THE REPUBLIC OF UGANDA

Ministry of Education and Sports

HOME-STUDY LEARNING

SENIOR 4

MATHEMATICS

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This material has been developed as a home-study intervention for schools during the lockdown caused by the COVID-19 pandemic to support continuity of learning.

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FOREWORD

Following the outbreak of the COVID-19 pandemic, government of Uganda closed all schools and other educational institutions to minimize the spread of the coronavirus. This has affected more than 36,314 primary schools, 3129 secondary schools, 430,778 teachers and 12,777,390 learners.

The COVID-19 outbreak and subsequent closure of all has had drastically impacted on learning especially curriculum coverage, loss of interest in education and learner readiness in case schools open. This could result in massive rates of learner dropouts due to unwanted pregnancies and lack of school fees among others.

To mitigate the impact of the pandemic on the education system in Uganda, the Ministry of Education and Sports (MoES) constituted a Sector Response Taskforce (SRT) to strengthen the sector's preparedness and response measures. The SRT and National Curriculum Development Centre developed print home-study materials, radio and television scripts for some selected subjects for all learners from Pre-Primary to Advanced Level. The materials will enhance continued learning and learning for progression during this period of the lockdown, and will still be relevant when schools resume.

The materials focused on critical competences in all subjects in the curricula to enable the learners to achieve without the teachers' guidance. Therefore effort should be made for all learners to access and use these materials during the lockdown. Similarly, teachers are advised to get these materials in order to plan appropriately for further learning when schools resume, while parents/guardians need to ensure that their children access copies of these materials and use them appropriately.
I recognise the effort of National Curriculum Development Centre in responding to this emergency through appropriate guidance and the timely development of these home study materials. I recommend them for use by all learners during the lockdown.

Alex Kakooza
Permanent Secretary
Ministry of Education and Sports

ACKNOWLEDGEMENTS

National Curriculum Development Centre (NCDC) would like to express its appreciation to all those who worked tirelessly towards the production of home–study materials for Pre-Primary, Primary and Secondary Levels of Education during the COVID-19 lockdown in Uganda.

The Centre appreciates the contribution from all those who guided the development of these materials to make sure they are of quality; Development partners - SESIL, Save the Children and UNICEF; all the Panel members of the various subjects; sister institutions - UNEB and DES for their valuable contributions.

NCDC takes the responsibility for any shortcomings that might be identified in this publication and welcomes suggestions for improvement. The comments and suggestions may be communicated to NCDC through P.O. Box 7002 Kampala or email admin@ncdc.go.ug or by visiting our website at http://ncdc.go.ug/node/13.

Grace K. Baguma
Director,
National Curriculum Development Centre

ABOUT THIS BOOKLET

Dear learner, you are welcome to this home-study package. This content focuses on critical competences in the syllabus.

The content is organised into lesson units. Each unit has lesson activities, summary notes and assessment activities. Some lessons have projects that you need to carry out at home during this period. You are free to use other reference materials to get more information for specific topics.

Seek guidance from people at home who are knowledgeable to clarify in case of a challenge. The knowledge you can acquire from this content can be supplemented with other learning options that may be offered on radio, television, newspaper learning programmes. More learning materials can also be accessed by visiting our website at www.ncdc.go.ug or ncdc-go-ug.digital/. You can access the website using an internet enabled computer or mobile phone.

We encourage you to present your work to your class teacher when schools resume so that your teacher is able to know what you learned during the time you have been away from school. This will form part of your assessment. Your teacher will also assess the assignments you will have done and do corrections where you might not have done it right.

The content has been developed with full awareness of the home learning environment without direct supervision of the teacher. The methods, examples and activities used in the materials have been carefully selected to facilitate continuity of learning.

You are therefore in charge of your own learning. You need to give yourself favourable time for learning. This material can as well be used beyond the home-study situation. Keep it for reference anytime.

Develop your learning timetable to ca ter for continuity of learning and other responsibilities given to you at home.

Enjoy learning

CLASS: SENIOR FOUR

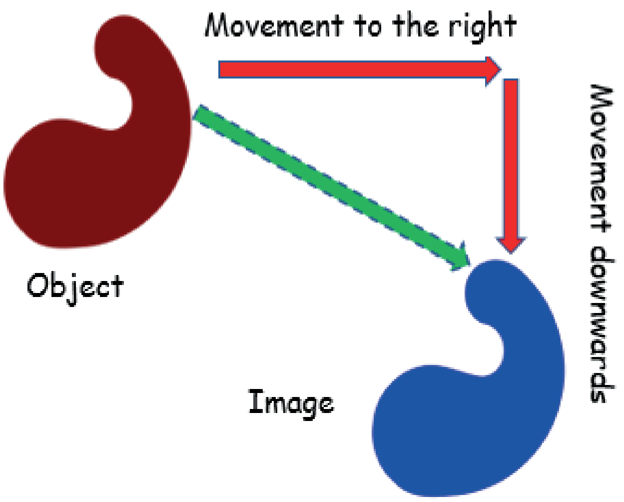
Dear Learner welcome to use this study material. As you prepare to start these activities, remember that you are studying from home due to the Covid-19 pandemic. It is therefore important that you keep safe by doing the following

1. Regularly wash your hands with soap and running water or use a sanitizer to sanitize your hands.
2. Always wear a face mask when you are in a crowded place and
3. keep a distance of 2 metres away from other people

Topic: MATRICES OF TRANSFORMATION

Lesson One: Determine and state matrices for the transformation: Translation

Materials Required:
Squared paper, pencil and a mathematical set
Knowledge you require: You need to revise topics on cartesian plane, vectors, matrices
Introduction: You will learn the meaning of matrices of transformation for reflection
In S.3 you learnt that a **Transformation** involves a change. Hence, a geometric transformation would mean to make some changes in any given geometric shape. The transformations you learnt include translation, reflection, rotation and enlargement,
Translationis the process of moving a shape.



From the diagram; $\frac{\boldsymbol{n}}{\boldsymbol{mo}}$ Translation= $\frac{\boldsymbol{itive})}{\boldsymbol{egative})}$

Learning Tips
Translations are described using vectors $\binom{x}{y}$,where the top value represents the movement parallel/along the $x-axis$(positive means right, negative means left), and the bottom value represents the movement parallel/along the $y-axis$(positive means up, negative means down).

Exercise

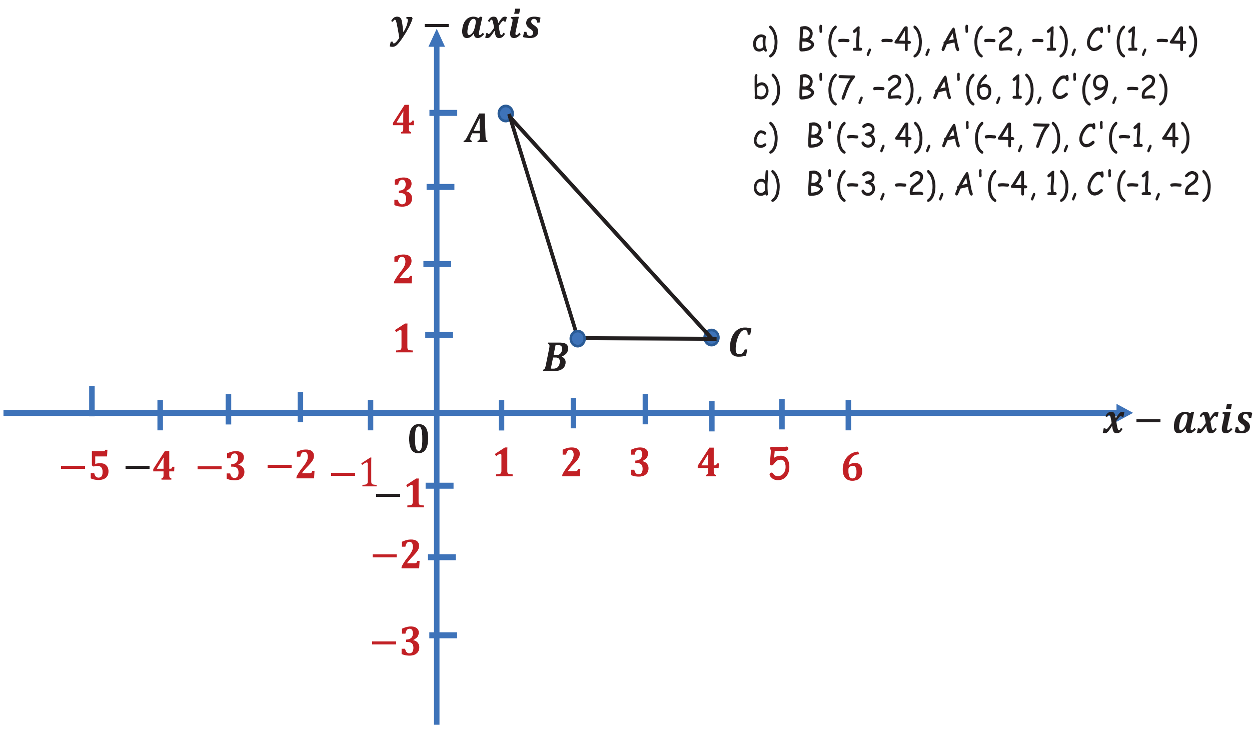
1. A transformation in which an object or geometric figure is picked up and moved to another location without any change in size or direction is

