QN	P425/1 PURE MATHS MARKING GUIDE. UTEC	Comments
1	$1-\cos 2\theta = \sin\theta$ $\Leftrightarrow 2\sin^2\theta - \sin\theta = 0$ M1 $\Leftrightarrow \sin\theta(2\sin\theta - 1) = 0$ M1 $\Rightarrow \sin\theta = 0$ or $\sin\theta = \frac{1}{2}$ $\Rightarrow \theta = 180°$ A1 $\Rightarrow \theta = 30°, 150°$ A1 A1	
2.	$(2+i)^2 = 4+4i+i^2 = 3+4i$ B1 $\Rightarrow \frac{50}{(2+i)^2} \Leftrightarrow \frac{50(3-4i)}{(3+4i)(3-4i)}$ M1 $\Leftrightarrow 2(3-4i)$ $= 6+(-8)i$ A1 $\Rightarrow a = 6$ A1, $b = -8$ A1	
3.	$y^2 = x^2+2xy+8 \Rightarrow 2y\frac{dy}{dx} = 2x+2\left(y+x\frac{dy}{dx}\right) = 0$ M1 i.e. $(y-x)\frac{dy}{dx} = x+y$ $\therefore \frac{dy}{dx} = \frac{x+y}{y-x}$ B1 Now: $\frac{dy}{dx} = 0 \Rightarrow x+y=0$ or $y=-x$. A1 Using $y=-x$ in $y^2 = x^2+2xy+8$ $\Rightarrow x^2 = x^2+8-2x^2$ $\Leftrightarrow x^2 = 4$ $\therefore x = 2$ A1 or $x = -2$. A1	

QN | SOLUTIONS | Comments

4

$$2y - x - 3 = 0 \iff y = \tfrac{1}{2}x + \tfrac{3}{2} \Rightarrow \text{grad. } m_1 = \tfrac{1}{2} \quad B_1$$

$$y = px + 3 \Rightarrow \text{grad. } m_2 = p$$

Using $\tan\theta = \pm\left(\dfrac{m_2 - m_1}{1 + m_1 m_2}\right)$

$$\Rightarrow \tan 45^\circ = \pm\left(\frac{p - \frac{1}{2}}{1 + \frac{1}{2}p}\right) \quad M_1$$

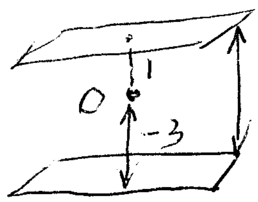
$$\frac{1}{1} = \frac{\pm(2p-1)}{p+2} \iff \pm(2p-1) = p+2 \quad B_1$$

$$\Rightarrow p = 3 \; A_1 \text{ or } p = -\tfrac{1}{3} \; A_1$$

5

Distance from 1st plane: $d_1 = \dfrac{3(0) - 4(0) + 12(0) + 13}{\sqrt{3^2 + (-4)^2 + 12^2}}$

$= 1$ unit



Distance from 2nd plane: $d_2 = \dfrac{3(0) - 4(0) + 12(0) - 39}{\sqrt{3^2 + (-4)^2 + 12^2}}$

$= -3$ units.

required distance $= 1 + |-3|$
$= 4$ units.

6.

$$\int_0^{1/2} \frac{4x}{4 - x^2}\,dx \iff -2\int_0^{1/2} \frac{-2x}{4 - x^2}\,dx \quad M_1$$

$$= -2\left[\ln(4 - x^2)\right]_0^{1/2} \quad M_1 \; B_1$$

$$= -2\left(\ln\tfrac{15}{4} - \ln 4\right) \quad B_1$$

$$= 2\ln\left(\tfrac{16}{15}\right) \simeq 0.12908 \quad A_1$$

QN	SOLUTIONS	Comments
7.	Let a, d be the 1st term and the common diff.	
	The G.P. $= a + (a+d) + (a+3d) + \cdots$	
	Common ratio: $\frac{a+d}{a} = \frac{a+3d}{a+d}$ M_1	
	$\Rightarrow a^2 + 3ad = a^2 + 2ad + d^2$	
	$\Rightarrow d^2 - ad = 0$ B_1	
	$d(d-a) = 0 \Rightarrow d = 0$ (ignore)	
	$\Rightarrow d = a$ B_1 $\Rightarrow$ common ratio $= \frac{a+d}{a} = \frac{2a}{a}$ M_1	
	$= 2.$ A_1	
8	(a) $V = \frac{1}{3}\pi r^2 h$; $h = 2r \therefore r = \frac{h}{2}$	
	$\therefore V = \frac{\pi}{3} h \left(\frac{h}{2}\right)^2$ M_1 B_1 $\Rightarrow V = \frac{\pi}{12} h^3$ A_1	Note
	(b) $\frac{dV}{dh} = \frac{\pi}{4} h^2$ M_1, when $h = 4$, $\frac{dV}{dh} = 4\pi \text{ cm}^2$ A_1	part (b) was corrected to "V changes with height"

