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PURE MATHEMATICS
PAPER 1

POST MOCK A

SECTION A(40marks)

1. The pressure – volume curve is of the form $pv^n = c$, a constant. If $p = 90$ when $v = 4$ and $p = 40$ when $v = 6.2$, find the values of n and c.

2. Find the term independent of x in the expansion of $\left(x + \frac{1}{x}\right)^6 \left(x - \frac{1}{x}\right)^8$.

3. Find the area contained between the curve $xy = 4$, the $x-$ axis and the lines $x = 2$ and $x = 3$.

4. A geometrical sequence has the first term 16 and common ratio $\frac{3}{4}$. If the sum of the first n terms 60, find the least possible value of n.

5. Evaluate: $\int_{-1}^{-\frac{1}{2}} \frac{(4x+2)}{(x-1)^4(x+2)^4}\,dx$

6. Solve the equation: $16\sin\theta\cos\theta = \tan\theta + \cot\theta$, for $0^o \le x \le 180^o$

7. Show that the line $2x - 3y + 26 = 0$ is a tangent to the circle $x^2 + y^2 - 4x + 6y - 104 = 0$, and find the diameter through the point of contact.

8. Find the tangent of the acute angle between the following pair of lines. $3x - y + 2 = 0, x - 2y - 1 = 0$.

SECTION B

9a) Given that $2 + 3i$ is a root of the equation $z^4 - 5z^3 + 18z^2 - 17z + 13 = 0$, find the other roots.

b) Given that $\left|\frac{z-1}{z+1-i}\right| = \frac{2}{3}$, find the Cartesian equation of the locus of the complex number.

10a) Evaluate: $\int_0^{\sqrt{\pi}} x^3 \sin x^2 \, dx$

b) Find $\int \frac{(1+x^4)}{(x+1)(1-x)(1+x^2)}dx$

11a) Solve for x and y for 0^o and 360^o given $\sin x + \sin y = \frac{\sqrt{3}}{2}$, and $\cos x + \cos y = \frac{1}{2}$

b) Solve the equation: $(3x+1)^{1/2} + (x-1)^{1/2} = (7x+1)^{1/2}$

12. A right circular cone of vertical angle 2θ is inscribed in a space of a fixed radius a with its vertex and rim of its base on the sphere. Show that its volume is $\frac{8}{3}\pi a^3 \sin^2\theta\cos^4\theta$, and hence, find the maximum volume of the cone.

13a) Given that $a\cos^2\theta + b\sin^2\theta = c,$ prove that $\tan^2\theta = \frac{c-a}{b-c}$, hence, solve for θ, in the equation $6\cos^2\theta + 2\sin^2\theta = 5$, where θ is acute.

b) Prove that: $8\cos 3\theta\cos 2\theta\cos\theta - 1 = \frac{\sin 7\theta}{\sin\theta}$

14. Given that $y = \frac{x^2+3}{x-1}$, show that for real values of x, y cannot lie between -2 and 6. Find the turning points and sketch the curve.

15. A curve whose equation has the form $y = x(x-2)(ax+b)$ touches the $x-$ axis at the point where $x = 2$ and the line $y = 2x$ at the origin. Find the values of a and b, sketch the curve and prove that the area enclosed by an arc of the curve and the segment of the line $y = 2x$ is $32/3$.

16a) Show that the equation $y^2 - 4y = 4x$ represents a parabola sketch the parabola.

b) Prove that the line y = x + 2 is a tangent to the parabola $y^2 = 8x$, hence determine the coordinates of the point of contact.

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POST MOCK B

SECTION A(40marks)

1. Solve for x and y for 0^o and 360^o given $\sin(x+y)=\frac{1}{2}$, and $\cos(x-y)=-\frac{\sqrt{3}}{2}$

2. Find the area enclosed between the curves $y^2=4(x-2)$ and $y^2=2x$.

3. Water is poured into a vessel, in the shape of a right circular cone of vertical angle 60^o, with the axis vertical, at a rate of $8\,m\,s^{-1}$. At what rate is the water surface rising when the depth of the water is $4\,m$?