Your answer--

2. How do you represent a translation algebraically?

   a. (x, y) b. $\frac{x}{y}$ c. $\begin{pmatrix} x \\ y \end{pmatrix}$ d. $\frac{y}{x}$

3. The coordinates of point G are (–5, 7). What would be the coordinates of point G' after translating it $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$?
4. Translate triangleABC,5 units left and 3 units down.



   a) B'(-1, -4), A'(-2, -1), C'(1, -4)
   b) B'(7, -2), A'(6, 1), C'(9, -2)
   c) B'(-3, 4), A'(-4, 7), C'(-1, 4)
   d) B'(-3, -2), A'(-4, 1), C'(-1, -2)

5. Point T(8,4) is translated to $T^1(5, -1)$. What is the translation that was used to map point T to T^1?

   a. $\begin{pmatrix} -3 \\ -3 \end{pmatrix}$ b. $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$ c. $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$ d, $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$

Learning Tip

Given the object and Translation, you can find the image using the rule;

Object coordinate=Translation image coordinates

Lesson Two: Determine and state matrices for the transformation: Reflection

Here are examples of a reflection as a transformation illustrated in the diagrams below. Notice that the object has been **changed (transformed)**into an image.

Figure 1

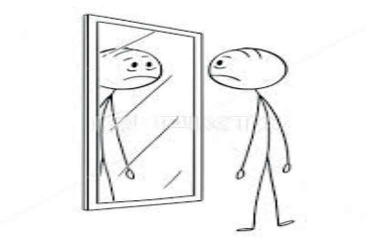


Figure 2

TRANSFORMATION USING MATRICES

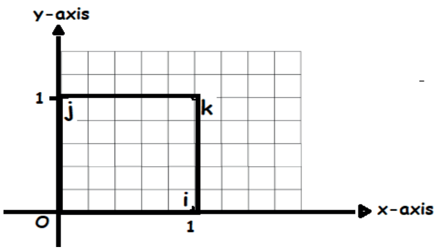
Matrix multiplication can be used to transform points in a plane.

Transformations can be represented by 2 X 2 matrices, and ordered pairs (coordinates) can be represented by 2 X 1 matrices.

Special Matrices

Transforming the unit square

The square with coordinates **O**(0, 0), **I** (1, 0), **J** (0, 1) and **K** (1, 1) is called the unit square. Unit square because the dimension of the square is 1 unit.



Recall, an identity;

Figure 3

- ✓ matrix is a square matrix, i.e. the number of rows is equal to the number of columns.
- ✓ The elements in the leading diagonal (in red) are all equal to 1 and every other element in the matrix is$\begin{pmatrix} \mathbf{1} & 0 \\ 0 & \mathbf{1} \end{pmatrix}$.

Therefore, in this topic you will use a 2x2 identity matrix

From the unit square only coordinates I (1,0) and J (0,1) are used to write out the identity matrix as$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Notice K (1,1) is not used because the coordinates do not lead to an identity matrix.

Explore

Transform I (1,0) and J (0,1)

using the transformation matrix$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Plot the image of the unit square on same graph

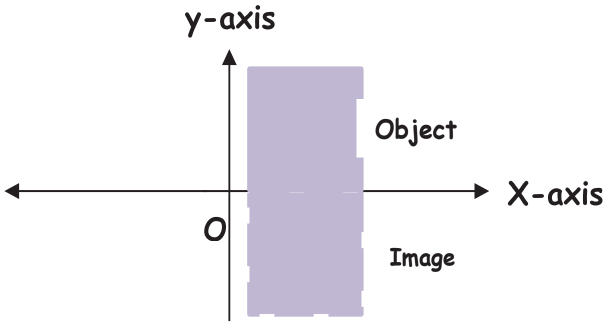


Figure 4

From the rule;

Transformation matrix × object=image

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Hence I^1=(1,0)

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

Hence J^1 = (0, -1)

Learning Tip: From the square grid it shows that transformation matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is of **Reflection** in the x-axis (y=0)

Activity

Fill in the table with the correct information

Object coordinate	**Matrix Transformation**	**Image coordinates**	**Describe the transformation**
I (1,0) J (0,1)	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	I^1 =(1,0) J^1 = (0, -1)	Reflection in the x-axis (y=0)
I (1,0) J (0,1)	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$		
I (1,0) J (0,1)	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$		Reflection in y=x (y-x=0)
I (1,0) J (0,1)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$		

Exercise

1. Which of the following is **NOT** a coordinate of the unit square?
   a. (1,1) b. (1,0) c. (0,1) d. (-1, -1)
2. Which of the following order of matrix shows a reflection?
   a. 2×2 (b) 1x2 (c) 2x1 (d) 2x3
3. The following is the correct rule of obtaining the image of a coordinate given the object coordinate and the transformation
   a. Object ×transformation=image c) Transformation x image=object
   b. Transformation ×object=image d) object× image=transformation
4. What is the image of the point (2,2) under the transformation $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

a. (-2,2) b. (0,2) c. (-2,0) d. (2, -2)

5. In each of the questions 5, use the object coordinates A (1,1), B (3,1), C (2,3) to obtain the respective image coordinates using the transformations given. Illustrate and relate the object and image on the same square grid. Describe the transformation.

   (i) Transformation $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} \end{pmatrix}$ **(iii)** Transformation $\begin{pmatrix} \mathbf{0} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{0} \end{pmatrix}$

   (ii) Transformation $\begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}$

6. In each of the following diagrams identify and describe the transformation used

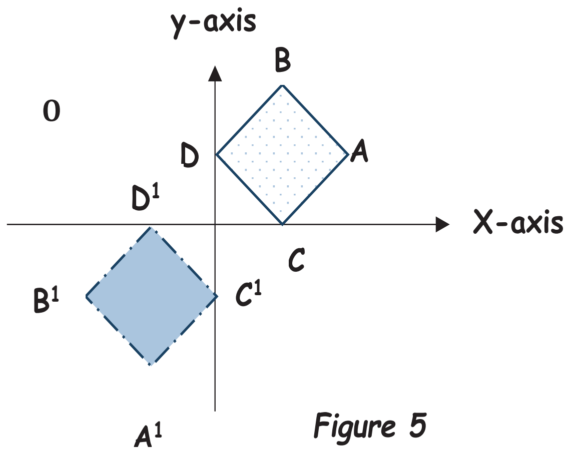


Figure 5

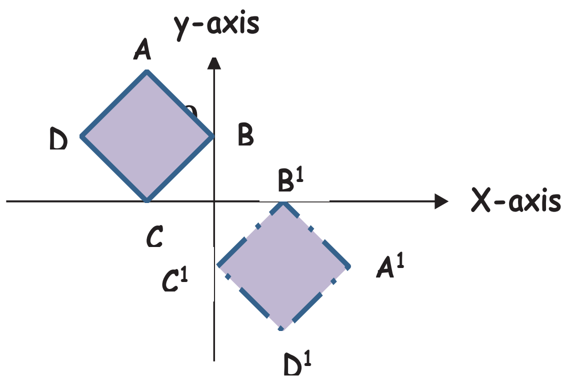


Figure 6

Your Answer--------------------------

Your Answer--------------------------

object when the image is given.

Recall: Given the matrixA= $\begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$, and the inverse A^{-1}= $\begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix}$then

$$A \times A^{-1} = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix}$$

$$A \times A^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

When $AA^{-1}=A^{-1}A=I$, then A^{-1} is said to be the *inverse* of A. Therefore, A is the object and A^{-1} is the image.

