

STANDARD HIGH SCHOOL ZZANA

S.6 SUBMTC WORK

Instructions. – forward scanned answers to stahiza2020@gmail.com

SECTION A (Attempt all questions from this section)

1. Solve for x in the equation $2 \log_3 x - \log_3 4 - 2 = 0$ (5 marks)
2. Find the coordinates of the stationary points of the curve $y = x^2 + \frac{16}{x^2}$ (5 marks)
3. If $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} + 4\mathbf{j}$, find the;
(i) Magnitude of \mathbf{a} and \mathbf{b}
(ii) Angle between \mathbf{a} and \mathbf{b} (5 marks)
4. Solve the equation $\tan x = \sec x$ for values of x between 0° and 360° (5 marks)
5. A bag contains 5 red, 3 yellow and 2 green balls. Three balls are picked at random without replacement. Find the probability that 2 are of the same colour (5 marks)
6. A discrete random variable has a probability function $f(x)$ given by
$$f(x) = \begin{cases} \frac{2x-1}{k} & \text{for } x = 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$
Where k is a constant
Find the; (i) value of k
(ii) expectation of x (5 marks)
7. The prices of five commodities A, B,...,E in shillings for the years 2010 and 2012 with the corresponding weights are given in the table below

Year	2010	2012	weight
Commodity	Cost per kg	Cost per kg	
A	2500	3000	4
B	1500	1200	3
C	1000	1400	5
D	500	700	6
E	x	1800	2

Given the average weighted index for 2012 using 2010 as the base year is 125, find the value of x
(5marks)

8. A particle of mass 2kg rest on a rough horizontal plane .A force of 6.5N inclined at an angle of $\sin^{-1} \frac{5}{13}$ to the horizontal acts upwards on the particle.If the particle is about to slide, find the coefficient of friction between the particle and the plane .
(5marks)

9. The table below shows the frequency distribution of the marks scored by 40 pupils in a test marked out of 50

Marks	Number of pupils
0 - 4	1
5 - 9	4
10 - 14	6
15 - 19	8
20 - 24	10
25 - 29	9
30 - 34	2

- a) Use suitable assumptions to estimate the ;
 - i) Percentage mean mark
 - ii) Standard deviation (10marks)
- b) Calculate the range of the middle 50% of the distribution (5marks)

10. The table below shows the overtime hours (x) and the corresponding earnings (y) in thousands for shillings for 9 employees of a certain factory

X(hours)	79	81	84	52	66	56	57	35	57
Y ('000')	70	84	84	53	47	63	66	51	34

- a) i) Draw a scatter diagram for the data
 ii) Comment on the relationship between the earnings and the time spent on the job
 b) i) Calculate the rank correlation coefficient
 ii) Comment on your results (15 marks)

11. The table below shows the quarterly sales in millions of shillings of a certain dairy farm for the years 2010, 2011 and 2012

Sales for year	2010	2011	2012
First quarter	122	144	160
Second quarter	191	214	225
Third quarter	328	375	397
Fourth quarter	281	308	322

- a) Calculate a four-quarterly moving average
 b) Plot on the same graph the original data and the moving average
 c) Find the income expected in the first quarter of 2013 (15 marks)

12. A curve is such that $\frac{dy}{dx} = 3 - 2x$ and a point P(1,0) lies on the curve

- a) Find the
 i) Equation of the curve (3 marks)
 ii) Coordinates and the points where the curve meets the x-axis (5 marks)
 iii) Coordinates and the nature of the stationary points (5 marks)
 b) Sketch the curve in (a) above and find the area enclosed by the curve and the x-axis (2 marks)

13. A company that manufactures three types of radios requires diodes (D), valves (V), transistors (T) and capacitors (C). Sony requires 4D, 3V, 5T and 2C. Panasonic requires 4V, 6T and 1C while Phillips requires 2D, 8T and 5C. The cost of each diode, valve, transistor and capacitor in thousands of shillings is 15, 5, 9 and 12 respectively

- a) Write down;
 i) 3 x 4 matrix for the requirements of the radios
 ii) 4 x 1 matrix for the cost of the accessories

- iii) Use matrix multiplication to find the cost of manufacturing each radio
- b) If Sony radio, panasonic and phillips are sold at shs. 200,000,150,000 and 160,000 each respectively .Use matrix method to find the percentage profit made by the company from sales of 20 sony , 25 panasonic and 15 phillips

(15mks)

14. A train starts from station P and accelerates uniformly for 2 minutes reaching a speed of 72kmh^{-1} .It continues at this speed for 5 minutes and then is retarded uniformly for a further 3 minutes coming to rest at station Q .