Note: In case the correction was not made:

From $V = \frac{\pi}{12} h^3 \Rightarrow \frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$ $M_1 A_1$

When $h = 4$; $\frac{dV}{dt} = 4\pi \frac{dh}{dt}$ (end of solution.

QN	Solutions	Comments.

SECTION B

9 (a) $x = t^2 - 2t - 3 \Rightarrow \dfrac{dx}{dt} = 2t - 2$ (M1)

$y = t^2 + 2t - 3 \Rightarrow \dfrac{dy}{dt} = 2t + 2$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{t+1}{t-1} \text{ (M1)(A1)}$$

$$\frac{d^2y}{dx^2} = \frac{(t-1).1 - 1(t+1)}{(t-1)^2} \cdot \frac{1}{2(t-1)} \text{ (M1)}$$

$$= \frac{-1}{(t-1)^3} \text{ (A1)}$$

$\dfrac{dy}{dx} = 0 \Rightarrow t = -1 \Rightarrow x = 0, y = -4$

The stationary point is $(0, -4)$ (A1)

At $t = -1$, $\dfrac{d^2y}{dx^2} = \dfrac{1}{8} > 0 \Rightarrow (0, 4)$ is a minimum turning point. (M1) (B1)

(b) At the y-axis, $x = 0 \Rightarrow t^2 - 2t - 3 = 0$

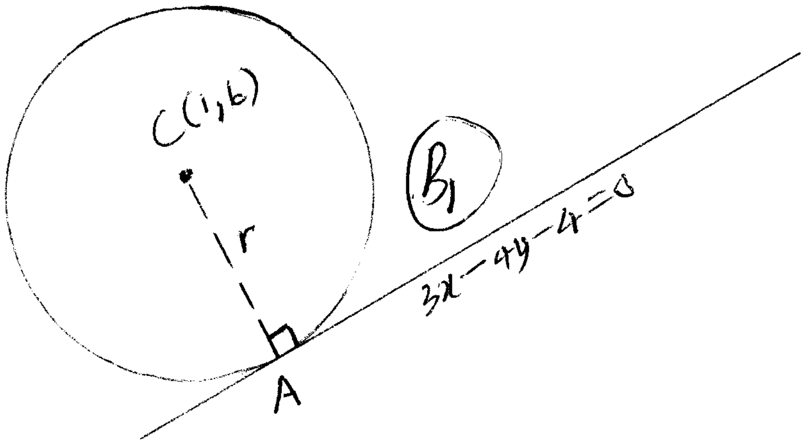
$(t-3)(t+1) = 0 \Rightarrow t = 3, -1$ (M1)

When $t = -1$, y is negative; but $t = 3$, $y = 12$ (positive) (B1)

At $t = 3$, $\dfrac{dy}{dx} = 2 \Rightarrow$ equation of tangent at $(0, 12)$ (M1)

is $y = 2x + 12$. (using $y = mx + c$)

(A1)

No	SOLNS.	COMMENTS
10	 B1	$y=\frac{3}{4}x-1$ $\Leftrightarrow 4y=3x-4$ $\Leftrightarrow 3x-4y-4=0$ M1

(a) radius, $r = \dfrac{|3(1)-4(6)-4|}{\sqrt{3^2+(-4)^2}}$ M1B1

$= 5$ units. A1

Equation of the circle: $(x-1)^2+(y-6)^2=25$ A1

(b) Gradient of tangent $= 3/4 \Rightarrow$ grad. of normal AC $= -4/3$ M1

Eqn of AC: $\dfrac{y-6}{x-1} = -4/3 \Leftrightarrow 4x+3y=22$ M1

At point A; solve: $4x+3y=22$ ---(1)
with: $3x-4y=4$ ---(2) simultaneously.

$4\times$ eqn(1): $16x+12y=88$
$3\times$ eqn(2): $9x-12y=12$ M1

$\Rightarrow 25x=100 \therefore x=4$ (B1)

$\Rightarrow y=2$ (B1) $\Rightarrow A(4,2)$ (A1)

AN | Solutions | Comments.