Explore

The inverse matrix ofmatrix B = $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ isB^{-1} = $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.Given that a triangle with A(1,1), B (2,1) and C (2,2) is transformed using a reflection in the x-axis. Find the coordinates of the image. Use the inverse of the transformation matrix to find the coordinated of the image

Using the matrix transformation

B = $\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$ Is the transformation matrix

T × A = A^1

$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}1\\-1\end{pmatrix} Hence A^1 = (1,-1)$

T ×B = B^1

$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix}=\begin{pmatrix}2\\-1\end{pmatrix} Hence B^1 = (2,-1)$

T ×C = C^1

$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}2\\2\end{pmatrix}=\begin{pmatrix}1\\-1\end{pmatrix} Hence C^1 = (2,-2)$

Using the Inverse of the matrix transformation

B^{-1} = $\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$ Is the transformation matrix

T ×A^1 = A

$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix} Hence A = (1,1)$

T ×B^1 = B

$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}2\\-1\end{pmatrix}=\begin{pmatrix}2\\1\end{pmatrix} Hence B = (2,1)$

T ×C^1 = C

$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}2\\-2\end{pmatrix}=\begin{pmatrix}2\\2\end{pmatrix} Hence C = (2,2)$

Learning Tip: In any dimension, a reflection has the property that if you do it twice in the same mirror line, you get back to where you started*(Refer to figure 4, 5 and 6)*. In the exploration you notice the inverse matrix transforms the image coordinates ($A^1B^1C^I$) back to the object coordinates (ABC). Hence, the product of a reflection is the identity.

Exercise

1. The inverse transformation for a reflection about y=x is

   Your Answer--
2. The inverse transformation for a reflection about the y-axis axis

   Your Answer--------------------------------- ------

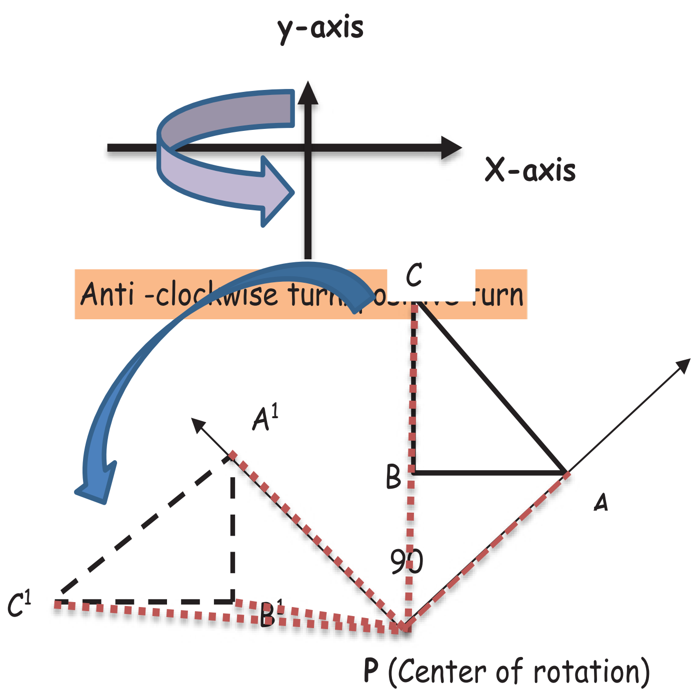
3. The vertices of triangle $A^1B^1C^1$ are $A^1(0, 3)$, $B^1(6, 3)$ and $C^1(6, 0)$. Find the coordinates of the object triangle ABC of the vertices under the inverse reflection matrix for the line $y = x$
4. The vertices of triangle $D^1E^1F^1$ are D^1 (-6,0), E^1 (-6, -3) and F^1(-2,0).Find the coordinates of the images of the vertices under theinverse reflectionthe line $y=-x$

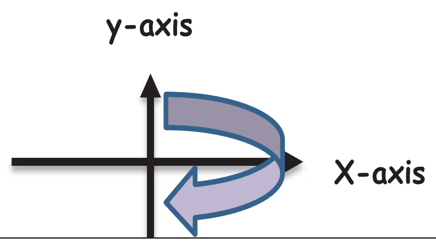
Further Activity

Look around your home/community and identify a situation where the property of reflection is used. Write about it.

Lesson Three: Determine and state matrices for the transformation: Rotation

A rotation is a transformation in a plane that turns every point of a preimage through a specified angle and direction about a fixed point. The direction is either anti clockwise or clockwise turn. The fixed point is called the centre of rotation.The amount of rotation is called the angle of rotation and it is measured in degrees.





Procedure to follow when rotation an object:

1. Join point A, B and C to the center of rotation
2. Use a protractor, draw a line 90° anticlockwise from the line *PA*. Mark on the line the point A^1 such that the line of $AP = PA^1$

Repeat steps 1 and 2 for point *B and C*. Join the points $A^1B^1C^1$ to form the image of triangle ABC.

Activity

From the unit square use coordinates I (1,0) and J (0,1) to find the images of I^1 and J^1geometrically. In each case;

i. write the coordinates of that image as a 2x2 matrix.
ii. Use the table below to fill in the correct 2x2 matrix.
iii. What do you notice about the 2x2 matrices you have obtained?

Angle of Rotation	Centre of Rotation	Matrix of transformation
Anticlockwise 90^0	**O** (0,0)	
Clockwise 90^0	**O** (0,0)	
Anticlockwise 180^0	**O** (0,0)	
Clockwise 180^0	**O** (0,0)	
Anticlockwise 270^0	**O** (0,0)	
Clockwise 270^0	**O** (0,0)	

Further Activity

Compare the results obtained in the table with those you would obtain if you used trigonometry where θ is the angle of rotation. The canter of rotation is at origin.

$$\begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$$

Exercise:

