LINEAR MOTION

1. A body moves along a straight line uniformly increasing its velocity from 2 mls to 18 mls in 10 s. Find the acceleration of the body during this time and the distance traveled.
2. A train starts from rest and accelerates uniformly, at 1.5 mls. Until it attains a speed of 30 mls. Find the distance traveled and the time taken
3. A train is uniformly retarded from 35 mls to 21 mls over a distance of 350 m. Calculate

(i) The retardation
(ii) The total time taken under this retardation to come to rest from a speed of 35 mls.

4. A car increases its speed from 18 km/h to 72 kmh in a distance of 50 m, the acceleration being uniform. Find
(i) The acceleration
(ii) The speed when the car has covered 25 m of the distance.

5. A train approaching a station does two successive half kilometers in 16 s and 20 s respectively. Assuming the retardation to be uniform, fins the further distance the train runs before coming to rest.
Solution

6. A,B and C are three points which lie in that order on a straight road with AB = 95 m and BC = 80 m. A car is traveling a long the road in the direction ABC with constant acceleration a ms^{-2}. The car passes through A with speed u mls, reaches B 5s later and C 25 after that. Find the values of U and a.

7. A particle traveling in a straight line with a constant acceleration covers distances P_1 and P_2 in the third and fourth seconds of its motion respectively.
Show that its initial speed is given by $U = ½ (7 P_1 – 5 P_2)$. Find also its initial acceleration.

8. A car moving on a straight road with uniform acceleration travels 11 m in the 5^{th} second of its motion and 23 m in the 11^{th} second.
Find its initial speed and acceleration
Solution:

9. A particle moves in a straight line with an initial velocity and a uniform acceleration. Find how far it travels in 125, given that it travels 48 m in the first 6s and 32 m in the last 25. Find also the initial velocity.

10. A body moves with uniform with uniform acceleration for 3s and describes 27m, it then moves with uniform velocity and during the next 5s describes 60 m.
Find its initial velocity and acceleration

11. A car A, traveling at a constant velocity of 25 mls, over takes a stationary car B. two seconds later car B sets off in pursuit, acceleration at a uniform 6 mls^{-2}
How far does B travel before catching up with A?

12. Two cars, A and B are traveling along parallel straight paths. The cars are observed to be side by side when A is at a point P of its path and again when at another point Q.
Assuming that A and B move with uniform accelerations α and β prove that, if their velocities at P are U_1 and U_2 respectively, the distance

PQ = $\dfrac{2(U_1 - U_2)(\beta U_1 - \alpha U_2)}{(\beta - \alpha)^2}$

13. Two cars A and B are traveling in the same direction along straight tracks, as they pass a fixed point P. their speeds are U ms^{-1} and V mls respectively, where U>U. Car A is moving with constant acceleration a mls^2 B with constant retardation a ms^{-2}. Show that A will overtake B after $\underline{V-U}$ seconds, and find their distance from P at this moment.

14. A point moving in a straight line is retarding uniformly and travels distances a and b in successive time intervals t_1 and t_2. Prove that the retardation is $\dfrac{2\left(a/t_1 - b/t_2\right)}{t_1 - t_2}$

15. Two cars A and B are initially at rest side by side. A sets off and moves with constant acceleration of 1/3 ms^{-2}, and 5 s later B sets off in the same direction moving with a constant acceleration of ¾ ms^{-2}. Find the time for which B has been in motion when it overtakes A, and show that in this time B travels 37 ½ m.

16. A car A, moving with uniform velocity QA along a straight road, passes a point X a car B, moving in the same direction with UB and uniform acceleration. b. B overtakes A at a point Y.

(i) Prove that the time taken to reach Y is $\dfrac{2(U_A - U_B)}{b}$

(ii) Find the distance XY

(iii) After passing X but before reaching Y, find the distance between the cars at time T and prove that the maximum distance between them is $\dfrac{(U_A - U_B)^2}{2b}$

17. P and Q are 2 points 5 metres apart on a straight road. A car X initially at rest leaves Q and travels away from P with a constant acceleration of f_1 ms^{-2}; at the same time as X leaves Q, another car y, passes P with a speed of U ms^{-2} and continues to move with a constant acceleration of f_2 ms^{-2} so as to catch up with X after t seconds.
Given that $f_2 < f_1$, and that U has the minimum possible value, determine the expression for U and t in terms of f_1, f_2 and s.