11 (a) Using $t = \tan\frac{1}{2}\theta \Rightarrow \sin\theta = \frac{2t}{1+t^2}$ and $\cos\theta = \frac{1-t^2}{1+t^2}$

$$\Rightarrow 3\left(\frac{2t}{1+t^2}\right) - \frac{4(1-t^2)}{1+t^2} = 4 \quad M1$$

$$\Leftrightarrow 6t - 4 + 4t^2 = 4 + 4t^2$$

$$6t = 8 \quad \therefore t = \frac{4}{3} \quad B1 \quad \text{or} \quad t = \infty \quad B1$$ (since the t^2-term disappears)

$$\text{ie, } \tan\frac{1}{2}\theta = 4/3 \Rightarrow \frac{1}{2}\theta = 53.13^\circ \quad \therefore \theta = 106.26^\circ \quad A1$$

$$\tan\frac{1}{2}\theta = \infty \Rightarrow \frac{1}{2}\theta = 90^\circ \quad M1 \quad \therefore \theta = 180^\circ \quad A1$$

Alternatively use the R-formula, and follow through

(b) $$L.H.S = \frac{2\cos\frac{3A+A}{2}\sin\frac{3A-A}{2}}{2\sin\frac{5A+3A}{2}\cos\frac{5A-3A}{2}} \quad M1 \quad M1$$

$$= \frac{\cos 2A \sin A}{\sin 4A \cos A}$$; set $\sin 2A = 2\sin A\cos A$

$\sin 4A = 2\sin 2A\cos 2A$

$$= \frac{\cos 2A \sin A}{2\sin 2A\cos 2A\cos A} \quad M1$$

$$= \frac{(\cos 2A)\sin A}{4(\cos 2A)\sin A\cos^2 A} \quad M1$$

$$= \frac{1}{4\cos^2 A} = \frac{1}{4}\left(\frac{1}{\cos^2 A}\right) \quad B1$$

$$= \frac{1}{4}\sec^2 A \quad A1$$

$$= R.H.S.$$

QN	SOLUTIONS	Comments.

12 (a) $(4-3x)^{\frac{1}{2}} = 4^{\frac{1}{2}}\left(1-\frac{3x}{4}\right)^{\frac{1}{2}}$ B1

$= 2\left\{1 + \frac{1}{2}\left(\frac{-3x}{4}\right)^1 + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(\frac{-3x}{4}\right)^2 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(\frac{-3}{2}\right)}{3!}\left(\frac{-3x}{4}\right)^3 + \dots\right\}$ M1 B1

$= 2 - \frac{3}{4}x - \frac{9}{64}x^2 - \frac{27}{512}x^3 + \dots$ A1

When $x=1$, Exact Value $= \sqrt{4-3\times1}$

$= 1$ B1

Approximate value $\simeq 2 - 3/4 - 9/64 - \frac{27}{512}$

$\simeq 1.056641$ B1

Hence the error $= 1.056641 - 1$

$\simeq 0.0566$ (4 dps) A1

(b) $\sqrt{61} = \sqrt{64-3}$

$= \sqrt{64(1 - 3/64)}$

$= 8\left[1 - \frac{3}{4}\left(\frac{1}{16}\right)\right]^{1/2}$ M1 set $x = \frac{1}{16}$ in the expansion

$= 8\left\{1 - \frac{3}{8}\left(\frac{1}{16}\right) - \frac{9}{32}\left(\frac{1}{16}\right)^2 - \frac{27}{256}\left(\frac{1}{16}\right)^3 + \dots\right\}$ B1

$\simeq 8(1 - 0.0234375 - 0.001098633 - 0.000025749)$ B1

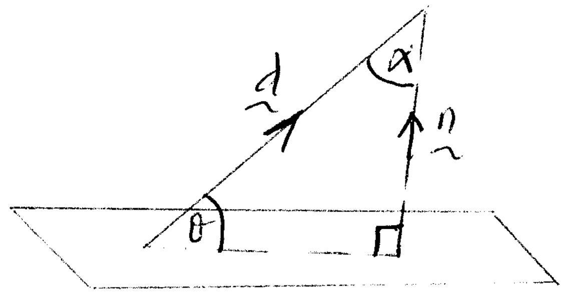
$\simeq 8 \times 0.975438118$

$\simeq 7.803505$ A1

$\simeq 7.8035$ (4 dps) B1

QN	Solutions	Comments.
14	(a) Using $x = 5+2\lambda$, $y = -2-\lambda$, $z = 4+3\lambda$ in (B1)	
	$3x+2y+z=29$ we've: $15+6\lambda-4-2\lambda+4+3\lambda=29$ (M1 B1)	
	$\Rightarrow 7\lambda = 14 \therefore \lambda = 2$ (B1)	
	$\Rightarrow x = 5+4$, $y = -2-2$; $z = 4+6$ (B1)	
	$\Rightarrow A(9,-4,10)$ (A1)	