1. A turn is also called a __________.

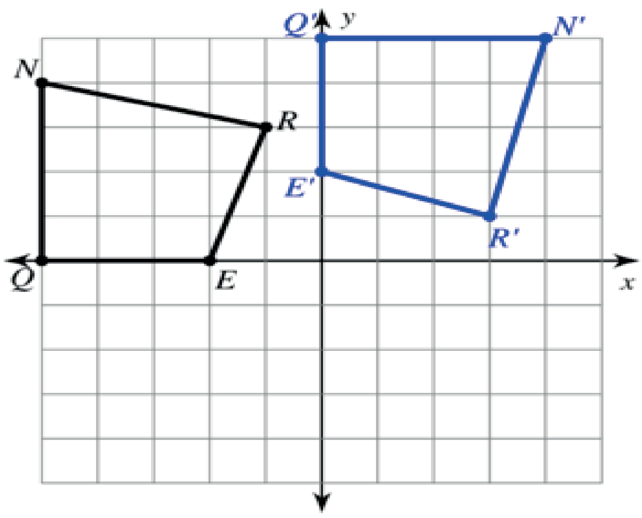
a. Translation

b. Reflection

c. Rotation

d. Transformation

2. How many degrees was the figure rotated about the origin (0,0)?



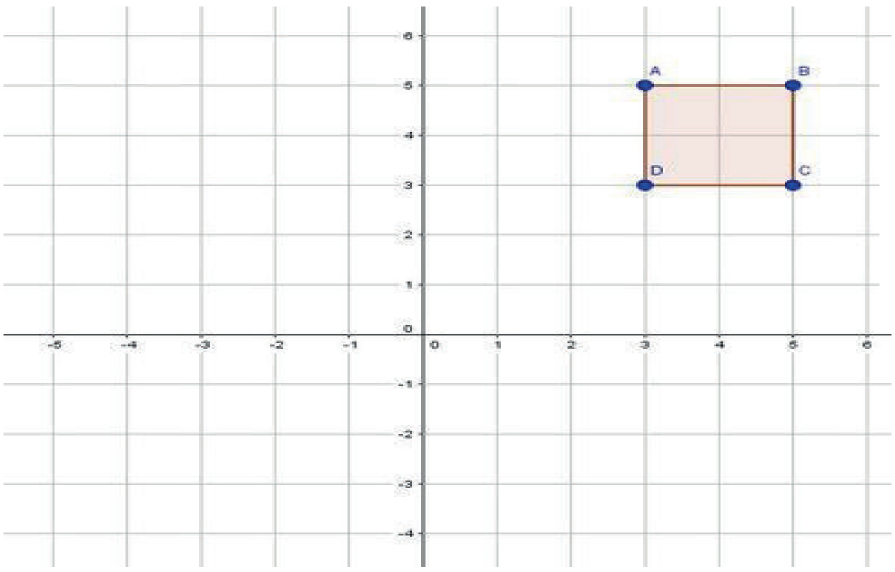
a. 90^0 clockwise

b. 90^0 anticlockwise

c. 180^0

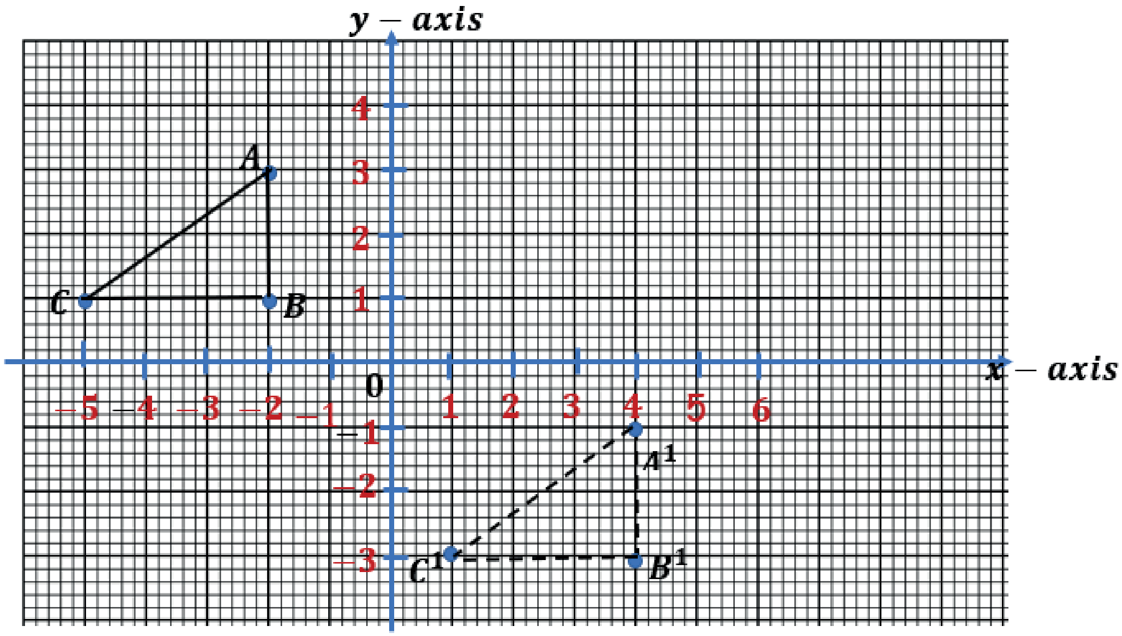
d. 270^0 anticlockwise

3. If you were to rotate ABCD 90 anti-clockwise about the origin, what would the coordinate of A' be?



a. (-3,5)
b. (-3,3)
c. (-5,3)
d. (-5,5)

4. Refer to the diagram below. The image A'B'C' is a rotation of ABC
   a. True
   b. False



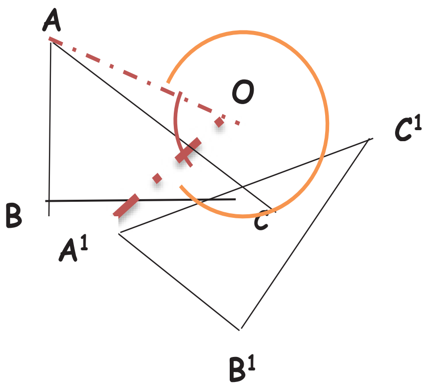
5. Triangle ABC with coordinates A (2,3), B(2, -2) and C (-1, -4) is rotated through 90° clockwise about the origin. What are the coordinates of $A^1B^1C^1$?

Lesson four: Determining Angle and Center of Rotation

Given an object, its image and the center of rotation, we can find the angle of rotation using the following steps.

Example

Find the angle of rotation. Given that $A^1B^1C^1$ is the image of ABC. *And O* is the center of rotation.



Procedure:

1. Choose any point in the given figure;e.g. A and join the chosen point to the center of rotation.
2. Find the image of the chosen point e.g. $A^1$and join it to the center of rotation.
3. Measure the angle between the two lines using a protractor.

Note: The sign of the angle depends on the direction of rotation.

- ✓ Anti-clockwise rotation is positive which is $A\grave{O}A^1$ indicated by the red arc or
- ✓ Cclockwise rotation is negative indicated by the green arc which is $A\overset{\frown}{0}A^1$.

Task: Check for the angle of rotation when you join B and B^1 to center of rotation and C to C^1. What do you notice?

Further Activity

Use a protractor and compass to obtain the image of the triangle whose coordinates are A (2,3), B (2, -2) and C (-1, -4) at a fixed-point O (0,0) through an angle of 90^0?

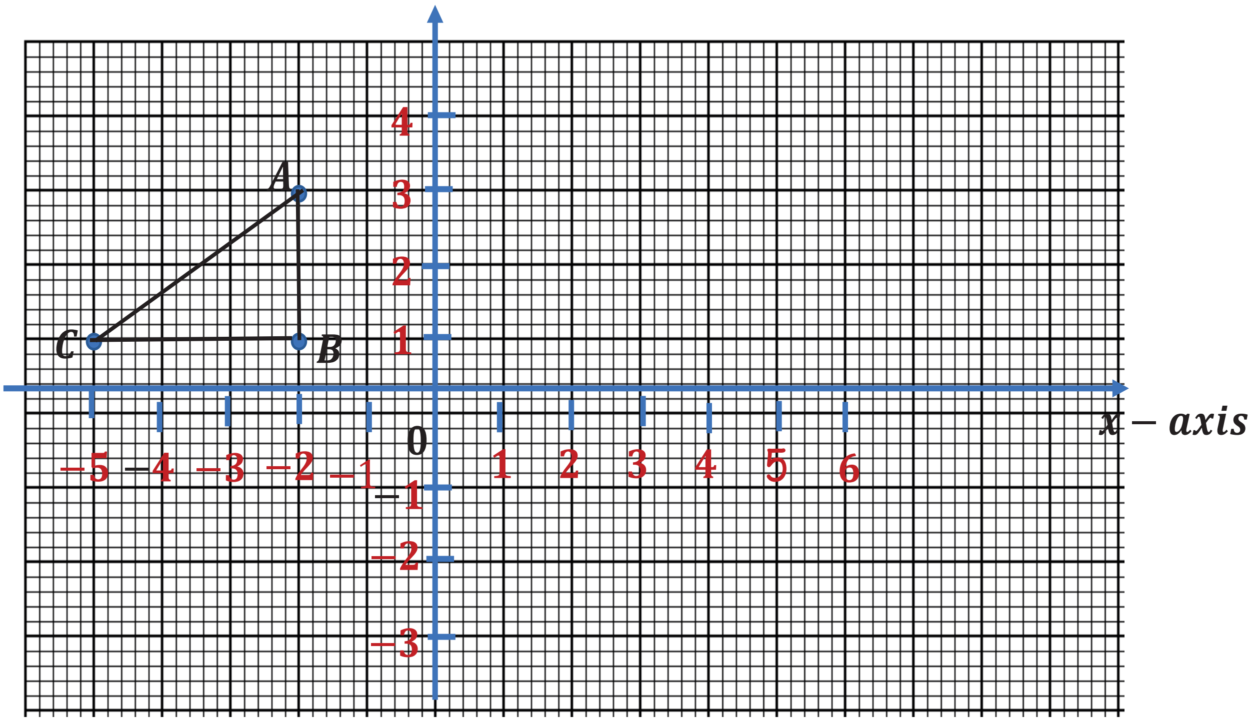
Exercise:

1. A point at (-4, 7) is rotated to point (7,4). What is the angle and center of rotation?

   a. Angle of rotation -90^0, center of rotation O (0,0)
   b. Angle of rotation 90^0, center of rotation O (0,0)
   c. Angle of rotation -270^0, center of rotation O (0,0)
   d. Angle of rotation -90^0, center of rotation (1,0)

2. The vertices of triangle $B^1C^1D^1$ are B^1 (-6,0), C^1 (-6, -3) and D^1 (-2,0). Find the coordinates of the images of the vertices under the inverse rotation of a 90^0 clockwise turn about the origin.

3. Rotate triangle ABC through a negative quarter turn about the point O. Label the new triangle $A^1B^1C^1$, reflect triangle $A^1B^1C^1$ through the x-axis to obtain a new triangle labeled $A^{11}B^{11}C^{11}$.



