(d) Given that the roots of the equation; $x^2 - bx + c = 0$ are $\sqrt{\alpha}$ and $\sqrt{\beta}$. Show that;

(i) $\alpha + \beta = b^2 - 2c$

(ii) $\alpha^2 + \beta^2 = (b^2 - 2c - \sqrt{2}c)(b^2 - 2c + \sqrt{2}c)$

2. (a) Find the solution set for which $\log_2 x - \log_x 4 \leq 1$.

(b) Solve the simultaneous equations; $2a - 3b + c = 10$,

$$a + 4b + 2c + 3 = 0,$$
$$5a - 2b - c = 7$$

(c) A geometric progression has the first term 10 and sum to infinity of 12.5. How many terms of the progression are needed to make a sum which exceeds 10?

(d) Given that the equations $y^3 - 2y + 4 = 0$ and $y^2 + y + c = 0$ have a common root, show that $c^3 + 4c^2 + 14c + 20 = 0$.

3. (a) Simplify $(2 + 5i)^2 + 5\left(\frac{7+2i}{3-4i}\right) - i(4 - 6i)$ expressing your answer in the form $a + bi$.

(b) The roots of the equation $3x^2 + 2x - 5 = 0$ are α and β. Find the value of $\alpha^4 + \beta^4$.

(c) Solve the equation $\sqrt{x+5} + \sqrt{x+21} = \sqrt{6x+40}$.

(d) Given that $\log_5 21 = m$ and $\log_9 75 = n$, show that $\log_5 7 = \frac{1}{2n-1}(2nm - m - 2)$

4. (a) Expand $(1 - x)^{\frac{1}{3}}$ as far as the term in x^3. Use your expansion to deduce $\sqrt[3]{24}$ correct to three s.f.

(b) In the expansion of $(1 + ax)^n$, the first three terms are $1 - \frac{5}{2}x + \frac{75}{8}x^2$. Find n and a, state the range of values for which the expansion is valid.