4. Without using tables or calculators, show that $\tan 15^0 = 2-\sqrt{3}$

5. Differentiate w.r.t x: $y=\left(\frac{x^2-1}{x^2+1}\right)^{1/4}$

6. The point $C(a,\ 4,\ 5)$ divides the line joining $A(1,\ 2,\ 3)$ and $B(6,\ 7,\ 8)$ in the ratio $\lambda : 3$. Find a and λ.

7. Given that $a\cos^2\theta + b\sin^2\theta = c$, prove that $\tan^2\theta = \frac{c-a}{b-c}$, hence, solve for θ, in the equation $6\cos^2\theta + 2\sin^2\theta = 5$, where θ is acute.

8. Prove that $\frac{2}{27} \le \frac{x^2-2x+2}{x^2+3x+9} \le 2$

SECTION B (Attempt any 3 questions from this section)

9a) If $x^2+2xy+3y^2=1$, prove that $(x+3y)^3\frac{d^2y}{dx^2}+2=0$

b) The curve is given parametrically by the equations $x=\frac{t^2}{1+t^3}$, $y=\frac{t^3}{1+t^3}$ show that $\frac{dy}{dx}=\frac{3t}{2-t^3}$ and that $\frac{d^2y}{dx^2}=48$ at a point $(\frac{1}{2},\frac{1}{2})$.

10a) If $\tan\alpha=p$, $\tan\beta=q$, $\tan\gamma=r$, prove that $\tan(\alpha+\beta+\gamma)=\frac{p+q+r-pqr}{1-qr-rq-pq}$

Hence, deduce a relation between p,q and r in each of the following cases:

i) $(\alpha+\beta+\gamma)=0$ ii) $\alpha+\beta+\gamma=\frac{\pi}{2}$

b) Prove that, if $\tan A=\frac{3+4x}{4-3x}$ and $\tan B=\frac{6+7x}{7-6x}$, the value of $\tan(A-B)$ is independent of x.

11a)i) Find the equation of a line through the points A and B with position vectors $3i-j+4k$ and $j-4k$.

ii) A point C has position vector $6i+4j+5k$. Find the perpendicular distance of C from the line in (i) above.

b) The position vectors of three points A, B, C are p, $3q-p$ and $9q-5p$ respectively. Show that the points are collinear.

12. Evaluate: $\int_1^2 \frac{3x+2}{(2x-1)^2(3-x)}dx$

13a) Express $1.2\cos x+1.6\sin x$ in the form $R\cos(x-\alpha)$, and hence solve $1.2\cos x+1.6\sin x=1.5$ for $0^0\le x\le 180^0$.

b) Prove that $\frac{\sin 4A+\sin 2A}{\cos 4A+\cos 2A}=\tan 3A$

c) Solve the equation $\tan 4x+\tan 2x=0$ for $0^0\le x\le 360^0$

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PURE MATHEMATICS

POST MOCK C

SECTION A(40marks)

1. Solve the equation: $\log_{10} e.In\left(x^2+1\right)-2\log_{10} e.Inx=\log_{10} 5$

2. Find the values of x for which $\dfrac{x^2+2x-19}{x-4}\rangle 4$

3. Differentiate $y=(x-2)^{-1/2}$ from first principles.

4. Determine n if in the expansion of $(2+3x)^n$ in ascending powers of x, the coefficient of x^{12} is four times that of x^{11}.

5. Show that: $\int_0^{\pi/4}\dfrac{\cos x-\sin x}{\cos x+\sin x}dx=\dfrac{1}{2}In2$

6. Prove that: $\cos A=\dfrac{s(b-a+c)}{bc}-1$

7. Prove that the circles $x^2+y^2+10(x+y)+25=0$ touches the x and y axes and find the points of contact.

8. Evaluate: $\int_0^{\frac{\pi}{2}}\dfrac{dx}{2+\cos x}$.

SECTION B

9a) Given that $z+2i$ is a factor of $z^4+2z^3+7z^2+8z+12$, solve the equation $z^4+2z^3+7z^2+8z+12=0$.

b) Given that $z=\cos\theta+i\sin\theta,$ show that $\dfrac{z-1}{z+1}=i\tan\dfrac{\theta}{2}$.

