

S5 PHYSICS

LESSON 7

TOPIC: KINETIC THEORY OF GASES

OBJECTIVE: By the end of this lesson, you should derive expression for pressure exerted by a gas in a container.

INTRODUCTION

Gases are composed of molecules. If we could see the molecules in a gas, we would see that they are separated from one another by distances which are large compared with their actual sizes. Their speeds differ and they move about in random directions. They continually collide with one another and with the container walls.

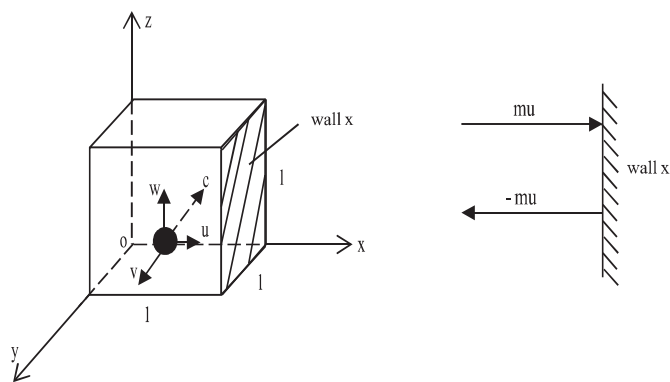
We make assumptions in the simple kinetic theory when deriving pressure of a gas.

- The intermolecular forces between the molecules are negligible except during a collision.
- The volume of molecules is negligible compared to the volume occupied by the gas.
- The molecules are perfectly elastic spheres so that after collision with the walls of the container, there is no energy lost that is; kinetic energy is conserved.
- The duration of a collision is negligible compared to the time between collisions.

DERIVATION OF THE PRESSURE EXERTED BY A GAS

We consider a cube of side, l , containing n molecules of a gas each of mass m . Suppose, c is the velocity of a molecule at any instant such the u , v and w are components of c in ox , oy and oz respectively as shown below;

$$\text{i.e. } c^2 = u^2 + v^2 + w^2$$



Considering a molecule colliding with wall x , with a velocity u .

$$\text{Change in momentum} = mu - (-mu) = 2mu.$$

$$\text{Time taken for the molecule to move to the opposite face and back to } x = \frac{2l}{u}.$$

$$\text{Rate of change of momentum} = 2mu \div \frac{2l}{u} = \frac{mu^2}{l}$$

$$\therefore \text{Force exerted} = \frac{mu^2}{l},$$

From Newton's 2nd law of motion, rate of change of momentum = Force applied.

$$\text{From Pressure} = \frac{\text{Force}}{\text{Area}},$$

$$\Rightarrow \text{Pressure } P = \frac{mu^2}{l} \div l^2$$

$$\text{Pressure } P = \frac{mu^2}{l^3}$$

For N molecules moving with different speeds; u_1, u_2, \dots, u_N the total pressure is;

$$P = \frac{mu_1^2}{l^3} + \frac{mu_2^2}{l^3} + \dots + \frac{mu_N^2}{l^3}$$

$$P = \frac{m}{l^3} (u_1^2 + u_2^2 + \dots + u_N^2) \dots\dots\dots \otimes$$

Suppose \bar{u}^2 is the mean value of all the squares of velocities along the ox direction, then;

$$\bar{u}^2 = \frac{u_1^2 + u_2^2 + \dots + u_N^2}{N}$$

$$\Rightarrow u_1^2 + u_2^2 + \dots + u_N^2 = \bar{u}^2 N$$

Substituting in equation \otimes

$$P = \frac{m}{l^3} N \bar{u}^2$$

$$P = \frac{mN\bar{u}^2}{l^3} \dots\dots\dots 1$$

For molecules in random motion and with no preferred direction, then;

$$u^2 = v^2 = w^2$$

$$\text{but } c^2 = u^2 + v^2 + w^2$$

$$\Rightarrow \bar{c}^2 = \bar{u}^2 + \bar{v}^2 + \bar{w}^2$$

$$\text{But for } u^2 = v^2 = w^2$$

$$\Rightarrow \bar{u}^2 = \bar{v}^2 = \bar{w}^2$$

$$\Rightarrow \bar{c}^2 = \bar{u}^2 + \bar{u}^2 + \bar{u}^2$$

$$\Rightarrow \bar{c}^2 = 3\bar{u}^2$$

$$\Rightarrow \bar{u}^2 = \frac{1}{3} \bar{c}^2$$

Substituting this in equation 1 above, we obtain;

$$P = \frac{1}{3} \frac{Nm}{l^3} \bar{c}^2; \text{ Where } N \text{ is the number of molecules and } m \text{ is the mass of a molecule.}$$

l^3 – is the volume v of the cube or gas.

$$\Rightarrow P = \frac{1}{3} \frac{Nm}{v} \bar{c}^2; \text{ Where } Nm \text{ is Total mass.}$$

$$\frac{Nm}{v} - \text{density} = \rho$$

$$\Rightarrow P = \frac{1}{3} \rho \bar{c}^2 \dots\dots\dots \otimes$$

LESSON 8

OBJECTIVE: By the end of this lesson, you should derive the relationship between kinetic energy (mean kinetic energy) of molecules of an ideal gas and temperature using kinetic theory.

From the kinetic theory;

$$P = \frac{1}{3} \rho \bar{c}^2$$

$$\rho = \frac{Nm}{v}$$

$$P = \frac{1}{3} \left(\frac{Nm}{v} \right) \bar{c}^2$$

$$Pv = \frac{1}{3} Nm \bar{c}^2$$

$$Pv = \frac{2}{3} \left(\frac{1}{2} Nm \bar{c}^2 \right) \dots\dots\dots 2$$

From the equation of state;

$$Pv = nRT \dots\dots\dots 3$$

Where n is the number of molecules of a gas.

From 2 and 3;

$$Pv = \frac{2}{3} \left(\frac{1}{2} Nm \bar{c}^2 \right) = nRT$$

$$\frac{1}{2} Nm \bar{c}^2 = \frac{3}{2} nRT \dots\dots\dots \oplus$$

Where $\frac{1}{2} Nm \bar{c}^2 = \frac{3}{2} nRT$ is the mean translational kinetic energy of molecules for n moles of a gas.

Using equation \oplus we can get the mean kinetic energy of a molecule by dividing through by N.

From $\frac{1}{2} Nm \bar{c}^2 = \frac{3}{2} nRT$, dividing through by N gives;

$$\frac{1}{2} m \bar{c}^2 = \frac{3}{2} \frac{n}{N} RT \dots\dots\dots w$$

Where $\frac{1}{2} m \bar{c}^2$ is the mean kinetic energy of a molecule.