18. A car A moves with a constant acceleration f from rest up to its maximum speed U. just as it starts it is overtaken by a car B moving with constant speed V. given that A catches up with B in a distance a (> $\dfrac{u^2}{2f}$), show the A has been traveling with speed u for a time $\left(\dfrac{a}{v} - \dfrac{u}{f}\right)$. Show also that u^2v-2afu + 2afv = 0. hence find U in terms of a + v, explaining carefully how you decide which route to choose of the quadratic equation.

19. A particle travels in a straight linr with constant acceleration a. if its displacement after a time t is X and after a time 2 t is X +y, prove that a = $\dfrac{y-x}{t^2}$ and that the speed after time 2t is

$\dfrac{3y-x}{2t}$

Motion under gravity:

Show that

1. Time taken to reach the highest point , t = $\frac{u}{g}$

2. The greatest height reached S = $\frac{u^2}{2g}$

3. Time of flight T= $\frac{2u}{g}$

Examples;

1) An object falls from a height of 3m. Find the time taken and the velocity on reading the ground.
2) A stone is thrown vertically upwards with an initial speed 28 mls. Find the time taken to reach the greatest height that it attains above the point of projection, and find this height.
3) A stone is dropped from a tower and falls to the ground below. If the stone hits the ground with a speed of 14 ms^{-1}, find the height of the tower.
4) A stone is thrown vertically upwards with a speed of 7 ms^{-1} from the top of a cliff which is 70 m above sea level. Find the time at which the stone hits the sea.
5) A ball is projected vertically upwards with an initial velocity. Find in terms of u and g, the times when it is at a height of ${4u^2}/{9g}$
6) A particle is projected vertically upwards with a velocity of u ms^{-1} and after an interval of ts another particle is projected upwards from the same point and with the same initial velocity. Prove that they will meet at a height of $\frac{4u - g^2t^2}{8g}m$
7) A particle is projected vertically upwards, with a velocity of U ms^{-1} and after ts another particle is projected upwards from the same point with an initial velocity 2u ms^{-1}. Prove that the particles will meet after a lapse of $\frac{4ut - gt^2}{2(u + gt)}$

8) A stone is dropped out of a tall building at the same instant as a ball is thrown vertically upwards from the ground. If the stone and the ball have the same speed when they collide, prove that the ball has traveled three times as far as the stone.

9) A stone is released from rest and allowed to fall vertically. After Ts another stone is projected vertically downwards from the same point, with an initial velocity u. this stone overtakes the first after a further Ts

Show that U = $\left(\frac{3gT}{2}\right)$

10) A particle of mass m is projected vertically upwards with speed u and when it reaches its greatest height, a 2nd particle of mass 2m is projected vertically upwards with speed 2 u from the same point as the first. Prove that the projection time log between the second particle and its collision with the first particle is $\frac{u}{4g}$

11) A particle X is projected vertically upwards from the ground with a velocity of 80 mls. Calculate the maximum height reached by X. A particle Y is held at a height of 300 m above the ground. At the moment when X has dropped 80 m from its maximum height Y is projected down wards with a velocity of V mls. The particles reach the ground at the same time. Calculate the velocity of V.

12) A particle is projected from a point A with a velocity Ums^{-1} vertically upwards. Show that for it to pas through a point B at h metres above o, $\frac{u^2}{g} \geq 2h.$

13) A particle of is projected upwards from a point O with a speed $^4/_3$ V. After it has traveled a distance $^2/_5{}^x$ above O, on its upward motion, a 2nd particle is projected vertically upwards from the same point and with the same initial speed. Given that the particles collide at a height of $^2/_5{}^x$ above O, x and V being constant, show that (i) at maximum height, H, $8v^2 = 9gH$
(ii) when the particles collide 9x = 20H

14) A particle is dropped from rest from the top of a cliff H m high. When it has fallen a distance D m, a second particle is dropped from the point from the top of the cliff. Show that when the first particle hits the ground, the second particle is at a height $2\sqrt{HD} - D$ above the first particle.

15) A ball is thrown vertically upwards with a velocity of 20mls 3s later, a 2nd ball is dropped from the same point. If the balls meet 1m above the ground.

Determine;
i) The time taken by the first ball to meet the second one.
ii) The height of the point of projection above the ground.

Motion graphs.

1. A train is brought to rest from a velocity of 24mls by a constant acceleration of -0.8 mls^2. Drawing v-t graph and find the distance covered by the train while it is decelerating.