10a) Evaluate: $\int_0^{\sqrt{\pi}} x^3\sin x^2\,dx$

b) Find $\int \frac{(1+x^4)}{(x+1)(1-x)(1+x^2)}dx$

11a) Solve the equations:

$x^2 + y^2 + z^2 + x + 2y + 4z - 6 = 0$, $\frac{x-y+z}{2} = x = \frac{x+y}{3}$

b) Solve the equation: $(3x+1)^{1/2} + (x-1)^{1/2} = (7x+1)^{1/2}$

12. A right circular cone of vertical angle 2θ is inscribed in a space of a fixed radius a with its vertex and rim of its base on the sphere. Show that its volume is $\frac{8}{3}\pi a^3 \sin^2\theta \cos^4\theta$, and hence, find the maximum volume of the cone.

13a) Given that $a\cos^2\theta + b\sin^2\theta = c$, prove that $\tan^2\theta = \frac{c-a}{b-c}$, hence, solve for θ, in the equation $6\cos^2\theta + 2\sin^2\theta = 5$, where θ is acute.

b) Prove that: $8\cos 3\theta \cos 2\theta \cos\theta - 1 = \frac{\sin 7\theta}{\sin\theta}$

14. Given that $y = \frac{x^2+3}{x-1}$, show that for real values of x, y cannot lie between -2 and 6. Find the turning points and sketch the curve.

15a) Evaluate: $\int_1^2 \frac{(x^4-1)^2}{x^2}dx$

b) Find the ratio of the volumes formed by rotating the area enclosed by the curve $y = x^4$, the line $x = 1$ and the $x-$ axis, i) about the $x-$ axis and ii) about the $y-$ axis.

16a) Obtain the equation of the tangent at the point $(3t^2, \ 2t^3)$ to the curve $27y^2 = 4x^3$ and find the coordinates of the point where this tangent meets the curve again.

b) Obtain the coordinates of the point of intersection of the tangents to the curve $y^2 = x^2(25 - x^2)$ at the points $(4, \ 12)$ and $(3, \ 12)$.

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POST MOCK D

SECTION A(40marks)

1. Solve the equations the ratio $x:y:z$, given that $2x+3y-z=0$, $3x-2y+4z=0$

2. Differentiate $y=(x-2)^{-1/2}$ from first principles.

3. Evaluate: $\sum_{r=1}^{10} 3\left(\frac{3}{4}\right)^r$

4. Evaluate: $\int_0^{\frac{\pi}{6}} \frac{\cos x}{1+\sin x}dx$

5. Prove that $\cos\frac{Q}{2}=\sqrt{\frac{s(s-q)}{pr}}$.

6. Find the equations to the lines through the point (2, 3) which makes angles of 45^{o} with the line $x-2y=1$.

7. Show that the lines with vector equations $\mathbf{r}=2\mathbf{i}-\mathbf{j}+2\mathbf{k}+\lambda(\mathbf{i}-2\mathbf{j}-2\mathbf{k})$ and $\mathbf{r}=-4\mathbf{i}-4\mathbf{j}+2\mathbf{k}+\mu(4\mathbf{i}+7\mathbf{j}+4\mathbf{k})$ meet and state the coordinates of the points of intersection.

8. On a certain curve for which $\frac{dy}{dx}=x+\frac{a}{x^2}$, the point $(2,\ \ 1)$ is a point of inflection. Find the value of a and the equation of the curve.

SECTION B

ATTEMPT ANY (FIVE) QUESTIONS FROM THIS SECTION

9a) Given that $z+2i$ is a factor of $z^4+2z^3+7z^2+8z+12$, solve the equation $z^4+2z^3+7z^2+8z+12=0$.

b) Given that $z = \cos\theta + i\sin\theta$, show that $\frac{z-1}{z+1} = i\tan\frac{\theta}{2}$.

10a) If $y = a\cos^2\theta + b\sin^2\theta$, Prove that $\frac{d^2y}{d\theta^2} + 4y = 2(a+b)$.

b) Sand falls on to horizontal ground at the rate of $9m^3$ per minute and forms a heap in the shape of a right circular cone with vertical angle 120^o. Show that $20\sqrt{3}$ seconds after sand begins to fall, the rate which the radius of the base of the pile is increasing is $\frac{\sqrt{3}}{\pi^{1/3}} m.\min^{-1}$.