(b)



$\underset{\sim}{d} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ (B1)

$\underset{\sim}{n} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

$\underset{\sim}{d} \cdot \underset{\sim}{n} = |\underset{\sim}{d}||\underset{\sim}{n}| \cos\alpha$; but $\alpha = 90° - \theta$

$\Rightarrow \cos\alpha = \cos(90° - \theta)$

$= \sin\theta$; where θ is the required angle.

$\Rightarrow \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \sqrt{4+1+9} \times \sqrt{9+4+1} \sin\theta$ (M1 B1)

$6-2+3 = \sqrt{14} \times \sqrt{14} \sin\theta$ (B1)

$7 = 14 \sin\theta$ (B1)

$\sin\theta = \frac{1}{2} \therefore \theta = \sin^{-1}\frac{1}{2}$

$= 30°$. (A1)

QN | SOLUTIONS | COMMENTS

13 (a)(i) $y = x^3 \Rightarrow \frac{dy}{dx} = 3x^2$ (M1)

At $A(-1,-1)$; grad. $= 3$ (B1) ; eqn of tangent: $\frac{y-(-1)}{x-(-1)} = 3$ (M1)

or $y = 3x + 2$. (A1)

(ii) At point B, $x^3 = 3x + 2 \iff x^3 - 3x - 2 = 0$

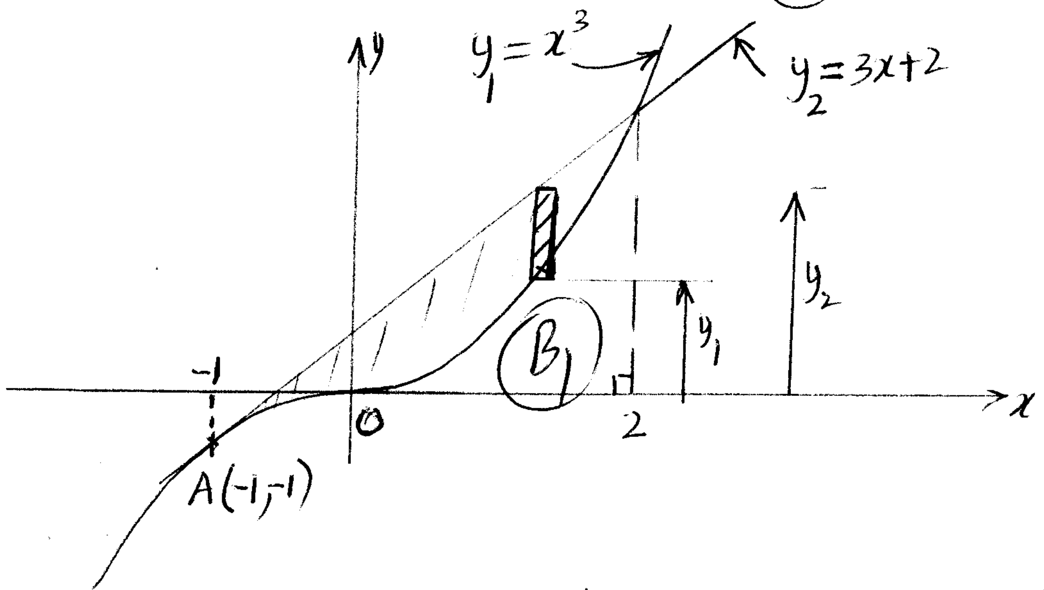
$x = -1$ is a repeated root $\Rightarrow (-1) + (-1) + x = 0$ (M1)

$\Rightarrow x = 2 \Rightarrow y = 8$ (B1)

$\therefore B(2,8)$ (A1)

Or $x = -1$ $\Rightarrow x+1$ is a factor, perform long division to find the other factors.