$\frac{N}{n}$ – is the number of molecules per mole

$\frac{N}{n} = N_A$ is called the Avogadro's number and it is a constant.

$$\Rightarrow \frac{n}{N} = \frac{1}{N_A}$$

Equation w becomes;

$$\frac{1}{2} m \bar{c}^2 = \frac{3}{2} \frac{1}{N_A} RT \therefore \text{Mean kinetic energy of a molecule is } \frac{1}{2} m \bar{c}^2 = \frac{3}{2} \frac{R}{N_A} T \dots\dots\dots z$$

But $\frac{R}{N_A}$ is a constant = K_B

K_B is called Boltzmann's constant

Equation z becomes;

$$\frac{1}{2} m \bar{c}^2 = \frac{3}{2} K_B T \dots\dots\dots s$$

$$\text{Where } K_B = \frac{R}{N_A} = \frac{8.31}{6.02 \times 10^{23}}$$

$$K_B = 1.38 \times 10^{-13} \text{ JK}^{-1}$$

$$\Rightarrow \frac{3}{2} K_B \text{ is a constant}$$

$$\Rightarrow \text{Mean translational kinetic energy} = \left(\frac{1}{2} m \bar{c}^2 \right) \propto \text{absolute temperature}$$

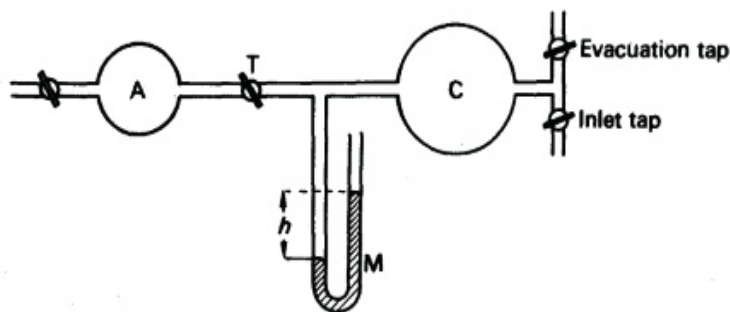
LESSON 9

SUBTOPIC: DALTON'S LAW OF PARTIAL PRESSURE.

OBJECTIVE: By end of this lesson you should derive the expression for relating pressure of a mixture of gases and state Dalton's law of partial pressure.

INTRODUCTION

The figure below shows an apparatus with which we can study the pressure of a mixture of gases. A is a bulb of volume V_1 containing air at atmospheric pressure, P_1 . C is another bulb of volume V_2 , containing carbon dioxide at a pressure P_2 . The pressure P_2 is measured on the manometer M.



When the bulbs are connected by opening the tap T, the gases mix, and reach the same pressure, p ; this pressure is given by the new height of the manometer. Its value is found to be given by

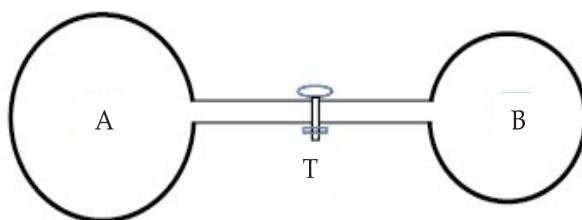
$$p = p_1 \frac{V_1}{V_1 + V_2} + p_2 \frac{V_2}{V_1 + V_2}.$$

Dalton's law of partial pressure states that the pressure of a mixture of gases is the sum of the partial pressures of the individual gases.

Partial pressure of a gas is the pressure the gas would have if it occupied the whole volume of the mixture alone.

EXAMPLE

Two bulbs A of volume 100 cm^3 and B of volume 50 cm^3 are connected to a tap, T which enables them to be connected with each other after being filled with gas or evacuated.



Initially bulb A is filled with an ideal gas at 10°C to a pressure of $3.0 \times 10^5 \text{ Pa}$ and bulb B is filled with an ideal gas at 100°C to a pressure of $1.0 \times 10^5 \text{ Pa}$. The two bulbs are connected with A maintained at 10°C and B at 100°C until equilibrium is established. The volume of the tubes is negligible. Calculate the pressure at equilibrium.

$$n_A = \frac{3 \times 10^5 \times 100}{283R} = \frac{3 \times 10^7}{283R}$$

$$n_B = \frac{1 \times 10^5 \times 50}{373R} = \frac{5 \times 10^6}{373R}$$

Total number of moles, n initially = $n_A + n_B$

$$n = \frac{3 \times 10^7}{283R} + \frac{5 \times 10^6}{373R}$$

Finally

$$n_A = \frac{P \times 100}{283R} = \frac{100P}{283R}$$

$$n_B = \frac{P \times 50}{373R} = \frac{50P}{373R}$$

Total number of moles, n finally = $n_A + n_B$

$$n = \frac{100P}{283R} + \frac{50P}{373R}$$

$$\frac{3 \times 10^7}{283R} + \frac{5 \times 10^6}{373R} = \frac{100P}{283R} + \frac{50P}{373R}$$

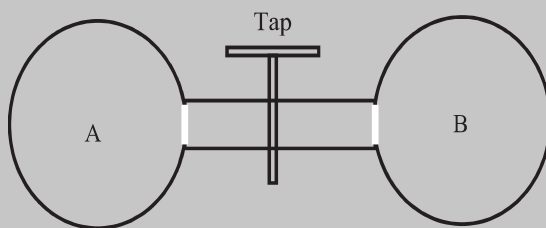
Simplifying gives,

$$0.48741P = 119411.893$$

$$P = 244995.14 \text{ Pa}$$

ASSIGNMENT

- Two cylinders A and B of volumes v and $3v$ respectively are separated and filled with a gas. They are then connected as shown in the figure below with the tap closed



The pressures of the gas in A and B are P and $4P$ respectively. When the tap is opened, the common pressure becomes 60 Pa . Assuming isothermal conditions, find the value of P . ($P = 18.46 \text{ Pa}$)

LESSON 10

TOPIC: THERMODYNAMICS

OBJECTIVES: By the end of this lesson, you

- should state the laws of thermodynamics
- Derive the expression for the work done when a gas expands

INTRODUCTION

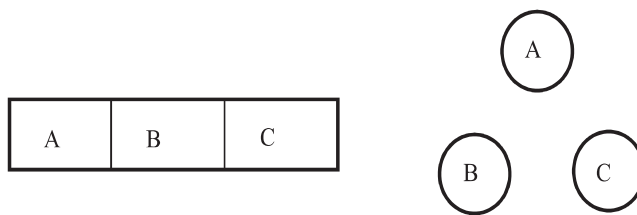
Let us now deal with the relation between work and heat energy. This branch of Physics, called thermodynamics, is widely used by engineers in their researches and studies into engines. The word 'thermodynamics' is composed of two words, namely 'Thermo' and 'dynamics'. 'Thermo' stands for heat while 'dynamics' is used in connection with mechanical motion which involves 'work' done. Thus, thermodynamics is the branch of physics which deals with process involving heat, work and internal energy.

FIRST LAW

It states that heat supplied to a system is equal to the increase in internal energy plus the external work done by the system to the surrounding. i.e.

$$\Delta Q = \Delta U + \Delta W$$

For any three bodies A, B and C arranged as shown below,



If A is in thermodynamic equilibrium with B and B is in thermodynamic equilibrium with C, it implies that A is in thermodynamic equilibrium with C. **This is the zeroth law of thermodynamics.**

Note

- The internal energy of the system is the sum of the kinetic energy and potential energy of the molecules of the system.
- From the first law of thermodynamics; $\Delta Q = \Delta U + \Delta W$, If ΔQ is positive, the system gains heat and if ΔQ is negative, the system loses heat.