Draw a velocity –time graph and use it to find the;

- i) Distance PQ in Km
- ii) Average speed of the train
- iii) Acceleration in ms^{-2}
- iv) Time taken to cover the distance between P and Q

(15mks)

SET 2

1. Evaluate $\frac{\log_6 216 + \log_2 64}{\log_3 243 - \log_{10} 0.1}$ [5mks]
2. The table below shows the ranks of marks awarded by Judge 1 (R_x) and Judge 2 (R_y) to 7 choir groups A to G.

Choir	A	B	C	D	E	F	G
Rank Judge 1 (R_x)	2	4	6	1	5	3	7
Rank Junk 2 (R_y)	2	3	5	1	6	4	7

Calculate Spearman's rank correlation coefficient between the marks awarded by the two judges.
Comment on your results. [5mks]

3. Solve the equation $3 \sin^2 \theta + \cos \theta + 1 = 0$ for values of θ from 0° to 180° inclusive.

[5mks]

4. A committee of 5 people is to be formed from a group of 6 men and 7 men.

- a. Find the number of possible committees. [2mks]
- b. What is the probability that there are only 2 women on the committee?

[3mks]

5. Find the gradient of the curve $y = 4x^2 (3x + 2)$ at the point (1, 20) [5mks]

6. Three events A, B and C are such that $P(A) = 0.6$, $P(B) = 0.8$, $P(B/A) = 0.45$ and $P(B \cap C) = 0.28$. Find;

- a. $P(A \cap B)$ [3mks]

- b. $P(C/B)$ [2mks]

7. The matrix $A = \begin{pmatrix} 2 & 1 \\ -3 & 0 \end{pmatrix}$ and I is a 2×2 identity matrix. Determine the matrix B such that $A^2 + \frac{1}{2} B = I$

[5mks]

8. A bullet of mass 50g is fired towards a stationary wooden block and enters the block when travelling horizontally with a speed of 500ms^{-1} . The wooden block provides a constant resistance of 36,000N. Find how far into the block the bullet will penetrate.

[5mks]

9. The table below shows the number of students and the marks scored in a test.

Marks	Number of students
0 – 4	10
5 – 9	7
10 – 14	5
15 – 19	3
20 – 24	7
25 – 29	11
30 – 34	37
35 – 39	20

- a. i) Draw a cumulative frequency curve (Ogive) for the data.

- ii) Use the Ogive to estimate the median mark.

[6mks]

- b. Calculate the:

- i) Mean mark

- ii) Standard deviation

[9mks]

10. A student attempts 20 objective questions by mere guess work. Each question has got four possible alternatives out of which only one is correct. Find the probability the he gets;

- Exactly 9 correct answers
- At least 12 correct answers
- At most 6 correct answers
- Between 6 and 14 correct answers inclusive
- At least 8 incorrect answers

11. The table below shows the prices (in Ug shs) of some food items in January, June and December together with the corresponding weights.

Item	Price (in Ug shs)			Weight
	January	June	December	
Matooke (1 bunch)	15,000	13,000	18,000	4
Meat (1 kg)	6,500	6,000	7,150	1
Posho (1kg)	2,000	1,800	1,600	3
Beans (1kg)	2,200	2,000	2,860	2

Taking January as the base month, calculate the:

- a. Simple aggregate price index for June. [5mks]
Comment on your result.

- b. Weighted aggregate price index for December
Comment on your result. [10mks]

12. The roots of the equation $2x^2 - 6x + 7 = 0$ are α and β . Determine the:

- a. Values of $(\alpha - \beta)^2$ and $\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2}$ [12mks]
b. Quadratic equation with integral coefficient whose roots are $(\alpha - \beta)^2$ and $\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2}$

[3mks]

13. A continuous random variable X has a probability density function given by;

$$f(x) = \begin{cases} \frac{kx}{6}, & 1 \leq x \leq 2 \\ 0, & \text{otherwise,} \end{cases}$$

Where k is a constant

- a. Find
i) The value of k [4mks]
ii) $P(X \geq 1.5)$ [4mks]
iii) The mean of X , $E(X)$ [3mks]
b. Sketch the graph of $f(x)$ [4mks]

14. A motorist moving at 90kmh^{-1} decelerates uniformly to a velocity $V \text{ ms}^{-1}$ in 10 seconds. He maintains this speed for 30 seconds and then decelerated uniformly to rest in 20 seconds.

- a. Sketch a velocity –time graph for the motion of the motorist. [6mks]
b. Given the total distance travelled is 800m, use your graph to calculate the value of V .

