P425/1

PURE MATHEMATICS

KING'S COLLEGE – BUDDO

INTERNAL MOCK EXAMINATION 2020

Uganda Advanced Certificate of Education

PURE MATHEMATICS

P425/1

3 Hours

INSTRUCTIONS TO CANDIDATES

Answer **all** the eight questions in section **A** and any **five** from section **B**

Any addition question(s) answered will **not** be marked

All necessary working **must** be clearly shown

Begin each answer on a fresh sheet of paper

Silent, non – programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS).

Attempt all the questions in this section

1. If α^2 and β^2 are the roots of $x^2 - 21x + 4 = 0$ and that α and β are both positive, find an equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ (5marks)

2. A cylinder has radius r and height h. The sum of r and h is 2. Find the maximum volume of the cylinder in terms of π (5marks)

3. Evaluate $\int_{In2}^{In5} \frac{2}{e^x - e^{-x}} dx$ (5marks)

4. The first term of a geometric progression is 18 and the sum to infinity is 20.Find the common ratio and the sum of the first 6 terms. (5marks)

5. Find the area bounded by the curve $y = 5x - x^2$ and the line $y = x$ (5marks).

6. Determine the angle between the line $r = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and the plane $2x - y + z = 4$

7. Prove that $(\cos ecA - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$ (5marks)

8. The line $y = 3x - 4$ is a tangent to the circle whose centre is the point $(5,2)$.find the radius of the circle (5marks)

SECTION B (60marks)

Attempt any 5 questions from this section. All questions carry equal marks.

9. Given the lines l_1 and l_2 are $l_1 : r = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ and $l_2 ; \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}$ intersect

a). Find the point of intersection of the lines (6marks)

b). Find a vector equation of the plane containing the line in (a) above (6marks)

10a).Solve the equation $2\cos\theta\cos 2\theta + \sin 2\theta = 2(\cos^3\theta - \cos\theta)$ for $0^0 \leq \theta \leq 360^0$ (7marks)

b). Prove that $\tan^{-1} x + \tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{\pi}{4}$ (5marks)

11a). The complex number z satisfies the equation $2z\overset{*}{z} - 4z = 3 - 6i$,where $\overset{*}{z}$ is a complex Conjugate of z. Find the possible values of z in the form $x + iy$ (6marks)

b) Use Demoivres theorem to find the four roots of the equation $z^4 - \sqrt{3} + i = 0$

(6marks)

12a). Given that $\log_9 xy = 6$, prove that $\log_3 x + \log_3 y = 12$. Hence solve the simultaneous equations $\log_9 xy = 6$ and $(\log_3 x)(\log_3 y) = 20$ (7marks)

b). If $y = \frac{x^2 + 3}{x - 1}$, where x is real , show that y cannot take any value between -2 and 6 (5marks)

13 a). The surface area of a cube is increasing at a rate of $10cms^{-1}$. Find the rate of increase of the Volume of the cube when the edge is of length $12cm$ (6 marks)

b). Prove that $\int_0^{\frac{\pi}{2}} x^2 \sin x \cos x dx = \frac{\pi^2}{16} - \frac{1}{4}$ (6marks)

14a).Use the substitution $t = \tan x$ to find the integral $\int \frac{1}{\cos 2x - 3\sin^2 x} dx$ (6marks)

b). Show that the integral $\int \frac{x}{2x^2 - x + 1} dx = \frac{1}{4}\log_e\left(2x^2 - x + 1\right) + \frac{1}{2\sqrt{7}}\tan^{-1}\left(\frac{4x-1}{\sqrt{7}}\right) + c$

(06 marks)

15a). Prove that the line $5y - 4x = 25$ touches thecurve $9x^2 + 5y^2 = 225$(5marks)

b). Show that the equation of the tangent to the curve $bx^2 + ay^2 = a^2b^2$ at the point with parametric equations $x = a\cos\theta, y = b\sin\theta$ is $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$. This tangent meets the $x-$axis at A and $y-$axis at B. Find the area of the triangle OAB (7marks)

16 A hot body of temperature of $80^0 C$ is placed in a room of temperature $22^0 C$, $12^0 C$ minutes later its temperature is $72^0 C$

i) Form a differential equation to represent the rate of change of temperature, θ of the body with time, t (9marks).

ii) Determine the temperature of the body after $30^0 C$ minutes (3 marks)

END