- If ΔU is positive, there is increase in internal energy. If it is negative then there is a decrease in internal energy.
- If ΔW is positive, the system does work against the surroundings (i.e. it expands).
- If ΔW is negative, work is done on the system (i.e. compressed).

For real gases, the internal energy consists of kinetic energy due to the motion of its molecules and potential energy due to the intermolecular forces.

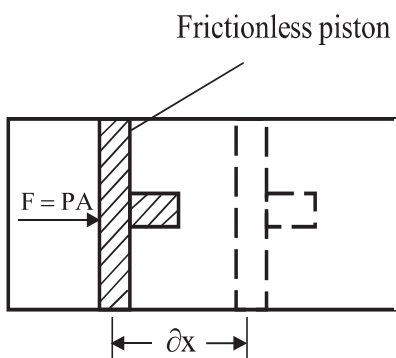
For ideal gases the internal energy consists of only the kinetic energy of its molecules and there is no potential energy since there are no intermolecular forces, i.e. for an ideal gas;

ΔU = translational (mean) kinetic energy

$$\Delta U = \frac{3}{2}nRT$$

WORK DONE BY A GAS, ΔW WHEN EXPANDING

Consider a mass of a gas at a pressure, P and volume, v enclosed in a cylinder by a frictionless piston of cross-sectional area A , under a force F on the piston.



Suppose the gas expands and pushes the piston through a distance ∂x such that the volume increases by ∂v , then,

$$\partial A = A\partial x$$

Since ∂x is so small, then the pressure is considered to remain constant.

Therefore, work done by the gas is;

$$\partial W = F \partial x$$

$$\Rightarrow \partial W = PA \partial x$$

$$\partial W = P \partial v, \quad \text{since } A \partial x = \partial v$$

The total work done, W in changing the volume of the gas from v_1 to v_2 is given by,

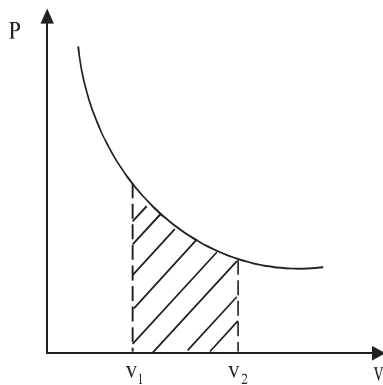
$$\int dW = \int_{v_1}^{v_2} P dv$$

$$W = P \int_{v_1}^{v_2} dv$$

$$W = P[v_2 - v_1] \dots\dots\dots \oplus$$

Equation \oplus is the expression of work done by the gas if P is constant.

Work done W is the same as the area under a P - v curve from v_1 to v_2 . i.e.



If the pressure P is not constant, then we use the equation of state, $Pv = nRT$

$$\Rightarrow P = \frac{nRT}{v}$$

$$\text{From; } W = \int_{V_1}^{V_2} P dv$$

$$\Rightarrow W = \int_{V_1}^{V_2} \frac{nRT}{v} dv$$

$$W = nRT \int_{V_1}^{V_2} \frac{1}{v} dv$$

$$W = nRT [\ln v]_{V_1}^{V_2}$$

$$W = nRT [\ln v_2 - \ln v_1]$$

$$W = nRT \ln \left(\frac{v_2}{v_1} \right) \dots\dots\dots \otimes$$

Equation \otimes is the external work done by the gas if P is not constant.

LESSON 11

SUBTOPIC: MOLAR HEAT CAPACITIES OF GASES

OBJECTIVE

By end of this lesson, you should define molar heat capacities derive a relation between the molar heat capacities

Molar heat capacity at constant volume (C_v)

It is heat energy required to change the temperature of one mole of a gas by one kelvin at constant volume.

Molar heat capacity at constant pressure (C_p)

It is heat energy required to change the temperature of one mole of a gas by one kelvin at constant pressure.

Note

For one kilogram of a gas, the heat capacities at constant pressure and constant volume are called **specific heat capacities**. i.e.

Specific heat capacity at constant volume $c_v = \frac{C_v}{m}$,

Specific heat capacity at constant pressure, $c_p = \frac{C_p}{m}$, where m is the mass of 1 mole.

RELATIONSHIP BETWEEN C_p AND C_v

Consider one mole of an ideal gas in a cylinder. Let a quantity of heat, δQ be supplied to the gas which is allowed to expand reversibly at constant pressure, p.

Suppose the volume of the gas increases from v to v + δv and the temperature from T to T + δT .

From the first law of thermodynamics, $\delta Q = \delta U + \delta W$ (i)

At constant pressure, $\delta Q = C_p \delta T$ (ii)

The increase of internal energy, δU for an ideal gas at constant volume by a temperature, δT is given by $\delta U = C_v \delta T$ (iii)

The external work done by the gas is given by $\delta W = p \delta v$ (iv)

Combining equations (i), (ii), (iii), (iv) gives

$$C_p \delta T = C_v \delta T + p \delta v \text{(m)}$$

For one mole of an ideal gas, $p v = R T$ (v)

And $p (v + \delta v) = R (T + \delta T)$

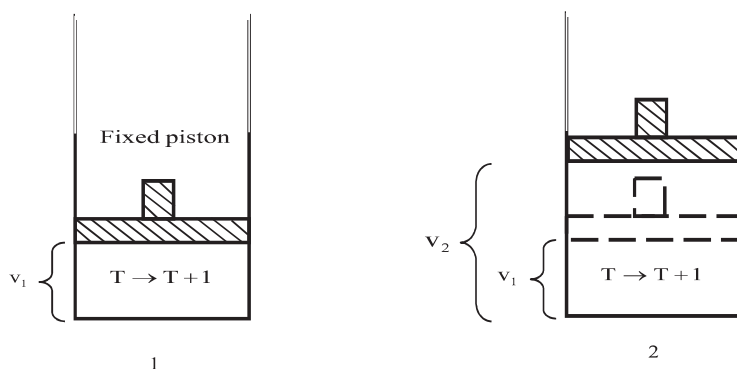
$$p v + p \delta v = R T + R \delta T \text{(vi)}$$

Combining (v) and (vi) gives $p \delta v = R \delta T$ (viii)

Combining (viii) and (m) gives $C_p \delta T = C_v \delta T + R \delta T$

Which gives $C_p - C_v = R$

Alternatively;



1 mole of a gas warmed at 1 mole of a gas warmed at
Constant volume by 1 K (C_v) constant pressure by 1 K (C_p)

In the first case, we consider 1 mole of a gas warmed at a constant volume through 1K such that the heat required is C_v , which is used only to increase the internal energy ∂U of the gas. (No external work done since volume is constant i.e. $\partial W = 0$)

$$\Rightarrow C_v = \partial U \dots\dots\dots 1$$

Secondly, we consider one mole of a gas warmed at a constant pressure through 1 K such that its volume increases from v_1 to v_2 and the heat supplied is C_p which is used to increase the internal energy ∂U and also do external work, ∂W .