[5mks]

- c. Determine the two decelerations. [4mks]

SET 3

1. Evaluate: $\int_0^3 (2x^2 + 3x - 5)dx$. (05 marks)

2. Determine the determinant and therefore the inverse of the matrix
 $M = \begin{pmatrix} 2 & 6 \\ 1 & 4 \end{pmatrix}$ (05 marks)

- 3a) A box contains 3 red pens and 4 blue pens. A man picks 2 pens randomly one at a time without replacement.
i) Make a probability free diagram for the picking of 2 pens.

ii) Find probability the pens picked are of same colour. (05 marks)

- b) Study the following sequence arithmetic in nature.
 $2 + 7 + 13 + \dots + 47$.

Determine: i) Common difference
ii) number of terms if 47 is the last term. (05 marks)

4. Prove that: $\frac{\cot^2 x}{1 + \cot^2 x} = \cos^2 x$. (05 marks)

5. Find: $\frac{dy}{dx}$ if $y = x^2(x - x^3)$ by chain rule and form equation of tangent at A (2, -24). (05 marks)

6. Six students of Joy High School were graded in the following two subjects Mathematics and Economics as follows:

Maths	A	B	C	D	F	E
Economics	C	F	B	A	D	E

Calculate rank correlation coefficient and comment on the results. (05 marks)

7. Given that $\mathbf{a} = 3\mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{i} + \mathbf{j}$ and $\mathbf{c} = 5\mathbf{i} - \mathbf{j}$. Determine $\mathbf{a} + \mathbf{b} + \mathbf{c}$ and its magnitude. (05 marks)

8. A particle is projected vertically upwards with an initial speed of 40m/s, acceleration due to gravity is 10m/s^2 . Determine the maximum height reached and the time taken when it is in space. (05 marks)

- 9a) Given that α and β are roots of a quadratic function $6x^2 + 2x - 3 = 0$

Find: i) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ and $\alpha^3 + \beta^3$
ii) quadratic function whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. (10 marks)

- b) Given that the two roots of the equation $ax^2 + bx + c = 0$ differ by 3, show that $b^2 = 9a^2 + 4ac$. (05 marks)

- 10a) Given that A and B are acute angles such that $\cos A = \frac{4}{5}$ and $\tan B = \frac{5}{12}$

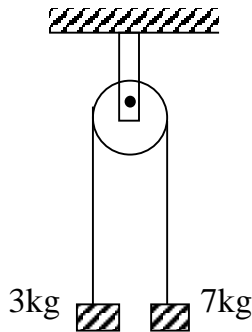
Find: i) $\sin(A + B)$
ii) $\tan(A - B)$

- b) Find x in the range $0 \leq x \leq 36$ if $\sec^2 x - 3\tan x + 1 = 0$. (15 marks)
(hint: $1 + \tan^2 x = \sec^2 x$)

11. The times taken by a group of students to solve a Physics problem are given below.

Time (Min) (x)	Number of students (f)
5 – 9	05
10 – 14	14
15 – 19	30
20 – 24	17
25 – 29	11
30 – 34	03

- How many students took part in the exercise.
 - Make a histogram on a graph paper and use it to estimate mode time for solving a problem.
 - Calculate the mean time and median using suitable formulars. (15 marks)
- 12a) A body of mass 3kg and 7kg are connected by a light non elastic string which passes over a smooth fixed pulley as shown.



The system is set free.

Determine:

- Common acceleration
 - tension in the string
(leave of in your answers)
- b) A body accelerated uniformly at 8m/s^2 from rest for 5 seconds. The velocity acquired was kept constant (maintained) for 20 seconds. It later slowed down and came to rest with further 10 seconds.
- Draw a velocity – time graph for the above motion.
 - Determine total distance covered.
 - Find the retardation in the last part of its Motion. (15 marks)
- 13 A random variable has a probability density function given by

$$f(y) = \begin{cases} K(4 - y), & 1 \leq y \leq 3 \\ 0, & \text{Otherwise} \end{cases}$$

- Find:
- the constant K
 - $P(y \leq 2)$
 - Sketch the function $f(y)$ on vertical and y (horizontal)
 - Determine the mean value of y, $E(y)$ (15 marks)

14. The table below shows the prices of some selected items consumed around Kampala City in 2010 and 2014 with weights attached.

