 36.955\,m/s$ B1

$V_y = 40\sin 22.5 - 9.8(4.69)$ M1

$V_y = -30.655\,m/s.$

$V = \sqrt{36.955^2 + (-30.655)^2}$ M1

$V = 48.01\,m/s.$ A1

$\theta = 39.7^\circ$ B1 below the horizontal.

12. $K.E = \frac{1}{2}mv^2$ B1

$E + \Delta E = \frac{1}{2}(m+\Delta m)(v+\Delta v)^2$ B1

$\Delta E = \frac{1}{2}(m+\Delta m)(v^2 + 2v\Delta v + (\Delta v)^2) - \frac{1}{2}mv^2$ B1

$\Delta E = \frac{1}{2}(mv^2 + 2mv\Delta v + m(\Delta v)^2 + v^2\Delta m + 2v\Delta m\Delta v + \Delta m(\Delta v)^2 - \frac{1}{2}mv^2$

$(\Delta v)^2 \approx 0, \quad \Delta m\Delta v \approx 0$ B1

$\Delta E = \frac{1}{2}(2mv\Delta v + v^2\Delta m)$ B1

$\frac{\Delta E}{E} = \frac{mv\Delta v}{\frac{1}{2}mv^2} + \frac{\frac{1}{2}v^2\Delta m}{\frac{1}{2}mv^2}$ B1

$\frac{\Delta E}{E} = 2\frac{\Delta v}{v} + \frac{\Delta m}{m}$ B1

Since $\left|\frac{\Delta E}{E}\right| = \left|2\frac{\Delta v}{v} + \frac{\Delta m}{m}\right| \le \left|2\frac{\Delta v}{v}\right| + \left|\frac{\Delta m}{m}\right|$ B1

Let $A = \dfrac{4.25}{3.152 - 2.4}$

$Max(A) = \dfrac{4.255}{3.1575 - 2.45}$ B1

$= 6.06557$ A1

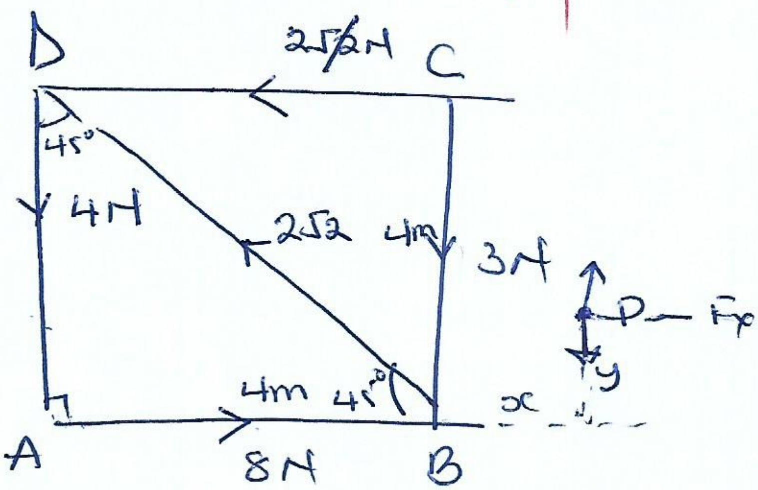
$Min(A) = \dfrac{4.245}{3.1525 - 2.35}$ B1

$= 5.28971$ A1

$\frac{12}{12}$

$[5.28971 \cdot 6.06557]$.
B1

13.



Force missing

$$\underset{\sim}{F} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -3 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -4 \end{pmatrix} + \begin{pmatrix} -2\sqrt{2}\cos 45 \\ 2\sqrt{2}\sin 45 \end{pmatrix}$$

$$\underset{\sim}{F} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$$

$\overset{\frown}{A}$

$4y + 5x = -(2\sqrt{2}\sin 45)4 + \underline{3}(4) - 2(4)$

$$4y + 5x = -4$$

$$y = 0$$
$$x = -4/5$$

It cuts It at 0.8m behind A.