11a) Solve the equations: $x^2 + 4xy + y^2 = 13$ and $2x^2 + 3xy = 8$ using $y = mx$.

b) When a polynomial $f(x)$ is divided by $(x-2)$ the reminder is -2, and when it is divided by $(x+3)$ the remainder is 6 and leaves no remainder when divided by $(x-1)$. Find the remainder when $f(x)$ is divided by $(x-2)(x+3)(x-1)$.

12a) A wire of length $28cm$ is to be cut and bent into a triangle whose sides are in the ratio $3:4:5$ and a square. Find the length of the side of the square for which the sum of the areas of the two figures is least.

b) Find the nature of the turning points of the curve $(x^2 - 2x - 2)e^x$, sketch the curve.

13a) In a triangle ABC, prove that if the internal bisector of angle A meets BC at D, the $AD = \frac{2bc}{b+c}\cos\frac{A}{2}$.

b) A man walking along a path sees a tree in a direction making 16^o with the path and the angle of elevation of the top of the tree is 5^o. After walking $100m$ along the path, he sees the tree in a direction making 25^o with the path. Calculate the height of the tree.

14. If x is real and $y = \frac{5x^2 + 8x + 4}{x^2 + x}$, prove that y cannot lie between -4 and +4. Find the turning points and sketch the graph from -3 to +3.

15a) Find the point of intersection of the line $\frac{x}{5} = \frac{y+2}{2} = \frac{z-1}{4}$ with the plane $3x + 4y + 2z = 25$.

b) Show that the line $\frac{x-2}{2} = \frac{y-2}{-1} = \frac{z-3}{3}$ is parallel to the plane $4x - y - 3z = 4$.

c) Find the perpendicular distance from the point $A(4, -3, 10)$ to the line $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$.

16. A circle A passing through the point $(t+2,\ 3t)$ has its centre at $(t,\ 3t)$. Another circle B of radius 2 units has its centre at $(t+2,\ 3t)$.

i) Determine the equations of the circles in terms of t.

ii) If $t = 1$, find the points of intersection of the two circles.

iii) Show that the common area of intersection of the circles is given by $8\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$.

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POST MOCK E

SECTION A(40marks)

1. Solve the equation $12x^3 - 4x^2 - 5x + 2 = 0$ given that it has a repeated root.

2. The sides AB and BC of a parallelogram $ABCD$ have equations $y + 3x = 1$ and $y = 5x - 7$ respectively. If the coordinates of D are $(5, 10)$, find the coordinates of A, B, C.

3. Without using tables or calculators, simplify $\left(5^{\log_{10} 4}\right)\left(50^{\log_{10} 25}\right)$

4. Find the mean value of a function $f(x) = \dfrac{x}{\sqrt{2x^2 + 1}}$ over the interval $0 \le x \le 2$.

5. Solve the equation for $0^o \le \theta \le 180^o$, given $\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$

6. Given that the circles $x^2 + y^2 + ax + by + c = 0$ and $x^2 + y^2 + bx + ay + c = 0$, are orthogonal, prove that $ab = 2c$.

7. The angle between r_1 and r_2 is $\cos^{-1}\left(\frac{4}{21}\right)$. If $r_1 = 6i + 3j - 2k$ and $r_2 = -2i + \lambda - 4k$, find the possible values of λ.

8. Evaluate: $\displaystyle\int_0^{\pi/6} \cos 5x \cos 2x \, dx$

SECTION B

ATTEMPT ANY (FIVE) QUESTIONS FROM THIS SECTION

9a) Differentiate w. r. t. x :

i) $y = 2x^{\cos x}$ ii) $y = \dfrac{e^{\sin x}}{\tan^{-1} x}$

b) If $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$, prove that $\frac{dy}{dx} = \cot\frac{1}{2}\theta$ and $\frac{d^2y}{dx^2} = -\frac{1}{4a}\text{cosec}^4\frac{1}{2}\theta$.

10. The point $P\left(2p, \frac{2}{p}\right)$ lies on the rectangular hyperbola A with equation $xy = 4$.

a) Find the equation of the normal to A at P.

b) If the normal at P meets A again at the point Q and the midpoint of PQ is M, find in cartesian form the equation of the locus of M as P varies.

