DIFFERENTIATION TEST 2022

UGANDA ADVANCED CERTIFICATE OF EDUCATION

PURE MATHEMATICS

DURATION: 3 hours

INSTRUCTIONS: *Attempt all the questions in section A and only five questions in section B.*

SECTION A: (40 MARKS)
Attempt all questions in this section.

1. Find the equation of the normal to the curve $y^2+3xy=2x^2-1$ at the point $(2,1)$ *(05 marks)*

2. Find the stationary points of the curve$y=\frac{x+3}{\sqrt{x+1}}$ and distinguish between them. *(05 marks)*

3. Using calculus of small changes, find the approximate value of $\tan 44.2^0$. *(05 marks)*

4. In a right pyramid with a square base, the sum of its heigh and the perimeter of its base is 36 m. Find the maximum height of the pyramid. *(05 marks)*

5. If $ye^{2x}=A\cos 3x+B\sin 3x$ where A and B are constants, show that $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+13y=0$.

6. Differentiate $x^2 + cos2x$ from first principles. *(05 marks)*

7. Given $2x = t + \frac{1}{t}$ and $2y = t - \frac{1}{t}$, show that $\frac{d^2y}{dx^2} = (\frac{2t}{t^2-1})^3$. *(05 marks)*

8. For an expression $R = kT^{\frac{2}{3}}$ where k is constant of proportionality.Calculate the percentage error in R if the error made in T is 2%. *(05 marks)*

SECTION B: (60 MARKS)

Attempt only five questions from this section.

9. (a) Show that $e^x \cos x$ has two turning points in the interval $0 \leq x \leq \pi$. *(06 marks)*

   (b) A large container in the shape of a right circular cone of height $10m$ and base radius $1m$ is catching drips from a tap leaking at a rate of $0.1m^3s^{-1}$. Find the rate at which the surface area of the water is increasing when the water is half way up the cone. *(06 marks)*

10. (a) Show that if$y = x + log_e[\frac{(1+x)e^{-2x}}{(1-x)}]^{\frac{1}{2}}$ then $\frac{d^2y}{dx^2} = \frac{2x}{(1-x^2)^2}$.
    . *(07 marks)*

    (b) One of the stationary points of the curve $y = \frac{ax+b}{x^2+1}$ is $(2, 1)$. Find the values of a and b.
    . *(05 marks)*

11. (a) Find the equation of the normal to the curve $\frac{y}{x+siny} = 3$ at the point where $y = \pi$.
    . *(06 marks)*

(b) A cylinder of maximum volume V is to be cut from a solid sphere of radius R. Prove that $V = \frac{4\sqrt{3}\pi R^3}{9}$. *(06 marks)*

12. (a) Given that $y = \sqrt{\frac{1-\cos x}{1+\cos x}}$, show that $\frac{dy}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$.
. *(06 marks)*

(b) Find the equation of the tangent and normal to the curve $y = ln(3x-5)$ at the point where $x = 2$. *(06 marks)*

13. (a) Given $y^3 = \frac{x-1}{(x^2-1)^2}$, show that $\frac{dy}{dx} = \frac{1-3x}{3(x+1)^{\frac{5}{3}}(x-1)^{\frac{4}{3}}}$.
. *(06 marks)*

(b) Find and classify the stationary points of the curve $y = 3x^4 - 4x^3$. Hence sketch the curve .
. *(06 marks)*

14. (a) A closed hollow right circular cone has internal height a and internal radius a. A solid circular cylinder of height h just fits inside the cone with the axis of the cylinder lying along the axis of the cone. Show that the volume of the cylinder is $\pi h(a-h)^2$. If a is fixed and h may vary, find h in terms of a when the volume of the cylinder is maximum. *(06 marks)*

(b) Differentiate the following with respect to x

(i) $(lnx)^x$ *(03 marks)*

(ii) $10^{\sqrt{1-x^2}}$ *(03 marks)*

15. (a) A square of side x cm is cut from each of the corners of a rectangular piece of cardboard 15 cm by 24 cm. The cardboard is then folded to form a box of depth x cm. Find the value of x for which the volume of the box is maximum. *(06 marks)*

(b) Differentiate with respect to x:

(i) $\frac{(x-1)^2 e^{4x}}{(x+1)^3}$ *(04 marks)*

(ii) $\sin^2(\sqrt{1-x^2})$ *(02 marks)*

16. (a) A particle moves in a straight line so that after$t$ seconds, its distance from a fixed point O is S metres where $S = t^2 e^{2-t}$. Find the distance of the particle from point O when it first comes to rest and its acceleration at that point. *(06 marks)*

(b) If $y^2 - 2y\sqrt{1+x^2} + x^2 = 0$. Show that $\frac{dy}{dx} = \frac{x}{\sqrt{1+x^2}}$. Hence find $\frac{d^2y}{dx^2}$ ·

. *(06 marks)*