$$\Rightarrow C_p = \partial U + \partial W, \text{ for } P = \text{constant}$$

$$\partial W = P[v_2 - v_1]$$

$$C_p = \partial U + P[v_2 - v_1]$$

From equation 1;

$$C_v = \partial U$$

$$\Rightarrow C_p = C_v + Pv_2 - Pv_1 \dots\dots\dots 2$$

Using the equation of state for one mole, $n = 1$

$$\Rightarrow Pv = RT$$

Suppose v_1 and T_1 are the values of volume and temperature before warming and v_2 and T_2 being the values after warming.

$$\Rightarrow T_1 = T_2$$

Equation of state becomes

$$Pv_1 = RT_1 \dots\dots\dots \otimes$$

And,

$$Pv_2 = RT_2 \dots\dots\dots \#$$

$$\text{But } T_2 = T_1 + 1$$

$$\Rightarrow Pv_2 = R(T_1 + 1)$$

$$\Rightarrow Pv_2 = RT_1 + R$$

$$Pv_2 = RT_1 + R \dots\dots\dots \otimes \otimes$$

Substituting \otimes and $\otimes \otimes$ in 2, we obtain;

$$C_p = C_v + RT_1 + R - RT_1$$

$$C_p = C_v + R$$

$$C_p - C_v = R$$

C_p is greater than C_v . This is because when the gas is at constant volume, the heat required C_v is used to increase the internal energy only but when the gas is heated at a constant pressure, the heat supplied C_p is used to increase the internal energy and do external work since there is an increase in volume.

LESSON 12

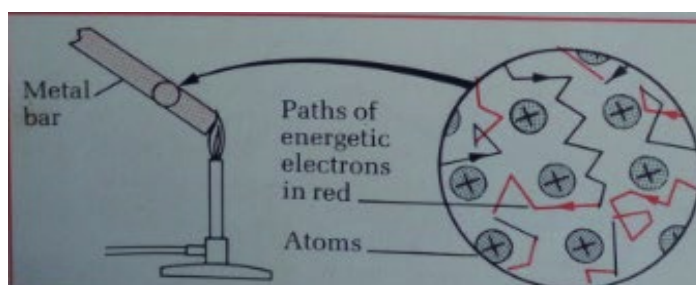
TOPIC: HEAT TRANSFER (THERMAL CONDUCTION)

OBJECTIVES: By the end of this lesson, you should:

- Describe the mechanism of conduction in solids.
- Describe experiments to determine the coefficient of thermal conductivity of a material.

INTRODUCTION

METALS are good conductors of heat and electricity. The reason is the presence of electrons inside the metal. At absolute zero, these electrons are attached to individual atoms in the outer shells. When the metal is supplied with energy to raise its temperature above absolute zero, the outer electrons of each atom break free as the atoms vibrate. Therefore, any metal above absolute zero contains lots of free electrons moving about inside the metal in a haphazard way. Free electrons collide with one another and with the atoms of the metal as they move about. Therefore energy is transmitted through the metal by movement of free electrons even though the electrons only travel short distances.



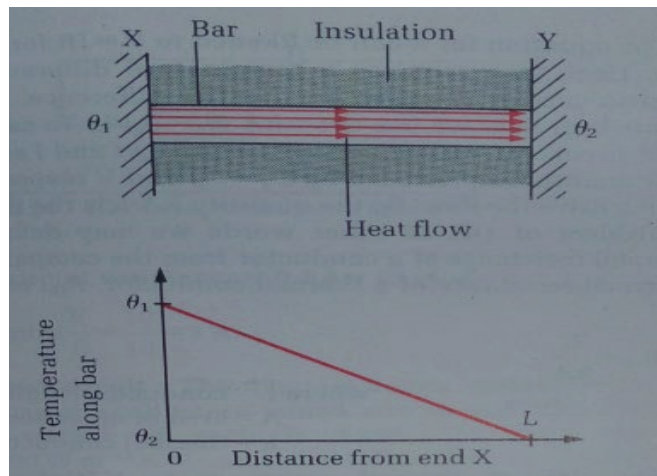
Insulators are poor thermal conductors because they have no free electrons. All the electrons are firmly attached to individual atoms, even at high temperatures. So insulators cannot conduct electricity at all. Yet they do conduct thermally, even if very poorly. Insulation to prevent heat loss from houses does not cut out heat losses altogether. The reason is that the vibrations of the atoms transmit energy through the material. When part of the material is heated, the atoms in that part vibrate more. The increased vibrations make neighbouring atoms vibrate more which in turn make other atoms further away vibrate more. So energy passes to other parts of the material. This mechanism is present in metals but energy transfer due to electrons is much greater than that due to atomic vibrations.

Gases and liquids conduct heat energy. When heated, molecules gain extra kinetic energy which is transferred to other molecules as a result of collisions between molecules. The mechanism is not unlike what happens in metals due to free electrons, but here the molecules, rather than free electrons, carry the energy. Gases and liquids are poor thermal conductors, in general, compared with metals. The reason is that molecules in a gas or liquid move much more slowly than the free electrons in a metal. So energy transfer in a liquid or gas is much slower. One type of liquid, though, is an excellent thermal conductor: liquid metals such as mercury. The reason is the presence of free electrons.

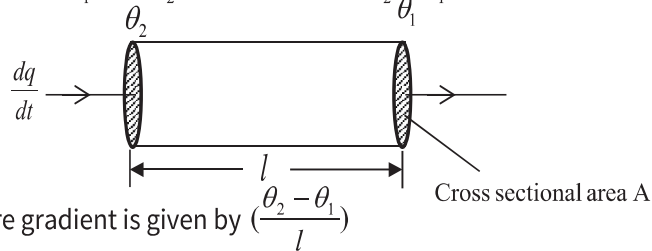
THE COEFFICIENT OF THERMAL CONDUCTIVITY

Consider a uniform bar which is insulated along its sides. Suppose one end is at a constant temperature θ_1 and the other end at a lower constant temperature θ_2 . Heat is conducted from the hotter end to the other end at a steady rate. Because the sides are perfectly insulated, the heat energy per second reaching the cold end is equal to the heat energy per second from the hot end. Therefore, the heat energy per second passing any position along the bar is the same.

The temperature varies along the bar as shown below. The temperature fall per unit length along the bar is the same from one end to the other.



The temperature gradient at any position along the bar is defined as the change of temperature per unit distance along the bar. Considering a conductor of length, l and cross sectional area A , having temperatures θ_1 and θ_2 at its ends with $\theta_2 > \theta_1$ as shown below.



The temperature gradient is given by $(\frac{\theta_2 - \theta_1}{l})$

The rate of heat flow $\frac{dq}{dt}$ along the bar depends on:

- The cross sectional area, A ,
- The temperature gradient, $(\frac{\theta_2 - \theta_1}{l})$

- The nature of the material.

Therefore, $\frac{dq}{dt} \propto A$ (i) And $\frac{dq}{dt} \propto \left(\frac{\theta_2 - \theta_1}{l}\right)$ (ii)

Combining (i) and (ii) $\frac{dq}{dt} = kA \left(\frac{\theta_2 - \theta_1}{l}\right)$.. (iii)

Where k is a constant of a given material, which depends on the nature of the material and it is called thermal conductivity or coefficient of thermal conductivity.