11a) Prove by induction that: $1\times 1! + 2\times 2! + 3\times 3! + \ldots + n\times n! = (n+1)! - 1$

b) Expand $(8+3x)^{1/3}$ in ascending powers of x as far as the term in x^3, hence, obtain the approximate value for $\sqrt[3]{8.72}$.

12a) Find the maximum and minimum values of the function $6\cos^2\theta + 8\sin\theta\cos\theta$ and the corresponding value of θ, hence solve $6\cos^2\theta + 8\sin\theta\cos\theta = 4$.

b) If $x = \tan\theta + \sin\theta$, $y = \tan\theta - \sin\theta$, prove that $(x^2 - y^2)^2 = 16xy$.

13a) Prove that $\int_1^3 \left(\frac{3-x}{x-1}\right)^{1/2} dx = \pi$ (Use the substitution $x = 3\sin^2\theta + \cos^2\theta$)

b) Differentiate w.r.t. x expressing your result as simple as possible. $y = \cos^{-1}\left[\frac{2+3\sin x}{3+2\sin x}\right]$

14. Sketch the curve $y + 3 = \frac{6}{x-1}$ and calculate the area of the region enclosed between the curve $y + 3 = \frac{6}{x-1}$ and the line $y + 3x = 9$.

15a) Solve the simultaneous equations:

$$z_1 + z_2 = 8$$
$$4z_1 - 3iz_2 = 26 + 8i$$

b) If $z = \cos\theta + i\sin\theta$, solve the equation $z^{4/3} = i$.

16a) A and B are points whose position vectors are $a = 2i + 5k$, $b = -i + 3j + k$ respectively, and the equations of the line L are $\frac{x-3}{2} = \frac{y-2}{2} = \frac{z-2}{1}$. Determine

i) the equation of the plane π which contains A and is perpendicular to L, and verify that B lies in the plane π.

ii) the angle between the plane π above and the line $r = 3i + j - 3k + t(i - 2j - 4k)$

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POST MOCK F

SECTION A (40 MARKS)

1. The points A, B, C have coordinates $(-3, 2)$, $(-1, -2)$ and $(0, k)$ respectively, where k is a constant. Given that $\overline{AC} = 5\overline{BC}$, find the possible values of k. (5)

2. The distance of the point (2, – 1) from the line $y = \frac{3}{4}x + c$ is twice its distance from the line $y = -4/3\, x$. Find the value of c. (5)

3. The curve C is given by $y = ax^2 + b\sqrt{x}$ where a and b are constants. Given that the gradient of C at the point $(1, 1)$ is 5, find a. (5)

4. The 3^{rd}, 5^{th} and 8^{th} terms of an A.P are $3x+8$, $x+24$ and x^3+15 respectively. Find the value of x and the common difference. (5)

5. Find the area enclosed between the curve $y = x(8-x)$ and the line $y = 12$. (5)

6. Solve the equation $\sin^2 x + \sin x = 1 - \sin 3x$, for $0^o \le x \le 360^o$ (6)

7. Given that $xIny + 2y = 3$, show that $\frac{dy}{dx} = \frac{y(2y-3)}{x(2y+x)}$ (4)

8. Two lines l and m have vector equations $r_1 = (2-3\lambda)i + (1+\lambda)j + 4\lambda k$ and $r_2 = (-1+3\mu)i + 3j + (7-\mu)k$, respectively, find;
   i) the position vector of their common point.
   ii) the angle between the lines. (5)

SECTION B (60 MARKS)

9a) Find the maximum and minimum values of the function $\frac{1}{3+\sin\theta - 2\cos\theta}$ stating clearly the values of θ.

b) Prove the identity: $\frac{\sin(A+B)}{\cos(A-B)}+1=\frac{(1+\tan B)(1+\cot A)}{\cot A+\tan B}$

10a) Given that $(1-2x)^5(2+x)^6 \equiv a+bx+cx^2+dx^3+\ldots$, find $a,\ b,\ c\ \& d$.

b) Given that x is so small that x^3 and higher powers of x can be neglected, show that $\frac{1}{\sqrt{1+x}}=1-\frac{x}{2}+\frac{3x^2}{8}$. By letting $x=\frac{1}{4}$, find a rational approximation of $\sqrt{5}$.

