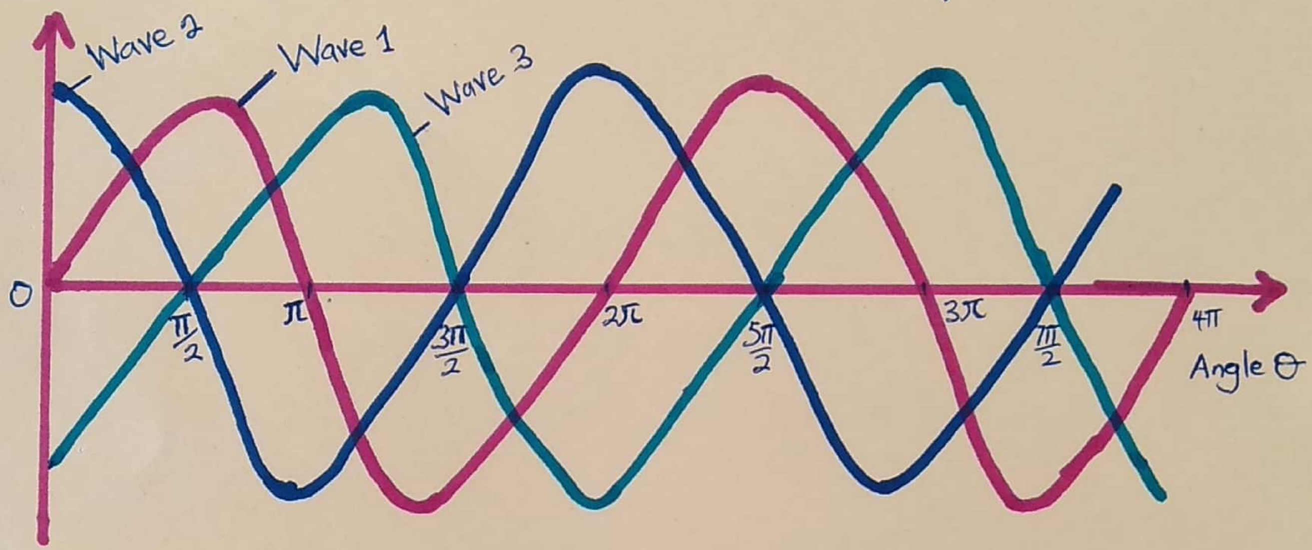
Standard High School- Zzana

Phase difference

This is the angle by which a wave leads or lags behind another wave.

Consider waves represented by the following sketches.



All the waves are not in phase.

- Wave 2 and 3 are said to be completely out of phase.
- Wave 2 leads wave 1 by an angle $\theta = \frac{\pi}{2}$ radians or 90°
- Wave 1 lags behind wave 2 by $\frac{\pi}{2}$ radians or 90°
- The angle $\frac{\pi}{2}$ radians or 90° is the phase difference between the two