Learning Tip:

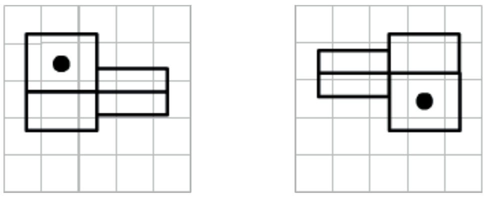
A rotation is a transformation: the original figure and the image are congruent/similar. The orientation of the image also stays the same. To perform a geometry rotation, we first need to know the point of rotation, the angle of rotation, and a direction (either clockwise or anti-clockwise.

The matrices of transformation for rotation can only be obtained if the center of rotation is at O (0,0)

Note. The center of rotation does not necessarily have to be at origin of the cartesian plane. It can be at any other point.

Revision Exercise

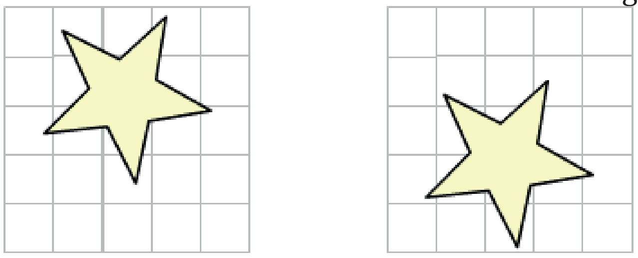
1. Which transformation is shown in the given image?



A B

a. Reflection
b. Translation
c. Rotation

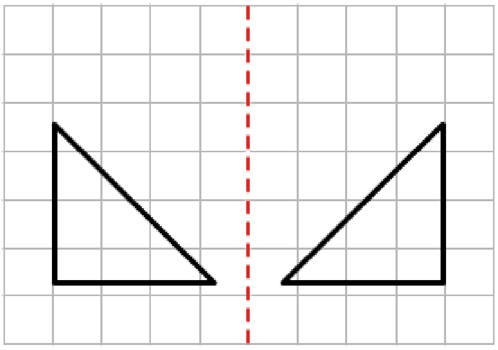
2. Which transformation is shown in the given image?



A B

a. Translation
b. Reflection
c. Rotation

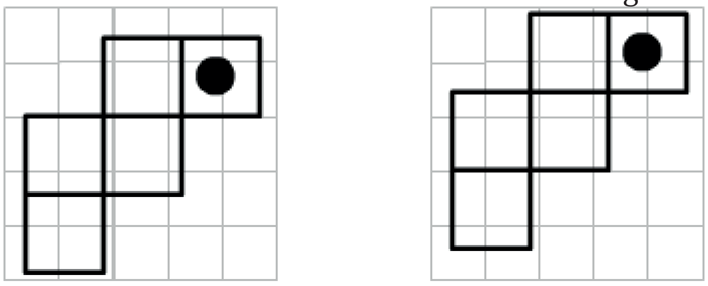
3. Which transformation is shown in the given image?



A B

a. Rotation
b. Translation
c. Reflection

4. Which transformation is shown in the given image?



A B

a. Rotation
b. Translation
c. Reflection

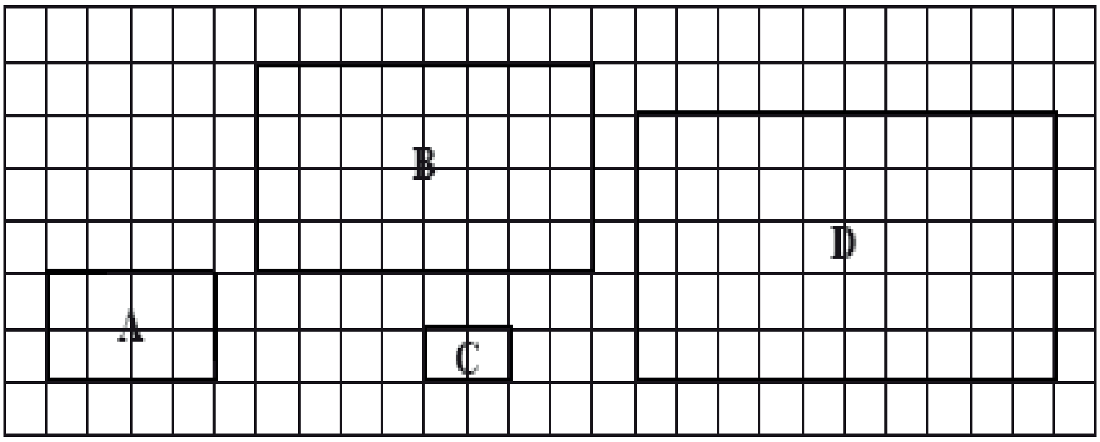
Lesson five: Determine and state matrices for the transformation: enlargement.
Activity
What can you say about the pictures?

Recall: Case 1: In the grid, rectangle A has been transformed into three other rectangles (B, C and D) of different sizes.

i. Compare the lengths of the rectangles

ii. Compare the width of all the rectangles

	observation		**observation**
Length of A Length of B		Width of A Width of B	
Length of A Length of C		Width of A Width of C	
Length of A Length of D		Width of A Width of D	



You will notice that when you compare the ratio of the corresponding sides of two figures, you obtain a constant ratio i.e.

$$\frac{Length of A}{Length of B} = \frac{4}{8} = \frac{1}{2} \ \mathbf{OR} \ \frac{Length of B}{Length of A} = \frac{8}{4} = 2,$$

$$\frac{Width of A}{Width of B} = \frac{2}{4} = \frac{1}{2} \ \mathbf{OR} \ \frac{Width of B}{Width of A} = \frac{4}{2} = 2$$

Learning Tip

The constant ratio obtained in comparing corresponding sides is referred to as a ***scale factor***. ***In this situation the scale factor is positive***

This mean that Rectangle B has been reduced by a factor $\frac{1}{2}$ to transform rectangle A, or rectangle A has been increased by a factor of 2 to transform rectangle B.Reducing and increasing the size of a figure is what is referred to as Enlargement.

Case 2: In the grid, rectangle A has been transformed into rectangleB.

iii. Compare the lengths of the rectangles

iv. Compare the width of all the rectangles

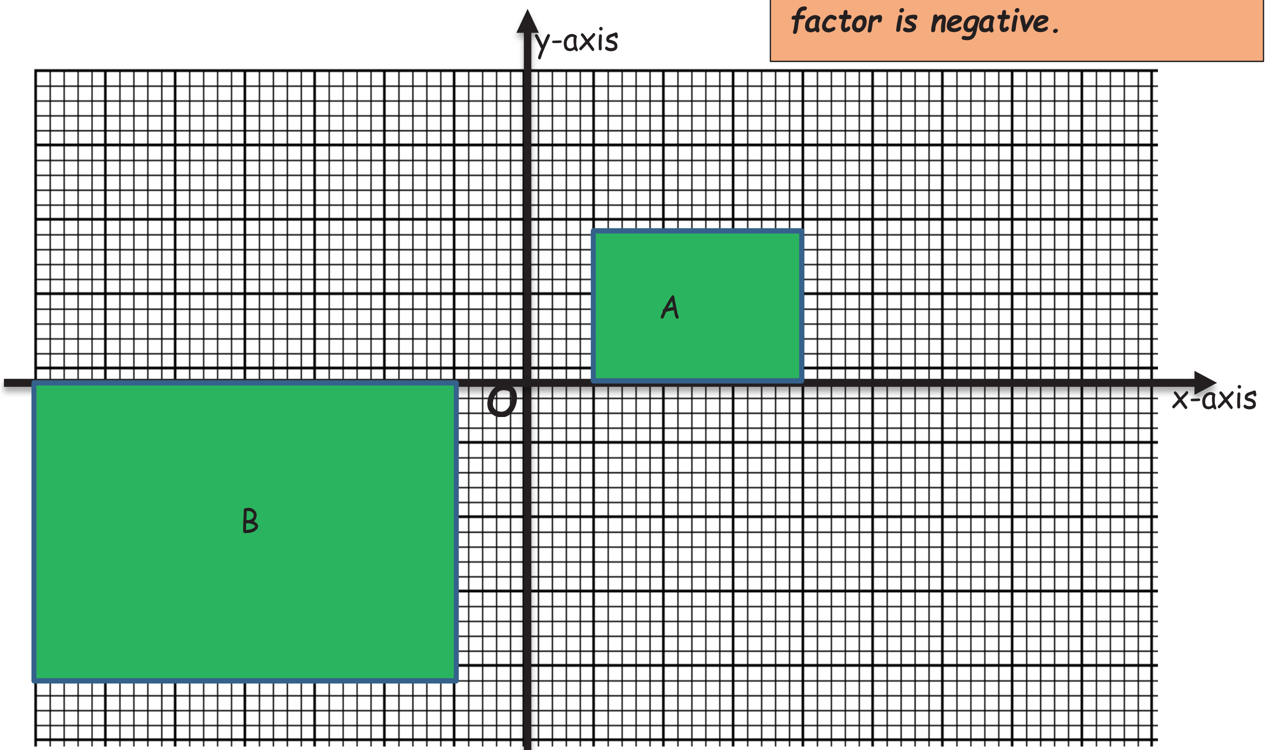
The ratio of the corresponding sides of two figures;