11a) Express in partial fractions: $f(x)=\frac{x^2+6x+7}{(x+2)(x+3)}$ and hence, show that

$\int_0^2 f(x)\,dx=2+In\frac{25}{18}$

b) Show that $\int_2^4 x\,In\,x\,dx=7In\,4\ -\ 3$

12a) A curve is given parametrically by the equations $y=2\sin^3 t$ and $x=2\cos^3 t$. Find the equation of the normal to the curve at a point where $t=\frac{\pi}{6}$.

b) Find the equation of a curve given that it passes through the point $(1,\ 0)$ and that its gradient at any point $(x,\ y)$ is equal to $x(y-1)^2$

13a) Find the equation of a circle which passes through the points $A(1,\ 2)$, $B(2,\ 5)$ and $C(-3,\ 4)$.

b) Given that the line $y=mx+c$ is a tangent to the circle $(x-a)^2+(y-b)^2=r^2$, show that $(1+m^2)r^2=(c-b+am)^2$.

14a) Use the substitution $u=\sqrt{1+x^2}$ to show that $\int_0^{\sqrt{3}} x^3\sqrt{1+x^2}=3\tfrac{13}{15}$.

b) Use the substitution $t=e^x$ to find $\int \frac{e^x}{e^x+e^{-x}}\,dx$

15a) Express $(1-i\sqrt{3})$ in the form $r(\cos\theta+i\sin\theta)$, hence or otherwise, express $(1-i\sqrt{3})^4$ in the form $a+bi$.

b) Describe the locus of a point defined by $|Z-1+2i|=3$.

c) Evaluate: $\left(1+i\sqrt{3}\right)^{2/3}$

16a) The position vectors of points A,B, C are $\frac{1}{4}\left(\underset{\sim}{a} + 3\underset{\sim}{b}\right)$, $\frac{1}{2}\left(3\underset{\sim}{a} - \underset{\sim}{b}\right)$ and $\frac{1}{8}\left(3\underset{\sim}{a} + 5\underset{\sim}{b}\right)$ respectively. Prove that the points lie in a straight line and determine the ratio $\overline{AB} : \overline{BC}$.

b) The distance of the point A(4, –1, 2) from a plane is $\sqrt{3}$. Given that the vector $\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$ is a normal to the plane find the Cartesian equation of the plane.

END

PURE MATHEMATICS
PAPER 425/1

FINAL REVISION QUESTIONS

SECTION A

1. Find the condition that the equations $x^2 + 2px + q = 0$ and $x^2 + 2Px + Q = 0$ must have a common root.

2. Solve the differential equation: $(x+2)\frac{dy}{dx} = (2x^2 + 4x + 1)(y-3)$ given that $x = 0$, $y = 7$.

3. Evaluate: $\tan^{-1}\frac{\sqrt{3}}{2} + \tan^{-1}\frac{\sqrt{3}}{5}$ leaving π in your answer.

4. Find the position vector of the point where the line $\mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}$ meets the plane $\mathbf{r} \,.\, \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = 12$.

5. Given the hyperbola $9x^2 - 16y^2 = 144$, find the eccentricity and coordinates of the foci.

6. If $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, show that $\frac{dy}{dx} = \frac{-2}{1+x^2}$

7. Prove by induction given that $\sum_{r=1}^{n}\frac{r}{2^r} = 2 - \frac{n+2}{2^n}$

8. Find the equation of the normal to the curve $y = In\left(\frac{x-1}{x+1}\right)$ at the point where $x = 3$.

9. Given that the equation $12x^3 - 52x^2 + 35x + 50 = 0$ has a repeated root, solve the equation.

10. Find the slope of the of the tangent to the curve $xy^3 - 2x^2y^2 + x^4 - 1 = 0$ at the point $(1,\ 2)$.

11. Find the equation to the circle which passes through the origin, and the points $2(,\ 0)$ and $(3,\ -1)$.

12. Find three numbers in a geometrical progression such that their sum is 39 and their product is 729.

13. If $y = \tan^2 x$, prove that $\frac{d^2y}{dx^2} = 2(1+y)(1+3y)$.

14. If $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, show that $\frac{dy}{dx} = \frac{2}{1+x^2}$.

15. Solve the differential equation, given $\frac{dy}{dx} = 3x\sin^2 y$ given that $y = -\frac{\pi}{4}$ when $x = \frac{1}{\sqrt{3}}$.

