

# KIIRA COLLEGE BUTIKI

## *Uganda Advanced Certificate of Education*

### PURE MATHEMATICS

#### Paper 1

#### LOCK DOWN REVISION QUESTIONS

#### SECTION A ( 40 marks)

1. Solve the inequality

$$\frac{x(x+2)}{x-3} \leq x + 1 \quad (5 \text{ marks})$$

2. Show that the line  $\frac{x-2}{2} = \frac{y-2}{-1} = \frac{z-3}{3}$

Is parallel to the plane  $4x - y - 3z = 4$  and find the perpendicular distance of the line from the plane. (5 marks)

3. Solve the equation

$$2\tan x - 3\cot x = 1$$

For  $0^\circ \leq x \leq 360^\circ$  (5 marks)

4. Calculate the co-ordinates of the point of the intersection of the curve

$$\frac{x}{y} + \frac{6y}{x} = 5 \text{ and } 2y = x - 2 \quad (5 \text{ marks})$$

5. The tangent to the curve  $y = 2x^2 + ax + b$  at the point  $(-2, 11)$  is perpendicular to the line  $2y = x + 7$ . Find the value of  $a$  and  $b$ . (5 marks)

6. Evaluate  $\int_0^{\frac{\pi}{3}} \cos 3x \cos 2x dx$  (5 marks)

7. Given that  $\varphi$  is a root of the equation  $x^2 - 2x + 3 = 0$  show that  $\varphi^3 = x - 6$  (5 marks)

8. A spherical balloon is being inflated by gas being pumped at the constant rate of  $200\text{cm}^3$  per second. What is the rate of increase of the surface area of the balloon when its radius is  $100\text{cm}$ ? (5 marks)
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### SECTION B (60 MARKS)

9. (a) If  $(x + 1)^2$  is factor of  $2x^4 + 7x^3 + 6x^2 + Ax + b$ , find the value of A and B. (5 marks)
- (b) Prove that, if the equations  $x^2 + ax + b = 0$  and  $cx^2 + 2ax - 3b = 0$  have a common root and neither a and b is zero, then
- $$b = \frac{5a^2(c-2)}{(c+3)^2} \quad (7 \text{ marks})$$
10. (a) Given that  $y = \log_e\left(\frac{3+4\cos x}{4+\cos x}\right)$  find  $\frac{dy}{dx}$  in the simplest form. (7 marks)
- (b) If  $y = e^{4x}\cos 3x$ , prove that  $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 25y = 0$  (7 marks)
11. (a) Given that  $z = \cos\theta_{\sin\theta}$ , where  $\theta \neq \pi$ , show that  $\frac{2}{1+z} = 1 - i\tan\frac{1}{2}\theta$ . (6 marks)
- (b) The polynomial  $p(z) = z^4 - 3z^3 + 7z^2 + 21z - 26$  has  $2 + 3i$  as one of the roots. Find the other three roots of the equation  $p(z) = 0$  (6 marks)
12. (a) A right circular cone with semi vertical angle  $\theta$  is inscribed in a sphere of radius  $\gamma$ , with its vertex and rim of its base on the surface of the sphere. Prove that its volume is  $\frac{8}{3}\pi r^3 \cos^4\theta \sin^2\theta$ . (6 marks)
- (b) If r is constant and  $\theta$  varies, show that the limits within which this volume must lie is  $0 < v < \frac{32\pi r^3}{81}1$  (6 marks)
13. (a) In any triangle ABC, prove that  $\tan\frac{1}{2}(B - C) = \left(\frac{b-c}{b+c}\right) \tan\frac{1}{2}(B + C)$  (6 marks)
- (b) In a particular triangle the angle A is  $51^\circ$  and  $b=3c$ . Find the angle B to the nearest degree. The area of this triangle is  $0.47\text{m}^2$ . Find side a to three decimal places.
14. (a) The points A and B have position vector  $i-2j+k$  and  $2ijk$  respectively. Given that  $OC = \lambda OA + \mu OB$  and  $OC$  is perpendicular to  $OA$ , find the Ratio of  $\lambda$  to  $\mu$ .

Write down the vector equation of the line, L through A which is perpendicular to OA. Find the position vector of P, the point of intersection of L and OB. (12 marks)

15. (a) Determine the equation of the normal to the ellipse  $x \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  at a point  $p(a \cos \theta, b \sin \theta)$ . (6 marks)
- (b) If the normal at p meets the x – axis at A and the y – axis at B, Find the locus of the midpoint of AB. (6 marks)
16. (a) Solve the differential equation  $x \frac{dy}{dx} = y + x^2(\cos x + \sin x)$ , given that  $y = 0$  when  $x = \frac{\pi}{2}$  (5 marks)
- (b) The rate of decay of a radioactive substance is proportional to the amount A remaining at any time t. If initially the amount was  $A_0$  and if the time taken for the amount of substance to become  $\frac{1}{2} A_0$  is T, find A at that time.

Find the time taken for the amount remaining to be reduced to  $\frac{1}{20} A_0$  (7 marks)