(b)



$\Delta A = (y_2 - y_1)\Delta x$

$\Rightarrow$ Area $= \int_{-1}^{2} (y_2 - y_1)\,dx$ (M1)

$= \int_{-1}^{2} (3x + 2 - x^3)\,dx$

$= \left[\frac{3}{2}x^2 + 2x - \frac{x^4}{4}\right]_{-1}^{2}$ (M1)

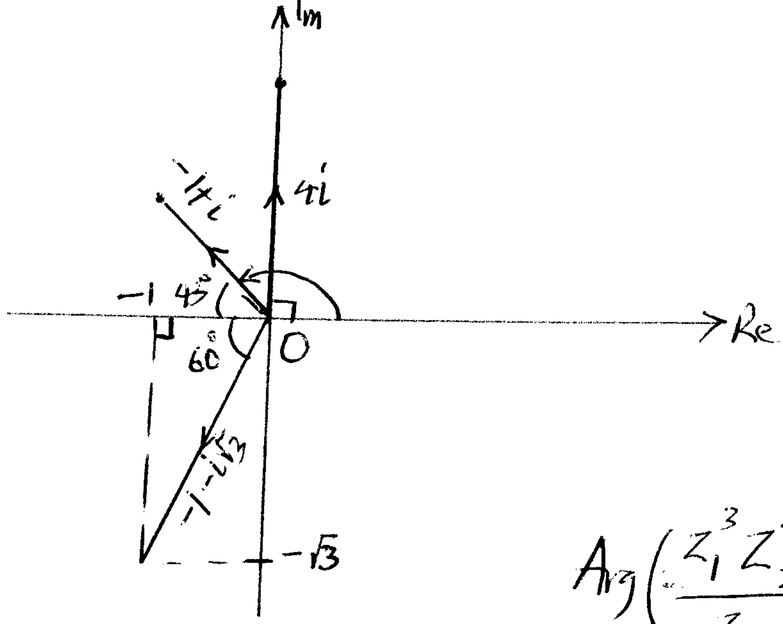
$= (6 + 4 - 4) - (\frac{3}{2} - 2 - \frac{1}{4})$ (B1)

$= 27/4$ sq. units.

$= 6\,3/4$ sq. units (A1)

4N | SOLUTIONS | Comments

15 (a)



$Arg(4i) = 90°$ (B1)

$Arg(-1+i) = 135°$ (B1)

$Arg(-1-i\sqrt{3}) = -120°$ (B1)

$$Arg\left(\frac{Z_1^3 Z_2^2}{Z_3}\right) = 3Arg(Z_1) + 2Arg(Z_2) - Arg(Z_3)$$

$= 3 \times -120° + 2 \times 135° - 90°$ (M1)

$= -360° + 270° - 90°$

$= 0° - 90° - 90°$ (B1)

$= -180°$ (not principal)

$\Rightarrow$ principal Argument is $180°$ (A1)

(b) Let $Z = \sqrt[3]{-8} \Leftrightarrow Z^3 + 8 = 0$ (B1)

$\Leftrightarrow (Z+2)(Z^2 - 2Z + 4) = 0$ (M1)

$\Rightarrow Z + 2 = 0 \therefore Z = -2$ (A1) and $Z^2 - 2Z + 4 = 0$

$Z = \frac{2 \pm \sqrt{4-16}}{2}$ (M1)

$\Rightarrow Z = 1 + i\sqrt{3}$ (A1)

and $Z = 1 - i\sqrt{3}$ (A1)

16 (a) $\sin x \dfrac{dy}{dx} + y\cos x = \tan 3x$

$\Leftrightarrow \dfrac{d}{dx}(y\sin x) = \tan 3x$. M1 B1

$\Rightarrow y\sin x = \int \tan 3x \, dx$

$= \dfrac{1}{3}\int \dfrac{\sec 3x \tan 3x}{\sec 3x} dx$ M1 B1

$\therefore y\sin x = \dfrac{1}{3}\ln(\sec 3x) + c$. A1

(b) $\dfrac{dP}{dt} \propto P \Rightarrow \dfrac{dP}{dt} = kP$ M1

$\Rightarrow \int \dfrac{1}{P} dP = \int k \, dt$

$\ln P = kt + c$ B1

Let $P = P_0$ at $t=0$, $c = \ln P_0 \Rightarrow \ln P = kt + \ln P_0$

set $P = 2P_0$ at $t = 10$, $\Rightarrow \ln 2P_0 = 10k + \ln P_0$ M1

$\Rightarrow k = \dfrac{\ln 2}{10}$ B1

Set $t = 20 \Rightarrow \ln P = \dfrac{20\ln 2}{10} + \ln P_0$

$\ln P = \ln 4P_0 \Rightarrow P = 4P_0$ B1

%-age increase $= \dfrac{4P_0 - P_0}{P_0} \times 100$ M1

$= 300\%$ A