Definition of k (thermal conductivity)

From equation (iii)

$$k = \frac{\frac{dq}{dt}}{A \left(\frac{\theta_2 - \theta_1}{l}\right)}$$

Where $\frac{dq}{dt}$ - rate of heat flow per second

A – unit cross sectional area

$\left(\frac{\theta_2 - \theta_1}{l}\right)$ - unit temperature gradient.

Units of k

$$k = \frac{\frac{dq}{dt}}{A \left(\frac{\theta_2 - \theta_1}{l}\right)} = \frac{\text{Js}^{-1}}{\text{m}^2 \frac{\text{K}}{\text{m}}} = \frac{\text{Js}^{-1}}{\text{mk}} = \text{Js}^{-1}\text{m}^{-1}\text{K}^{-1}$$

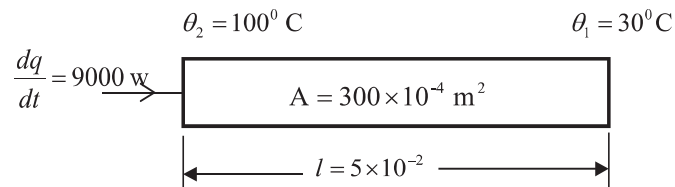
$$k = \frac{\text{Js}^{-1}}{\text{mk}} \text{Js}^{-1}\text{m}^{-1}\text{K}^{-1}, \text{ but } \text{Js}^{-1} = \text{watts}$$

$$k = \text{wm}^{-1}\text{K}^{-1}$$

EXAMPLE

An aluminum plate of area 300 cm^2 and thickness 5.0 cm has one side maintained at 100°C by steam and another side at 30°C . Energy passes through the plate at a rate of 9 kw . Calculate the coefficient of thermal conductivity of aluminum.

Solution



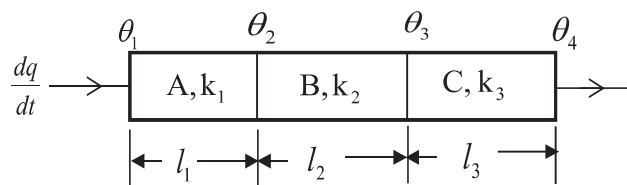
$$9000 = k(3 \times 10^{-2}) \left(\frac{100 - 30}{5 \times 10^{-2}} \right)$$

Using $\frac{dq}{dt} = kA(\theta_2 - \theta_1)$, $450 = k(3 \times 10^{-2})(70)$

$$k = 214.3 \text{ Js}^{-1}\text{m}^{-1}\text{K}^{-1}$$

HEAT FLOW THROUGH SEVERAL PLATES**(a) Plates in series**

Consider three plates A, B and C of thermal conductivities, k_1 , k_2 and k_3 respectively whose ends are maintained at temperatures θ_1 , θ_2 , θ_3 and θ_4 as shown below:



Note:

$$\theta_1 > \theta_2 > \theta_3 > \theta_4 .$$

In this case we assume that the cross sectional area, A is constant and the rate of heat flow through the plates is the same.

$$\frac{dq}{dt} \text{ for A, } \frac{dq}{dt} = k_1 A \left(\frac{\theta_1 - \theta_2}{l_1} \right)$$

$$\frac{dq}{dt} \text{ for B, } \frac{dq}{dt} = k_2 B \left(\frac{\theta_2 - \theta_3}{l_2} \right)$$

$$\frac{dq}{dt} \text{ for C, } \frac{dq}{dt} = kC \left(\frac{\theta_3 - \theta_4}{l_3} \right)$$

Since $\frac{dq}{dt}$ is the same throughout

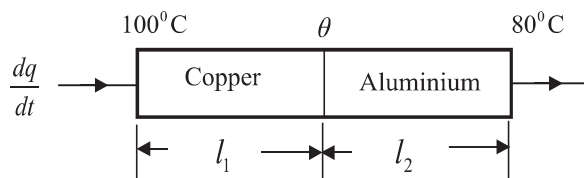
$$k_1 A \left(\frac{\theta_1 - \theta_2}{l_1} \right) = k_2 B \left(\frac{\theta_2 - \theta_3}{l_2} \right) = k_3 C \left(\frac{\theta_3 - \theta_4}{l_3} \right)$$

EXAMPLE

Heat is conducted through a wall consisting 5 mm thickness of copper and 10 mm thickness of aluminium. The temperature of 100°C and 80°C are maintained on the outside of copper and aluminium respectively.

- Determine the temperature between the copper and aluminium junction.
 - Determine the rate of heat flow through the wall if its cross sectional area is 20 cm^2 .
- (Thermal conductivities of copper and aluminium are $100\text{ W m}^{-1}\text{K}^{-1}$ and $200\text{ W m}^{-1}\text{K}^{-1}$ respectively.)

SOLUTION



Let the temperature of the junction be θ

$$l_1 = 5 \times 10^{-3} \text{ m}$$

$$\text{For copper; } \frac{dq}{dt} = k_{\text{cu}} A \left(\frac{100 - \theta}{l_1} \right)$$

$$l_2 = 10 \times 10^{-3} \text{ m}$$

$$\text{For Aluminium, } \frac{dq}{dt} = k_{\text{Al}} A \left(\frac{\theta - 80^\circ}{l_2} \right)$$

$$\left(\frac{dq}{dt}\right)_{cu} = \left(\frac{dq}{dt}\right)_{Al}, \text{ since Copper and Aluminium are in series.}$$

$$k_{cu} A \left(\frac{100 - \theta}{l_1} \right) = k_{Al} A \left(\frac{\theta - 80}{l_2} \right)$$

$$100 \left(\frac{100 - \theta}{5 \times 10^{-3}} \right) = 200 \left(\frac{\theta - 80}{10 \times 10^{-3}} \right)$$

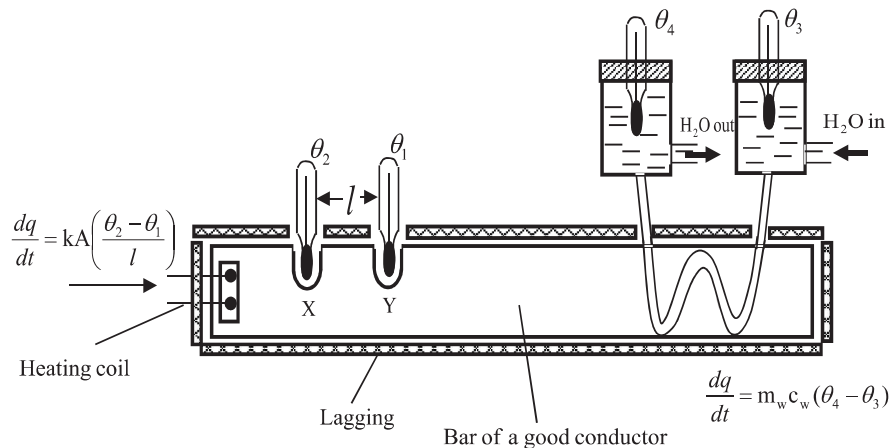
$$\theta = 90^\circ \text{C}$$

(b) For plates in parallel

For plates in parallel the rates of heat flow are different and the total heat flowing per second is equal to the sum of the individual rates of heat.