SECTION B

16a) The curve with equation $y = e^{3x} + 1$ meets the line $y = 8$ at the point $(h,\ 8)$.

i) Find h.

ii) Show that the area of the finite region enclosed by the curve with the equation $y = e^{3x} + 1$, the $x-$ axis, the $y-$ axis and the line $x = h$ is $2 + \frac{1}{3}In7$.

b) Find and classify the stationary points on the curve $y = x^2e^x$, hence, sketch the curve.

17a) Use Maclaurin's theorem to expand $In(2+3x)$ as far as the term in x^4, and hence, evaluate $In(2.03)$ to 4 d.p.

b) Find the mean value of $y = 4e^{2x} - 3e^x$ between $x = 1$ and $x = 2$.

18a) Solve the equation: $x^4 - 3x^3 + 4x^2 = 8$

b) The remainder when the expression $x^3 - 2x^2 + ax + b$ is divided by $(x-2)$ is five times the remainder when the same expression is divided by $(x-1)$, and 12 less than when the same expression is divided by $(x-3)$. Find the values of a and b.

19. Express as partial fractions: $\frac{x^3 - 5x^2 + 6x - 5}{(x-1)(x-4)}$ and hence evaluate $\int_5^9 \frac{x^3 - 5x^2 + 6x - 5}{(x-1)(x-4)}dx$

20a) Find the general solution of the d.e: $2(x^2+1)\frac{dy}{dx} = x(4 - y^2)$ given $y = 1$ when $x = 0$.

b) The rate at which the height h of a certain plant increases is proportional to the natural logarithm of the difference between its present height and its final height H.

21) Prove that: $\frac{\tan A - 3\tan 3A}{\cot A - 3\cot 3A} = \frac{\tan A - 3\cot A}{\cot A - 3\tan A}$

b) Prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. Deduce that
$\sin^3\theta + \sin^3(120^o + \theta) + \sin(240^o + \theta) = -\frac{3}{4}\sin 3\theta$

22a) The points A, B, C have position vectors $(-2i + 3j), (i - 2j), (8i - 5j)$ respectively.

i) Find the vector equation of line AC.

ii) Determine the coordinates of D, if $ABCD$ is a parallelogram.

iii) Write down the vector equation of the line through point B perpendicular to AC and find where it meets AC.

23. Draw on the same axes graphs of $f(x) = 4 + 3x - x^2$ and $y = \dfrac{1}{f(x)}$ and state the coordinates of the points of intersection.

24a) Find the equation of the tangents drawn from the point $(1,\ 3)$ to the parabola $y^2 = -16x$.

b) Prove that the tangents to the parabola $y^2 = 4ax$ at the points $P(ap^2,\ 2ap)$ and $Q(aq^2,\ 2aq)$ intersect at the point $R(apq,\ a(p+q))$.

25.a) Show that $\int_0^{\pi/4} x\sec^2 x\, dx = \dfrac{\pi}{4} - \dfrac{1}{2} In\ 2$

b) A curve is given parametrically by $x = 7t^{1/2} + 2$, $y = t^{1/2}(t+1)$. Show that its gradient function is given by $\dfrac{3(x-2)^2}{343} + \dfrac{1}{7}$.

26a) The sides a, b, c of a triangle ABC are in the ratio $3:6:5$, find the smallest angle of the triangle.

b) Given that A, B, C are angles of a triangle, prove that
$\cos A + \cos B + \cos C - 1 = 4\sin\frac{1}{2}A\sin\frac{1}{2}B\sin\frac{1}{2}C$.

27. A right circular cone of vertical angle 2θ is inscribed in a space of a fixed radius a with its vertex and rim of its base on the sphere. Show that its volume is $\dfrac{8}{3}\pi a^3\sin^2\theta\cos^4\theta$, and hence, find the maximum volume of the cone.