2. A particle moves from rest in a straight line with an acceleration of 4ms^{-2} for 3s. It maintains a uniform velocity for 6s and is then brought to rest again in a time of 4s with a uniform retardation. Draw a velocity – time graph and find the final acceleration and distance of the particle from its starting point.

3. Two stations A, B are 8.8 km apart. A train starts from rest at A and is uniformly accelerated until its speed is 64 km/hr. it travels for a certain time at this speed and is then uniformly retarded to rest at B; the magnitude of the retardation being twice that of the acceleration. The time taken is 10.5 mins. Sketch a velocity – time graph and hence find the number of minutes during which the speed is 64 km/hr.

4. A cyclist accelerates at a rate of a ms^{-2} for 10 s, then travels at constant speed for 20 s and then stops with a retardation of 2a ms^{-2}. Given that the total distance traveled is 550 m, find the value of a and the maximum speed.
5. A train starts from rest with constant acceleration a, until it reaches a speed w. it then travel at a constant speed w before it decelerates uniformly to rest, the retardation having magnitude f. the average speed for the whole Journey is 5/8 w.
    a) Show that the train travels at a constant speed for one quarter of the total time.
    b) Find what fraction of the journey is covered at constant speed.

6. A motorist starting a car from rest accelerates uniformly to speed for another 50s and then applies brakes and decelerates uniformly to rest. His deceleration is numerically equal to three times his previous acceleration.
    i) Sketch a velocity time graph
    ii) Deceleration takes place
    iii) Given that the total distance moved is 840 m, calculate the vulue of V
    iv) Calculate the initial acceleration

7. A car is traveling on a straight road with a constant acceleration a ms^{-2}. at a point A where the speed of the car is u ms^{-1}, the driver sees an

obstacle ahead. The car continues to accelerate for Ts and then moves with a constant retardation r ms^{-2} until it comes to rest at B. show that the distance d m between A and B is given by

$2rd = u^2 + 2 (a + r) UT + a (a+ r)T^2$

8. Two points P and Q are x metres apart in the same straight line. A particle starts from rest at P and moves directly towards Q with acceleration a ms^{-2} until it acquires a speed V ms^{-1}. It maintains this speed for a time T seconds and is then brought to rest at Q under retardation a ms^{-2}. prove that

   T = $\frac{x}{v} - \frac{v}{\alpha}$

9. A car moves from rest with constant acceleration 3f from w to x, then continues from x to y with acceleration f, from y to z with constant speed V. if the times taken for wx, XY and yz are each equal to T; find
   a) T in terms of V and f
   b) the ratio of the distances WX: XY:YZ
   c) the retardation if the car comes to rest in a further time, T
   d) the total distance traveled in terms of V and f

10. A lift descends with uniform acceleration a, followed with uniform retardation 2a. if it begins and ends at rest, descending a distance S, in time t, prove that S = $\frac{1}{3}at^2$

11. A train takes a time T to perform a journey from rest to rest. It accelerates uniformly from rest for a time PT and retards uniformly to rest at the end of the journey for a time QT; during the intermediate time it travels uniformly with speed V. prove that the average speed for the journey is ½ V(2 – p – q)

PROJECTILE MOTION

- A projectile is a particle given an initial velocity at an angle to the horizontal and moves entirely under the action of its own weight. (Air resistance negligible)
- Vertical acceleration = g (acceleration due to gravity)
- Horizontal acceleration = 0 (constant velocity
- The usual equations of motion i.e.
- $V = u + at$, $S = ut + ½ at^2$ and $V^2 = u^2 + 2as$ apply.

PROJECTION FROM A HORIZONTAL PLANE

Consider a body which is projected with speed u from a point o at an angle θ to the horizontal.

- U = velocity /speed of projection
- θ= angle of projection /elevation
- R = Horizontal range
- H = maximum height
- Path OAB = TRAJECTORY-path a projectile follows.