DETERMINING COEFFICIENT OF THERMAL CONDUCTIVITY OF A GOOD CONDUCTOR

Experiment for determining thermal conductivity of a metal of a good conductor using Searle's apparatus



- The conductor is heated by steam or heating coil H and cooled by circulating water.
- It is also heavily lagged to prevent heat losses to the surroundings.
- When the apparatus has been running for some time, a steady state is attained where temperatures θ_1 , θ_2 , θ_3 and θ_4 become steady and they are recorded.
- A distance between X and Y is measured and the mass rate of flow m_w is determined.
- If A is the cross sectional area of the bar, then the rate of heat flow throughout is given by;

$$\frac{dq}{dt} = kA \left(\frac{\theta_2 - \theta_1}{l} \right) \dots\dots\dots (1)$$

- The heat in equation (1) is carried away by the cooling water per second.
- If c_w is the specific heat capacity of water then the heat carried away by water per second is;

$$\frac{dq}{dt} = m_w c_w (\theta_4 - \theta_3) \dots\dots\dots (2)$$

Therefore,

$$kA \left(\frac{\theta_2 - \theta_1}{l} \right) = m_w c_w (\theta_4 - \theta_3) \dots\dots\dots \otimes$$

And thermal conductivity, k , can be determined from equation \otimes .

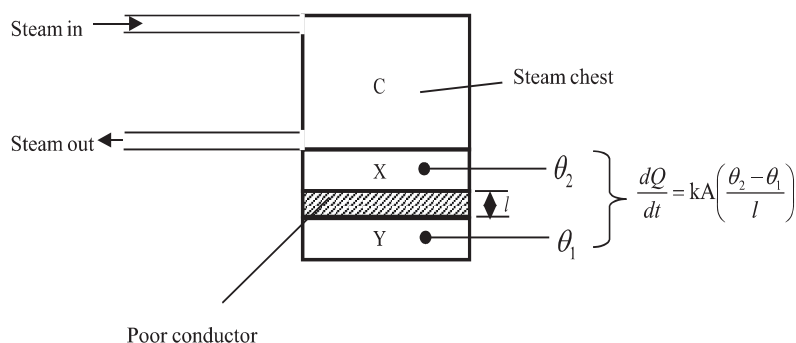
Note. When determining the thermal conductivity of a metal by this method the following conditions are necessary:

- Heat flow through the conductor should be steady.
- The temperature gradient should be steep.

A steady heat flow can be obtained by lagging the conductor and the problem of getting a steep temperature gradient can be solved by using a bar which is long compared to its diameter.

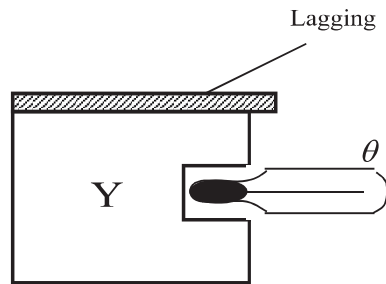
DETERMINATION OF THERMAL CONDUCTIVITY OF A POOR CONDUCTOR E.G. GLASS

Let X and Y be brass slabs.

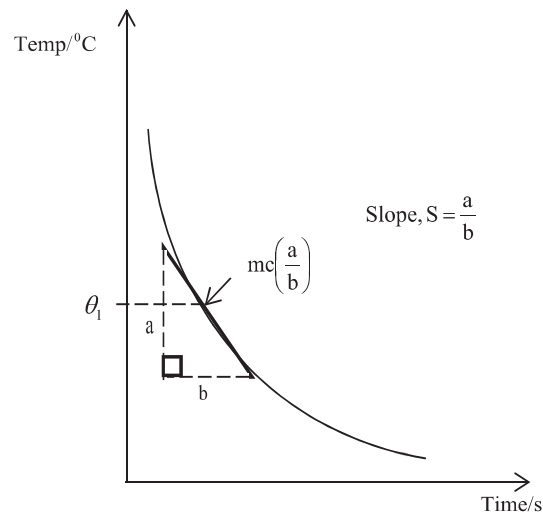


- X and Y are brass slabs.
- A thin sample of known diameter, d_1 and thickness l is placed between two brass slabs X and Y

- Steam is passed through the chest C until the temperature θ_1 and θ_2 are steady. θ_1 and θ_2 are recorded.
- A poor conductor is removed and Y is heated directly by steam chest until temperature rises by about 10°C above θ_1 .
- A lagging is put on top of Y and a temperature of Y is recorded every after suitable time interval until it falls by about 10°C below θ_1 .



- A cooling curve is plotted and its gradient, S of the tangent to the curve at θ_1 is formed.



Then thermal conductivity k can be determined from;

$$kA \left(\frac{\theta_2 - \theta_1}{l} \right) = mc \left(\frac{a}{b} \right)$$

Where,

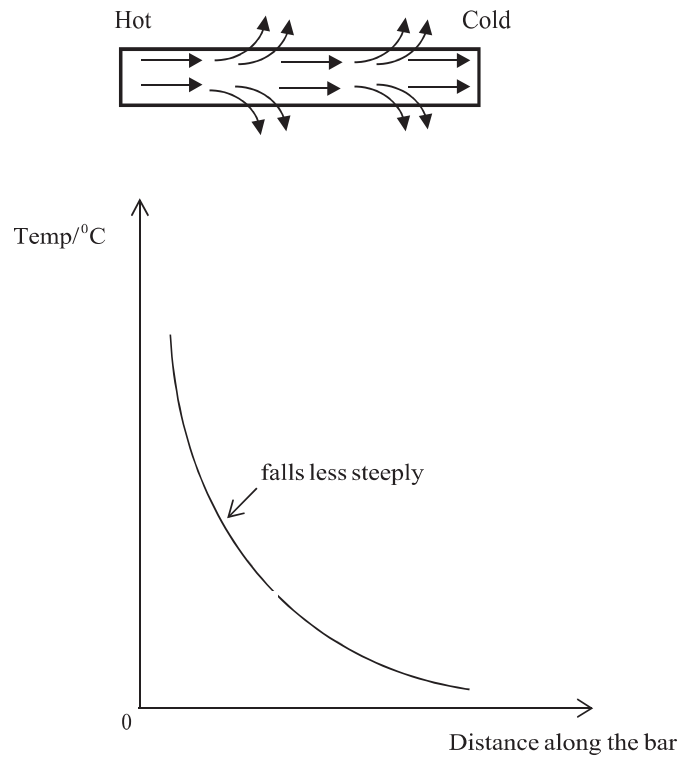
m - mass of Y

c - specific heat capacity of Y

A - cross sectional area of Y

Temperature distribution along an unlagged bar

For an unlagged bar temperature falls less and less steeply from the hot end to the cold end as shown in the figure below. This is because if the bar is unlagged, heat escapes from sides to the surroundings.



ASSIGNMENT

A sheet of rubber and a sheet of cardboard each 2 mm thick are pressed together and their outer faces are maintained respectively at 0°C and 25°C . If the thermal conductivities of rubber and the cardboard are respectively 0.13 and $0.05 \text{ W m}^{-1}\text{K}^{-1}$, find the quantity of heat which flows in one hour across the composite sheet of area 100 cm^2 .