Wave 1 can be represented by

$$y = \sin\theta \;,\; y = \sin\omega t$$

but $\theta = \omega t$

wave 2 can be represented by

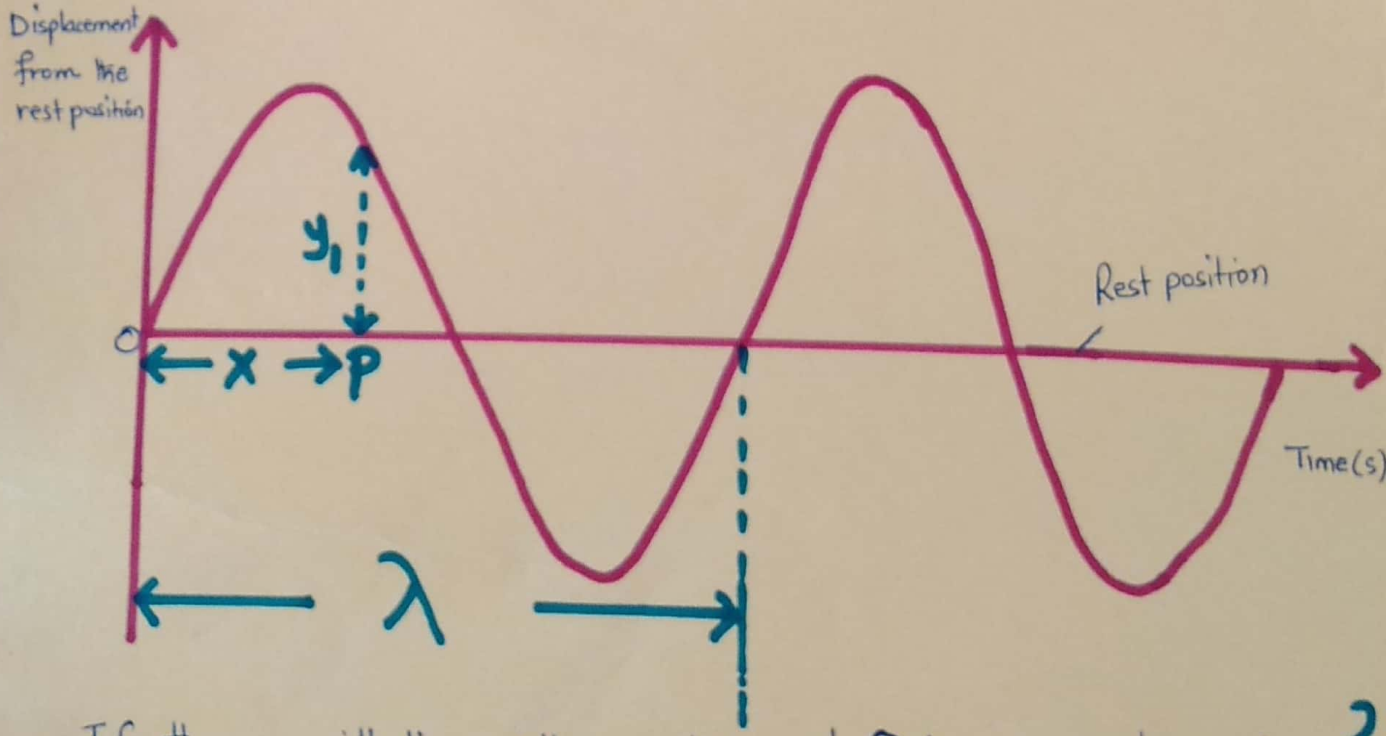
$$y = \sin\left(\omega t + \frac{\pi}{2}\right)$$

wave 3, the equation is

$$y = \sin\left(\omega t - \frac{\pi}{2}\right)$$

Equation of a progressive wave

Consider a waveform below.



If the oscillation of the particle at O is simple harmonic with frequency, f and angular velocity, ω, then the displacement y with time is given by

$$y = a\sin\omega t \longrightarrow (i)$$

Suppose the wave generated travels towards the right, the particle at P a distance x from O will lag behind by a phase angle ϕ

$$y_1 = a\sin(\omega t - \phi) \text{ ——— (ii)}$$

From the figure above the Phase angle, $2\pi = \lambda$ and the Phase angle $\phi = x$

$$2\pi = \lambda \text{ —— (iii)}$$

$$\phi = x \text{ —— (iv)}$$

eqn (iv) $\div$ eqn (iii)

$$\frac{\phi}{2\pi} = \frac{x}{\lambda}$$

$$\Rightarrow \boxed{\phi = \frac{2\pi x}{\lambda}} \text{ ——— } *$$

Substitute * in eqn (ii)

$$y_1 = a\sin\left(\omega t - \frac{2\pi}{\lambda}x\right)$$

But $\omega = 2\pi f = \dfrac{2\pi}{T}$

So, $y_1 = a\sin\left(\dfrac{2\pi}{T}t - \dfrac{2\pi}{\lambda}x\right)$

$$y_1 = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$ — ** Factor out 2π

Generally for a wave travelling to the right the equation of a progressive wave is

$$\boxed{y = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)}$$

<u>Note</u> If the wave is travelling to the left it arrives at P before O. This makes the vibration at P to lead the vibration at O and its equation is given by

$$\boxed{y = a\sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)}$$

Worked Examples.

1. A displacement of travelling wave in the x direction ~~x~~ is given by
$$y = a\sin 2\pi\left(\frac{t}{0.5} - \frac{x}{0.2}\right) \text{m}$$
Find the speed of the wave.