(b) For a couple: $\underset{\sim}{F} = 0$.

$$\underset{\sim}{P} + \underset{\sim}{F} = 0$$
$$\underset{\sim}{P} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$

$$|\underset{\sim}{P}| = \sqrt{4^2+5^2} = \sqrt{16+25}$$
$$= \cancel{2+}\sqrt{41}\,N.$$

(14) $$f(x) = \begin{cases} \frac{2}{13}(x+1), & 0 \le x \le a \\ 2/13(5-x); & a \le x \le 3 \\ 0; & \text{Else where.} \end{cases}$$

$$f(a) = 2/13(a+1) = \frac{2}{13}(5-a).$$

$$2a = 5-1$$
$$a = \frac{4}{2} = 2$$
or

$$\int_0^a \frac{2}{13}(x+1)\,dx + \int_a^3 \frac{2}{13}(5-x)\,dx = 1 \text{ B1}$$

$$\frac{2}{13}\left[\frac{x^2}{2}+x\Big|_0^a + 5x-\frac{x^2}{2}\Big|_a^3\right] = 1. \text{ B1}$$

$$\overline{3.152 - 2.4}$$

$$\text{Max}(A) = \frac{4.255}{3.1575 - 2.45} \quad \text{B1}$$

$$= 6.06557 \quad \text{A1}$$

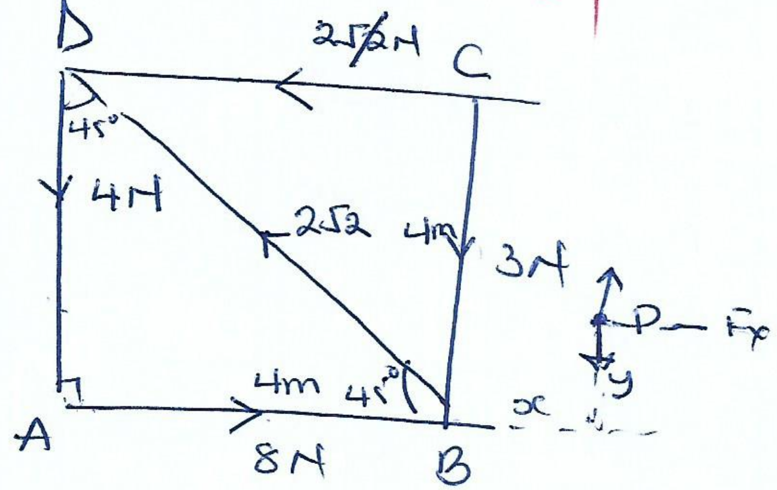
$$\text{Min}(A) = \frac{4.245}{3.1525 - 2.35} \quad \text{B1}$$

$$= 5.28971 \quad \text{A1}$$

$\frac{12}{12}$

$[5.28971 \, , \, 6.06557]$. B1

13.



Force missing

$$\underset{\sim}{F} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -3 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -4 \end{pmatrix} + \begin{pmatrix} -2\sqrt{2}\cos 45 \\ 2\sqrt{2}\sin 45 \end{pmatrix}$$

$$\underset{\sim}{F} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$$

$\overset{\frown}{A}$.

$$4y + 5x = -(2\sqrt{2}\sin 45)4 + \underline{3}(4) - 2(4)$$

$$4y + 5x = -8 + 12 - 8$$

$$\frac{2}{13}\left[\frac{a^2}{2}+a+\left(15-\frac{a}{2}\right)-\left(5a-\frac{a^2}{2}\right)\right]=1$$

B1 M1

$$a^2-14a+\frac{21}{2}=\frac{13}{2}$$

M1

$$a^2-14a=-\frac{8}{2}$$

$$a^2-14a+4=0.$$

$$a=2.$$ A1

(b) $$P(X<2.5)=\int_0^2 \frac{2}{13}(x+1)\,dx+\int_{2.0}^{2.5}\frac{2}{13}(5-x)\,dx$$

M1 B1

$$=\frac{2}{13}\left[\frac{x^2}{2}+x\Big|_0^2+5x-\frac{x^2}{2}\Big|_2^{2.5}\right]$$