LESSON 13

TOPIC: THERMAL RADIATION

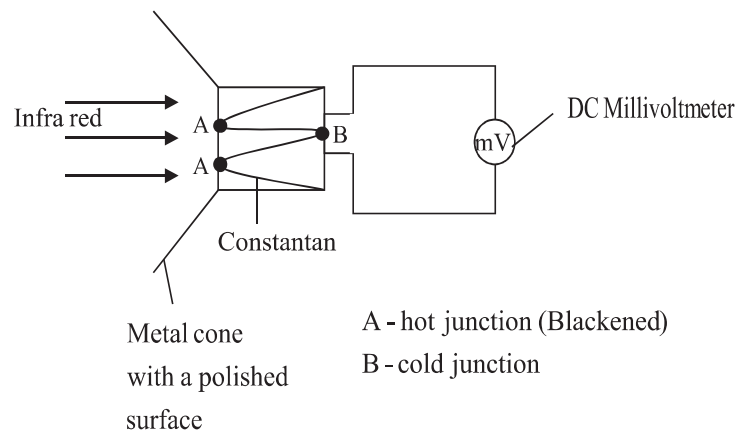
OBJECTIVES: By end of this lesson, you should describe radiation as a means of heat transfer and solve related numerical problems.

INTRODUCTION

All objects emit thermal radiation. The hotter an object is, the more energy per second is carried away from it by thermal radiation. Thermal radiation consists of electromagnetic waves with a range of wave length covering the infra-red and visible regions of the electromagnetic spectrum. When thermal radiation is directed at a surface, some of the radiation is absorbed by the surface and some is reflected. Some of the radiation may pass through the surface and be transmitted through the material to pass out through the side. Shiny silvered surfaces are the best reflectors, whereas dull black surfaces are very good absorbers. Surfaces which appear black daylight do not reflect any light. They absorb all the light which falls on them. Such surfaces are good absorbers of radiation.

Detection of infra-red

This can be done by using a thermopile as shown below;

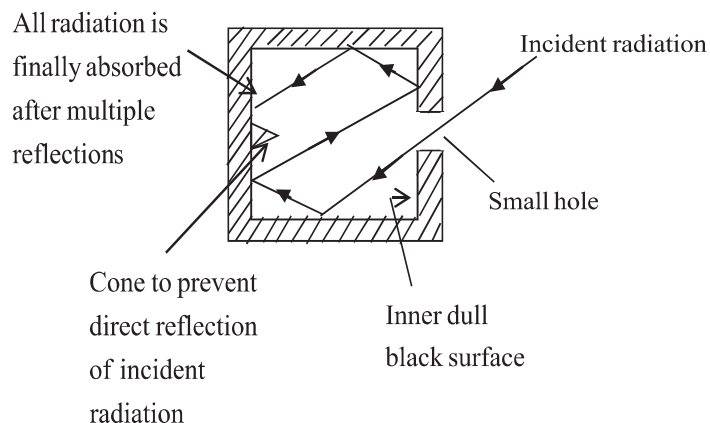


A metal cone concentrates the infra-red radiation onto the blackened metal junctions, A which get warmed up. As a result an e.m.f is set up between the hot and cold junctions and can be measured by a mill voltmeter. The magnitude of e.m.f depends on the intensity of incident radiation.

BLACK BODY RADIATION

An object with surfaces which absorb thermal radiation of all wave lengths is called a **black body** because it reflects no light. **A black body is defined as a body which absorbs all incident radiation falling on it and reflects none.**

APPROXIMATION OF A BLACK BODY



The absorber which approximates to a black body is made by punching a very small hole in an enclosure e.g. a tin whose inside walls are rough and blackened. All the radiation is finally absorbed and none is reflected after undergoing multiple reflections.

STEFAN'S LAW

This states that the total energy E , radiated per unit surface area per unit time is directly proportional to the fourth power of the absolute temperature T of the black body.

i.e. $E \propto T^4$. Hence $E = \sigma T^4$

Where, σ is Stefan's constant,

$$E = \sigma T^4 \quad \sigma = 5.7 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}$$

$$\text{But } E = \frac{\text{energy}}{\text{Area} \times \text{time}} = \sigma T^4$$

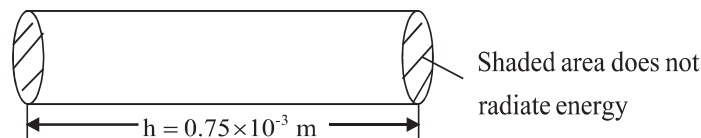
$$\text{Power radiated by the black body} = A\sigma T^4$$

Where; A is the surface area of the body.

Note. For a non-black body, we consider; $E = \varepsilon \sigma T^4$ where; ε is the emissivity of the non-black body.

EXAMPLE. A cylinder has a radius of 10^{-2} m and a height of 0.75 mm. Calculate the temperature of the cylinder if it's assumed to be a lamp of power 1 KW and acting as a black body. (Stefan's constant $\sigma = 5.7 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}$)

Solution



$$\text{Surface area } A = \pi r h$$

$$\text{For two surfaces; } A = 2\pi r h$$

$$\text{Power radiated by the black body} = A \sigma T^4$$

$$(1 \times 10^3) = (2\pi r h) \sigma T^4$$

$$T^4 = \frac{1000}{2\pi r h \sigma} = \frac{1000}{2 \times 3.14 \times 10^{-2} \times 0.75 \times 10^{-3}}$$

$$T^4 = 3.7248 \times 10^{14}$$

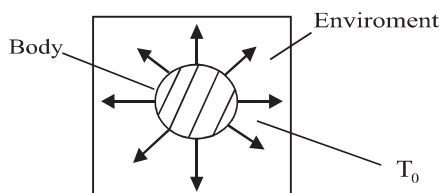
$$T = 4393.2 \text{ K}$$

PREVOST'S THEORY OF HEAT EXCHANGE

Prevost's theory of heat exchange states that a body emits radiation at a rate which depends on the nature of its surface and its temperature and absorbs heat at a rate which is determined by the nature of its surface and the temperature of the surrounding. Hence a body at the same temperature as that of the surroundings is in the state of thermodynamic equilibrium with its surroundings.

Note: From the above theory we deduce that a good absorber of radiation must also be a good emitter of radiation. Experiments show that a dull black surface is a good absorber and emitter of radiation while a shiny surface is a poor absorber and emitter of radiation.

If a black body of surface area A is at absolute temperature, T in an environment which is at a lower temperature T_0 , then the body emits heat to the surrounding environment.