Soln

Given. $y = a\sin 2\pi\left(\frac{t}{0.5} - \frac{x}{0.2}\right) \text{m}$

Compare with $y = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

$\Rightarrow T = 0.5\,\text{s}$, $\lambda = 0.2\,\text{m}$

$f = \frac{1}{T}$

$f = \frac{1}{0.5}$, $f = 2\,\text{Hz}$

But $V = f\lambda$

$V = 2 \times 0.2$

$V = 0.4\,\text{ms}^{-1}$

Remember the unit for the displacement is is metres, so you don't change to any other units.

2. A sound wave propagating in the x-direction is given by

$$y = 0.4 \sin\left[10\left(200t - \frac{x}{100}\right)\right] m$$

Find the speed of the wave.

Soln

Given $y = 0.4 \sin\left[10\left(200t - \frac{x}{100}\right)\right] m$

Enter the 10 inside the bracket

$$y = 0.4 \sin\left(2000t - \frac{10x}{100}\right)$$

Compare with eqn **

$$y = a \sin\left(\frac{2\pi}{T}t - \frac{2\pi x}{\lambda}\right)$$

$\Rightarrow$ $a = 0.4m$

$$\frac{2\pi}{T} = 2000$$

$$T = \frac{2\pi}{2000}$$

$$T = 3.14 \times 10^{-3} s$$

$$f = \frac{1}{T}$$

$$f = \frac{1}{3.14 \times 10^{-3}}$$

$$f = 318.3 \text{ Hz}$$

Also, $\frac{2\pi}{\lambda} = \frac{10}{100}$ $\left(\frac{2\pi}{\lambda}\cancel{x} = \frac{10}{100}\cancel{x} \right)$

$$\frac{2\pi}{\lambda} = \frac{1}{10}$$

$$\lambda = 10 \times 2\pi$$

$$\lambda = 62.8m$$

From $V = f\lambda$

$$V = 318 \times 62.8$$

$$\underline{V = 2.0 \times 10^{4} ms^{-1}}$$

3. The displacement y in meters of a plain progressive wave is given by

$$y = a \sin 2\pi \left(100t - \frac{10x}{17} \right).$$

find the wavelength of the wave and the speed of the wave.

Soln

Given $y = a\sin 2\pi\left(100t - \frac{10}{17}x\right)$

Compare with $y = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

$\Rightarrow$

$$100 = \frac{1}{T}$$

$\left(100\cancel{t} = \frac{\cancel{t}}{T}\right)$

$\leftarrow 100 = \frac{1}{T}$

$$T = \frac{1}{100}$$

$$T = 0{\cdot}01\,s$$

$$f = \frac{1}{T}$$

$$f = \frac{1}{0{\cdot}01}$$

$$f = 100\,Hz$$

Also, $\frac{10}{17} = \frac{1}{\lambda}$ $\leftarrow\left(\frac{10}{17}\cancel{x} = \frac{x}{\lambda}\right)$

$$\lambda = \frac{17}{10}$$

$$\lambda = 1{\cdot}7\,m$$

from $V = f\lambda$

$V = 100 \times 1{\cdot}7 = \underline{170\,ms^{-1}}$

Exercise

1. The displacement of a particle in a progressive wave is
$$y = 2\sin[2\pi(0.25x - 100t)],$$
where x and y are in cm and t is in seconds. Calculate
(i) wavelength
(ii) Velocity of propagation of the wave
(An. $\lambda = 4.0\,cm$, $v = 4\,ms^{-1}$)

2. The displacement y given of a wave travelling in the x-direction at time t is
$$y = a\sin 2\pi\left(\frac{t}{0.1} - \frac{x}{2.0}\right) \text{meter}$$
Find (i) the velocity of the wave
(ii) The period of the wave

3. A plane progressive wave is given by

$$y = a\sin\left(100\pi t - \frac{10}{9}\pi x\right)$$

where x and y are in millimeters and t is in seconds.

Calculate the

i) wavelength

ii) Velocity of propagation

iii) Period T of the wave.

4. The displacement in metres of a plane progressive wave is given by the equation

$$y = 0.5\sin\left[\pi\left(200t - \frac{20}{17}x\right)\right]$$

Find :

i) Wavelength

ii) Speed of the wave.