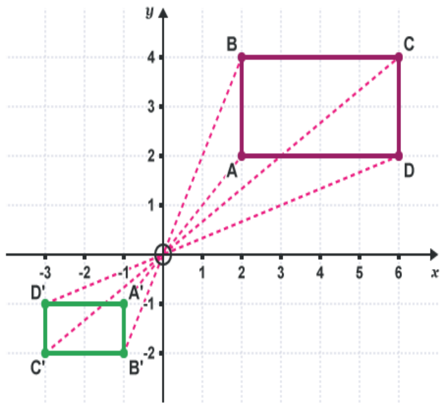
$\frac{Length of A}{Length of B} = -\frac{3}{6} = -\frac{1}{2}$ **OR** $\frac{Length of B}{Length of A} = -\frac{6}{3} = -2,$

$\frac{Width of A}{Width of B} = -\frac{2}{4} = -\frac{1}{2}$ **OR** $\frac{Width of B}{Width of A} = \frac{-4}{2} = -2$

Learning Tip

The constant ratio obtained in comparing corresponding sides is referred to as a ***scale factor. In this case the scale factor is negative.***





Learning Tip:

From the diagram, we describe an **enlargement** using a **negative scale factor** which causes the **enlargement** to appear on the other side of the **centre of enlargement O (0,0);**

The image will be inverted (upside down).

The shape will also change size depending on the value of the **enlargement**.

Scale factor=$\frac{image\ distance}{object\ distance}$

It is important to notice two things about the scale factor:

i. The scale factor from the object to the image is always the reciprocal of the scale factor from the image figure to the object
ii. If you begin with the smaller figure, your scale factor will be less than one. If you begin with the larger, your scale factor will be greater than one.

Lesson six: state matrices for the transformation: enlargement.

The general matrix for an enlargement is $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$.Where k is the scale factor for length.

Rule

Scale factor × identity matrix

K ×$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$., for only when the center of enlargement is at O (0.0)

e.g. $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$represents an enlargement, centre (0, 0) of scale factor 2.

Exercise

1. A cube has sides of length 4 cm. Its sides are increased by scale factor 2. What is the length of the sides of the new cube?
   a. 8cm b. 2cm c. 6cm

2. Plot the following points in order then join them up in order to make an irregular hexagon. Enlarge the hexagon using a scale factor of -2 about a center of enlargement of (-3, -4). Write down the coordinates of the image of the Hexagon.

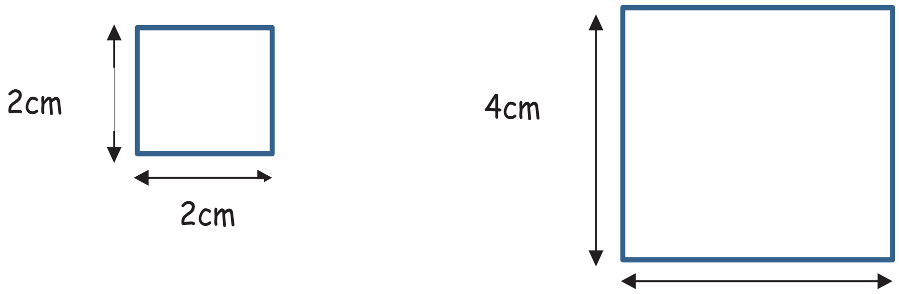
$$(-3, -1), (-2, -2), (-1, -2), (0, -1), (-1, -4), (-2, -4)$$

3. Enlarge the triangle A(0, -1) B (2, -2) and C (0, -4) from centre (0,0) by scale factor 0.5. Give the coordinates of the image point $A^1B^1C^1$

4. Enlarge the triangle A (0, -1) B (2, -2) and C (0, -4) from centre (0,0) by scale factor -3. Give the coordinates of the image point $A^1B^1C^1$

Lesson Seven: Identify the relationship between area scale factor and determinant of the transformation matrix.

Recall: What is the area of the squares? Relate the area of each squares to the length of each square.



Learning Tips:

The **lengths** of the larger square are 2 times longer than the smaller square.

The length scale factor is 2.

The **area** of the smaller square is 4 cm^2. The area of the larger square is 16 cm^2.

The area scale factor is 4. This is the length scale factor squared.

Generally, if the length scale factor is k, the area scale factor is k^2.

Using Matrices, given the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, the Scale factor is 2.

Area Scale factor= determinant of the matrix

=(2x2) -(0x)

=4

Exercise

1. A square has an area of 9 cm^2. Its sides are enlarged by scale factor 3. What is the length of the sides of the enlarged square?
2. A shape has an area of 3 cm2. Its lengths are enlarged by scale factor 5. What is the area of the new shape?
3. A rectangle ABCD has area 6 square units. It is transformed using the matrix $\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$.

   What is;

   a. The determinant of the matrix and the area scale factor of the transformation?

   b. the area of the image of rectangle ABCD

Lesson 8:Determine and identify a single matrix for successive transformations.

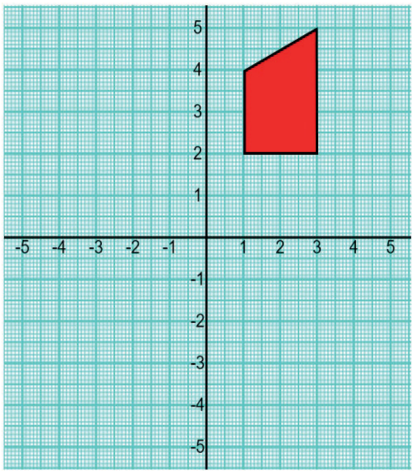
1. A triangle with vertices P (0, 2), Q (1, 4) and R (2, 2) is mapped on its image $P^1Q^1R^1$ by the matrix transformation $T_1=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Triangle $P^1Q^1R^1$ is then mapped onto $P^{11}Q^{11}R^{11}$ by another matrix transformation $T_2=\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$. Find the:

(i) Coordinates of $P^1Q^1R^1$ and $P^{11}Q^{11}R^{11}$

(ii) Matrix transformation that will map $P^{11}Q^{11}R^{11}$ back to PQR

(iii) Ratio of the area of triangle PQR to that of triangle $P^{11}Q^{11}R^{11}$

2. The diagram shows a red trapezium drawn on a grid.



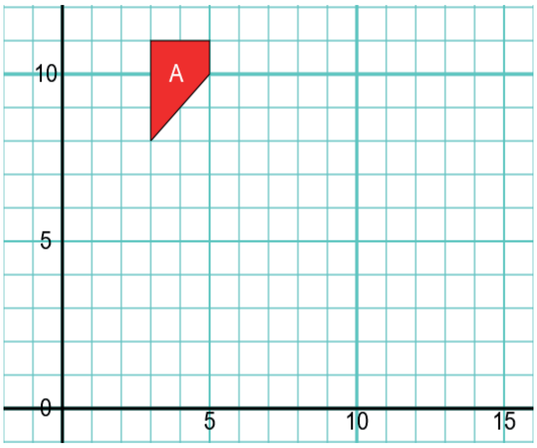
The trapezium is subjected to two transformations, one after the other.

One transformation is a reflection in the line y=x.

The other transformation is a reflection in the y-axis.

Does it matter in which order these transformations are made? Explain your answer.

3. The shape A is drawn on the coordinate grid as shown below.



Opio and Hasifa each transform the shape A onto shape B.

Opio uses a reflection in the line y = 7 followed by a rotation of 90° anticlockwise about the point (9,9).
Hasifatransforms shape A first with a reflection in the line y=xfollowed by his favourite transformation.

(a) Draw and label shape B.

(b) Describe fully Hasifa'sfavourite transformation.