T - temperature of the body

T_0 - temperature of the environment

Using, Power radiated by the black body = $A\sigma T^4$

$$\text{Net power radiated} = A\sigma(T^4 - T_0^4)$$

EXAMPLE. A solid copper sphere of diameter 10 mm is cooled to a temperature of 150 K and then placed in an enclosure maintained at 290 K. Assuming that all interchange of heat is by radiation, calculate the initial rate of rise of temperature of the sphere. The sphere may be assumed as a black body. (Density of copper = $8.9 \times 10^3 \text{ kg/m}^3$, specific heat capacity of copper = $3.7 \times 10^2 \text{ J/kg/K}$, Stefan's constant $\sigma = 5.7 \times 10^{-8} \text{ W/m}^2/\text{K}$)

SOLUTION

Net power radiated = $A\sigma(T^4 - T_0^4)$, Surface area of a sphere $A = 4\pi r^2$

Net power radiated = $4\pi r^2 (T^4 - T_0^4)\sigma$, Net power radiated 0.1176 W

But heat energy = $mc\theta$, Power absorbed = $mc\left(\frac{\theta}{t}\right)$

Power absorbed = (density \times volume) $\times c \times \left(\frac{\theta}{t}\right)$

Power absorbed = $\rho \times \frac{4\pi r^3}{3} \times c \times \left(\frac{\theta}{t}\right)$

Power absorbed = 0.1176 = $(8.9 \times 10^3 \times \frac{4\pi}{3} \times 3.7 \times 10^2) \times \frac{\theta}{t}$

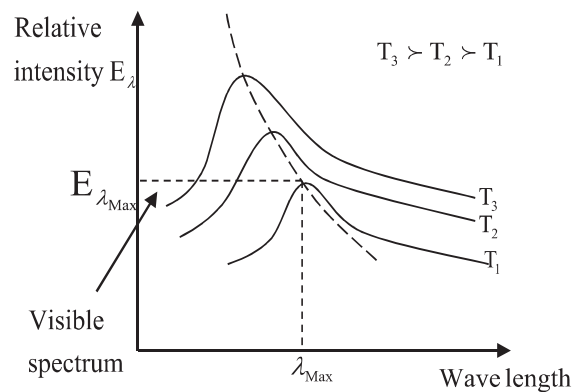
$$\frac{\theta}{t} = \frac{3 \times 0.1176}{4 \times 4\pi \times 8.9 \times 10^3 \times \left(\frac{10}{2} \times 10^{-3}\right)^3 (3.7 \times 10^2)}$$

$$\frac{\theta}{t} = 0.068 \text{ K s}^{-1}$$

ASSIGNMENT. A metal sphere of diameter 1×10^{-2} m at a temperature of 240 K is placed in an enclosure at a temperature of 100 K. Assuming that all interchange of heat is by radiation. Calculate the initial rate of temperature rise of the sphere if it is assumed to behave like a black body. (Density of the sphere $= 7.2 \times 10^3$ kg/m³, specific heat capacity of the sphere $= 3.5 \times 10^2$ J/kg/K, Stefan's constant $\sigma = 5.7 \times 10^{-8}$ Wm⁻²K⁻¹)

ENERGY DISTRIBUTION IN A SPECTRUM OF A BLACK BODY

The spectral curves show the variation of relative intensity (energy distribution) emitted by a black body with wave length as temperature rises.

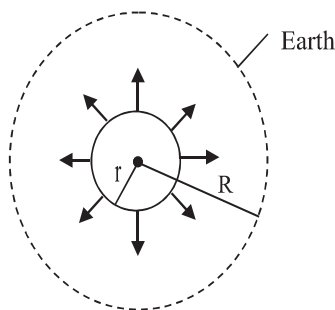


As the temperature rises the intensity of every wavelength increases. At a particular temperature there is a wavelength which is more intensely emitted than all the others.

As the temperature rises, the intensity of shorter wavelengths increases more rapidly. This is why when the temperature of the source is progressively raised, the radiation is first red, becomes less red, tending to white.

SOLAR CONSTANT

Solar constant is the energy per unit area per second incident on the earth's surface from the sun i.e.



Total energy per second (power) radiated by the sun $= A_s \sigma T_s^4$

Where, A_s – Surface area of the sun $= 4\pi r^2$

T_s = Surface temperature of the sun

Solar energy $= 4\pi r^2 \sigma T_s^4$

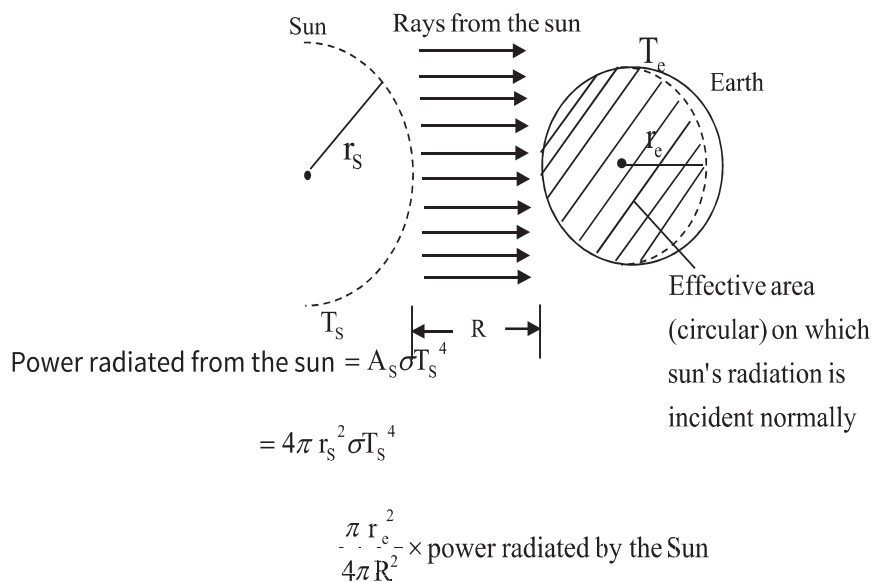
Energy arriving per square metre per second on the earth (solar constant) $= \frac{4\pi r^2 \sigma T_s^4}{4\pi R^2}$

Energy constant $= \frac{r \sigma T_s^4}{R^2}$.

Where, r – radius of the sun

R – Radius of the Earth.

RADIATIVE EQUILIBRIUM OF THE SUN AND THE EARTH



Power received by the Earth

Where, $4\pi R^2$ is the total area available over which the sun radiates at a distance R from the earth.

Power radiated by the Earth $= A_e \sigma T_e^4$, Where, A_e is the surface area of the effective part of the Earth that radiates.

$$A_e = 4\pi r_e^2$$

$$\text{Power radiated by the Earth} = 4\pi r_e^2 \sigma T_e^4$$

Assuming radiative equilibrium;

Power radiated by the Earth = Power received by the Earth

$$4\pi r_e^2 \sigma T_e^4 = \frac{\pi r_e^2}{4\pi R^2} \times (4\pi r_s^2 \sigma T_s^4)$$

$$T_e^4 = T_s^4 \times \left(\frac{r_s^2}{4R^2} \right)$$

Note. We assumed that the sun and Earth are black bodies.

ASSIGNMENT

- (a) Explain the mechanism of heat conduction in solids.
- (b) Describe a method of determining the thermal conductivity of cork in the form of a thin sheet.
- (c) A window of height 1.0m and width 1.5m contains a double glazed unit consisting of two single glass panes, each of thickness 4.0mm separated by an air gap of 2.0 mm. Calculate the rate at which heat is conducted through the window if the temperatures of the external surfaces of glass are 20°C and 30°C respectively. [Thermal conductivities of glass and air are $0.72 \text{ Wm}^{-1}\text{K}^{-1}$ and $0.025 \text{ Wm}^{-1}\text{K}^{-1}$ respectively]