Components of U

$U_x = U \cos \theta$ = Horizontal component (constant)

$U_y = U \sin \theta$ = vertical component, (reduced by g)

Equations

a) **Greatest (minimum) Height (H)**
b) Time to reach maximum Height
c) **Time of flight (T)**
d) **Horizontal Range (R)**
e) For maximum Range (R max)
f) **Direction of motion and velocity after a given time t**

Worked Examples

1. A projectile is fired with a velocity of 320m/s at an angle of 30^0 to the horizontal. find
   I. The time to reach its greatest height
   II. The maximum height reached
   III. Its horizontal range
   IV. With the same velocity , what is the max possible range
   V. The velocity and direction of motion after 10s
2. The maximum range of a gun is 150m. What is the muzzle velocity and what is the greatest height reached by the shot?
3. A particle is projected with speed V at an angle α to the horizontal from a point X on a horizontal plane. It hits the plane again at a point A. Find the range XA. If the range XA = the greatest height reached by the particle, show that $\tan \alpha = 4$. hence determine V given that the greatest height

$= \left(\frac{1000}{34}\right)m$ (g = 10 m s^{-2})

4. A man throws a ball at an initial speed of 50ms^{-1} , show, by taking g=10ms^{-2}, that the times of flight corresponding to a horizontal range of 100m are the positive roots of the equation $T^4 - 100T^2 + 400 = 0$
5. A canon has a maximum range R. Prove that the height reached in such a case is $\frac{1}{4}R$ and that the time taken to reach the maximum height is $\left(\frac{R}{2g}\right)^{\frac{1}{2}}$.
6. A stone thrown at an angle θ to the horizontal takes T seconds in its flight and makes R m as the horizontal range. Show that $2R\tan\theta = gT^2$.
7. A particle is projected from level ground towards a vertical pole, 4m high and 30m away from the point of projection. It just passes the pole in one second. Find
   (i). its initial speed and angle
   (ii) The distance beyond the pole where the particle will fall.

<u>EQUATION OF TRAJECTORY</u>

1. A ball is thrown with an initial velocity of 30 m/s at 30^o to the horizontal. It just clears a wall, the foot of which is 25m from the point projection. Find the height of the walls.
2. A particle is projected at 20^0 to the horizontal and just clears a wall which is 10m high and 30m from the point of projection, find the initial speed of the particle.
3. A squash player strikes a ball giving it a speed of 20m/s , towards a vertical wall, which is at a horizontal distance of 10m . The ball hits the wall at a point which is 5m above the point of projection. Find the 2 possible angles of projection
4. A particle is fired with initial velocity $\sqrt{2ga}$ to hit a target at a point $\left(a, \frac{a}{2}\right)$ Find the possible angles of projection and the ratio of the times of the times of flight along the 2 possible paths.
5. An object is thrown upwards at an angle α to the horizontal with a speed of U ms^{-1} and just clears a vertical wall 4m high and 10m from the point of projection when travelling horizontally. Find the values of α and U.
6. A mortar fires shells at different angles of projection from point O. If the speed of Projection is $\sqrt{50g}$, where g is the acceleration due to gravity , and the shell passes through point B(10,20).

(i) Find the possible angles of projection

(ii) Deduce that the difference between the corresponding times taken to travel from O to B is $\frac{10 - 2\sqrt{5}}{\sqrt{g}}$

PROJECTION FROM A CLIFF

1. A ball is thrown forward horizontally from the top of a cliff with a velocity of 10m/s. the height of the cliff above the ground is 45m . Calculate
   (i) The time to reach the ground
   (ii) The distance from the cliff of the ball on hitting the ground
   (iii) The velocity and direction of motion of the ball to the horizontal just before it hits the ground.
2. A particle is thrown from the edge of a vertical cliff with a velocity of 50m/s at an angle $\tan^{-1}(^{7}/_{24})$ above the horizontal. The particle strikes the sea at a point 240m from the foot of the cliff. Find the time for which the particle is in air and the height of the cliff. Find also the velocity and direction of motion as it strikes the sea
3. A stone is thrown up at an angle of $30^{0}$to the horizontal with a speed of $30ms^{-1}$ from the edge of the cliff 50m above sea level. If the stone lands in the sea , calculate
   (a). how long it is in the air
   (b). how far from the base of the cliff it lands
   (c). The speed and direction of the stone as it hits the water

4. A stone is thrown horizontally with a speed u from the edge of a vertical cliff of height h. The stone lands on the ground at a point which is a distance d horizontally from the base of the cliff. Show that $2hu^2 = gd^2$

A particle is projected with a speed of $10\sqrt{2g}$ $ms^{-1}$from the top of a cliff 50m high.

The particle hits the sea at a distance 100m from the vertical below the point of projection. Show that there two possible angles of projection which are perpendicular. Find the time taken from the point of projection in each case