Topic: LINES AND PLANES IN THREE DIMENSIONS

Lesson One: Apply Pythagoras theorem to calculate the distance between two points

Materials Required:

Note book, pencil, calculator and a mathematical set

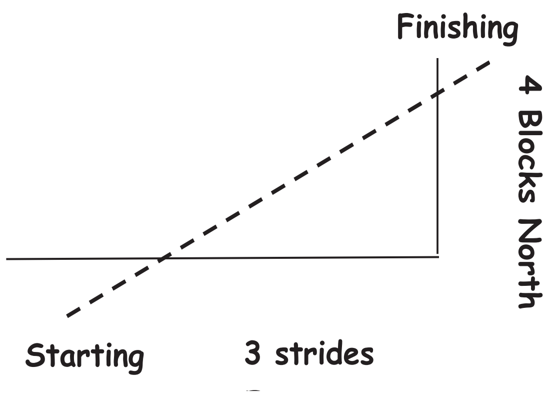
Knowledge you require: Stating Pythagoras theorem, trigonometry and basic mathematical operations

Introduction

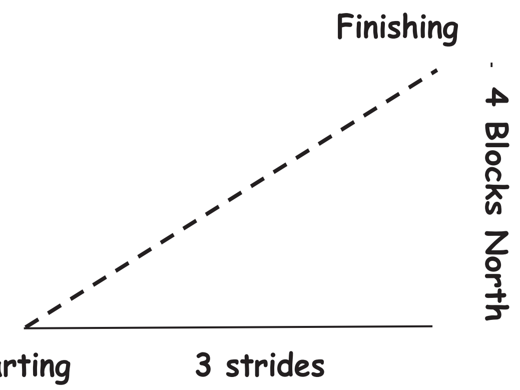
In S.2 you learnt about Geometry, Nets, Length and properties and Area properties of different solids.

Activity:

If I walk 3 strides East and 4 strides North, what would be my shortest from the starting point?



You may determine the shortest distance from the starting point to the finishing point by counting the strides.



Use Knowledge of Pythagoras Theorem;

Shortest Distance from starting point

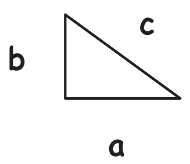
$=\sqrt{3^2+4^2}$

$=\sqrt{9+16}$

$=\sqrt{25}$

$= 5$ strides

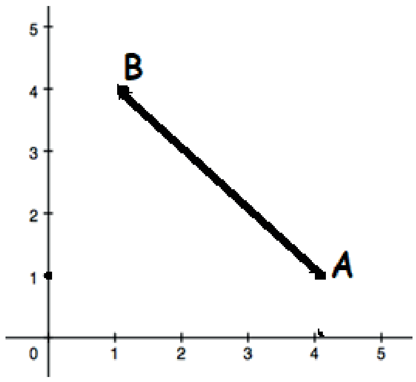
Learning Tip: Pythagoras theorem is used in a right -angled triangle as indicated in the diagram.



$$c^2 = a^2 + b^2$$

Exercise

1. Calculate the shortest distance between A and B by drawing a line connecting them and using the Pythagorean Theorem.



a. $\sqrt{18}$
b. $\sqrt{12}$
c. 12
d. None of the above

2. What is the distance between the points (-2, 5) and (4, -3)?

a. $2\sqrt{10}$ b. $2\sqrt{2}$ c. $2\sqrt{17}$ d. 10

3. Given A (-3, -2), B (x,3),C (4,5) and AB=BC, what is the value of coordinate x?

a. x=1 b. x=-1 c. $x = 7$ d. x=0

Lesson Two: Thsion Geometry

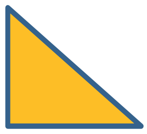
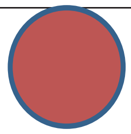
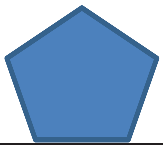
Introduction:

In this lesson, you will learn the meaning of three-dimensional objects. It is important to recall the meaning of two-dimensional objects. You need to use the objects within your own environment.

Imagine you are living in a two-dimensional plane, and in this world,there is no height. You could travel around measure distances and angles. You could move fast or slow, forward and backward or sideways. You could move in straight lines, circles, or anything so long as you never go up or down.

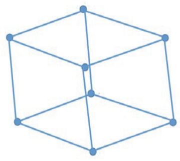
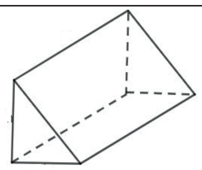
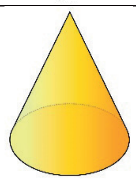
What would be your life like living in two dimensions plane? Well, for me it's impossible to imagine. And that is the reason why Three Dimension Geometry is important and necessary to learn their properties. In the real world, everything you see is in a three-dimensional shape, it has length, breadth, and height. Just simply look around your homestead and observe. Even a thin sheet of paper has some thickness if you look at it sideways.

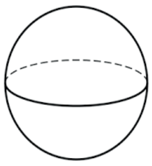
Activity: Fill in the missing information

Name of Shape	Picture of Shape	Characteristics
Square		Faces - 1 Edges - 4 Vertices - 4
Triangle		Faces - 1 Edges - 3 Vertices - 3
Circle		Faces ---------------- Edges - -------------- Vertices -------------
Pentagon		Faces ----------------- Edges ---------------- Vertices ----------------

> **Learning Tip:**
>
> Two-dimensional or 2-D shapes do not have any thickness.

Activity 2: Fill in the missing information

Name of Shape	Picture of Shape	Characteristics
Cube		Faces - 6 Edges - 12 Vertices –8
Triangular Prism		Faces - 5 Edges - Vertices -
Cone		Faces ---------------- Edges - -------------- Vertices -------------

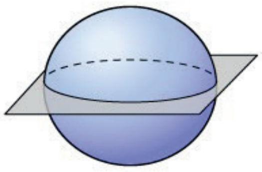
Sphere		Faces ---------------- Edges --------------- Vertices ---------------

Learning Tip:

In geometry, a three-dimensional shape can be defined as a solid figure or an object or shape that has three dimensions – length, width and height. Three-dimensional shapes havethickness or depth.

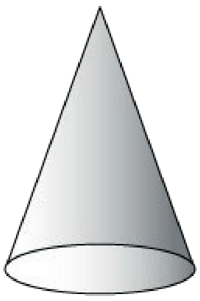
Exercise

1. How many faces or planes does a cube have?
   a. 4 b. 6 c. 8 d. 10
2. How many faces does a triangular pyramid have?
   a. 5 b. 4 c. 3 d. 3
   b. 4
3. How many edges does a rectangular prism have?
   a. 12 b. 8 c. 6
4. Which types of three-dimensional figures have no vertices?
   a. cylinders and spheres c. circles and cones
   b. cylinders and pyramids d. circles and cubes
5. Which of the words best describes the two-dimensional shape created by the cross-section shown on the sphere?



a. Oval
b. Circle.
c. Ellipse
d. sphere

6. What two-dimensional shape would be created by slicing this cone parallel to the base?



a. Square
b. Circle.
c. Rectangle
d. Triangle

Lesson Three: Identifying and working out the angle between a line and a plane

Learning Tip:

Look at the tree in the compound. The ground where the tree grows is **a plane**.

The tree has a shadow to the same ground (plane).

There is **a point of intersection** between the tree and its shadow.

The angle between a line and the plane is the angle between the tree and its shadow on the same ground.

The shadow line is what is referred to as the **PROJECTION** in that ground which is a plane.

Activity.

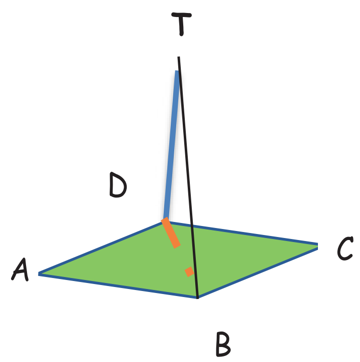
Materials required: Stick, pencil, pen, a piece of paper or anything that can act as a line and another surface that will act as a plane.

Task: Use stick or a pencil incline the stick or pencil at an angle to the flat surface.

i. What do you notice on the flat surface?
ii. Trace and draw out the shadow of the stick
iii. Identify and label the angle between the stick and the shadow.

Example

In the diagram, ABCD is a rectangular plane with length 4cm and width 3cm. A line TD6cm is inclined at an angle to the plane.