Item	Beans	Maize	Clothing	Oil	Sugar
Price in 2010	1200	500	15000	4500	4000
Price in 2014	1800	4500	25000	5000	3200
Weights	2	3	1	4	5

Taking 2010 as the base year, calculate and comment on the:

- Simple price index for the items.
- Simple aggregate price index.
- Weighted price index for the items.
- Weighted aggregate price index for the items. (15 marks)

SET 4

- Three matrices P , Q and I are such that $P = \begin{pmatrix} a & a+1 \\ a-1 & a+2 \end{pmatrix}$ is singular and I is an identity matrix. Find the value of a and hence the matrix Q if $P + I = Q$. (05marks)
- Given that $A(1,2)$, $B(4,3)$ and $C(5, -1)$ are vertices of a triangle ABC , find angle ABC .
(05marks)
- If $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $4x^2 - 8x + 1 = 0$, find the equation whose roots are α and β .
(05marks)
- Two bags contain similar balls. Bag **A** contains 4 red and 3 white balls while bag **B** contains 3 red and 4 white balls. A bag is selected at random and a ball is drawn from it. Find the probability that a red ball is drawn.
(05marks)
- When a polynomial $g(x)$ is divided by $x^2 + 2x - 3$, the remainder is $2x - 2$. find the remainder when $g(x)$ is divided by;

$$\begin{array}{l} x - 1 \\ x + 3 \end{array}$$
(03marks)
(02marks)
- The table below shows the price per kg of three food crops.

Item	Price per kg (shs)		Weights
	2000	2010	
Beans	4000	5000	3
Millet	3000	4000	3
Maize	2500	3000	4

- Calculate the price index of each item for 2010 basing on 2000. (03marks)
 - Calculate the weighted price index for 2010. (02marks)
- The number of computers sold by **JA** Company in a period of 8 months is as shown below.

No. of computers	250	200	220	270	220	260	300	240
Month	Jan	Feb	Mar	April	May	Jun	Jul	Aug

Calculate the four point moving averages for the data. (05marks)

8. Three forces of magnitudes 5N, 12N and 10N on bearings of 060° , 210° and 330° respectively act on a particle. Find the resultant of the system of forces. (05marks)

9. The table below shows the cumulative frequency distribution of marks of 800 candidates who sat a national mathematics contest.

Mark(%)	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
F	30	80	180	330	480	610	700	760	790	800

- Calculate the mean and standard deviation (08marks)
- Construct an Ogive for the data and use it to estimate the;
 - Median mark (04marks)
 - Quartile deviation (02marks)
- Proportion of candidates that failed if the pass mark was 50% (01mark)

10. A quadratic curve has gradient function $(k - 2x)$ and is such that when $x = 1$, $y = 2$ and when $x = 1$, $y = 0$.

Candidates	A	B	C	D	E	F	G	H	I	J
Math (M)	35	56	65	78	49	82	20	90	77	35
Physics (P)	57	75	62	75	53	100	38	82	82	20

Find the value of k and state the equation of the curve. (07marks)

Sketch the curve. (05marks)

Find the area bounded by the curve and the x-axis. (03marks)

11. The table below gives marks obtained in mathematics examination (M) and physics Examination (P) obtained by 10 candidates.

- Draw a scatter diagram and comment. (07 marks)
- Find the score in mathematics by a candidate who scored 82 in physics. (02marks)
- Calculate the rank correlation coefficient and comment on your result. (06marks)

12. a) A and B are events such that $P(A) = \frac{1}{3}$, $P(A \text{ or } B \text{ but not both}) = \frac{5}{12}$ and $P(B) = \frac{1}{4}$. Calculate:

$P(A \cup B)$	(04marks)
$P(A' \cap B)$	(02marks)
$P(B'/A)$	(02marks)

- (a) Two men fire at a target. The probability that Allan hits the target is $\frac{1}{2}$ and the probability that Bob does not hit the target is $\frac{1}{3}$. Allan fires at the target first followed by Bob. Find the probability that:

Both hit the target (02marks)

Only one hits the target (03marks)

None of them hits the target. (02marks)

13. a) Given that $2\sin(A-B) = \sin(A+B)$

Show that $\tan A = 3\tan B$. (03marks)

Hence determine the possible values of A between -180° and 180° when $B=30^\circ$.

(03marks)

- (b) Solve the equation $\sin 2x - \cos 2x = 1$ for $0^\circ \leq x \leq 360^\circ$. (06marks)

- (c) Without using tables or calculators, show that $\cos 75^\circ = \frac{\sqrt{2}(\sqrt{3}-1)}{4}$.

(03marks)

14. a) Bodies of mass 6kg and 2kg are connected by a light inextensible string passing over a smooth fixed pulley with the masses hanging vertically. Find the acceleration of the system when released from rest. (05marks)

- (b) A body of mass 2kg moves along a smooth horizontal surface with speed of 2ms^{-1} . It then meets a rough horizontal surface whose co-efficient of friction is 0.2. Find the horizontal distance it travels on the rough surface before it comes to rest. (05marks)

- (b) A particle of mass 5kg rests on a smooth surface of a plane inclined at angle 30° to the horizontal. When a force X acting up the plane is applied to the particle, it rests in equilibrium. Find the normal reaction and force X . (05marks)

END