B1

$$=\frac{2}{13}\left[4+\left(12.5-\frac{6.25}{2}\right)-(10-2)\right]$$

B1

$$=\frac{2}{13}\left[-\cancel{8}\,4+12.5-3.125\right]$$

$$=\frac{11}{52}=\cancel{0.2115}$$ B1

$$\frac{43}{52}=0.5269.$$

$\frac{12}{12}$

15. $P(H)=\frac{3}{4}$

$P(T)=\frac{1}{4}$ B1

$p=\frac{3}{4}$

$q=\frac{1}{4}$

$n=48$

$X\sim N(np,\ npq)$ B1

$np=48\times\frac{3}{4}=36$ B1

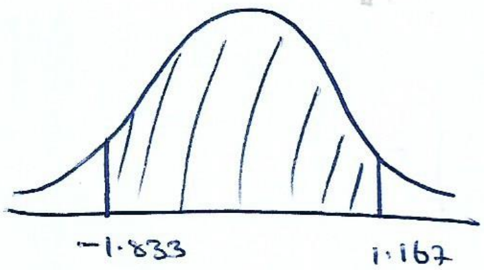
12

$P(30 \le X < 40) = P(30.5 < X < 39.5) =$ B1

$P\left(\frac{30.5-36}{3} < Z < \frac{39.5-36}{3}\right) =$

$P(-1.833 < Z < 1.167) = \left(\begin{matrix}0.4664\\0.0002\end{matrix}\right) + \left(\begin{matrix}0.3772\\0.0015\end{matrix}\right)$ B1

$= 0.8453$ M1 A1

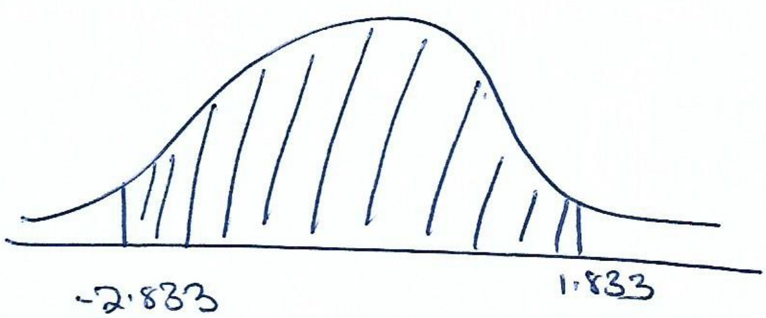


(b) $P(28 \le X < 42) = P(27.5 < X < 41.5)$ B1

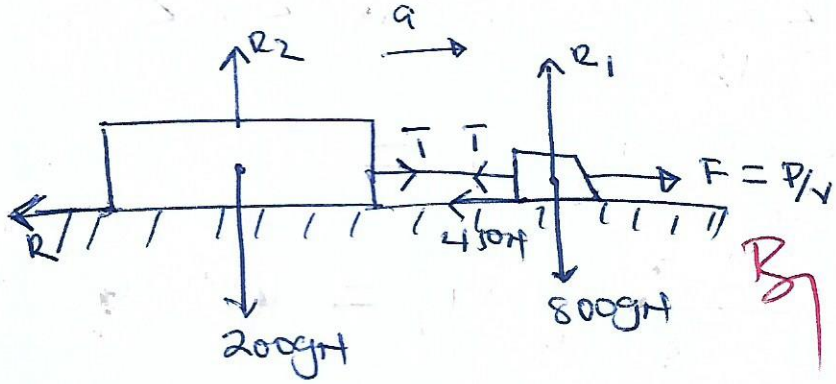
$= P\left(\frac{27.5-36}{3} < Z < \frac{41.5-36}{3}\right) =$

$= P(-2.833 < Z < 1.833) = 0.4666 + 0.4977$ M1 B1

$= 0.9643.$ A1



16:



$$F = P/V = \frac{9.75\times10^3}{\left(\frac{54\times1000}{60\times60}\right)}$$ B1

$$F = \frac{9.75\times10^3 \times 60\times60}{54000}$$

$$F = \frac{9.75\times3600}{54} = 650N$$ A1

$$650 - (450 + T) = 0$$ B1

$$T = 650 - 450$$ M1

$$T = 200N$$ A1

$$T - R = 0$$ M1

$$T = 200 = R$$

$$\therefore R = 200N$$ A1

$$F - (450 + T) = 8000a$$ B1

$$\frac{9.75\times10^3}{\left(\frac{72\times1000}{3600}\right)} - (450 + 200) = 8000a$$ M1

$$487.5 - 650 = 8000a$$ M1

$\frac{12}{12}$