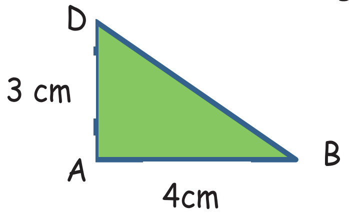
i. What is the projection of line TB to the plane ABCD?
ii. What is the angle between the line TB and plane ABCD?
iii. Calculate the length of line DB and TB
iv. Calculate the angle between the line TB and the plane ABCD.

Solution

i. The projection of line TB to the plane ABCD is DB or DB

ii. The angle between the line TB and plane ABCD is $T\dot{B}D$ or $D\dot{B}T$

iii. Calculate the length of line DB and TB



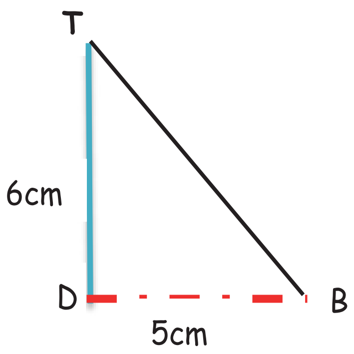
ABD **or** BCD IS Right angled triangle, therefore use Pythagoras theorem to find the dimension DB

$AB^2+AD^2=DB^2$

$4^2+3^2=DB^2$

$16+9=DB^2$ Taking square roots to both sides;

$\sqrt{25}=\sqrt{DB^2}$



TDB **or** BDT IS Right angled triangle, therefore use Pythagoras theorem to find the dimension TB

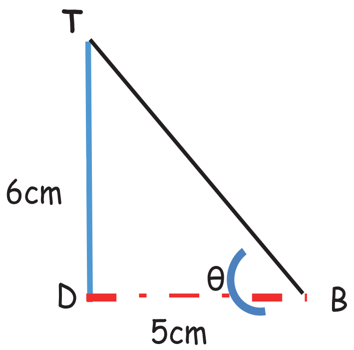
$BD^2+DT^2=TB^2$

$5^2+6^2=TB^2$

$25+36=TB^2$ Taking square roots to both sides;

$\sqrt{61}=\sqrt{TB^2}$

iv. Calculate the angle between the line TB and the plane ABCD.



To calculate the angle between TB and ABCD, you need to identify the projection of TB to the plane ABCD. The project is BD. Therefore, the angle is TBD labelled θ

Using Trigonometrical functions

Tan θ= $\frac{Opposite\ side}{adjacent\ side}$

Tan θ= $\frac{DT}{DB}$

Tan θ= $\frac{6}{5}$

Tan θ=1.2

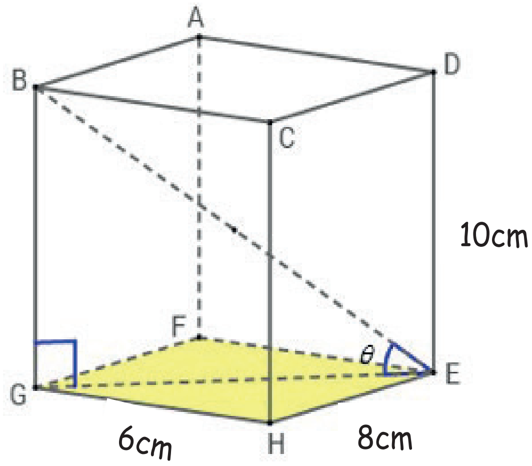
$\Theta=tan^{-1}1.2$

Learning Tip: The angle between a line and a plane is the angle between the line and its projection to the plane.

Exercise

1. A ________ is an endless flat surface.
   a. Point b. line c. plane
2. A vertex is
   a. a straight line.
   b. a squiggle line.
   c. a point where two lines meet to form an angle.

For questions 3-5 use the 3-dimensional shape of a cuboid ABCDEFGH.



3. The projection for BE to the plane EFGH is;

a. FE b. HE c. GE

4. The projection for AE to the plane EFGH is;

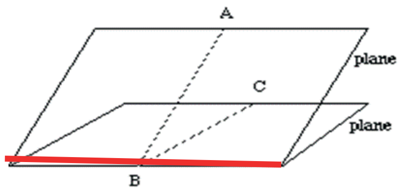
a. FE b. HE c. GE

5. Calculate the following
   i. The angle between the line BE and the plane EFGH
   ii. The angle between the line AE and the plane ADEF
   iii. The angle between the line AC and the plane ABGF
6. A room is in the shape of a cuboid. Its floor measures 7.2m by 9.6m and its height is 3.5m.Calculate the length of;
   i. AC
   ii. AG
   iii. Calculate the angle that AG makes with the floor.

Lesson Four: Identifying and working out the angle between two planes

Activity: Draw a picture of your house at home. Identify any two planes and the angle between the two planes.

Look at the two planes below



The two planes have a **Common Line** marked red

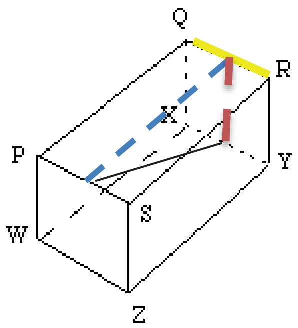
AB and BC meet at **common point** B on the **common line.**

Draw lines on each plane that meet on, and are perpendicular to, the common line in order to find the angle between the two planes

The angle between the two planes is $A\dot{B}C$ or $C\dot{B}A$

Example

The figure shows a cuboid.



What is the angle between plane QRYX and QRPS?

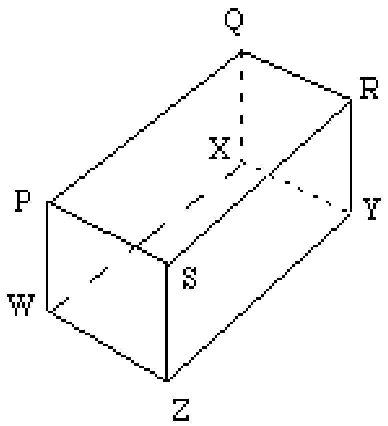
Solution

Common line between the planes QRYX and QRPS is the line marked yellow which is QR

The angle between plane QRYX and QRPS is 90^0

Exercise

1. The figure above is a cuboid.

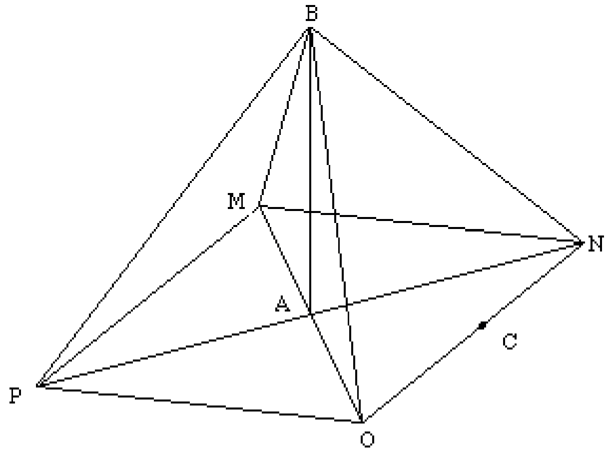


The angle between planes QWZR and QXYR is

a. angle ZQX
b. angle ZRX
c. angle ZQY
d. angle ZRY

2. The figure shows a pyramid on a horizontal square base MNOP.

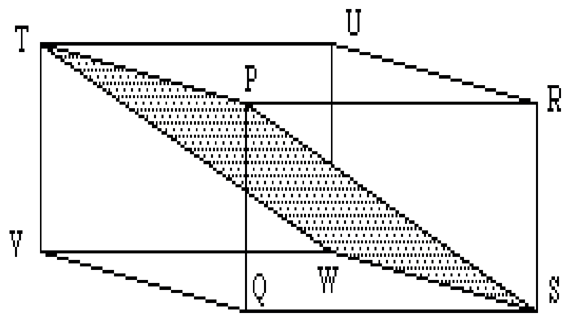
AB is vertical and C is the midpoint of NO.



Copy the figure.

i. Draw lines on the figure to show the angle between plane BNO and plane MNOP.
ii. Label this angle y
iii. If AB is 9 cm and BP is 11 cm, find the length of AP.
iv. If BOA is 67° and BO is 11 cm, find the length of OA.

3. The figure shows a rectangular box.



i. Name the angle between plane PTWS and QVWS?
ii. What is the intersection of plane PTUR and plane TUVW?
iii. What is the angle between plane PSWT and plane TUWV?

Learning Tip

Two planes intersect at a common line. To find the angle between two planes, draw lines on each plane that meet on the common line, and are perpendicular to, the line of intersection (common line).

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